An incremental meter placement method for state estimation considering collinear measurements and high leverage points

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Abstract

The performance of the power system state estimation (SE) is influenced by the configuration of the meters and measurement redundancy. Therefore, the measurement set needs to be updated by installing new SCADA meters and phasor measurement units for improving the quality of the SE solution. However, the potential inconsistency between the existing meters and the new meters should be addressed. Otherwise, the additional meters may lead to numerical problems such as collinearity (linear dependence due to duplicated measurements) and the existence of high leverage points (HLPs) (influential measurements). Hence, this paper proposes an incremental meter placement method. The proposed method utilizes the HLPs and aims to improve the numerical performance of the SE and facilitate the elimination of bad data. The cuckoo search optimization is used for selecting the optimal locations and the numbers of the new meters. The performance of the proposed algorithm is tested on UK 18-bus, the IEEE 30-bus, and 118-bus systems and simulation results show improvements in the quality of the SE solution.

Keywords

Cuckoo search, Collinearity, High leverage points, Phasor measurement unit (PMU), State estimation.

The conventional state estimation (SE) is formulated as an iterative weighted least-squares (WLS) problem (Monticelli, 2000; Chakrabarti et al., 2010; Biswal et al., 2012). The WLS SE is a nonlinear regression operation that deduces the response (state variables) from the observation set (measurement set) (Liu et al., 2014). The meters that are associated with the power system SE includes synchronized phasor measurement units (PMUs) and SCADA meters. The SCADA meters which are the conventional meters in the power systems include: voltage magnitudes meters, active/reactive power flows and power-injections meters, and the ampere meters (Chakrabarti et al., 2010; Biswal et al., 2012). However, the WLS SE is vulnerable to be ill-conditioned (Liu et al., 2014) and even unsolvable (Abood et al., 2016) when the gain matrix is singular or close to the singularity. The ill-conditioned SE results in significant deviations in the final solution even with a small perturbation in the input data. This is mainly because the state estimator relies on the accuracy and redundancy (Gu et al., 1983; Abood et al., 2019; Baldwin et al., 1993) of the existing measurements. The sufficiency of the measurements and measurement redundancy is required to achieve the system's observability in contingency cases (Manousakis et al., 2012). The accuracy and the numerical stability of the SE mainly rely on the types and locations of the meters and the diversity of the measurement set (Biswal, 2016; Abood and Sreeram, 2014). Therefore, utility companies started equipping the EMS with the advanced PMUs that can accurately measure the complex voltages and transmit data with high timing sampling with the aid of GPS (Saleh et al., 2017). However, the problem of meter placement

has deviated to become "PMU placement" as installing the relatively expensive PMUs with their communication channels must follow strategic rules for obtaining efficient configurations to justify the cost (Biswal, 2016; Abood and Sreeram, 2014).

In the context of the SE, the configuration of the PMUs and the conventional meters (SCADA meters) should contribute to an accurate and stable SE in addition to achieving observability (Liu et al., 2014; Biswal, 2016; Celik and Liu, 1995). Nevertheless, a clear majority of the PMU placement methods (Biswal, 2016; Abood and Sreeram, 2014) address only the observability problem and aim to design virtual measurement systems that are based on PMUs only. Unfortunately, this choice is likely infeasible for two reasons: the large-scale power systems require numerous PMUs that may not be affordable, and most of the power systems are already observable using the conventional meters (Celik and Liu, 1995; Rosli et al., 2014). Therefore, the preferred solution by utilities is incremental meter placement methods that use minimal PMUs (Aam et al., 1983). Nevertheless, numerical problems may arise due to the excessive number of metering and the existence of repeated measurements for the same quantities. The linear dependence (also called collinearity or multicollinearity) among the measurements yields an unsolvable SE in the case of perfect collinearity and a rank-deficient Jacobian matrix or ill-conditioned SE in a weak collinearity case (Stewart, 1987). Another measurement-based problem that affects SE is the presence of high-influential measurements (Rousseeuw and Van Zomeren, 1990) that attract the regression/estimation solution toward them much more than other regular observations. Among the various types of influential observations, the high leverage points (HLPs) gained the attention of the researchers in statistics (Saleh et al., 2017; Rosli et al., 2014; Majumdar and Pal, 2016). However, even though the HLPs are related to the process of bad data detection (BDD) of the SE solution (Rousseeuw and Van Zomeren, 1990; Majumdar and Pal, 2016), there is ambiguity in identifying and utilizing the good-leverage points that can enhance the quality of the state vector (Chen and Abur, 2006; Benedito et al., 2014). The existing meter placement methods (Rosli et al., 2014; Bretas et al., 2011) either discard the measurements of HLPs or consider them as critical measurements that must be avoided in the placement procedure. However, the analogy with the critical measurements cannot reflect the entire numerical features of the influential measurements as the HLPs can be beneficial to the accuracy of the SE. Furthermore, the multicollinearity and HLPs are

addressed separately in (Abood et al., 2016; Saleh et al., 2017; Rosli et al., 2014; Chen and Abur, 2006) ignoring the mutual numerical relationship.

