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MOD Natural Neutrosophic Subset Topological Spaces and Kakutani's Theorem

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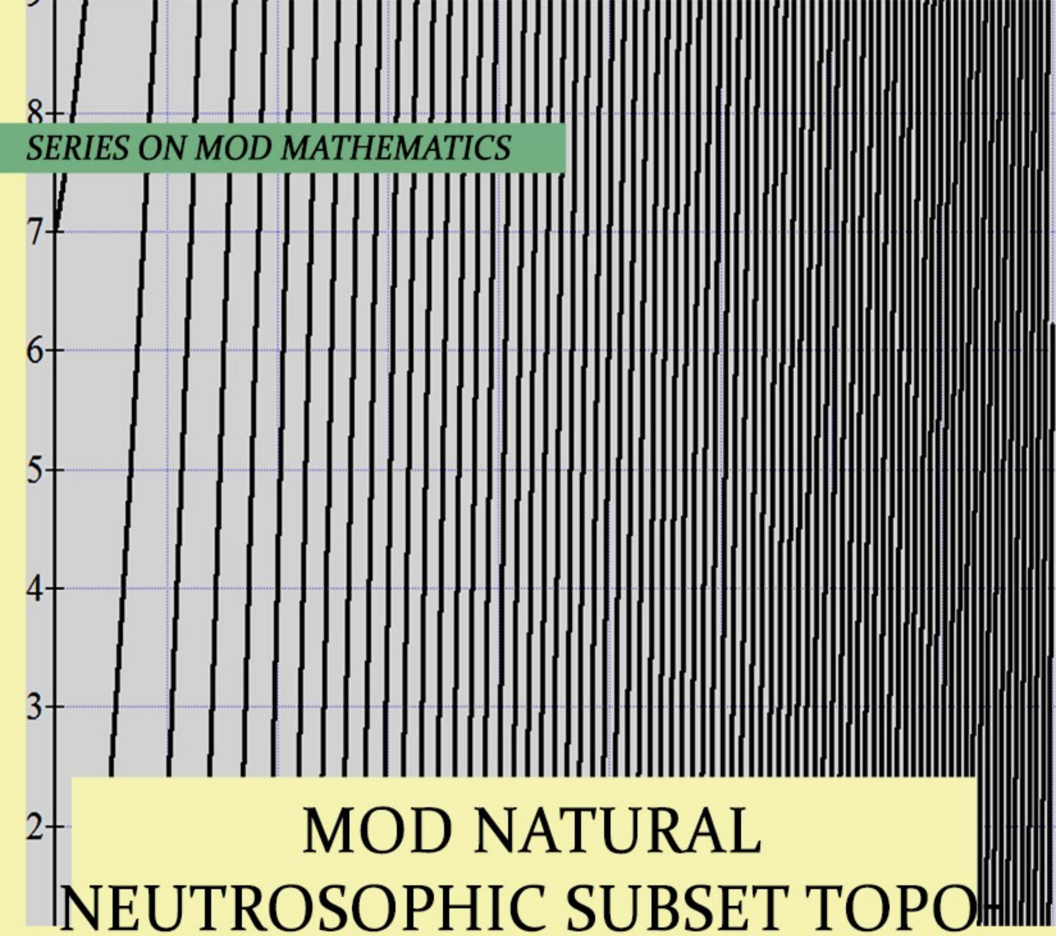

SERIES ON MOD MATHEMATICS

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CONTENTS

PREFACE

In this book authors for the first time develop the notion of MOD natural neutrosophic subset special type of topological spaces using MOD natural neutrosophic dual numbers or MOD natural neutrosophic finite complex number or MOD natural neutrosophic-neutrosophic numbers and so on to build their respective MOD semigroups.

Later they extend this concept to MOD interval subset semigroups and MOD interval neutrosophic subset semigroups. Using these MOD interval semigroups and MOD interval natural neutrosophic subset semigroups special type of subset topological spaces are built.

Further using these MOD subsets we build MOD interval subset matrix semigroups and MOD interval subset matrix special type of matrix topological spaces.

Likewise using MOD interval natural neutrosophic subsets matrices semigroups we can build MOD interval natural neutrosophic matrix subset special type of topological spaces.

We also do build MOD subset coefficient polynomial special type of topological spaces.

The final chapter mainly proposes several open conjectures about the validity of the Kakutani's fixed point theorem for all MOD special type of subset topological spaces.

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Chapter One

BASIC CONCEPTS

In this chapter we just mention how the MOD subset special type of topological spaces, MOD natural neutrosophic special type of topological spaces, MOD interval subset special type of topological spaces and MOD interval natural neutrosophic subset special type of topological spaces are built using Z_n, $\langle Z_n \cup I\rangle$, $\langle Z_n \cup g\rangle$, $\langle Z_n \cap h\rangle$, $\langle Z_n \cup k\rangle$ and $C(Z_n)$, $\langle Z_n \cup g\rangle_I$, Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup k\rangle_I$, $\langle Z_n \cup I\rangle_I$ and $\langle Z_n \cup h\rangle_I$, $[0, n)$ $C([0, n))$, $\langle [0, n) \cup g\rangle$, $\langle [0, n) \cup h\rangle$, $\langle [0, n) \cup I\rangle$ and $\langle [0, n) \cup k\rangle$ and ${}^I[0, n)$, $\langle [0, n) \cup g\rangle_I$, $\langle [0, n) \cup h\rangle_I$, $\langle [0, n) \cup k\rangle_I$, $C^I([0, n))$ and $\langle [0, n) \cup I\rangle_I$ respectively are used in building these MOD special type of topological spaces.

We can built MOD topological spaces using $\cup$ and $\cap$ which can be of finite or infinite order. The validity of the Kakutani theorem remains a open problem in these spaces also.

On these MOD interval subset special type of topological spaces using $P([0, n))$ or $P({}^I[0, n))$ or $P(\langle [0, n) \cup g\rangle)$ or $P(\langle [0, n) \cup g\rangle_I)$ or $P(\langle [0, n) \cup I\rangle_I)$ or $P(\langle [0, n) \cup I\rangle)$ or $P(\langle [0, n) \cup k\rangle_I)$ or $P(\langle [0, n) \cup h\rangle)$ or $P(\langle [0, n) \cup h\rangle_I)$ a study of Kakutani's fixed point theorem is suggested.

For in many cases one is forced to accept the existence of fixed points however in some cases the other properties of Kakutani's theorem many not be true in general.

In fact the study or verification of Kakutani's theorem in case of these MOD subset special type of topological spaces constructed in this book happens to be a challenging problem for these spaces not only use $\cup$ or $\cap$ which is used in the classical topological spaces but these MOD subset special type of topological spaces makes use of the operations + and $\times$, + and $\cup$, + and $\cap$, $\times$ and $\cup$, $\times$ and $\cap$ apart from $\cup$ and $\cap$. So several structural changes occur for all operations but do not in general yield idempotents.

What we are satisfied with is the closure axiom and the basic definition for the collection of subsets to be topological spaces, which is why we call these topological spaces as MOD subset special type of topological spaces.

Finally we do not always use subsets from reals or complex or rational numbers they can be matrix subsets or matrix with subset entries or it can be polynomial subsets or polynomials with subset coefficients/ entries from the above said test. However under the two operations taken from the set of operation $\{+, \cup, \cap, \times\}$ we see they are MOD subset special type of topological spaces.

If Kakutani's theorem or a modified form of Kakutani's theorem is proved for the MOD subset special type of topological spaces for certain they will hold good in case of MOD matrices subset special type of topological spaces and MOD polynomial subset special type of topological spaces for when we say subsets it can be anything need not always take values from R or C or Q and further the operations need not be $\cup$ or $\cap$ it can be + or $\times$ also or a combination for any two from the four operations. So such study is interesting and innovative, throw many challenges to researchers in topology.

Chapter Two

MOD SUBSET SEMIGROUPS UNDER $\cup$ AND $\cap$

In this chapter for the first time we introduce the operations of '$\cup$' the union or '$\cap$' the intersection of the MOD subsets of $S(Z_n)$ (or $S(C(Z_n))$ or $S(Z_n^I)$ or $S(C^I(Z_n))$ or $S(\langle Z_n \cup g\rangle)$ or $S(\langle Z_n \cup g\rangle_I)$ or $S(\langle Z_n \cup h\rangle)$ or $S(\langle Z_n \cup h\rangle_I)$ or $S(\langle Z_n \cup k\rangle)$ or $S(\langle Z_n \cup k\rangle_I)$ or $S(\langle Z_n \cup I\rangle)$or $S(\langle Z_n \cup I\rangle_I)$).

We will first illustrate this situation by examples. However if $\cap$ is to be a closed operation on $S(Z_n)$, we have to induct the element ϕ, for if A, B $\in S(Z_n)$ then if $A \cap B = \phi$ then to have $\{S(Z_n, \cap\}$ to be a semigroup so we have to induct ϕ with $S(Z_n)$ then only $\{S(Z_n) \cup \phi, \cap\}$ is a semigroup.

We will illustrate this by examples.

Example 2.1. $G = \{S(Z_4), \cup\}$ is defined as the MOD semigroup. Let $A = \{0, 2, 3\}$ and $B = \{1, 2\} \in G$, $A \cup B = \{0, 2, 3\} \cup \{1, 2\} = \{0, 1, 2, 3\} = Z_4$.

Let $X = \{0\}$ and $Y = \{1, 3\} \in G$, $X \cup Y = \{0\} \cup \{1, 3\} = \{0, 1, 3\}$.

Thus G is a semigroup or a semilattice.

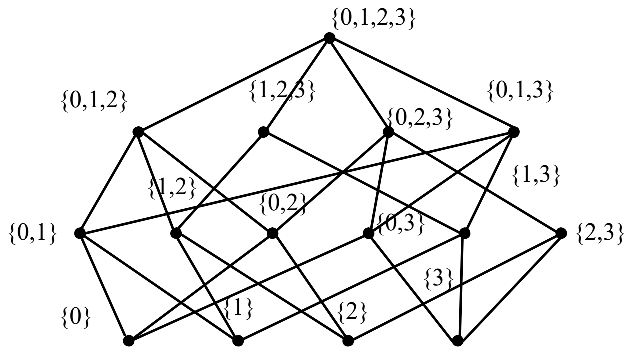

Figure 2.1

So G is a semilattice given in figure 2.1.

Example 2.2. Let H = {S(Z_3), ∪} be the semigroup.

o(H) = $2^3 - 1$.

H = {{0}, {1}, {2}, {0, 1}, {0, 2},{1, 2}, {1, 2, 3}}.

H is infact a semilattice.

Example 2.3. Let W = {S(Z_{43}), ∪} be the MOD semigroup (or semilattice) of order $2^{43} - 1$.

This has subsemigroups for take A = {3}, then A ∪ A = {3}. Thus every singleton set is a subsemigroup.

Let B = {0, 5} ∈ W; B ∪ B = {0, 5} = B. Hence every element of W is a subsemigroup of order one.

Let A = {0, 2, 5} and B = {6, 9, 8, 14, 42} ∈ W. A ∪ B = {0, 2, 5} ∪ {6, 9, 8, 14, 42} = {0, 2, 5, 6, 8, 9, 14, 42}. Hence P = {A, B, A ∪ B} is a subsemigroup of order three.

Thus every pair of subsets A, B in $S(Z_{43})$ with $A \not\subset B$ or $B \not\subset A$ generates a subsemigroup of order three given by $\{A, B, A \cup B\}$; this is true for every $A, B \in S(Z_{43})$.

Let A = {0, 5, 7, 8, 9, 12, 34} and B = {0, 5, 7, 8, 9, 12, 34, 40, 41, 25, 28, 39} $\in S(Z_{43})$. $A \subseteq B$; so $A \cup B = B$. Thus $M = \{A, B\} \subseteq W$ is a subsemigroup of order two.

Hence one can get subsemigroups (subsemilattices under $\cup$) of any desired order bounded by $2^{43} - 1$.

However if ϕ is not included in $S(Z_{43})$ we will not be in a position to define the operation $\cap$. Thus $S(Z_{43}) \cup \phi = P(Z_{43})$.

We can define $\cap$ and under $\cap$, $P(Z_n)$; $2 \leq n < \infty$ is a semigroup or a semilattice.

We will give one or two examples of them before we make a comparison between $(P(Z_n), \cup)$, $\{P(Z_n), \cap\}$, $\{P(Z_n), +\}$ and $\{P(Z_n), \times\}$ as semigroups.

Example 2.4. Let $S = \{P(Z_{10}), \cap\}$ be the MOD semigroup which is also a semilattice.

We see every element x in S is a subsemigroup of cardinality one.

For if $x \in S$; $x \cap x = x$ hence the claim.

Let A = {3, 0, 2, 5} and B = {4, 6, 8} $\in$ S.

$A \cap B = \phi$, however {A, B} is not a subsemigroup for $\phi \notin \{A, B\}$. But $P = \{\phi, A, B\} \subseteq S$ is a subsemigroup of S of order 3.

Let $A = \phi$ and B = {9, 8, 6, 0} $\in$ S; clearly $A \cap B = \phi$, so {A, B} is a MOD subsemigroup of order two.

Let P = {0, 2, 4, 6, 8} and Q = {4, 5, 6, 9, 0} $\in$ S. P $\cap$ Q = {0, 4, 6} $\in$ S. Only {P, Q, P $\cap$ Q} is a MOD subsemigroup of S.

Let A = {4, 5, 2, 0}, B = {2, 4, 9, 6} and C = {2, 7, 6, 3, 1} $\in$ S. We see M = {A, B, C} is not a subsemigroup of S.

We now find A $\cap$ B = {4, 2}, A $\cap$ C = {2} and B $\cap$ C = {2, 6}. Thus M is not a subsemigroup however (A $\cap$ C) $\cap$ {B $\cap$ C} = {2} = (B $\cap$ C) $\cap$ (A $\cap$ B) and {A $\cap$ B} $\cap$ {A $\cap$ C} = {2}.

Thus we can complete a subset of S which is not a subsemigroup into a subsemigroup [ ].

Here we denote by M^C = {A, B, C, A $\cap$ B, A $\cap$ C, B $\cap$ C, (A $\cap$ B) $\cap$ (A $\cap$ C), (A $\cap$ B) $\cap$ (B $\cap$ C), (A $\cap$ C) $\cap$ (B $\cap$ C), (A $\cap$ C) $\cap$ (B $\cap$ C)} = {{2}, {2,4}, {2,6}, {2, 4, 5, 0}, {2, 4, 6, 9}, {2, 7, 6, 3, 1}}.

We call M^C the MOD subsemigroup completion of the subset M [,].

We see it is always possible to complete any subset into a subsemigroup after a finite number of steps.

In view of all these we have the following result.

Theorem 2.1. *Let $P(Z_n) = \{S \cup \phi\}$ be the power set of Z_n. Let $S = \{P(Z_n), \cap\}$ be the MOD semigroup.*

i) $o(S) = 2^n$.
ii) *S has MOD subsemigroups of all orders t; $1 \leq t < 2^n$.*
iii) *All subsets of S can be completed to get a subsemigroup in a finite number of steps.*

Proof is direct and hence left as exercise to the reader.

Next we show how to describe + and × on $P(Z_n)$, the power set of Z_n by some examples.

Example 2.5. Let $\{P(Z_9), +\} = S$ be the MOD semigroup. We define $\phi + A = A$. Clearly $\phi \not\in A$ for any A. So $0 \in Z_9$ is different from ϕ for 0 can be in some of the subsets of Z_9. Hence by no means we can call ϕ the additive identity.

ϕ is only a notation to show a set is empty. When ϕ is added to A, that is nothing is added to A, nothing happens to A remains as A.

Also $\phi + \phi = \phi$ with this $\{S(Z_9), +\}$ is a semigroup of order 2^9. If $A = \{2, 3, 0, 5\}$ and $B = \{1, 4, 6, 5\} \in S$, ten

$$A \cup B = \{0, 1, 2, 3, 5, 4, 6\} \qquad \text{I}$$

$$A \cap B = \{5\} \qquad \text{II}$$

$$A + B = \{3, 4, 1, 6, 7, 8, 2, 5\} \qquad \text{III}$$

We see all the three equations I, II and III are different. Hence all the MOD semigroups $(S, +)$, $(S, \cup)$ and $(S, \cap)$ are different semigroups with same cardinality.

Infact $\{S, \cup\} = \{P(Z_9), \cup\}$ and $\{S, \cap\} = \{P(Z_9), \cap\}$ are both idempotent semigroups.

Now we proceed onto define on $P(Z_n)$ the product operation we define $A \times \phi = A$ for when A is product with nothing. A remains unaffected.

It is important to keep on record that 0 is the nothing of the real number system likewise ϕ the empty set is the nothing of subsets of any universal set U.

So it is left for one to define $0 \times \phi = 0$ if we work in the subsets of numbers and $\{0\} + \phi = \{0\}$.

In this book we use only this convention. For basically MOD number are from Z_n, Z_n^I or $[0,n)$ or ${}^I[0, n)$; $2 \leq n < \infty$.

Keeping this in mind we give examples of MOD subset semigroups under product on $P(Z_n)$.

Example 2.6. Let $S = \{P(Z_{15}), \times\}$ be the MOD subset semigroup under product.

If $A = \{0, 5, 10, 3, 7\}$ and $B = \{4, 5, 9, 2, 11, 7, 1\} \in S$ to find $A \times B$ and compare it with $A + B$, $A \cap B$ and $A \cup B$.

Now $A \times B = \{0, 5, 10, 3, 7\} \times \{1, 2, 4, 5, 7, 9, 11\}$

$= \{0, 5, 10, 3, 7, 6, 14, 12, 13, 4, 2\}$ I

$A \cap B = \{0, 5, 10, 3, 7\} \cap \{1, 2, 4, 5, 7, 9, 11\}$

$= \{5, 7\}$ II

$A \cup B = \{0, 5, 10, 3, 7\} \cup \{1, 2, 4, 5, 7, 9, 11\}$

$= \{0, 5, 10, 3, 7, 1, 2, 4, 9, 11\}$ III

$A + B = \{0, 5, 10, 3, 7\} + \{1, 2, 4, 5, 7, 9, 11\}$

$= \{1, 2, 4, 5, 7, 9, 11, 6, 12, 14, 0, 8, 10, 3\}$ IV

It is clear the four equations I, II, III and IV are distinct and thus the four MOD semigroups are different in their algebraic structures.

Further it is important to keep on record that for a topological space in general we need only two closed binary operations on the collection of subsets. Also the subsets collection in our case is finite. Thus we can have 6 distinct MOD topological spaces using the power set; $P(Z_n)$; $2 \leq n < \infty$.

We can have

$$\{P(Z_n), \cup, \cap\} = T_o,$$
$$\{P(Z_n), \cup, +\} = T_{+}^{\cup},$$
$$\{P(Z_n), \cup, \times\} = T_{\times}^{\cup},$$
$$\{P(Z_n), \cap, +\} = T_{+}^{\cap},$$
$$(P(Z_n), \cap, \times\} = T_{\times}^{\cap} \text{ and}$$
$$T_{\times}^{+} = \{P(Z_n), +, \times\}$$

which we choose to call as special type of MOD subset topological spaces or MOD subset topological spaces of special type.

All the six topological spaces are distinct further $o(P(Z_n)) = 2^n$ only a finite collection of subsets. We have defined special type of subset topological spaces [13].

When we say they are subsemiring of type I. It is important to note that as we do not need any relation between the operators on a topological spaces as we do not find relation connecting + and × or ∪ and ∩ or + and ∩ or × and ∩ and so on. In case of type II subset semirings subsets are taken from semirings and in case of type I semirings subsets are from a ring.

