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Modelling Dialogues for Optimal Legislation

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ABSTRACT

This paper presents a framework for modelling legislative deliberation in the form of dialogues. Roughly, in legislative dialogues coalitions can dynamically change and propose rule-based theories associated with different utility functions, depending on the legislative theory the coalitions are trying to determine.

1 INTRODUCTION

This paper investigates how to model legislative deliberation involving coalitions which express public interests. We follow [3], which proposed a framework for moral dialogues, and we show how that work can be easily extended to legislation procedures.

We assume that the legislative procedure can be analysed into two different components: deliberation—the preparatory process of legislation, which runs in the form of a dialogue involving coalitions of agents—and voting [for a critique of this distinction, see 12]. Informally, the idea of legislative dialogue is thus the following:

- Given an initial theory $\mathcal{T}_0$—i.e., the current legislative corpus or a part of it—coalitions propose the theory that amends $\mathcal{T}_0$ and that they would prefer;
- Each theory is associated with an utility that measures the impact of the proposed changes given the utility of $\mathcal{T}_0$; the intended reading is in terms of the consequence for the society if all agents would conform to such norms [8];
- Coalitions deliberate in a different way depending on which of the above theories are employed to compute the utility;
- We may have more rounds in which coalitions amend theories proposed earlier;
- Coalitions are not fixed during the debate.

Several rationality criteria can guide the legislative dialogue and the amendments proposed by coalitions. In addition to those considered in [3]—such as maximin principle and Pareto efficiency—we show how to model Kaldor-Hicks optimality and two constraints for dialogues: maximising majority in coalitions, and minimising changes in the revision of the initial theory.

While there is a large literature using argumentation for modelling joint deliberation among agents [for an overview, see 1], to the best of our knowledge no systematic investigation has been developed combining means-ends rationality principles, theory revision in the law and formal dialogues. The proposal of Shapiro and Talmon [12] is a recent exception, which shares with us the idea that the legislative process proceeds in rounds of deliberation focused on editing a legal text, but the authors do not consider utility criteria guiding the procedure; on the contrary, they analyse voting outcomes—which we do not discuss here—upon a range of conditions, including reaching consensus, a Condorcet-winner, a time limit, or a stalemate.

The layout of the paper is the following. Section 2 introduces to basic concepts, such as legislative theory, legislative coalition, and the coalition utility. Section 3 offers an analysis of some rationality criteria guiding legislative dialogues. Section 4 shows how legislative dialogues work. Section 5 illustrates additional constrains such as minimising legislative revision and maximising majorities.

2 BASICS

A corpus of legislative provisions in a given legal system can be defined as a set of legislative rules, a set which we call a *legislative theory*. In line with acknowledged literature (for a survey, see [2, 5, 11]), we assume a logic language from which it is possible to build legislative theories. A legislative theory is thus made of a set rules and a superiority relation over the rules.

Definition 1. *A* ***legislative theory*** *is a tuple* $\mathcal{T} = \langle \mathcal{R}, \succ \rangle$ *where* $\mathcal{R}$ *is a set of rules, and* $\succ \subseteq \mathcal{R} \times \mathcal{R}$ *is a superiority relation over the rules.*

Rules have the form $r : \psi_1, \ldots, \psi_n \Rightarrow \phi$ where r is a label identifying the rule. When needed, we adopt the following convention: for any rule r, $A(r)$ and $C(r)$ denote respectively the set of antecedents of r and its consequent. In the rest of the paper, a set of legislative theories is denoted by $\mathfrak{T}$, and we may just say theory instead of legislative theory.

The legislative deliberation process involves a legislative body of lawmakers (such as the members of a parliament), which we generically call legislative agents, in short *agents*. During the deliberation process, agents can dynamically form coalitions. Typically, at the beginning of the deliberation, coalitions correspond to political-party groups in the legislative body.

Definition 2. *Let Ag be a finite set of agents. A* ***legislative coalition*** *in Ag is a subset of agents in Ag. The set 2^{Ag} of all coalitions is denoted by C.*

For brevity we will often speak of coalitions instead of legislative coalitions.

When legislative agents, i.e., the members of the legislative body, argue about theories to govern their own society, they form coalitions proposing theories that represent social interests corresponding to the utility resulting from such theories.