In contrast, statistical studies (Rousseeuw and Van Zomeren, 1990; Bagheri et al., 2012; Midi and Mohammed, 2015) have achieved crucial steps in identifying the HLPs and the sources of multicollinearity. In this context, several authors (Bagheri et al., 2012; Midi and Mohammed, 2015; Nurunnabi et al., 2014) proved that the HLPs are the main source for multicollinearity and defined a new group of observations, which is named as the leverage collinearity-influential observations. To the best of the authors' knowledge, this paper is the first study, which employs the collinearity-influential measurements in a meter placement algorithm for improving the numerical performance of the state estimator using minimal PMUs.

An incremental meter placement algorithm that includes both the PMUs and the conventional power meters is proposed in this paper to enhance the quality of the state vector based on the state-of-the-art diagnostic techniques (Bagheri et al., 2012; Midi and Mohammed, 2015; Nurunnabi et al., 2014). This paper identifies the high leverage measurements and presents a meter placement strategy that employs the HLPs for improving the SE accuracy, facilitating the BDD, and decreasing the multicollinearities. The metaheuristic cuckoo search optimization (CSO) (Yang and Deb, 2013) is used to find the optimal locations of the incremental meters.

The rest of the paper is organized as follows: the mathematical formulation of the SE, the numerical problems, and the metrics of assessing the numerical stability and accuracy of the SE are given in the "Numerical performance of SE" section, the proposed method with the CSO algorithm is presented in "The incremental meter placement" section, simulation tests and comparisons are provided in the "Case studies and simulation results" section followed by the conclusions in the "Conclusion" section.

Numerical performance of SE

WLS state estimator

If the measurement set (z) of m measurements is used for estimating the state vector (x) of n variables, the SE is formulated as a WLS problem that minimizes the following objective function:

$$J(x) = \left[z - h(x) \right]^T R^{-1} \left[z - h(x) \right] \tag{1}$$

where R^{-1} is a diagonal matrix that has inverse of the measurement error variances on its diagonal. If the

system is completely observable, the Gauss-Newton update of (1) for the kth iterations is as follows:

$$\Delta x^k = \left(H^T R^{-1} H_x\right)^{-1} H^T R^{-1}\left[z - h\left(x^k\right)\right]$$

where $H_{ij} = \partial h_i / \partial x_j$ is the measurements Jacobian matrix with $i = 1, 2, …, m$, $j = 1, 2, …, n$ and $\Delta x^k = x^{k+1} - x^k$. The final expression of the WLS estimator is as follows:

$$G\left(x^k\right)\Delta x^k = H^T R^{-1}\left[z - h\left(x^k\right)\right] \qquad (2)$$

where $G(x^k) = H_x^T W H_x$ is the gain matrix (Xu and Abur, 2004; Schweppe, 1970) which is a square and sparse matrix. The solution of (2) exists only if the gain matrix is invertible (Gu et al., 1983), i.e. the measurement Jacobian and the gain matrix must be full-rank matrices (Monticelli, 2000). However, this is necessary but insufficient condition as the performance of the state estimator and the accuracy of the SE solution require additional/more conditions.

Quality of the SE solution

The assessment of the SE stability is based on the value of the condition number. The condition number is a measure of the sensitivity of the system to erroneous measurements. The condition number of the state estimator (Belsley et al., 2005) is:

$$\kappa\left(G\right) = \lambda_{\max} / \lambda_{\min} \qquad (3)$$

where λ_{max} and λ_{min} are the maximum and minimum singular values of the gain matrix, respectively. The numerical sensitivity/stability of the state estimator depends on the variation in magnitude of the maximum singular value to the minimum singular value. The condition number of a well-conditioned SE must be close to unity.

On the other hand, the accuracy of the SE is associated with the variances of the state variables. The SE errors represent the variances between the estimated states and the true values. The SE variances are deduced from the gain matrix as follows (Abood et al., 2016; Bretas et al., 2011; Jiang and Vittal, 2006):

$$SE_{\mathrm{var}} = diag\left(G^{-1}\right)$$

Hence, the more accurate state variables, the smaller are the variances.

Collinearity of the measurements

Installing new meters may readily reinforce collinearity (or multicollinearity) problems when there is dependence among the measurements of the existing meters with that of the new meters (Abood et al., 2016; Belsley et al., 2005). Therefore, an analytical procedure should be implemented prior to allocating new meters to detect the multicollinearity and identify the buses with dependent measurements. Though the significant achievements of the researchers in statistics and economic in identifying the source of data collinearities, it is rare to find an analytical-based SE study. However, PMU placement of Schweppe (1970) follows a statistics-based method (Abood et al., 2016), which is known as the variance decomposition, is used in Schweppe (1970) to identify the multicollinearities among the measurements. The method of Liu et al. (2014) and Belsley et al. (2005) decomposes the SE variances into the variance decomposition proportions (VDPs) so that each one of the VDPs is associated with only one singular value. The values of VDPs range from 0 to 1. The individual proportions P_{ij} are computed as follows:

$$P_{ij} = \frac{\rho_{ji}}{\rho_j}, \quad i, j = 1, 2, …, m \qquad (5)$$

where $\rho_{ij} = v_{kj}^2 / \lambda_i^2$ and $\rho_j = \sum_{i=1}^{N_s} \rho_{ij}$.

