Oct 26th, 11:00 AM - 12:30 PM

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Abstract. Optimization approaches and models are developed for job shop scheduling problems over the last decades, particularly the most attempts have been done in industry and considerable progress has been made on an academic line. The Job-shop scheduling considered the most significant industrial activities, mostly in manufacturing. The JSSP (Job Shop Scheduling Problems) is typical NP-hard problem. To solve this problem, we have used the linear programming approach. Real data have been taken from the company of the metalworking industry. The model has been created, then it was analyzed using Spreadsheet – Excel Solver. The appropriate sequence has been obtained and the results shown that it is possible to achieve the minimum completion time compared to other sequence combination

Keywords: Job Shop Scheduling, Linear Programming, Optimization, Spreadsheet – Excel Solver

Introduction

During the last few years' research into scheduling, especially in its most everyday industrial form of job shop scheduling, has risen in significance due to the request of industry. Although much enhancement has been made on an academic line, doubts stay over the transfer of the technology to fit the flexibility demands of modern production facilities. After some visits through Kosovo enterprises , we have noticed that the planning and scheduling tasks in industries are generally done by experienced workers with manual paperwork and graphical presentation and also by some industrial databases, the same principle we have found in literature [1][2]. A numerous of analytical techniques such as "Branch and Bound" and linear programming or heuristic approaches like priority rules and neighborhood methods were investigated. Recently, most of the studies have to do with the solving techniques such as Artificial Intelligence, Tabu Search, Genetic Algorithm and Simulation Techniques. Above methods mentioned and other techniques have a significant role in moving forward in solving problems on scheduling. The goal of this article is to use a linear programming algorithm for the job shop scheduling problem for optimization makespan and total completion flow time by spreadsheet- Excel Solver.

Problem description, assumptions and presentations

In this part of article has been determined the problem on the JSSP (Job Shop Scheduling Problem) in a metalworking company, scheduling process will be investigated where jobs should be proceeding to machines for processing, activities are presented as a jobs and resources are represented by machines and one machine can be processed only one job at the same time and also, the job should visit one machine only once. All jobs having a particular order of processing. Our goal will be focused on the minimization of the makespan. Below are presented some notations which throughout the case are used [3]:

- Number of machines are indexed with j= machine (j=1, 2, …, m),
- The transfer time will not be taken in account,
- P – is processing time,
- W – is waiting time,
- Ci – is time completion of job i.
- Fi=Ci-ri - is flow time of job i, when ri - is release date
- Li=Ci-di – is lateness of job i, di - is due date
- Ti = max {0; Li}, Li>0 – is tardiness of job i,
- Ei=max{0, -Li} – is earliness of job i,
- Ni – is tardy jobs number.

Now, JSSP – the job-shop scheduling problem is described as below:

1) Is given a set of n jobs $J = \{J_i : i \in \{1, . . . , n\}\}$ and a parameter $e_i \in R+$ (positive real numbers) represents the size of the job J_i. Job
2) Is given a set of m machines $M = \{M_j : j \in \{1, . . . , m\}\}$. Then, machine sets
3) Processing time for each Job
4) Transfer time, but in this case transfer time will not be in consideration.

Also, jobs have to be processed across the machines in a particular sequence or also recognized as technological constraint. The makespan is a maximal time that is required to complete processes for all operations, while mean flow time is the average time which is required for all operations. Our main reason is to minimize the makespan value $(C_{max} = \max\{C_i\}, i = 1,2,3, \dots, n)$ and making some solutions under constrains.

In our problem the mathematical description includes the following elements:

- Set of machines
- Set of Jobs

By selecting a proper process plan and also machining resource, the minimize of makespan is the aim of process planning and scheduling or any other objective function for each job along with a complete schedule that satisfy all precedence constraints.

Fellow are shown some assumptions used in this research:

1) The jobs and machines are independent.
2) Each machine should be processed only one operation at the same time.
3) Every machines are disposable at time zero.
4) Each operation should to be processed in the course of a continuous period on given machine.
5) The release date (r_i) for jobs may be different, in our case all jobs starting at time zero.
6) The setup times are independent of the jobs sequence and are included in the processing times.
7) The transfer time between machines will be not taken in account.
8) There are no interruptions or machine breakdowns on the shop floor.

Mathematical formulation

We suppose that any consecutive operations of the same job are executed on different machines. A schedule of tasks is an allocation of the operations for intervals of time on the machines. The problem in scheduling and planning is to find a schedule which can optimize a given objective. The formulation begins with the defining of the objective functions for scheduling.