It is important to note that these MOD special type of topological spaces are of finite cardinality.

Such types of special type of subset topological spaces are discussed elaborately in [13].

We have also given subspaces using matrix subsets.

Hence we proceed to describe MOD natural neutrosophic special type of subset topological spaces using subsets of Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup k\rangle_I$ and $\langle Z_n \cup h\rangle_I$.

On these subsets we can define four operations + or × or ∪ or ∩ and under all these operations these subsets are only semigroups.

We using these subset semigroups build S.

MOD natural neutrosophic special type of subset topological spaces is defined in an analogous way as $S(C(Z_n))$ or $S(\langle Z_n \cup g\rangle)$ or $S(\langle Z_n \cup h\rangle)$ or $S(\langle Z_n \cup k\rangle)$ or $S(\langle Z_n \cup I\rangle)$.

We will illustrate this situation by examples.

Example 2.7. Let S = {collection of all subsets from the MOD natural neutrosophic semigroup of Z_{10}^I, ×} in which usual zero is dominated that is $0 \times a = 0$ for all $a \in Z_{10}^I$.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$.

$\{S', \cap, \times\} = T_\cap^\times$, $T_\cup^\times = \{S', \cup, \times\}$ and $T_o = \{S', \cup, \cap\}$ are the three MOD natural neutrosophic subset special type of topological spaces associated with the semigroup $\{Z_{10}^I, \times\}$.

Let $A = \{0, I_5^{10}, I_4^{10} + 3, I_2^{10} + 1, 2, 8, I_6^{10} + 6\}$ and $B = \{1, I_2^{10}, I_8^{10} + 9, 8, 6, 2\} \in T_o$.

$A \cup B = \{0, I_5^{10}, I_4^{10}+3, I_2^{10}+1, 2, 8, I_6^{10}+6\} \cup \{1, I_2^{10}, I_8^{10}+9, 8, 6, 2\} = \{0, I_5^{10}, I_4^{10}+3, I_2^{10}+1, 2, 8, I_6^{10}+6, 1, I_2^{10}, I_8^{10}+9, 6\}$ I

$A \cap B = \{0, I_5^{10}, I_4^{10}+3, I_2^{10}+1, 2, 8, I_6^{10}+6\} \cap \{1, I_2^{10}, I_8^{10}+9, 8, 6, 2\} = \{8, 2\}$ II

Take the same A, B $\in T_{\cap}^{\times}$.

$A \times B = \{0, I_5^{10}, I_4^{10}+3, I_2^{10}+1, 2, 8, 6+I_6^{10}\} \times \{1, I_2^{10}, I_8^{10}+9, 8, 6, 2\} = \{0, I_5^{10}, I_4^{10}+3, I_2^{10}+1, 2, 8, 6+I_6^{10}, 8+I_6^{10}, I_0^{10}, I_8^{10}+I_2^{10}, I_4^{10}+I_2^{10}, I_2^{10}, 4, 2, I_2^{10}+I_4^{10}+I_8^{10}+7, I_6^{10}+I_2^{10}+I_8^{10}+9, 6, I_4^{10}+4, I_2^{10}+8, 8+I_6^{10}, 8+I_4^{10}, 6+I_2^{10}, I_4^{10}+6, 2+I_2^{10}, 2+I_6^{10}$ III

Thus all the three equations are distinct so these MOD natural neutrosophic special type of subset topological spaces are also distinct.

We see the only common property between these MOD topological subset spaces is that they have same cardinality apart from that all the three are distinct. T_o being the ordinary MOD natural neutrosophic topological space.

It is important to note that T_o is unaffected by any Z_n^I we use, $2 \leq n < \infty$ the only factor being that if n is a prime the only natural neutrosophic element is I_0^n, but in case when n is not a prime we have several elements which are MOD natural neutrosophic elements and they are associated with $T_o, T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ further the product operation also contributes to many other properties.

We give an example in which many MOD natural neutrosophic elements are present.

Example 2.8. Let S = $\{S(Z_{12}^I), \times\}$ be the MOD natural neutrosophic subset semigroup under the MOD natural neutrosophic zero dominated product.

Let $S' = S \cup \{\phi\}$; $T_o = \{S', \cup, \cap\}$, $T_{\cup}^{\times} = \{S', \cup, \times\}$ and $T_{\cap}^{\times} = \{S', \cap, \times\}$ be the three MOD natural neutrosophic special type of subset topological spaces but using the MOD natural neutrosophic under product × subset semigroup, which is MOD natural neutrosophic zero dominated.

Let A = $\{ I_8^{12} + I_6^{12} + 2, 4 + I_0^{12}, 2 + I_6^{12}, 6, 10, 0\}$ and B = $\{ I_0^{12} + I_4^{12} + I_6^{12} + 3, 3 + I_3^{12}, 2 + I_6^{12}, I_6^{12} + I_8^{12}, I_9^{12} + 2, 3, 2, 6, 10, 1\} \in S$.

A ∪ B = $\{ I_8^{12} + I_6^{12} + 2, 4 + I_0^{12}, 2 + I_6^{12}, 6, 10, 0\} \cup \{2, 3, 1, 6, 10, 2 + I_9^{12}, I_6^{12} + I_8^{12}, 2 + I_6^{12}, 3 + I_3^{12}, 3 + I_0^{12} + I_4^{12} + I_6^{12}\} = \{0, 1, 6, 2, 3, 10, 2 + I_8^{12} + I_6^{12}, 4 + I_0^{12}, 2 + I_9^{12}, I_6^{12} + I_8^{12}, 3 + I_3^{12}, 3 + I_0^{12} + I_4^{12} + I_6^{12}\}$ I

A ∩ B = $\{2 + I_6^{12} + I_8^{12}, 4 + I_0^{12}, 2 + I_6^{12}, 6, 10, 0\} \cap \{2, 3, 1, 6, 10, 2 + I_9^{12}, I_6^{12} + I_8^{12}, 2 + I_6^{12}, 3 + I_3^{12}, 3 + I_0^{12} + I_4^{12} + I_6^{12}\} = \{6, 10, 2 + I_6^{12}\}$ II

A × B = $\{2 + I_6^{12} + I_8^{12}, 4 + I_0^{12}, 0, 6, 10, 2 + I_6^{12}\} \times \{1, 2, 3, 6, 10, I_9^{12} + 2, 2 + I_6^{12}, 3 + I_3^{12}, I_6^{12} + I_8^{12}, 3 + I_0^{12} + I_4^{12} + I_6^{12}\} = \{2 + I_6^{12} + I_8^{12}, 4 + I_0^{12}, 0, 10, 2 + I_6^{12}, 4 + I_6^{12} + I_8^{12}, 8 + I_0^{12}, 8, 4 + I_6^{12}, 8 + I_6^{12}, 6 + I_6^{12} + I_8^{12}, I_0^{12}, 6, 6, I_6^{12}, I_6^{12} + I_8^{12}, I_6^{12}, 8 + I_6^{12} + I_8^{12}, 6 + I_0^{12}, 4 + I_9^{12} + I_6^{12} + I_8^{12}, 8 + I_9^{12} + I_0^{12}, I_9^{12}, 8 + I_9^{12}, 4 + I_9^{12} + I_6^{12}, 4 + I_6^{12} + I_0^{12} + I_8^{12}, 8 + I_0^{12} + I_6^{12}, I_6^{12}, 8 + I_6^{12}, 4 + I_6^{12} + I_0^{12}, 6 + I_3^{12} + I_6^{12} + I_8^{12} + I_0^{12}, I_0^{12} + I_3^{12}, 6 + I_3^{12}, I_3^{12}, 6 + I_3^{12}, 6 + I_3^{12} + I_6^{12}, I_6^{12} + I_8^{12} + I_0^{12} + I_4^{12}, I_6^{12} + I_8^{12} + I_0^{12}, I_6^{12} + I_8^{12}, 6 + I_0^{12} + I_8^{12} + I_6^{12} + I_4^{12}, I_0^{12} + I_4^{12} + I_6^{12}, 6 + I_0^{12} + I_4^{12} + I_6^{12}, 6 + I_0^{12} + I_4^{12} + I_6^{12}\}$

III

Clearly I, II and III are distinct hence the three MOD natural neutrosophic special type of subset topological spaces related to the MOD natural neutrosophic subset semigroup under MOD natural neutrosophic zero dominated product are distinct.

Now we find A × B for the above said A and B in which the product is the usual zero dominated product.

A × B = $\{I_8^{12} + I_6^{12} + 2, 4 + I_0^{12}, 2 + I_6^{12}, 6, 10, 0\} \times \{I_0^{12} + I_4^{12} + I_6^{12} + 3, 3 + I_3^{12}, 2 + I_6^{12}, I_6^{12} + I_8^{12}, 2 + I_9^{12}, 2, 3, 6, 10, 1\} = \{I_0^{12} + I_6^{12} + I_0^{12} + I_8^{12} + I_4^{12} + 6, I_0^{12} + I_4^{12} + I_6^{12}, 6 + I_6^{12} + I_0^{12} + I_4^{12}, 6 + I_0^{12} + I_4^{12} + I_6^{12}, 0, 6 + I_8^{12} + I_6^{12} + I_3^{12} + I_0^{12}, I_3^{12} + I_0^{12}, 6 + I_0^{12} + I_3^{12}, 6 + I_3^{12}, 4 + I_6^{12} + I_8^{12} + I_0^{12}, 8 + I_0^{12} + I_6^{12}, 4 + I_0^{12} + I_6^{12}, I_6^{12}, 8 + I_6^{12}, I_0^{12} + I_6^{12} + I_8^{12} + I_0^{12}, I_6^{12} + I_8^{12} + I_0^{12}. I_6^{12} + I_8^{12} + I_0^{12}, I_6^{12} + I_8^{12}, 4 + I_9^{12} + I_6^{12} + I_8^{12} + I_0^{12}, I_9^{12} + I_0^{12} + 8, 4 + I_6^{12} + I_9^{12}, I_9^{12}, 8 + I_9^{12}, 4 + I_8^{12} + I_6^{12}, 8 + I_0^{12}, 4 + I_6^{12}, 8, 6 + I_8^{12} + I_6^{12}, I_0^{12}, 6 + I_6^{12}, 6, 10, I_8^{12} + I_6^{12} + 2, 4 + I_0^{12}, 2 + I_6^{12}, 8 + I_8^{12} + I_6^{12}, 8 + I_6^{12}, 4\}$ IV

It is easily verified III and IV are distinct. Hence the product in which 0 is dominated gives a different value from the product in which MOD natural neutrosophic zero is dominated.

Next we proceed onto express the MOD natural neutrosophic special type of a subset topological spaces using the MOD subset semigroup under + given by $\{S(Z_n), +\}$; $2 \leq n < \infty$ by some examples.

Example 2.9. Let S = {elements from the MOD natural neutrosophic subset semigroup $S(Z_{18}^I), +\}$. Let $S' = \{S \cup \{\phi\}\}$. $T_o = \{S', \cup, \cap\}$, $T_\cup^+ = \{S', \cup, +\}$ and $T_\cap^+ = \{S', \cap, +\}$ be the MOD natural neutrosophic special type of subset topological spaces associated with the MOD natural neutrosophic subset semigroup $\{S(Z_{18}), +\}$.

Let $A = \{5 + I_9^{18}, I_0^{18} + I_3^{18} + 4, 0, 2, 5, 3 + I_{12}^{18}\}$ and $B = \{0, 1, 2, I_0^{18}, 10, 8, I_{10}^{18} + 5, I_6^{18} + I_{16}^{18} + 9\} \in T$.

A + B = {5 + I_9^{18}, $I_0^{18} + I_3^{18}$ + 4, 0, 2, 5, I_{12}^{18} + 3} + {0, 1, 2, 10, 8, I_0^{18}, $I_6^{18} + I_{16}^{18}$+9, I_{10}^{18}+5} = {5 + I_9^{18}, $I_{10}^{18} + I_3^{18}$ + 4, 0, 2, 5, I_{12}^{18} + 3, 7 + I_9^{18}, 6 + $I_0^{18} + I_3^{18}$, 4, 7, 5 + I_{12}^{18}, 15 + I_9^{18}, 14 + $I_0^{18} + I_3^{18}$, 10, 12, 15, 13 + I_{12}^{18}, 13 + I_9^{18}, 12 + $I_0^{18} + I_3^{18}$, 8, 13, 11 + I_{12}^{18}, $I_0^{18} + I_9^{18}$ +5, $I_0^{18} + I_3^{18}$ + 4, I_0^{18}, 2 + I_0^{18}, 5 + I_0^{18}, 5 + I_0^{18}, 3 + $I_0^{18} + I_{12}^{18}$, 14 + $I_9^{18} + I_6^{18} + I_{16}^{18}$, 13 + I_6^{18} + 9, 11 + $I_6^{18} + I_{16}^{18}$, 14 + $I_6^{18} + I_6^{18}$, 12 + I_{12}^{18} + $I_6^{18} + I_{16}^{18}$, 10 + $I_9^{18} + I_{10}^{18}$, 9 + $I_0^{18} + I_3^{18} + I_{10}^{18}$, 5 + I_{10}^{18}, 7 + I_{10}^{18}, 10 + I_{10}^{18}, 8 + $I_{10}^{18} + I_{12}^{18}$} I

A ∩ B = {5 + I_9^{18}, $I_0^{18} + I_3^{18}$ + 4, 0, 2, 5, 3 + I_{12}^{18}} ∩ {0, 1, 2, 10, 8, I_0^{18}, I_{10}^{18}+5} = {0, 2} II

A ∪ B = {5 + I_9^{18}, I_0^{18} + I_3^{18} + 4, 0, 2, 5, 3 + I_{12}^{18}} ∪ {0, 1, 2, 10, 8, I_0^{18}, I_{10}^{18} + 5} = {0, 1, 2, 10, 8, 5, 5 + I_9^{18}, $I_0^{18} + I_3^{18}$ +4. 3 + I_{12}^{18}, I_0^{18}, I_{10}^{18} + 5} III

We see all the three equations I, II and III are distinct so the three spaces T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$ are also distinct.

Example 2.10. Let S = {Elements from the MOD natural neutrosophic subset semigroup {S(Z_{43}^{I}), +}} be the MOD natural neutrosophic special type of subset topological spaces using the MOD natural neutrosophic subset semigroup under +. S′ = {S ∪ {ϕ}. T_o = {S′, ∪, ∩}.

Let A = {I_0^{43} + 8, I_0^{43}, 4, 5, 8, I_0^{43} + 19, 0, 1, 42} and B = {I_0^{43} + 27, I_0^{43} + 10, 10, 12, 1, 5, 6, 40, I_0^{43} + 2} ∈ T_o.

A ∪ B = {8 + I_0^{43}, I_0^{43}, 4, 5, 8, 0, 1, 42, 19 + I_0^{43}} + {I_0^{43} + 27, I_0^{43} + 10, 10, 12, 1, 5, 6, 40, I_0^{43} + 2} = {8 + I_0^{43}, I_0^{43}, 4, 5, 8, 0, 1, 42, 19 + I_0^{43}, 27 + I_0^{43}, I_0^{43} +10, 10, 12, 5, 6, 40, I_0^{43} + 2} I

$A \cap B = \{8 + I_0^{43}, I_0^{43}, 4, 5, 8, 0, 1, 42, 19 + I_0^{43}\} \cap \{I_0^{43} + 27, 10 + I_0^{43}, 10, 12, 1, 5, 6, 40, I_0^{43} + 2\} = \{1\}$ II

Let A, B $\in T_{\cup}^{+}$ (or $T_{\cap}^{+}$).

$A + B = \{8 + I_0^{43}, I_0^{43}, 4, 5, 8, 0, 1, 42, 19 + I_0^{43}\} + \{27 + I_0^{43}, 10 + I_0^{43}, 10, 12, 1, 5, 6, 40, 2 + I_0^{43}\}$

$= \{35 + I_0^{43}, I_0^{43} + 27, 31 + I_0^{43}, 32 + I_0^{43}, 39, 40 + I_0^{43}, 2 + I_0^{43}, 28 + I_0^{43}, 26 + I_0^{43}, 3 + I_0^{43}, 18 + I_0^{43}, 21 + I_0^{43}, 10 + I_0^{43}, 14 + I_0^{43}, 15 + I_0^{43}, 7, 2, 11 + I_0^{43}, 9 + I_0^{43}, 29 + I_0^{43}, 40, 16 + I_0^{43}, 7 + I_0^{43}, 2 + I_0^{43}, 6 + I_0^{43}, 14, 15, 18, 10, 11, 24 + I_0^{43}, 1, 9, 29 + I_0^{43}, 5, 6, 25 + I_0^{43}, 20 + I_0^{43}, 12 + I_0^{43}, 16, 17, 20, 12, 13, 14, 41, 4, 11, 31 + I_0^{43}, 9 + I_0^{43}, 1 + I_0^{43}, 5, 6, 9, 5 + I_0^{43}, 1, 2, 0, 20 + I_0^{43}, 13 + I_0^{43}, 5 + I_0^{43}\}$ III

Clearly I, II and III are distinct, hence the three MOD natural neutrosophic special type of subset topological spaces associated with the MOD natural neutrosophic subset semigroup $\{S(Z_{43}^{I}), +\}$ under +, viz; T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$ are distinct.

Thus if $S = \{S(Z_n^I), +, \cup, \cap, \times\}$ be a MOD natural neutrosophic subset under operations $\cup$ or $\cap$ or + or $\times$.

Then associated with $S' = S \cup \{\phi\}$ we get the following MOD natural neutrosophic special type of subset topological spaces of all types using the MOD natural neutrosophic subset semigroup on $\{S, \cup\}$ or $\{S, \cap\}$ or $\{S, +\}$ or $\{S, \times\}$.

Let $T_o = \{S', \cup, \cap\}$, $T_{\cup}^{+} = \{S', +, \cup\}$, $T_{\cup}^{\times} = \{S', \times, \cup\}$, $T_{\cap}^{+} = \{S', +, \cap\}$, $T_{\cap}^{\times} = \{S', \times, \cap\}$ and $T_{\times}^{+} = \{S', +, \times\}$ be the six distinct MOD natural neutrosophic special type of subset topological spaces associated with semigroups under the operations + or × or ∪ or ∩ respectively.

We will illustrate this by some examples.

Example 2.11. Let S = {S(Z_{15}^{I}), +} (or {S(Z_{15}^{I}), ×} or {S(Z_{15}^{I}), ∩} or {S(Z_{15}^{I}), ∪}) be the MOD natural neutrosophic subset semigroups under + or × or ∩ or ∪ respectively. Let S′ = {S ∪ {ϕ}}; T_o = {S′, ∪, ∩}, $T_{\cup}^{+}$ = {S′, ∪, +}, $T_{\cap}^{+}$ = {S′, ∩, +}, $T_{\cup}^{\times}$ = {S′, ∪, ×}, $T_{\cap}^{\times}$ = {S′, ∩, ×} and $T_{\times}^{+}$ = {S′, +, ×} be the six MOD natural neutrosophic special type of subset topological spaces associated with the MOD natural neutrosophic subset semigroup using S(Z_{15}^{I}).

We show all the four operations are distinct hence the related 6 topological spaces are also distinct.

Let A = {I_0^{15} + I_3^{15} + 5, 2, 10, 3, I_6^{15} + 8, I_{10}^{15} + I_9^{15}, 11} and B = {I_5^{15} + 4, 6, 3, 9, 12, 0, 2, 10, I_6^{15} + 8, I_3^{15}, I_5^{15}} ∈ S′.

