

FACTA UNIVERSITATIS

Series: **Automatic Control and Robotics** Vol. 18, N° 3, 2019, pp. 163 - 175<https://doi.org/10.22190/FUACR1903163K>

CONTROL OF MHD MICROPOLAR FLUID FLOW

UDC (681.5.01:621.313.5)

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Abstract. *In this paper, the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid through a parallel plate channel is investigated. The upper and lower plate have been kept at the two constant different temperatures and the plates are electrically insulated. The applied magnetic field is perpendicular to the flow, while the Reynolds number is significantly lower than one i.e. the considered problem is in induction-less approximation. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. The influences of each of the governing parameters on velocity, heat transfer on the plates (Nusselt number), flow rate and skin friction are discussed with the aid of graphs.*

Key words: magnetohydrodynamic, micropolar, heat transfer

1. INTRODUCTION

The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators and flowmeters and it has applications in nuclear reactors, filtration, geothermal systems and others.

The interest in the outer magnetic field effect on heat-physical processes appeared seventy years ago. Blum et al. [1] carried out one of the first works in the field of heat and mass transfer in the presence of a magnetic field. The research in MHD flows was stimulated by two problems: the protection of space vehicles from aerodynamic overheating and destruction during the passage through dense layers of the atmosphere; the enhancement of the operational ability of constructive elements of high temperature MHD generators for direct transformation of the heat energy into electric. The first

Received February 19, 2019

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problem showed that the influence of a magnetic field on ionized gases is a convenient control method for mass, heat and hydrodynamic processes.

The flow and heat transfer of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates has been studied by many researchers [2-4] due to its important applications in the further development of MHD technology. The MHD devices for liquid metals attracted the attention of metallurgist [5]. It was shown that the effect of the magnetic field could be very helpful in the modernization of technological processes. The increasing interest in the study of MHD phenomena is also related to the development of fusion reactors where plasma is confined by a strong magnetic field [6]. Many exciting innovations were put forth in the areas of MHD propulsion [7], remote energy deposition for drag reduction [8], MHD control of flow and heat transfer in the boundary layer [9,10].

All the studies cited above are limited to classical Newtonian fluids. There are many fluids important from the industrial point of view, and they display non-Newtonian behaviour. Due to the complexity of such fluids, several models have been proposed but the micropolar model is the most prominent one.

Eringen [11] initiated the concept of micropolar fluids to characterize the suspensions of neutrally buoyant rigid particles in a viscous fluid. The micropolar fluids exhibit microrotational and microinertial effects and support body couple and couple stresses. It may be noted that micropolar fluids take care of the microrotation of fluid particles by means of an independent kinematic vector called the microrotation vector.

According to the theory of micropolar fluids proposed by Eringen [11] it is possible to recover the inadequacy of the Navier–Stokes theory to describe the correct behaviour of some types of fluids with a microstructure such as animal blood, muddy water, colloidal fluids, lubricants and chemical suspensions. In the mathematical theory of micropolar fluids there are, in general, six degrees of freedom, three for translation and three for microrotation of microelements. Extensive reviews of the theory and applications can be found in the review articles [12,13] and in the recent books [14,15].

The research interest in the MHD flows of micropolar fluids has increased substantially over the past decades due to the occurrence of these fluids in industrial and magneto-biological processes. These flows take into account the effect arising from the local structure and micro-motions of the fluid elements, and are able to describe the behaviour of the polymeric additives, animal blood, lubricants, liquid crystals, dirty oils, solutions of colloidal suspensions, etc. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Chamkha et al. [16] and Bachok et al. [17].

Basic ideas and techniques for both steady and unsteady flow problems of Newtonian and non-Newtonian fluids are given by Ellahi [18]. The basic equations governing the flow of couple stress fluids are non-linear in nature and even of higher order than the Navier Stokes equations. Thus an exact solution of these equations is not easy to find. Different perturbation techniques and a reasonable simplification are commonly used for obtaining solutions of these equations.

In the present paper, the hydromagnetic flow and heat transfer characteristics of a viscous electrically conducting incompressible micropolar fluid in a parallel plate channel is considered.

