# Multiple Intelligences in the Mathematics Classroom: A Curriculum Project on Linear Equations and Inequalities in One Variable 

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[^0]Multiple Intelligences in the Mathematics Classroom: A Curriculum Project on Linear Equations and Inequalities in One Variable

Angelina Rocchio


#### Abstract

This curriculum project explores the utilization of Howard Gardener's Multiple Intelligences theory in the mathematics classroom. There are eight distinct multiple intelligences that can be found in heterogeneous classrooms of students and often students have a blend of these eight intelligences. This curriculum project discusses different methods for integrating certain multiple intelligences into the mathematics classroom. The multiple intelligences that have been included in this curriculum project are Verbal/Linguistic, Logical/Mathematical, Visual/Spatial, Kinesthetic, Interpersonal, and Intrapersonal. The Algebra unit this curriculum project focuses on is Linear Equations and Inequalities in One Variable. The curriculum provided includes a variety of ways to incorporate the multiple intelligences theory throughout each of the thirteen lessons within the unit.


## Introduction

Every student in a mathematics class has a distinct learning style. Many times mathematics instruction does not reflect the diversity in learning styles among students. The theory of Multiple Intelligences developed by Howard Gardner is an adept model for mathematics education. One of the most difficult challenges for a teacher in a classroom, especially a mathematics teacher, is being able to utilize the different intelligences to accommodate each student's unique learning style. Implementing these different intelligences in the classroom can lead to a deeper understanding and also enhance the learning experience of every student.

Multiple Intelligences is a theory that was formulized by Howard Gardner in 1983. His book, Frames of Mind, challenge the way educators view student learning styles. Gardner identifies seven different intelligences in the book, amending the eighth intelligence as he continued his research. Before individually examining each intelligence, it is essential to understand how Gardner defines intelligence. From the perspective of Gardner, intelligence is defined as "a bio psychological potential to process information that can be activated in a cultural setting to solve problems or create products that are of value in a culture" (Gardner, 1999, p. 33). Gardner emphasizes the fact that each individual is born with the potential for all intelligences. This potential can then be "realized to a greater or lesser extent as a consequence of the experiential, cultural, and motivational factors that affect each person" (Gardner, 1999, p. 82). In essence, Gardner is conveying is that everyone has their own unique mix of intelligences. While one person has cultivated strengths in one intelligence, someone else has strengths in another. A persons strengths and weaknesses are similar to that of a fingerprint. Each
person has a mix of intelligences that is uniquely his or hers. While some students may share similarities, every student is different. As we've now become familiar with what multiple intelligences are, we can now investigate each individual type of intelligence.

## Literature Review

Howard Gardner is a cognitive psychologist from Harvard University who has developed a theory based on multiple skills and abilities; this theoretical perspective is known as Multiple Intelligence (MI). His theory is centered on the premise that there are many different types of talents or knowledge that can enrich one's life. Each intelligence is a unique blend of characteristics. It is crucial that teachers recognize individual differences and strengths when it comes to multiple intelligences. For that reason, it is important to understand the makeup of each intelligence. The end product of his research are the eight intelligences: (1) visual-spatial- capacity to perceive the visual-spatial world accurately and to modify or manipulate one's initial perceptions (2) bodily-kinestheticabilities to control one's body movements and to handle objects skillfully (3) musical-rhythmical-abilities to produce and appreciate rhythm, pitch, and tone, and appreciation of the forms of musical expressiveness (4) interpersonal-capacities to discern and respond appropriately to the moods, temperaments, motivations, and desires of other people (5) intrapersonal- knowledge of one's own feelings, strengths, weaknesses, desires, and the ability to draw upon this knowledge to guide behavior (6) logical-mathematical- the abilities to discern logical or numerical patterns and to handle long chains of reasoning and (7) verbal- linguistic-sensitivity to the sounds, rhythms, and meanings of words; sensitivity to the different functions of language (8) naturalistic- the potential for discriminating among plants, animals, rocks, and the world around us, as used in
understanding nature, making distinctions, identifying flora and fauna (Gardner, 1993). In light of this, the application of the theory supports instructional techniques that align with the standards and practices of MI.

Students should be taught based on their ability and ways of learning; active and involved teaching is a step towards students' academic success. MI asks the question, in what ways do students naturally learn. Teachers generally carry the belief that all students are capable of learning. Multiple Intelligences Theory offers different instructional practices that can aid teachers in realizing students' potential, abilities, and talents while addressing the standards of the curriculum. The standard this project applies is: A.REI.3: Reasoning With Equations And Inequalities- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Below, each intelligence included in this curriculum is discussed specifically around the instruction and learning of the A.REI. 3 Standard.

## Logical-Mathematical

Logical-Mathematical intelligence is most closely related to the study of mathematics. Mathematics is a subject that requires abstract thought in high school mathematics. Gardner notes "while the products fashioned by individuals gifted in language and music are readily available to a wide public, the situation with mathematics is at the opposite extreme. Except for a few initiates, most of us can only admire from afar the ideas and works of mathematicians" (Gardner, 1983, p. 136). It is understandable then why many students struggle with mathematics. It is a hard subject to grasp, especially if the logic-mathematical is not one's predominant intelligence. The logicalmathematical intelligence "underlies the deductive methods of science and law. Its way
of working is not just through symbols and syllogisms, however, but is often initially non-verbal - a sense of how causes are related though a series of steps" (Wahl, 1999, p. 3). Logical-mathematical intelligence is more than mechanics; it involves intuition. Students that excel in logical-mathematical intelligence often are "fascinated with patterns in numbers or with science and will pursue ideas far beyond their apparent utility." (Wahl, 1999, p. 3). Students high in this intelligence are able to internalize the intricacies of logic and mathematical reasoning. (Gardner, 1983, p.139)

## Linguistic

Gardner writes that the core of linguistic intelligence is sensitivity to the meaning of words; the order of words; to the sounds, rhythms, inflections, and meters of words; and to the different functions of language all exemplify this intelligence (Gardner, 1983, p.77). Each person has a varying degree of such sensitivities. A person that is extremely skilled in all would be an exceptional poet. Gardner isolates four aspects of linguistic intelligence. The first is rhetoric of language, or the ability to persuade others through language. The second is the mnemonic aspect with the ability to remember information. Third is the role of language in explanations, the aspect a teacher uses the most often. The final and fourth aspect is using language to talk about language, much as this paragraph has been doing. (Gardner, 1983, p.78). This intelligence is "the strength of many people who are comfortable in the universe of words, reading, and writing ... A linguistically strong child with average math skills may do fairly well on an IQ test" (Wahl, 1999, p. 4). Students with linguistic skills are able to function exceptionally well in an education system that deals with words. These students are adept in the realm of language.

## Spatial Intelligence

Spatial Intelligence is directly correlated to the visual realm. Spatial intelligence is neatly described by Gardner in saying that, "central to spatial intelligence are the capacities to perceive the visual world accurately, to perform transformations and modifications upon one's initial perceptions, and to be able to re-create aspects of one's visual experience, even in the absence of relevant physical stimuli" (Gardner, 1983, p.173). Spatial intelligence involves students visualizing pictures in their minds. People that excel in the spatial realm are adept at converting problems from the linguistic form into a spatial idea. In the classroom, students that are high in spatial intelligence learn well from visual aides, such as diagrams, flow charts and manipulatives. Spatial intelligence is exceedingly important since "about 40\% of children are visual learners, thus elevating the spatial intelligence to a prime position in communicating and understanding" (Wahl, 1999, p. 15).

## Bodily - Kinesthetic Intelligence

Bodily - Kinesthetic Intelligence is exactly what the name implies: it is the intelligence that uses body movement. The people with a high level of bodily intelligence are "individuals - like dancers and swimmers - who develop keen mastery over the motions of their bodies, as well as those individuals - like artisans, ballplayers, and instrumentalists - who are able to manipulate objects with finesse" (Gardner, 1983, p.207). Those with a highly developed sense of bodily intelligence can then have both mastery of their entire body, and also mastery over the objects used by the body. This is evident in the pianist's rapid finger movements and the gymnast's body manipulations in floor routine. Bodily-kinesthetic intelligence is "used by many to gain insight, solve
problems, and process information in other areas. For instance, a violinist uses it to control the bow even though melodic decisions are made by his or her musical intelligence" (Wahl, 1999, p. 3).

Students that excel in bodily-kinesthetic intelligence are often the restless students, who cannot seem to sit still while in class. In fact, "many students who get in trouble in the school setting are kinesthetic. They need movement, action, and physical content. When most teachers don't provide it, they create it by falling off chairs [and] horse-playing with classmates" (Wahl, 1999, p. 20). Teaching activities that involve body movement, as well as touching and manipulating objects can tap into this intelligence. They will learn by "being 'walked though' math procedures, having a 'hands-on experience,' getting a 'gut feeling,' and 'going through the motions" (Wahl, 1999, p. 3). Often learning the material through these bodily ideas can lead to deeper understanding in one of the other intelligences.

## Interpersonal Intelligence

The first of the two people intelligences, interpersonal intelligence is the capacity to interact well with other people. The core capacity of interpersonal intelligence is "the ability to notice and make distinctions with other individuals and, in particular, among their moods, temperaments, motivations and intentions" (Gardner, 1983, p.239). The people that are adept in interpersonal intelligence are very skilled at reading other people and perceiving their needs and moods. Every single person has interpersonal intelligence. Most of our lives are "filled with interpersonal relationships; some of us are more agile at problem solving in this area than others" (Wahl, 1999, p. 3) People highly skilled in interpersonal intelligence are often found in political or religious arenas. In the
classroom, these students thrive on group projects. They excel in cooperative learning. They may "demonstrate leadership, and be continually drawn into social situations. They may acquire friends easily and even obligingly alter their personae with each" (Wahl, 1999, p. 4). Interpersonal intelligence often can be easy to spot in students, and it can help foster learning within an entire group.

## Intrapersonal Intelligence

In contrast to interpersonal intelligence, intrapersonal intelligence looks inward, at oneself. The core capacity of intrapersonal intelligence is "access to one's own feeling life - one's range of affects or emotions: the capacity instantly to effect discriminations among these feelings and, eventually, to label them, to enmesh them in symbolic code, to draw upon them as a means of understanding and guiding one's behavior (Gardner, 1983, p.139). A common word applied to those strong in this intelligence is introspective. These students may seem thoughtful and sensitive. While the students may be strong in internalizing answers about themselves, they may not have the linguistic or interpersonal skills to effectively communicate. This intelligence is strongly shown in philosophers that wonder about their meaning in life, or religious theologians.

## Classroom Implementation

Before effectively using multiple intelligences in the classroom, it is important to use some steps for establishing an environment in the classroom around multiple intelligences. Gardner gives a list of some practices that are useful for the teacher to employ before implementing multiple intelligences in the classroom:

1. Learn more about the MI theory and practices.
2. Form study groups.
3. Visit institutions that are implementing MI ideas.
4. Attend conferences that feature MI ideas.
5. Join a network of schools.
6. Plan and launch activities, practices, or programs that grow out of immersion in the world of MI theory and approaches. (Gardner, 1983, p.145)

Instead of diving into the idea of a multiple intelligence classroom, it may be better to start with spicing up the everyday uniform curriculum. Teachers may "enhance the flavor of your usual daily math instruction, (or any other subject for that matter) with Multiple Intelligences seasoning and you'll notice your students beginning to perk up. That in turn will energize you, your class preparation, and your teaching day. You'll get used to that extra spicy edge." (Wahl, 1999, p.11) Start slowly by working one intelligence at a time into a lesson plan. Look to see what the predominate intelligences used in the class are and use those intelligences in a lesson. Teachers should start slowly, and pick up momentum as they become comfortable with seasoning lessons with different intelligences. Many multiple intelligence handbooks do not even use all of the eight intelligences. The key point to remember when adding multiple intelligences to the classroom is to start small. Start small, and when the teacher is comfortable with seasoning with multiple intelligences, then the teacher should simply add more. Clearly all of the practices may not be available for all schools, but for teachers seeking to connect at a deeper level with the cultural and learning styles of their students, the above provide idea of where to begin. Of the steps above, learning about the theory and having a system of collaboration provide a natural place to implement multiple intelligences into the mathematics classroom. To effectively teach using the MI theory, an educator must
be fluent in the theory itself, and understand its implications and limitations. Secondly, an educator must have a support system, where he or she can talk about success and failures regarding planning and implementation guided by MI theory. The support system should also give the educator new ideas to try and encourage the MI classroom.