The entries v_{kj}^2 are computed based on the singular value decomposition. According to Abood et al. (2016) and Bretas et al. (2011), the measurements with VDPs larger than 0.5 and a condition index higher than 30 are identified as collinear measurements. However, the VDPs cannot diagnose the main source beyond the multicollinearity and the relationship with the high-influential observations (Majumdar and Pal, 2016; Nurunnabi et al., 2014).

Outliers and HLPs

The observation set of any regression has one or more observation that influences the final solution more than others (Belsley et al., 2005; Hadi, 1992). The influential measurements may alter the results if they are excluded from the regression process (e.g. in the BDD processing) (Chen and Abur, 2006). The outliers are located away from the bulk of the data either in the observations' space (i.e. X-outliers that are also known as leverage points) or in the space of the response (i.e. Y-outliers or the vertical outliers). However, Figure 1 provides a scatter pattern for the main types of observations around a virtual fitted curve.

The leverage measurements can be good or bad based, in that they can enhance or deteriorate the SE accuracy. The bad-leverage points mislead
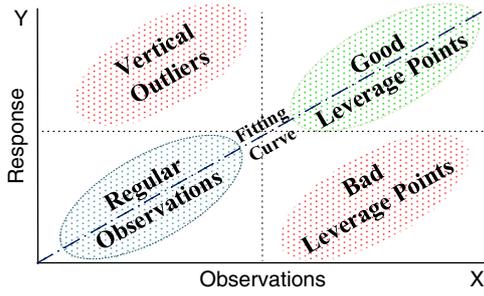
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Figure 1: Scatter plot of main types of observations.

the interpretation of the SE solution and disrupt the elimination of bad data. However, detecting the HLPs in an overdetermined estimation with many redundant measurements is not a straightforward process due to the potential multiple HLPs.

The traditional measures (Hadi, 1992; Habshah et al., 2009) are based on the so-called Hat matrix that is shown in (6):

$$T = H\left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \qquad (6)$$

where T is the Hat matrix that has diagonal entries τ_{ii}. The diagonal entries range from 0 to 1. The conventional BDD is based on the τ_{ii} values of (6). Regarding the cut-off values, Hadi (1992) suggested $\tau_{ii} \geq 0.5$ to be associated with HLPs and the twice-the-mean value ($2m/n$) is used by Celik and Liu (1995) as a cut-off value. However, the traditional metric of (6) and all the measures that are based on the Hat matrix such as Chen and Abur (2006), Nurunnabi et al. (2014), Habshah et al. (2009) and Bagheri and Midi (2015), and the BDD may fail to detect multiple HLPs in large-scale systems and suffer from swamping and masking effects. The swamping effect occurs when non-influential observations are identified as influential, whereas the masking effect occurs when an influential point cannot be detected (Chen and Abur, 2006). The observation being masked cannot be identified as an influential point unless the masking points are deleted from the measurement set. Moreover, Zhao et al. (2016) shows that leverage points' distance can be greater than unity. Therefore, the improved generalized measure of the observations' influences (Bagheri et al., 2012; Nurunnabi et al., 2014) is:

$$\omega_{ii}^* = \begin{cases} \dfrac{\tau_{ii}}{1 - \tau_{ii}}, & i \in E \\ \dfrac{1}{\tau_{ii}}, & i \in D \end{cases}$$

where D is a deleted set of observations and E is the remaining subset (a sequential elimination process is embedded above). Regarding the cut-off value of the X-axis that identifies the HLPs, is computed in Habshah et al. (2009) as:

$$cut\left(\omega_{ii}^*\right) > Median\left(\omega_{ii}^*\right) + c\,MAD\left(\omega_{ii}^*\right) \qquad (7)$$

where factor c is a small integer number (2 or 3) and:

$$MAD\left(\omega_{ii}^*\right) = \frac{Median\left\{\left|\omega_{ii}^* - Median\left(\omega_{ii}^*\right)\right|\right\}}{0.6747}$$

The above cut-off criterion is known as the diagnostic robust generalized potential (DRGP), which is proved to be a robust measure for threshold values (Zhao et al., 2016). However, Majumdar and Pal (Rousseeuw and Van Zomeren, 1990) used the robust Mahalanobis distance with the generalized studentized residuals to identify the low leverage and the HLPs.

The incremental meter placement

Effective meter placement algorithms need to be compatible with the numerical characteristics of the existing measurement set and the numerical performance of the SE. Nevertheless, most of the existing meter placement algorithms lack comprehensive algorithms due to: employed only one type of meter (either PMUs of conventional meters); addressing only one of the numerical problems of the "Numerical performance of SE" section; adopting one objective, which is almost the observability of the system. To the best knowledge of the authors', all the paper under the title of optimal PMU placement is based on the connectivity matrix, which boosts the buses with the highest number of branches to be the candidates to host the PMUs.

Celik and Liu (1995) implemented a placement method for conventional meters using a multi-step heuristic placement algorithm that consumes time and computational efforts due to the numerous set of potential solutions and the presence of subjective selection metrics. However, Saleh et al. (2017) is the pioneering paper to refer to the leverage points as an influential measurement to consider in the meter placement. Nonetheless, the placement algorithm of Saleh et al. (2017) suggests avoiding the HLPs. Further, Bretas et al. (2011), Majumdar and Pal (2016) and Ahmadi et al. (2011) treated the PMUs and conventional meters separately using a sequential procedure for meters allocation, where Saleh et al.

