Space-Time-Frequency Machine Learning for Improved 4G/5G Energy Detection

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Abstract—In this paper, the future Fifth Generation (5G New Radio) radio communication system has been considered, coexisting and sharing the spectrum with the incumbent Fourth Generation (4G) Long-Term Evolution (LTE) system. The 4G signal presence is detected in order to allow for opportunistic and dynamic spectrum access of 5G users. This detection is based on known sensing methods, such as energy detection, however, it uses machine learning in the domains of space, time and frequency for sensing quality improvement. Simulation results for the considered methods: k-Nearest Neighbors and Random Forest show that these methods significantly improves the detection probability.

Keywords—spectrum sensing, cognitive radio, machine learning, energy detection, 4G, LTE, 5G, k-nearest neighbors, random forest

I. INTRODUCTION

FREQUENCY spectrum is a scarce resource of high value for contemporary and future radio communication networks with ever-growing traffic for ubiquitous Internet access. Nowadays, effective (possibly broadband) spectrum access faces a major problem of limited resources, on one hand, and inefficient usage of the ones already licensed, on the other. The basic issue is how to maintain guaranteed quality of services while a number of spectrum users and the generated traffic are exponentially growing [1]. As a solution to this problem, the idea of cognitive radio was proposed. Cognitive radio is a concept of the intelligent radio network and devices that acquire awareness on their radio environment, and dynamically adopt their communication parameters to the available transmission opportunities (spacial, spectral and timing conditions) [2]. Moreover, cognitive radio is expected to learn from its past actions by assessing the quality of the decisions taken. A cognitive-radio user (called secondary user – SU) is a radio device, that is able to determine the current state of the spectrum occupancy, and to transmit and receive the data, keeping the interference generated to the licensed (incumbent) systems (called primary users – PUs) at the acceptable level. For this spectral awareness, spectrum sensing (SS) techniques are essential. SS allows SU to decide whether the spectrum is occupied or not. If the spectrum is idle, SU can transmit without disturbing PU. On the other hand, if PU is active (transmitting), SU should detect this transmission, and wait until PU releases the spectrum.

Common detection methods include, the energy detection method (ED) [3], matched filtering [4], cyclostationarity detection [5], waveform based sensing [6], wavelet transform based detection [7] and other methods. In this paper, the focus is put on ED method. ED is very simple, and is based on the received signal energy calculation. It does not require any knowledge on signal's properties, however, the noise-level cognition is essential [8]. The noise-power level respective to the detected signal-power level, as well as the noise power estimation error significantly impact the ED-based SS performance.

Thus, the goal of this paper is to examine, how the cognitive-radio introduced intelligence, and in particular learning ability, can increase the probability of SS and the radio-environment awareness. The applied machine learning (ML) methods are introduced to support standard ED-based sensing of resource blocks (RBs) that are being used by the Long-Term Evolution (LTE) system base station (eNodeB) in the downlink transmission. The ultimate goal of the applied methods is to achieve high LTE-RBs sensing performance to enable the fourth generation system (4G) and the fifth generation system (5G) New Radio opportunistic (cognitive) communication by utilizing sensed unused RBs. This 5G communication should be particularly effective (spectrum-wise) as it can use the orthogonal frequency division multiplexing (OFDM) subcarriers and RBs orthogonal to sensed LTE downlink signal.

The ML algorithms can significantly improve signal detection techniques. ML applied in SS can be used directly on signal energy values as discussed in [9]–[11]. Moreover, ML methods can use different sets of subject-signal features in order to classify it, e.g., ED decisions, calculated energy-values, location in space, time and frequency, etc., for classification of the PU's signal, namely PU's transmission presence or absence thereof [12]–[14].

Here below, LTE downlink signal detection is discussed, which applies the ED method. Two ML methods for ED performance improvement are considered, namely k-nearest neighbors (kNN) and random forest (RF). Detection is performed for every LTE resource block (LTE-RB), with consideration of interrelationships between LTE-RBs. The LTE signal shows correlation in time, which reflects daily traffic changes. The considered signal is also correlated in frequency and in space (spacial location of SU). Frequency correlation reflects typical resource (channels) management among the LTE-system cells, while spacial correlation reflects shadowing effect in the radio communication channel.