A ∪ B ={I_0^{15} + I_3^{15} + 5, 2, 10, 3, 8, I_6^{15}, I_{10}^{15} + I_9^{15}, 11} ∪ {4 + I_5^{15}, 6, 3, 9, 12, 0, 2, 10, 8 + I_6^{15}, I_3^{15}, I_5^{15}}

= {I_0^{15} + I_3^{15} + 5, 2, 3, 10, 8 + I_6^{15}, 11, I_{10}^{15} + I_9^{15}, 4 + I_5^{15}, 6, 9, 12, 0, I_3^{15}, I_5^{15}} I

Next we find A ∩ B = {I_0^{15} + I_3^{15} + 5, 2, 10, 3, 8 + I_6^{15}, I_{10}^{15} + I_9^{15}, 11} ∩ {4 + I_5^{15}, 6, 3, 9, 12, 0, 2, 10, 8 + I_6^{15}, I_3^{15}, I_5^{15}} = {2, 3, 8 + I_6^{15}} II

A + B = {I_0^{15} + I_3^{15} + 5, 2, 10, 3, 8 + I_6^{15}, I_{10}^{15} + I_9^{15}, 11} + {4 + I_5^{15}, 6, 3, 9, 10, 12, 2, 10, 8 + I_6^{15}, I_3^{15}, I_5^{15}}

$= \{9 + I_0^{15} + I_3^{15} + I_5^{15}, 6 + I_5^{15}, 14 + I_5^{15}, 7 + I_5^{15}, 3 + I_5^{15}, 12 + I_5^{15} + I_6^{15}, 4 + I_{10}^{15} + I_9^{15} + I_5^{15}, 15 + I_5^{15}, 10 + I_5^{15}\ 11 + I_0^{15} + I_3^{15}\ 8, 1, 9, 14 + I_6^{15}, 6 + I_{10}^{15} + I_9^{15}, 2 + I_5^{15}, 2, 8 + I_0^{15} + I_3^{15}, 5, 13, 6, 11 + I_6^{15}, I_{10}^{15} + I_6^{15} + 11, 8 + I_5^{15} + I_6^{15}, 14 + I_0^{15} + I_3^{15}, 11, 4, 12, 2 + I_6^{15}, 9 + I_{10}^{15} + I_9^{15}, 8 + I_6^{15} + I_3^{15}, 5, 2, 10, 3, 8 + I_6^{15}, I_{10}^{15} + I_9^{15}, 11, 5 + I_{10}^{15} + I_3^{15}, 11 + I_5^{15}, 2 + I_0^{15} + I_3^{15}, 14, 7, 0, 5 + I_6^{15}, 12 + I_{10}^{15} + I_9^{15}, I_{10}^{15} + I_9^{15} + I_5^{15}, 8, 7 + I_0^{15} + I_3^{15}, 4, 10 + I_6^{15}, I_{10}^{15} + I_9^{15} + 2, 8 + I_5^{15} + I_6^{15}, 13, I_0^{15} + I_3^{15}, 13 + I_6^{15}, 10 + I_{10}^{15} + I_9^{15}, 1 + I_6^{15}, 6, 13 + I_0^{15} + I_3^{15}, 10 + I_6^{15}, 3 + I_6^{15}, 11 + I_6^{15}\}$ III

Now we find

$A \times B = \{I_0^{15} + I_3^{15} + 5, 2, 10, 3, 8 + I_6^{15}, I_{10}^{15} + I_9^{15}, 11\} \times \{4 + I_5^{15}, 6, 3, 9, 12, 0, 2, 10, 8 + I_6^{15}, I_3^{15}, I_5^{15}\}$

$= \{5 + I_5^{15} + I_0^{15} + I_3^{15}, 8 + I_5^{15}, 5 + I_5^{15}, 10 + I_5^{15}, 12 + I_5^{15}, 2 + I_6^{15} + I_5^{15} + I_0^{15}, 14 + I_5^{15}, I_{10}^{15} + I_9^{15} + I_5^{15} + I_0^{15}, I_0^{15} + I_3^{15}, 12, 3, 0, 3 + I_6^{15}, I_{10}^{15} + I_9^{15}, 6, I_0^{15} + I_3^{15}, 9, 9 + I_6^{15}, 3, 12, 12 + I_6^{15}, 6, 6 + I_6^{15}, 10 + I_0^{15} + I_3^{15}, 4, 5, 1 + I_6^{15}, 7, 5 + I_6^{15}, 10, 5 + I_0^{15} + I_3^{15}, 10 + I_0^{15} + I_3^{15} + I_6^{15}, 1 + I_6^{15}, 5 + I_6^{15}, 9 + I_6^{15}, 4 + I_6^{15}, I_{10}^{15} + I_9^{15} + I_0^{15}, 13 + I_6^{15}, I_{10}^{15} + I_9^{15} + I_3^{15}, I_3^{15}, I_5^{15} + I_0^{15}, I_0^{15} + I_{12}^{15}, I_5^{15}, I_5^{15} + I_0^{15}\}$ IV

Clearly all the four equations are distinct and hence all the six MOD topological spaces are distinct.

Next we proceed onto define the substructures like MOD topological subset subsemigroups, MOD topological strong subset ideals and MOD topological subset ideals.

Definition 2.1. *Let $\{S', \cup, \cap\}$(where $S' = (S(Z_n^I) \cup \{\phi\})$ (or $\{S', \cup, +\}$ or $\{S' \cup, \times\}$ or $\{S', \cap, +\}$ or $\{S', \cap, \times\}$) be the MOD special type of topological subset spaces.*

A non empty subset $\{P, \times\}$ $(P \subset S')$ (or $\{P, +\}$ or $\{P, \cap\}$ or $\{P, \cup\}$) is defined as a MOD subset topological subsemigroup space if $\{P, \times\}$, $\{P, +\}$, $\{P, \cap\}$ and $\{P, \cup\}$ are subsemigroups.

We will first illustrate this situation by some examples.

Example 2.12. Let S = $\{\{S(Z^{I}_{20}), X\}$ or $\cup$ or $\cap$ or $+\}$ be the MOD natural neutrosophic subset semigroup under $\times$ {or $\cup$ or $\cap$ or +). Let $S' = S \cup \{\phi\}$ and $T_o = \{S', \cup, \cap\}$, $T^{+}_{\cup} = \{S', +, \cup\}$, $T^{+}_{\cap} = \{S', +, \cap\}$, $T^{\times}_{\cup} = \{S', \times, \cup\}$, $T^{\times}_{\cap} = \{S', \times, \cap\}$ and $T^{+}_{\times} = \{S', +, \times\}$ be the 6 MOD natural neutrosophic special type of subset topological spaces associated with any two of the MOD natural neutrosophic subset semigroups.

Consider $P_o = \{S(Z_{20}) \cup \{\phi\}, \cap, \cup\} \subseteq T_o$ is a MOD subset natural neutrosophic special type of topological subspace of T_o. This is a very well known fact.

But $M = \{\phi, \{0, 5, 6, 8, 9\}, \{4, 19, 18, 1, I^{20}_{5}\}, \cap\} \subseteq T_o$ is a MOD natural neutrosophic special type of subset topological space subsemigroup under $\cap$.

Clearly M is not a MOD natural neutrosophic special type of subset topological space subsemigroup under $\cup$ or $\times$ or +.

Let $L = \{A = \{0, I^{20}_{4} + 5, 10, I^{20}_{2} + I^{20}_{5} + I^{20}_{15} + 12, 14, 16, 3, 19\}, B = \{4, I^{20}_{6} + 1, I^{20}_{18} + I^{20}_{8} + I^{20}_{10} + 15, 14, I^{20}_{0} + 19\}, C = \{0, I^{20}_{4} + 5, 10, I^{20}_{2} + I^{20}_{5} + I^{20}_{15} + 12, 1, 3, 16, 19, 4, I^{20}_{6} + 1, I^{20}_{18} + I^{20}_{10} + 15, 14, I^{20}_{0} + 19\}, \cup\} \subseteq T_o$ be a MOD natural neutrosophic special type of subset topological space subsemigroup under $\cup$ and not a MOD natural neutrosophic special type of subset topological space subsemigroup under $\cap$ or + or $\times$.

Consider B = {A = { I_0^{20}, 5, 6, 16, 0, I_6^{20}, I_{16}^{20}}, D = { I_5^{20}, 6, 16, I_0^{20}}, C = { I_5^{20}, 10, I_{10}^{20}, 0 I_0^{20}, 16, 6, I_6^{20}, I_{16}^{20}}} $\subseteq$ $T_{\cup}^{\times}$ is a MOD natural neutrosophic special type of subset topological subsemigroup under ×, clearly B is not a MOD natural neutrosophic special type subset topological space subsemigroup under + or ∪ or ∩.

Thus these MOD natural neutrosophic special type of subset topological spaces subsemigroup under each of the operations.

Now having seen examples of MOD natural neutrosophic special type of subset topological space subsemigroups we now proceed onto define MOD natural neutrosophic special type of subset topological space strong subsemigroup in the following.

Definition 2.2. *Let S = {S(Z_n^I)} be the MOD natural neutrosophic subset collection. Let S′ = S ∪ {φ}, then {S′, ∪}, (or {S′, ∩} or {S′, +} or {S′, ×}) are MOD natural neutrosophic subset semigroups.*

T_o = {S′, ∪, ∩}, $T_{\cup}^{\times}$ = {S′, ∪, ×}, $T_{\cup}^{+}$ = {S′, ∪, +}, $T_{\cap}^{+}$ = {S′, ∩, +}, $T_{\cap}^{\times}$ = {S′, ×, ∩} and $T_{\times}^{+}$ = {S′, +, ×} be the MOD natural neutrosophic special type of subset topological spaces P ⊆ S′ be a MOD natural neutrosophic special type of subset topological space subsemigroup under +, ×, ∩ and ∪ then we define P to be the MOD natural neutrosophic special type of subset topological strong subsemigroup.

We will give first example of this situation.

Example 2.13. Let S = {S(Z_{12}^I) be the MOD natural neutrosophic subset collection.

Let $S' = S \cup \{\phi\}$ and T_o, $T_\times^+$, $T_\cup^+$, $T_\cap^+$, $T_\cup^\times$ and $T_\cap^\times$ be the MOD natural neutrosophic special type of subset topological spaces.

Let $B = \{S(Z_{12}) \cup \{\phi\}\}$ = {collection of al subsets of Z_{12} with $\phi\} = P(Z_{12}) \subseteq S'$.

Clearly B is a MOD natural neutrosophic special type of subset topological strong subsemigroup of T_o, $T_\times^+$, $T_\cup^+$, $T_\cap^+$, $T_\cup^\times$ and $T_\cap^\times$.

Let $W = \{A = \{6 + I_0^{12}, 6, I_6^{12}, I_0^{12}, I_6^{12} + I_0^{12} + 6, I_6^{12} + 6, 0, I_6^{12} + I_0^{12}\}, \{0, I_0^{12}\} = C, B = \{0, I_0^{12}\ I_6^{12}, I_6^{12} + I_0^{12}\}\} \subseteq S'$.

Clearly $W + W = W$, $W \times W = W \cap W = W$. $W \cup W = W$.

So W is a MOD natural neutrosophic subset special type of topological subspaces of T_o, $T_\times^+$, $T_\cup^+$, $T_\cup^\times$, $T_\cap^+$ and $T_\cap^\times$.

W is clearly MOD natural neutrosophic special type of topological spaces strong subsemigroups under $+$, $\times$, $\cup$ and $\cap$.

In view of all these we have the following theorem.

Theorem 2.2. *Let $S = \{S(Z_n^I)\ ;\ 2 \leq n < \infty\}$ be the collection of MOD natural neutrosophic subsets of Z_n^I. Let $S' = S \cup \{\phi\}$. $T_o = \{S', \cup, \cap\}$, $T_\times^+ = \{S', +, \times\}$, $T_\cup^+ = \{S', \cup, +\}$, $T_\cap^+ = \{S', \cap, +\}$, $T_\cup^\times = \{S', \cup, \times\}$ and $T_\cap^\times = \{S', \cap, \times\}$ be the MOD natural neutrosophic special type of subset topological spaces.*

$W \subseteq S'$ is a MOD natural neutrosophic special type of subset topological space strong subsemigroup if and only if W is

a MOD natural neutrosophic special type of subset topological subspaces of T_o, $T_{\times}^{+}$, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$.

Proof is direct and hence left as an exercise to the reader.

However we supply an hint to the proof of the theorem.

If $W \subseteq S'$ is a MOD natural neutrosophic special type ofstrong topological space strong subsemigroup then by definition we know {W, +}, {W, ×}, {W, ∪} and {W, ∩} are all MOD natural neutrosophic subset special topological space subsemigroups.

So {W, ∪, ∩}, {W, +, ×}, {W, +, ∪}, {W, ×, ∪}, {W, ∩, +} and {W, ∩, ×} are all MOD natural neutrosophic special type of topological subset subspaces of T_o, $T_{\times}^{+}$, $T_{\cup}^{+}$, $T_{\cap}^{\times}$, $T_{\cap}^{+}$ and $T_{\cup}^{\times}$ respectively. The converse follows from the similar argument.

Next we proceed onto describe and define MOD natural neutrosophic special type of ¾ strong topological spaces.

Definition 2.3. *Let $S = \{S(Z_n^I)\}$ be the MOD natural neutrosophic subset collection. $S' = S \cup \{\phi\}$. $T_o = \{S', \cup, \cap\}$, $T_{\cup}^{+} = \{S', \cup, +\}$, $T_{\cap}^{+} = \{S', \cap, +\}$, $T_{\cup}^{\times} = \{S', \cup, \times\}$, $T_{\cap}^{\times} = \{S', \cap, \times\}$ and $T_{\times}^{+} = \{S' +, \times\}$ be the six distinct MOD natural neutrosophic special type of subset topological spaces.*

Let $P \subseteq S'$, if P is a MOD natural neutrosophic subset subsemigroup under any three of the operations {+, ∪ and ∩} or {×, ∪ and ∩} or {+, × and ∪} or {×, + and ∩,. then we define P to be the MOD natural neutrosophic special type of ¾ strong subsemigroups subset topological space.

We will illustrate this situation by some examples.

Example 2.14. Let S = S(Z_{24}^{I}) be the MOD natural neutrosophic subset. Let S′ = S ∪ {ϕ} be the set with empty set. T_o = {S′, ∪, ∩}, $T_{\cup}^{+}$ = {S′, +, ∪}, $T_{\cap}^{+}$ = {S′,+, ∩}, $T_{\cup}^{\times}$ = {S′, ×, ∪}, $T_{\cap}^{\times}$ = {S′, ×, ∩} and $T_{\times}^{+}$ = {S′, ×, +} be the MOD natural neutrosophic subset special type of topological spaces related with the MOD natural neutrosophic subset semigroups {S′, +}, {S′, ∪}, {S′, ∩} and {S′, ×}.

Let W = {{$I_0^{24} + I_{12}^{24} + 12$, I_0^{24}, 0}, {$I_0^{24} + I_{12}^{24}$, I_0^{24}, 0}, {I_0^{24}, 0}}.

{W, ∩} is a MOD natural neutrosophic special type of topological space subset subsemigroup.

{W, ×} is a MOD natural neutrosophic special type of topological space subset subsemigroup.

{W, +} is a MOD natural neutrosophic special type of topological space subset subsemigroup.

However {W, ∪} is not a MOD natural neutrosophic subset special type of topological space subsemigroup.

Hence W is only a MOD natural neutrosophic special type of ¾ strong topological space subset subsemigroup of S′.

Let P = {I_0^{24}, I_6^{24}, I_{12}^{24}, 0, 12, $I_0^{24} + 12$, $12 + I_6^{24}$, $12 + I_{12}^{24}$, $I_0^{24} + I_6^{24}$, $I_0^{24} + I_{12}^{24}$, $I_6^{24} + I_{12}^{24}$, $I_0^{24} + I_6^{24} + I_{12}^{24}$, $12 + I_0^{24} + I_6^{24}$, $12 + I_0^{24} + I_{12}^{24}$, $12 + I_6^{24} + I_{12}^{24}$, $12 + I_0^{24} + I_6^{24} + I_{12}^{24}$, 0} ⊆ S′ be the MOD natural neutrosophic subset of S′.

{P, +}, {P, ∪} and {P, ∩} are MOD natural neutrosophic special type topological space subset ¾ strong subsemigroup of S′. Clearly P and W are different.

Interested reader can find other two types of MOD natural neutrosophic special type of ¾ strong topological space subsemigroups.

Now we see W is a MOD natural neutrosophic subset special type of topological subspaces of $T_{\times}^{+}$, $T_{\cup}^{+}$ and $T_{\cup}^{\times}$ associated with the MOD natural neutrosophic subset special type ¾ strong topological space subsemigroup.

In view of all these we have the following theorem.

Theorem 2.3. *Let $S = (S(Z_n^I))$ be the MOD natural neutrosophic subset collection, $2 \leq n < \infty$.*

Let $S' = S \cup \{\phi\}$; $T_o = (S', \cup, \cap)$, $T_{\cup}^{+} = \{S', +, \cup\}$, $T_{\cap}^{+} = \{S' +, \cap\}$, $T_{\cup}^{\times} = \{S', \times, \cup\}$, $T_{\cap}^{\times} = \{S', \times, \cap\}$ and $T_{\times}^{+} = \{S', +, \times\}$ be the MOD natural neutrosophic special type of subset topological spaces associated with the MOD natural neutrosophic subset semigroups $\{S', \cup\}$, $\{S', \cap\}$, $\{S', +\}$ and $\{S', \times\}$. $W \subseteq S'$ is a MOD natural neutrosophic subset special type of ¾ strong topological space subsemigroup under any of the three triples binary operations $\{\cup, \cap, +\}$ (or $\{\cup, +, \times\}$ or $\{\cap, +, \times\}$, or $\{\cup, \cap, \times\}$) if and only if W is a MOD natural neutrosophic special type of subset topological subspaces such as $\{W, \cup, \cap\}$, $\{W, \cup, +\}$ and $\{W, \cap, +\}$ if the operations $(\cup, \cap, +\}$ is taken (likewise for other triples).

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto define the notion of MOD natural neutrosophic special type of subset ½ strong topological space subsemigroups under only any of the pair of operations $\{\cup, \cap\}$ (or $\{\cup, +\}$ or $\{\cup, \times\}$ or $\{\cap, \times\}$ or $\{\cap, +\}$ or $\{+, \times\}$).

We will first illustrate this situation by some examples.

Example 2.15. Let S = $\{S(Z_6^I)\}$ be the MOD natural neutrosophic subset collection. S′ = S ∪ {ϕ}, T_o = {S′, ∪, ∩}, $T_\cup^+$ = {S′ +, ∪}, $T_\cap^+$ = {S′, +, ∩}, $T_\cup^\times$ = {S′, ×, ∪}, $T_\cap^\times$ = {S′, ×, ∩} and $T_\times^+$ = {S′, ×, +} be the MOD natural neutrosophic special type of subset topological spaces associated with the MOD natural neutrosophic subset semigroups {S′ ∪}, {S′, ∩}, {S′,+} and {S′, ×}.

Let V ⊆ S′, if V is a MOD subset natural neutrosophic semigroup under any two of the operations then V is a MOD natural neutrosophic subset special type of ½ strong topological space subsemigroups.

Take V = $\{\{3 + I_3^6, 5 + I_2^6, I_0^6 + I_4^6, 1 + I_2^6 + I_4^6\}\}$ ⊆ S′.

