Oppel-Kundt illusion balance

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Perceptual estimates of spatial dimensions of visual objects depend on their shape and surface attributes. The present psychophysical study emphasizes two main contributors to the Oppel-Kundt illusion: the outline of the filled space and the mode of filling. In past experiments, both factors have been considered significant. Our experiments were performed by using combined stimuli of the Oppel-Kundt figures and supplementary objects situated within the empty intervals of the figures. Line segments, empty and filled rectangles, blurred contours, and grey and color images were used for the supplementary stimuli role. The experimental data demonstrated an innate property of the objects to balance the illusion of distance if they were placed within the Oppel-Kundt figure and to create an illusion of extent when compared with an empty space interval. Both the balance magnitude and the induced illusion strength varied depending on the objects' spatial structure. The supplementary objects showed a tendency to differ from each other by their functional capacity and were ranked from lowest to highest: a line segment, a solid bar with a blurred outline, a contour of a rectangle, a solid fill rectangle, greyscale patterns, and color pictures. The experimental findings provided support for an explanation of the Oppel-Kundt illusion in terms of the spatial-temporal summation of excitations representing the object outline and surface attributes at the lower cortical levels of the visual system. Along with the facts already established in current literature, the experimental data gave rise to the assumption that any visual object could appear larger than its occupied area, and that the Oppel-Kundt illusion could become a separate case in the common sensory phenomenon of object size illusion.

Key words: Oppel-Kundt illusion, combined stimuli, balance of illusions, object size

INTRODUCTION

The Oppel-Kundt illusion (IO-K) refers to the difference in visual estimation of equidistant distorted and undistorted extents: the linear space filled with uniform elements appears longer than the empty space; the apparent length first increases with the number of filling elements, but then, after a maximum, slightly decreases. The misperception studies started with J.J. Oppel's report (1854-1855) on undivided lines or areas appearing smaller than those that were divided (Wade et al., 2017). An accompanying illustration of the distorted/undistorted configuration was not given in the report, but further examination (Hering, 1861; Kundt, 1863; Helmholtz, 1867) revealed that either part of the suggested stimulus, a subdivided or undivided line or area, might also produce the expansion effects

when compared with an empty space. A great variety of filled/empty patterns comprising spots, stripes, or other geometric shapes have been designed and experimentally justified, thus the original Oppel's stimulus became a combination of two inducers, unequal in strength. During the length matching procedure, the expansion effect of the undivided inducer partly balanced that of the subdivided one. The author discerned the residue.

A complete theory on the neural mechanisms of the expansion effect has not yet been proposed (Wackerman, 2017). Originally, the illusion of interrupted extent was explained by the retina curvature (Hering, 1861; Kundt, 1863). Later studies invoked various other rationale: eye movements and muscles fatigue (Wundt, 1898; Lewis, 1912), attractive and repulsive forces in the cortical representations (Spiegel, 1937), perspective depth

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cues in the stimulus (Tausch, 1954), varying distribution of attention in the two halves of the filled/unfilled pattern (Piaget, 1969), and differences in the averaged contour density and corresponding zero-crossing numbers within a range of spatial scales contributing quantitatively to the illusory effect (Craven and Watt, 1989; Watt, 1990). Regrettably, numerical studies of the stimuli with high numbers of filling elements fusing into a solid bar did not receive much attention and remained a matter for further interpretation.

According to experiments that grouped adjacent stripes in the filled/unfilled pattern (Noguchi et al., 1990), the illusion was related to higher neural levels generating size constancy or figural segregation. Contrarily, a later experimental study (Bulatov et al., 1997) specified spatial filtering processes in early visual pathways as inducers of overestimation of the filled interval length. Current concepts have explained the perceptual overestimate effect as being caused by spatial anisotropy (Wackerman, 2012), terminal repulsion (Mikellidou, 2012, Mikellidou and Thompson, 2014), spatiotemporal integration along the continuous excitation path produced by the stimulus contours (Bertulis et al., 2014; 2018), or context-evoked excitation augmentation (Bulatov et al., 2017). Computational modelling (Erdfelder and Faul, 1994; Wackermann and Kastner, 2010; Bulatov et al., 2017) provided an adequate description of certain illusory effects which could be related to different neural correlates in subcortical and cortical visual networks. Evidently, the Oppel-Kundt (O-K) phenomenon has been represented as a multi-factorial event.

The purpose of this study was to highlight the two main triggers that contribute to IO-K: the empty or filled space borders and the filling mode. Both factors were perceptually significant. Importance of the contours was emphasized during testing of the superposed O-K figures comprising a regular sequence of uniform filling stripes, a contour rectangle, or a solid block (Bertulis et al., 2014), and also in an examination of the expansion effect produced by the Müller-Lyer wings attached to the O-K pattern with either spots or high stripes (Bertulis et al., 2018). The next excitation factor, the filling mode, mattered to the illusion as well. The factor was suggested early on (Messer, 1876) but a convincing illustration of the filling role in the illusion genesis was only advanced later. It was a non-monotonic dependence of the illusion magnitude on the number of subdividing elements (Knox and Watanabe, 1894; Obonai, 1933; Spiegel, 1937; Piaget and Osterrieth, 1953). Many elements close together may be less effective than a few wider spaced ones and more effective than a solid bar. Some other spatial parameters of the stimulus, e.g., appearance of the filling and limiting elements, also influenced the form of the O-K illusion curve (Spiegel, 1937; Wackerman and Kastner, 2009).

In the present experiments, we matched the equidistantly distorted and undistorted space intervals in distance. We observed changes in IO-K strength when supplementary stimuli appeared in the undistorted stimulus portion. For this purpose, the modified O-K figures resembling Oppel's combined stimulus were designed. The two-part O-K stimulus (Fig. 1A) served as the basis for the modifications (Fig. 1B). The supplementary objects: a uniformly filled rectangle (B1), the contour of a rectangle (B2), a block with blurred contours (B3), a line segment (B4) of varying positions (a, b, c, and others) in the stimulus, a birch rind (B5), a column of bricks (B6), a bridge section (B7), stained glass (B8), and a sequence of vertical stripes (B9) were situated in the traditionally empty stimulus interval. Three supplementary objects: B6, B7, and B8, were shown in color. Supplementary object B9 was used to check the balance between the two expansion effects produced by the distorted intervals.

In parallel, we recorded the illusion of extent induced by the supplementary stimuli when compared to an empty space interval.

METHODS

Stimuli and apparatus

The O-K type stimuli (Fig. 1) formed of uniform vertical stripes with luminance of 50 cd/m^2, 3.6 arc min in width, and 36 arc min in height were presented on the monitor against 0.1 cd/m^2 luminance backgrounds. The stimuli intervals filled with stripes were considered the referential distances, and those filled with the supplementary objects (Fig. 1; B1-B9) were the test cases. The references were fixed in length at 200 arc min, and the test items were varied by the subjects when making the perceived length of the test to be equal to that of the reference in the experiments.

When the testing intervals were changing in length, the supplementary blocks (B1, B2, and B3), lines of varying positions (B4), or stripes (B9) got longer or shorter by moving accurately together with the terminal stripe, therefore, no structure changes occurred and no gaps or crossings appeared at the interval ends. The remaining four inducers (B5, B6, B7 and B8) also did not vary in structure when the subjects were manipulating the panel keys. The computer program moved the lateral stimulus limiter only to the right or to the left and the movement steps opened or closed the stimulus periphery making the visible object longer or shorter.

The stimuli were drawn by the Cambridge Research Systems VSG 2/3 and displayed on an EIZO T562-T, CRT monitor (800 × 600 pixels) calibrated and gamma corrected using a Cambridge Research Systems OptiCAL photometer. The Psychophysical Experiment Toolbox in the MathWorks Matlab software platform controlled the presentations of the stimuli, introduced changes according to the subject's command, and recorded the subject's responses.