Definition 3. *Let $\mathfrak{T}$ be a set of theories, $\mathbb{V}$ an ordered set of values (on which the social utility functions are computed), and C the set of all legislative coalitions. A* ***coalition social theory utility distribution*** *is a function*

$$U : \mathfrak{T} \to \prod_{0}^{|C|} \mathbb{V}.$$

Given a theory $\mathcal{T}$ and n agents, the function returns a vector of $2^n + 1$ values, which define the value of the theory for each possible coalition in Ag and where the first value, conventionally, indicates the aggregated welfare for all coalitions. Thus, the overall coalitions' utility corresponds in the vector to projection $\pi_0(U(\mathcal{T}))$, while the value of the theory for any specific coalition i corresponds to the projection on the i-th element of the vector, $U_i(\mathcal{T}) = \pi_i(U(\mathcal{T}))$.

In the remainder, $U_i(\mathcal{T})$ denotes the utility of any coalition $i \in C$. Also, we abuse notation and write $U_C(\mathcal{T})$ to denote the overall coalitions' utility, i.e., $U_j(\mathcal{T})$ where $j = \bigcup_{k \in C} k$. Accordingly, the overall coalitions' utility corresponds in the vector to projection $\pi_0(U(\mathcal{T}))$.

In line with ideas developed e.g. by rule utilitarianism, we can determine what is the value of a theory (for each coalition, in our case, and based on the context in which the theory is used) with respect to some inference mechanism [see 7].

In particular, an approach to articulate the way in which utility springs from any theory $\mathcal{T}$ can be based on the utility of conclusions that follow from arguing on $\mathcal{T}$.

Let us first give a basic language setting. A literal is a propositional atom or the negation of a propositional atom. For each literal l in a set Lit of literals and given a (possibly different) set of literals $\{l_1, \ldots, l_n\}$, we can define a function λ that assigns for each coalition i in C an utility value, i.e., the utility that the state of affairs denoted by l brings to i in a context described by $l_1, \ldots, l_n$.

Definition 4. *Let C and $\mathbb{V}$ be, respectively, a set of coalitions and an ordered set of values. A* ***coalition literal valuation*** *is a function*

$$\lambda : C \times Lit \times \text{pow}(Lit) \to \mathbb{V}.$$

If $E(\mathcal{T}) = \{c_1, \ldots, c_m\}$ is the set of conclusions of a theory $\mathcal{T}$, then a coalition utility can be given by agglomerating the values of all conclusions:

$$U_i(\mathcal{T}) = \underset{\forall l \in E(\mathcal{T})}{F^i} \lambda(i, l, E(\mathcal{T})). \tag{1}$$

where F^i is a function/operator that agglomerates the individual values with respect to a coalition i into a single value.

The agglomeration function F can simply correspond to the sum of individual valuations with respect to any coalition i [8]:

$$U_i(\mathcal{T}) = \sum_{l \in E(\mathcal{T})} \lambda(i, l, E(\mathcal{T})). \tag{2}$$

3 OBJECTIVES OF LEGISLATION

As any theory can be associated with a utility, we may identify particular theories. For example, one may consider agents' utility optimal theories, i.e., theories maximising the coalitions' utility, or (strong) 'Pareto optimal theories', i.e., theories for which no coalition can be made better off by making some coalitions worse off, or 'maximin optimal theories', i.e., theories maximising the utility of the worst off coalitions.

Thus we may assume that any legislative debate has the objective of leading at the end (for the voting stage) to the best theory according to some rational standard based on utility considerations.

The following definitions adapt the intuition of [3] to the case of legislative coalitions.

Definition 5. *Let C be a set of coalitions. A theory $\mathcal{T}^*$ is a* ***coalitions' utility optimal theory*** *in a set of theories $\mathfrak{T}$ iff there is no theory $\mathcal{T} \in \mathfrak{T}$ such that $U_C(\mathcal{T}) > U_C(\mathcal{T}^*)$.*[1]

Definition 6. *Let C be a set of coalitions. A theory $\mathcal{T}^*$ is a* ***Pareto optimal theory*** *in a set of theories $\mathfrak{T}$ iff there is no theory $\mathcal{T} \in \mathfrak{T}$ such that $U_i(\mathcal{T}^*) \leq U_i(\mathcal{T})$ for all $i \in C$ and $U_j(\mathcal{T}^*) < U_j(\mathcal{T})$ for some $j \in C$.*