Note that in [15], we have considered some ED-based SS methods with ML, however there, the ML algorithms have been trained separately for the selected points in space, so that the location coordinates (the space-dimension) have not been used as features in the ML training and testing. This approach has lower computational complexity, since the most suitable subset of training data is selected at the beginning of the algorithm. However, ML would never be performed for exactly the same spatial coordinates as in the testing phase. This introduces the need for preliminary calculations in order to choose the nearest point in space. In consequence, a point chosen this way might not indicate the similar (correlated) propagation conditions as the other points in close proximity. In the approach proposed in this paper below, the above mentioned problem is solved by treating the space coordinates as two additional ML features. The main contribution of this paper can be then summarized as:

- ML-based SS algorithm development and formulation of the space-time-frequency feature dataset used for ML training;
- Obtainment of a trained ML model (via computer simulations), that uses information of SU's location as feature input data, and thanks to that it can improve detection performance;
- Verification of the method and comparison of the results for various modulations used in LTE downlink.

The rest of the paper is organized as follows. In section II, the ED method is described in detail. In section III, the proposed ML algorithms are discussed, as well as their application to ED-based SS. In section V, the simulation setup is described, and in section VI, all of the results are shown and discussed. Finally, in section VII, conclusions are drawn.

II. SIGNAL ENERGY DETECTION

Detecting signals comes down to deciding, which hypothesis is more probable, hypothesis $\mathcal{H}_0$ or hypothesis $\mathcal{H}_1$. Hypothesis $\mathcal{H}_0$ means that signal has not been transmitted, and that the received signal $y(t)$ consists of just noise $n(t)$. Hypothesis $\mathcal{H}_1$ denotes that the received signal consists of noise and the transmitted signal $s(t)$. Thus, hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are defined as follows [16]:

$$\begin{aligned} \mathcal{H}_0 &: y(t) = n(t), \\ \mathcal{H}_1 &: y(t) = h(t) * s(t) + n(t), \end{aligned} \tag{1}$$

where $h(t)$ denotes the channel impulse response, and $*$ is a linear convolution. The performance indicators of a detection method are: the probability of detection P_{d} (required to be high) and the probability of false alarm P_{fa} (required to be low). P_{d} is a probability of correctly deciding that hypothesis $\mathcal{H}_1$ is true, while P_{fa} is a probability of wrongly deciding that hypothesis $\mathcal{H}_1$ is true.

As mentioned above, in this paper, ED is considered for the signal presence detection. This method requires the knowledge about the noise power level or its estimation. On the other hand, no signal properties knowledge required. Here, the energy of the received signal is calculated, and is considered

as the so-called test function. Test function $T(y)$ calculated over N signal's samples $y(n)$ is calculated as follows [17]:

$$T(y) = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2. \tag{2}$$

In order to decide, whether the signal is present, the value of the test function is compared with threshold λ defined as [17]:

$$\lambda = \sigma_{\mathrm{n}}^2 \left(Q^{-1}(\bar{P}_{\mathrm{fa}}) \sqrt{\frac{1}{N}} + 1 \right), \tag{3}$$

where σ_{n} denotes the noise power, $\bar{P}_{\mathrm{fa}}$ is an assumed maximum level of P_{fa} and Q^{-1} denotes the inverse Q function, which is described using equation:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du, \tag{4}$$

where u is an integration variable. If value of the test function is higher than threshold, it is assumed that the spectrum is occupied. Otherwise, it is assumed that the spectrum is idle. Therefore the probabilities of detection and false alarm can be described as:

$$\begin{aligned} P_{\mathrm{d}} &= \Pr\{T(y) > \lambda | \mathcal{H}_1\}, \\ P_{\mathrm{fa}} &= \Pr\{T(y) > \lambda | \mathcal{H}_0\}. \end{aligned} \tag{5}$$

The probability of detection is calculated as the probability of the test function value being higher than the threshold, while hypothesis $\mathcal{H}_1$ is true. The probability of false alarm is the probability of the test function value exceeding the threshold value, while hypothesis $\mathcal{H}_0$ is true.