Subjects watched the monitor screen monocularly. A chin/forehead support limited head movement. The distance between the monitor screen and the subject's eye was 100 cm, which caused the screen elements to subtend 1.2 × 1.2 arc min.

Subjects

Eight University students (seven female and one male; mean age 20.7 years) and three University lecturers (one female and two males, mean age 55 years) took part in the experiments. All subjects were normally sighted or were wearing their usual optical corrections. None of the students had practiced any similar procedure before and were naïve in respect to the goals of the study. The lecturers were well-informed about the purpose and methods of

the investigation. A difference between the results of the two participant groups was not observed.

All subjects gave informed consent prior to the experiment in accordance with the Helsinki

Declaration

The authors have received written approval of the Kaunas Regional Biomedical Research Ethics Commitee.

Procedure

The experiments were carried out in a dark room. The method of adjustment was used to establish the functional dependence of illusion strength on spatial parameters of the stimuli. Biases in the judgment criteria, an inherent characteristic of the method, were reduced by randomizing stimuli with different parameters in the presentation sequences.

Subjects were asked to adjust the test interval length (by two pixels at a time) to make it equal to that of the referential stimulus. The initial length of the test interval was randomized and distributed evenly within a range of 200 ± 10 arc min. No instructions concerning gaze fixation were given, and observation time was effectively
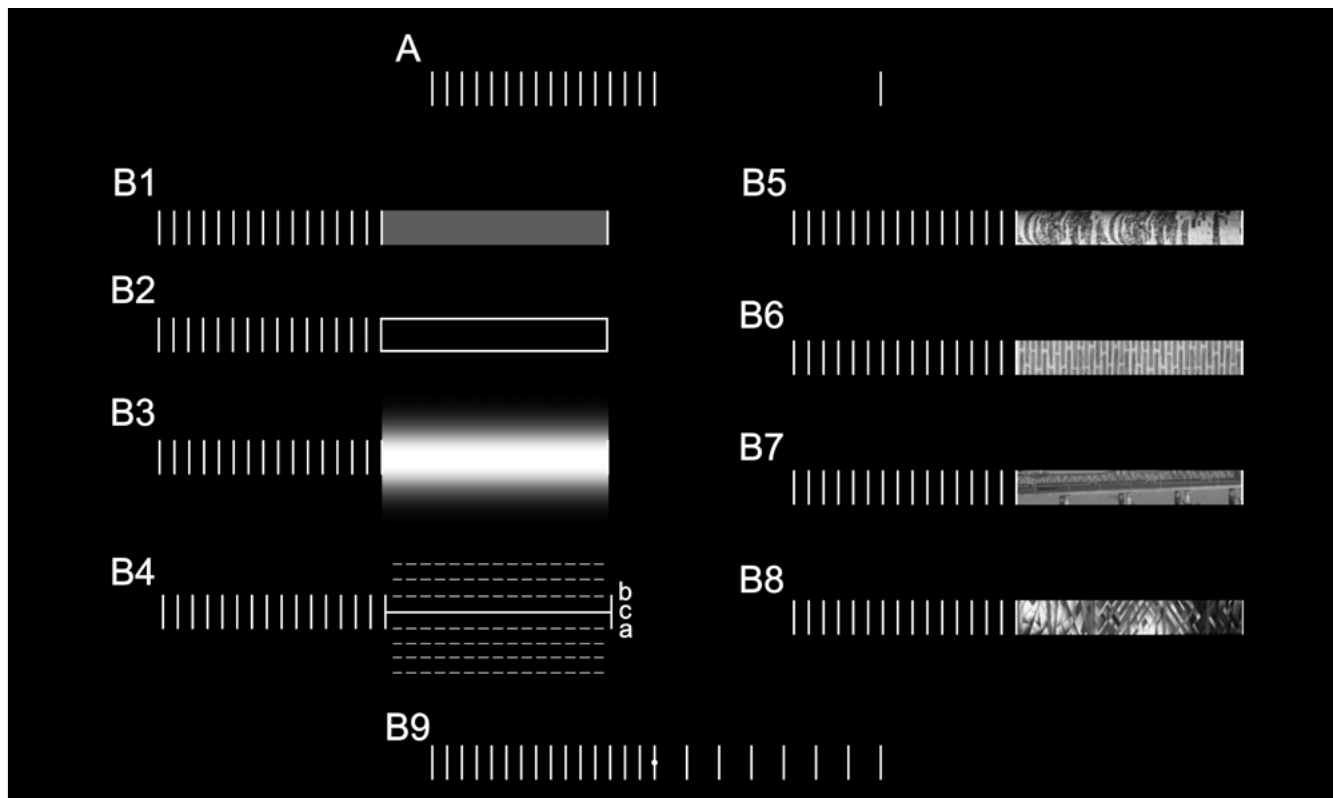

Fig. 1. Facsimiles of the stimuli. (A) O-K figure. (B) Combined patterns B1–B9 with supplementary objects in the test part of the stimulus. See the main text for a detailed explanation.

unlimited. The errors made by the subjects were considered the illusion strength values. Over the course of the experiment, the subjects carried out forty experimental sessions with twelve stimuli per session and two to three trials for each stimulus. Each data point represents the average of ten trials. The subjects were not given any additional practice or knowledge of the results during the experimental sessions. As a whole, two experimental series were carried out: Experiment 1 with stimuli A, B1, B2, B3, and B4; and Experiment 2 with stimuli B5, B6, B7, B8, and B9. Ten of eleven subjects participated in Experiment 1, and four subjects participated in Experiment 2.

Analysis

Matlab MathWorks was used for the data analysis. The Kolmogorov-Smirnov with Lilliefors corrections distribution normality test was applied. For all cases, the normality of frequency distribution was not rejected. Parametric methods were used. One-way repeated measures ANOVA was applied to the data. In *post hoc* multiple pairwise comparisons, a two-tailed Tukey's honest significant difference criterion was used.

RESULTS

Experiment 1

Ten subjects established a typical non-monotonic illusion IO-K strength dependence on the number of filling stripes, *n*. The average group response curve
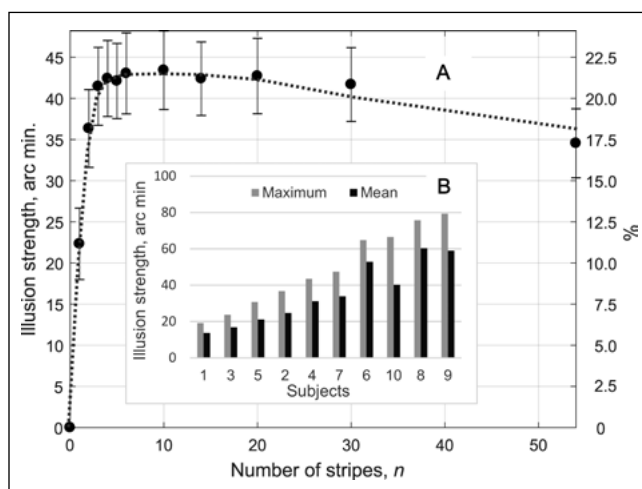

Fig. 2. IO-K as a function of the number of filling stripes. (A) 10 subject group response curve. Displayed are grand means (circles) ± standard errors of the means (bars). Grand means are based on the individual response data. Dashed line, a smoothing curve. (B) Rank ordering of the illusion maximum and mean.

(Fig. 2A) started from 0 arc min at $n=0$ because individual left/right anisotropy errors seen in some subjects' data were excluded. With an increasing number of filling stripes, the curve reached a maximum of about 43.5 arc min (21.8% of the reference length; SD=19.6) at $n=10$, remaining at practically the same level until $n=30$, and declined to 35 arc min (17.5%; SD=18.7) at the solid filling. The illusion mean was about 35.4 arc min (17.7% of the reference length). Among subjects, the illusion maximum and mean ranged from 19 to 79 arc min and from 14 to 60 arc min, respectively (Fig. 2B). The individual IO-K functions (Fig. 3) exposed a common non-monotonic profile as well, irrespective of wide dispersion in the illusion strength, which may have primarily been caused by the individual properties of the visual signal processing (Bertulis et al., 2018).