Clearly V is a MOD natural neutrosophic subset special type of ½ strong topological space subsemigroups given by {V, ∩) and {V, ∪}. However {V, +} and {V, ×} are not MOD natural neutrosophic subset subsemigroups.

Let W = $\{A = \{I_0^6 + 3, I_2^6 + I_4^6 + I_0^6, 3\}, B = \{I_0^6, I_2^6 + I_4^6 + I_0^6, 0\}\}$; W is a MOD natural neutrosophic special type of subset ½ strong topological space subsemigroup as {W, +} and {W, ×} are MOD natural neutrosophic subset subsemigroups of {S′,+} and {S′, ×} respectively.

Clearly W is not a MOD natural neutrosophic subset subsemigroup under ∪ or ∩.

In view of this we have the following theorem.

Theorem 2.4. *Let S = $\{S(Z_n^I)\}$ be the MOD natural neutrosophic subset collection. S′ = S ∪ {ϕ}; let T_o = {S′, ∪, ∩}, $T_\cup^+$ = {S′, +, ∪}, $T_\cap^+$ = {S′, +, ∩}, $T_\cup^\times$ = {S′,∪, ×}, $T_\cap^\times$ = {S′, ∩,×} and $T_\times^+$ = {S′, +, ×} be the MOD natural neutrosophic* *special type of subset 6 topological spaces. $V \subseteq S'$, be a MOD natural neutrosophic special type of ½ strong subset topological subsemigroup if and only if V is a MOD natural neutrosophic special type of subset topological subspace of T_o (or $T_{\cup}^{+}$ or $T_{\cap}^{+}$ or $T_{\cup}^{\times}$ or $T_{\cap}^{\times}$ or $T_{\times}^{+}$; or used in the naturally exclusive sense.*

Proof is direct and hence left as an exercise to the reader.

We have seen examples of all types of MOD natural neutrosophic special type of subset topological spaces.

We next define topologically strong ideals of MOD natural neutrosophic subset special type of topological subspaces of T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and $T_{\times}^{+}$.

Definition 2.4. *Let $S = (S(Z_n^I))$ be the MOD natural neutrosophic subsets collection $S' = \{\phi\} \cup S$ be the power set of Z_n^I. $T_o = \{S', \cup, \cap\}$, $T_{\cup}^{+} = \{S', \cup, +\}$, $T_{\cap}^{+} = \{S', \cap, +\}$, $T_{\cup}^{\times} = \{S', \cup, \times\}$, $T_{\cap}^{\times} = \{S', \cap, \times\}$ and $T_{\times}^{+} = \{S', +, \times\}$ be the six MOD natural neutrosophic special type of topological subset spaces using the MOD natural neutrosophic subset semigroups $\{S', +\}$, $\{S', \cup\}$, $\{S', \cap\}$ and $\{S', \times\}$.*

Let $W \subseteq S'$ be the MOD natural neutrosophic special type of subset strong topological subspacesof MOD natural neutrosophic subset semigroups.

- *i) W is a MOD natural neutrosophic ideal of the MOD natural neutrosophic special type of subset topological spaces if W is an ideal with respect all four products. This is possible only when $W = S'$.*
- *ii) W is a MOD natural neutrosophic ¾ super strong ideal if W is a strong subset semigroup and W is an ideal, only under any of the three*

products from $\cup$, $\cap$, + and $\times$ this happens only when W = S′.

iii) *W is a MOD natural neutrosophic super ½ strong ideal if W is a strong subset semigroup and W is an ideal when under any two of the products $\cup$, $\cap$, + and $\times$.*

iv) *W is a MOD natural neutrosophic just an ideal if W is a strong or ¾ strong or ½ subsemigroup but W is only an ideal in one of the products +, $\times$, $\cup$ and $\cap$.*

We will illustrate this situation by some examples.

Example 2.16. Let S = $\{S(Z_6^I)\}$ be the MOD natural neutrosophic subset topological spaces using $S' = S \cup \{\phi\}$; $T_o = \{S', \cup, \cap\}$, $T_\cap^+ = \{S' \cap, +\}$, $T_\cup^+ = \{S', \cup, +\}$, $T_\cup^\times = \{S', \cup, \times\}$, $T_\cap^\times = \{S', \cap, \times\}$ and $T_\times^+ = \{S', +, \times\}$.

Let W = {Collection of all subsets from $\{\langle 0, 3, 6, 9\rangle \cup \langle I_0^{12}, I_6^{12}, I_4^{12}, I_8^{12}, I_{10}^{12}, I_9^{12}, I_3^{12}\rangle\}\}$, $\{W, \cup, \cap\} = W_o$, $W_\cup^+ = \{W, \cup, +\}$, $W_\cap^+ = \{W, \cap, +\}$, $\{W, \cap, \times\} = W_\cap^\times$, $W_\cup^\times = \{W, \cup, \times\}$ and $W_+^\times = \{W, +, ,\times\}$ are all MOD natural neutrosophic subset strong special type of topological subspaces associated with the MOD natural neutrosophic subsemigroups $\{W, \cup\}$, $\{W, \cap\}$, $\{W, +\}$ and $\{W, \times\}$.

Cleaarly W_o, $W_\cup^+$, $W_\cap^+$, $W_\cup^\times$, $W_\cap^\times$ and $W_\times^+$ are all MOD natural neutrosophic subset special type of topological subspace ideals under $\times$ and $\cap$ that is $A \times \{W_o\} \subseteq W_o$.

$(A \times W_\cup^+) \subseteq W_\cup^+$,

$A \times W_\cap^+ \subseteq W_\cap^+$

$A \times W_\cup^\times \subseteq W_\cup^\times$

$A \times W_{\cap}^{\times} \subseteq W_{\cap}^{\times}$

and $A \times W_{\times}^{+} \subseteq W_{\times}^{+}$ for all $A \in S'$.

Similarly $A \cap W_{\times}^{+} \in W_{\times}^{+}$, $A \cap W_{\cup}^{\times} \in W_{\cup}^{\times}$,

$A \cap W_{\cap}^{\times} \in W_{\cap}^{\times}$, $A \cap W_o \in W_o$, $A \cap W_{\cap}^{+} \in W_{\cap}^{+}$,

and $A \cap W_{\cup}^{+} \in W_{\cup}^{+}$ for all $A \in S'$.

However $A \cup W_o \notin W_o$ in general for all $A \in S'$.

$A \cup W_o \notin W_o$.

For take $A = \{5, 7, 4, 11\}$ and $B = \{3, 9 + I_9^{12}, 6 + I_4^{12}, 0\}$.

$A \cup B = \{5, 7, 4, 11\} \cup \{3, 9 + I_9^{12}, 6 + I_4^{12}, 0\}$

$= \{4, 5, 7, 11, 3, 9 + I_9^{12}, 6 + I_4^{12}, 0\} \notin W_o$.

Hence the claim.

Consider for the same A and B; $A + B = \{5, 7, 4, 11\} + \{3, 9 + I_9^{12}, 6 + I_4^{12}, 0\} = \{8, 2 + I_9^{12}, 11 + I_4^{12}, 5, 10, 4 + I_9^{12}, 2, 11, 4, 1 + I_2^{12}, 7, 1 + I_9^{12}, 10 + I_2^{12}, 8 + I_9^{12}, 5 + I_4^{12}\} \notin W_o$.

However $A \cap B = \{5, 7, 4, 11\} \cap \{3, 9 + I_9^{12}, 6 + I_4^{12}, 0\} = \phi \in W_o$.

$A \times B = \{5, 7, 4, 11\} \times \{3, 9 + I_9^{12}, 6 + I_4^{12}, 0\} = \{0, 3, 9 + I_9^{12}, 6 + I_4^{12}, 9, 3 + I_9^{12}, I_9^{12}, I_4^{12}, 9\} \in W_o$.

Thus $A \times W_o \in W_o$ for all $A \in S'$ and $A \cap W_o \in W_o$ for all $A \in S'$. Hence W_o is a MOD natural neutrosophic special type of topological subset subspace ideal only under $\times$ and $\cap$.

Consider $W_{\times}^{+}$ the MOD natural neutrosophic special type of topological subset subspace of $S_{\times}^{+}$.

Now we see $W_\times^+$ is also only a MOD natural neutrosophic special type of topological subset subspace ideal under $\times$ and $\cap$.

$A = \{5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10\} \in S'$. Let $B = \{3 + I_2^{12}, 0, I_8^{12} + I_4^{12} + 9\} \in W_\times^+$.

We first find $A + B = \{5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10\} + \{3 + I_2^{12}, 0, 9 + I_8^{12} + I_4^{12}\}$

$= \{8 + I_3^{12} + I_2^{12}, 10 + I_2^{12}, 5 + I_2^{12}, I_2^{12}, 4 + I_2^{12}, 11 + I_2^{12} + I_6^{12}, 7 + I_2^{12}, 1 + I_2^{12}, 5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10, 2 + I_8^{12} + I_4^{12}, 4 + I_8^{12} + I_4^{12}, 11 + I_8^{12} + I_4^{12}, 6 + I_8^{12} + I_4^{12}, 10 + I_8^{12} + I_4^{12}, 5 + I_8^{12} + I_4^{12} + I_6^{12}, 1 + I_8^{12} + I_4^{12}, 7 + I_8^{12} + I_4^{12}\} \notin W_\times^+$ so in general $A + W_\times^+ \not\subset W_\times^+$ So $W_\times^+$ cannot be a MOD natural neutrosophic special type of topological subspace subset ideal of S' under +.

Consider for the same $A \in S'$ and $B \in W_\times^+$. $A \cup B = \{5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10\} \cup \{3 + I_2^{12}, 0, I_8^{12} + I_4^{12} + 9\} = \{5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10, 3 + I_2^{12}, 0, I_8^{12} + I_4^{12} + 9\} \notin W_\times^+$.

Thus $W_\times^+$ is not a MOD natural neutrosophic special type of topological subset subspace ideal of S' under $\cup$.

Consider for the same A in S' and in $W_\times^+$, $A + B = \{5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10\} + \{3 + I_2^{12}, 0, 9 + I_8^{12} + I_4^{12}\} = \{8 + I_3^{12} + I_2^{12}, 10 + I_2^{12}, 5 + I_2^{12}, 1 + I_2^{12}, 4 + I_2^{12}, 11 + I_6^{12} + I_4^{12} + I_2^{12}, 5 + I_3^{12}, 7, 2, 9, 1, 8 + I_6^{12}, 4, 10, 2 + I_8^{12} + I_4^{12}, 4 + I_8^{12} + I_4^{12}, 11 + I_8^{12} + I_4^{12}, 6 + I_8^{12} + I_4^{12}, 10 + I_8^{12} + I_4^{12}, 5 + I_8^{12} + I_4^{12} + I_6^{12}, 1 + I_8^{12} + I_4^{12}, 7 + I_8^{12} + I_4^{12}\} \notin W_\times^+$.

Thus we see $W_{\times}^{+}$ is not a MOD natural special type of topological subspace subsets ideal under +.

In view of this we prove for S = $\{S(Z_n^I)\}$ and S′ = S ∪ {ϕ}, it is impossible to have MOD natural neutrosophic special type of subset topological subspaces which can be super ideals.

Hence we conjecture does there exist any special type of topological space using a universal power set or otherwise say P such that {P, ∪, ∩}, {P, +, ∪}, {P, ×, ∪}, {P, +, ∩}, {P, +, ×} and {P ×, ∩} the six distinct topological spaces to have a nontrivial proper subset M such that {M, ∪, ∩}, {M, ∪, +}, {M, ∪, ×}, {M, ∩, +}, {M, ∩, ×} and {M, +, ×} are subspaces and M is to be a super ideal.

Clearly we cannot have this using the power set of $P(Z_n^I)$; $2 \leq n < \infty$.

Now we proceed onto built MOD natural neutrosophic special type of topological subset spaces using $P(C(Z_n^I))$, $P(\langle Z_n \cup g\rangle_I)$, $P(\langle Z_n \cup h\rangle_I)$, $P(\langle Z_n \cup I\rangle_I)$ and $P(\langle Z_n \cup k\rangle_I)$ and study the special features associated with them by appropriate examples.

Example 2.17. Let V = $P(C^I(Z_{10}))$ be the MOD natural neutrosophic finite complex number subsets, that is power set of $C^I(Z_{10}) = \{a + bi_F + I_t^{10} \,/\, a, b \in Z_{10};\ i_F^2 = 9$ and t is a zero divisor or idempotent or nilpotent in $C(Z_{10})\}$.

{V, ∪, ∩} = V_o is the MOD natural neutrosophic special type of ordinary topological subset space of finite complex numbers.

Clearly M = P (Z_{10}) and N = $P(C(Z_{10}))$ are MOD natural neutrosophic finite complex number subset special type of topological subspace of V.

M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$ are MOD natural neutrosophic finite complex number subset special type of topological subspaces of M.

Similarly N_o, $N_{\times}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cap}^{\times}$ and $N_{\cup}^{\times}$ are MOD natural neutrosophic finite complex number subset special type of topological subspaces of M.

Thus both M and N are MOD natural neutrosophic finite complex number special type of subset strong topological subsemigroups of V.

Consider B = $\{\{i_F, 9, 5 + I_5^C, 6 + I_0^C + I_2^C + I_4^C + I_6^C + I_8^C\}, \{9, 1, 5 + I_5^C, 6 + I_0^C + I_2^C + I_4^C + I_6^C + I_8^C, I_0^C + I_5^C, 9i_F, 6i_F + I_0^C + I_2^C + I_4^C + I_6^C + I_8^C, 4 + I_0^C + I_6^C + I_8^C, 4i_F + I_0^C + I_2^C + I_6^C + I_8^C, I_0^C\}, \{9, 6 + I_0^C + I_2^C + I_4^C + I_6^C + I_8^C\}, \{1, I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + 6\}, \{I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + 6\}, \ldots\}$.

Find $\langle B, \times\rangle$, the MOD natural neutrosophic finite complex number subset subsemigroup.

Then can we say $\{\langle B, \times\rangle, \cup\}$ is a MOD natural neutrosophic finite complex number subset subsemigroup?

Let D = $\{\{6 + I_0^C, 6, I_4^C + I_2^C + I_8^C + I_6^C + I_0^C\}$ be a subset of V and $\{D, \cup\}$, $\{D, \cap\}$ and $\{D, \times\}$ are MOD natural neutrosophic finite complex number ¾ strong subset subsemigroup.

Hence, $D_o = \{D, \cup, \cap\}$. $D_{\cup}^{\times} = \{D, \cup, \times\}$ and $D_{\cap}^{\times} = \{D, \cap, \times\}$ are all three MOD natural neutrosophic finite complex number special type of topological subset subspaces of V.

Consider D + D = $\{6 + I_0^C, 6, I_4^C + I_2^C + I_6^C + I_8^C + I_0^C\}$ + $\{6, 6 + I_0^C, I_4^C + I_2^C + I_8^C + I_0^C\}$ = $\{2 + I_0^C, 2, 6 + I_4^C + I_2^C + I_6^C + I_8^C + I_0^C\} \notin$ D. Thus D is not closed under +.

Let L = $\{5, 0, 5 + I_0^C, I_0^C, 5 + I_5^C, 5 + I_0^C + I_5^C, I_5^C + I_0^C\} \in$ V.

Clearly L + L = L,

$L \cap L = L$,

$L \cup L = L$ and

$L \times L = L$.

Thus L is a MOD natural neutrosophic finite complex number strong subsemigroup.

$L_o = \{L, \cup, \cap\}$, $L_{\cup}^{+} = \{L, +, \cup\}$, $L_{\cap}^{\times} = \{L, \times, \cap\}$, $L_{\cap}^{+} = \{L, +, \cap\}$, $L_{\cup}^{\times} = \{L, \times, \cup\}$ and $L_{\times}^{+} = \{L, +, \times\}$ are MOD natural neutrosophic finite complex number special type of subset topological subsapces of V.

We see none of them are ideals.

Take A = {5, 3} $\in$ V,

$A \cap L = \{5\} \notin L$.

$A \cup L = \{5, 3\} \cup L \notin L$.

$A + L = \{5, 3\} + L$

$= \{3, 5, 8, 8 + I_0^C, 3 + I_0^C, \ldots\} \notin L$.

$A \times L = \{3, 5\} \times L$

$= \{5, 0, 5 + I_0^C, I_0^C + 5 + I_5^C, I_5^C, 5 + I_0^C + I_5^C,$

$I_5^C + I_0^C\} = L$.

So only under $\times$ with this A, it is closed.

Let B = $\{I_2^C, I_4^C\} \in V$, $B \times L = \{I_2^C, I_2^C + I_0^C, I_0^C, I_4^C, I_4^C + I_0^C\} \not\subseteq B$, so for this B, $B \times L \not\subseteq B$. Thus L cannot be an ideal more so a super strong ideal of V.

In some cases even just an ideal of V.

Hence it is left as an open conjecture to find condition on subsets $W \subseteq S'$ to be

i) just an ideal.
ii) ½ strong ideal.

(for ¾ strong ideal and strong ideal situations are ruled out).

The existence of ¾ strong ideals and strong ideals can be studied in case of other topological spaces.

The striking feature about these MOD natural neutrosophic finite complex modulo integer special type of topological spaces of subsets using $P(C^I(Z_n)) = S(C^I(Z_n)) \cup \{\phi\}$ can be analysed by any interested researcher.

However some facts about this structure is mentioned below.

Let M = $P(C^I(Z_n))$ be the MOD natural neutrosophic finite complex number subset collection.

$M_o = \{M, \cup, \cap\}$, $M_\times^+ = \{M, +, \times\}$, $M_\cup^+ = \{M, +, \cup\}$, $M_\cap^+ = \{M, +, \cap\}$, $M_\cup^\times = \{M, \times, \cup\}$ and $M_\cap^\times = \{M, \times, \cap\}$ are MOD natural neutrosophic finite complex number special type of topological subset space using subsets of $P(C^I(Z_n))$.

We list out the properties enjoyed by these six spaces.

Property 2.1. S = {P(Z_n)) $\subseteq$ M is always a subspace of all the six MOD natural neutrosophic finite complex number special type of topological subset spaces, M_o, $M_{\times}^{+}$, $M_{\cap}^{+}$, $M_{\cap}^{\times}$, $M_{\cup}^{\times}$ and $M_{\cup}^{+}$.

Property 2.2. R = {P(C(Z_n))} $\subseteq$ M is always a subspace of all the six MOD natural neutrosophic finite complex number special type of topological subset spaces M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$ $M_{\cup}^{+}$ and $M_{\cap}^{\times}$.

Property 2.3. W = P($Z_n i_F$) where $Z_n i_F$ = {ai_F / a $\in$ Z_n}} $\subseteq$ M is only a subspace of M_o, $M_{\cup}^{+}$ and $M_{\cap}^{+}$.

Clearly W is not a subspace of $M_{\times}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$ as {W, ×} is not even a closed set under ×.

Property 2.4. M has no strong ideals but has strong subsemigroups.

Next we proceed onto study the properties associated with MOD natural neutrosophic-neutrosophic subset special type of topological space using P($\langle Z_n \cup I\rangle_I$) by examples.

Example 2.18. W = {P($\langle Z_6 \cup I\rangle_I$) be the MOD natural neutrosophic-neutrosophic subset.

W_o = {W, $\cup$, $\cap$}, $W_{\cup}^{+}$ = {W, $\cup$, +}, $W_{\cap}^{+}$ = {W, $\cap$, +}, $W_{\cup}^{\times}$ = {W, $\cup$, ×}, $W_{\cap}^{\times}$ = {W, $\cap$, ×} and $W_{\times}^{+}$ = {W, +, ×} be the MOD natural neutrosophic-neutrosophic special type of subset topological space using subset semigroups (W, +),{W, ×}, {W, $\cup$} and {W, $\cap$}.