Once the educator has thoughtfully considered using MI as a theoretical perspective supporting instructional practices, and is ready to implement it into the classroom, there are many methods for the implementation. Each of the eight intelligences can be expressed in the classroom using different activities and teaching methods. A brief idea of how this can be done is shown in Table 1 below. Table 1 provides ideas for how to implement multiple intelligences in the classroom. These examples are general ideas for how a subject can be taught using a specific intelligence. For example, if in a literature class the teacher was trying to use a spatial intelligence, they may use mind mapping for the plot in a novel. The third column is examples of teaching materials. This is a short list of materials an educator can use to help key into a

Table 1: Summary of the "Seven Ways of Teaching" (Armstrong, 1994, p. 52)

| Intelligence | Examples of Possible <br> Teaching Activities | Examples of Teaching <br> Materials | Examples of <br> Possible <br> Instructional <br> Strategies |
| :--- | :--- | :--- | :--- |
| Linguistic | Lectures, discussions, <br> word games, story <br> telling, choral reading, <br> journal writing, etc. | Books, tape recorders, <br> typewriters, stamp sets, <br> books on tape, etc. | Read about it, talk <br> about it, listen to it |
| Logical- | Brain teasers, problem <br> Mathematical | Calculators, math <br> solving, science | manipulatives, science |
|  | experiments, mental <br> calculations, number <br> games, critical <br> thinking, etc. | equipment, math <br> games, etc. | conceptualize it, about it, |
|  | Visual presentations, <br> art activities, <br> imagination games, | Graphs, maps, videos, <br> LEGO sets, art | See it, draw it, <br> materials, optical <br> illusions, cameras, <br> mind-mapping, |
| metaphor, visualization, <br> etc. | picture library, etc. |  |  |

specific intelligence. The last column gives specific verbs for the teacher to use for a subject that will tie the subject into different intelligences. If a teacher wants to use a specific intelligence for a concept in school, the teacher can look for a verb that matches with the intelligence and use that verb in relation to the concept. This table is only a short list of the different methods used to incorporate multiple intelligences in the classroom; many other examples can be easily thought of by the educator. Further on in this paper, multiple activities related to mathematics will be presented and discussed.

While most educators will agree that the idea of personalizing education to every individual student is an estimable goal, the actual method of individualization becomes overwhelming. The subject of mathematics falls neatly into the intelligence titled "logical/mathematical." However, it is important to remember when teaching mathematics that not every student excels in the mathematical intelligence. It is therefore important to teach students mathematics using different intelligences. This is not to say that there are certain subjects or content that should not be taught because it is not the predominate intelligence of students. Gardner neatly states, "All young people should study the history of their country, the principles of algebra and geometry, and basic laws that govern living and nonliving objects. A commitment to some common knowledge does not mean that everyone must study these things in the same way and be assessed in the same way." (Gardner, 1983, p.152). Instead, each concept in mathematics should be presented to best capitalize on students' predominate intelligences. An example of how this can be done is found in Table 2 on the next page. (Kuzniewski, 2006, p. 59) This table shows different ways of teaching mathematics using seven different intelligences. Each section has five specific activities that can help a mathematics educator brainstorm
methods of teaching concepts without relying heavily on the mathematical intelligence.
This chart does not show the naturalist intelligence.
Table 2: Seven Ways of Teaching Mathematics (Kuzniewski, 2006, p. 59)

| Logical/Mat hematical | Verbal/ <br> Linguistic | Visual/ Spatial | Bodily/ Kinesthetic | Interpersonal | Intrapersonal |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Find unknown quantities/ entities in a problem | Write a series of story problems for others to solve | Do a survey of students likes/dislikes, then graph the results | Use different parts of the body to measure things | Solve complex story problems in a group | Track thinking patterns for different math problems |
| Teach how to use a calculator for problem solving | Explain how to work a problem to others while they follow | Estimate measurement by sight and by touch | Add and subtract members to and from a group to learn about fractions | Do a statistical research project and calculate percentages | Bridge math concepts beyond school (what? So what? Now what?) |
| Create number sequences and have a partner find the pattern | Make up puns using math vocabulary terms | Add, subtract, multiply, and divide using various manipulatives | Design something that requires applying math concepts | "Each one teach one" new math processes/ope rations | Use guided imagery to see complex story problems |
| Mind-map proofs for geometry theorems | Solve problems with a partner-one solves and one explains process | Imagine using a math process successfully, then really do it | Create and act out a play in which the characters are geometric shapes | Describe everything you do to solve a problem with a partner | Evaluate your strengths/weak nesses in understanding math |
| Design classification charts for math formulas and operations | Create poems telling when to use different math operations | Learn metric measurement through visual equivalents | Make up a playground game that uses math concepts operations | Have teams construct and solve problems linking many math operations | Watch mood changes as you do math problems-note causes |

## Curriculum Design

This Curriculum Project was created for students in Algebra 1. The unit is intended for instruction during the first quarter of the school year. A Unit Overview is provided as well as a Unit Timeline for teachers to be able to plan ahead and manage their time. This suggested 13-day timeline is based on a class period of 45 minutes. If the class period is shorter, the teacher can adjust the lesson to fit the needs of their students, taking the amount of class time into consideration. This curriculum design was created to align to the New York State Common Core Standards. The instructional materials are provided in the Appendix. In addition to the necessary worksheets and activities, each lesson plan includes the goal for the lesson, the instructional outcomes, and the New York State Common Core learning standards that are addressed.

## Unit: Linear Equations and Inequalities in One Variable

## Unit Overview:

This unit focuses on the algebra of equations and inequalities in one variable. Students deepen their understanding of the properties of equality and how the basic mathematical operations can be used to transform expressions, equations, and inequalities. These understandings form a basis for students to solve equations and inequalities in one variable and to rearrange formulas to isolate a specific quantity. Students use algebraic tools and concepts to investigate and interpret equations in a useful way. There are multiple intelligences that are highlighted within each lesson in this unit.

Below you will find the layout of how the lesson integrates MI into mathematics instruction of the A.REI. 3 Standard (Reasoning with equations and inequalities- solve linear equations and inequalities in one variable, including equations with coefficients
represented by letters). In the appendix you will find worksheets used as well as key
solutions for all materials.

## Integration of MI into Instruction

Multiple Intelligences used in this unit: Logical-Mathematical Intelligence, Interpersonal Intelligence, Bodily-Kinesthetic Intelligence, Linguistic Intelligence, Intrapersonal Intelligence, and Spatial Intelligence

## Essential Questions:

- How can we represent information symbolically?
- How are different representations related to each other?
- How do we prove that our solutions to problems are correct?


## Standards:

A.REI.3: Reasoning With Equations And Inequalities- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

## Unit Structure:

Lesson 1: Writing Equations and Inequalities (Day 1)
Lesson 2: Writing Equations and Inequalities (Day 2)
Lesson 3: Writing Equations and Inequalities (Day 3)
Lesson 4: Writing Equations and Inequalities (Day 4)
Lesson 5: Spotting Solutions (Day 1)
Lesson 6: Spotting Solutions (Day 2)
Lesson 7: Spotting Solutions (Day 3)
Lesson 8: Spotting Solutions (Day 4)
Lesson 9: Strategies for Solving Equations
Lesson 10: Matching Equivalent Representations
Lesson 11: Matching Strategies to Equations
Lesson 12: Solve Literal Equations
Lesson 13: Comparing Graphs and Equations
*Materials \& Handouts: Appendix pages 42-91
*Answer Keys: Appendix pages 92-125

| Lesson | Writing Equations and Inequalities (Day 1) |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Use functions to write equations that represent a specific case. |
| Materials | - Student handout <br> - Document camera <br> - See Appendix Pages 41-44 |
| Warm Up | 1. Individual Work: Students will use what they have learned in previous units to write a function representing a given situation. <br> 2. Whole class discussion: Students share out their functions and second representation. Listen for students who prefer a table of values compared to a graph to represent the function, and students who use the language of domain/range or input/output to describe the situation. |
| Exploration | Use the Stronger and Clearer Each Time routine <br> 1. Individual Work Time: Give students about 5 minutes of independent think time to answer the questions. <br> 2. Partner Work: Have students share their thinking with a partner. Then, pair students for Round 1 to make their explanations stronger. Have them find another partner for Round 2 to make their explanations clearer. Afterwards, have students return to their first partner to share out their new statement. <br> Circulate to observe student approaches. Observe how students are making sense of the situation, equation, and table of values, and listen for opportunities to elicit academic vocabulary. |
| Discussion | Select which responses to focus on during discussion. Select a response that highlights the representations for input and output values. <br> Prompts for discussion: <br> - What are the quantities or relationships in this situation? |


|  | - What's unknown? <br> - What quantities do you consider when deciding how to write a function? (Note: answers will vary) <br> - What changes and what stays the same in the situation? |
| :---: | :---: |
| Application/ Reflection | 1. Individual Work: Give students a minute to read over the problem sets before they work with a partner on the left column. <br> Tip: Do the math. Consider how students may answer the questions. For example, in Problem Set 1, students could write the equation 60 $=12 \mathrm{x}-24$ or $5=\mathrm{x}-2$, depending on the quantity converted, and in the Your Turn, students could write $1.29+0.99(x-1)$ or $0.3+$ $0.99 x$. <br> 2. Partner Work: Have students work in partners to make sense of the problems on the left column and answer them together before working independently on the Your Turn problems on the right column. <br> 3. Share out: If there is time, identify one or two pairs to share out and annotate their work |
| Regents connection | 1. Algebra I CC NYS Regents August 2016 Q. 16 \& January 2017 Q. 30 were modified and used in Problem Set 1. <br> 2. Algebra I CC NYS Regents June 2015 Q. 26 was modified and used in Problem Set 2. |


| Lesson | Writing Equations and Inequalities (Day 2) |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | - Analyze graphical representation of an equation in one variable. |
| Materials | - Student handout <br> - Matching cards and Key |


|  | - Document Camera <br> - See Appendix Pages $45-48,80$ |
| :---: | :---: |
| Warm Up | 1. Individual Work: Students explain how the equation and question match, using information about the situation and table of values. <br> 2. Whole Class Discussion: Students share out their explanations. Listen for students who make observations about what is the input value and what is the output value. Annotate the table or situation to match what students say about the equation and question. |
| Exploration | 1. Individual Work: Students match an equation to the graph based on information presented in the situation and table. <br> 2. Partner Work: In anticipation of the full class discussion, give pairs of students time to share and discuss their work. Ask pairs to compare the equations and consider the similarities and differences. <br> 3. Circulate to observe student approaches. Observe how students are making sense of the representations, and listen for opportunities to elicit academic vocabulary. |
| Discussion | Select which situations/questions to focus on during discussion based on common answer, areas of misconception, etc. <br> Promps for discussion: <br> - What do you look at first when analyzing a graph? (Note: Answers will vary) <br> - What similarities and differences do you notice between the equations? How did you interpret those similarities and differences using the graph/table of values? <br> - Compare the term that represents rates. What do you notice? (Note: This is an opportunity to assess students' understanding of rate) <br> - For reach representation: What is the domain and range? Name the independent and dependent variables. |
| Application/ Reflection | 1. Individual Work: Students complete the Application questions by matching a situation, graph, or table to one of the equations and write and explanation for two of thes matches. Then they write a response to the reflection question. <br> 2. Pair/Share: If time allows, ask students to share their responses. As |


|  | you circulate, look for explanations that connect the graphs to other <br> representations for input/output and a specific case. |
| :--- | :--- |


| Lesson | Writing Equations and Inequalities (Day 3) |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Write Equations in One Variable |
| Materials | - Student handout <br> - Connecting Representations <br> - Document Camera <br> - See Appendix Pages 49-51 |
| Warm Up | 1. Individual Work: Venn Diagram-Students analyze the examples of functions and equations before writing down characteristics that overlap or are unique to each. (Tip: In anticipation of the whole class discussion, listen for students who make connections to prior learning about equations that represent relations, and variables representing unknowns. <br> 2. Whole Class Discussion: Students share out what they know and possibly wonder about functions compared with equations. |
| Exploration | 1. Connecting Representations: Students make connections between equations and sentences in an instructional routine called Connecting Representations. <br> a. Individually Think: Students individually look for connections. ( 15 seconds) <br> b. Make Connections: Students share connections with a partner, while the teacher circulates to select which pairs will share. (3-4 min) <br> c. Share and study connections: Students discuss connections as a group. (6-8 min) <br> d. Create a representation: Students create a representation |