(2017) proposed a PMU placement method using an evolutionary algorithm. Hence, the proposed method aims to include both the conventional meters and the PMUs and to consider both the problem of multicollinearity and the HLPs, simultaneously. Therefore, a metaheuristic technique, which is the CSO, employs for the proposed algorithm to reduce the computational efforts by allocating the new meters using in a unified placement procedure. Likewise, for reducing the potential solutions fed to the CSO-based placement algorithm, a combined analytical process is implemented to identify the buses with the measurements that influence both multicollinearities and HLPs simultaneously to avoid the squaretail or multi-step procedures (Gu et al., 1983; Biswal, 2016; Rosli et al., 2014). The collinearity-influential observations and the cuckoo search are described prior to the proposed CSO-based meter placement method.

High leverage collinearity-influential measurements

For addressing the numerical problem of collinearity and the HLPs using a unified approach, the researcher adopts state-of-the-art statistical studies (Benedito et al., 2014; Bagheri et al., 2012) that point out the mutual relationships based on the numerical characteristics of the measurement set. Recent studies of Chen and Abur (2006) and Midi and Mohammed (2015) identify a set of observations, which is named as the collinearity-influential observations that can change the multicollinearity pattern by creating or hiding collinear measurements in the measurements set (Benedito et al., 2014; Bagheri et al., 2012; Midi and Mohammed, 2015). Changing the multicollinearity pattern via the HLPs is the main task of the proposed method of meter placement.

Based on their influence on the collinearity, the high-leverage collinearity-influential observations are divided into collinearity-enhancing and collinearity-reducing observations, and both have good and bad observations (Bagheri et al., 2012). That means the deletion of the collinearity-reducing set deteriorates the SE by promoting multicollinearities among the remaining set. However, the criterion of Bagheri et al. (2012) performs well only in detecting a single collinearity-influential observation. Therefore, this paper adopts the latest studies of Bagheri and Midi (Midi and Mohammed, 2015; Bagheri and Midi, 2015) which propose a detailed measure for the case of multiple collinearity-influence observations that is defined as μ_{ii}:

$$\mu_{ii}^{D} = \begin{cases} log \dfrac{\kappa_{(D)}}{\kappa_{(D-i)}}, & i \in D \text{ and } \neq \{D\} \neq 1 \\[2mm] log \dfrac{\kappa_{(i)}}{\kappa}, & \neq \{D\} \text{ and } D = i, i = 1, 2,.., m \\[2mm] log \dfrac{\kappa_{(D+1)}}{\kappa_{(D)}}, & i \in E \end{cases} \tag{8}$$

where D refers to the suspected group of multiple-leverage points, and E is the remaining "good" observations. Negative and positive values of μ_{ii} represent collinearity-enhancing and collinearity-reducing observations, respectively. Thus, the cut-off values of the leverage collinearity-enhancing (LCE) observations and the leverage collinearity-reducing observations are cut^1 and cut^2, respectively (Zhao et al., 2016):

$$cut^1(LCE) = Median\left(\mu_{ii}^{D}\right) - cMad\left(\mu_{ii}^{D}\right) \tag{9}$$

$$cut^2(LCR) = Median\left(\mu_{ii}^{D}\right) + cMad\left(\mu_{ii}^{D}\right) \tag{10}$$

The above is the pair of the X-axis, whereas the boundaries of Y-axis, which are associated with the residuals of the measurements, are based on the standardized least trimmed squares residuals (LTSR) with $\pm\sqrt{\chi^2_{1,0.975}}$ as the cut-off values (Bagheri et al., 2012; Bagheri and Midi, 2015). Thus, the identification process of the influential observations requires a diagnostic plot with double cut-off values; one pair indicates the DRGP and another refers to the residuals of the collinearity-influential observations/measurements.

The Venn diagram of Figure 2 shows the updated classification of the significant observations. Thus, the cut-off values of the double-side diagnostic plot are
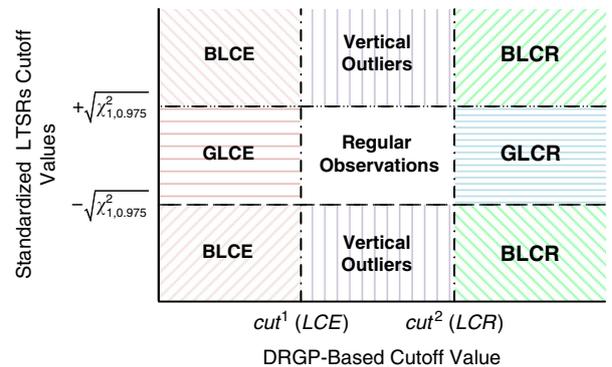

Figure 2: Residual pattern of the regression observations.

shown in the Venn diagram of Figure 2. The updated classification includes nine categories which are the good-leverage collinearity-enhancing observations (GLCE), good-leverage collinearity-reducing observations (GLCR), bad-leverage collinearity-enhancing observations, and bad-leverage collinearity-reducing observations (BLCR). Hence, the measurements with residuals higher than LTSR and $\mu_{ii} > cut^2$ (LCR) are the BLCR measurements, whereas the GLCR measurements have μ_{ii} similar to that of the BLCR but with lower residuals.