Objective function

The aim or objective in many scheduling problems is to minimize various functions of the completion times of the tasks subject to several constraints [4][5]. The objective function in most cases can be simply a function of one or more measures of performance. Here, we have written the objective function as a linear combination of the decision variable, F = $f\ (x)$ and after that we will develop technical constraints for the job shop scheduling situations [4].
Let F be an objective function for the ensuing criteria: Minimize function F. Optimization of the makespan or to optimize the maximum completion time (the criteria which are the most commonly used to estimate the scheduling algorithms and heuristics in the literature) the objective was to minimize the completion time of all jobs on the last phase of processing. In this formulation the objective function is the minimization of completion time for the last process among all jobs without breaking any constraints. Therefore, if F is the makespan's value, it must be equal than or larger to the completion time C_i of all jobs $i = 1,2,3, \dots, n$, on the last stage S_{im} of processing time by their respective last set of machines M. Then, we have:

$$\mathbf{F \geq C_{max} = max(C_i)/n\ , i = 1, 2, 3, \dots, n} \quad \textbf{(2.1)}$$

Criteria of the mean completion time, can be modelled as:

$$\mathbf{F \geq C_{max} = max(C_i)/n\ , i = 1, 2, 3, \dots, n} \qquad \textbf{(2.2)}$$

Constrains for the job shop model

Linear programming (LP) is the most recognized technique of operational research, it is created for models with constraint functions and linear objective. An LP model may be created and solve to decide the best courses of action as in the product combination subject to the possible constraints [6]. The job shop scheduling problem is a typical linear programming. As a result, the objective function constraint and let S_{ik} means the start processing time of jth step of the ith job, below are some parameters of JSSP mathematical model:

m – is the number of machines,
n – is the number of jobs,
$O_{i,k}$ – is the operation k of the job i.
$S_{i,k}$ – is the start time of the processing of operation $O_{i,k}$,
$t_{i,k}$ – is the processing time of operation $O_{j,k}$.
Now we can write the main sufficient constrain below for the JSS problem:

$$\mathbf{S_{ik} - S_{i,k-1} + t_{ik} \leq 0, 1 \leq i \leq n; 1 \leq k \leq k_i} \qquad \textbf{(2.3)}$$

$$\mathbf{S_{i1} \geq 0, 1 \leq i \leq n} \qquad \textbf{(2.4)}$$

$$\mathbf{S_{ik} - S_{i,p} + t_{ik} \leq 0, 1 \leq i, j \leq n; 1 \leq k, p \leq k_i} \qquad \textbf{(2.5)}$$

In the constrain, equation (2.3) means that process (i,k) must be processed after process (i,k-1) and (2.4) means that the start processing time must be no less than zero, and (2.5) means that a certain machine only can process one part at the same time, which can eliminate conflict of two jobs.

Whereas:

t_{ik} – is processing time of process (i,k)

m_{ik} – is the machine number of process (i,k)

(i,k), m(i,k) means that the k^{th} step of the i^{th} part is processed by the $m(i,k)^{th}$ machine, and k_i means the last step of the i^{th} part. The objective of scheduling was to minimize the processing time of all tasks, that is to say that let all parts (jobs) completed as fast as possible.

Computational results and analyses

Using Excel Solvers for Optimal jobs processing scheduling

The Linear Programing Algorithm for model 4Jx4M problem were verified through solving and optimizing objective function. Optimizers or solvers, are program tools which help users to locate the most ideal approach to distribute resources.

For the given problem, we have formulated a mathematical model which describes the problem situation. Objective function, decision variables, and constraints are the main components which are included on the model. The model is called a linear programming model, if it consists of linear constraints and the linear objective function in decision variables. A linear programming (LP) is a method used to solve models with linear objective function and linear constraints [7]. Dantzig in 1963 has developed the simplex Algorithm to solve linear programming problems. By using this technique, we can solve problems with two or more dimensions. Excel Solver and spreadsheet we have used for solving our linear programming problem.

The cell G13 is the overall process time duration.

The processing time of jobs is referenced in cells: F11:F19 – “J1”,

F20:F23 – “J2”,

F24:F27 – “J3”,

F28:F31 – “J4”, table 2.1.

In the cells E16:E31 is entered the sequence of operations for each job accordingly in table 2.1.

Table 2.1. Sequence of operations given in gama cells E16:E31

Job processing	Machine	Processing Time, Time “Days”
J1	M2	9
	M1	8
	M4	4
	M3	6
J2	M1	7
	M4	6
	M2	5
	M3	8
	M3	7
	M2	4

J3	M4	5
	M1	5
J4	M3	6
	M2	9
	M4	7
	M1	5

The cells H15:AA15 are reserved for the decision variables $S_{11}, S_{12}, S_{13}\ S_{14}$, $S_{21}, S_{22}, S_{23}\ S_{24}$, $S_{31}, S_{32}, S_{33}\ S_{34}$, $S_{41}, S_{42}, S_{43}\ S_{44}$, C_1, C_2, C_3, C_4 and the cells H14:AA14 for their values.
The restrictions (2.3) coefficient matrix is entered in the table area G16:AA31 (table 2.2).