Definition 7. *Let C be a set of coalitions. A theory $\mathcal{T}^*$ is a* ***maximin optimal theory*** *in a set of theories $\mathfrak{T}$ iff there is no theory $\mathcal{T} \in \mathfrak{T}$ such that $\min_{i \in C} U_i(\mathcal{T}) > \min_{i \in C} U_i(\mathcal{T}^*)$.*

Other notions of efficiency can be introduced in addition to those in [3], such as Kaldor-Hicks efficiency [9, 10], which is very relevant in domains such as law and economics. As is well known, this notion claims to be more realistic than Pareto efficiency, since it is extremely difficult to make any change without making at least one coalition worse off. Under the Kaldor-Hicks efficiency, thus, a theory is efficient if those coalitions which are made better off could in theory compensate those which are made worse off and so produce a Pareto efficient outcome. This means that Kaldor-Hicks efficient theories are Pareto optimal, but the reverse is not true. Our formalism does not allow us to explicitly express the idea of compensation, but if one Pareto optimal theory $\mathcal{T}^*$ exceeds the utility for some (but not all) coalitions j with respect to another Pareto optimal theory $\mathcal{T}$[2], then one could view $\mathcal{T}^*$ as compensating a loss for some j in $\mathcal{T}$:

Definition 8. *Let C be a set of coalitions. A theory $\mathcal{T}^*$ is a* ***Kaldor-Hicks optimal theory*** *in a set of theories $\mathfrak{T}$ iff, for each Pareto optimal theory $\mathcal{T}^* \in \mathfrak{T}$, there is a coalition i such that $U_i(\mathcal{T}^*) > U_i(\mathcal{T})$.*

[1] Equivalently, we can say that a theory $\mathcal{T}^*$ is coalitions' utility optimal in a set of theories $\mathfrak{T}$ iff for all $\mathcal{T} \in \mathfrak{T}$ $U_C(\mathcal{T}) \leq U_C(\mathcal{T}^*)$.

[2] If $\mathcal{T}^*$ would exceed the utility for all coalitions, then $\mathcal{T}^{**}$ would not be optimal.

Clearly, each Kaldor-Hicks optimal theory is Pareto optimal.

We can now formulate the general problem of a legislative theory elicitation.

Given: a set of coalitions C and a set of theories $\mathfrak{T}$;
Find: the best legislative theory $\mathcal{T}$ in $\mathfrak{T}$.

The problem can be specified. In fact, one may seek a coalitions' utility optimal theory, a Pareto optimal theory, a Maximin optimal theory, or a Kaldor-Hicks optimal theory.

4 LEGISLATIVE DIALOGUES

A legislative dialogue is the process through which coalitions propose their normative theories with the aim to improve on the current legislative corpus of provisions. The normative system resulting from the dialogue is taken to be justified and so it is suitable for the voting stage.

Let us define two simple operations for amending legislative theories. Here we consider two very basic operations [4, p. 165ff.], but more refined revisions can be adopted without affecting our overall framework [see 6].

Definition 9. *Let $\mathcal{T} = \langle \mathcal{R}, \succ \rangle$ be a legislative theory.*
The ***contraction*** *of $\mathcal{T}$ with respect to a set R of rules is:*

$$(\mathcal{T})^{-R} = \langle \mathcal{R} - R, \succ' \rangle$$

where $R \subseteq \mathcal{R}$ and $\succ' = \succ - \{(r, s) \mid r \in R \text{ or } s \in R\}$.
The ***expansion*** *of $\mathcal{T}$ with respect to a set R of rules is:*

$$(\mathcal{T})^{+R} = \langle \mathcal{R} \cup R, \succ' \rangle$$

where $\succ' = \succ \cup \{(r, s) \mid r \in R, s \in \mathcal{R} \text{ and } C(s) = \neg C(r)\}$.

Definition 9 identifies the legal ways through which legislative theories can be amended: coalitions propose possible amendments in dialogues.