Viscous dissipation and Joule heating effects have also been taken into account. The effects of the governing parameters on the flow and heat transfer aspects of the problem are discussed.

2. PHYSICAL AND MATHEMATICAL MODEL

The problem of the laminar MHD flow and heat transfer of an incompressible electrically conducting micropolar fluid between parallel plates is considered. MHD channel flow analysis is usually performed assuming the fluid constant electrical conductivity and treating the problem as a one-dimensional one: with these two main assumptions, the governing equations are considerably simplified and they can be solved analytically.

The physical model shown in Figure 1, consists of two infinite parallel plates extending in the x and z -direction. A fully developed flow takes place between parallel plates that are at a distance h , as shown in Figure 1. Electrically conductive fluid flows through the channel due to the constant pressure gradient and the applied magnetic field. A uniform magnetic field of the strength B is applied in the y direction. The upper and lower plate have been kept at the two constant temperatures T_1 and, T_2 respectively.

The fluid velocity v and the magnetic field B are:

$$\mathbf{v} = u\vec{i}, \quad (1)$$

$$\mathbf{B} = B\vec{j}. \quad (2)$$

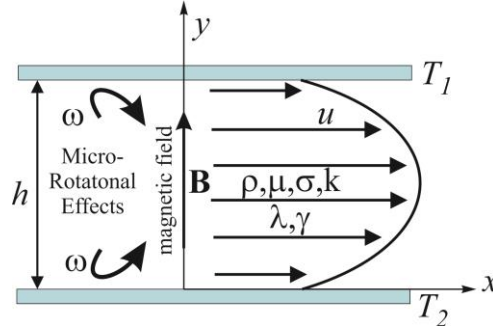


Fig. 1 Physical model and coordinate system

The described laminar MHD flow and heat transfer is mathematically presented with following equations:

$$(\mu + \lambda) \frac{d^2 u^*}{dy^{*2}} + \lambda \frac{d\omega^*}{dy^*} - \sigma B^2 u^* - \frac{dp}{dx} = 0, \quad (3)$$

$$\gamma \frac{d^2 \omega^*}{dy^{*2}} - \lambda \frac{du^*}{dy^*} - 2\lambda \omega^* = 0, \quad (4)$$

$$k \frac{d^2 T^*}{dy^{*2}} + (\mu + \lambda) \left(\frac{du^*}{dy^*} \right)^2 + \sigma B^2 u^{*2} = 0. \quad (5)$$

The no slip conditions require that the fluid velocities are equal to the plate's velocities, boundary conditions for the temperature are isothermal conditions and there is no microrotation near plates. The fluid and thermal boundary conditions for this problem are represented by equations:

$$\begin{aligned} u^* = 0, \omega^* = 0, T^* = T_2 \quad \text{for } y^* = 0, \\ u^* = 0, \omega^* = 0, T^* = T_1 \quad \text{for } y^* = h. \end{aligned} \quad (6)$$

In these general equations and boundary conditions, the used symbols are common for the theory of MHD flows.

Now the following transformations have been used to transform equations given above into a nondimensional form:

$$\begin{aligned} U = \frac{h^2 P}{\mu}, P = -\frac{\partial p}{\partial x} = \text{const}, \theta = \frac{T - T_2}{T_1 - T_2}, K = \frac{\lambda}{\mu}, \\ \Gamma = \frac{\gamma}{\mu h^2}, Ha = Bh \sqrt{\frac{\sigma}{\mu}}, Pr = \frac{\mu c_p}{k}, Ec = \frac{U^2}{c_p (T_1 - T_2)}. \end{aligned} \quad (7)$$

The equations (3-5) get the following form:

$$(1 + K) \frac{d^2 u}{dy^2} + K \frac{d\omega}{dy} - Ha^2 u + 1 = 0, \quad (8)$$

$$\Gamma \frac{d^2 \omega}{dy^2} - K \frac{du}{dy} - 2K\omega = 0, \quad (9)$$

$$\frac{d^2 \theta}{dy^2} + (1 + K) Pr Ec \left(\frac{du}{dy} \right)^2 + Ec Pr Ha^2 u^2 = 0. \quad (10)$$