We see M = {P(Z_6)} is a MOD natural neutrosophic-neutrosophic special type of subset topological subspaces given by $M_o = \{M, \cup, \cap\}$, $M_{\times}^{+} = \{M, +, \times\}$, $M_{\cup}^{+} = \{M, \cup, \times\}$ and $M_{\cap}^{\times} = \{M, \cap, \times\}$. None can contribute to ideals.

Infact M is a strong subset subsemigroup of W.

Consider $N = P(\langle Z_6 \cup I \rangle)\} \subseteq W$, N is also a MOD natural neutrosophic-neutrosophic special type of subset topological subspaces; $N_o = \{N, \cup, \cap\}$, $N_{\cup}^{+} = \{N, \cup, +\}$, $N_{\cap}^{+} = \{N, +, \cap\}$ $N_{\cup}^{\times} = \{N, \times, \cup\}$, $N_{\cap}^{\times} = \{N, \times, \cap\}$ and $N_{\times}^{+} = \{N, +, \times\}$.

N is only a MOD natural neutrosophic-neutrosophic strong subset subsemigroup and is not an ideal.

However $M \subseteq N$, proper containment.

In view of this we can define the notion of restricted ideals or restricted quasi vector spaces.

We see N is an ideal over M infact a strong ideal. Also N can be viewed as a quasi special vector space over M where M is not the field which the classical definition of vector space requires.

Thus if $L = \{S(\{2, 0, 4\})\} \cup \{\phi\}$. Then L is a MOD natural neutrosophic-neutrosophic special type of topological subset subspaces given by $L_o = \{L, \cup, \cap\}$, $L_{\times}^{+} = \{L, +, \times\}$, $L_{\cup}^{+} = \{L, \cup, +\}$, $L_{\cap}^{+} = \{L, \cap, +\}$, $L_{\cup}^{\times} = \{L, \cup, \times\}$ and $L_{\cap}^{\times} = \{L, \cap, \times\}$.

Infact both M and N are MOD natural neutrosophic-neutrosophic special quasi vector spaces over L.

Similarly if $T = P(\{0, 3\}) \subseteq W$, then T is a MOD natural neutrosophic-neutrosophic special type of subset topological subspaces associated with T given by $T_o = \{T, \cup, \cap\}$, $T_{\times}^{+} = \{T,$

$+, \times\}$, $T_{\cup}^{+} = \{T, +, \cup\}$, $T_{\cap}^{+} = \{T, +, \cap\}$, $T_{\cup}^{\times} = \{T, \cup, \times\}$ and $T_{\cap}^{\times} = \{T, \cap, \times\}$. Clearly L, M and N are MOD natural neutrosophic-neutrosophic special quasi vector spaces over T.

However T is a not a MOD natural neutrosophic-neutrosophic special quasi vector space over L or M or N.

Consider $B = \{P(Z_6(I)\} \subseteq W$, is again a MOD natural neutrosophic-neutrosophic subset special type topological subspace of W.

Clearly B is not an ideal.

Let $D = \{P(\langle Z_6 I \cup I_t^I \rangle) / t \in \langle Z_6 \cup I \rangle$ is an idempotent or nilpotent or a zero divisor$\} \subseteq W$ be a MOD natural neutrosophic-neutrosophic special type of subset topological subspace associated with strong subset semigroup D.

Clearly D is not a strong ideal of W.

However D is a ¾ strong ideal of W.

$A + W \not\subseteq W$ in general for $A \in W$.

We have following properties associated with MOD natural neutrosophic-neutrosophic special type of topological subset spaces associated with the subset semigroups built using $W = P(\langle Z_n \cup I \rangle_I)$, $W_o = \{W, \cup, \cap\}$, $W_{\cup}^{+} = \{W, +, \cup\}$, $W_{\cap}^{+} = \{W, +, \cap\}$, $W_{\times}^{+} = \{W, +, \times\}$, $W_{\cup}^{\times} = \{W, \times, \cup\}$ and $W_{\cap}^{\times} = \{W, \times, \cap\}$.

Property 2.5. Let $W = \{P(\langle Z_n \cup I \rangle_I)\}$ be the six topological spaces mentioned above. W has MOD natural neutrosophic-neutrosophic special type of subset strong subsemigroup P. Associated with P we have the six MOD natural neutrosophic-

neutrosophic special type of subset topological subspaces, P_o, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\times}^{+}$, $P_{\cup}^{\times}$ and $P_{\cap}^{\times}$.

Property 2.6. W has no MOD natural neutrosophic-neutrosophic strong ideals.

Property 2.7. Let W = $(P\langle Z_n \cup I\rangle_I)$. All the associated MOD natural neutrosophic special type of subset topological spaces has MOD topological subspaces B which are MOD quasi special vector spaces over certain other subspaces C such that $C \subseteq B$.

Property 2.8. E = $\{P(\langle Z_nI \cup I_t^I\rangle) \,/\, t$ is an idempotent or nilpotent or a zero divisor in $\langle Z_n \cup I\rangle\} \subseteq W$ is such that E is ½ strong ideal of W.

Next we proceed onto describe by examples MOD natural neutrosophic dual number special type of topological subset spaces using $P(\langle Z_n \cup g\rangle_I)$.

Example 2.19. Let Z = $\{P(\langle Z_{12} \cup g\rangle_I)\}$ be the MOD natural neutrosophic dual number special type of subset topological space associated with MOD natural neutrosophic dual number subset semigroup $\{Z, \cup\}$, $\{Z, \cap\}$, $\{Z, +\}$ and $\{Z, \times\}$.

Let $Z_o = \{Z, \cup, \cap\}$, $Z_{\times}^{+} = \{Z, +, \times\}$, $Z_{\cap}^{+} = \{Z, \cap, +\}$, $Z_{\cup}^{\times} = \{Z, +, \cup\}$, $Z_{\cup}^{+} = \{Z, \cup, \times\}$ and $Z_{\cap}^{\times} = \{Z, \cap, \times\}$ be the MOD natural neutrosophic dual number subset special type of topological subset spaces associated with Z.

Let A = $\{4+ g, 8 +3g, 5 +4g, I_3^g, I_{2g}^g, 2g, 4g, 11g\}$ and B = $\{2g + 4, 5g, I_0^g, 9g, I_g^g, 3 +2g\} \in Z$.

We will find $A + B$, $A \cup B$, $A \cap B$ and $A \times B$.

$A + B = \{4 + g, 8 + 3g, 5 + 4g, I_3^g, I_{2g}^g, 2g, 4g, 11g\} + \{2g + 4, 5g, I_0^g, 9g, I_g^g, 3 + 2g\} = \{8 + 3g, 0 + 5g, 9 + 7g, 2g + 4 + I_3^g, 4 + 2g + I_{2g}^g, 4 + 4g, 6g + 4, g + 4, 4 + 5g, 8 + 8g, 5 + 9g, 5g + I_3^g, 5g + I_{2g}^g, 7g, 9g, 4g, 4 + g + I_0^g, 8 + 3g + I_0^g, 5 + 4g + I_0^g, I_3^g + I_0^g, I_{2g}^g + I_0^g, 2g + I_0^g, 4g + I_0^g, 11g + I_0^g, 4 + 10g, 8, 5 + g, I_3^g + 9g, I_{2g}^g + 9g, 11g, g, 8g, I_g^g + 4 + g, I_g^g + 8 + 3g, I_g^g + 5 + 4g, I_g^g + I_3^g, I_g^g + I_{2g}^g, I_g^g + 2g, 4g + I_g^g, 11g + I_g^g, 7 + 3g, 11 + 5g, 8 + 6g, I_3^g + 3 + 2g, 3 + 4g, I_{2g}^g + 3 + 2g, 3 + 6g, 3 + g\}$ I

$A \cup B = \{4 + g, 8 + 3g, 5 + 4g, I_3^g, I_{2g}^g, 2g, 4g, 11g\} \cup \{2g + 4, 5g + I_0^g, 9g, I_g^g, 2g, 4g, 11g\} = \{4 + g, 8 + 3g, 5 + 4g, I_3^g, I_{2g}^g, 2g, 4g, 11g, 2g + 4, 5g, I_0^g, I_g^g, 9g, 2g, 4g, 11g\}$ II

$A \cap B = \{4 + g, 8 + 3g, 5 + 4g, I_3^g, I_{2g}^g, 2g, 4g, 11g\} \cap \{2g + 4, 5g, I_0^g, 9g, I_g^g, 3 + 2g\} = \phi$ III

$A \times B = \{4 + g, 8 + 3g, 4g + 5, I_3^g, I_{2g}^g, 2g, 4g, 11g\} \times \{2g + 4, 5g, I_0^g, 9g, I_{2g}^g, 3 + 2g\} = \{4, 4g + 8, 8 + 2g, I_3^g, I_{2g}^g, 8g, 4g, g, I_0^g, 0, I_g^g, I_{3g}^g, 11g, 3 + 10g, 6g, 9g\}$ IV

Clearly the equations I, II, III and IV are different hence all the six MOD natural neutrosophic dual number subset special topological spaces $Z_o = \{Z, \cup, \cap\}$, $Z_\times^+ = \{Z, +, \times\}$, $Z_\cap^+ = \{Z, +, \cap\}$, $Z_\cap^\times = \{Z, \times, \cap\}$, $Z_\cup^+ = \{Z, +, \cup\}$ and $Z_\cup^\times = \{Z, \cup, \times\}$ all are distinct.

One can have subspaces associated with all these spaces.

For instance W = (P(Z_{12}) $\subseteq$ Z = P($\langle Z_{12} \cup g\rangle_I$)} $\subseteq$ Z is a proper subset of Z and W_o = {W, $\cup$, $\cap$}, $W_\times^+$ = {W, +, $\times$}, $W_\cup^+$ = {W, +, $\cup$}, $W_\cap^+$ = {W, +, $\cap$}, $W_\cup^\times$ = {W, $\times$, $\cup$} and $W_\cap^\times$ = {W, $\times$, $\cap$} are all MOD subset special type of topological subspaces of Z.

However W is not an ideal of Z.

Consider V ={P(Z_{12}g)} $\subseteq$ Z. We see V is a MOD special type of subset topological subspace; V_o ={V, $\cup$, $\cap$}, $V_\times^+$ = {V, +, $\times$}, $V_\cap^+$ = {V, +, $\cap$}, $V_\cup^+$ ={V, +, $\cup$}, $V_\cup^\times$={V, $\times$, $\cup$} and $V_\cap^\times$= {V, $\times$, $\cap$} are all distinct MOD natural neutrosophic subset special type of topological subspaces of Z and none of them are ideals of Z.

Consider B ={P($\langle Z_{12}$g $\cup$ $I_t^g\rangle$) where t $\in$ $\langle Z_{12} \cup g\rangle$ is a zero divisor or an idempotent or a nilpotent element of $\langle Z_{12} \cup$ g$\rangle\rangle$)} $\subseteq$ Z.

Clearly B is a MOD natural neutrosophic dual number subset special type of topological subspaces given by B_o = {B, $\cup$, $\cap$}, $B_\times^+$ = {B, +, $\times$}, $B_\cap^+$= {B, +, $\cap$}, $B_\cap^\times$= {B, $\times$, $\cap$}, $B_\cup^+$ = {B, +, $\cup$} and $B_\cup^\times$ = {B, $\cup$, $\times$}.

Clearly B is an not ideal under all four operations. B is an ideal only under $\times$ and $\cap$ operations. Clearly under $\cup$ and + they are not ideals.

Interested reader can prove that there is no nontrivial MOD subset natural neutrosophic dual number special type of topological subspace which is an ideal under + and $\cup$.

For it can be easily established.

Next we proceed onto describe by an example the MOD natural neutrosophic special dual like number special type of topological subset space using $P(\langle Z_n \cup h\rangle_I)$ where $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ and $\langle Z_n \cup h\rangle_I$ = {Collection of all MOD natural neutrosophic special dual like numbers} = $\{a + bh + I_t^h /$ a, b $\in Z_n$ and t is a nilpotent or a zero divisor or an idempotent of $\langle Z_n \cup h\rangle\}$; $2 \leq n < \infty$.

Example 2.20. Let $S = \{P(\langle Z_{10} \cup h\rangle_I\}$ be the MOD natural neutrosoophic special dual like number subsets collection including ϕ. $\{S, \cup\}$, $\{S, \cap\}$, $\{S, \times\}$ and $\{S, +\}$ are all MOD natural neutrosophic special dual like number subset semigroups.

Hence we obtain the corresponding six MOD natural neutrosophic special dual like number special type of subset topological spaces.

$S_o = \{S, \cup, \cap\}$, $S_\times^+ = \{S, +, \times\}$, $S_\cap^+ = \{S, +, \cap\}$, $S_\cup^+ = \{S, +, \cup\}$, $S_\cup^\times = \{S, \times, \cap\}$ and $S_\cap^\times = \{S, \times, \cap\}$.

All the six spaces are distinct evident from the following.

Let $A = \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, 5 + I_2^h, I_5^h\}$ and $B = \{I_5^h, 2h, 2 + I_4^h, 4h + 2h + I_6^h, I_0^h + 3h\} \in S$.

We find $A \cup B$, $A \cap B$, $A + B$ and $A \times B$.

$A \cup B = \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, 5 + I_2^h, I_5^h\} \cup \{I_5^h, 2h, 2 + I_4^h, 4h + 2 + I_6^h, I_0^h + 3h\}$

$= \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, 5 + I_2^h, I_5^h, I_4^h + 2, 2 + 4 + I_6^h, I_0^h + 3h\}$ I

$A \cap B = \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, 5 + I_2^h, I_5^h\} \cap \{I_5^h, 2h, 2 + I_4^h, 4h + 2h + I_6^h, I_0^h + 3h\} = \{I_5^h, 2h\}$ II

$A + B = \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, I_5^h, 5 + I_2^h\} + \{I_5^h, 2h, 2 + I_4^h, 2 + 4h + I_6^h, I_0^h + 3h\}$

$= \{3 + 5h + I_5^h, 2h + I_5^h, 9h + 1 + I_5^h, 2 + I_0^h + I_5^h, I_5^h, 5 + I_2^h + I_5^h, 2h + I_5^h, 4h, 2 + 2h + I_4^h, 2 + 6h + I_6^h, 5h + I_0^h, 3 + 7h, h + 1, 2 + 2h + I_0^h, 2h + 5h + I_5^h, 3 + 7h, 5 + I_4^h + 5h, 5 + 9h, 2 + 6h + I_6^h, 5h + I_0^h, 9h + 1 + I_5^h, h + 1, 3 + 9h + I_4^h, 3h + 3 + I_6^h, I_0^h + 2h + 1, 2 + I_0^h + I_5^h, 2 + 2h + I_0^h, 4 + I_0^h + I_4^h, 4 + 4h + I_0^h + I_6^h, 2 + 3h + I_0^h, 2h + I_5^h, 2 + I_5^h + I_4^h, 2 + 4h + I_6^h + I_5^h + 3h + I_0^h + I_5^h, 5 + I_5^h + I_5^h, 5 + 2h + I_2^h, 7 + I_4^h + I_2^h, 7 + 4h + I_2^h + I_6^h, 5 + 3h + I_0^h + I_2^h\}$ III

We find $A \times B = \{3 + 5h, 2h, 9h + 1, 2 + I_0^h, I_5^h, I_2^h + 5\} \times \{I_5^h, 2h, 2 + I_4^h, 4h + 2 + I_6^h, I_0^h + 3h\}$

$= \{I_5^h, I_5^h + I_0^h, 6h, 4h, 0, 4h + I_0^h, I_2^h, 6 + I_6^h, 4h + I_4^h, 2 + 8h + I_4^h, 5h + I_0^h + I_2^h, 4 + I_0^h + I_4^h, 4 + 8h + I_0^h + I_6^h, 6h + I_0^h, I_2^h + I_5^h, I_4^h + I_2^h\}$ IV

Clearly the equations I, II, III and IV are distinct hence $S_o = \{S, \cup, \cap\}$, $S_\times^+ = \{S, +, \times\}$, $S_\cap^+ = \{S, +, \cap\}$, $S_\cap^\times = \{S, \times, \cap\}$, $S_\cup^+ = \{S, +, \cup\}$ and $S_\cup^\times = \{S, \times, \cup\}$ all are distinct MOD natural neutrosophic special dual like number special type of subset topological spaces built using $\{S, \cup\}$, $\{S, \cap\}$, $\{S, +\}$ and $\{S, \times\}$ the four MOD natural neutrosophic special dual like number subset semigroup.

Next we proceed onto describe MOD natural neutrosophic special quasi dual number subset special type of topological spaces using $P(\langle Z_n \cup k\rangle_I)$ where $\langle Z_n \cup k\rangle = \{a + bk /$ $a, b \in Z_n, k^2 = (n – 1)k\}$ and $\langle Z_n \cup k\rangle_I = \{a + bk + I_t^k$ $/ a, b \in Z_n$, t is an idempotent or nilpotent or zero divisor of $\langle Z_n \cup k\rangle\}$.

We will describe this situation by some examples.

Example 2.21. Let $B = \{P(\langle Z_4 \cup k\rangle_I)\} = \{a + bk + I_t^k / a, b \in Z_4, k^2 = 3k, t \in \langle Z_4 \cup k\rangle = \{a + bk / a, b \in Z_4\}$ and t is an idempotent or a nilpotent or a zero divisor}, be the collection of MOD natural neutrosophic special quasi dual number subsets from $\langle Z_4 \cup k\rangle_I$.

We see $\{B, \cup\}$, $\{B, \cap\}$, $\{B, \times\}$ and $\{B, +\}$ are MOD natural neutrosophic special quasi dual number subset semigroups.

We first show all the four MOD natural neutrosophic quasi dual number subset semigroups are distinct.

Take $A = \{2 + 2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k + I_2^k + I_{2k}^k\}$ and $B = \{2, k, 3 + 2k, 2k + 1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, I_0^k + I_k^k + 2k + I_2^k\} \in B$.

$A \cap B = \{2 + 2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k + I_2^k + I_{2k}^k\} \cap \{2, k, 3 +2k, 2k +1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, I_0^k + I_k^k + 2k + I_2^k\} = \{2, k\}$ I

$A \cap B = \{2 +2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k, I_2^k + I_{2k}^k\} \cup \{2, k, 3 + 2k, 2k +1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, I_0^k + I_k^k + 2k + I_2^k\} = \{2 + 2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k + I_2^k + I_{2k}^k, 3 + 2k, 2k + 1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, I_0^k + I_k^k + 2k + I_2^k\}$ II

$A + B = \{2 + 2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k + I_2^k, I_{2k}^k\} + \{2, k, 3 + 2k, 2k +1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, I_0^k + I_k^k + 2k + I_2^k\} = \{2k, I_0^k, 2 + I_2^k, 1 + I_{2k}^k, 0, 2 + k, 3 + 3k + I_2^k + I_{2k}^k, 2 + 3k, 2 + I_0^k + k, k + I_k^k, 3 + k + I_{2k}^k, 2 + k, 2k, 1 + I_2^k + I_{2k}^k, 1, 1 + 2k, I_0^k, 3 + 2k + I_k^k, 2 + 2k + I_{2k}^k, 1 + 2k, 3 + 3k, k + I_2^k + I_{2k}^k, 3, 2k + 3 + I_0^k, 2k + 1 + I_k^k, 2k + I_{2k}^k, 2k + 3, 3k +1, 2 + k + I_2^k + I_{2k}^k, 2k + I_0^k + I_{2k}^k, I_{2k}^k + I_0^k, I_k^k + I_{2k}^k + I_0^k + 2, I_{2k}^k + 1 + I_0^k, I_{2k}^k + I_0^k + k + 2, 3 + 3k + I_2^k + I_{2k}^k + I_0^k, 2, 2 + 2k + I_0^k, 2k + I_k^k, 3 + 2k + I_{2k}^k, 2 + 2k, 3k, 1 + k + I_2^k + I_{2k}^k, 2 + 2k + I_0^k, 2 + I_0^k, I_0^k + I_k^k, 3 + I_{2k}^k + I_0^k, 2 + I_0^k, k + I_0^k, 1 + I_0^k + I_2^k + I_{2k}^k + 3k, 2 + I_0^k + I_k^k + I_2^k, 2 + 2k + I_0^k + I_2^k + I_{2k}^k, I_0^k + I_k^k + 2k + I_2^k, 3 + 2 + I_0^k + I_k^k + I_2^k + I_{2k}^k, 2 +2k + I_0^k + I_k^k + I_2^k, 3k + I_0^k + I_k^k + I_2^k, 1 + k + I_2^k + I_0^k + I_k^k + I_{2k}^k\}$ III

We next find A × B.

