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A New Optimized Reduced Order Model of High-order Discrete Time Systems and Design

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Abstract
This paper presents a method of designing the Controller for large scale discrete time systems. The Controller is designed via a reduced order model for a given high order system. In the proposed reduction method, the numerator coefficients are obtained using Interpolation criteria while the denominator polynomial is obtained by using, one of the stability preserving methods, the Dominant pole method. An optimised reduced order model is derived with minimum ISE. It has been shown that the control designed for the reduced order model, when applied to the higher order system, improves the performance of the controlled system. The method has been tested by considering typical numerical examples, available in the literature, and the results are found to be satisfactory.
Index Terms: Interpolation criterion; control; order reduction; controller simplification
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I. INTRODUCTION:

In discrete time control, the control input is calculated once in every sampling interval and is held constant during this period. Due to the finite sampling frequency, it may happen in discrete time controller that the system state trajectory is unable to move along the surface. Controller for any system has a complexity proportional to the number of states in the system. Hence, the simulation and design of controller for higher order system is a difficult problem. The exact analysis of high order models is both complex and costly. Hence, it is always desirable to reduce high order system to lower order system.

Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. To overcome these problems model order reduction techniques are implemented. It is desirable to reduce higher order transfer function to low order model which are expected to approximate the performance of original high order system.

A mixed method is proposed for the reduction of high order continuous systems. This method is developed from the method [1, 8] available in literature and it overcomes the limitations and drawbacks of some existing methods. In the present paper, Interpolation method is used for obtaining the numerator and dominant pole method is used for obtaining the denominator of the reduced order model. Then an optimum model (with minimum ISE) is obtained by varying the interpolation points. Using the proposed method a controller is designed for the high order original systems. The purpose of this paper is to show that control design can also be done via reduced order model. It will be also shown that if a sliding mode control is designed from the reduced order model and if applied to the higher order system by aggregation, it results in controller designed for the high order system.

II. REDUCTIONPROCEDURES

Consider an n^{th} order linear time invariant discrete system represented by

$$G_n(z) = \frac{p_0 + p_1 z + \ldots + p_{n-1} z^{n-1}}{q_0 + q_1 z + \ldots + q_n z^n} = \frac{p_n(z)}{q_n(z)}$$

Where d_i, $i=0, 1 \ldots n$-1and e_i, $i=0, 1 \ldots n$ are constants.

For the high order system a reduced k^{Th} order model is proposed as given below,

$$\widehat{G}_k(z) = \frac{\widehat{p}_0 + \widehat{p}_1 z + \ldots + \widehat{p}_{k-1} z^{k-1}}{\widehat{q}_0 + \hat{q}_1 z + \ldots + \hat{q}_k z^k} = \frac{\hat{p}_k(z)}{\hat{q}_k(z)}$$

Where the a_i, $i=0, 1 \ldots k-1$ and b_i, $i=0, 1 \ldots. k$ are constants.

Reduced order denominator:

Step 1: The denominator b_k (s) of reduced model can be obtained from the Dominant pole method of the denominator of the original system.

Reduced order numerator:

Step 1: Choose $2k$ point's $z_0, z_1 \ldots z_{2k-1}, z_{2k} \in C$(they can be multiple) from the location of the poles of the original systems and obtain $g(z)$ as given below:

$$\begin{aligned} g(z) &= (z - z_0)(z - z_1)....(z - z_{2k-1}) \\ &= (z - z_0)^{k_0}(z - z_1)^{k_1}....(z - z_j)^{k_j} \\ &= z^{2k} + g_{2k-1}z^{2k-1} + ... + g_1 z + g_0 \end{aligned}$$

Step 2: Compute $p\hat{q}$ and $\hat{p}q$, respectively [10],

$$\begin{aligned} p\hat{q} &= (p_{n-1}z^{n-1} + \ldots + p_0)(\hat{q}_k z^k + \ldots + \hat{q}_0) \\ &= c^{(0)}_{k+n-1}z^{k+n-1} + \ldots.. + c^{(0)}_1 z + c^{(0)}_0 \end{aligned}$$

$$c^{(0)}_0 = p_0\hat{q}_0,$$

$$c^{(0)}_1 = p_0\hat{q}_1 + p_1\hat{q}_0,$$

....

$$c^{(0)}_{k+n-2} = \hat{q}_k p_{n-2} + \hat{q}_{k-1}p_{n-1},$$

$$c^{(0)}_{k+n-1} = \hat{q}_k p_{n-1},$$

and

$$\begin{aligned} q\hat{p} &= (q_n z^n + \ldots + q_0)(\hat{p}_{k-1}z^{k-1} + \ldots + \hat{p}_0) \\ &= d^{(0)}_{k+n-1}z^{k+n-1} + d^{(0)}_{k+n-2}z^{k+n-2} + \ldots \\ &\quad + d^{(0)}_1 z + d^{(0)}_0, \end{aligned}$$

$$d^{0}_0 = q_0\hat{p}_0$$

$$d^{(0)}_1 = \hat{p}_0 q_1 + \hat{p}_1 q_0,$$

....