The boundary dimensionless conditions for the equations given above are:

$$\begin{aligned} u = 0, \omega = 0, T = T_2 \quad \text{for } y = 0, \\ u = 0, \omega = 0, T = T_1 \quad \text{for } y = h. \end{aligned} \quad (11)$$

After the basic mathematical transformations from equations (8) and (9), the equation for velocity is:

$$u^{iv} - au'' + bu - d = 0, \quad (12)$$

where:

$$\begin{aligned} a = B^* + E - AD^*, b = EB^*, d = EC, A = \frac{K}{1 + K}, \\ B^* = \frac{Ha^2}{1 + K}, C = \frac{1}{1 + K}, D^* = \frac{K}{\Gamma}, E = \frac{2K}{\Gamma}. \end{aligned} \quad (13)$$

The solution of the equation (12), yields three possible cases:

$$u = C_1 \exp(\delta_1 y) + C_2 \exp(\delta_2 y) + C_3 \exp(\delta_3 y) + C_4 \exp(\delta_4 y) + \frac{d}{b}, \quad (14)$$

$$u = (C_5 + C_6 y) \exp(\xi_1 y) + (C_7 + C_8 y) \exp(\xi_2 y) + \frac{d}{b}, \quad (15)$$

$$u = [C_9 \cos(\beta_1 y) + C_{10} \sin(\beta_1 y)] \exp(\alpha_1 y) + [C_{11} \cos(\beta_1 y) + C_{12} \sin(\beta_1 y)] \exp(-\alpha_1 y) + \frac{d}{b}. \quad (16)$$

And the solutions of the equations (9) and (10) are, respectively:

$$\omega = C_1 \mathcal{T}_1 \exp(\delta_1 y) + C_2 \mathcal{T}_2 \exp(\delta_2 y) + C_3 \mathcal{T}_3 \exp(\delta_3 y) + C_4 \mathcal{T}_4 \exp(\delta_4 y), \quad (17)$$

$$\begin{aligned} \theta = & -Pr Ec \left[\frac{1}{4\delta_1^2} {}^1C \exp(2\delta_1 y) + \frac{1}{4\delta_2^2} {}^2C \exp(2\delta_2 y) + \frac{1}{4\delta_3^2} {}^3C \exp(2\delta_3 y) + \right. \\ & + \frac{1}{4\delta_4^2} {}^4C \exp(2\delta_4 y) + \frac{1}{2} {}^1D y^2 + \frac{{}^2D}{(\delta_1 + \delta_3)^2} \exp((\delta_1 + \delta_3)y) + \\ & + \frac{{}^3D}{(\delta_1 + \delta_4)^2} \exp((\delta_1 + \delta_4)y) + \frac{{}^1E}{(\delta_2 + \delta_3)^2} \exp((\delta_2 + \delta_3)y) + \\ & + \frac{{}^2E}{(\delta_2 + \delta_4)^2} \exp((\delta_2 + \delta_4)y) + \frac{1}{2} {}^3E y^2 + \frac{1}{\delta_1^2} {}^1F \exp(\delta_1 y) + \\ & \left. + \frac{1}{\delta_2^2} {}^2F \exp(\delta_2 y) + \frac{1}{\delta_3^2} {}^3F \exp(\delta_3 y) + \frac{1}{\delta_4^2} {}^4F \exp(\delta_4 y) + \frac{1}{2} {}^5F y^2 + {}^1H_1 y + {}^1H_2 \right], \end{aligned} \quad (18)$$

$$\omega = (E_{13} + E_{14} y) \exp(\xi_1 y) + (E_{15} + E_{16} y) \exp(\xi_2 y), \quad (19)$$