$A \times B = \{2 + 2k, 2 + I_0^k, I_k^k, 3 + I_{2k}^k, 2, k, 1 + 3k + I_2^k + I_{2k}^k\} \times \{2, k, 3 + 3k, 2k + 1, I_{2k}^k + I_0^k + 2, 2k, I_0^k, 2k + I_2^k + I_0^k + I_k^k\} = \{0, I_0^k, I_k^k, 2 + I_{2k}^k, 2k, 2 + 2k + I_2^k + I_{2k}^k, 2k + I_0^k, I_k^k, 3k + I_{2k}^k, 3k, 2k + I_2^k + I_{2k}^k, 2 + 2k, 2 + 2k + I_0^k, I_k^k, 1 + k + I_{2k}^k, I_2^k + I_{2k}^k + 3 + k, 2 + I_0^k, 3 + 2k + I_{2k}^k, 2, 1 + 3k + I_2^k + I_{2k}^k, I_{2k}^k + I_0^k, I_0^k + I_k^k + I_{2k}^k, 2 + I_{2k}^k + I_0^k, 2k + I_0^k + I_{2k}^k, 2 + 2k + I_{2k}^k + I_2^k + I_0^k, I_0^k, I_2^k + I_0^k + I_k^k, I_{2k}^k + I_0^k + I_k^k + I_{3k}^k\}$ IV

All the four equations are distinct hence the MOD subset natural neutrosophic special quasi dual number semigroups associated with them are also distinct.

Now we proceed onto describe the 6 new special type of topological spaces associated with them.

$B_o = \{B, \cup, \cap\}$, $B_{\cup}^{+} = \{B, +, \cup\}$, $B_{\cap}^{+} = \{B, +, \cap\}$, $B_{\cup}^{\times} = \{B, \times, \cup\}$, $B_{\cap}^{\times} = \{B, \times, \cap\}$ and $B_{\times}^{+} = \{B, \times, +\}$ are the six distinct MOD natural neutrosophic special quasi dual number subset special type of topological space associated with the MOD natural neutrosophic subset special quasi dual number subset semigroups, $\{S, \cup\}$, $\{S, \cap\}$, $\{S, +\}$ and $\{S, \times\}$.

It is pertinent to keep on record that none of these MOD natural neutrosophic subset special type of topological spaces have MOD strong subspaces which are not MOD strong ideals.

They can never be a strong ideal, to be more precise they cannot be ideals under + and $\cup$, they can be ideals only under $\cap$ and $\times$.

In all cases they are ideals under $\cap$ but only in few cases they are ideals under $\times$.

Interested reader can analyse this situation.

Next we proceed onto construct MOD matrix subset special type of topological spaces using subsets of a MOD matrix semigroups under $\cup$ or $\cap$ or + or $\times$.

We will first illustrate this situation by some examples.

Example 2.22: Let $M = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle/ a_i \in Z_{12}, 1 \leq i < 8\right\}$ be the

collection of all MOD 4×2 matrices. Let R = P(M) = {collection of all subsets of M}. {P(M), $\cap$} be a MOD subset matrix semigroup under $\cap$ associated with M.

{P(M), ∪} is again a MOD matrix subset semigroup under ∪ associated with M.

{P(M), $\times_n$} is a MOD matrix subset semigroup under $\times_n$ {P(M), +} is a MOD matrix subset semigroup under +.

We shall show all the four MOD matrix subset semigroups are distinct.

Let A = $\left\{ \begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix} \right\}$ and B = $\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix} \right\} \in P(M)$.

$$A + B = \left\{ \begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix} \right\} + \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 4 & 2 \\ 9 & 1 \\ 7 & 7 \\ 0 & 9 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 3 & 4 \\ 7 & 11 \\ 7 & 11 \end{bmatrix} \right\} \qquad \text{I}$$

$$A \cup B = \left\{ \begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix} \right\} \cup \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix} \right\}$$

$$= \{\begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix}\} \qquad \text{II}$$

$$A \cap B = \{\begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix}\} \cap \{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix}\} \qquad \text{III}$$

$$= \{\phi\}$$

$$A \times_n B = \{\begin{bmatrix} 3 & 0 \\ 6 & 9 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & 5 \\ 7 & 4 \end{bmatrix}\} \times_n \{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 0 & 7 \end{bmatrix}\}$$

$$= \{\begin{bmatrix} 3 & 0 \\ 6 & 0 \\ 10 & 6 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 2 \\ 0 & 0 \\ 10 & 6 \\ 0 & 4 \end{bmatrix}\} \qquad \text{IV}$$

All the four equations are distinct, hence the four MOD subset matrix semigroups are different.

Now using these four MOD subset matrix semigroups build the six different MOD subset special type of topological spaces.

$R_o = \{R, \cup, \cap\}$ the MOD ordinary subset special type of matrix topological space.

$R_{\cup}^{+}$ = {R, +, ∪} is a MOD subset special type of matrix topological space different from R_o.

$R_{\cap}^{+}$ = {R, +, ∩} is also a MOD subset special type of matrix topological space different from R_o and $R_{\cup}^{+}$.

$R_{\times_n}^{+}$ = {R, +, $\times_n$} is a MOD subset special type of matrix topological space different from R_o, $R_{\cup}^{+}$ and $R_{\cap}^{+}$.

$R_{\cup}^{\times_n}$ = {R, ∪, $\times_n$} is a MOD subset special type of matrix topological space different from R_o, $R_{\cup}^{+}$, $R_{\cap}^{+}$ and $R_{\times_n}^{+}$.

$R_{\cap}^{\times_n}$ = {R, ∩, $\times_n$} is a MOD subset special type of matrix topological space different from R_o, $R_{\cup}^{+}$, $R_{\cap}^{+}$, $R_{\times_n}^{+}$ and $R_{\cup}^{\times_n}$.

Thus it is a matter of routine to prove all the six MOD subset special type of topological spaces are different in their properties.

Example 2.23. Let W = {(a_1, a_2, a_3, a_4) / $a_i \in Z_{15}$; $1 \leq i \leq 4$} be the collection of all MOD row matrices with entries from Z_{15}.

Let S = P(W) = {collection of all matrices subsets from W} be the collection of all subsets of W including the empty set.

S_o ={S, ∪, ∩}, $S_{\times}^{+}$ = {S, +, ×}, $S_{\cap}^{+}$ = {S, +, ∩}, $S_{\cup}^{+}$ = {S, +, ∪}, $S_{\cup}^{\times}$ = {S, ×, ∪} and $S_{\cap}^{\times}$= {S, ×, ∩} are all six distinct MOD subset matrix special type of topological spaces.

Clearly S_o, $S_{\times}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ have MOD subset special type of strong matrix topological subspaces which are not strong ideals.

This work is also a matter of routine so left as an exercise to the reader.

Now we proceed onto show we can have MOD subset special type of matrix topological spaces which can be non commutative. Such type is possible only in case of MOD matrix subset topological spaces of special type.

We will illustrate this situation by an example or two.

Example 2.24. Let B = $\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ / $a_i \in Z_6$; $1 \leq i \leq 4\}$ be the MOD square matrices. Let P = S(B) ={collection of all matrix subsets from B} be the MOD matrix subset collection including the empty set.

$P_o = \{P, \cup, \cap\}$, $P_{\times}^{+} = \{P, +, \times\}$, $P_{\cup}^{+}=\{P, +, \cup\}$, $P_{\cap}^{+} = \{P, +, \cap\}$, $P_{\cup}^{\times} = \{P, \times, \cup\}$ and $P_{\cap}^{\times} = \{P, \times, \cap\}$ be the six distinct MOD matrix subsets special type of topological spaces using P = S(B).

Clearly $P_{\times}^{+}$, $P_{\cup}^{\times}$ and $P_{\cap}^{\times}$ are the three MOD matrix subsets noncommutative special type of topological spaces.

Let A $=\{\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\}$ and

B = $\{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\} \in P_{\times}^{\times}$ (or $P_{\cup}^{\times}$ or $P_{\cap}^{\times}$).

Consider A × B $=\{\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\} \times \{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix},$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\} = \{\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \times$$

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\}$$

$$= \{\begin{bmatrix} 0 & 2 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}\}$$

I

Consider

$$B \times A = \{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\} \times \{\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\}$$

$$= \{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\}$$

$$= \{\begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 4 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 3 & 4 \end{bmatrix}\} \quad \text{II}$$

Clearly I and II are different, that is $A \times B = B \times A$. Hence we see of the six MOD matrix subset special type of topological spaces three are non commutative and three are commutative and three are commutative.

The non commutative topological spaces can occur only when the MOD matrices under consideration are square matrices and the product is taken as the usual matrix product $\times$.

However if $\times$ is replaced by $\times_n$, then the all the six MOD subset matrix special type of topological spaces would only be commutative.

One can get strong subspaces and subspaces and ideals.

All these are left as an exercise to the reader.

Next we proceed onto describe MOD dual number subset matrix special type of topological spaces by some examples.

Example 2.25. Let $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \;/\; a_i \in \langle Z_{16} \cup g\rangle;\ 1 \leq i \leq 5\right\}$ be the collection of all 5×1 matrices. S(W) be the collection of subsets of W.

Let S(W) = D, $\{D, \cup\}$, $\{D, \cap\}$, $\{D, +\}$ and $\{D, \times_n\}$ are four distinct MOD subset semigroups.

We see $D_o = \{D, \cup, \cap\}$, $D_{\times}^{+} = \{D, +, \times\}$, $D_{\cap}^{+} = \{D, +, \cap\}$, $D_{\cup}^{\times} = \{D, \times, \cup\}$, $D_{\cap}^{\times} = \{D, \times, \cap\}$ and $D_{\cup}^{+} = \{D, +, \cup\}$ are the MOD six distinct subset matrix topological spaces associate with D.

Finding MOD topological subspaces and ideals happens to be a matter of routine so left as an exercise to the reader.

Consider E = {subsets of matrix from P = $\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$ / $a_i \in$ Z_{16} g; $1 \leq i \leq 5\}\} \subseteq$ D is such that E_o = {E, $\cup$, $\cap$}, $E_{\times}^{+}$ = {E, +, $\times$}, $E_{\cup}^{+}$ = {E, +, $\cup$}, $E_{\cap}^{+}$ = {E, +, $\cap$}, $E_{\cup}^{\times}$ = {E, $\times$, $\cup$} and $E_{\cap}^{\times}$ = {E, $\times$, $\cap$} are MOD subset dual number special type of topological subspaces and none of them are strong ideals.

They are all ideals only with respect to the two operations $\times$ and $\cap$ and not under the operations + and $\cup$.

The speciality about this E is such that E $\times$ E = $\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\}$.

Thus we call E the MOD dual number special type of subset topological subspace whose product yields zero subset matrix.

This is the speciality enjoyed only by the MOD dual number matrix subset special type of topological subspace of D.

Next we proceed onto describe MOD finite complex number matrix subset special type of topological spaces by some examples.

Example 2.26. Let S = { $\begin{bmatrix} a_1 & a_5 & a_9 & a_{13} & a_{17} & a_{21} \\ a_2 & a_6 & a_{10} & a_{14} & a_{18} & a_{22} \\ a_3 & a_7 & a_{11} & a_{15} & a_{19} & a_{23} \\ a_4 & a_8 & a_{12} & a_{16} & a_{20} & a_{24} \end{bmatrix}$ / $a_i \in$

$\mathbb{C}(Z_{15})$ = {a + bi$_F$ / a, b ∈ Z_{15}, i_F^2 = 14}, 1 ≤ i ≤ 24} be the MOD finite complex number matrices.

P(S) = V = {collection of all subset matrices of S}. V is a MOD subset finite complex number matrix semigroups under ∪ (or ∩ or × or +).

We can have the corresponding MOD finite complex number subset matrix special type of topological spaces viz., V_o ={V, ∪, ∩}, $V_\times^+$ = {V, +, ×}, $V_\cup^+$ = {V, +, ∪}, $V_\cap^+$ = {V, +, ∩}, $V_\cup^\times$ = {V, ×, ∪} and $V_\cap^\times$ = {V, ×, ∩}.

Consider P = { $\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{bmatrix}$ / $a_i \in$

Z_{15}, i_F = {ai$_F$ / a ∈ Z_{15}}; 1 ≤ i ≤ 18]; S(P) = {collection of all matrix subsets from P}.

Let S(P) = L; L_o = {L, ∪, ∩}, $L_\cup^+$ = {L, +, ∪} and $L_\cap^+$ = {L, +, ∩} are MOD finite complex number subset matrix special type of topological subspaces of L_o, $L_\cup^+$ and $L_\cap^+$ respectively.

However under $\times_n$ neither P nor S(P) = L is even a closed set.

L_o, $L_{\cup}^{+}$ and $L_{\cap}^{+}$ are ideals under $\cap$ only. That is if $A \in V$ and $B \in L_o$ or $L_{\cup}^{+}$ and $L_{\cap}^{+}$ then $A \cap B \in L_o$ (or $L_{\cup}^{+}$ and $L_{\cap}^{+}$) hence the claim.

So here we can have the main feature associated with MOD finite complex number subset matrices.

Theorem 2.5. *Let $M = \{s \times t$ matrices with entries from $C(Z_n)$; $s \neq t$, $2 \leq s, t < \infty\}$. $V = P(M)=\{$collection of all subsets of M that is power set of M$\}$.*

i) *$\{V, \cup\}$, $\{V, \cap\}$, $\{V,+\}$ and $\{V, \times_n\}$ are the four distinct MOD finite complex number subset matrix semigroups.*

ii) *$V_o= \{V, \cup, \cap\}$, $V_{\times_n}^{+} = \{V, +, \times_n\}$, $V_{\cap}^{+} = \{V, +, \cap\}$, $V_{\cup}^{+} = \{V, +, \cup\}$, $V_{\cup}^{\times_n} = \{V, \times, \cup\}$ and $V_{\cap}^{\times_n} = \{V, \cap, \times_n\}$ are the six different MOD finite complex number subset matrix special type of topological spaces.*

iii) *If $P(L) = \{$Power set of L where $L = \{s \times t$ matrices with entries from $Z_n i_F$, $s \neq t\} \subseteq M\} = W$, then W is only a MOD finite complex number subset matrix topological subspaces $W_{\cup}^{+} \subseteq V_{\cup}^{+}$, $W_{\cap}^{+} \subseteq V_{\cap}^{+}$ and $W_o \subseteq V_o$ and L as well as P(L) is not even closed under $\times_n$.*

iv) *W_o, $W_{\cup}^{+}$ and $W_{\cap}^{+}$ are ideals only under $\cap$.*

Proof is left as an exercise to the reader.

Next we proceed onto describe MOD neutrosophic subset matrix special type of topological spaces by some examples.

Example 2.27. Let $S = \{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} / a_i \in \langle Z_6 \cup I\rangle; 1 \leq i$

$\leq 8\}$ be the MOD neutrosophic 2×4 matrix collection with entries from S.

Let M = P(S) = {collection of all subsets of S or power set of S} be the MOD neutrosophic subset matrix.

We can have six distinct MOD neutrosophic matrix subset special type of topological spaces given by $M_o = \{M, \cup, \cap\}$, $M^{+}_{\times_n} = \{M, +, \times_n\}$, $\{M, +, \cup\}, = M^{+}_{\cup}$, $M^{+}_{\cap} = \{M, +, \cap\}$, $M^{\times_n}_{\cup} = \{M, \times_n, \cup\}$ and $M^{\times_n}_{\cap} = \{M, \times_n, \cap\}$. All of them are of finite cardinality.

Let N = P(T) = {collection of all subset matrices from

$$T = \{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} \text{ where } a_i \in Z_6 I;\ 1 \leq i \leq 8\}\} \subseteq M.$$

Clearly $N_o = \{N, \cup, \cap\}$, $N^{+}_{\times_n} = \{N, +, \times_n\}$, $N^{+}_{\cup} = \{N, +, \cup\}$, $N^{+}_{\cap} = \{N, +, \cap\}$, $N^{\times_n}_{\cup} = \{N, \times_n, \cup\}$ and $N^{\times_n}_{\cap} = \{N, \times_n, \cap\}$ are the 6-distinct MOD neutrosophic matrix subset special type of topological subspaces of M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ respectively.

Clearly all the six spaces are ideals only under the operation $\times_n$ and $\cap$ and not under + and $\cup$.

Thus MOD neutrosophic subset matrix special type of topological spaces has no MOD neutrosophic subset matrix special type of topological space ideals only ideals under $\times_n$ and $\cap$.

Next we proceed onto describe MOD special dual like number matrix subset special type of topological spaces using $t \times s$ – matrix collection with entries from $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ by some examples.

Example 2.28. Let L = $\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix}$ / $a_i \in \langle Z_{16} \cup h\rangle$; $h^2 = h$; $1 \leq i \leq 6\}$ be the collection of all 3×2 matrices with entries from $\langle Z_{16} \cup h\rangle$. P(L) = W = {Power set of L that is collection of all subsets from L including the empty set}.

Define the four operations +,. $\times_n$, $\cup$ and $\cap$ on W so that {W, +},{W, $\times_n$}, {W, $\cap$} and {W, $\cup$} are all MOD special dual like number subset matrix semigroups.

All the four semigroups are distinct.

Consider W_o = {W, $\cup$, $\cap$}, $W^{+}_{\times_n}$ = {W, +, $\times_n$}, $W^{+}_{\cup}$ = {W, +, $\cup$}, $W^{+}_{\cap}$ = {W, +, $\cap$}, $W^{\times_n}_{\cup}$ = {W, $\times_n$, $\cup$} and $W^{\times_n}_{\cap}$ = {W, $\times_n$, $\cap$} all the six are distinct MOD special dual like number subset matrix special type of topological spaces associated with the four semigroup.

The reader is left with the task of finding subspaces and ideals.

Let V = P(T) = {power set of T where

T = $\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix}$ / $a_i \in Z_{16}$ h, $1 \leq i \leq 6\} \subseteq$ W.

We see V_o = {V, $\cup$, $\cap$}, $V^{+}_{\times_n}$ = {V, +, $\times_n$}, $V^{+}_{\cup}$ = {V, +, $\cup$}, $V^{+}_{\cap}$ = {V, +, $\cap$}, $V^{\times_n}_{\cup}$ = {V, $\times_n$, $\cup$} and $V^{\times_n}_{\cap}$ = }V, $\times_n$, $\cap$}

are the six distinct MOD special dual like number subset matrix special type of topological subspaces of W_o, $W^{+}_{\times_n}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times_n}_{\cup}$ and $W^{\times_n}_{\cap}$ respectively.