The O-K stimuli combined with the supplementary objects B1, B2, and B3 produced relatively weak (balanced) distortions of perceived length, Bso (Fig. 3). The Bso curves were situated clearly below IO-K and appeared to be flatter. Within the 0 to 5 filling stripes interval, most curves ascended alongside the negative scale in the Y axis, and some showed negative values within the next interval, $n<5$. Presumably, the supplementary objects: a uniform block (B1), the contour of a rectangle (B2), and blurred contours (B3) balanced the overestimate of the stripe sequence (IO-K) by inducing their own illusory effect (Iso). The balance magnitude, MBso=IO-K - Bso of a single supplementary object varied greatly among subjects (one-way repeated measures ANOVA, $F_{2,9}=19.6$, $P<0.01$). For instance, the solid block (B1) reduced the IO-K mean to 2.9, 4.9, 5.7, and 13.6 arc min for subjects 7, 9, 4, and 8, respectively (Table I). In other cases (subjects 1, 2, 5, 6, 10, and, especially, 3), the balance was much stronger and Bso achieved rather low negative values. On average, object B1 decreased the IO-K mean by 35.4 + 4.1=39.5 arc min value (112 %, $t_9=8.9$, $P<0.05$). The residue was -2.1% (Table I).

The supplementary objects B2 and B3 appeared less inducible than B1 (Table I). The contour of a rectangle (B2) decreased the IO-K mean by 35.4 - 1.0=34.4 arc min (97%; $t_9=7.9$, $P<0.05$), and the solid filling with blurred contours (B3) balanced the IO-K mean by 35.4 - 3.6=31.8 arc min (90%; $t_9=8.8$, $P<0.05$). The residue was 0.5% for B2 and 1.8% for B3.

Basically, the MBso values (39.5 arc min, 34.4 arc min, and 31.8 arc min; one-way repeated measures ANOVA, $F_{2,29}=5.8$, $P<0.05$) suggested the B1 > B2 > B3 rank ordering in the supplementary objects' activity (one-way repeated measures ANOVA, $F_{2,29}=4.9$, $P<0.05$), although MBso values varied among subjects. In the data from subjects 2, 3, 5, and 8, the three curves stayed apart in a clear top-down sequence: B3, B2, and B1 (Fig. 3), but they varied in the vertical parting. For
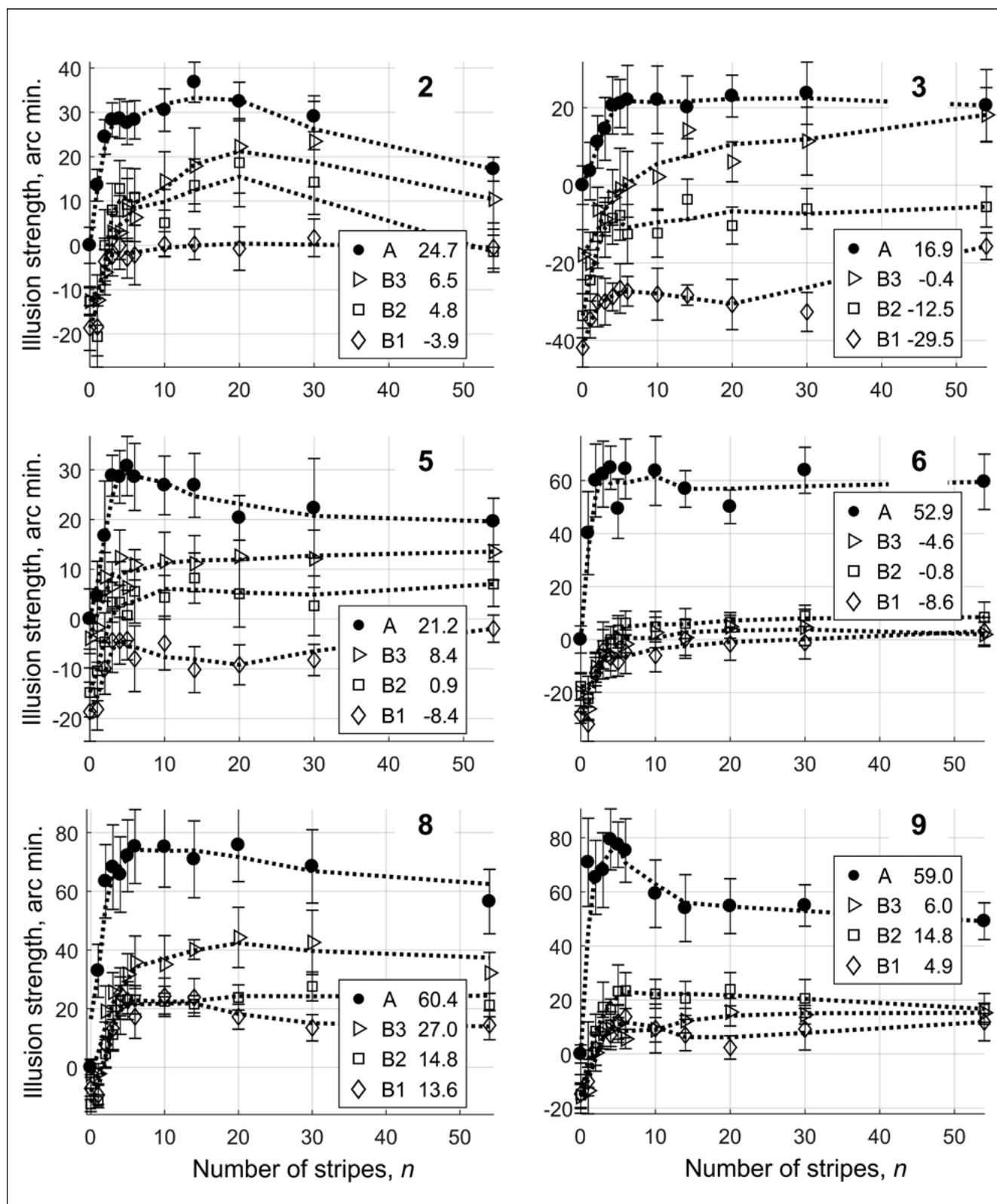
Fig. 3. IO-K as a function of the filling stripe number in the absence of the supplementary objects (black circles) and Bso in the presence of the uniform block B1 (diamonds), the contour frame B2 (rectangles), or solid filling with blurred edges B3 (triangles) within the empty O-K stimulus interval. Vertical bars, 95% confidence intervals. Dashed lines, smoothing curves. In the frames, the illusion means values, arc min. Data for six subjects (2, 3, 5, 6, 8, and 9).

subject 8, curves B2 and B3 diverged at $n=14$. For subjects 6 and 9, the three curves run close to one another, though the mean values were not the same (difference not significant).

The IO-K balance coefficient, $k_b=1$ - Bso /IO-K illustrated variability in the stimuli effectiveness as well (Table I): object B1 was more effective than B2 or B3 for nine subjects; B2 exceeded B3 in the six subjects' data; the B1 > B2 > B3 rank ordering was established by four persons. Still, the k_b data average supported the B1 > B2 > B3 sequence by giving the 1.12 : 0.97 : 0.9 proportion (Table I; one-way repeated measures ANOVA, $F_{2,29}=4.5$, $P<0.05$).

Moreover, the supplementary objects' ability to evoke the illusion of extent I_{so} in Experiment 1 (Fig. 4) legitimated the B1 > B2 > B3 rank ordering in the stimuli effectiveness: (B1) 18.2 arc min; 51.4% (SD=11.5): (B2) 16.7 arc min; 47.2% (SD=8.0): (B3) 11.9 arc min; 34% (SD=8.0; one-way repeated measures ANOVA, $F_{2,29}=17.6$, $P<0.05$).