$$d^{(0)}_{k+n-2} = \hat{p}_k q_{n-1} + \hat{p}_{k-2}q_n,$$

$$d^{(0)}_{k+n-1} = a_{k-1}e_n.$$

*Step 3:(1)*Divide $p\hat{q}$ by *g(z)* to get *f(z):*

$$\begin{array}{r} c^{(0)}_{k+n-1}z^{n-k-1} + c^{(1)}_{k+n-2}z^{n-k-2} + \ldots \\ z^{2k} + g_{2k-1}z^{2k-1} + \ldots + g_1 z + g_0 \,\big)\, \overline{c^{(0)}_{k+n-1}z^{k+n-1} + c^{(0)}_{k+n-2}z^{k+n-2} + \ldots + c^{(0)}_{n-k-1}z^{n-k-1} + \ldots + c^{(0)}_0} \\ \underline{c^{(0)}_{k+n-1}z^{k+n-1} + g_{2k-1}c^{(0)}_{k+n-1}z^{k+n-2} + \ldots + g_0 c^{(0)}_{k+n-1}z^{n-k-1}} \\ c^{(1)}_{k+n-2}z^{k+n-2} + \ldots + c^{(1)}_{n-k-2}z^{n-k-3} + \ldots + c^{(1)}_0 \\ \underline{c^{(1)}_{k+n-2}z^{k+n-2} + \ldots + g_0 c^{(1)}_{k+n-2}z^{n-k-2}} \\ c^{(2)}_{k+n-3}z^{k+n-3} + \ldots + c^{(2)}_0 \\ \underline{\ldots \quad \ldots \quad \ldots \quad \ldots} \\ c^{(n-k)}_{2k-1}z^{2k-1} + \ldots + c^{(n-k)}_0 \end{array}$$

Thus get the recursive relation

$$c^{(1)}_i = c^{(0)}_i - c^{(0)}_{k+n-1}g_{i+k-n+1},$$

$$i = 0,1,\ldots, k + n - 2,$$

$$c^{(2)}_i = c^{(1)}_i - c^{(1)}_{k+n-2}g_{i+k-n+2},$$

$$i = 0,1,\ldots, k + n - 3,$$

$$c^{(3)}_i = c^{(2)}_i - c^{(2)}_{k+n-3}g_{i+k-n+3},$$

$$i = 0,1,\ldots, k + n - 4,$$

....

$$c^{(l)}_i = c^{(l-1)}_i - c^{(l-1)}_{k+n-l}g_{i+k-n+l},$$

$$c_i^{(n-k)} = c_i^{(n-k-1)} - c_{2k}^{(n-k-1)} g_{i,}$$
$$i = 0,1,\ldots,2k-1,$$

When $k<0$, let $g_o=0$. In the above the recursive relations, the superscript n in $c_i^{(n)}$ represent the coefficients which are obtained after carrying out the algorithm n steps. And the subscript i in $c_i^{(n)}$ represents the corresponding degree about the variable z.

(2) Divide $q\hat{p}$ by *g(z)* to get *h(z),* [10]

Step 4: According to $f(z) \equiv h(z)$, get a linear system with *(2m+1)* unknowns and *2m* equations. Let $\hat{q}_0 = 1$ *and solve* $\hat{p}_0,\ldots,\hat{p}_{k-1,}\ \hat{q}_{0,\ldots,}\ \hat{q}_k$ by using cramer rule.

As before, in step *3*and step *4*will be explained as follows. In fact, in the computation process of in step *3*, the remainder function can be written as

$$f(z) = c_{2k-1}^{n-k} z^{2k-1} + c_{2k-1}^{n-k} z^{2k-2} + \ldots + c_0^{n-k}$$

Because $\{c_i^{(0)}\}_{i=0,\ldots,k=n-1}$ are some linear combinations of $\hat{q}_{0,\ldots,}\ \hat{q}_k$, and $\{c_i^{(1)}\}_{i=0,\ldots,k+n-2}$ are also some linear combinations of $\hat{q}_{0,\ldots,}\ \hat{q}_k$, keeping it up, it is found that the coefficients $\{c_i^{(n-k)}\}_{i=0,\ldots,2k-1}$ of *f (z)* are linear combinations of $\hat{q}_{0,\ldots,}\ \hat{q}_k$. Similarly, divide $q\hat{p}$ by *g(z)* to get *h(z),*

$$g(z) = d_{2k-1}^{n-k} z^{2k-1} + d_{2k-1}^{n-k} z^{2k-2} + \ldots + d_0^{n-k}$$

And find that the coefficients $\hat{p}_0,\ldots,\hat{p}_{k-1.}$

III.DESIGN OF CONTROLLER

In general, series controllers are preferred over feedback controllers because of the higher order systems require a large number of state variables i.e., large number of transducers to sense during feedback. The parameters of the controller are tuned to get a response, meeting the desired specifications. The tuned parameters are introduced into the higher order system for stabilization processes.