All of them are ideals under $\times_n$ and $\cap$ however more of them is an ideal under + and or ($\cup$).

So V cannot contribute for a strong ideal though V is a strong subsemigroup hence a strong MOD special dual like number matrix subset special type of topological subspaces.

Interested reader can study the special features associated with them.

Next we proceed onto describe by example the MOD special quasi dual number subset matrix special type of topological spaces using

$$\langle Z_n \cup k\rangle = \{a + bk / a, b \in Z_n, k^2 = (n-1) k\}.$$

Example 2.29. Let S = P(W) = {Power set of W where

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} / a_i \in \langle Z_{12} \cup k\rangle, k^2 = 11k, 1 \leq i \leq 6\right\}\}$$ be the

MOD special quasi dual number subset matrices with entries from $\langle Z_{12} \cup k\rangle$.

{S, +}, {S, $\times_n$}, {S, $\cup$} and {S, $\cap$} are MOD special quasi dual number subset matrix semigroups. All the four semigroups are distinct. We see all the six MOD special quasi dual number matrix subset special type of topological spaces.

$S_o = \{S, \cup, \cap\}$, $S^{+}_{\times_n} = \{S, +, \times_n\}$ and $S^{\times_n}_{\cap} = \{S, \cap, \times_n\}$ are all distinct.

Finding ideals, subspaces and strong subspaces is a matter of routine so left as an exercise to the reader.

Next we proceed onto describe MOD natural neutrosophic subset matrix special type of topological spaces built using $Z^I_n = \{a, b + I^n_t$ / a, b $\in Z_n$, t is an idempotent or zero divisor or nilpotent element of Z_n; that is elements in Z_n which are not invertible in Z_n by examples.

Example 2.30. Let V = P(S) = {Power set of S where S = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in Z^I_4; 1 \leq i \leq 5\}$ be the MOD natural neutrosophic subset matrix special type of topological spaces associated with the MOD natural neutrosophic subset matrix semigroups $\{V, +\}$, $\{V, \times_n\}$, $\{V, \cup\}$ and $\{V, \cap\}$. The related MOD natural neutrosophic subset matrix special type of topological spaces are $V_o = \{V, \cup, \cap\}$, $V^{+}_{\times_n} = \{V, +, \times_n\}$, $V^{+}_{\cup} = \{V, \cup, +\}$, $V^{+}_{\cap} = \{V, +, \cap\}$, $V^{\times_n}_{\cup} = \{V, \times_n, \cup\}$ and $V^{\times_n}_{\cap} = \{V, \times_n, \cap\}$.

We can find ideals, subspaces and other substructures.

Let A = {(2, 0, 2, 2, 2), (2, 2, 2, 2, 2)} and B = {(0, 0, 0, 2, 0), (2, 0, 0, 0, 2), (0, 2, 2, 2, 0)} $\in$ V.

We see A × B = {(2, 0, 2, 2, 2,), (2, 2, 2, 2, 2)} × {(0, 0, 0, 2, 0), (2, 0, 0, 0, 2), (0, 2, 2, 2, 0)} = {(0, 0, 0, 0, 0)}.

Thus A and B is a MOD subset matrix topological zero divisor.

Let A = {(2, 2, 0, 2, 2), (2, 2, 2, 2, 0)} $\in$ V, we see A $\times$ A = {(0, 0, 0, 0, 0)}; thus A is a MOD subset matrix topological nilpotent element of V.

Let $D = \{(I_0^4, 1, I_0^4, 1, 0)\} \in V$

$$D \times D = \{(I_0^4, 1, I_0^4, 1, 0)\} \times \{(I_0^4, 1, I_0^4, 1, 0)\}$$
$$= \{(I_0^4, 1, I_0^4, 1, 0)\}$$

is the MOD subset matrix topological idempotent of V.

We define the following.

Definition 2.5 *Let P(S) = V = {Power set of S where S = {s $\times$ t matrices with entries from Z_n (or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ or $C(Z_n)$) MOD subset matrices $2 \leq n, s, t < \infty$.*

V be the MOD subset matrix special top ological spaces of 6 different types.

We say A, B $\in$ V is a MOD subset matrix special type of topological zero divisor pair if A $\times_n$ B = {(0)} matrix.

We say A $\in$ V is a MOD subset matrix special type of topological idempotent if A $\times_n$ A = A.

We define A $\in$ V to be a MOD subset matrix special type of topological nilpotent if A $\times_n$ A ={(0)}.

We will first illustrate this by some examples.

Example 2.31. Let V = {(P(M), the collection of all subsets from M where M = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle/ a_i \in Z_{12}, 1 \leq i \leq 5\right\}$ be the MOD subset matrix collection $V_o = \{V, \cap, \cup\}$, $V^{+}_{\times_n} = \{V, +, \times_n\}$, $V^{+}_{\cap} = \{V, +, \cap\}$, $V^{+}_{\cup} = \{V, +, \cup\}$, $V^{\times_n}_{\cup} = \{V, \times_n, \cup\}$ and $V^{\times_n}_{\cap} = \{V, \times_n, \cap\}$ be the MOD subset matrix special type of topological spaces.

Let A = $\left\{\begin{bmatrix} 3 \\ 0 \\ 6 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \\ 6 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 9 \\ 3 \\ 3 \end{bmatrix}\right\}$ and B = $\left\{\begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 8 \\ 8 \\ 4 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 0 \\ 8 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 8 \\ 8 \\ 0 \end{bmatrix}\right\} \in V$. We see A $\times_n$ B = $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}$. Thus A, B in V is a MOD subset matrix special type of topological zero divisor pair.

Let B $= \left\{\begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix}\right\} \in V$. We find B $\times_n$ B;

$$B \times_n B = \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \times_n \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix} \times_n \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \times_n \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix}\right.$$

$$\left. \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

This is the way product operation $\times_n$ is performed on V.

The set $\left\{ \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \right\} = W$ in V is such that

$$W \times_n W = \left\{ \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ 0 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 9 \\ 9 \end{bmatrix} \right\} = W.$$

Thus W is a MOD idempotent matrix subset.

Similarly if $T = \{\begin{bmatrix}4\\4\\0\\4\\0\end{bmatrix}, \begin{bmatrix}4\\4\\4\\4\\0\end{bmatrix}, \begin{bmatrix}4\\4\\0\\0\\0\end{bmatrix}\}$

and $S = \{\begin{bmatrix}6\\6\\6\\0\\0\end{bmatrix}, \begin{bmatrix}0\\6\\6\\6\\0\end{bmatrix}\} \in V$ then $T \times_n S = \{\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}\}$ is a MOD subset matrix zero divisor.

Interested reader can study MOD subset matrix idempotents, MOD subset matrix nilpotents and MOD subset matrix zero divisors of the MOD subset matrix special type of topological spaces.

This study can also be extend to MOD natural neutrosophic subset matrix special type of topological spaces built using P(M) where M is a $n \times t$ matrix with entries from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$.

Such study is left as an exercise to the reader.

Finding MOD natural neutrosophic subset matrix special type of topological zero divisors, nilpotents and idempotents.

Further the study of MOD subset special type of topological spaces using W = S(P[x]) where $P[x] = \{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in Z_n$ or $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ or Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup h\rangle_I$ where $\{W, \cup\}$, $\{W, \cap\}$, $\{W, \times\}$ and $\{W, +\}$ are MOD subset polynomial semigroups and W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ are MOD subset polynomial special type of topological spaces.

We will provide one or two examples of them. All these MOD topological special type of spaces are of infinite order.

Example 2.32. Let S = {P(M[x]) where $M[x] = \{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle Z_{124} \cup g\rangle\}\}$ = {collection of all subsets from M[x]}, $\{S, +\}$, $\{S, \cup\}$, $\{S, \cap\}$ and $\{S, \times\}$ are all MOD subset dual number polynomials S_o, $S_{\times}^{+}$, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ are MOD subset dual number polynomial special type of topological spaces, built using the MOD subset dual number polynomial semigroups.

Let $A = \{3x^2 + 4gx + 1, (5g + 3)x + 2g\}$ and $B = \{2gx^3 + 1, (5g + 3)x + 2g\} \in S$.

$A \cup B = \{3x^2 + 4gx + 1, (5g + 3)x + 2g, 2gx^3 + 1\}$ I

$A \cap B = \{(5g + 3)x + 2g\}$ II

$A + B = \{2gx^3 + 3x^2 + 4gx +2, (5g + 3)x + 2gx^3 + 2g + 1, (9g +3)x + 2g + 1 + 3x^2, (10g + 6)x + 4g\}$ III

$A \times B = \{6gx^5 + 2gx^3 + 3x^2 + 4gx + 1, 6gx^4 + (5g + 3)x + 2g + 2gx^3 (15g +9)x^3 + 12gx^2 + (5g +3)x + 6gx^2 + 2g, (30g + 9)x^2 + 12gx\}$ IV

All the four equations are distinct so the four semigroups are distinct hence the six MOD special type of subset polynomial dual number topological spaces are distinct.

Finding MOD dual number subset topological spaces zero divisors, nilpotents and idempotents happens to be a matter of routine.

Example 2.33. Let R = {P(T[x]) where T[x] = $\{\sum_{i=0}^{\infty} a_i x^i /$ $a_i \in \langle Z_{47} \cup g\rangle_I\}\}$ = {collection of all MOD natural neutrosophic dual number subset polynomial collection). {R, $\cup$}, {R, $\cap$}, {R, $\times$} and {R, +} be the MOD subset natural neutrosophic dual number semigroups.

Using them we built R_o, $R_{\cup}^{+}$, $R_{\cap}^{+}$, $R_{\times}^{+}$, $R_{\cup}^{\times}$ and $R_{\cap}^{\times}$ be the MOD natural neutrosophic dual number polynomial subset special type of topological spaces.

The reader is left as an exercise to find MOD natural neutrosophic dual number subset polynomial topological spaces zero divisors, nilpotents and idempotents.

Example 2.34. Let W = {P(S[x])} = {collection of all subsets polynomial from S[x] = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle Z_{12} \cup I\rangle_I\}\}$ be the MOD natural neutrosophic-neutrosophic subset polynomial collection. W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ are the MOD natural neutrosophic-neutrosophic subset special type of topological polynomial spaces built using {W, +}, {W, $\cap$}, {W, $\cup$} and {W, $\times$} the MOD natural neutrosophic subset polynomial semigroups.

Finding MOD natural neutrosophic-neutrosophic subsets polynomial special type of topological space idempotents, nilpotents and zero divisors are left as exercise to the reader.

Example 2.35. Let S ={(P(M[x]$_9$) = {collection of all subsets from M[x]$_9$ = $\{\sum_{i=0}^{9} a_i x^i$ / $a_i \in Z_{12}$; $x^{10} = 1\}\}$ be the MOD subset finite degree polynomial collection {S, +}, {S, ∪}, {S, ∩} and {S, ×} be the MOD subset polynomial semigroups, S_o, $S_\times^+, S_\cup^+$, $S_\cap^+$, $S_\cup^\times$ and $S_\cap^\times$ be the MOD subset polynomial special type of topological space.

All properties can be studied by the interested reader.

Example 2.36. Let T = {S(P[x]$_{18}$)} = {Collection of all subsets from P[x]$_{18}$ = $\{\sum_{i=0}^{18} a_i x^i$ / $a_i \in \langle Z_{19} \cup h\rangle_I$; $h^2 = h$, $x^{19} = 1$ }} be the MOD natural neutrosophic special dual like number subset polynomials collection T_o, $T_\times^+, T_\cup^+$, $T_\cap^+$, $T_\cup^\times$ and $T_\cap^\times$ be the MOD natural neutrosophic special dual like number subset polynomial special type of topological spaces built using MOD natural neutrosophic special dual like number subset polynomial semigroups {T, ∪}, {T, ∩}, {T, ×} and {T, +}.

Studying the MOD natural neutrosophic special dual like number subset polynomial special type of topological space zero divisors, idempotents and nilpotents is considered as a matter of routine and hence left as an exercise to the reader. Also order of these special type of topological spaces are of finite order.

Example 2.37. Let B = {P(S[x]$_5$} = {collection of all subsets from S[x]$_5$ = $\{\sum_{i=0}^{5} a_i x^i$ / $a_i \in C^I(Z_{10})$; $x^6 = 1$ } be the MOD natural neutrosophic finite complex modulo coefficient polynomial subset collection} be the MOD natural neutrosophic finite complex modulo integer subset polynomial collection. {B, +}, {B, ∪}, {B, ∩} and {B, ×} be the MOD natural neutrosophic finite complex number subset polynomial semigroups.

Let B_o, $B^{+}_{\times}$, $B^{+}_{\cup}$, $B^{+}_{\cap}$, $B^{\times}_{\cup}$ and $B^{\times}_{\cap}$ be the MOD natural neutrosophic finite complex number subset polynomial special type of topological space associated with these MOD semigroups.

The reader is expect to find MOD natural neutrosophic finite complex number subset polynomial special topological space zero divisors, nilpotents and idempotents.

The reader is left with the task of finding order of B.

Next we proceed onto give several problem for the interested reader.

PROBLEMS

1. Study the MOD subset semigroup S ={P(Z_n) under $\cup$}.

   i) Can S have MOD subset ideals?

   ii) Can S have MOD subset subsemigroups?

   iii) Can we say if n is a composite number S will have more ideals?

   Is (iii) true? Justify your result.

   iv) Can we say if n is a prime S will have less number of ideals?

   Is (iv) true? Justify your claim.

   v) Prove every subset is a MOD subset semigroup and is not an ideal.

2. Let B = {P(Z_{48}), $\cup$} be the MOD subset semigroup.

   Study questions (i) to (v) of problem (1) for this B.

3. Let D = {P(Z_{53}), $\cup$} be the MOD subset semigroup.

   Study questions (i) to (v) of problem (1) for this D.

4. Can there be any structure difference between B and D of problems 2 and 3 respectively (as the operation is only $\cup$}?

5. Let M = {P(Z_{128}), $\cup$} be the MOD subset semigroup.

   Study questions (i) to (v) of problem (1) for this M.

6. Can there be structural difference between B in problem (2) and this M in problem (5)?

7. Can there be any structural difference between D in problem (3) and M is problem (5)?

8. Can we say what ever be n ($2 \leq n < \infty$) in $P(Z_n)$ under the binary operation '$\cup$' they behave uniformly immaterial of n odd or n even or n prime or $n = p^t$; $t \geq 2$ and p a prime? Prove your claim.

9. Let $W = \{P(Z_n), \cap\}$ be the MOD subset semigroup under the operation $\cap$.

   Study questions (i) to (v) of problem (1) for this W.

10. Let $V = \{P(Z_{16}), \cap\}$ be the MOD subset semigroup under the operation $\cap$.

    Study questions (i) to (v) of problem (1) for this V.

11. Let $R = \{P(Z_{47}), \cap\}$ be the MOD subset semigroup.

    Study questions (i) to (v) problem (1) for this R.

12. Let $E = \{P(Z_{484}), \cap\}$ be the MOD subset semigroup under $\cap$.

    Study questions (i) to (v) of problem (1) for this E.

13. Compare R of problem (11) with V of problem (10).

14. Find the difference if it exist in structure between R of problem (11) and E of problem (12).

15. Compare the MOD subset semigroups in problems (10) and (12).

16. Let $T = \{P(Z_{15}), +\}$ be the MOD subset semigroup.

    i) Show T has only few MOD subset subsemigroups.

    ii) Is T a S-semigroup?

    iii) Can T have S-subsemigroups?

iv) Can T have S-ideals?

v) Obtain any other special feature associated with T.

17. Compare the MOD subset semigroups $\{P(Z(_n), \cup\}$, $\{P(Z_n), \cap\}$ and $\{P(Z_n), +\}$ among themselves.

Prove $\{P(Z_n), +\}$ has the minimum number of MOD subset subsemigroups in comparison with $\{P(Z_n), \cup\}$ and $\{P(Z_n), \cap\}$.

18. Let N = $\{P(Z_{79}), +\}$ be the MOD subset semigroup.

i) Study questions (i) to (v) of problem (16) for this N.

ii) Prove N has less number of MOD subset subsemigroups in comparison with T.

19. Let U = $\{P(Z_{243}), +\}$ be the MOD subset semigroup.

i) Study questions (i) to (v) of problem (16) for this U.

ii) Compare U with N of problem (18).

iii) Compare U with T of problem (16).

20. Let Y = $\{P(Z_{28}), \times\}$ be the MOD subset semigroup under product;

i) Is Y a S-subset semigroup?

ii) Can Y have MOD subset zero divisors?

iii) Can Y have MOD subset S-zero divisors?

iv) Can Y have MOD subset idempotents?

v) Can Y have MOD subset S-idempotents?

vi) Does Y contain MOD subset nilpotents?

vii) Find all MOD subset subsemigroups of Y which are not ideals of Y.

viii) Find all MOD subset ideals of Y.

ix) Can Y have MOD subset S-ideals?

x) Obtain any other special feature associated with Y.

21. Let C = $\{P(Z_{43}), \times\}$ be the MOD subset semigroup.

i) Study questions (i) to (x) of problem (20) for this C.

ii) Prove C is a S-subset semigroup.

iii) Prove C has less number of MOD subset subsemigroups and ideals in comparison with Y of problem (20).

iv) Prove C has no nontrivial MOD subset zero divisors, idempotents and nilpotents.

v) Obtain any other special feature associated with C.

22. Let G = $\{P(Z_{5^8}), \times\}$ be the MOD subset semigroup.

i) Study questions (i) to (x) of problem 20 for this G.

ii) Prove G has more MOD subset nilpotents than Y in problem 20 and C in problem 21.

iii) Prove G has more ideals than C in problem 21.

iv) Compare Y of problem 20 with this G.

v) Compare C of problem 21 with this G.

23. Let P = $\{P(Z_n);\ 2 \leq n < \infty\}$ be the power set of Z_n. $\{P, \cup\}$, $\{P, \cap\}$, $\{P, +\}$ and $\{P, \times\}$ are the four distinct MOD subset semigroups of P.

Let $P_o = \{P, \cup, \cap\}$, $P_{\times}^{+} = \{P, +, \times\}$, $P_{\cup}^{+} = \{P, +, \cup\}$, $P_{\cap}^{+} = \{P, +, \cap\}$, $P_{\cup}^{\times} = \{P, \times, \cup\}$ and $P_{\cap}^{\times} = \{P, \times, \cap\}$ be the six MOD subset special type of topological spaces.

i) Which MOD subset special type of topological space has maximum number of MOD subset special type of topological subspaces?

ii) Which of them given by (i) are MOD subset ideals?

iii) Find all MOD subset special type of topological zero divisors, nilpotents and idempotents of P_o, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\times}^{+}$, $P_{\cap}^{\times}$ and $P_{\cap}^{\times}$.

iv) Can P have MOD subset ideals?

24. Let $V = \{P(Z_{24})\}$ be the MOD subset collection. V_o, $V_{\times}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{+}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be the MOD subset special type of topological spaces.

i) Study questions (i) to (iv) of problem (23) for this V.

ii) Obtain any other special feature associated with V.

25. Let $W = \{P(Z_{53})\}$ be the MOD subset collection. Let W_o, $W_{\times}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ be the MOD subset special type of topological spaces associated with W.

i) Study questions (i) to (iv) of problem (23) for this W.

ii) Compare W with V of problem (24).

iii) Obtain all special features associated with W.