According to the Fig. 4 data, the contour of a rectangle (B2) and the body of the blurred outline (B3) individually manufactured the illusions of a certain size, but the contour and filling combined together (B1) did not reach the sum of the two results: B1 < B2 + B3. Only the data of subject 4 served as an exception.

It can be noted, also, that the abilities of objects B1, B2, and B3 to balance IO-K and to induce the illusion of extent,

Iso, looked divergent. MBso: 39.5, 34.4, and 31.8 arc min were more than two times greater than the Iso strength: 18.2, 16.7 and 11.9 arc min, correspondingly ($t_9=5.3$, $t_9=4.1$, $t_9=4.5$, for all three $P<0.05$). Overall, Iso<MBso.

A single line segment (supplementary object B4) balanced the O-K illusion (Fig. 5) in a similar manner to the two identical line segments in the contour of a rectangle (object B2). The stimulus effectiveness varied among subjects again. In the diagrams of subject 2, the balance curves B2 and B4 practically coincided, and the mean values were alike: 4.8 arc min (B2) and 4.2 arc min (B4). MBso were close: 19.9 arc min (B2) and 20.5 arc min (B4). I_{so} by B2 and B4 were equal in strength, 12.6 arc min. Yet, the B2 curves of subjects 8 and 10 were below B4, and the Bso mean were different: 14.8/27.8 arc min ($t_{11}=8.6$, $P<0.05$) and -13.1/-6.2 arc min ($t_{11}=6.3$, $P<0.05$), respectively. So, MBso by a single line looked less than that produced by the contour of a rectangle: 32.6/45.6 arc min (for subject 8, $t_{11}=8.6$, $P<0.05$) and 46.5/53.4 arc min (for subject 10, $t_{11}=6.3$, $P<0.05$). The Iso strength by B4 tended to become less than that induced by B2: 2.2/12.5 arc min for subject 8, ($t_{20}=4.1$, $P<0.05$) and 18.8/22.1 (for subject 10, difference not significant). Like the previous three supplementary objects, the line segment (B4) was less potent in producing I_{so} than balancing IO-K : 12.6/20.5 arc min (for subject 2 $t_{20}=12.6$, $P<0.05$); 2.2/32.6 arc min (for sub-

Table I. The I_{O-K} balance (B_{so}) in the presence of supplementary objects B1, B2, and B3.

Subjects	I_{O-K} mean	B_{so} mean, arc min (I_{O-K} in the presence of objects B1, B2, and B3) and balance coefficient, k_b						k_b: 1 > 2 or 3	k_b: 2 > 3	k_b: 1 > 2 > 3
		Solid fill B1		Empty contour B2		Blurred edges B3				
	arc min	arc min	k_b	arc min	k_b	arc min	k_b			
1	13.7	-7.1	1.52	-7.3	1.53	-4.2	1.31	+	+	−
2	24.7	-3.9	1.16	4.9	0.80	6.5	0.74	+	+	+
3	16.9	-29.5	2.75	-12.5	1.74	-0.4	1.03	+	+	+
4	31.2	5.7	0.82	8.5	0.73	5.4	0.83	+	−	−
5	21.2	-8.4	1.40	0.9	0.96	8.4	0.61	+	+	+
6	52.9	-8.6	1.16	-0.8	1.01	-4.6	1.09	+	−	−
7	33.9	2.9	0.91	-0.5	1.01	7.2	0.79	+	+	−
8	60.4	13.6	0.78	14.8	0.76	27.0	0.55	+	+	+
9	59.0	4.9	0.92	14.8	0.75	6.0	0.90	+	−	−
10	40.3	-10.9	1.27	-13.1	1.32	-14.8	1.37	−	−	−
Average	35.4	-4.1	1.12	1.0	0.97	3.6	0.9	9:1	6:4	4:6
Reference length ratio	17.7%	-2.1%	−	0.5%	−	1.8%	−			

ject 8, t_{20}=6.9, *P*<0.05); and 18.8/46.5 arc min (for subject 10, t_{20}=12.9, *P*<0.05).

On average, the supplementary line (B4) induced 27% and balanced 79% of the referential illusion strength (Fig. 5). This way, Iso<MBso. The first three supplementary objects exceeded B4 according to their effectiveness. B1, B2, and B3 induced 34%, 47%, and 51% and balanced 90%, 97%, and 112% of the IO-K strength, correspondingly.

The spatial dynamics of MBso and induced illusion Iso were observed in the experiments with varying positions of the line segment (Fig. 1, B4) inside and outside the empty interval in the O-K stimulus. The number of filling stripes in the referential stimulus interval did not vary and was fixed at 14 or 0. Three positions of the line segment within the empty interval limits (a, b, and c) yielded the Iso strength maximum of approximately 20 arc min, when *n* was 0, and showed the Bso maximum at 20 arc min on average, when *n* was 14 (Fig. 6). In the positions above and below the test interval, the line segment had no touch with the vertical terminators and seemed to be less influential. With an increasing shift downward or upward, both the balance MBso and the illusion Iso strength receded gradually (Fig. 6). If the line was shifted away from position *c* to the 31 arc min position, MBso decreased by 3-4 arc min and the Iso strength descended by 12-13 arc min. At the 47 arc min shift from position *c*, MBso was reduced by 10-14 arc min while the Iso strength fell by 17-19 arc min. At the 62 arc min shift, the balance effect was still discernible (about 2 arc min), but the illusion manifestation hardly registered. The balance effect faded and IO-K was restored to the 40±2 arc min level only when the supplementary line

was about 77 arc min away from position *c*. A symmetric profile of the Bso dynamics (stars) was comparable with that of the Iso curve (circles) in shape, but the MBso mean was easily greater than the I_{so} mean (-11.4 arc min over - 6.5 arc min; t_{10}=25, *P*<0.05). Again, Iso<MBso.

Taken together, the Experiment 1 findings suggested three regularities: the supplementary objects possessed a property to balance the illusory percept in the O-K figure and to produce an illusion of extent in presentations together with a free space interval, MBso was evidently greater than Iso strength, and an assumed rank ordering of the supplementary objects' capability in the balancing IO-K and in inducing Iso was: B1 > B2 > B3 > B4.

Experiment 2

Patterns B5, B6, B7, and B8 balanced IO-K demonstrably (Fig. 7) like the other supplementary objects in Experiment 1 (Fig. 3). The data varied among subjects as well. For subject 2, the birch rind (B5) reduced the IO-K mean to -3.9 arc min (Fig. 7; B5) and was as effective as B1: the mean values were equal to -3.9 arc min, in spite of the curves diverging at *n* >14. The color stimuli B6, B7, and B8 appeared to be more influential than the greyscale images B1 and B5 (t_{11}=11.8, *P*<0.05). The bridge pattern, B7 minimized the IO-K mean to -10.2 arc min (t_{11}=3.6, *P*<0.05), and the bricks, B6 or stained glass, B8 diminished it to -13.8 and -13.9 arc min (t_{11}=8.1, *P*<0.05 and t_{11}=7.7, *P*<0.05, respectively).