Definition 10. *A* ***legislative dialogue*** *(henceforth, dialogue) is a sequence of legislative theories $(\mathcal{T}_k)_{k=0,\dots K}$ such that*

- *theory $\mathcal{T}_0$ is the initial theory;*
- *for every $\mathcal{T}_k$, $k > 0$, there is a set of theories $\mathfrak{T}^k = \{\mathcal{T}^k_{i_1}, \dots, \mathcal{T}^k_{i_n}\}$ where $\{i_1, \dots, i_n\} \subseteq C$ (i.e., theories individually proposed by coalitions $i_1, \dots, i_n$) such that each $\mathcal{T}^k_{i_j} = (\mathcal{T}_{k-1})^{\pm R}$ $(1 \leq j \leq n)$ for some set R of rules;*
- *theory $\mathcal{T}_{k+1} = Choice(\mathfrak{T}^k)$, where Choice is a function that selects theory $\mathcal{T}_{k+1}$ out of a non-empty set $\mathfrak{T}^k$;*
- *theory $\mathcal{T}_K$ is terminal iff $\mathfrak{T}^K = \emptyset$.*

Definition 11. *The set of theories $\mathfrak{T}^d$ proposed in a dialogue $d = (\mathcal{T}_k)_{k=0,\dots K}$ is $\bigcup_{k \in \{0,\dots K\}} \mathfrak{T}^k$.*

We can note that theory $\mathcal{T}_k$ may be included in $\mathfrak{T}^k$, possibly leading to some sort of equilibrium. However, we are not interested in computing *equilibria* as we deal with principles and not with *moves* as in standard game theoretic approaches. For this reason, we rely on dialogues and not on games, though our dialogues may be seen as *mirroring* such games.

A dialogue is sound if, and only if, the choice function is sound. We concentrate on a few sound *Choice* functions, each of them combining a well established rational criterion with legal ways in which legislation can be amended. Rational criteria may include global utility maximisation (following rule utilitarianism), other choices are maximising coalitions' utility choice, a Pareto choice, or Kaldor-Hicks choice.

Definition 12. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,\dots K}$ is a* ***coalitions' utility maximising choice function*** *iff any theory $\mathcal{T}_k$ $(2 \leq k)$ is a coalitions' utility optimal theory in the set of theories $\mathfrak{T}^{k-1}$.*

Definition 13. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,\dots K}$ is a* ***Pareto choice function*** *iff any theory $\mathcal{T}_k$ $(2 \leq k)$ is a Pareto optimal theory in the set of theories $\mathfrak{T}^{k-1}$.*

Definition 14. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,\dots K}$ is a* ***maximin choice function*** *iff any theory $\mathcal{T}_k$ $(2 \leq k)$ is a maximin optimal theory in the set of theories $\mathfrak{T}^{k-1}$.*

Definition 15. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,\dots K}$ is a* ***Kaldor-Hicks choice function*** *iff any theory $\mathcal{T}_k$ $(2 \leq k)$ is a Kaldor-Hicks optimal theory in the set of theories $\mathfrak{T}^{k-1}$.*

Example (Running example). *Let us consider three fixed coalitions: coalition i_1 representing people with high incomes because of their high salary, coalition i_2 representing those with high incomes because of tax evasion, and coalition i_3 representing those with low incomes. Suppose the initial theory $\mathcal{T}_0$ comprises the following:*

$$\begin{aligned}\mathcal{R} = \{& r_1 : UpperClass \Rightarrow RaiseTax,\\ & r_2 : TaxEvader \Rightarrow SeverePunishment,\\ & r_3 : LowerClass \Rightarrow Subsidies,\\ & r_4 : LowerClass, TaxEvader \Rightarrow \neg Subsidies,\\ & r_5 : TaxEvader \Rightarrow PoorCountry,\\ & r_6 : \ \Rightarrow LowerClass,\\ & r_7 : \ \Rightarrow TaxEvader\}\\ \succ = \{& \langle r_4, r_3 \rangle\}\end{aligned}$$

The conclusions of $\mathcal{T}_0$ are the following:

$$E(\mathcal{T}) = \{SeverePunishment, \neg Subsidies, PoorCountry, LowerClass, TaxEvader\}.$$

Consider, for example, coalition i_2 and assume that the λ function is defined as follows (we omit the literals that are not logically derived):

$$\begin{aligned}\lambda(i_2, SeverePunishment, E(\mathcal{T})) &= -10\\ \lambda(i_2, \neg Subsidies, E(\mathcal{T})) &= -5\\ \lambda(i_2, PoorCountry, E(\mathcal{T})) &= -2\\ \lambda(i_2, LowerClass, E(\mathcal{T})) &= 0\\ \lambda(i_2, TaxEvader, E(\mathcal{T})) &= 18.\end{aligned}$$

Hence, the overall utility of $\mathcal{T}_0$ for i_2 is 1. Similarly, we could assume that λ works for coalitions i_1 and i_3 such that the overall utility for the former is 3 and 1 for the latter. If the global utility is the sum of individual coalitions utility, the utility distribution for $\mathcal{T}_0$ is $[5, 3, 1, 1]$.