Referring to Figure 2, there are seven out of the nine types of observations that have a bad or no influence on the estimation. Moreover, only the LHS observations of Figure 2 are associated with the goal of reducing the collinearities. Nonetheless, this classification is discarded in existing SE studies of meter placement, which indicates a research gap.

Heuristic meter placement algorithm

As shown in Figure 2, only the GLCR and GLCE measurements have a good collinearity-influence on the SE solution. However, the GLCR are the only measurements that contribute to the reduction of collinearities using good observations. Hence, the placement algorithm can be implemented efficiently by considering the GLCR observations as its placement priority. Therefore, high influential-collinearity with bad-leverage points (BLCR) is involved in the proposed algorithm to facilitate the BDD processing. The core objective of the proposed placement strategy is to install the candidate PMUs and conventional meters into the buses of GLCR and BLCR measurements respectively to avoid the creation of more leverage points and collinear measurements. The proposed objective function of the meter placement algorithm is:

$$OF(x) = \left(\sum_{i=1}^{N} \mu_{ii} \Big|_{cond1} \right) + \left(\sum_{i=1}^{N} \mu_{ii} \Big|_{cond2} \right) \quad (11)$$

where μ_{ii} is the collinearity-influential index of (13) such that cond1 and cond2 refer to the conditions of (8) for the GLCR and BLCR measurements, respectively. The first and second part of (11) indicates the locations of the PMUs and the conventional meters, respectively.

To select only one meter for each bus, the symbol +, in the objective function, represents "OR" operator. However, the above algorithm resembles that of previous incremental placement studies such as in Abood et al. (2016) and Saleh et al. (2017). Although the proposed heuristic algorithm is straightforward as

it depends directly on the results of the collinearity-influential measurements, it requires additional computations and subjective decisions in the case of large-scale systems with terminal buses and adjacent HLPs measurements. Therefore, the following constraints should be considered in the fitness function of the optimal placement algorithm:

- Avoid the terminal buses from the list of buses host PMUs. However, terminal buses are not excluded from the candidate list of conventional meters (to increasing local redundancy of critical buses and reducing/eliminating critical measurements).
- Avoid the adjacent buses as the PMU can measure the currents of all the incident branches on its bus.

The constraints must distinguish between the two parts of (11) in regard the configuration of the power system. Hence, the proposed method of this paper is based on CSO-based, metaheuristic techniques for optimizing the locations of the candidate buses.

Cuckoo search optimization

The CSO (Yang and Deb, 2013) is a swarm population-based nature-inspired algorithm that emulates the parasitic breeding of the cuckoo birds which exploit nests of other birds in its environment to lay its eggs. The CSO algorithm is based on a three-stage procedure:

- Cuckoos lay their eggs in the nests of the host bird, and the cuckoo eggs should resemble the eggs of the host birds' to be undetectable.
- The new eggs will be hatched into chicks by the host birds' breeding (if the host birds could not detect and remove them).
- The next generation contains the nest with the high-quality eggs.

The host bird may discover the number of eggs laid by a cuckoo with a simple probability function of ($P = \{0,1\}$), i.e. either to be detected or not. When generating new solution x_i^{t+1}, the random-walk of cuckoo search is enhanced by the so-called Lévy flight, as follows:

$$x_i^{t+1} = x_i^t + \alpha \, Levy(s,\lambda) \quad (12)$$

where $\alpha > 0$ is the step size and:

$$Levy(s,\lambda) \sim s^{-\lambda}, (1 < \lambda \leqslant 3)$$

The candidate nests, the eggs, and the selection techniques are adopted for optimizing the configuration of the meter using incremental meters. A discrete CSO version (Ouaarab et al., 2014) is followed by the proposed meter placement algorithm as it performs more efficiently using fewer iterations and avoids trapping in local minima. The algorithm of the discrete CSO has the following steps:

1. **Initializing:** Generate initial population of n host nests and Set the objective function $f(x), x = (x_1, ..., x_d)^T$
2. **Start fitness process**
   a. **While** ($t <$ Max. Generation) or (stop criterion) **do:**
   b. Searching with p_c group (smart cuckoos)
   c. Start random Lévy flight according to (12)
   d. Evaluate the fitness F_i
   e. Choose a nest among n (say, j) randomly
   f. **if** ($F_i > F_j$) **then**
      replace j be the new solution
   g. **end if**
   h. Abandon fraction p_a of worse nests and build new ones;
   i. Keep the best solutions (or nests with quality solution
   j. Rank the solutions and find the best
3. **end the fitness loop**

The adoption of the CSO for the proposed meter placement algorithm is implemented mainly by using the fitness function of Steps 2.d and 2.f for optimizing the meters locations and the host nests refers to the host buses.

Outliers and HLPs

For this purpose of optimizing the meters locations, the connectivity matrix Λ, which is based on the bus-impedance matrix, is used to reflect the system's configuration in the following fitness function:

$$\text{FF}(x) = \left(w_1 \left(\sum_{i=1}^{N} \mu_{ii} \Big|_{cond1} \right) \cdot \Lambda \right) + \left(w_2 \left(\sum_{i=1}^{N} \mu_{ii} \Big|_{cond2} \right) \right) \quad (13)$$

where $w_2 < w_1 \leq 1$ are weight factors used to normalize the selection values. The fitness function given in (13) is used to guide the discrete CSO. Thus, the proposed placement algorithm includes the following steps:

1. Prepare the measurements set, the connectivity matrix, and the measurements Jacobian matrix.
2. Solve the SE problem of (2).