In the data for subject 10, the color stimuli also appeared more influential than the greyscale ones (Ta-
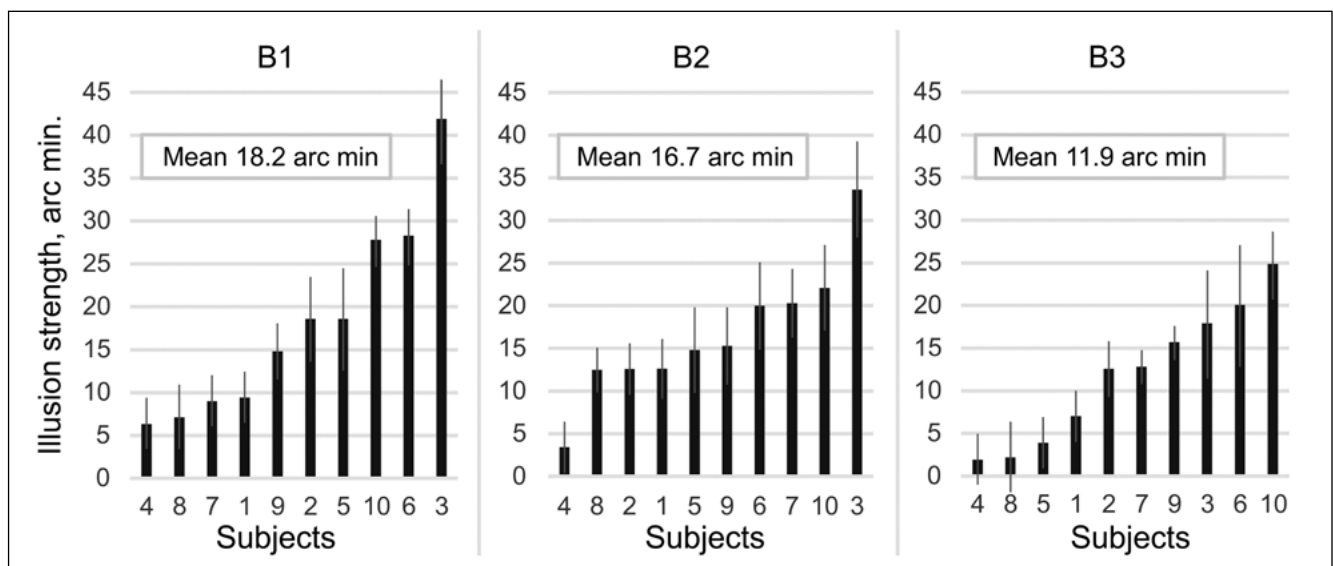

Fig. 4. Rank ordering of the Iso strength induced by the supplementary objects: filled rectangle, B1; contour of a rectangle, B2; and block with blurred contours, B3.
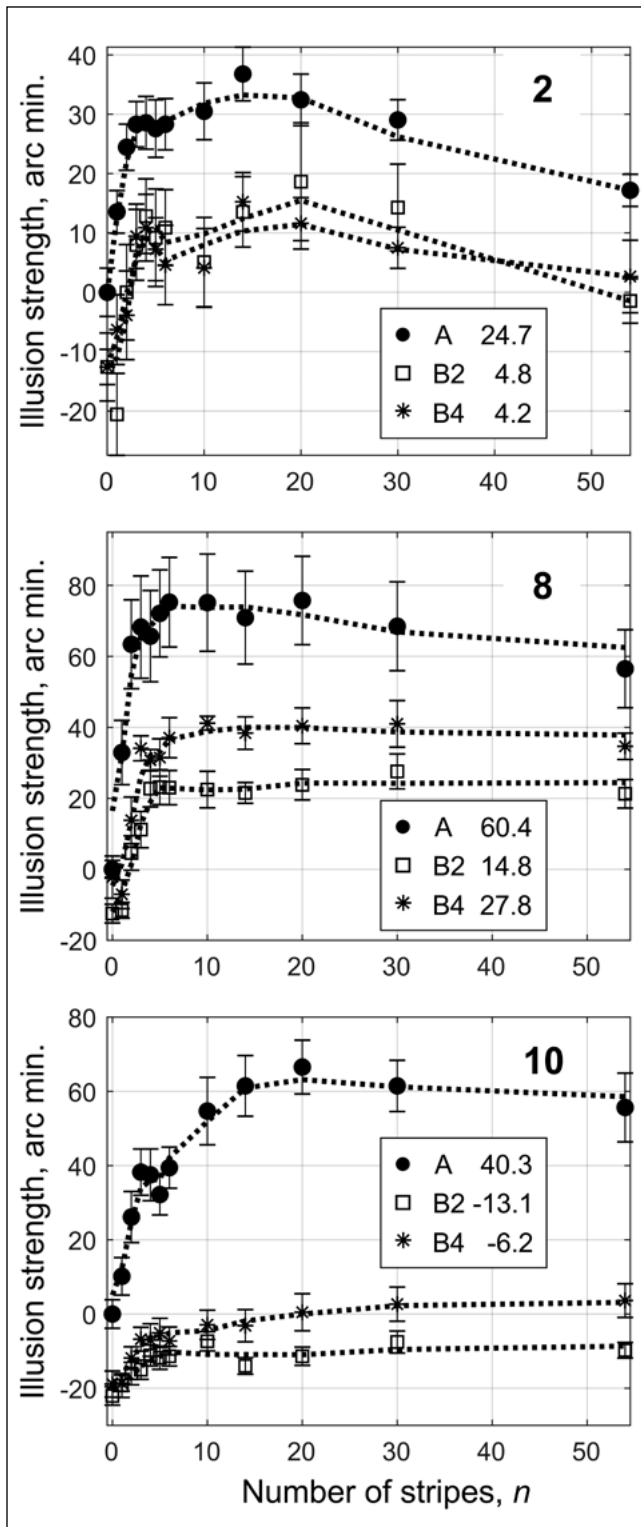
Fig. 5. IO-K as a function of the filling stripe number (circles) and Bso at the presence of the contour of a rectangle, B2 (squares) or supplementary line, object B4 (stars), in the combined stimulus empty interval (in position a or b; Fig. 1 B4). Vertical bars, 95% confidence intervals. Dashed lines, smoothing curves. In the frames, the illusion means values, arc min. Subjects 2, 8, and 10.

ble II). The balance coefficient, k_b was lower for the objects B1 and B5 (1.21 and 1.27) than for the patterns B6, B7, and B8 (1.29, 1.37, and 1.40), respectively (t_{11}=4.4, $P<0.05$). Though subject 8 (Table II) provided the k_b values that were quite similar for the grey (0.78; 1.00) and color (0.75; 0.77; 0.96) objects (t_{11}=1.9, $P>0.05$). Taken together, a tendency of k_b to appear lower for the grey (1.09) than for color patterns (1.23) could be seen. Consequently, one might witness a tendency of MBso for the greyscale objects (45.6 arc min; 109%) to become lower than that for the color objects (51.4 arc min; 123%). To explain, the average of the IO-K means for subjects 2, 8, and 10 (Table I) was: (24.7 + 60.4 + 40.3) / 3=41.8 arc min. Then, MBso for the greyscale was: 41.8 arc min × 1.09=45.6 arc min and for the color: 41.8 arc min × 1.23=51.4 arc min.

Iso strength was larger for the chromatic stimuli than for achromatic ones in all cases (Table II). On average, it was 24.4 arc min and 16.3 arc min, respectively. For the chromatic objects, Iso strength (24.4 arc min, 58%) was approximately two times smaller than MBso (51.4 arc min, 123%; t_{115}=3.9, $P<0.05$). Repeatedly, Iso<MBso.

A reliable rank ordering of the chromatic objects according their capacity to balance IO-K and induce Iso could not be suggested because of an insufficient experimental sample. For the two achromatic objects B1 and B5, both the k_b and Iso values (Table II) were also similar, thus, the stimuli efficiency was approximately equal, B1 ≈ B5.
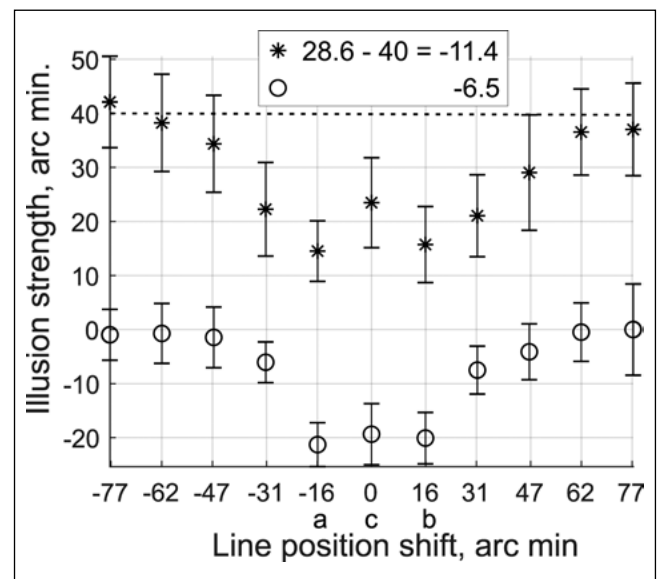

Fig. 6. Illusion of extent, Iso induced by the line segment (B4, circles) and IO-K balance, B_{so} in the combined stimulus (stars) as functions of the line position shift from the stimulus axis. Vertical bars, 95% confidence intervals. Dashed horizontal line at the 40 arc min level, the IO-K strength. In the frame, the mean values of MBso and Iso, arc min. Subject 2.