$$\begin{aligned} \theta = & -Pr Ec [(\Omega_{28} + \Omega_{29} y + \Omega_{30} y^2) \exp(2\xi_1 y) + (\Omega_{31} + \Omega_{32} y + \Omega_{33} y^2) \exp(2\xi_2 y) + \\ & + (\Omega_{34}^* + \Omega_{35}^* y) \exp(\xi_1 y) + (\Omega_{36}^* + \Omega_{37}^* y) \exp(\xi_2 y) + \Omega_{38}^* y^2 + \Omega_{39}^* y^3 + \\ & + \Omega_{40}^* y^4 + {}^2H_1 y + {}^2H_2], \end{aligned} \quad (20)$$

$$\begin{aligned} \omega = & [P_3^* \sin(\beta_1 y) + P_4^* \cos(\beta_1 y)] \exp(\alpha_1 y) + \\ & + [P_5^* \sin(\beta_1 y) + P_6^* \cos(\beta_1 y)] \exp(-\alpha_1 y), \end{aligned} \quad (21)$$

$$\begin{aligned}
\theta = -\text{Pr Ec} & \left\{ \left[\frac{1}{2\alpha_1} \Omega_{45} + \frac{1}{2} (\chi_1 \Omega_{47} - \chi_2 \Omega_{49}) \cos(2\beta_1 y) + \right. \right. \\
& + \frac{1}{2} (\chi_2 \Omega_{47} + \chi_1 \Omega_{49}) \sin(2\beta_1 y) \left. \right] \exp(2\alpha_1 y) + \\
& + \left[\frac{1}{2\alpha_1} \Omega_{46} - \frac{1}{2} (\chi_1 \Omega_{48} + \chi_2 \Omega_{50}) \cos(2\beta_1 y) + \right. \\
& * \frac{1}{2} (\chi_2 \Omega_{48} - \chi_1 \Omega_{50}) \sin(2\beta_1 y) \left. \right] \exp(-2\alpha_1 y) - \\
& - \frac{1}{2\beta_1} \Omega_{51} \sin(2\beta_1 y) - \frac{1}{2\beta_1} \Omega_{52} \cos(2\beta_1 y) + [(\Omega_{53} \chi_1 - \Omega_{55} \chi_2) \cos(\beta_1 y) + \\
& + (\Omega_{53} \chi_2 + \Omega_{55} \chi_1) \sin(\beta_1 y)] \exp(\alpha_1 y) + [(\Omega_{54} \chi_1 - \Omega_{56} \chi_2) \cos(\beta_1 y) - \\
& - (\Omega_{54} \chi_2 + \Omega_{56} \chi_1) \sin(\beta_1 y)] \exp(-\alpha_1 y) + \frac{1}{2} \Omega_{57} y^2 + {}^3H_1 y + {}^3H_2 \left. \right\}. \tag{22}
\end{aligned}$$

With the aid of the expressions for velocity, micro-rotation and temperature, we now derive the following important characteristics of the flow.

The flow rate:

$$Q = \int_0^1 u(y) dy. \tag{23}$$

The shear stress:

$$\tau = (\mu + \lambda) \left[\frac{du}{dy} \right]_{y=1,0} + \lambda [\omega]_{y=1,0}. \tag{24}$$

Nusselt number:

$$\begin{aligned}
Nu &= -\frac{\rho c_p h^2}{k} \frac{U}{\theta(T_1 - T_2) + T_2 - T_i} \frac{d\theta}{dy} = k_i \frac{d\theta}{dy}, \\
Nu_{up} &= k_1 \frac{d\theta}{dy} \Big|_{y=1}, \quad Nu_{lp} = k_2 \frac{d\theta}{dy} \Big|_{y=0}. \tag{25}
\end{aligned}$$

3. RESULTS AND DISCUSSION

In the section 2, the mathematical model for the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid between two infinite horizontal parallel plates under a constant pressure gradient and the applied magnetic field is defined. The influences of the Hartmann number, the coupling parameter and the spin-gradient viscosity parameter on velocity, heat transfer on the plates (Nusselt number), flow rate and skin friction are discussed with the aid of graphs.

The first three figures, 2 to 4, show the influence of the characteristic parameters on velocity profiles.