3. Compute the μ_{ii} values of (8) and identify the GLCR and the GLCE observations based on the cut-off values of (9) and (10).
4. Select the parameters of the CSO: the nest's number, number of eggs, and the number of maximum iterations (t_{max}).
5. Start the discrete CSO algorithm using the fitness function given in (13).
6. Update the best eggs and the best host nests.
7. Extract the position vector x_i^t that indicates the locations of the new meters by the host buses.

The proposed algorithm discards the critical measurement analysis, the residual sensitivity matrix, and the individual SE errors from being constraints to the placement algorithm. However, even though the critical measurements and BDD are not included in the proposed algorithm explicitly, increasing local redundancy addresses these problems. Moreover, the new PMUs contribute drastically to the SE accuracy, and there is no need to remove them when applying BDD processing as they are highly accurate. The conventional meters are utilized to increase measurement redundancy and produce a flexible BDD analysis as they are more affordable than the PMUs.

The proposed placement algorithm includes four stages of reducing the number of incremental meters: the first reduction is by switching from a candidate list that requires meters be connected in such a way that covers all the grid of the power system (Manousakis et al., 2012; Abood and Sreeram, 2014; Aminifar et al., 2010; Chakrabarti et al., 2009), the second stage is to reduce the candidate list to include only the buses of outliers measurements (Saleh et al., 2017; Celik and Liu, 1995), the third is limited to the HLPs, the fourth is associated with the buses of collinearity-reducing measurements, which are very limited in the power systems. The reduction rate can be significant in large-scale power systems. Therefore, the upper bound to the number of incremental measurements is not necessary for the proposed method.

Case studies and simulation results

The performance of the proposed incremental meters' placement algorithm is evaluated using three test systems: the UK 18-bus (Ahmadi and Green, 2009), the IEEE 30-bus (Christie, 2000), and the IEEE 118-bus (Christie, 2000) systems. The IEEE systems are transmission systems with a mesh-shaped grid, whereas the 18-bus system is a radial distribution system. The UK 18-bus system is employed to

demonstrate the influence of the distribution grids' configuration on the meter placement algorithms. The measurement sets are obtained from the load-flow studies by adding Gaussian distributed noises to the results to simulate the real-time measurements.

The proposed CSO-based algorithm is executed for multiple iterations to determine the optimal meter configuration meet the optimization constraints. The number of the eggs is assigned to be twice the number of the buses of each system, and the maximum iteration is 500 each. MATLAB R2018a is used for implementing the algorithm.

IEEE 30-bus system

The topological diagram of the IEEE 30-bus system is shown in Figure 3 to illustrate the connectivity of the system and to compare with the placement studies that concern only the buses with the highest number of branches such as buses 6, 10, and 12. The measurement set contains 78 measurements including 15 pairs of power-injections measurements (active and reactive), 23 pairs of power-flow measurements (active and reactive), and one voltage measurement (Zhao et al., 2016).

Despite the reasonable global redundancy of the system, which is 1.57, the local redundancy of buses 1, 3, 6, 12, 22, 26, and 30 are very low. However, the SE solution of the test system reveals a multicollinearity problem due to the high local

redundancies of few buses. The highest condition index in the Jacobian matrix is determined by (5) to be 323, which indicates perfect multicollinearities since it is higher than the threshold value of 30. The SE is executed along with the leverage points' identification of (8). The outliers, leverage points, and the buses of the highest SE variances are provided in Table 1.

The meter placement scenarios include: First, adopting the existing optimal PMU placement methods which nominate the buses 6, 10, 12 to host the PMUs since they have more braches than others (a necessary condition for observability but not sufficient to the SE performance). Alternatively, the SE variances can nominate the buses of the highest SE variances discarding the presence of multicollinearities. Third, employing the heuristic procedure of the "Heuristic meter placement algorithm" section, which selects the two RHS lists of Table 1 (the shaded columns of the GLCR and BLCR measurements) without any modification for practical consideration. Fourth, the proposed CSO-based method of the "Outliers and HLPs" section, which applies modifications to the RHS lists.

The proposed placement method selects six incremental meters for improving the SE (three PMUs and three Conv. Meters), which represents only 20% of the total bus number. The selected three PMUs are installed at buses 4, 9, and 18, and three injection-power meters at buses 22, 23, and 25. Both the list of the outliers in Table 2 and Figure 4 shows that the HLPs measurements are more than six. However, the proposed placement algorithm excluded the terminal buses (e.g. bus 11), the adjacent buses (e.g. 21, 22, 24, 25, and 26) for achieving optimal meter location.