The supplementary object B9 also possessed the property of being able to balance IO-K and to produce Iso, but the difference between MBso and Iso significantly decreased or disappeared completely
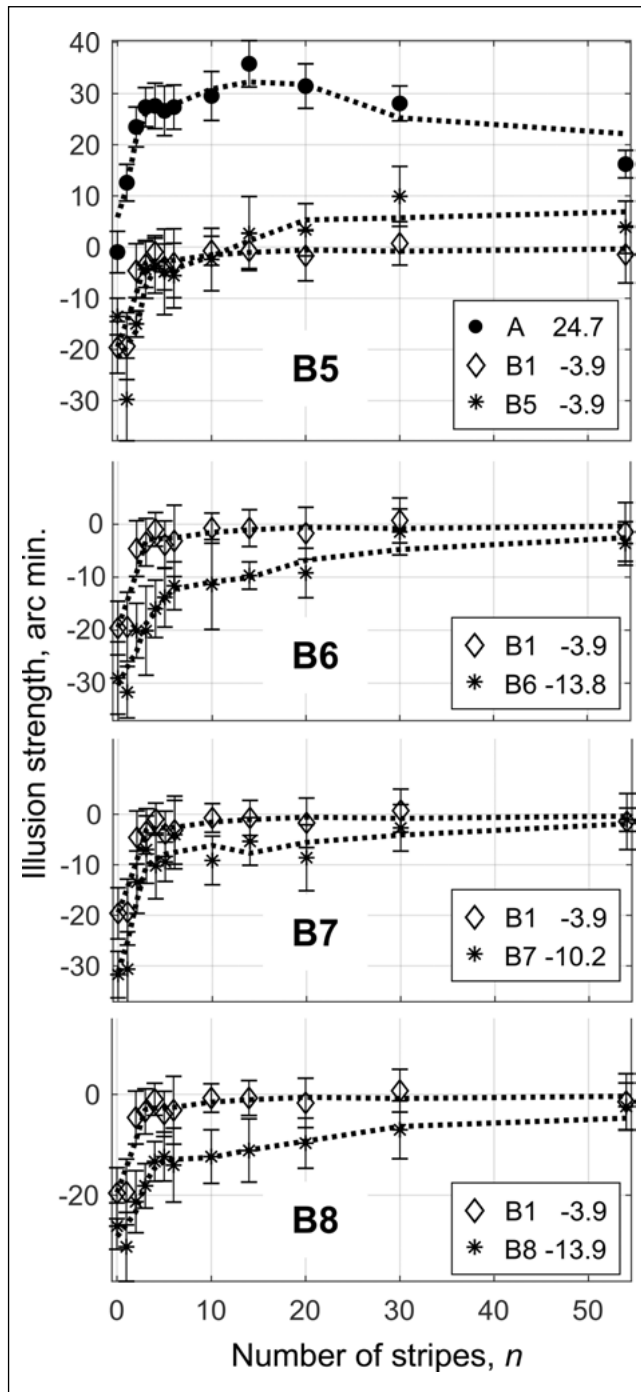

Fig. 7. IO-K as a function of the filling stripe number (circles) and Bso at the presence of the supplementary object B1 (diamonds), B5, B6, B7, or B8 (stars) in the stimulus empty interval. Vertical bars, 95% confidence intervals. Dashed lines, smoothing curves. In the frames, the mean values of the curves, arc min. Subject 2.

(Fig. 8). In the data for subject 8, MBso (44.9 arc min + 5.1 arc min=50.0 arc min) was greater than the I_{so} strength (26.6 arc min) by 1.9 times. But it was a smaller difference compared to the data given by subject 8 in Experiment 1: for stimuli B1, B2, and B3, the MBso average (42.3 arc min) exceeded the Iso strength (7.6 arc min) by 5.6 times (t_{17}=4.6, $P<0.05$). For subject 10, MBso (20.3 arc min - 5.7 arc min=14.6 arc min) in Experiment 2 was 1.6 times greater than the Iso strength (9.0 arc min), while, in Experiment 1, MBso (53.19 arc min) exceeded Iso (17.3 arc min) by 3.1 times (differences among ratios were significant, t_{17}=4.3, $P<0.05$). Yet more convincing data was revealed in the absence of predominance of MBso over Iso by a single arc min for subjects 2 and 11. In Experiment 2, MBso for subject 2 (24.7 arc min + 0.6 arc min=25.3 arc min) was not bigger but even smaller (ratio 0.88) than the Iso strength (28.6 arc min), while, in Experiment 1, MBso for subject 2 (22.3 arc min) exceeded Iso (14.6 arc min) by 1.5 times (differences among ratios were significant, t_{22}=4.4, $P<0.05$). For subject 11, MBso (24.8 arc min + 13.3 arc min=38.1 arc min) was again smaller (ratio 0.9) than the Iso strength (42.2 arc min) in Experiment 2.

Five vertical stripes were present in the test interval of the combined stimulus B9 shown for subject 2, but four stripes were given for subject 10, and fourteen stripes for subjects 8 and 11.

In sum, Experiment 2 supported the assumption that supplementary objects combined in the O-K figure balanced the illusory percept due to them inducing a self-dependent illusion of extent. MBso could appear rather close to the illusion Iso strength when the referential and test objects were alike in their spatial structure. This is, Iso ≈ MBso. In experiment 2, the color objects appeared more inducible than the greyscale ones.

DISCUSSION

The present study validated an inherent property of the visual objects to balance the illusion of distance when presenting them within the O-K figure and, also, to create an illusion of extent when compared with an empty space interval. Both MB_{so} and I_{so} strength varied in dependence on the objects' spatial structure. The objects' impact could be ranked in a suggested order: B4<B3<B2<(B1 and B5)≤(B6, B7, and B8). A simple geometric shape, the horizontal line segment (object B4) connecting the vertical terminators, balanced 79% of the O-K illusion mean. The two parallel line segments forming a contour of a rectangle (object B2) were more effective and balanced 97% of the mean. The efficiency of the filled rectangle with sharp contours (object B1) was also higher, at 112%. The supplementary object

B3 was not well defined in terms of shape because of the blurred outline and appeared to be less effective (about 90%) than the filled area with sharp contours (B1) despite the fact that B3 was about two times bigger in area and luminance than B1. Concerning the illusion I_{so} strength, a single line segment showed the lowest result again, about 27% of the IO-K mean value. The block with blurred contours made up 34%, the contour of a rectangle provided 47%, and the solid block accounted for 51% of the O-K manifestation.