The effect of the Hartmann number on the velocity is shown in Figure 2. It can be seen from figure that the velocity, as expected, becomes small for large values of Ha . This happens because of the imposing of a magnetic field normal to the flow direction which creates a Lorentz force opposite to the flow direction. This fact is used to manage the fluid flow.

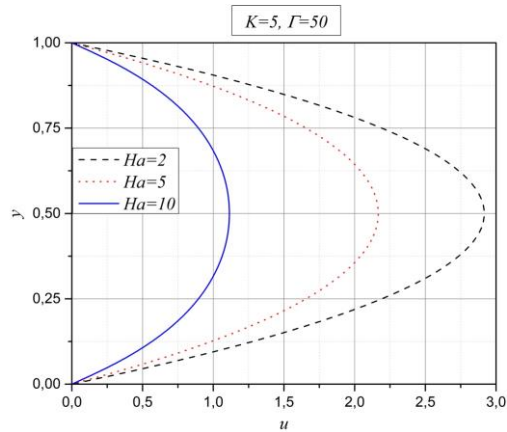


Fig. 2 Velocity profiles for different values of Hartmann number

Figure 3 shows the effect of the spin-gradient viscosity parameter on velocity, which predicts that the velocity increases as the spin-gradient viscosity parameter decreases. This fact leads to the conclusion that the increase of the gyro-viscosity γ reduces the flow compared to the viscous fluid case.

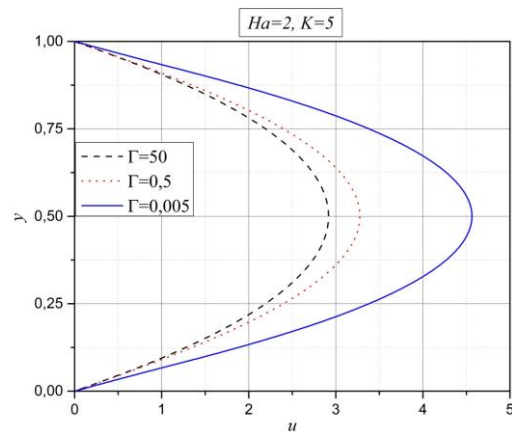


Fig. 3 Velocity profiles for different values of the spin-gradient viscosity parameter

The next figure, Figure 4, shows the effect of the coupling parameter on velocity. From Figure 4, it can be observed that the increase in coupling parameter K decreases the velocity which means, as expected, that the resistance of the fluid increases with the increase of K . In the limit $K \rightarrow 0$, the results correspond to the case of viscous fluid.

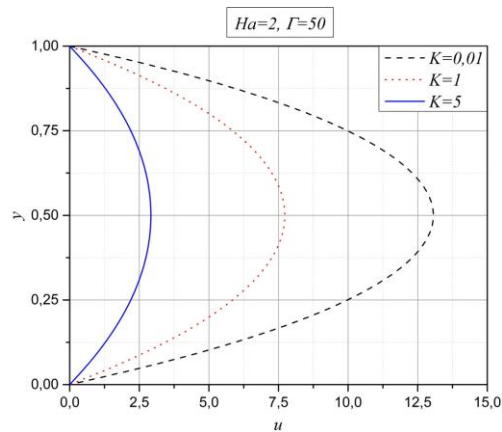


Fig. 4 Velocity profiles for different values of the coupling parameter

The influence of the Hartmann number, the coupling parameter and the spin-gradient viscosity parameter on the heat transfer on the plates (Nusselt number) are presented in the figures 5, 6 and 7. With expression (25), the Nusselt number is defined as a product of the constant (k_i) and the heat transfer. The next three figures will be used to describe a change of the heat transfer on the plates in function of characteristic parameters, neglecting the constant (k_i).