III. MACHINE LEARNING FOR ENERGY DETECTION IMPROVEMENT

ML techniques are methods of finding patterns and similarities in the input data (set of features) in order to categorize that data into predefined or initially unknown output values. The function of mapping input data into output values is unknown, and the task of ML is to find out the best way of this function approximation.

In this paper, two ML methods are being considered, namely the kNN and the RF methods. Both of these algorithms belong to the category of supervised algorithms, which implies that in the training phase, the target values of outputs (labels) are known for a given training dataset.

1) k-NN algorithm: For a given input data point, the kNN algorithm consists in choosing k nearest neighboring training data points (feature data points), by calculating their distance to the input point [18]. The output value of an input point is then decided as an output value that is the most common (representative) among those closest k neighboring points. The distance value is usually calculated as the Euclidean distance [19].

The algorithm is easy to analyze. However, the drawback is that the kNN requires storing of all of the training data to perform the future-data prediction. This implies that for large training datasets prediction can be extremely time and memory consuming.

In the case considered in this paper, transmission using various LTE-RBs is to be detected. For each LTE-RB the ED decision is made. Then, the kNN algorithm is supposed to learn dependencies between LTE-RBs detected as occupied. This is possible, because LTE-RBs are usually transmitted in bunches, so the probability of a LTE-RB being occupied is higher, the more of the closest LTE-RBs are considered as occupied.

More details on the kNN application for the LTE-RBs detection can be found in section IV.

2) RF algorithm: The RF algorithm is also a supervised algorithm. It is a combination of several, slightly different decision tree (DT) algorithms [20]. The DT algorithm builds a decision tree based on the training data. For each branch of this tree, the input data division into categories is made. For full depth DT, all of the training data space gets correctly divided into categories matching their output labels. This means that though all of the training data points are categorized correctly, DT may overfit on new data point outside of the training data. The RF algorithm is a way of minimizing overfitting by averaging operation of several DTs, that are performed on randomly chosen data subsets.

We believe that RF should perform well in the considered LTE scenario. A fundamental advantage of this algorithm is that once the algorithm is trained, it does not require storing all of the training data. However, for a large number of DTs used, the RF algorithm can be computationally complex.

IV. ALGORITHM DESCRIPTION

We propose that the sensing algorithm consists of three main steps:

- Collecting the ED decisions regarding a single LTE-RB in the current time slot.
- Features calculation for each LTE-RB, both for LTE-RBs recognized as occupied and for LTE-RBs recognized as idle.
- ML algorithm application on calculated features, trained on the collected training data.

The first step is to use ED on the received signal samples. For an assumed level of $\bar{P}_{\text{fa}}$ the threshold λ is calculated. Then, the test function value, i.e., the signal's energy is obtained in a given time slot, and for a given set of subcarriers, that form one LTE-RB. If the test function value for this LTE-RB is higher than the threshold, the LTE-RB is recognized as occupied, otherwise it is decided that the LTE-RB is idle.

At this point, the second phase of the sensing procedure begins, which constitutes of the features calculation for each LTE-RB. Some of the features are calculated based on ED decisions, the rest of the features are measured to provide information on time, frequency and space-location of transmission. The used features are listed below:

- the LTE-RB frequency identifier - a number that unequivocally determines a set of subcarriers on which the given LTE-RB is being detected;
- the time slot identifier - a number that corresponds to the time of LTE-RB detection;
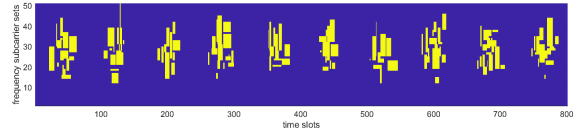

Fig. 1. Sample LTE resources allocation in time and frequency.