What should coalition i_2 do? Although it represents tax evaders (leading for them to a significant positive utility: 15) and their being free-riders, which makes poor the country, only slightly impacts on them personally (−2), the overall utility is positive but small. Hence, coalition i_2 knows that $\mathcal{T}_0$ can be improved. This can be done, for example, by directly working on rules leading to negative utilities, i.e., rules r_2, r_4, r_5 and r_6. For instance, i_2 could propose to amend *theory $\mathcal{T}_0$ by contraction $(\mathcal{T}_0)^{-\{r_4\}}$, i.e., by removing rule r_4. Hence, the overall utility of the new theory would be 6 for i_2.*

Of course, this is i_2's view but the other coalitions play in the debate and work differently. Assume that the new theory $\mathcal{T}_1$ resulting from the debate involving all coalitions goes against the interests of coalition i_2, since the final utility distribution is $U(\mathcal{T}_1) = [8, 2, 0, 6]$ (i.e., taxes are slightly raised for upper classes, tax evasion is more severely punished, and public subsidies are raised for lower classes).

Suppose other revisions of $\mathcal{T}_0$ are obtained, such as $\mathcal{T}_2$ and $\mathcal{T}_3$ where $U(\mathcal{T}_2) = [6, 2, 2, 2]$ (i.e., taxes slightly raised for upper classes together with public subsidies for lower classes and imprisonment for tax fraud is lowered from 5 years to 3 years), and $U(\mathcal{T}_3) = [7, 3, 3, 1]$ (i.e., a tax evasion amnesty is proposed). If the coalitions' utility maximising choice is adopted then $\mathcal{T}_1$ is elicited, while the maximin choice yields $\mathcal{T}_2$, and the Pareto choice results into $\mathcal{T}_3$. No If revision $(\mathcal{T}_3)^{\pm R}$ is chosen and leads to $\mathcal{T}_4$, where $U(\mathcal{T}_3) = [7, 4, 3, 1]$, then we would have a Kaldor-Hicks optimal result.

5 CONSTRAINTS ON LEGISLATION

In Section 3 we have identified some objectives of the legislative procedure, if coalitions are assumed to adopt some type of means-ends rationality. However, deliberative procedures usually also assume that some basic constraints apply to them. We focus on the principle of majority-driven debate and of minimal revision.

5.1 Majority-driven Coalitions in Legislation

We should notice that Definition 10 does not require that coalitions are fixed in the dialogue, but simply that at each turn in the dialogue some coalitions individually propose some revised theories. Hence, if the legislative body works on the basis of the *majority principle* as applied to the agents forming the coalitions, it is obvious that such coalitions could change during the dialogue.

Definition 16. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,...K}$ is a **coalitions' majority optimal choice function** iff any theory $\mathcal{T}_k = \mathcal{T}_{i_j}^k$ $(2 \le k)$ in the set of theories $\mathfrak{T}^{k-1}$ is such that $|i_j| > |Ag|/2$.*

In other words, a coalitions' majority optimal choice ensures that each theory selected at each turn is proposed by a majoritarian coalition in Ag (since the size of the coalition i_j must exceed the half of the size of the set of agents). Definition 16 works with simple majority, but other requirements such as supermajority or unanimity can be easily implemented.

Example (Running example). *Let Ag be the set of agents:*

$$Ag = \{ag_1, ag_2, ag_3, ag_4, ag_5\}$$

with the following coalitions:

$$i_1 = \{ag_1\} \qquad i_2 = \{ag_2, ag_3\}$$
$$i_3 = \{ag_1, ag_2, ag_3, ag_4, ag_5\} \qquad i_4 = \{ag_1, ag_2, ag_3, ag_4\}$$

Assume four additional theories with the following utility vectors:

$$U(\mathcal{T}_0) = [5, 3, 1, 1] \quad U(\mathcal{T}_{i_1}) = [8, 2, 0, 6] \quad U(\mathcal{T}_{i_2}) = [6, 2, 2, 2]$$
$$U(\mathcal{T}_{i_3}) = [7, 3, 3, 1] \quad U(\mathcal{T}_{i_4}) = [7, 4, 3, 1]$$

If Pareto choice and coalitions' majority optimal choice *are jointly adopted, the dialogue could run as follows:*

- *step 0:* *initial theory $\mathcal{T}_0$;*
- *step 1:* $\mathfrak{T}^1 = \{\mathcal{T}_{i_1}, \mathcal{T}_{i_2}, \mathcal{T}_{i_3}, \mathcal{T}_{i_4}\}$;
- *step 2:* $\mathcal{T}_2 = Choice(\mathfrak{T}^1) = \mathcal{T}_{i_4}$.