Comparing the two, MBso exceeded Iso strength by more than two times. A similar proportion was established in the experiments with the chromatic patterns B6, B7, and B8: MBso was more than two times greater compared with Iso strength, 123% and 58%. Hence, con-

frontation of two illusions of extent did not seem like an algebraic sum of two misperceptions. Presumably, the stimuli processing dynamics acted on each other in the length matching task and not only the output signals. Processes of spatial extent encoding have been recognized as a dynamic time-evolving phenomenon (Russo and Dellantonio, 1989; VanRullen and Thorpe, 2001). Encoding of the reference and supplementary stimuli when presented simultaneously in the experiments might evolve over time in individual ways because of the differences of the stimuli in the spatial structure. One of the processes could become more powerful than the other, for example, the development of the supplementary illusion Iso might partially mask the progress of the referential effect, IO-K. Indeed, the dominance of
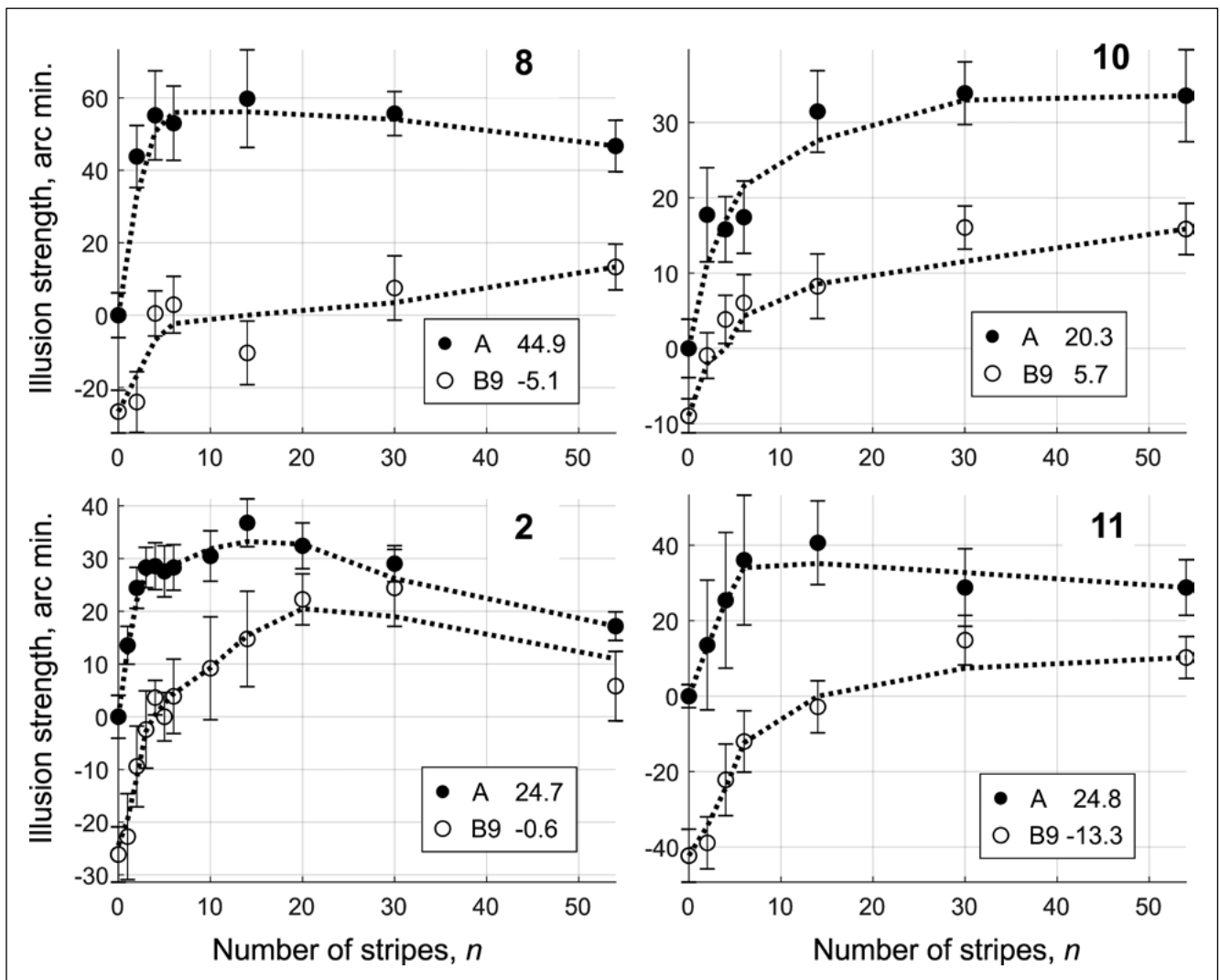

Fig. 8. IO-K as a function of the filling stripe number (dark symbols) and illusion balance, Bso at the presence of the supplementary object B9 (bright symbols). Vertical bars, 95% confidence intervals. Dashed lines, smoothing curves. In the frames, the mean values of the curves, arc min. Five vertical stripes were present in the test interval of the combined stimulus B9 shown for subject 2, but four stripes were given for subject 10, and fourteen stripes for subjects 8 and 11.

Table II. IO-K balance coefficient, k_b and Iso strength for the greyscale and color supplementary objects. See the main text for a detailed explanation.

Subjects	Symbols	Supplementary objects						
		Greyscale			Color			
		B1	B5	Average	B6	B7	B8	Average
2	k_b	1.16	1.16	1.16	1.56	1.41	1.56	1.51
	Iso $_{arc\ min}$	18.6	12.6	15.6	28.1	30.8	25.2	28.0
8	k_b	0.78	1.00	0.89	0.75	0.77	0.96	0.83
	Iso $_{arc\ min}$	7.2	6.1	6.6	13.3	19.8	16.5	16.5
10	k_b	1.27	1.21	1.24	1.37	1.40	1.29	1.35
	Iso $_{arc\ min}$	27.8	25.4	26.6	27.3	31.7	27.1	28.7
Average	k_b	1.07	1.12	1.09	1.23	1.19	1.27	1.23
	Iso $_{arc\ min}$	17.9	14.7	16.3	22.9	27.4	22.9	24.4

the supplementary illusion I_{so} over the reference illusion IO-K in a combined stimulus dropped significantly or disappeared completely when two similar spatial structures, two sequences of stripes, were opposing each other. In this case, MBso and the supplementary illusion Iso strength, when measured separately, became less different or about the same. In other words, the I_{O-K} balance approached the algebraic sum of the two misperceptions: Bso ≈ IO-K - Iso. Consequently, the final score of the interaction of two illusory processes depended on the structure of the competing stimuli.

The data on functional interaction of the supplementary objects and O-K stimulus pointed out the object contour and surface importance in the illusion emergence. The contour defined the object's integrity, shape, and spatial positions. Even a horizontal line segment, while connected with the vertical terminal stripes, referred to an integral object being able to balance the stripe sequence illusion and to induce a self-dependent illusion. A line detached from the terminals and pushed aside from the O-K stimulus still became a simpler structure but remained an influential object. The line segments showed a relatively weak effect on stripe sequence overestimation IO-K because they had perceptually small areas, if any. A tangible area, when present, highlighted the object's unity and structure, thus contributed to the object's identification. A composition of four segments forming, for instance, the contour of a rectangle with an empty area inside showed an increased effectiveness. The area filling produced further strengthening of the stimulus's functional effects. The contents of the filling also affected the stimulus impact because it helped in the object's identification, for instance, the surface properties like granularity provided information about the material characteristics of a visual object (Adelson and

Bergen, 1991; Zhang and Tan 2002). A solid filling was less inducible than the optimal number of equidistantly distributed stripes. The greyscale views were less inductive than the colored images perhaps because of lower informativeness (Table II).

Obviously, the object outline, and surface were the complementary and competing factors that laid the ground for the IO-K process. The empty contour and the body with a blurred outline separately evoked illusions of a similar strength, but the contour and filling when combined together into a single stimulus did not provide the sum of the two results.

Livingstone and Hubel (1987; 1988) proposed separate cortical channels for the processing of form, color, movement, and depth of visual stimuli. Biologically realistic models (Marr, 1982; Biederman and Ju, 1988; Grossberg, 1994) incorporated separate processing modules responsible for determining the existence and location of outer edges or contours of objects and for filling in and the surface properties within the boundaries. The separate neural pathways processing the form and surface attributes of visual objects appeared distinguishable in temporal dynamics (Breitmeyer and Tapia, 2011). At subconscious levels (earlier cortical areas), form processing proceeded faster than surface processing, whereas, in contrast, at conscious levels form processing proceeded slower than surface processing. Thus, synthesis of the object shape that started at unconscious levels (Breitmeyer, 2007; VanRullen, 2007) could be completed in the higher cortical areas together with or after the surface processing ending.