- the first coordinate of the sensor (SU sensing the PU signal) position;
- the second coordinate of the sensor position;
- the ED decision regarding considered LTE-RB;
- the number of adjacent neighbouring LTE-RBs recognized as occupied;
- the number of diagonally neighboring LTE-RBs recognized as occupied;
- the history coefficient with the forgetting factor.

The history coefficient is defined as:

$$\phi_{ED}(m,l) = ED(m,l) + \alpha \cdot \phi_{ED}(m-1,l), \qquad (6)$$

where α is the forgetting factor in the range of $[0,...,1]$, $ED(m,l)$ is the ED decision for the LTE-RB of time slot identifier m and subcarriers identifier l.

The above mentioned features are supposed to provide sufficient information to the ML algorithm, so that it can correct the ED results. The use of ML is the last part of the algorithm.

V. EXPERIMENT SETUP

The considered signal has been simulated as a downlink LTE signal of bandwidth 10 MHz. OFDM signal is transmitted over 12 subcarriers and every LTE-RB has the bandwidth of 180 kHz. One LTE-RB lasts 0.5 ms. To generate every OFDM symbol, FFT of order 1024 has been used, and the cyclic prefix of the length of 144 samples has been added. The QPSK modulation has been considered in most results. For comparison also other LTE downlink modulation have been considered, namely 16-QAM and 64-QAM.

The aim of the experiment was to evaluate how the ML algorithms can improve the ED method, if correlation of the received signal ED detection in time, frequency and space is in place. The correlation in frequency has been simulated by introducing the Gaussian distribution in the probability of choosing the frequency channels (frequencies for LTE-RB). The probability of choosing some center frequencies is higher than choosing other frequencies. This approach can be justified as a way of avoiding interference from outside of used frequency band. The correlation in time corresponds to the daily changes in communication traffic. At some periods of a day, more resources are occupied, and at other periods (especially at night hours) the traffic is much lower. This has been simulated using the Gaussian distribution of the time-slots transmission probability. Figure 1 shows an example resource allocation in time and in frequency. The correlation in space is introduced by the shadowing effect in the channel-model simulation.

Finally, in the radio channel model, Rayleigh fast fading effect has also been simulated. For this purpose, the extended

pedestrian A Model (EPA) [21], [22] has been used. Additive White Gaussian Noise (AWGN) has been added to the signal. The Signal-to-Noise power Ratio (SNR) has been calculated relative to the power of one LTE-RB.

Figure 2 presents the SNR values distribution in the two-dimensional space. It shows that the SNR values are spatially correlated; for a given point in the considered space, the other close points have similar propagation (attenuation) conditions. The SU experiences different received signal attenuation, depending on its location. Thus, the SU can learn through ML algorithm to recognize areas of similar SNR levels and use this information to improve the PU's signal (transmission) presence detection. As the path loss has not been implemented, the units of x and y axis on Fig. 2 are not specified. It is assumed though that the distances between every two closest location points are distant enough that for every location point, different fading channel impulse response has been generated. Note that in Fig. 2, the eNodeB location is not marked, because in this context, the knowledge of the eNodeB location is not important and it could be assumed to be anywhere in the given space, or even outside of the presented space borders. The antenna transmission model has been assumed to be omnidirectional, and all of the SNR differences in space result from shadowing effect only.