5.2 Minimal Revisions in Legislation

Finally, another constraint could also require to minimise the revision of the initial theory, in order to keep legislative revision as simple as possible. In theory revision for the legal domain the idea of minimal change has been widely investigated [for an overview, see 4, 6]. Here we simply focus on minimal contraction or expansion with respect to the set R of rules that are removed from, or added to $\mathcal{T}_0$:

Definition 17. *The choice function of a dialogue $(\mathcal{T}_k)_{k=1,...K}$ is a **minimal revision choice function** iff any theory $\mathcal{T}_k = \mathcal{T}_{i_j}^k$ $(2 \le k)$ in the set of theories $\mathfrak{T}^{k-1} = \{\mathcal{T}_{i_1}^k, \ldots, \mathcal{T}_{i_n}^k\}$ is such that*

$$\mathcal{T}_{i_1}^k = (\mathcal{T}_0)^{\pm R_{i_1}}, \ldots, \mathcal{T}_{i_n}^k = (\mathcal{T}_0)^{\pm R_{i_n}}$$

and there is no $R_{i_m} \in \{R_{i_1}, \ldots, R_{i_n}\}$ such that $|R_{i_m}| \le |R_{i_j}|$.

Example. *Consider $\mathcal{T}_0$ as in Example 4. For the sake of simplicity, assume that that we only have two coalitions, i_1 and i_2 and that the λ function is defined as follows for all coalitions: for $i \in \{i_1, i_2\}$*

$$\begin{array}{lcl}
\lambda(i, SeverePunishment, E(\mathcal{T})) & = & -10 \\
\lambda(i, \neg Subsidies, E(\mathcal{T})) & = & -5 \\
\lambda(i, PoorCountry, E(\mathcal{T})) & = & -2 \\
\lambda(i, LowerClass, E(\mathcal{T})) & = & 0 \\
\lambda(i, TaxEvader, E(\mathcal{T})) & = & 18.
\end{array}$$

The utility distribution is thus $[2, 1, 1]$,

Suppose that coalitions are modelled through a coalitions' utility maximising choice function and imagine that $\mathfrak{T}^{k-1} = \{\mathcal{T}_{i_1}^k, \mathcal{T}_{i_2}^k\}$. Consider that $\mathcal{T}_{i_1}^k$ is $(\mathcal{T}_0)^{-R_{i_1}}$ such that $R_{i_1} = \{r_4\}$. Here the utility distribution is $[12, 6, 6]$.

Also, consider that $\mathcal{T}_{i_2}^k$ is $(\mathcal{T}_0)^{-R_{i_1}}$ such that $R_{i_1} = \{r_2, r_4\}$. In this second case, the utility distribution is $[32, 16, 16]$. *If we just apply coalitions' utility maximising choice function to both coalitions, it is trivial to conclude that $\mathcal{T}_k = \mathcal{T}_{i_2}^k$. However, minimal change, if adopted rules out this option, thus $\mathcal{T}_k = \mathcal{T}_{i_1}^k$.*

6 OPTIMISING LEGISLATIVE DIALOGUES

The use of dialogues and their iterative nature suggests some different (search) strategies to find an optimal theory in a set of theories.

6.1 Coalitions' Utility Optimising Dialogues

For the terminal theory of a dialogue to be coalitions' utility optimal in the theories proposed in the dialogue, it is sufficient that the dialogue has a coalitions' utility maximising choice function whose output theory $\mathcal{T}_k$ is always included in the proposed theories $\mathfrak{T}^k$.

Proposition 1. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,...K}$ with a coalitions' utility maximising choice function is coalitions' utility optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue if for any $\mathcal{T}_k, \mathcal{T}_k \in \mathfrak{T}^k$.*

However, the terminal theory may not be a strict 'improvement' of $\mathcal{T}_0$. Therefore, one may consider dialogues to elicit coalitions' utility optimal theories based on the idea of improving theories.