Visual awareness of the object form was associated with activity at or beyond the anterior region (area TE) of inferior temporal (IT) cortex (Bar and Biederman, 1999). One would expect the surface filling-out processes also to be completed at or beyond that level (Breit-

meyer and Tapia, 2011). Conscious perceptual report of visual stimuli could be associated with the dorsal prefrontal and parietal areas (Beck et al., 2001; Dehaene et al., 2001; Lumer et al., 1998), which were functionally connected with the surface-processing and form-processing regions.

Although many visual features have been suggested to be important during object recognition (e.g., color and shape), the size of the object was not listed as one of these features. Contrarily, the processing of familiar size was found to be mediated by underlying image properties of the object (Long and Konkle, 2017). Surface properties could be taken into account by subjects to judge object size (Gibson, 1950). Textural statistical properties influenced the size of a visual object (Giora and Gori, 2010). It appeared as though the object's perception provided a sense of scale. Neither a distance nor a size category could be invented in an empty vision space with an absence of objects. Of course, a familiar size processing started at subconscious levels similarly to the shape encoding around 100 ms after stimulus onset (Fisher, 2018). The perceived size differences, like those produced by visual illusions, originated from early visual areas (Weidner and Fink, 2007). But judgments on the perceived distance or size could likely be made after the entire shape was recognized: the brain activity in response to familiar compared objects was observable within the first 200 ms in monkeys (Peissig et al., 2007) and also in human infants (Carver et al., 2006); behavioral reaction times for unfamiliar image presentations were about 250 ms in monkeys (Fabre-Thorpe et al., 1998) and about 350 ms in humans (Thorpe et al., 1996; Rousselet et al., 2002); the simple reaction time to familiar small *vs.* big objects was about 330 ms in adults and about 530 ms in children (Fisher, 2018); and the time required to process different semantic sizes was longer – up to 700 ms duration of stimulus presentation was needed for full-scaled O-K illusion development in psychophysical experiments with the length matching procedure (Bertulis et al., 2014).

We considered the contour and surface encoding the prerequisite of the perceptual expansion of distance and size. The computational modelling of information integration (Erdfelder and Faul, 1994) predicted the IO-K behavior for different variations of stimulus spatial parameters rather well. Supporting the modelling, the information integration processes could be explained in terms of the spatial-temporal summation of excitations representing the object outline and surface attributes at the subconscious levels in the vision modality. Summations in receptive fields as the Gaussian components produced positional shifts in the excitation profiles, inevitably. Next, the visual spatial attention amplified neural responses. Attention fields as Gaussians attracted receptive fields throughout visual cortex (Klein et al., 2014) resulting in biases of the positions of stimuli outside the focus of attention away from the attended locations (Suzuki and Cavanagh, 1997; Shim and Cavanagh, 2005; Cutrone et al., 2018). Cognitively, visual awareness of an object provided a sense of continuity with locations of the structural parts and relationships of distances among them. Perceptually, the object-specific positional shifts turned into misestimating of the perceived distances, which could not be weighty in an ordinary everyday practice but could be easily fixed in situations with a length comparison task.

In the present study, subjects were asked to match the referential and supplementary objects in one aspect, the length. Any stimulus triggered a false sensation of extension and balanced each other's one-dimensional effect while being presented side by side. The result was the same as in the IO-K measurements, in which a distance expanded only perpendicularly to the stripes, and as in the Helmholtz (1867) illusion in which a square enlarged orthogonally to the filling lines' orientation. However, the two-dimensional effect of the area expansion has become known and studied after Oppel's report. A surface containing a random distribution of simple geometric figures like squares, triangles, and circles looked larger than a uniform surface (Verrillo and Graeff, 1970). The textured rectangular patterns compared to uniform greyscale areas of the same shape and size also appeared larger (Giora and Gori, 2010). In the latter experiments, subjects were clearly instructed to gauge the square areas considering both their dimensions. A non-monotonic function similar to the O-K illusion dependence on the number of the filling stripes was established. The enlargement effect of the filled area increased with an increase of the number of the white and the black sub squares (with decrease of their size and, consequently, with increase of the fundamental spatial frequency) until it reached the maximum and decreased afterwards with further growth of the filling units' number. The experimental curves for a check board and random arrangements run in parallel, but the check board patterns provided a higher illusory effect. The curves for the random board were significantly below those of the strictly organized patterns in an analogous way to IO-K, which was also lower for the filling elements of inhomogeneous distribution than for those of equidistant spacing (Nogouchi et al., 1990). Also, the panels with sharp contours of the sub squares provided a higher illusory effect than the textures composed by the blurred border units (Giora and Gori, 2010) like the stimuli B2 and B3 in the experiments of the present study.

Based on the provided experimental and literature data, we assume that the O-K expansion effect, also named the illusion of interrupted extent (Sanford, 1903), the filled space illusion (Lewis, 1912), or the filled/empty illusion (Wackerman and Kastner, 2010), is to be acknowledged as an individual case of a common sensory phenomenon, the object's size illusion. It can be stated that any visual object appears larger than its occupied area. The object boundaries produce an expansion of both the perceived length and height, and, consequently, size (Makovski, 2017). Disconnected or missing boundaries produce greater illusions than continuous and fully closed ones. The object's size illusion is an effortless, subconscious, multilevel, and time-consuming event. It is a slowly growing process in comparison to the famous illusion of extent, the Müller-Lyer effect.

The striking features of the O-K stimulus are the illusory contours and the equidistantly dispersed filling. The stimulus illusory outline is as effective as a vivid edge in the illusion's genesis, but the subdivided area is a functionally prominent attribute due to alternative reasons: attractive/repulsion interactions between the filling elements (Ericson, 1970), spatial frequency extraction (Bulatov et al., 1997), terminal repulsion of the penultimate element (Mikellidou 2012), spatial anisotropy of the expansion effect (Wackermann et al., 2012), and context-evoked excitation augmentation (Bulatov et al., 2017; 2019).

The starting neural mechanism of the object size illusion, including the O-K effect, is associated with the object-specific integrating procedures at the cortical subconscious levels. It may indeed be that the positional shifts in the earlier excitation profiles are those irreparable inducers of the perceptual expansions of the visual objects' size. The sensory manifestation of the O-K event is to be attributed to the high-level visual processes, such as the object and scenery cognition and attention allocation on the positions selected for distance and size comparisons in accordance to the given task.

CONCLUSIONS

The Oppel-Kundt illusion decrease was examined by presence of supplementary objects in the undistorted stimulus part. In parallel, a self-dependent illusion of extent produced by a supplementary object and an empty space interval exposed simultaneously next to each other were examined.

The score of interaction of the two illusory processes depended on the spatial structure of the competing stimuli. The supplementary objects could be lined up according to their inductivity representing contribu-

tion of their outline and surface attributes to processes of the misperception of extent.

The line segments demonstrated the weakest illusory and balancing effects. The contour of a rectangle with an empty area inside showed an increased efficiency. The filled in area produced further strengthening of the stimulus's functional impact. The mode of filling mattered to the object's capacity. A solid fill was less inducible than the optimal number of equidistantly distributed stripes. The greyscale fillings were less inductive than the color patterns.

The experiment's findings supported an explanation of the Oppel-Kundt illusion in terms of the spatial-temporal summation of excitations representing the object outline and surface attributes at the subconscious cortical levels in the visual system. Also, the data provided an assumption that any visual object appeared larger than its occupied area, and the Oppel-Kundt expansion effect exhibited an individual case of a common sensory phenomenon, the objects' size illusion.

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