The considered PU's signals have been generated using MATLAB software (R2018a version 9.4, MathWorks, Natick, MA, USA). Moreover, the ED algorithm has been performed using MATLAB. The examined ML algorithms have been implemented in Python programming language using scikit-learn library [23]. Finally, it should be mentioned that the α coefficient used to calculate the history coefficient has been assumed to be equal to 0.9.
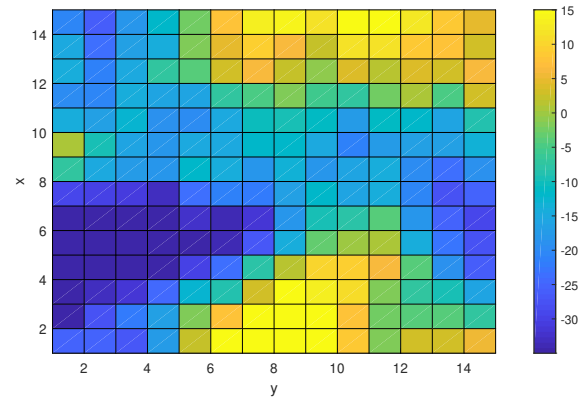
VI. RESULTS

The first step in our sensing algorithm is to apply the ED method for the LTE downlink transmission signal and collect (store) the results. Figure 3 presents the P_d and P_fa for three values of the assumed level of $\bar{P}_\mathrm{fa}$ equal to 10%, 5% and 1%. It can be observed that for higher $\bar{P}_\mathrm{fa}$, the method performs better in terms of P_d, but as it can be expected, P_fa is also higher and close to assumed $\bar{P}_\mathrm{fa}$ level. It can be also observed that even for very low SNR, P_d and P_fa obtain approximately equal values, so P_d is never equal to zero. This is due to ED method, where the value of λ threshold depends on noise power and on the assumed $\bar{P}_\mathrm{fa}$ level, so for the worst channel state, the percentage of noise (or noisy transmitted signal) exceeding the threshold is equal to $\bar{P}_\mathrm{fa}$.

Higher P_fa cause higher probabilities of making a mistake by the ED method, i.e., deciding that a given LTE-RB is occupied, when it is actually free. Higher P_fa can also be a source of misleading information for the ML algorithm.

The first examined ML algorithm is the kNN method. It has been tested for different values of parameter k which stands for the number of neighboring data points taken into account in the prediction. The values of k parameter are as follows; $k = 1$, $k = 3$, $k = 5$ and $k = 9$. For all of the kNN results, feature datasets for 40000 LTE-RBs have been used
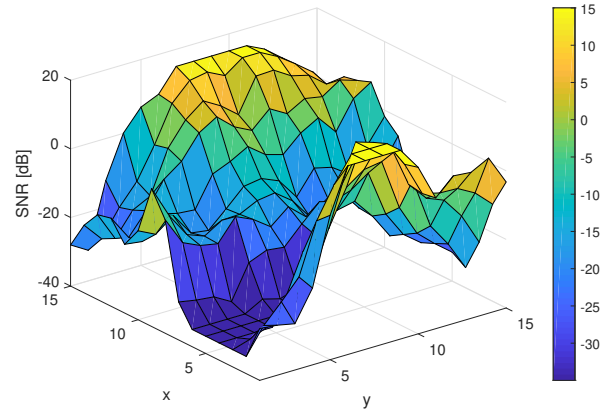
(a) SNR distribution in space.

(b) SNR distribution in space seen from above.
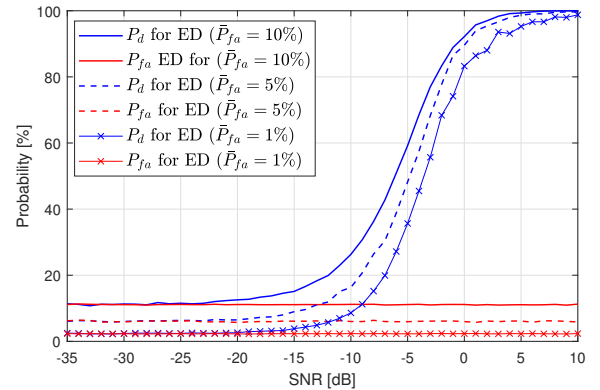
Fig. 2. Shadowing SNR representation in space.