|  | with a partner and a few partner pairs will share their work with the whole group (6-8 min) <br> e. Reflect: Students reflect on learning and the teacher selects a few students to read their reflections with the whole group (3-5 min) <br> 2. Circulate to observe student approaches. Observe how students are making sense of the representations, and listen for opportunities to support students in making connections between the representations. |
| :---: | :---: |
| Discussion | 1. Select the examples to focus on during the "share and study connections" time, based on common answers, areas of misconception, etc. <br> 2. Promps for discussion: <br> a. What are the quantities in the situation (described in the sentence)? <br> b. How can one quantity be changed to represent the other quantity? <br> c. How can the relationship between the two quantities be represented? |
| Application/ Reflection | 1. Individually: Students practice connected Regents problems. <br> 2. Pair/Share: As you circulate, look for connections students are making about content from this week to answer these questions. |
| Regents connection | 1. Algebra 1 CC NYS Regents August 2015 Q.8; Key: 4 <br> 2. Algebra 1 CC NYS Regents August 2016 Q.14; Key: 3 <br> 3. Algebra 1 CC NYS Regents August 2016 Q.16; Key: 3 <br> 4. Algebra 1 CC NYS Regents January 2015 Q.23; Key: 4 <br> Tip: Highlight quantities/ relationships students' focus on (not just the numbers or given values). Call these out by having students annotate their own work, or share their approaches to the class. |
| Lesson | Writing Equations and Inequalities (Day 4) |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence |


|  | - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| :---: | :---: |
| Objectives | Write Inequalities in one variable |
| Materials | - Student handout <br> - Card Sort (print friendly version) \& key <br> - Document Camera <br> - See Appendix Pages 52-54, 81 |
| Warm Up | 1. Individual Work: Compare and Contrast. Students compare the two sets of representations to determine what makes them different. <br> 2. Whole Class Discussion: Students share out. Listen for students who make observations about the "at least" and shading. |
| Exploration | Card Sort Guide <br> 1. Individual Work time: Students sort the cards <br> 2. Partner Work: In anticipation of the full class discussion, give pairs of students time to share and discuss their work. Ask pairs to compare equations and inequalities, and consider the similarities and differences. <br> 3. Circulate to observe student approaches. Observe how students are making sense of the situations, and listen for opportunities to push for academic vocabulary and make connections to prior learning. Identify work that highlights the focus on units. |
| Discussion | 1. Select which equations/inequalities to focus on during discussion based on common answer, areas of misconception, etc. <br> 2. Promps for discussion: <br> a. What values would make the statement "at least"/"at most"/"maximum", etc true? <br> b. What does it mean to represent a value with a closed dot or open dot on a number line? |
| Application/ Reflection | 1. Individual Work: Students complete the questions and then write a response to one of the reflection questions. If time allows, review |


|  | distractors with the full class. <br> 2. Pair/Share: As you circulate, look for approaches students take to <br> identifying how to write the inequality. |
| :--- | :--- |
| Regents <br> connection | Algebra 1 CC NYS Regents August 2015 Q:5; Key: 4 |


| Lesson | Spotting Solutions (Day 1) |
| :---: | :---: |
| Intelligences Used | - Musical Intelligence <br> - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Model a situation using linear equation |
| Materials | - Student handout <br> - Dan Meyer's Three Acts: 25 Billion Apps folder of materials <br> - Document Camera <br> - See Appendix Pages 55-58 |
| Warm Up | 1. Individual Work: Students analyze worked examples to answer questions about the process and use their understanding to answer another problem. <br> 2. Whole Class Discussion: Students share out their responses. Listen for students who describe the process of solving using language that describes the properties of equality or combining like terms or distributive property. |
| Exploration | Three Acts Description \& Dan Meyer's 25 Billion Apps <br> 1. Individual Work Time: Students watch a video, record what they notice and wonder about the context, and consider questions to answer as a class. <br> 2. Partner Work: In anticipation of partner work, ask students, "what information would be useful to know here?" Provide information. <br> 3. Whole Class Discussion: Reveal the image for Act Three. Students reflect on their model and share out what is important to consider when creating a model. |


|  | Tip: This is the second Three Acts of the course. Focus on the approaches and thinking that support modeling. There is more structure to ease students into the process of mathematical modeling. Focus on one question: "When should you start bombarding the App Store with purchases if you want to win?" Provide the table of values based on requests or description for how they would gather data. Before the Reveal, remind students that mathematical models are created to make sense of the world and make predictions (they are not always exact). If students switch from trying to graph to using a table or writing an equation, listen for scale as a reason. |
| :---: | :---: |
| Discussion | Select which models/questions to focus on during discussion based on approaches, misconceptions, etc. <br> Prompts for discussion: <br> - During Act 1: What do you know or want to know? What question can we answer using math we have learned? <br> - During Act 2: What are the quantities/relationships in this situation? What quantities do you consider when deciding how to write a function? How would you use what changes in the situation to write an equation? <br> - During Act 3: What assumptions have you made in your model? Interpret the parameters in your model. What do the $y$ intercept/units of slope represent? What does the y-intercept represent? |
| Application/ Reflection | 1. Individual Work: Complete the questions in the Sequel. Ask, "What will you consider next time you create and use a model?" <br> 2. Share out: If there is time, identify two or three responses and reflections to share out. |


| Lesson | Spotting Solutions (Day 2) |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Solve Linear equations in 1 variable using an algebraic method |
| Materials | - Student handout; placemat |


|  | •Card Sort (KEY) <br> - <br> Warm Up <br> See Appendix Pages 59 - 63, 82 |
| :--- | :--- |
| 2. Individual work time: Notice and wonder, students record what |  |
| they notice and wonder about the given representations of |  |
| expressions. |  |


| Application/ <br> Reflection | 1. Partner work: Students work with partners to apply what they <br> learned about the Algebra Tiles. <br> Tip: Provide students with Algebra Tiles to manipulate- <br> there are many templates for cutting these pieces out if <br> purchasing this is not an option-or use an online tool such <br> as Virtual Algebra Tiles. Instead of printing, you can use <br> the placemat in plastic sleeves for students to reuse with <br> dry erase markers. |
| :--- | :--- |
| 2. Independent work: Students answer the reflection questions. |  |
| 3. Share out: If there is time, identify two or three reflections to share |  |
| out connecting to annotated work from earlier in the lesson. |  |


| Lesson | Spotting Solutions (Day 3) |
| :---: | :---: |
| Standards |  |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Solve linear inequalities in 1 variable using an algebraic method. |
| Materials | - Student handout <br> - Document camera <br> - See Appendix Pages 64-65 |
| Warm Up | 1. Individual Work: Students solve the given equation and represent each step using Algebra Tiles for the visual model. Students also provide justifications for each step. <br> 2. Whole Class Discussion: Students share out their approaches. Listen for students who make connections to the properties, especially zero pairs and inverse. |
| Exploration | Contemplate Then Calculate <br> 1. Launch the routine by explaining what, why, and how the routine will work. |


|  | 2. Flash an image ( $1 / 2$ second) <br> - Students share what they noticed about the image with a partner <br> - Students share what they noticed with the class while the teacher records <br> 3. Students find a shortcut based on the given image/ task. <br> - Teacher circulates to listen to student approaches, select presenters. |
| :---: | :---: |
| Discussion | Prompts for discussion: What are the chunks in the image? How are you using these chunks? What does your strategy work? <br> - Annotate to highlight different ways of chunking the square together, changing the form of the squares to us multiplication and area. <br> 4. Students share their strategies with the group <br> - After strategy is shared, another student restates the strategy why the teacher annotates the strategy on posters. <br> - Prompts for discussion: What is that strategy/ approach called? How does that connect what we have learned this week? <br> 5. Students select a sentence prompt and reflect on what they have learned. <br> Teacher records the reflections and prompts students to consider how they connect to what they initially notice and the strategies surfaced. |
| Application/ Reflection | 1. Individual Work Time: Students complete the Application questions, recording an annotation strategy that they want to remember from today's lessons as well as comparing inequalities to equations. <br> 2. Pair/Share: Identify students to share out their responses to highlight key similarities and differences between equations and inequalities. |


| Lesson | Spotting Solutions (Day 4) |
| :--- | :--- |
| Intelligences | $\bullet \quad$ Logical-Mathematical Intelligence |
| Used | $\bullet \quad$ Interpersonal Intelligence |


|  | - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| :---: | :---: |
| Objectives | Determine the solution to an equation or inequality and justify its validity. |
| Materials | - Student handout <br> - Matching Activity \& key <br> - Document camera <br> - See Appendix Pages 66-68, 83-84 |
| Warm Up | 1. Individual Work: Students analyze worked examples to answer questions about the process and use their understanding to answer another problem <br> 2. Whole Class Discussion: Students share out their responses. Listen for students who describe the process of solving inequalities using language that describe inequality sign. |
| Exploration | 1. Individual Work Time: Students identify which situation can be represented as an equation or inequality. Then writes the equation/inequality, solves, and matches to its solution or solution set. <br> 2. Partner Work: In anticipation of the full class discussion, give pairs of students time to share and discuss their work. Ask pairs to use Algebra Tiles if they need it to write the equations or inequalities. <br> 3. Prompt students to use the properties when they justify each step and have them discuss how they know the values are solutions to the equation or inequality Circulate to observe student approaches. Observe how students justify the steps of solving and how they describe solutions. |
| Discussion | Select which situations/ questions to focus on during discussion based on common answer, areas of misconception, etc. <br> Prompts For discussion: <br> - How do you if an equation or inequality describes the situation? <br> - How can you use the Algebra Tiles to help make sense of each step for solving? |


|  | • How do you know you have the problem? What are units of the <br> quantities and relationship |
| :--- | :--- |
| Application/ <br> Reflection | 1. Individual Work: Students complete Regents questions. <br> 2. Pair/ Share: If time allows, review distractors with the full class and <br> connect to what students have been working on this past week. |
| Regents <br> connection | 1. Algebra 1 CC NYS Regents August 2014 \#20; Key:1 <br> 2. Algebra 1 CC NYS Regents January 2015 \#13; Key:3 <br> 3. Algebra 1 CC NYS Regents June 2014 \#5; Key:1 <br> 4. Algebra 1 CC NYS Regents June 2014 \#27; CR |


| Lesson | Strategies for Solving Equations |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Use visual representations to support students in revisiting strategies do solving equations. <br> - Recognize similarities and differences between different strategies applied to the same equation and how the same strategy is applied to different equations. <br> - Solve an equation with a variable on one side of the equation. |
| Materials | - Student handout <br> - Connecting Representations <br> - Task <br> - Slides <br> - Posters <br> - See Appendix Pages 69 |

\(\left.$$
\begin{array}{|l|l|}\hline \text { Warm Up } & \begin{array}{l}\text { 1. Individual Work: Students solve the given equation using any } \\
\text { method. }\end{array}
$$ <br>
\hline 2. Pair-share: Have students share what they noticed about the graph <br>

with a partner.\end{array}\right]\)| Exploration |
| :--- |
| Use the connecting representations task to support students who need <br> additional time learning how to solve equations and to preview <br> representing solutions to inequalities on a number line. <br> Note that for this task, students are matching solution strategies <br> represented algebraically with solution strategies represented on <br> number lines, so the equations in both deliberately do not match. <br> 1. Launch the routine: Tell students what, why, and how they will <br> work today. |
| 2. Make Connections: Students share connections with a partner |
| while the teacher circulates to select which pairs will share (3-4 |
| mins) |



| Lesson | Matching equivalent representations |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Use visual representations to connect the different forms of the same expressions <br> - Algebraically prove equivalency by representing each equation identically <br> - Create equivalent algebraic expressions, equations, or inequalities <br> - Add or subtract polynomials expressions with fluency |
| Materials | - Student handout <br> - Document camera to display student work <br> - Copies of the matching activity for each pair of students <br> - See Appendix pages $85-87$ |
| Warm Up | 1. Individual work: Students describe similarities and differences between two different sequences. <br> 2. Whole class discussion: Share some similarities and differences students notice. Use annotation to make those similarities and differences visible for other students. |
| Exploration | Exploration: <br> 1. Individual work time: students look at the cards for 30 seconds or so without discussing what they notice about the cards with anyone. <br> 2. Partner work: Students sort out the cards into any categories they choose. |


|  | 3. Partner work: Students match the cards into equivalent groups. <br> Teacher circulates to observe student approaches, selects students work to display. |
| :---: | :---: |
| Discussion | Students and display student matches for discussion. Annotate the examples to highlight how students have used algebra title or area model relationships to make the matches. <br> Possible/ suggested prompts for discussion: <br> - What is the similar/different about the cards in this match? <br> - Why isn't [this card] a match for [this other card]? <br> - What shortcuts for determining equivalence do you see? <br> Tip: Remind students that each group only has 3 cards in it, one from each $A, B$, and $C$, and that this means that other cards are not a match. |
| Application/ Reflection | 1. Individual work: Students apply what they learned to solve an example regents problem ( $3-4 \mathrm{~min}$ ) then write a response to the reflection prompt (2-3 min). <br> 2. Reflection: <br> a. Students share reflections with a partner <br> b. Select 2-3 reflections to be read to the class. Highlight connections between reflections and the goal for today's lesson. |


| Lesson | Matching strategies to equations |
| :---: | :---: |
| Intelligences Used | - Musical Intelligence <br> - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |


| Objectives | Match worked examples to equations they best fit, then apply the strategies from the examples to equations <br> - Solve an equation or inequality with a common variable on both sides algebraically <br> - Analyze an equation or inequality use additive or multiplicative inverses, the zero product property, or the distributive property to identify a solution. |
| :---: | :---: |
| Materials | - Student handout <br> - Document camera to display student work <br> - Copies of the matching activity for each pair of students. <br> - See Appendix Pages 70-72, 88-89 |
| Warm Up | 1. Individual Work: Students look at the inequality solved two different ways to find an error in how inequalities were solved and to compare different strategies. <br> 2. Whole Class Discussion: Share student work from the practice problem with the class. Use annotation to make the connections the student describes to the worked example clear for other students. |
| Exploration | 1. Individual Work Time: Students look at the cards for 30 seconds or so without discussing what they notice about the cards with anyone. <br> 2. Partner Work: Students sort the cards into any categories they choose. <br> 3. Partner Work: Students resort the cards, matching the worked examples to equations the strategies could be applied. <br> 4. Partner Work: Students apply the strategies to the matched equations. <br> Teacher circulate to observe student approaches, selects student work to display |
| Discussion | Select and display student work from the card for discussion. <br> Annotate the examples to highlight connections between the work examples and practice problems <br> Possible/suggested prompts for discussion: <br> - Why did you match [this worked example] to [this equation]? <br> - What is similar and what is different about [this equation] to [this |


|  | worked example]? <br> Tip: Based on the circulation that you do, choose an area of solving equations to focus on. Also, we recommended noting other strategies that students use to solve the equations than ones provided with the worked examples. |
| :---: | :---: |
| Application/ Reflection | 1. Individual Work: Students apply what they learned to solve a pair of equations ( $3-4 \mathrm{~min}$ ) then write a response to the reflection prompt (2-3 min.) <br> 2. Reflection: <br> a. Students share reflections with partner. <br> b. Select 2-3 reflections to be read to the class. Highlight connections between reflections and the goal for today's lesson |


| Lesson | Solving Literal Equations |
| :---: | :---: |
| Intelligences Used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| Objectives | Match steps to solving literal equations to input equations and output equations <br> - Solve for one variable ( $x$ in terms if $y$, etc.) <br> - Algebraically to prove equivalency each equation identically |
| Materials | - Student Handout <br> - Document camera to display student work <br> - Copies of the matching activity for each pair of students. <br> - Copies of the place mat for each pair of students <br> - See Appendix Pages 73-77, 90 |
| Warm Up | 1. Individual Work: Students look at a guess and check strategy for solving an inequality and find the error in the work. |


|  | 2.Whole Class Discussion: Share a student example describing the <br> error for a brief discussion. Use annotation to make the error (x=2 <br> instead of x>2) clear for students. <br> Exploration <br> 1. Individual Work Time: Students look at the cards for 30 seconds <br> or so without discussing what they notice about the cards with <br> anyone. <br> 2. Partner Work: Students match the cards in groups with one A, B, <br> C, and D card in each group so that the equation B and equation C <br> represent steps between transforming equation A into equation D. |
| :--- | :--- |
| 3.Partner Work: Students apply the results of the matching activity <br> to solve literal equations for given variables within the given <br> context. |  |
| Teacher circulates to observe student approaches, selects student |  |
| work to display |  |


| Lesson | Comparing Graphs and Equations |
| :--- | :--- |
| Intelligences | $\bullet \quad$ Musical Intelligence |


| used | - Logical-Mathematical Intelligence <br> - Interpersonal Intelligence <br> - Bodily-Kinesthetic Intelligence <br> - Linguistic Intelligence <br> - Intrapersonal Intelligence <br> - Spatial Intelligence |
| :---: | :---: |
| Objectives | Explain how to use graphs to solve equations. <br> - Express an equation with a variable on both sides as two linear functions <br> - Use a graph to solve an equation with a common variable on both sides of the equation |
| Materials | - Student handout <br> - Document camera to display student work (optional) <br> - Connecting Representations <br> - Task <br> - Slides <br> - Posters (print as large possible) <br> - See Appendix Pages 78, 91 |
| Warm Up | 1. Individual Work: Students solve the given equation using any method. <br> 2. Pair-share: Have students share what they noticed about the graph with a partner |
| Exploration | Use this Connecting Representations task to introduce the idea of using graphs to solving equations. <br> Ideally students will match the graphs of the lines to each side of the given equations and recognize that one can quickly tell either the solution or lack of a solution from the graph. <br> 1. Launch the routine: Tell students what, why, and how they will work today. <br> 2. Make connections: Students share connections with a partner while |


|  | the teacher circulates to select which pairs will share (2-4 mins) <br> 3. <br> Share and study connections: Students discuss connections as a <br> group (15 mins). |
| :--- | :--- |
| Discussion | Annotate representations to highlight connections as students <br> describe their thinking; push for clarity and for consistent use <br> mathematical and academic language. |
| Prompts for discussion: <br> - How did you connect the two representations? |  |
| - How did you interpret the representations? |  |
| - How are the solutions (or lack of a solution) to these equations |  |
| represented in the graphs? |  |

## How This Works In The Classroom

This section presents how the author has used the work provided in her own classroom and provides first person voice to share her experience.

I work in an urban New York City school and we currently have a lab class for all $9^{\text {th }}$ grade students in Algebra. This allows for, and almost demands, utilizing different modalities to reach students. Our classes are comprised of students that may have never passed a single math class to students that excel and find the material very basic. Our job is to reach each student where they are. This is often a difficult feat given 29 students in a class.

Employing multiple intelligences throughout a lesson is one way I have ensured that each child is receives moments of comfort in their strength as well as challenged when their intelligence preference is not being utilized. This ensures a more equal footing in the classroom.

I use our first period of Algebra to introduce a lesson, delve into the intricacies, ask the probing questions and elicit feedback. Multiple intelligences are capitalized during that time especially Linguistic, Logical-Mathematical and Spatial. During the lab period, the students will often work in pairs. Sometimes they complete puzzles, scavenger hunts around the room and creating models. These activities utilize many of the other intelligences. This enables all students to be exposed to the material in their preferred intelligence and there is a better possibility of all students understanding the mathematics being taught.

One major hindrance to implementing MI in the classroom is our own preferences. I fight to overcome my own biases to expose my students to techniques that will work best for them even though they may not work for me. As a mathematics teacher who seeks to instruct for student understanding, that can be evidenced in performance on assessments, I have found the intelligences naturally cultivated in our classroom experiences, often without intent. I have found using the different approaches to be beneficial for our students motivation and learning. My major weakness within MI is with musical intelligence and I find I do not implement that well in my classroom. This is a huge disservice to my students as many of them have that intelligence as their strongest one.

Being aware and collaborating with other educators is key to student success. The more we share and develop our own strengths, the better each of our individual classrooms become. A weakness of mine is strength of a colleague. Just as our students need to be met at their level, we need to understand our preferences as well and force ourselves out of our comfort zones to better educate our students.

## Conclusion

Mathematics is a subject that many students find difficult. A major reason why students struggle with mathematics is that the material is not always being taught with the students' learning needs in mind. The major contribution of this curriculum project addresses this problem using the model of Gardner's theory of Multiple Intelligences. Each intelligence included into the curriculum has been described in detail. This project also develops a system for integrating multiple intelligences into the mathematics classroom. The major contribution is a compilation of mathematics activities that will use multiple intelligences to enhance a normal standard. Using the NCTM Standards as a frame, each activity will foster learning with numerous multiple intelligences. There are many ways for an educator to use multiple intelligences in the mathematics classroom, and activities are easy to find and devise. Using the eight different intelligences and methods of implementation outlined in this thesis, teachers can easily help develop better learning and thinking skills in their students.

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## Appendix

## Materials to use with lessons.

Name:
Class: $\qquad$ Date:
Warm Up: Work independently

## Directions:

- Write the function that represents the situation.
- Create a different representation of the input/output values for this situation.

Situation
Margaret works at a furniture store and is paid $\$ 185$ a week,
plus $3 \%$ of her total sales in dollars, $x$. What is her weekly pay?

Function: $\qquad$

Representation:

## Directions:

- Examine the problem below.
- Independently, answer the questions about the problem.

A student incorrectly explained the relationship between the table of values and the equation. Here is the student's work:


- What does the table of values represent in the situation?
- Without solving the equation, explain how you would find the value of $x$ and what that means in the situation.

Round 1 Partner Swap: Stronger

- Can you explain when you said...
- Can you expand on (elaborate)...
- You said ... what is your reasoning?
- Can you provide evidence?


## NOTES:

| When you said .... maybe say it like .... | NOTES: |
| :--- | :--- | :--- |
| You should expand on .... |  |
| When you said ... your reasoning was a |  |
| little unclear |  |
| State your evidence better when.... |  |

REVISION: Write an edited statement of your responses to share with a partner.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Reflection:

## Directions:

- With a partner, complete the left column of the problem sets.
- Independently, answer the Your Turn questions.


## Problem Set 1

A student incorrectly used a function to write the equation. Here is the student's work:

## Kendal bought $x$ boxes of cookies to bring to a party. fach box

 contains 12 cookies. She decides to keep two bowes for herselt She brings 60 cookles to the party. Write an equation that can be used to find the number of boxes, $x$, Kendal bought.$$
\begin{aligned}
& f(x)=12 x-2 \\
& 60=12 x-7
\end{aligned}
$$

- What information from the situation did the student forget to use?
- What should the equation look like?


## Your Turn!

Sandy programmed a website's checkout process with an equation to calculate the amount customers will be charged when they download songs

The website offers a discount. If one song is bought at the full price of $\$ 1.29$, then each additional song is $\$ .99$.

State an equation that represents the cost, $f(x)$, when $x$ songs are downloaded.

## Problem Set 2

A student used a function and table of values to write the equation correctly. Here is the student's work:



```
Hese thoen!
```



- What does the column of 所) represent in the situation?


## Your turnt

Alex is selling tickets to a school play. An aduit ticket costs $\$ 6.50$ and a student ticket costs $\$ 4.00$. Alex focuses on selling adult tickets, $x_{\text {, ol }}$ even though he's aiready collected $\$ 48$ from student ticket sales.

Wite an equation that can be used to find the number of adult tickets, x. Alex will need to sell to collect $\$ 308$ from all ticket sales.

- Why is the value of $f(f)$ replaced with the quantity 150 ?

Reflection: Paying attention to ... when writing an equation from a function/situation is helpful because ..

$\qquad$
$\qquad$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$ Warm Up: Work independently

## Directions:

- Analyze the situation with the given function and table of values.
- Explain how you know the given equation matches the possible question.


## Situation

| Margaret works at a furniture store and is paid $\$ 185$ a week |
| :--- |
| plus $3 \%$ of her total sales in dollars, $x$. |
| Function: $y=185+0.03 x$ |
| $\boldsymbol{x}$ $\boldsymbol{y}$ <br> 0 185 <br> 100 188 <br> 200 191 <br> 300 194 <br> 400 197 <br> 500 200 |


| Equation | Question | Explanation |
| :---: | :--- | :--- |
| $y=185+0.03(200)$ | How much will she make if <br> she sells $\$ 200$ worth of <br> furniture this week? |  |
| $200=185+0.03 x$ | How much furniture did <br> she sell to make a weekly <br> salary of $\$ 200 ?$ |  |

Directions: Match the graph with the correct equation for the given situation.
Situation with Table of Values
Rowan has $\$ 50$ in a savings jar and is putting in $\$ 5$ every week. He wants to buy shoes worth $\$ 200$.
When will he have enough to buy these shoes?

| $x$ | $y$ |
| :---: | :---: |
| 0 | 50 |
| 5 | 75 |
| 10 | 100 |
| 15 | 125 |
| 20 | 150 |
| 25 | 175 |



EQUATION 1
$200=50+5 x$

EQUATION 2
$y=50+5(200)$

This graph matches equation $\qquad$ because
$\qquad$
$\qquad$
$\qquad$

Equation $\qquad$ does not match the graph, because $\qquad$
$\qquad$
$\qquad$

## Reflection: Work independently

Directions:

- Match the equation to its graph, table of values, and/or situation represented on the card. NOTE: some equations may have more than one matching card.
- Place the card(s) with the equation it matches and explain why the match works.