Table 2.2. Matrix coefficients for restrictions

S_{12}	S_{11}	S_{14}	S_{13}	S_{21}	S_{24}	S_{22}	S_{23}	S_{33}	S_{32}	S_{34}	S_{31}	S_{41}	S_{42}	S_{44}	S_{43}	C_1	C_2	C_3	C_4
-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1

In the same way are entering the restrictions (2.5).
The objective function, its coefficients and overall processing time should be entered in cell G16, the formula is:
=SUMPRODUCT(H16:AA16,$H$14:$AA$14).
The coefficients of the objective function from (2.1) are entered in the row H44:AA44.
The cell H45 is reserved for the objective function value, calculated by the formula:
=SUMPRODUCT(H44:AA44,H14:AA14)
In cell G13, we enter the formula for calculating all jobs processing time duration, i.e., the biggest value of the Ci variables: =MAX(H14:AA14).

Results

Now, we can use the Excel Solver for optimization objective function, and let know which cells on the worksheet represent the objective function, the constraints and decision variables. The “Tools”- “Solver” dialog window the corresponding information is entered as in figure 3.1.

- $H$45 – for the objective function target cell,
- $H$14:$AA$9 – for the variables,
- $F$16:$F$43<=$G$16:$G$43 and $H$14:$AA$14>=0 – for the restrictions.

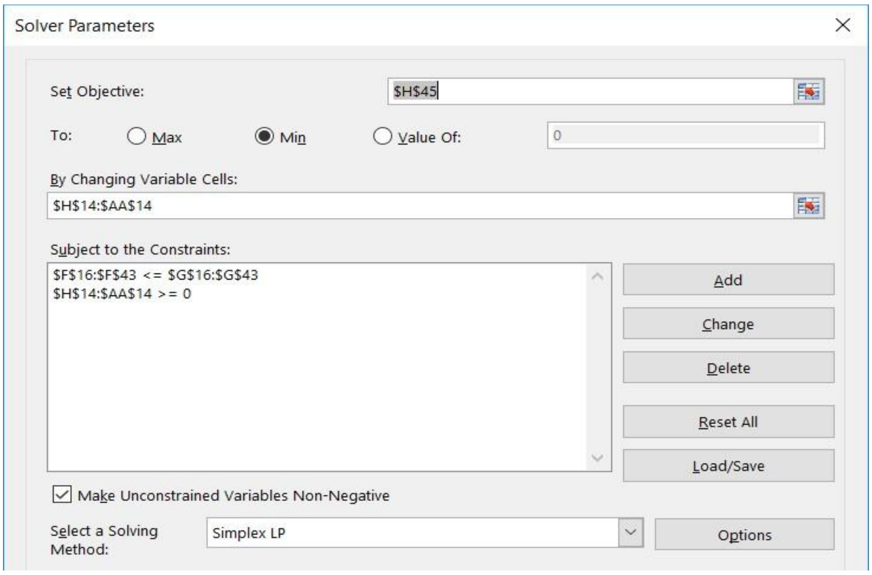

Fig. 3.1. Dialog window of Solver parameters

The button “solve” is activated to find the optimal solution for Job shop scheduling in our case of 4x4 model. After the solver is activated we obtain results for start and end time for each job in each machine table 2.3.

Table 2.3. The start and finish time for each job corresponding to each machine

	J1		J2		J3		J4	
	start	end	start	end	start	end	start	end
M1	9	17	0	7	26	31	34	40
M2	0	9	13	18	9	13	18	24
M3	21	27	27	35	0	7	12	17
M4	17	21	7	13	21	26	27	34

The total flow time is: 132, Makespan Cmax is 39. The Gantt Chart (figure 3.2) of solution problem have been calculated automatically based on values from table 2.3.

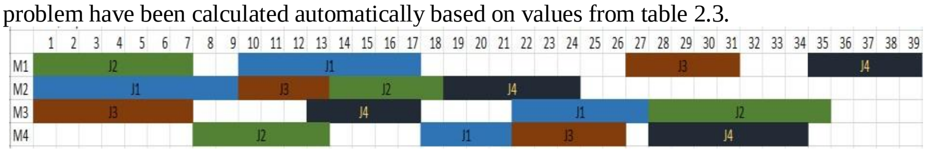

Figure 3.1. The Gantt Chart of JSSP for model 4x4

Conclusion

In this paper are presented the usage of linear programming algorithm to the job shop scheduling problem (JSSP). The real case study has been investigated, and we consider that is important to analyze scheduling models regard to objectives that are well suited for real cases and for which they are intended. For the given problem, we have formulated a mathematical model which has described the problem situation. Objective function, decision variables, and constraints were the main components which have been included in the model. The Linear Programing Algorithm for model

4Jx4M problem were verified through solving and optimizing an objective function. This model gives the solution of the optimization. It gives the optimal allocation of the start time and end-time for processing of each job and duration of the overall processing time, the optimal processing schedule. This approach has been tested in a metalworking manufacturing industry for defining the optimal scheduling process in a number of resources, it can be easily applied and modified for different practical demands. The numerical results show that our adaptation is competitive when compared with other existing methods in the literature. However, the model developed is only limited to the job shop scheduling problem case 4Jx4M instance, but it can be modified easily also for other objective functions and other instances.

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