Fig. 3. ED results comparison for different $\bar{P}_\mathrm{fa}$ values.

for training and 4000 LTE-RBs for testing. The results were averaged over 30 iterations. The kNN ML performance has been tested using group k-fold cross-validation method. The results are presented in Fig. 4. It can be observed that the best performance has been achieved for the $k = 1$. This means that the simplest, and least complicated algorithm, in terms of the calculations, is the best solution for the ED improvement in the considered scenario. For low SNR, the detection improvement
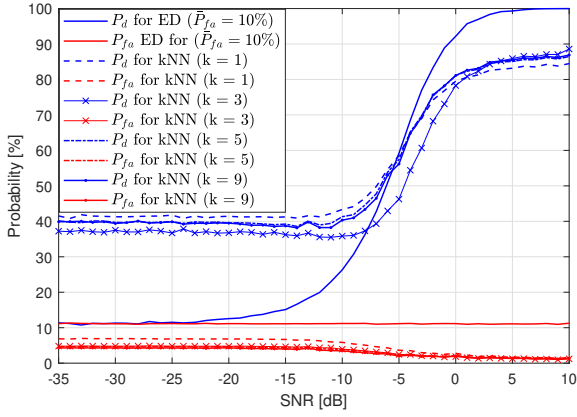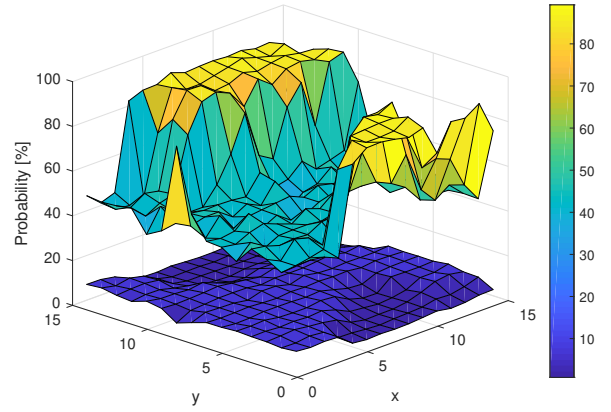
Fig. 4. Results of kNN algorithm for $\bar{P}_{fa} = 10\%$.

(a) P_{d} and P_{fa} distribution of the kNN in space.

(b) P_{d} distribution of the kNN in space seen from above.

Fig. 5. Probability representation in space of kNN results.

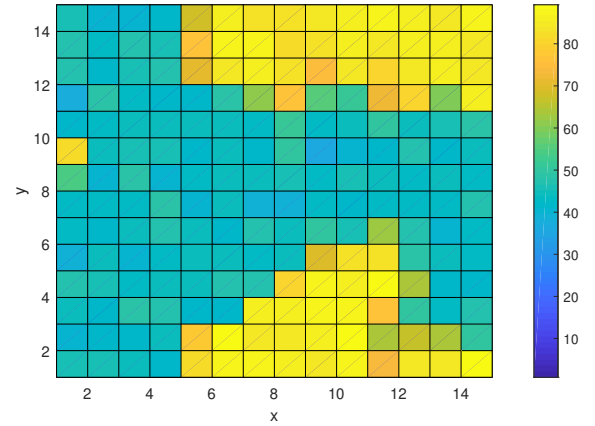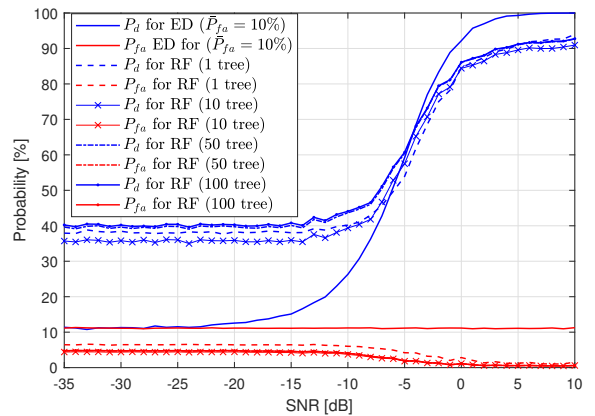

Fig. 6. Results of RF algorithm for $\bar{P}_{fa} = 10\%$.

is significant. On the other hand, it should be noted that P_{d} after the ML application is never equal to 100% even for high SNR. This is caused by high value of P_{fa}, which prevents ML from reaching the highest accuracy. This problem can be solved by using ML adaptively as needed. For high SNR, the ED is performing well and there is no need of using other algorithms, which can be demanding computationally. Another aspect worth noting, is that kNN algorithm lowers P_{fa} down from the assumed level of 10% for high SNR. The probability results for different SNRs were obtained by performing multiple simulations for every location point and by averaging the results for every SNR.

