

# Sensitivity Assessment for Projector Camera Geometry Reconstruction Systems

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## Abstract

The principal point is an important parameter in the characterisation of optical systems. We wish to better understand the optical system parameters and their sensitivity to a good or poor estimation of principal point, to which the focal length, in particular, can be highly sensitive, which this work seeks to understand.

## 1 Introduction

There are many circumstances where we need a camera to observe a projected display, particularly in the calibration of multi-projector systems, or to infer the three-dimensional shape being projected onto in single- or multi-projector displays. Where frequently a calibrated stereo camera pair would be used to determine 3D shape, with a data projector under our control we can accomplish 3D inference with a *single* camera (as illustrated in Fig. 1), but with the substantial added complication that the projector-camera optical relationship is not calibrated, or may even be changing over time. Calibration means estimating both intrinsic and extrinsic parameters of the camera and projector. As a result, repeated and accurate calibration is essential to our broader research objective of large-scale 3D inference.

Essentially what we require is a relatively precise characterization of the transformation between projector and camera domains; that is, to map the projected and observed pixels to one another. This field is very well established [1, 2], and recent work as in [3] has shown interest in tackling such calibration problem.

## 2 Method

As mentioned in [3], the workflow for calibration and 3D geometry inference involves estimating camera and projector principal points, collecting correspondences between points in the projector and camera views, estimating the *fundamental matrix*  $F$  by using the standard normalized 8-point algorithm with RANSAC [2], and then estimating the camera and projector focal lengths using the method introduced by Bougnoux [1]. Given the camera and projector principal points  $X_{p_1}$  and  $X_{p_2}$ , the fundamental matrix  $F$ , the epipolar skew symmetric matrix  $[e_2]_x$ , and the modified  $3 \times 3$  identity matrix  $I$ , which has its (3,3) component set to 0, then the focal length can be estimated as

$$f_1^2 = -\frac{X_{p_2}^\top [e_2]_x I F^\top X_{p_1} X_{p_1}^\top F X_{p_2}}{X_{p_2}^\top [e_2]_x I F^\top I F X_{p_2}} \quad (1)$$

It was determined that certain catastrophic failures or serious distortions in 3D inference were due to failures in focal length estimation (to the extent that the estimated focal length being complex / imaginary), themselves stemming from poorly estimated principal points. Because the principal point, particularly that of the projector, is not necessarily known, we wished to more deeply understand the impact of varying the principal point on estimating the focal length, a question so far not explored in the literature to the best of our knowledge.

We asserted projector principal points that are offset from the *correct* location, and for each such point the resulting focal length was computed. In our tests, pre-recorded data were modified with the offset principal point on each iteration, but all other measurements unchanged. Also, note that the camera principal point is assumed to be located in the center of the camera image, and only the projector principal point was offset.

## 3 Results

Fig. 2 illustrates the results of the experiment, where the asserted projector principal point was varied in both the  $x$  and  $y$  directions

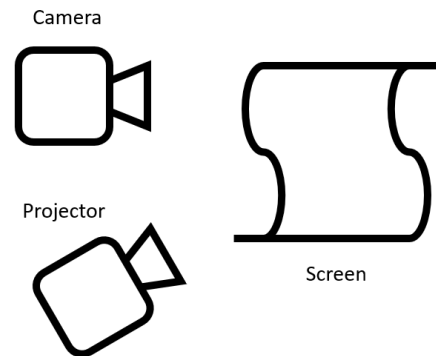


Fig. 1: Optical system setup: The output from a projector is observed by a camera. We seek to find a calibration of the combined system.

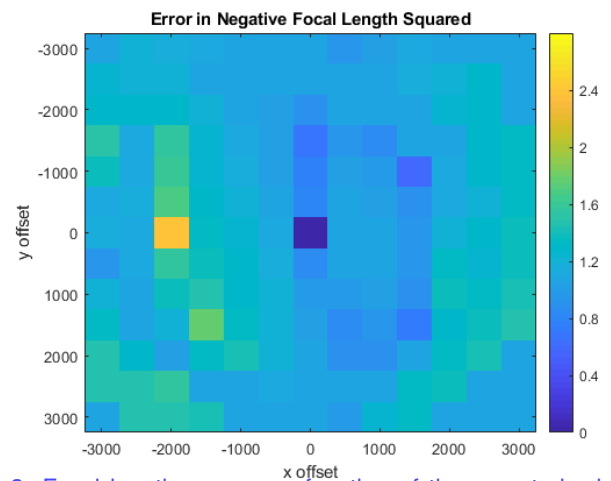


Fig. 2: Focal length error as a function of the asserted principal point. Note: the *correct* principal point is located at coordinate (0,0). The colourbar axis is a scaled false colour representation according to  $|f_1^2 - f_{correct}^2|^{1/8}$ ; any value smaller than 0.8 corresponds to an acceptable estimate focal length within a tolerance of  $\pm 15\%$ .

over a wide range, including extending outside of the projector's imaging range. The colourmap is a non-linear representation of error in focal length, emphasizing small errors. At the (0,0) position the projector principal point is *correct* and the focal length estimate has no error. We observe a rapid spatial divergence of focal length from truth, motivating a renewed attention on principle point estimation, based on focus-of-expansion approaches. Future work includes more densely sampling asserted principal point offsets and running similar analysis on more data sets.

## Acknowledgment

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## References

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