Table 2 lists the SE errors, the number of the outliers, and the condition numbers of the test system before and after incremental meters by the proposed method. The proposed method eliminates the critical-pair measurements indirectly by increasing the local redundancy on the buses of BLCR. For instance, the measurements of flow 21-22, flow 22-24, injection 23, and flow 25-26 are no longer critical pairs after the incremental meter placement. Thus, the total number of outliers is reduced, the total SE variances decreased, and the condition number is also dropped to 3.83×106 which is about half of that of the base case as shown in Table 2. Table 3 provides a comparison with the existing placement methods (Ahmadi et al., 2011; Aminifar et al., 2010; Chakrabarti et al., 2009). Note that the total SE variances which are computed as given in (14) used only for comparison purposes:
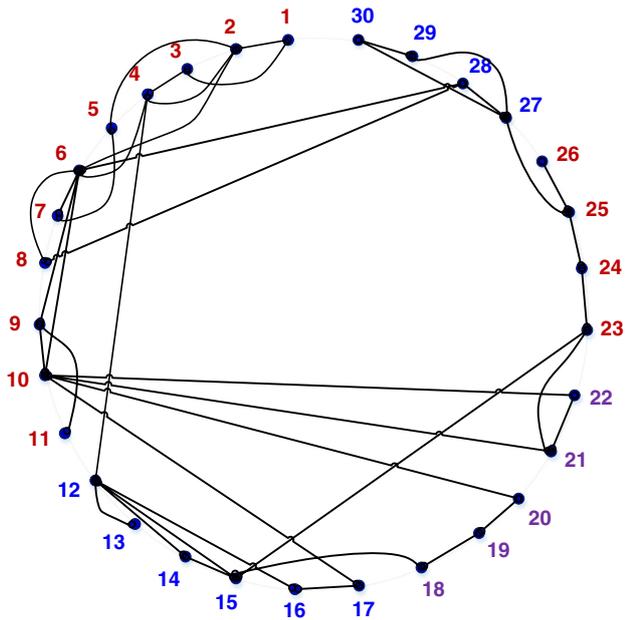

Figure 3: The topological diagram of the IEEE 30-bus system.

$$SE_{TotVar} = \sum_{i=1}^{n} SE_{var} \qquad (14)$$

Table 1. Numerical characteristic of the test systems.

| Test systems | High SE errors | Buses of | | |
		Outlier measurements	GLCR	BLCR
IEEE 30-bus	26, 29, 30	2, 4, 9, 10, 11, 12, 18, 19, 21, 22, 23, 24, 25, 26, 29	4, 9, 11, 18, 19	21, 22, 23, 24, 25, 26
UK 18-bus	13, 14	1, 5, 7, 9, 12, 14, 16, 17	1, 7	14, 16, 17
IEEE 118-bus	10, 35, 36, 60, 61, 64	1, 3, 5, 9, 12, 17, 31, 32, 34, 52, 54, 59, 61, 63, 64, 66, 78, 79, 86, 89, 99, 110, 113, 114, 116, 117	3, 12, 32, 54, 76, 96, 110, 114	9, 25, 34, 35, 52, 59, 64, 66, 68, 86, 113 (11 meters)

Table 2. Comparison of the numerical performance of the SE of the test systems.

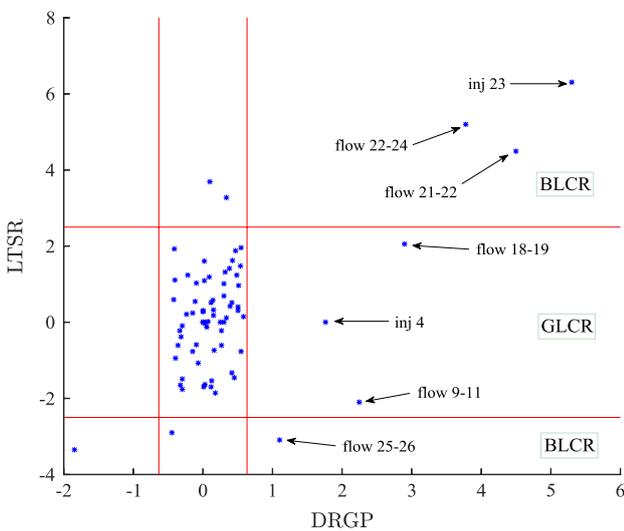| Test systems | Total SE errors | | Condition numbers | | No. of outliers | |
	Before	After	Before	After	Before	After
IEEE 30-bus	8.14×10^{-4}	5.06×10^{-4}	1.84×10^{6}	3.87×10^{4}	15	2
UK 18-bus	9.73×10^{-4}	6.26×10^{-4}	2.86×10^{7}	2.02×10^{4}	8	2
IEEE 118-bus	5.93×10^{-6}	9.48×10^{-4}	1.27×10^{17}	1.27×10^{9}	26	5

Figure 4: Scatter pattern of DRGP vs LTSR for 30-bus system.

UK 18-bus system

The radial network of the 18-bus system that is shown in Figure 5 requires eight or nine PMUs to achieve observability when employing the traditional PMU placement methods that are based on the connectivity matrix of the system. Hence, utilizing the conventional meters along with the PMUs in one-meter placement algorithm is necessary for the distribution systems to reduce the installation cost.

Table 1 displays the numerical characteristics of the 18-bus state estimator which refers to low accuracy and numerical stability. Further, the proposed placement algorithm allocates two PMUs in buses 1 and 6 and two power meters in buses 14 and 16, which is different from the list of GLCR and BLCR measurements. Noting that buses 1 and 14 are already connected to power generation sources which may justify the installation. However,

Table 3. Comparison of PMU placement methods.