PROCEDURAL STEPS FOR DESIGINING A CONTROLLER:

1. Assume the transfer function of the controller as [ ]

$$G_c(z) = \frac{G_{ref}(z)}{R(z)[1 - G_{ref}(z)]}$$

2. The overall closed loop transfer function of the controller and original system is derived as,

$$T(z) = \frac{G(z)G_c(z)}{1 + G(z)G_c(z)}$$

NUMERICAL EXAMPLES

Example 1: consider the Fifth order system as given

$$G(z) = \frac{3z^4 - 8.886z^3 + 10.0221z^2 - 5.0919z + 0.9811125}{z^5 - 3.7z^4 + 5.47z^3 - 4.037z^2 + 1.4856z - 0.2173} = \frac{p(z)}{q(z)}$$

A Second order reduced model is obtained for the above higher order system, in following steps, using the proposed method given in section-3.

$$R_2(s) = \frac{b_0 + b_1 z}{a_2 + a_1 z + z^2} = \frac{\hat{p}(z)}{\hat{q}(z)}$$

Step 1: Reduced Order denominator is obtained by using Dominant pole method as below:

$$q(z) = z^5 - 3.7z^4 + 5.47z^3 - 4.037z^2 + 1.4856z - 0.2173$$

Hence, the reduced order denominator is:

$$\hat{q}(z) = z^2 - 1.863z + 0.8754$$

Step 2: The numerator of reduced order model is obtained by the interpolation method as given in proposed procedure

For a 2nd order model the 4 required interpolation points are selected randomly as:

$$g(z) = z^4 - 2z^3 + 1.27z^2 - 0.252z$$

Where g(z) is the polynomial obtained by the selected interpolation points.

From original order numerator and reduced order denominator,

$$p(z)\hat{q}(z) = 3z^6 - 14.29z^5 + 28.48z^4 - 30.42z^3 + 18.36z^2 - 5.941z + 0.845$$

Divide $p(z)\hat{q}(z)$ by g (z)to get the quotient e(z) and the remainder f(z).

Thus,

$$f(z) = -2.936z^3 + 6.001z^2 - 3.903z + 0.8045 \quad (1)$$

From original order numerator and reduced order denominator,

$$q(z)\hat{p}(z) = (z^5 - 3.7z^4 + 5.41z^3 - 4.037z^2 + 1.4856z - 0.2173)(b_1 z + b_0)$$

$$q(z)\hat{p}(z) = b_1 z^6 - (3.7b_1 - b_0)z^5$$

$$+(5.41b_1 - 3.7b_0)z^4 - (4.037b_1 - 5.47b_0)z^3$$

$$+(1.4856b_1 - 4.037b_0)z^2 - (0.2173b_1 - 1.4856b_0)z - 0.2173b_0$$

Divide $q(z)\hat{p}(z)$ by g (z) to get the quotient l (z) and the remainder h(z).

Thus,

$$h(s) = (-0.74b_0 + 0.141b_1)z^3 - (1270.68a_0 + 8296.1a_1)z^2 + (876.91b_0 + 3935.228b_1)z - 0.2713b_0 \quad (2)$$

From equation (1) &(2), we get

$$b_0 = -1.064$$

$$b_1 = 1.313$$

Hence, the reduced model is:

$$R(z) = \frac{1.313z - 1.064}{z^2 - 1.8632z + 0.8754}$$

The corresponding ISE = 0.148

The procedure is repeated with another set of interpolation points and the process is repeated until optimum value of ISE is attained.

The optimum reduced order model is obtained as, [11]

$$R(z) = \frac{1.313z - 1.064}{z^2 - 1.8632z + 0.8754}$$

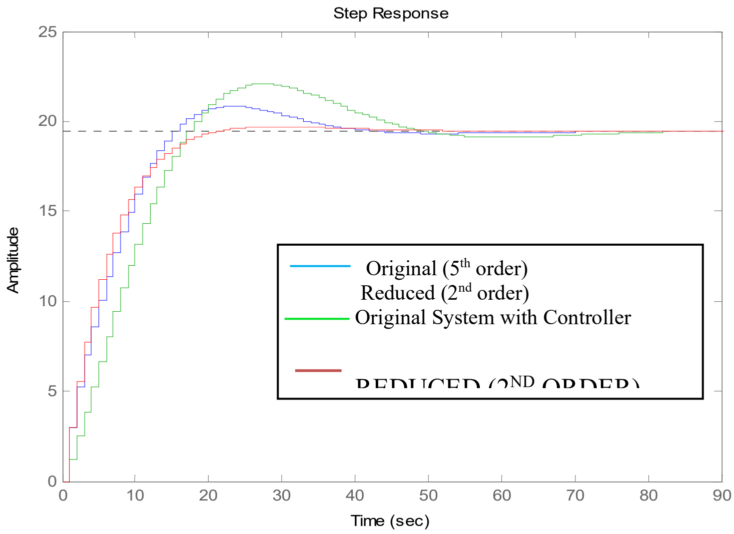

Fig.1. Step response of original and reduced order model

CONCLUSION

A method for order reduction of large scale discrete time system is proposed. The proposed method gives a stable model and by optimising the interpolation points yields a better approximation. A method for designing discrete-time control for higher order system via reduced order model is presented. This method has been tested on two numerical examples chosen from the literature and the step responses of the original, reduced and controlled system are compared. The results were observed to be satisfactory.

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