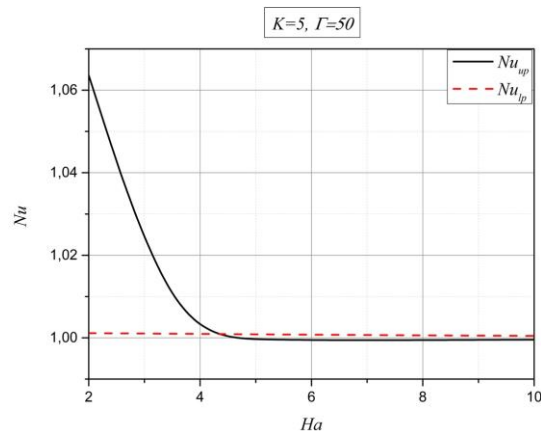


Fig. 5 Influence of Hartmann number on Nusselt number

The increase of the Hartmann number causes a decrease of the Nusselt number on the upper plate, while the Nusselt number on the lower plate remains constant with the change of the Hartmann number, which is shown in Fig. 5. The increase of the Hartmann number causes the decrease of velocity (Figure 2) and as the Hartmann number increases the velocity field gets the uniform form between the plates, in that case the temperature in fluid flow is constant and there is no more heat transfer on the plates.

In Fig. 6, the influence of the spin-gradient viscosity parameter on the Nusselt number is presented. From this figure it is obvious that for certain values of the spin-gradient viscosity parameter, as well as for other parameters, there is a maximum of heat transfer on the upper plate. This means that it is possible to control the heat transfer on the plates by changing the characteristics of the micropolar fluid.

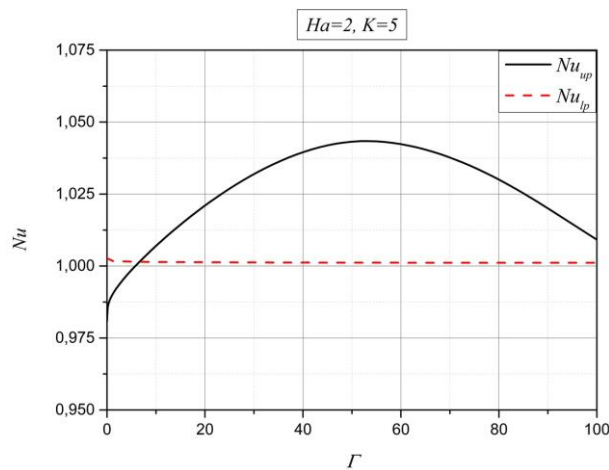


Fig. 6 Influence of the spin-gradient viscosity parameter on Nusselt number

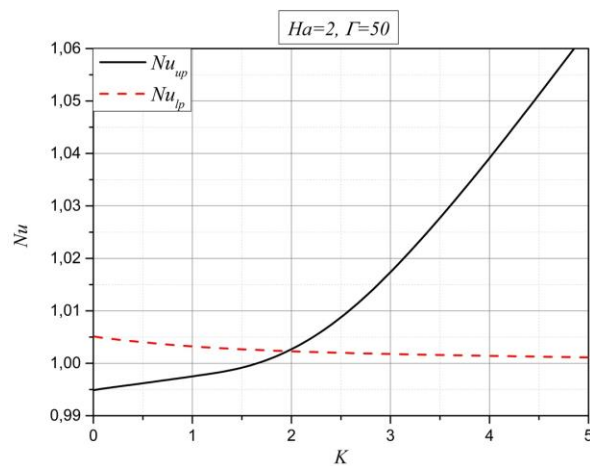


Fig. 7 Influence of the coupling parameter on Nusselt number

Fig. 7 shows the influence of the coupling parameter on the heat transfer on the plates. Increasing the coupling parameter K causes the increase of the heat transfer on the upper plate. While the coupling parameter K increases, i.e. the additional viscosity λ increases, the temperature in the fluid flow increases as a consequence of viscous heating and because of that there is an increase of the heat transfer on the upper plate.

The change of the fluid flow Q as a result of the change of characteristic parameters is shown in figures 8, 9 and 10.

From Figure 8, it can be noted that the fluid flow Q decreases while the Hartmann number increases. This is the consequence of the Lorentz force which is opposite to the flow direction and whose intensity increases with the increase in the magnetic field intensity, i.e. the Hartmann number.