Equation 3:
$500=15 x+50$

Equation 3 matches with card .... because ....
$\qquad$
$\qquad$
$\qquad$
$\qquad$


Reflection: Paying attention to $\qquad$
in a graph/table of values/situation/equation is helpful, because $\qquad$
$\qquad$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up:

## Directions:

- Analyze the examples of functions and of equations, then complete the Venn Diagram.

| Functions | Equations |
| :--- | :--- |
| Margaret works at a furniture store and is paid $\$ 185$ a <br> week plus $3 \%$ of her total sales in dollars, x . What is <br> her weekly pay? | Tom is 57 years old and has a son named James. <br> In three years Tom will be twice as old as James. <br> How old is James? |
| $\mathrm{f}(\mathrm{x})=0.5 \mathrm{x}-4$ | $10=0.5 \mathrm{x}-4$ |
| $\mathrm{y}=0.5 \mathrm{x}-4$ | $7(\mathrm{x}+2)=5 \mathrm{x}+10$ |
|  |  |



## What are the chunks of the equation? What are the chunks of the situation?

## Create a representation:

## Reflecting on Learning:

A. When interpreting situations to generate equations, it's important to pay attention to...
B. Looking for ... in a situation helps me create a rule because ...
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Reflection:

RREGENTS CONNECTION: Practice Problems
Answer the Regents problems, and explain why you selected the answer choice. Use the empty space to show your work.

1. A typical cell phone plan has a fixed base fee that includes a certain amount of data and an overage charge for data use beyond the plan. A cell phone plan charges a base fee of $\$ 62$ and an overage charge of $\$ 30$ per gigabyte of data that exceed 2 gigabytes. If $C$ represents the cost and $g$ represents the total number of gigabytes of data, which equation could represent this plan when more than 2 gigabytes are used?
(1) $C=30+62(2-g)$
(3) $C=62+30(2-g)$
(2) $C=30+62(g-2)$
(4) $C=62+30(g-2)$
2. A parking garage charges a base rate of $\$ 3.50$ for up to 2 hours, and an hourly rate for each additional hour. The sign below gives the prices for up to 5 hours of parking.

| Parking Rates |  |
| :---: | :---: |
| 2 hours | $\$ 3.50$ |
| 3 hours | $\$ 9.00$ |
| 4 hours | $\$ 14.50$ |
| 5 hours | $\$ 20.00$ |

Which linear equation can be used to find $x$, the additional hourly parking rate?
(1) $9.00+3 x=20.00$
(3) $2 x+3.50=14.50$
(2) $9.00+3.50 x=20.00$
(4) $2 x+9.00=14.50$
3. Kendal bought $x$ boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Which equation can be used to find the number of boxes, $x$, Kendal bought?
(1) $2 x-12=60$
(3) $12 x-24=60$
(2) $12 x-2=60$
(4) $24-12 x=60$
4. In 2013, the United States Postal Service charged $\$ 0.46$ to mail a letter weighing up to 1 oz . and $\$ 0.20$ per ounce for each additional ounce. Which function would determine the cost, in dollars, $c(z)$, of mailing a letter weighing $z$ ounces where $z$ is an integer greater than 1 ?
(1) $c(z)=0.46 z+0.20$
(3) $c(z)=0.46(z-1)+0.20$
(2) $c(z)=0.20 z+0.46$
(4) $c(z)=0.20(z-1)+0.46$

Name: Class: $\qquad$ Date: $\qquad$
Warm Up:

## Directions:

* Analyze the algebraic representation, graphical representation, and stuations below.
- Describe the similarlies and differences.


Directions: during Independent \& Pair Work

1. Decide whether the information on the card could represent (or be the solutions to) a linear equation or an inequality.
2. Write the equations or inequalities, identifying connections to the situations or solutions.

Directions: during Presentations (Whole Group)
3. Listen to the presenting group. Then revise and improve your work.

## Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

| Equations | Inequalities |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Reflection: Work independently on Regents practice.

PREGENTS CONNECTION: Answer the Regents problem and explain why you selected the answer choice.

The cost of a pack of chewing gum in the vending machine is $\$ 0.75$. The cost of bottle of juice in the same machine $\$ 1.25$. Julia has $\$ 22.00$ to spend on chewing gum and bottles of juice for her team and she must buy seven packs of chewing gum. If $b$ represents the number of bottles of juice, which inequality represents the maximum number of bottles she can buy?
(1) $0.75 b+1.25(7) \geq 22$
(3) $0.75(7)+1.25 b \geq 22$
(2) $0.75 \mathrm{~b}+1.25(7) \leq 22$
(4) $0.75(7)+1.25 \mathrm{~b} \leq 22$

Why isn't the representation an equation?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Reflection: Pick one, and write your reflection.
A. When deciding if a situation can be represented by an equation or inequality, it is helpful to pay attention to ... because ...
B. When interpreting situations to generate inequalities, it's important to pay attention to...
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up:

## Directions:

- Independently, complete the problem set.

Problem Set


THREE ACTS: 25 Billion Apps

## ACT 1: Understand

| What do you notice? |  |
| :--- | :--- |
|  |  |

Our Question: $\qquad$
$\qquad$
Too High: $\qquad$ Too Low: $\qquad$

ACT 2: Plan \& Solve
Identify quantities, relationships, and assumptions. Create, use, and revise your model.
Quantities: $\qquad$

Relationships: $\qquad$

Assumptions:


ACT 3: Interpret
Compare the result generated by your model to the actual result.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Reflection:

The Sequel
According to your linear model, when did the app store sell its first app? Calculate an answer mathematically then find the actual answer. If those answers are different, what could explain the difference?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

According to your parameters in your model, when did Apple have less than 25 billion downloads? Represent all values in your solution.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Reflection: Select one and respond
A. I learned ....about writing a model while working with ....It helps me learn when my classmates....
B. When interpreting situations to generate equations, it's important to pay attention to...
C. Looking for ... in a situation helps me make a prediction because ...

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently


Directions:

- Use Algebra Tiles and sketch the image.
- Write the equations and justification for each resulting equation.


## Directions: during Independent \& Pair Work

1. Decide which three cards (A, B, C) match together, or represent equivalent expressions.
2. Record your matches in the given table.
3. Annotate or discuss how you know these cards match.

Directions: during Presentations (Whole Group)
4. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

| EXAMPLE NOT IN CARD SET A1 $3 x+6$ | EXAMPLE NOT IN CARD SET <br> B1 | EXAMPLE NOT IN CARD SET <br> C2 $3(x+2)$ |
| :---: | :---: | :---: |
| EXAMPLE NOT IN CARD SET <br> A4 $3(x-2)$ | EXAMPLE NOT IN CARD SET B2 | EXAMPLE NOT IN CARD SET <br> C1 $3 x-6$ |

Reflection: Work independently
Directions:

- Use Algebra Tiles and sketch the image.
- Write the equations and justification for each resulting equation.





## Reflection: Paying attention to ... in an equation/visual model is helpful because

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently to solve the equation, give visual model, and justify.


Reflecting on Learning: pick one of the prompts below and write your response
A. Paying attention to ... is helpful because...
B. I can use the structure of ... to make ... easier.

## Reflection: Work independently

Directions:

- Use your favorite shortcut described today to determine the value of heart
- Annotate the picture to clearly show the strategy you used.


What values can $\geqslant$ be?

What are the similarities and differences in solving an inequality compared to an equation?

| Similarities | Differences |
| :--- | :--- |
|  |  |
|  |  |

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Analyze student work

## Directions:

- Independently, complete the problem set.

Problem Set

| Tyrese solved this inequality correctly. <br> Here is his work: |
| :---: |
| $x-3+1 \geq 9$ |
| $x-3+1 \geq 9$ |
| $x-2 \geq 9$ |
| $+2+2$ |
| $x \geq 11$ |

- Why did Tyrese combine -3 and +1 before adding something to both sides?

> Your Turn! Solve the inequality
> $0.75(7)+1.25 b \leq 22$

Maria didn't solve this inequality correctly. Here is her work:


- Maria wanted to write the $x$ first in her solution. What did she forget to change in order to keep the answer correct?
$12 \geq x$ and $x \geq 12$ mean two different things. Explain the meaning of both.


## Directions: during Independent \& Pair Work

1. Sort the situations as representing an equation or inequality.
2. Write an equation or inequality to represent the situation.
3. Solve.
4. Match the situation to its solution, and explain what the solution or solution set means in the context.
5. Record your work below.

## Directions: during Presentations (Whole Group)

6. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

| Situations |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |


|  |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |

## Reflection: Practice Regents questions

## - REGENTS CONNECTION:

1. What is the value of $x$ in the equation $\frac{x-2}{3}+\frac{1}{6}=\frac{5}{6}$ ?
(1) 4
(3) 8
(2) 6
(4) 11
2. Connor wants to attend the town carnival. The price of admission to the carnival is $\$ 4.50$, and each ride costs an additional 79 cents. If he can spend at most $\$ 16.00$ at the carnival, which inequality can be used to solve for $r$, the number of rides Connor can go on, and what is the maximum number of rides he can go on?
(1) $0.79+4.50 r<16.00 ; 3$ rides
(2) $0.79+4.50 r<16.00 ; 4$ rides
(3) $4.50+0.79 r<16.00 ; 14$ rides
(4) $4.50+0.79 r<16.00 ; 15$ rides
3. Which value of $x$ satisfies the equation $\frac{7}{3}\left(x+\frac{9}{28}\right)=20$ ?
(1) 8.25
(3) 19.25
(2) 8.89
(4) 44.92
4. Given $2 x+a x-7>-12$, determine the largest integer value of a when $x=-1$.

$\qquad$
$\qquad$

Name: Class: $\qquad$ Date:
Warm Up: Work independently, then discuss work with a partner.
Directions: Solve the equation below using any method. Show your work below.

$$
3 x+6=18
$$

## Reflection:

Directions: With your partner, create the missing representation.

|  |  | Bar Model: |
| :--- | :--- | :--- |
|  |  |  |

Explain how parts of the bar model you created connects to parts of the given equation.
$\qquad$
$\qquad$
$\qquad$Choose a Reflection Prompt:

- Paying attention to ... in an equation is helpful because...
- When interpreting a bar model, I learned to pay attention to...

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.
Directions: Look at the examples below and decide which strategy is correct and why then answer the questions below.

Urszula solves the same equation two different ways and gets different answers. One of her strategies has an error in it.

| Strategy 1 |  | Strategy 2 |  |
| :---: | :---: | :---: | :---: |
| $4 x-20 \leq-6 x$ | Add $6 x$ to both sides of the inequality. | $4 x-20 \leq-6 x$ | Subtract 4 x from both sides of the equation. |
| $10 x-20 \leq 0$ | Add 20 to both sides of the inequality. | $-20 \leq-10 x$ | Divide both side of the equation by 10 . |
| $10 x \leq 20$ | Divide both sides of the inequality by 10 . | $-2 \leq-1 x$ | Divide both sides of the equation by -1 . |
| $x \leq 2$ |  | $2 \leq x$ |  |

How are Urszula's two answers different from each other?
$\qquad$

How is Strategy 1 different from Strategy 2?
$\qquad$
$\qquad$

Which strategy has an error in it and what is the error?

Directions: during Independent \& Pair Work

1. Sort the worked examples into your own categories.
2. Describe how you sorted the worked examples
3. Match the worked examples to equations that the strategy seems ike it would be most helptil to use.

Directions: during Presentations (Whole Group)
4. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

Sort by your own categories
List the groups of cards you made.

Describe how you sorted the cards.

Match the solution strategies to the equations where that strategy will be most efficient to use. List the groups of cards you made.

Describe how you matched the worked examples to the equations.

Solve the given equations.

## Reflection:

Directions: Solve each equation or inequality from the Regents exam given below.

| Question 1 | Question 2 <br> $4(a+3)-3 a=25+3 a$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |

Reflection: Describe ways you can determine which strategy is most helpful for solving a given equation or inequality.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.

Directions: Look at the work below and then answer the questions that follow.

Rayen solved the inequality $3 x+6>12$. Their work is below.

```
3x+6>12
Try substituting x}=0.3(0)+6=0+6=6\mathrm{ but 6> 12
Try substituting }x=-2.3(-2)+6=-3+6=3\mathrm{ but 3>12
Try substituting x =1 3(1)+6=3+6=9 but 9>12
Try substituting x}=2.3(2)+6=6+6=12\mathrm{ and 12=12 so }x=2\mathrm{ is the solution.
```

Rayen has a small error in their work. Explain what the error is and how Rayen could improve their solution.
$\qquad$
$\qquad$
$\qquad$

## Directions: during Independent \& Pair Work

1. Match an A card with a B, C, and D card
2. Attach your matches to the placemat.
3. Write a justification explaining why the match makes sense.

## Directions: during Presentations (Whole Group)

4. Listen to the presenting group. Then revise and improve your work.

## Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

Sort the cards into groups of four where the steps to transform Equation $\mathbf{A}$ into Equation $\mathbf{D}$ are correct. List the groups of cards you made.

Direction: Use the appropriate row from the matches your group made to help you solve this problem.