Systems	IEEE 30-Bus	IEEE 118-Bus
GA (Xu and Abur, 2004)	7	29
IP (Liu et al., 2014)	7	29
BPSO (Ahmadi et al., 2011)	7	29
PSO (Saleh et al., 2017)	7	28
Proposed CSO-based	(3 PMU + 3 CM) = 6	(8 PMU + 11 CM) = 19

distribution systems need a more subjective decision regarding including the terminal buses in the presence of distributed generation (DG). The four incremental meters (equals 22.2% of the bus number) improve the quality of the state estimator as shown in Table 2. The 18-bus test system is not included in the comparison of Table 3 as it is not tested for meter placement methods.

IEEE 118-bus system

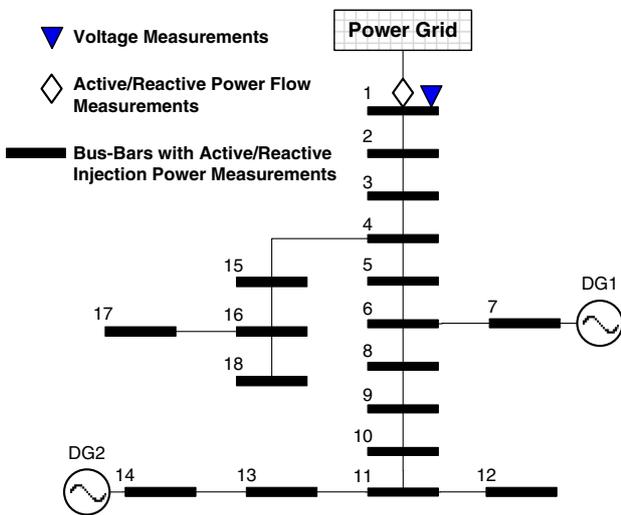The 118-bus system is used to verify the performance of the proposed method in large-scale power systems that have many influential and critical measurements. The implementation of the proposed CSO-based meter placement algorithm results in 19 meters (8 PMUs and 11 power meters), which is around 16% of the number of the buses. Table 1 provides the number and locations of the outliers' measurements. Moreover, Table 2 shows the numerical characteristics of the SE of the 118-bus system before and after adding the new meters which reveal significant improvements that are highlighted in italic.

Table 3 shows that the number of the incremental meters of the proposed CSO-based placement algorithm is much lower than the numbers of PMUs that have been assigned by well-known optimal placement methods (Liu et al., 2014; Saleh et al., 2017; Xu and Abur, 2004; Ahmadi et al., 2011; Chakrabarti et al., 2009). The proposed incremental meters improve the accuracy of the SE solution and increase the local redundancies that alleviate the problem of eliminating the bad data. The numerical results prove that the proposed method is more efficient in large-scale systems as the number of the total incremental meters is only 16% of the system's buses and the ratio of the PMUs is only 9.3%.

On the other hand, Figure 6 shows the performance of the CSO for the 118-bus meter placement compared to that using BPSO technique. However, the convergence rate is higher for the 18-bus and 30-bus systems.

Conclusion

In this paper, an incremental meters' placement algorithm is proposed to enhance the quality of
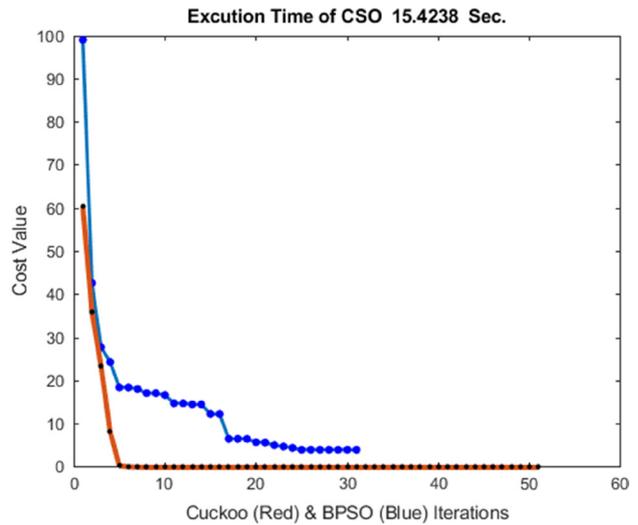

Figure 5: The UK 18-bus test system.

Figure 6: Convergence of the cuckoo search of the 118-bus SE.

the power system SE using minimal PMUs. The proposed algorithm utilizes the discrete CSO and the collinearity-HLPs analysis to build an efficient placement strategy. Employing the leverage collinearity-influential measurements is the main tool for the proposed method to accomplish meter placement that solves the multicollinearities and improves the quality of the SE solution. Furthermore, the proposed approach alleviates the undetectable bad data by reducing the number of outliers, which could be beneficial for avoiding the security attack. The case studies depict that the WLS SE can be enhanced using a lower number of meters by employing the proposed method without manipulating the existing set. It is observed that the number of PMUs and the total number of meters achieved by the proposed algorithm are much lower than that of the corresponding methods. Hence, a statistical analysis must be carried out to the measurement set before selecting the numbers and types of the new meters. The simulation results show that the proposed placement method performs well in the distribution systems and the large-scale systems and thus, it can be recommended to be employed for modern distribution grids.

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