The P_{d} and P_{fa} results can also be calculated for each tested location in space. Figure 5 presents such results. The higher surface represents P_{d} in space, and the lower surface represents P_{fa}.

Next, the RF algorithm has been examined. Similarly as for kNN, RF has been tested for different values of the main parameter. Here, the main parameter is the number of DTs used. The RF algorithm has been performed for 1 tree, 10, 50 and 100 trees. For all of the RF results, feature datasets for 24000 LTE-RBs have been used for training and 4000 LTE-RBs for testing. The results were averaged over 30 iterations. The RF ML performance has been tested using group k-fold cross-validation method as well. The results are presented in Fig. 6. In this case, the results are very similar. The RF for 100 trees performs the best, although the differences are not significant. In the rest of the paper, the RF algorithm for 1 tree will be considered. Similarly as in the previous example, the highest improvements are made for low SNR and for high SNR, P_{d} is lower than 100% and close to 90%.

Similarly as in the kNN algorithm's case, the probability surfaces of P_{d} and P_{fa} for RF results are shown in Fig. 7.

In order to compare, how the ML algorithms perform for ED results obtained for lower $\bar{P}_{\mathrm{fa}}$, two additional Figs. are presented. Figure 8 presents the ED performance for $\bar{P}_{\mathrm{fa}} = 5\%$ and kNN results obtained for $k = 1$, as well as the RF results obtained for 1 tree. Figure 9 presents the same ML algorithms and ED results, but for $\bar{P}_{\mathrm{fa}} = 1\%$. It can be observed that the lower the $\bar{P}_{\mathrm{fa}}$, the closer P_{d} gets to 100% for high SNRs.

To compare results of algorithm presented in this paper, and results obtained in [15], the Fig. 10 has been obtained. It can be observed that best results have been achieved for the RF algorithm with 1 tree, for ML trained separately for some chosen locations in space, and tested for the same locations. The kNN algorithms perform very similarly for
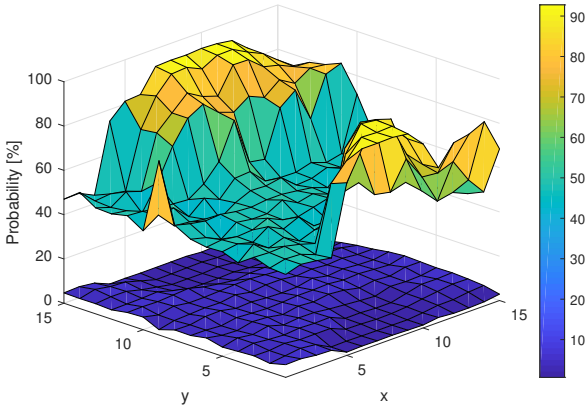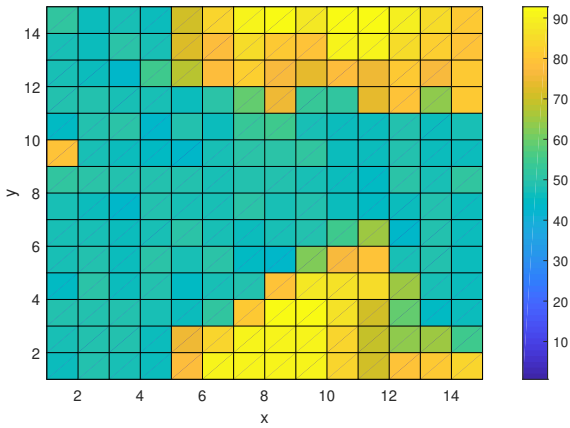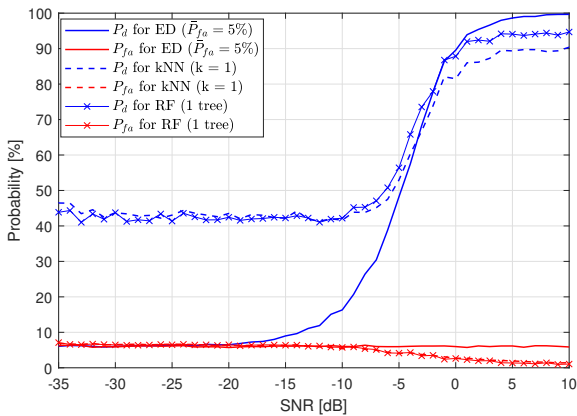
(a) P_d and P_fa distribution of the RF results in space.