Kostandin has a cylindrical watering with a radius of 10 centimeters and a height of 20 centimeters. He wants to write marks on the side of the can at different heights so that he knows how high to fill the can if he wants 1000 cubic centimeters (1 liter), 2000 cubic centimeters, or 3000 cubic centimeters. He knows that the formula for the volume of a cylinder is $V=\pi r^{2} h$.


How high do each of the marks have to be on the can? Explain your reasoning below.
$\qquad$
$\qquad$
$\qquad$

Directions: For each of the following problems

- rearrange the equation to isolate an appropriate variable
- use the rearranged equation to solve the question given


## Problem 1

Mwenye sells tickets for a play, with adult tickets costing $\$ 10$ and child tickets costing $\$ 5$ and she sells $\$ 2560$ worth of tickets. Unfortunately, she did not keep track of how many total tickets she sold but she does know that there were 32 children.

Mwenye writes the formula $10 x+5 y=T$ to help her solve the problem.
What do $x$ and $y$ represent in this equation?

Rearrange the equation to isolate $x$.

Use your new equation to find the total number of adult tickets.

## Problem 2

Margie stores her cereal in a cylindrical container with a height of 20 centimeters and a radius of 10 centimeters. She wants to make sure that the container has enough room for her cereal.

The equation for the volume of a cylinder is $V=\pi r^{2} h$.
In this context, what do $r$ and $h$ represent?

Rearrange the equation to isolate $h$.

Use your your new equation to find the height of 2000 cubic centimeters of cereal in Margie's cereal container. Will Margie's cereal fit in her container?

## Directions: Solve the Regents problem given below.

## Question 1

Michael borrows money from his uncle, who is charging him simple interest using the formula
$t=$ Prt. To figure out what the interest rate, $r$, is, Michael rearranges the formula to find $r$. His new formula is $r$ equals
(1) $\frac{t-P}{t}$
(3) $\frac{1}{P t}$
(2) $\frac{P-l}{t}$
(4) $\frac{P_{t}}{T}$

## Question 2

The formula for the area of a traperoid is $A=\frac{1}{2} h\left(b_{1}+b_{2}\right)$. Express $b_{1}$ in term of $A_{1} h_{\text {, and }} b_{2}$.

The area of a trapezoid is 60 square feet, its beight is 6 ft , and one base is 12 f . Find the number of feet in the other base.

Reflection:


Reflection: How is solving literal equations like $I=\operatorname{Prt}$ for $r$ similar to solving equations like $6 x=$ 18 ?

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.
Directions: Solve the equation below using any method. Show your work below.

$$
-2 x-3=2 x-3
$$

## Reflection:

Directions: With your partner, create the missing representation.

| Equation and Solution: |  | Graph |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | - |  | 1 | 1 |
|  |  |  |  | 8 | - |  |  |  |
|  |  | - |  | - | , |  |  |  |
|  |  |  |  |  | , |  |  | - |
|  |  |  |  |  |  |  |  | - |
|  |  |  |  |  |  |  |  |  |
|  | 2 | $-8$ | -4 | 0 |  | 4 | 8 |  |
|  |  |  |  |  | $\square$ |  |  |  |
|  |  |  |  |  | , |  |  | - |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | - |  |  | - |

Explain how parts of the graph you created connects to parts of the given equation and its solution.
$\qquad$
$\qquad$

Choose a Reflection Prompt:

- Paying attention to ... in an equation is helpful because...
- When interpreting a bar model, I learned to pay attention to...


## Additional Resources



| Card 1 | Card 2 |
| :---: | :---: |
| Rowan has $\$ 50$ in a savings jar and is putting in \$5 every week. He wants to buy shoes worth $\$ 250$. For how many weeks will Rowan need to save to buy these shoes? |  |
| Card 3 | Card 4 |
| Jim works in real estate sales. His annual salary is | Create your own |
|  |  |
| An online company lets you download songs for \$1 each after you have paid a $\$ 5$ membership fee. The maximum number of songs you can download in a month is 100 . How much do you pay? | An online company lets you downiload songs for \$1 each after you have paid a $\$ 5$ membership fee. You download 50 in a month. How much did you pay? |
| Card 7 | Card 8 |
| The length of a garden is 10 feet longer than its width. The perimeter of this rectangular garden is $\mathbf{4 4}$ yards. What is the width of the garden? <br> (NOTE: perimeter is the sum of the lengths and widths around the rectangular garden) | The cost of a bottie of water at a food stand is \$1 and the cost of an apple is $\$ 0.80$. Shanequa has $\$ 20$ to spend on bottles of water and apples for her teammates and she must by eight bottles of water. What is the maximum number of apples she can buy? |
| Card 9 | Card 10 |
| A sunflower grows at a rate of 4 cm per day. The maximum height of a sunflower is 4 meters. How many days does the sunflower grow? | The School Gaming Club has three times as many female students as male students. Every year, there are at least 20 students in the club. How many male students can there be in the club? |
| Card 11 | Card 12 |
| $\begin{array}{lllllllllll}-20 & -15 & -10 & -5 & 0 & 5 & 10 & 15 & 20 & 25 & 30\end{array}$ | $\begin{array}{lllllllllll} -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 & 0 \end{array}$ |

A1

A5
$2 x+3-3-x=-3$
$2 x-1=-2+3$

B2


B3


B7

C3

$$
7=5 x-3
$$

C7
$2 x-4$

C8
$2 x-1=1$

| Card 1 <br> Rowan has $\$ 50$ in a savings jar and is putting in $\$ 5$ every week. He wants to buy shoes worth $\$ 250$. For how many weeks will Rowan need to save to buy these shoes? | Card 2 <br> Jim works in real estate sales. His annual salary is $\$ 20,000$ plus $2.5 \%$ of his home sales. What was the total value of all homes he sold if his annual pay for the year was $\$ 120,000$ ? |
| :---: | :---: |
| Card 3 <br> An online company lets you download songs for $\$ 1$ each after you have paid a $\$ 5$ membership fee. The maximum number of songs you can download in a month is 100 . How much do you pay? | Card 4 <br> The length of a garden is 10 feet longer than its width. The perimeter of this rectangular garden is 44 yards. What is the width of the garden? <br> (NOTE: perimeter is the sum of the lengths and widths around the rectangular garden) |
| Card 5 <br> A sunflower grows at a rate of 4 cm per day. The maximum height of a sunflower is 4 meters. What is the height of the sunflower? | Card 6 <br> The cost of a bottle of water at a food stand is \$1 and the cost of an apple is $\$ 0.80$. Shanequa has $\$ 20$ to spend on bottles of water and apples for her teammates and she must by eight bottles of water. What is the maximum number of apples she can buy? |


| CarcA A |  |  |
| :--- | :--- | :--- |
|  | $4,000,000$ |  |
| Card C |  |  |
|  | 40 |  |
| Card B |  |  |


| A1 | A2 |
| :---: | :---: |
| $3 x^{2}+y^{2}+2 x^{2}-y^{2}$ | $3 x^{2}+2 y^{2}$ |
| A3 | A4 |
| $3 x^{2}+3 y^{2}$ | $3 x^{2}+3 y^{2}+9$ |
| A5 | A6 |
| $3 x^{2}+3 y^{2}-3$ | $3 x^{2}+y^{2}-2 x^{2}+y^{2}$ |
| A7 | A8 |
| $6+3 x^{2}+2 y^{2}-3-y^{2}$ | $3 x^{2}+3 y^{2}-3$ |



| C1 |  | C2 |  |
| :--- | :--- | :--- | :--- |
|  | $x^{2}+2 y^{2}$ |  | $3\left(x^{2}+y^{2}+3\right)$ |
| C3 |  | C4 |  |
|  | $3\left(x^{2}+y^{2}-1\right)$ |  | $3\left(x^{2}+y^{2}-1\right)$ |
| C5 |  |  |  |
| C7 |  |  |  |
|  | $3(5)^{2}+2(3)^{2}$ | C6 |  |



| Worked Example 2 | $\frac{3}{4} x+\frac{9}{2}=3 x$ <br> $4\left(\frac{3}{4} x+\frac{9}{2}=3 x\right)$ <br> $4 \times \frac{3}{4} x+4 \times \frac{9}{2}=4 \times 3 x$ <br> of thply both sides <br> common multiple of <br> the denomimators. |
| :--- | :--- |
| $\frac{12}{4} x+\frac{36}{2}=12 x$ | Distribute the <br> multiplication <br> through all terms. |
| $3 x+18=12 x$ | Reduce the <br> fractions to lowest <br> terms. |
| $18=9 x$ so $2=x$ | Divide both sides of <br> the equation by the <br> coefficient of $x$. |

## Worked Example 4

$$
4 x-6=2(x+2)
$$

Represent the equation using Algebra tiles.


Remove
identical
aigebra tles
from both
sides.

Divide the tiles
into equal groups.

## Worked Example 5

$$
-4(x+3)=0
$$

Either $-4=0$ or

$$
x+3=0
$$

$$
\text { Since }-4 \neq 0 \text { then }
$$

$$
x+3=0 \text { is true. }
$$

$$
x=-3
$$

Notice that the product of two factors is 0 .

The only way to get a product of 0 is if one or more of the factors is 0 .

If one of the factors cannot be zero, the other factor must be.

Solve the equation created by setting the factor equal to 0.

| Equation A | Equation $\mathbf{B}$ |
| :---: | :---: |
| $3(x-3)=2 x+4$ | $9 x-8=12 x-7$ |
| Equation C | Equation D |
| $-3(x+4)+8=8$ | $1-2 x=\frac{1}{3} x$ |
| Equation E | Equation F |
| $x^{2}-5 x=-5 x+25$ | $-12\left(x-\frac{1}{7}\right)=0$ |


| A1 $r=\sqrt{\frac{V}{\pi h}}$ | A2 $r=\frac{V}{2 \pi h}$ | A3 $r=\sqrt{V \pi h}$ | A4 $r=2 V \pi+2 h$ |
| :---: | :---: | :---: | :---: |
| B1 $V \pi+h=\frac{r}{2}$ | B2 $\frac{V}{\pi h}=r^{2}$ | B3 $\frac{V}{2 \pi}=r h$ | B4 $V \pi h=r^{2}$ |
| C1 $\frac{V}{\pi}=2 r h$ | $\mathrm{C} 2$ $V h=\frac{r^{2}}{\pi}$ | C3 $V \pi=\frac{r}{2}-h$ | $\mathrm{C} 4$ $\frac{V}{\pi}=r^{2} h$ |
| D1 $V=\frac{r^{2}}{\pi h}$ | D2 $V=\frac{1}{\pi}\left(\frac{r}{2}-h\right)$ | D3 $V=\pi r^{2} h$ | D4 $V=2 \pi r h$ |

$10-x=3 x+2$



Answer Key
Name:
Warm Up: Work independently
Directions:

- Write the function that represents the situation.
- Create a different representation of the input/output values for this situation.

Situation
Margaret works at a furniture store and is paid $\$ 185$ a week, plus $3 \%$ of her total sales in dollars, $x$. What is her weekly pay?

Function: $f(x)=185+0.03 x$


## Directions:

- Examine the problem below.
- Independently, answer the questions about the problem.

A student incorrectly explained the relationship between the table of values and the equation. Here is the student's work


- What does the table of values represent in the situation? In the table, $X$ represents the total sales, in dollars, and $Y$ represents her weekly pay.
- Without solving the equation, explain how you would find the value of $x$ and what that means in the situation.
Isolate the variable X in terms of Y by using subtraction and division. X will represent her total sales for the week.


## Round 1 Partner Swap: Stronger

| Can you explain when you said... | NOTES: |
| :--- | :--- |
| *Students answers will vary based on |  |
| Can you expand on (elaborate)... | (hou said ... what is your reasoning? |
| their discussions. |  |
| Can you provide evidence? |  |

## Round 2 Partner Swap: Clearer

- When you said ..., maybe say it like .... $\quad$ NOTES:
- You should expand on ...
- When you said ... your reasoning was a
"Students answers will vary based on little unclear
- State your evidence better when....

REVISION: Write an edited statement of your responses to share with a partner.
*Students answers will vary based on their discussions.
$\qquad$
$\qquad$
$\qquad$

Reflection:
Directions:

- With a partner, complete the left column of the problem sets.
- Independently, answer the Your Turn questions.


## Problem Set 1

A student incorrectly used a function to write the equation. Here is the student's work:

Kendal bought $x$ bones of cookies to bring to a party. fach box contains 12 cookies. She decides to keep two bowes for herselt She brings 60 cookles to the party. Witte an equation that can be uied to find the number of bores, $x$, Kendal beught.
$f(x)=12 x-2$
$60=12 x-9$

- What information from the situation did the student forget to use?
The student forgot to multiply 2 by 12 to determine how many cookies she kept.
- What should the equation look like? $12 x-24=60$


## Your Turn!

Sandy programmed a website's checkout process with an equation to calculate the amount customers will be charged when they download songs.