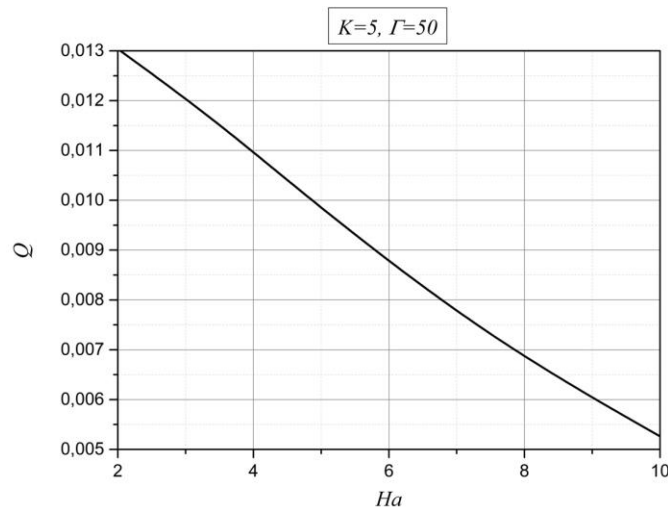


Fig. 8 Influence of the Hartmann number on the flow Q

The influence of the spin-gradient viscosity parameter and the coupling parameter on the fluid flow Q is shown in Figures 9 and 10. The tendency of change of the fluid flow Q is the same for changes in all characteristic parameters for micropolar fluid. In both cases the increase of characteristics for micropolar fluid causes a decrease in the fluid flow Q . In the case of the spin-gradient viscosity parameter there is the limit value after which the further increase of this parameter does not affect the fluid flow Q . But in the case of the coupling parameter, there is no limit value and the fluid flow Q always decreases as the coupling parameter increases.

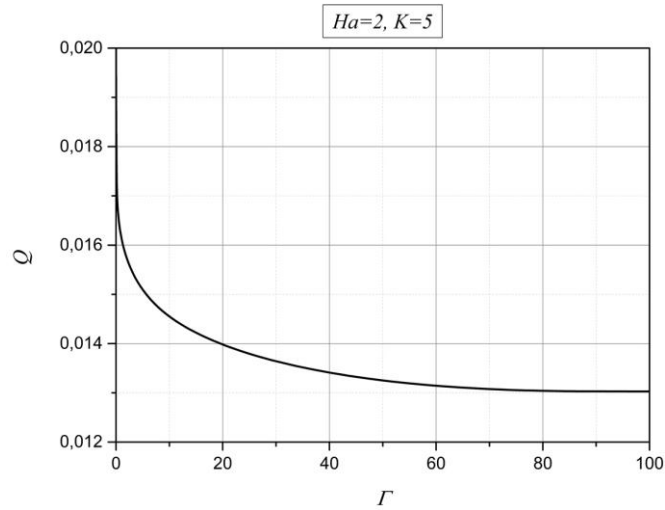


Fig. 9 Influence of the spin-gradient viscosity parameter on flow Q

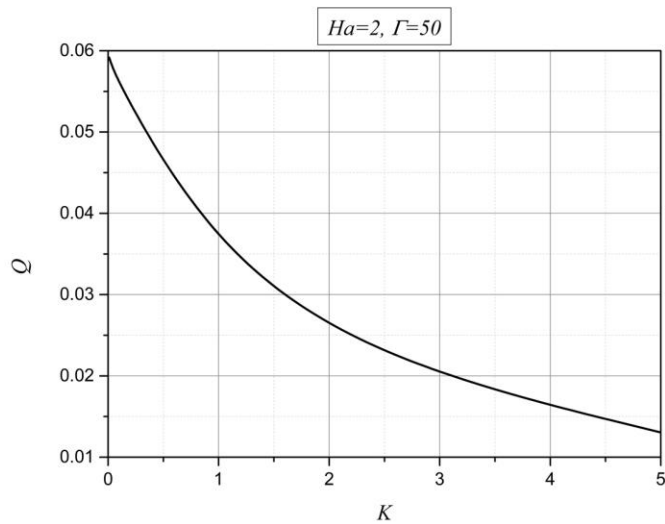


Fig. 10 Influence of the coupling parameter on the flow Q

The last two figures show the influence of the Hartmann number, the coupling parameter and the spin-gradient viscosity parameter on the skin friction.