(b) P_d distribution of the RF results in space seen from above.

Fig. 7. Probability representation in space of RF results.

Fig. 8. Results of kNN and RF algorithms for $\bar{P}_{fa} = 5\%$.

low SNR for both proposed solutions. The worst performance has been achieved for the RF algorithm trained on feature set including spatial coordinates. Nevertheless, the differences between performances of algorithms are small.

Except QPSK, other modulation methods than QPSK have been tested as well. As expected, higher-order modulations
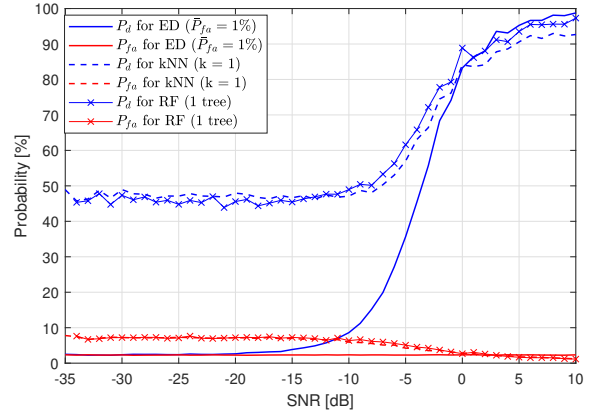
Fig. 9. Results of kNN and RF algorithms for $\bar{P}_{fa} = 1\%$.
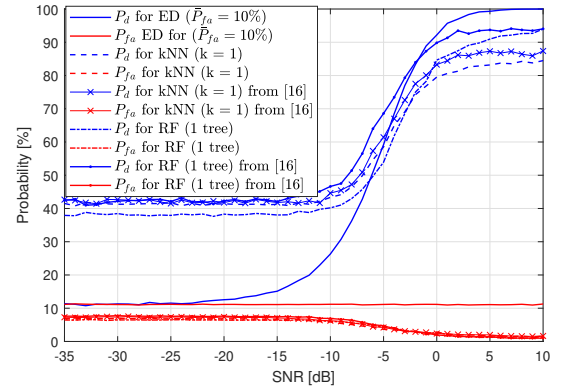
Fig. 10. Results of kNN and RF algorithms for $\bar{P}_{fa} = 10\%$.

used in the LTE downlink transmission, namely 16-QAM and 64-QAM, do not affect the ED results, and since ML algorithms are based on ED decision data, ML performs the same for every type of the signal modulation.

VII. CONCLUSION

In this paper, an algorithm for improving Energy Detection of LTE signal has been proposed. The received signal is considered in three domains, namely in time, frequency and space. Two Machine Learning algorithms has been tested for enhancing detection performance; the k-Nearest Neighbors algorithm and the Random Forest algorithm. This approach allows to significantly improve detection, especially for low Signal-to-Noise Ratio. The proposed approach of considering signal in space domain, allows to train the Machine Learning algorithms without any knowledge of Signal-to-Noise Ratio.

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