The website offers a discount. If one song is bought at the full price of $\$ 1.29$, then each additional song is $\$ .99$.

State an equation that represents the cost, $f(x)$, when $x$ songs are downloaded.

$$
f(x)=0.99 x+1.29
$$

Problem Set 2
A student used a function and table of values to write the equation correctly. Here is the student's work:

Rowan has $\$ 50$ in a saving jar and is putting in $\$ 5$ every week. He
wants to buy shoes worth $\$ 150$. When will he have enough to buy
thowe shoes?


- What does the column of $f(f)$ represent in the situation?
The column $f(t)$ represents how much money he will have after $t$ weeks.
- Why is the value of $f($ f) replaced with the quantity 150 ? $f(t)$ is replaced with $\$ 150$ because the question asks how many weeks it will take for Rowen to buy the $\$ 150$ shoes.
Your turn!
Alex is selling tickets to a school play. An
aduit ticket costs $\$ 6.50$ and a student ticket
costs $\$ 4.00$. Alex focuses on selling adult
tickets, $x$, even though he's already
collected $\$ 48$ from student ticket sales.
Write an equation that can be used to find
the number of adult tickets, $x$, Alex will need
to sell to collect $\$ 308$ from all ticket sales.
$6.50 x+48=308$

Reflection: Paying attention to ... when writing an equation from a function/situation is helpful because ... *Student responses will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently

## Directions:

- Analyze the situation with the given function and table of values.
- Explain how you know the given equation matches the possible question.


## Situation

Margaret works at a furniture store and is paid $\$ 185$ a week plus $3 \%$ of her total sales in dollars, $\boldsymbol{x}$.

Function: $y=185+0.03 x$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 0 | 185 |
| 100 | 188 |
| 200 | 191 |
| 300 | 194 |
| 400 | 197 |
| 500 | 200 |


| Equation | Question | Explanation |
| :---: | :--- | :--- |
|  | How much will she make if <br> she sells $\$ 200$ worth of <br> furniture this week? | In the equation, $x$ represents her <br> total sales, in dollars, so if she <br> sells $\$ 200$ worth, she will make <br> $\$ 191$. |
| $20185+0.03(200)$ |  |  |

Directions: Match the graph with the correct equation for the given situation.
Situation with Table of Values
Rowan has $\$ 50$ in a savings jar and is putting in $\$ 5$ every week. He wants to buy shoes worth $\$ 200$.
When will he have enough to buy these shoes?

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 0 | 50 |
| 5 | 75 |
| 10 | 100 |
| 15 | 125 |
| 20 | 150 |
| 25 | 175 |




EQUATION 2
$y=50+5(200)$

This graph matches equation 1 because the intercept is 50 in both the equation and the graph and both have a slope of 5 .

Equation 2 does not match the graph, because when this equation is simplified, it states
$y=1050$, which this graph does not show.



Equation 4:
$f(x)=15 x+50+500$

Equation 4 matches with card 3 ., because
$\qquad$
$\qquad$
$\qquad$
$\qquad$
in a graph/table of values/situation/equation is helpful, because $\qquad$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up:
Directions:

- Analyze the examples of functions and of equations, then complete the Venn Diagram.

| Functions | Equations |
| :---: | :---: |
| Margaret works at a furniture store and is paid $\$ 185 \mathrm{a}$ <br> week plus $3 \%$ of her total sales in dollars, $x$. What is <br> her weekly pay? | Tom is 57 years old and has a son named James. <br> In three years Tom will be twice as old as James. <br> How old is James? |
| $f(x)=0.5 x-4$ | $10=0.5 x-4$ |
| $y=0.5 x-4$ | $7(x+2)=5 x+10$ |



What are the chunks of the equation? What are the chunks of the situation?
Variable, coefficient, constant, and solution

## Create a representation:



Terms: $5 x, 7, \sqrt{2}$

## Reflecting on Learning:

A. When interpreting situations to generate equations, it's important to pay attention to..
B. Looking for ... in a situation helps me create a rule because ... Answers will vary.

## Reflection:

nREGENTS CONNECTION: Practice Problems
Answer the Regents problems, and explain why you selected the answer choice. Use the empty space to show your work.

1. A typical cell phone plan has a fixed base fee that includes a certain amount of data and an overage charge for data use beyond the plan. A cell phone plan charges a base fee of $\$ 62$ and an overage charge of $\$ 30$ per gigabyte of data that exceed 2 gigabytes. If $C$ represents the cost and $g$ represents the total number of gigabytes of data, which equation could represent this plan when more than 2 gigabytes are used?
(1) $C=30+62(2-g)$
(3) $C=62+30(2-g)$
(2) $C=30+62(g-2)$

2. A parking garage charges a base rate of $\$ 3.50$ for up to 2 hours, and an hourly rate for each additional hour. The sign below gives the prices for up to 5 hours of parking.

| Parking Rates |  |
| :--- | ---: |
| 2 hours | $\$ 3.50$ |
| 3 hours | $\$ 9.00$ |
| 4 hours | $\$ 14.50$ |
| 5 hours | $\$ 20.00$ |

Which linear equation can be used to find $x$, the additional hourly parking rate?
(1) $9.00+3 x=20.00$
(3) $2 x+3.50=14.50$
(2) $9.00+3.50 x=20.00$
(4) $2 x+9.00=14.50$
3. Kendal bought $x$ boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Which equation can be used to find the number of boxes, $x$, Kendal bought?
(1) $2 x-12=60$
(3) $12 x-24=60$
(2) $12 x-2=60$
(4) $24-12 x=60$
4. In 2013, the United States Postal Service charged \$0.46 to mail a letter weighing up to 1 oz , and $\$ 0.20$ per ounce for each additional ounce. Which function would determine the cost, in dollars, $c(z)$, of mailing a letter weighing $z$ ounces where $z$ is an integer greater than 1 ?
(1) $c(z)=0.46 z+0.20$
(3) $c(z)=0.46(z-1)+0.20$
(2) $c(z)=0.20 z+0.46$
(4) $c(z)=0.20(z-1)+0.46$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up:

## Directions:

- Analyze the algebraic representation, graphical representation, and situations below
- Describe the similarities and differences.

| 1) Rowan has $\$ 50$ in a savings jar and is putting in $\$ 5$ every week. He wants to buy shoes that cost $\$ 150$. When can he buy this pair of shoes? $150=50+5 x$ | 2) Rowan has $\$ 50$ in a savings jar and is putting in $\$ 5$ every week. He likes to buy shoes that are at least $\$ 150$. When can he buy his next pair of shoes? $150 \leq 50+5 x$ |
| :---: | :---: |
| He wants to buy shoes that cost $\$ 150$. "that cost" signifies an equation. $\mathrm{x}=20$ the graph has a closed circle on 20 only one solution. | He likes to buy shoes that are at least \$150. <br> "at least" signifies an inequality. $x \geq 20$ <br> the graph has a closed circle 20 and extends to positive infinity. Many solutio |



| Equations | Inequalities |
| :---: | :---: |
| Card 1 |  |
| Card 2 | Card 5 |
| Card 3 | Card 8 |
| Card 6 | Card 10 |
| Card 7 | Card 11 |
| Card 9 |  |
|  |  |
|  |  |
|  |  |

Reflection: Work independently on Regents practice.

PREGENTS CONNECTION: Answer the Regents problem and explain why you selected the answer choice.

The cost of a pack of chewing gum in the vending machine is $\$ 0.75$. The cost of bottle of juice in the same machine $\$ 1.25$. Julia has $\$ 22.00$ to spend on chewing gum and bottles of juice for her team and she must buy seven packs of chewing gum. If b represents the number of bottles of juice, which inequality represents the maximum number of bottles she can buy?
(1) $0.75 b+1.25(7) \geq 22$
(2) $0.75 b+1.25(7) \leq 22$

$$
\begin{aligned}
& \text { (3) } 0.75(7)+1.25 b \geq 22 \\
& \text { (4) } 0.75(7)+1.25 b \leq 22
\end{aligned}
$$

Why isn't the representation an equation?
This is not an equation because the problem is asking for the "maximum" number of bottles she can buy. She has $\$ 22$ that she is able to spend. She is able to spend less than $\$ 22$ but she cannot spend more than the amount she has.


Reflection: Pick one, and write your reflection.
A. When deciding if a situation can be represented by an equation or inequality, it is helpful to pay attention to ... because ...
B. When interpreting situations to generate inequalities, it's important to pay attention to... Answers will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up:

## Directions:

- Independently, complete the problem set.


## Problem Set

Ken solved this problem correctly.
Here is his work

| $6 x+5 x+3=11+14$ |
| :---: |
| $6 x+5 x+3=\\|+14$ |
| $11 x+3=25$ |
| -3 |
| $1 \mid x=22$ |
| $x 11$ |
| $x=2$ |

$x=2$

$$
\begin{aligned}
& \text { Your Turn! } \\
& \text { Solve for } g \text {, given the equation } \\
& 182=62+30(\mathrm{~g}-2) \\
& 182=62+30 g-60 \\
& 182=30 \mathrm{~g}+2 \\
& 180=30 \mathrm{~g} \\
& 6=g
\end{aligned}
$$

- In the first step, Ken combined $6 x$ and $5 x$. Why didn't he also add the 3 to get $14 x$ ?
$6 x$ and $5 x$ are like terms. The 3 does not have a variable, $x$, so it cannot be combined.

- Look at Jamila's work in the step marked with an arrow. Why did she multiply 5 by both $m$ and 3 ? Jamila cross multiplied to solve the proportion. $(m+3)$ is an expression in the ratio, so she must distribute the 5 to both the $m$ and the 3


ACT 1: Understand

| What do you notice? | What do you wonder? |
| :---: | :---: |
| Responses will vary | Responses will vary |
|  |  |
|  |  |