Definition 18. *Let C a set of coalitions. A theory $\mathcal{T}^*$ is a **coalitions' utility improvement** of a theory $\mathcal{T}$ iff $U_C(\mathcal{T}^*) > U_C(\mathcal{T})$.*

Proposition 2. *A theory is a coalitions' utility optimal theory in a set of theories $\mathfrak{T}$ iff there exist no coalitions' utility improvements in $\mathfrak{T}$ of the theory.*

An initial theory is not optimal if there exists an improvement.

Proposition 3. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a coalitions' utility maximising choice function is coalitions' utility optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue and it is a coalitions' utility improvement of the initial theory, if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$, and there exists a theory $\mathcal{T}_k$ which is a coalitions' utility improvement of $\mathcal{T}_{k-1}$.*

In other words, if there exists no improvement in a dialogue then the initial theory remains the optimal theory, and a legislative dialogue is not necessary to find the optimal theories.

6.2 Pareto and Kaldor-Hicks Optimising Dialogues

Dialogues can be similarly tuned to elicit Pareto optimal theories.

Proposition 4. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a Pareto choice function is Pareto optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$.*

As the terminal theory may not be an improvement of $\mathcal{T}_0$, we can consider Pareto improving theories, i.e., theories leading to a utility gain, without any coalitions being made worse off.

Definition 19. *Let C a set of coalitions. A theory $\mathcal{T}^*$ is a **Pareto improvement** of a theory $\mathcal{T}$ iff $U_i(\mathcal{T}^*) \geq U_i(\mathcal{T})$ for all $i \in C$ and $U_i(\mathcal{T}^*) > U_i(\mathcal{T})$ for some $i \in C$.*

Proposition 5. *A theory is a Pareto optimal theory in a set of theories $\mathfrak{T}$ iff there exist no Pareto improvements in $\mathfrak{T}$ of the theory.*

Proposition 6. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a Pareto choice function is Pareto optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue and it is a coalitions' utility improvement of the initial theory, if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$, and there exists a theory $\mathcal{T}_k$ which is a Pareto improvement of $\mathcal{T}_{k-1}$.*

Similar results can be given for Kaldor-Hicks optimality.

6.3 Maxmin Optimising Dialogue

Similarly to coalitions' utility and Pareto improving choice functions, maximin can be accommodated in dialogues.

Proposition 7. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a maximin choice function is maximin optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$.*

Definition 20. *Let C a set of coalitions. A theory $\mathcal{T}^*$ is a **maximin improvement** of a theory $\mathcal{T}$ iff $\min_{i \in C} U_i(\mathcal{T}) < \min_{i \in C} U_i(\mathcal{T}^*)$.*

Proposition 8. *A theory is a maximin optimal theory in a set of theories $\mathfrak{T}$ iff there exist no maximin improvements in $\mathfrak{T}$ of the theory.*

Proposition 9. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a maximin choice function is maximin optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue and it is a coalitions' utility improvement of the initial theory, if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$, and there exists a theory $\mathcal{T}_k$ which is a maximin improvement of $\mathcal{T}_{k-1}$.*

6.4 Improving Majorities

We have previously mentioned that dialogues aim at maximising majorities by reconfiguring coalitions during the debate.

Proposition 10. *The terminal theory of a dialogue $d = (\mathcal{T}_k)_{k=1,\dots K}$ with a coalitions' majority optimal choice is majority optimal in the set of theories $\mathfrak{T}^d$ proposed in the dialogue if for any $\mathcal{T}_k$, $\mathcal{T}_k \in \mathfrak{T}^k$.*

Definition 21. *Let C a set of coalitions and $i_j, i_k \in C$. A theory $\mathcal{T}^*_{i_j}$ is a **coalitions' majority improvement** of a theory $\mathcal{T}_{i_k}$ iff $|i_j| > |i_k|$.*

7 SUMMARY

In this paper we extended Governatori *et al.* [3]'s framework to the legal domain for modelling legislative deliberation. First of all, we assumed that the legislative procedure can be analysed into two different components: deliberation—the preparatory process of legislation, which runs in the form of a dialogue involving coalitions of agents—and voting—which was not discussed here.

The idea of legislative deliberation consists in revising the current legislative corpus or a part of it, where agents's coalitions propose in a dialogue legislative theories that amends such corpus. Each revision is associated with an utility that measures the impact of the proposed changes. Several rationality criteria have been described according to which coalitions deliberate.

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