Figure 11 shows the influence of the spin-gradient viscosity parameter and the Hartmann number on the skin friction. The increase of the spin-gradient viscosity parameter causes the increase of the skin friction absolute value, while the increase of the Hartmann number causes the decrease of the skin friction. In the case of the spin-gradient viscosity parameter there is a limit value after which the further increase in the spin-gradient viscosity parameter does not affect the intensity of the skin friction.

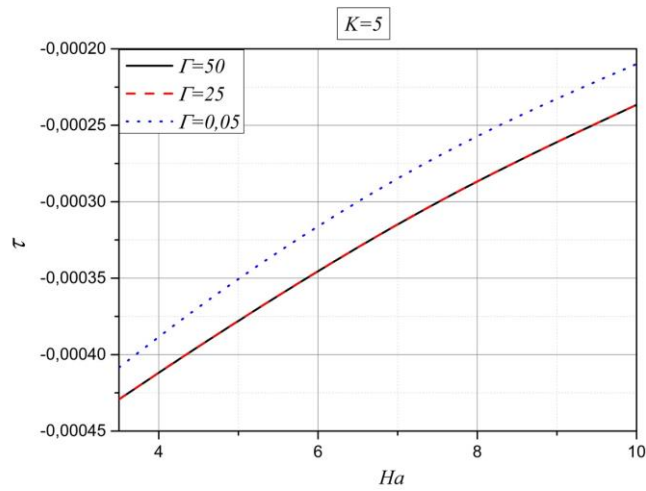


Fig. 11 Influence of the Hartmann number and of the spin-gradient viscosity parameter on skin friction

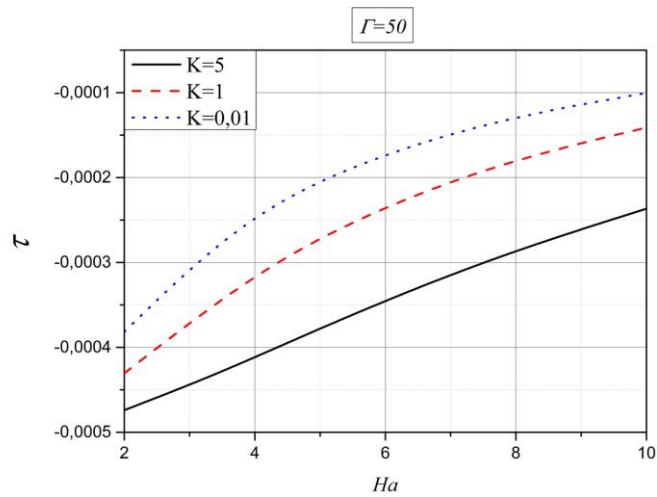


Fig. 12 Influence of the Hartmann number and of the coupling parameter on the skin friction

From Fig. 12, it can be noted that increase of the Hartmann number causes the decrease of the skin friction absolute value, but the increase of coupling parameter causes the increase of the skin friction. The increase of the coupling parameter is the consequence of the increase in the additional viscosity λ , and thus the increase in the total viscosity, therefore the results are as expected.

4. CONCLUSION

In this paper, the steady flow and heat transfer of an incompressible electrically conducting micropolar fluid between two infinite horizontal parallel plates under a constant pressure gradient or a constant flow rate has been considered. The upper and lower plates have been kept at the two constant different temperatures and the plates are electrically insulated. The applied magnetic field is perpendicular to the flow. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. The influences of each of the governing parameters on velocity, temperature, flow rate and shear stress are discussed with the aid of graphs. The obtained results show that the control of flow and heat transfer for observed case can be realized by changing the Hartmann number, spin-gradient viscosity parameter and coupling parameter.

Acknowledgement: *This paper is supported by the Serbian Ministry of Sciences and Technology Development (Project No. 35016; Research of MHD flow in the channels, around the bodies and application in the development of the MHD pump).*

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