## Our Question: When should I start bombarding the App store with purchases if I want to win?

```
Too High: Ex: 06/24/2025 To Low. Ex:
    06/24/2025 01/11/2020
~~~~~~~~~~~~~~~~
```

ACT 2: Plan \& Solve
Identify quantities, relationships, and assumptions. Create, use, and revise your model.
Quantities: video over a long period of time to collect data

| Relationships: | $\begin{array}{l}\text { month and day the video was taken, the time zone of the video } \\ \text { Assumptions: } \\ \text { the ability to slow it down (ex: minute by minute count) }\end{array}$ |
| :--- | :--- |



ACT 3: Interpret
Compare the result generated by your model to the actual result
Example answer: There is a constant rate of change, there should be a linear model,
the independent variable will be minutes and the dependent variable will be measured in the number of downloads, the rate of change is downloads/minute.

Reflection:

## The Sequel

According to your linear model, when did the app store sell its first app? Calculate an answer mathematically then find the actual answer. If those answers are different, what could explain the difference?

Answers will vary.
$\qquad$
$\qquad$

According to your parameters in your model, when did Apple have less than 25 billion downloads? Represent all values in your solution.

Answers will vary.
$\qquad$
$\qquad$
$\qquad$

Reflection: Select one and respond
A. I learned ....about writing a model while working with ....it helps me leam when my classmates....
B. When interpreting situations to generate equations, it's important to pay attention to..
C. Looking for ... in a situation helps me make a prediction because ..

Answers will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up; Work independently


| What do you notice? |
| :---: |
| Answers will vary. |
| What do you wonder? |
| Answers will vary. |

Directions: during Independent \& Pair Work

1. Decide which three cards (A, B, C) match together, or represent equivalent expressions.
2. Record your matches in the given table.
3. Annotate or discuss how you know these cards match.

Directions: during Presentations (Whole Group)
4. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

| EXAMPLE NOT IN CARD SET <br> A1 $3 x+6$ | EXAMPLENOTIN CARD SET <br> B1 | EXAMPLE NOT IN CARD SET <br> C2 $3(x+2)$ |
| :---: | :---: | :---: |
| EXAMPLE NOT IN CARD SET <br> A4 $3(x-2)$ | EXAMPLE NOT IN CARD SET <br> B2 | EXAMPLE NOT IN CARD SET <br> C1 $3 x-6$ |

Reflection: Work independently
Directions:

- Use Algebra Tiles and sketch the image.
- Write the equations and justification for each resulting equation.

$\qquad$
$\qquad$

Name: $\qquad$ Class: $\qquad$ Date:

Warm Up: Work independently to solve the equation, give visual model, and justify.


Reflecting on Learning: pick one of the prompts below and write your response
A. Paying attention to ... is helpful because..
B. I can use the structure of ... to make ... easier,

Answers will vary.
$\qquad$
$\qquad$
$\qquad$

Reflection: Work independently

## Directions:

- Use your favorite shortcut described today to determine the value of heart
- Annotate the picture to clearly show the strategy you used


What are the similarities and differences in solving an inequality compared to an equation?

| Similarities | Differences |
| :--- | :--- |
| 1. Inverse operations <br> 2. Can be graphed | 1. More than one solution <br> 2. Rules when multiplying/dividing <br> by a negative number. |
|  |  |

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Analyze student work

## Directions:

- Independently, complete the probiem set.

Problem Set

| Tyrese solved this inequality correctly Here is tis work: $\begin{array}{r} x-3+1 \geq 9 \\ x-3+1 \geq 9 \\ x-2 \geq 9 \\ +2+2 \\ x \geq 11 \end{array}$ <br> - Why did Tyrese combine -3 and +1 before adding something to both sides? <br> Before solving, check to see if you have to distribute and/or combine like terms to simplify the expression(s) first. <br> Maria didn't solve this inequality correctly. Here is her work: <br> - Maria wanted to write the $x$ first in her solution. What did she forget to change in order to keep the answer correct? <br> Maria forgot to change the inequality from $\geq$ to $\leq$. | Your Turn! Solve the inequality $0.75(7)+1.25 b \leq 22$ $b \leq 13.4$ |
| :---: | :---: |
| $12 \geq x$ and $x \geq 12$ mean two different things. Explain the meaning of both. <br> $12 \geq x$ means the value of $x$ any numb <br> $x \geq 12$ means the value of $x$ is any num | less than 12, including 12. <br> er greater than 12 , including |

Directions: during Independent \& Pair Work

1. Sort the situations as representing an equation or inequality

Write an equation or inequality to represent the situation.
. Solve.
4. Match the situation to its solution, and explain what the solution or solution set means in the context.
5. Record your work below

Directions: during Presentations (Whole Group)
6. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

| Situations | Solutions |
| :---: | :---: |
| $\begin{aligned} & \text { card } 1 \\ & 50+5 x=250 \end{aligned}$ | card c |
| $\begin{aligned} & \text { card } 2 \\ & 20000+0.025 x=120000 \end{aligned}$ | card a |
| $\begin{aligned} & \text { card } 3 \\ & 1 x-5<100 \end{aligned}$ | card d |
| card 4 | card c |
| $\begin{aligned} & \text { card } 5 \\ & 4 x<400 \end{aligned}$ | card b |
| card 6 $1(8)+0.80 x<20$ | card e |

## Reflection: Practice Regents questions

## Pregents connection:

1. What is the value of x in the equation $\frac{x-2}{3}+\frac{1}{6}=\frac{5}{6}$ ?

| (1) 4 | (3) 8 |
| :--- | :--- |
| (2) 6 | (4) 11 |

2. Connor wants to attend the town carnival. The price of admission to the carnival is $\$ 4.50$, and each ride costs an additional 79 cents. If he can spend at most $\$ 16.00$ at the carnival, which inequality can be used to solve for $r$, the number of rides Connor can go on, and what is the maximum number of rides he can go on?
(1) $0.79+4.50 r<16.00 ; 3$ rides
(2) $0.79+4.50 r<1600: 4$ rides

## (3) $4.50+0.79 r<16.00 ; 14$ rides

(4) $4.50+0.79 \mathrm{~F}<16.00 ; 15$ rides
3. Which value of $x$ satisfies the equation $\frac{7}{3}\left(x+\frac{9}{28}\right)=20$ ?

| (1) 8.25 | (3) 19.25 |
| :--- | :--- |
| (2) 8.89 | (4) 44.92 |

4. Given $2 x+a x-7>-12$, determine the largest integer value of a when $x=-1$.

2
$\qquad$
Reflection: Paying attention to
Answers will vary.
$\qquad$
$\qquad$

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.

Directions: Solve the equation below using any method. Show your work below.
$3 x+6=18$
$3 x=12$
$x=4$

Reflection:
Directions: With your partner, create the missing representation.

| Answers will vary. | Bar Model: |
| :--- | :--- |
| Equation Solving Strategy: |  |
|  |  |
|  |  |

Explain how parts of the bar model you created connects to parts of the given equation. Answers will vary.

## Choose a Reflection Prompt:

- Paying attention to ... in an equation is helpful because..
- When interpreting a bar model, I learned to pay attention to.. Answers will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.
Directions: Look at the examples below and decide which strategy is correct and why then answer the questions below.

Urszula solves the same equation two different ways and gets different answers. One of her strategies has an error in it.

| Strategy 1 | Strategy 2 |  |  |
| :---: | :--- | :--- | :--- |
| $4 x-20 \leq-6 x$ | Add $6 x$ to both sides of the <br> inequality. | $4 x-20 \leq-6 x$ | Subtract $4 x$ from both <br> sides of the equation. |
| $10 x-20 \leq 0$ | Add 20 to both sides of <br> the inequality. | $-20 \leq-10 x$ | Divide both side of the <br> equation by 10. |
| $10 x \leq 20$ | Divide both sides of the <br> inequality by 10. | $-2 \leq-1 x$ | Divide both sides of the <br> equation by -1. |

How are Urszula's two answers different from each other?
Strategy 1's answer states the value of $x$ is less than or equal to 2 . Strategy 2 states the value of x is greater than or equal to 2 .

## How is Strategy 1 different from Strategy 2?

Strategy 1 starts by adding $-6 x$ to both sides to make the inequality positive, strategy 2 begins by subtracting $4 x$ from both sides to make the inequality negative.

## Which strategy has an error in it and what is the error?

Strategy 2 has the error. She did not change the flip the inequality sign to greater than or equal to when dividing by -1 .

## Directions: during Independent \& Pair Work

1. Sort the worked examples into your own categories
2. Describe how you sorted the worked examples.
3. Match the worked examples to equations that the strategy seems like it would be most helpful to use.

Directions: during Presentations (Whole Group)
4. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

Sort by your own categories
List the groups of cards you made.

Describe how you sorted the cards.

Match the solution strategies to the equations where that strategy will be most efficient to use. List the groups of cards you made.

Describe how you matched the worked examples to the equations
$\qquad$
$\qquad$

Solve the given equations.

Reflection:
Directions: Solve each equation or inequality from the Regents exam given below.

| Question 1 | Question 2 |
| :--- | :--- |
| $4(a+3)-3 a=25+3 a$ <br> $4 \mathrm{a}+12-3 \mathrm{a}=25+3 \mathrm{a}$ <br> $1 \mathrm{a}+12=25+3 \mathrm{a}$ <br> $1 \mathrm{a}=13+3 \mathrm{a}$ <br> $-2 \mathrm{a}=13$ <br> $\mathrm{ax}-3) \geq a \mathrm{x}+7 \mathrm{~b}$ <br> $\mathrm{a}=-\frac{13}{2}$ | $\mathrm{bx} \geq \mathrm{ax}+10 \mathrm{~b}$ |
|  | $\mathrm{bx}-\mathrm{ax} \geq 10 \mathrm{~b}$ |
|  | $\mathrm{x}(\mathrm{b}-\mathrm{a}) \geq 10 \mathrm{~b}$ |
| $\mathrm{x} \leq \frac{10 \mathrm{~b}}{\mathrm{~b}}$ |  |

Reflection: Describe ways you can determine which strategy is most helpful for solving a given equation or inequality Answers will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.

Directions: Look at the work below and then answer the questions that follow.

Rayen solved the inequality $3 x+6>12$. Their work is below.

```
3x+6>12
Try substituting x =0. 3(0)+6=0+6=6 but 6>12
Try substituting }x=-2,3(-2)+6=-3+6=3\mathrm{ but 3> 12
Try substituting }x=1(3)(1)+6=3+6=9\mathrm{ but 9 > 12
Try substituting }x=2,3(2)+6=6+6=12 and 12=12 so x = 2 is the solution
```

Rayen has a small error in their work. Explain what the error is and how Rayen could improve their solution.
When Rayen substituted $x=-2$, she had $3(-2)+6=-3+6=3$. She should have gotten $-6+6=0$. Rayed could had solved this by using inverse operations to get $x>2$.

Directions: during Independent \& Pair Work

1. Match an A card with a B C, and D card
2. Attach your matches to the placemat
3. Write a justification explaining why the match makes sense.

Directions: during Presentations (Whole Group)
4. Listen to the presenting group. Then revise and improve your work.

Reminders:

- take turns, so that everyone participates
- take your time and do not rush
- explain your reasoning
- challenge each other when you disagree

Sort the cards into groups of four where the steps to transform Equation A into Equation D are correct. List the groups of cards you made.
$\mathrm{A} 1, \mathrm{~B} 2, \mathrm{C} 4, \mathrm{D} 3 \quad \mathrm{~A} 2, \mathrm{~B} 3, \mathrm{C} 1, \mathrm{D} 4 \quad \mathrm{~A} 3, \mathrm{~B} 4, \mathrm{C} 2, \mathrm{D} 1 \quad \mathrm{~A} 4, \mathrm{~B} 1, \mathrm{C} 3, \mathrm{D} 2$

Direction: Use the appropriate row from the matches your group made to help you solve this problem.
Kostandin has a cylindrical watering with a radius of 10 centimeters and a height of 20 centimeters. He wants to write marks on the side of the can at different heights so that he knows how high to fill the can if he wants 1000 cubic centimeters ( 1 liter), 2000 cubic centimeters, or 3000 cubic centimeters. He knows that the formula for the volume of a cylinder is $V=\pi r^{2} h$.


How high do each of the marks have to be on the can? Explain your reasoning below.
1000 cubic $\mathrm{cm}=$ height of the mark has to be at approx. 31.4 cm
2000 cubic $\mathrm{cm}=$ height of the mark has to be at approx. 62.8 cm
3000 cubic $\mathrm{cm}=$ height of the mark has to be at approx. 94.2 cm

## Directions: For each of the following problems

- rearrange the equation to isolate an appropriate variable
- use the rearranged equation to solve the question given


## Problem 1

Mwenye sells tickets for a play, with adult tickets costing $\$ 10$ and child tickets costing $\$ 5$ and she sells $\$ 2560$ worth of tickets. Unfortunately, she did not keep track of how many total tickets she sold but she does know that there were 32 children.

Mwenye writes the formula $10 x+5 y=T$ to help her solve the problem.

What do $x$ and $y$ represent in this equation?
$x$ is the number of adult tickets, $y$ is the number of child tickets.

Rearrange the equation to isolate $x$.

$$
x=T-5 y
$$

$$
10
$$

Use your new equation to find the total number of adult tickets.
240 adult tickets

## Problem 2

Margie stores her cereal in a cylindrical container with a height of 20 centimeters and a radius of 10 centimeters. She wants to make sure that the container has enough room for her cereal.

The equation for the volume of a cylinder is $V=\pi r^{2} h$.
In this context, what do $r$ and $h$ represent?
$r$ is the radius $(10 \mathrm{~cm})$ and $h$ is the height $(20 \mathrm{~cm})$

Rearrange the equation to isolate $h$.

$$
\frac{v}{\pi r^{2}}=h
$$

Use your your new equation to find the height of 2000 cubic centimeters of cereal in Margie's cereal container. Will Margie's cereal fit in her container?

The height is approx. 62.8 cm

## Reflection:

Directions: Solve the Regents problem given below
Question 1
Michael borrows money from his uncle, who is charging him simple interest using the formula
$I=$ Prt. To figure out what the interest rate, $r$, is, Michael rearranges the formula to find $r$. His new formula is $r$ equals
(1) $\frac{l-P}{t}$
(2) $\frac{P-I}{t}$
(3) $\frac{l}{P t}$
(4) $\frac{P t}{l}$

Question 2
The formula for the area of a trapezoid is $A=\frac{1}{2} h\left(b_{1}+b_{2}\right)$. Express $b_{1}$ in term of $A, h$, and $b_{2}$

$$
\frac{2 \mathrm{~A}}{\mathrm{~h}}-\mathrm{b}_{2}=\mathrm{b}_{1}
$$

The area of a trapezoid is 60 square feet, its height is 6 ft , and one base is 12 ft . Find the number of feet in the other base.

The other base is 8 ft

Reflection: How is solving literal equations like $I=\operatorname{Prt}$ for $r$ similar to solving equations like $6 x=$ 18 ? Answers will vary.

Name: $\qquad$ Class: $\qquad$ Date: $\qquad$
Warm Up: Work independently, then discuss work with a partner.

Directions: Solve the equation below using any method. Show your work below.

$$
\begin{aligned}
& -2 x-3=2 x-3 \\
& +3+3 \\
& -2 x=2 x+0 \\
& -2 x \quad-2 x \\
& -4 x=0 \\
& x=0
\end{aligned}
$$

Reflection:
Directions: With your partner, create the missing representation.


Explain how parts of the graph you created connects to parts of the given equation and its solution. Answers will vary.Choose a Reflection Prompt:

- Paying attention to ... in an equation is helpful because...
- When interpreting a bar model, I learned to pay attention to...

Answers will vary.


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