26. Let $R = \{P(Z_{64})\}$ be the MOD subset; R_o, $R_{\cup}^{+}$, $R_{\cap}^{+}$, $R_{\times}^{+}$, $R_{\cup}^{\times}$ and $R_{\cap}^{\times}$ be the MOD subset special type of topological spaces associated with R.

i) Compare R with W and V of problem (25) and (24) respectively.

ii) Study questions (i) to (iv) of problem (23) for this R.

27. Let $S_1 = \{P(\langle Z_{10} \cup I\rangle)\}$ be the MOD neutrosophic subset collection. $\{S_1, \cup\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) of problem (1) for this S_1.

ii) Find all special features associated with this S_1.

iii) How is this S_1 different from S in problem 1?.

28. Let $R_1 = \{P(\langle Z_{19} \cup I\rangle), \cup\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) of problem (1) for this R_1.

ii) Compare R_1 with S_1 of problem (27).

29. Let $D_1 = \{P(\langle Z_{64} \cup I\rangle), \cup\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) problem (1) for this D_1.

ii) Compare D_1 with R_1 of problem 28.

iii) Compare D_1 with S_1 of problem 27.

30. Let $L_1 = \{P(\langle Z_{16} \cup I\rangle), \cap\}$ be the MOD neutrosophic subset semigroup.

Study questions (i) to (v) of problem (1) for this L_1.

31. Let $X_1 = \{P(\langle Z_{29} \cup I\rangle), \cap\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) of problem 1 for this X_1

ii) Compare this X_1 with L_1 of problem 30.

32. Let $Y_1 = \{P(\langle Z_{210} \cup I\rangle), \cap\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) of problem (1) for this Y_1.

ii) Compare this Y_1 with X_1 of problem 31.

iii) Compare this Y_1 with L_1 of problem 30.

33. Let $E_1 = \{P(\langle Z_{24} \cup I\rangle), +\}$ be the MOD subset neutrosophic semigroup under +.

i) Study questions (i) to (v) of problem (16) for this E_1.

34. Compare $G = \{P(\langle Z_n \cup I\rangle), +\}$ with $E = \{P(\langle Z_n \cup I\rangle), \cap\}$ and $F = \{P(\langle Z_n \cup I\rangle), \cup\}$; $2 \leq n < \infty$.

35. Let $M_2 = \{P(\langle Z_{47} \cup I\rangle), +\}$ be the MOD subset neutrosophic semigroup.

Study questions (i) to (v) of problem 16 for this M_2.

36. Let $P_3 = \{P(\langle Z_{3^7} \cup I\rangle), +\}$ be the MOD neutrosophic subset semigroup.

i) Study questions (i) to (v) of problem(16) for this P_3.

ii) Compare this P_3 with M_2 of problem 35.

iii) Compare this P_3 with problem E_1 of 33.

iv) Compare problem E_1 of 33 with problem M_2 of 35.

37. Let $W_3 = \{P(\langle Z_{48} \cup I\rangle), \times\}$ be the MOD neutrosophic subset semigroup under $\times$.

    i) Study questions (i) to (x) of problem (20) for this W_3.

38. Let $M_5 = \{P(\langle Z_{17} \cup I\rangle), \times\}$ be the MOD neutrosophic subset semigroup.

    i) Study questions (i) to (x) of problem (20) for this M_5.

    ii) Compare this M_5 with W_3 of problem 37.

39. Let $N_8 = \{P(\langle Z_{5^{20}} \cup I\rangle), \times\}$ be the MOD neutrosophic subset semigroup.

    i) Study questions (i) to (x) of problem (20) for this N_8.

    ii) Compare this N_8 with M_5 of problem 38.

    iii) Compare this N_8 with W_3 of problem 37.

40. Let $T = \{P(\langle Z_n \cup I\rangle);\ 2 \leq n < \infty\}$ be the power set of $\langle Z_n \cup I\rangle$. Let $\{T, +\}$, $\{T, \times\}$, $\{T, \cup\}$ and $\{T, \cap\}$ be MOD neutrosophic subset semigroups. $T_o = \{T, \cup, \cap\}$, $T_{\times}^{+} = \{T, +, \times\}$, $T_{\cup}^{+} = \{T, +, \cup\}$, $T_{\cap}^{+} = \{T, +, \cap\}$, $T_{\cup}^{\times} = \{T, \times, \cup\}$ and $T_{\cap}^{\times} = \{T, \times, \cap\}$ be the MOD neutrosophic subset special type of topological spaces.

    Study questions (i) to (iv) of problem(23) for this T.

41. Let $W = \{P(\langle Z_{96} \cup I\rangle)\}$ be the MOD neutrosophic subsets of $\langle Z_{96} \cup I\rangle$, W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ be the MOD neutrosophic subset special type of topological spaces associated with the power set P(W).

    Study questions (i) to (iv) of problem (23) for this W.

42. Let J = $\{P(\langle Z_{41} \cup I\rangle)\}$ be the MOD neutrosophic subsets of $\langle Z_{41} \cup I\rangle$. J_o, $J_{\times}^{+}$, $J_{\cap}^{+}$, $J_{\cup}^{+}$, $J_{\cup}^{\times}$ and $J_{\cap}^{\times}$ be the MOD neutrosophic subset special type of topological spacesof the power set J.

    i) Study questions (i) to (iv) of problem (23) for this J.

    ii) Compare this J with W of problem 42.

43. Let K = $\{P(\langle Z_{29} \cup I\rangle)\}$ be the MOD neutrosophic subset special type of topological spaces constructed using K.

    i) Study questions (i) to (iv) of problem (23) for this K.

    ii) Compare K with J in problem (42).

    iii) Compare K with W in problem (41).

    iv) Compare J in problem (42) and W in (41).

44. Let H = $\{P(Z_{42})\}$ be the MOD finite complex number subsets collection. H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ be the MOD finite complex number subset special topological spaces.

    i) Study questions (i) to (iv) of problem (23) for this H.

    ii) Find all special features enjoyed by H.

    iii) Compare H with K of problem (43)

    iv) Compare H with J of problem (42).

45. Let B = $\{P(C(Z_{23}))\}$ be the MOD finite complex number subsets collection. B_o, $B_{\times}^{+}$, $B_{\cup}^{+}$, $B_{\cap}^{+}$, $B_{\cup}^{\times}$ and $B_{\cap}^{\times}$ be the MOD finite complex number subset special type of topological spaces using B.

    i) Study questions (i) to (iv) of problem (23) for this B.

ii) Compare this B with H of problem (44).

iii) Compare this B with K of 44.

46. Let D = $\{P(C(Z_{128}))\}$ be the MOD finite complex number subset collection. D_o, $D_\times^+$, $D_\cup^+$, $D_\cap^+$, $D_\cup^\times$ and $D_\cap^\times$ be the MOD finite complex number subset special topological spaces.

i) Study questions (i) to (iv) of problem (23).

ii) Compare D with B of problem (45)

iii) Compare D with H of problem (44).

47. Work with all properties of MOD subset finite complex number semigroups built using $P_1 = \{P(C(Z_{43}))\}$, $P_2 = \{P(C(Z_{24}))\}$, $P_3 = \{P(C(Z_{3^7}))\}$ under the $\cup$, $\cap$, $+$ and $\times$.

48. Let E = $\{P(\langle Z_n \cup g\rangle)\}$ be the MOD subset dual number collection from $\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$. $\{E, \cup\}$ be the MOD subset dual number semigroup.

Study questions (i) to (v) of problem (1) for this E.

49. Let V = $\{P(\langle Z_{13} \cup g\rangle), \cup\}$ be the MOD subset dual number semigroup.

i) Study questions (i) to (v) of problem (1) for this V.

ii) Compare this V with E of problem 48.

50. Let W = $\{P(\langle Z_6 \cup g\rangle), \cup\}$ be the MOD subset dual number semigroup.

Study questions (i) to (v) of problem 1 for this E.

51. Let P = $\{P(\langle Z_{7^6} \cup g\rangle), \cup\}$ be the MOD subset dual number semigroup.

i) Study questions (i) to (v) of problem 1 for this P.

ii) Compare this P with V of problem 49

iii) Compare this P with E of problem 48.

52. Let Z = {P ($\langle Z_n \cup g\rangle$), $\cap$} be the MOD subset dual number semigroup.

i) Study questions (i) to (v) of problem (1) for this Z.

ii) Compare Z with the MOD subset dual number semigroup in which '$\cap$' is replaced by $\cup$.

iii) Obtain all special features enjoyed by Z.

53. Let A = {P ($\langle Z_{42} \cup g\rangle$), +} be the MOD subset dual number semigroup.

i) Study questions (i) to (v) of problem (16) for this A.

ii) Compare A with the MOD semigroup M = {P ($\langle Z_{42} \cup g\rangle$), $\cup$}.

54. Let B = {P ($\langle Z_{53} \cup g\rangle$), +} be the MOD subset dual number semigroup

i) Study questions (i) to (v) of problem (16) this B.

ii) Compare B with A of problem 53.

55. Let C = {P ($\langle Z_{121} \cup g\rangle$), +} be the MOD subset dual number semigroup.

i) Study questions (i) to (v) of problem (16) for this C.

ii) Compare this C with A of problem 53.

iii) Compare this C with B of problem 54.

56. Let $D = \{P (\langle Z_n \cup g\rangle), \times\}$ be the MOD dual number subset semigroup.

    i) Study properties (i) to (x) of problem 20 for this D.

    ii) Compare the D with $D_1 = \{P (\langle Z_n \cup g\rangle), \cup\}$, $D_2 = \{P (\langle Z_n \cup g\rangle), \cap\}$ and $D_3 = \{P (\langle Z_n \cup g\rangle), +\}$.

57. Let $E = \{P (\langle Z_{40} \cup g\rangle), \times\}$ be the MOD subset dual number semigroup.

    i) Study properties (i) to (x) of problem (20) for this E.

    ii) Prove E has more numbers of zero divisors and nilpotent subsets than Y in problem 20.

    iii) Obtain any other special feature associated with E.

58. Let $F = \{P (\langle Z_7 \cup g\rangle), \times\}$ be the MOD dual number subset semigroup.

    i) Study questions (i) to (x) of problem (20) for this F.

    ii) Compare this F with E of problem 57.

    iii) Prove E in problem 57 has more zero divisor than F.

59. Let $G = \{P (\langle Z_{192} \cup g\rangle), \times\}$ be the MOD subset dual number semigroup.

    i) Study questions (i) to (x) of problem (20) for this G.

    ii) Compare this G with (F) of problem 58.

    iii) Compare this G with (E) of problem 57.

    iv) Compare this G with (D) of problem 56.

v) Prove G has more number of nilpotents than D, E and F of problems 56, 57 and 58 respectively.

60. Let H = {P (⟨$Z_n \cup g$⟩)} be the MOD dual number subset collection H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ be the MOD subset dual number special type of topological spaces.

i) Prove all the six spaces are distinct.

ii) Prove H has more number of MOD dual number subset topological nilpotents where every be n, $2 \leq n < \infty$.

iii) Prove H has more number of MOD dual number subset topological zero divisors what be n; $2 \leq n < \infty$.

iv) Study questions (i) to (iv) of problem (23) for this H.

61. Let I = {P (⟨$Z_{24} \cup g$⟩)} be the MOD dual number subset collection I_o, $I_{\times}^{+}$, $I_{\cup}^{+}$, $I_{\cap}^{+}$, $I_{\cup}^{\times}$ and $I_{\cap}^{\times}$ be the MOD dual number special type of topological spaces.

i) Study questions (i) to (iv) of problem (60) for this H.

ii) Prove H has more number of idempotents.

62. Let J = {P (⟨$Z_{47} \cup g$⟩)} be the MOD dual number collection J_o, $J_{\times}^{+}$, $J_{\cup}^{+}$, $J_{\cap}^{+}$, $J_{\cup}^{\times}$ and $J_{\cap}^{\times}$ be the MOD subset dual number special type of topological spaces.

i) Study questions (i) to (iv) of problem (60) for this J.

ii) Prove J has less number of MOD subset dual number topological idempotents, zero divisors and nilpotents than I in problem 61.

63. Let K = {P (⟨$Z_{144} \cup g$⟩)} be the MOD subset dual number associated topological spaces K_o, $K_{\cup}^{+}$, $K_{\cap}^{+}$, $K_{\times}^{+}$, $K_{\cup}^{\times}$ and $K_{\cap}^{\times}$.

   i) Study questions (i) to (iv) of problem (60) for this K.

   ii) Prove K has more number of MOD subset dual number topological nilpotents.

   iii) Compare K with J of problem 62.

   v) Compare K with I of problem 61.

64. Let L = {P (⟨$Z_n \cup h$⟩)} be the MOD special dual like number subsets from ⟨$Z_n \cup h$⟩ = {a + bh / a, b ∈ Z_n, $h^2 = h$}; $2 \leq n < \infty$.

   i) Prove {L, ∪}, {L, ∩}, {L, ×} and {L, +} are four distinct MOD special dual like number subset semigroups.

   ii) Prove $L_{\times}^{+}$ = {L,+, ×}, L_o = {L, ∪, ∩}, $L_{\cap}^{+}$ = {L, +, ∩}, $L_{\cup}^{+}$ = {L, +, ∪}, $L_{\cup}^{\times}$ = {L, ×, ∪} and $L_{\cap}^{\times}$ = {L, ×, ∩} are six distinct MOD special dual like number subset special type of topological spaces.

   iii) Study for this {L, ∪} and {L, ∩} questions (i) to (iv) of problem (1).

   iv) Study for {L, +} questions (i) to (v) of problem 16.

   v) Study questions (i) to (x) of problem (20) for this {L, ×}.

   vi) Find all MOD subset special dual like number special topological subspace zero divisors, idempotents and nilpotent subsets.

65. Let M = {P (⟨$Z_{20} \cup h$⟩)} be the MOD special dual like number subsets M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$ be the MOD special dual like number subsets special types of topological spaces associated with the MOD special dual like number subset semigroups {M, +}, {M, ×}, {M, ∪} and {M, ∩}.

   i) Study questions (i) to (vi) of problem 64 for this M.

   ii) Derive all special features associated with M.

66. Let N = {P (⟨$Z_{13} \cup h$⟩)} be the MOD special dual like number subsets N_o, $N_{\times}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times}$ and $N_{\cap}^{\times}$ be the MOD special dual like number subset special type of topological spaces associated with {N, ×}, {N, +}, {N, ∪} and (N, ∩}

   i) Study questions (i) to (vi) of problem 64 for this N.

   ii) Compare this N with M of problem (65).

   iii) Prove N has less number MOD topological idempotents zero divisors and nilpotents in comparison with M problem 65.

67. Let O = {P (⟨$Z_{2^3 3^5 5^2} \cup h$⟩)} be the MOD special dual like number subsets. Let O_o, $O_{\times}^{+}$, $O_{\cup}^{+}$, $O_{\cap}^{+}$, $O_{\cup}^{\times}$ and $O_{\cap}^{\times}$ be the MOD special dual like number subset special type of topological spaces associated with the MOD special dual like number subset semigroups {O,+}, {O, ×}, {O, ∩} and {O, ∪}.

   i) Study questions (i) to (vi) of problem (64) for this O.

   ii) Compare this O with N of problem (66).

iii) Compare this O with M of problem (65).

iv) Prove O has more number of MOD topological nilpotents than M and N in problems (66) and (66) respectively.

68. Let P = {P ($\langle Z_n \cup k\rangle$)} be the MOD special quasi dual number subsets from $\langle Z_n \cup k\rangle = \{a + bk / a, b \in Z_n, k^2 = (n-1)k\}$. {P, +}, {P, ×}, {P, ∪} and {P, ∩} be the MOD special quasi dual number subset semigroups. Let P_o, $P_{\times}^{+}$, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\cup}^{\times}$ and $P_{\cap}^{\times}$ be the MOD special quasi dual number subset special type of topological spaces.

i) Study questions (i) to (vi) of problem 64 for this P.

ii) Study questions to (iv) of problem (60) for this P.

iii) Obtain any other special feature associated with P.

69. Let Q = {P ($\langle Z_{12} \cup k\rangle$)} be the MOD special quasi dual number subset. {Q, +}, {Q, ×}, {Q, ∩} and {Q, ∪} be the MOD special quasi dual number subset semigroups. Let Q_o, $Q_{\times}^{+}$, $Q_{\cup}^{+}$, $Q_{\cap}^{+}$, $Q_{\cup}^{\times}$ and $Q_{\cap}^{\times}$ be the MOD special quasi dual number subset special type of topological space built using the MOD subset semigroups.

Study questions (i) to (iii) of problem (68) for this Q.

70. Obtain all special features associated with each of these $P(\langle Z_n \cup g\rangle_I)$ the collection of all subsets of MOD natural neutrosophic dual numbers.

71. Let R = {P ($\langle Z_5 \cup k\rangle$)} be the MOD special quasi dual number subsets of $\langle Z_5 \cup k\rangle$. {R, +}, {R, ×}, {R, ∪} and {R, ∩} be the MOD special quasi dual number subset semigroups. R_o, $R_{\times}^{+}$, $R_{\cup}^{+}$, $R_{\cap}^{+}$, $R_{\cup}^{\times}$ and $R_{\cap}^{\times}$ be the

MOD special quasi dual number subset special type of topological spaces associated with the four semigroups.

i) Study questions (i) to (iii) of problem (68) for this R.

ii) Obtain any other special feature associated with R.

iii) Compare R with problem Q of problem (69)

72. Let S = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in P(Z_{120}); 1 \leq i \leq 5\}$ be the MOD subset matrix collection. $\{S, \cup\}$, $\{S, \cap\}$, $\{S, +\}$ and $\{S, \times\}$ be the MOD subset matrix semigroups.

$S_o = \{S, \cup, \cap\}$, $S_{\times}^{+} = \{S, +, \times\}$, $S_{\cup}^{+} = \{S, +, \cup\}$, $S_{\cap}^{+} = \{S, +, \cap\}$, $S_{\cap}^{\times} = \{S, \times, \cup\}$ and $S_{\cap}^{\times} = \{S, \times, \cap\}$ be the MOD subset matrix special type of topological spaces associated with the MOD subset semigroups.

i) Can $\{S, \cup\}$, $\{S, \cap\}$, $\{S, \times\}$ and $\{S, +\}$ have ideals?

ii) Find all MOD subset matrix subsemigroups which are not ideals.

iii) Which of the four MOD subset semigroups are S-semigroup?

iv) What are the MOD subset topological zero divisors in $S_{\cup}^{\times}$, $S_{\cap}^{\times}$ and $S_{\times}^{+}$?

v) Find all MOD subset topological idempotents in $S_{\times}^{+}$.

vi) Can the MOD subset topological spaces have S-zero divisors? Justify!

vii) Can these MOD subset topological spaces have S-idempotents?

viii) Can the MOD subset special type of matrix topological spaces contain topological nilpotent elements?

ix) Prove S cannot have MOD matrix subset-strong topological ideals.

x) Prove S can have MOD matrix subset strong special type of topological subspaces.

73. Let $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle/ a_i \in P(Z_{13}), 1 \leq i \leq 6 \right\}$ be the matrix

subsets collection $\{W, \cup\}$, $\{W, \cap\}$, $\{W, \times_n\}$ and $\{W, +\}$ be the MOD matrix subset semigroups. S_o, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\times_n}^{+}$, $S_{\cup}^{\times_n}$ and $S_{\cap}^{\times_n}$ be the MOD matrix subset special type of topological spaces.

i) Study questions (i) to (x) of problem (72) for this S.

ii) Compare S in (72) with this W.

74. Let $P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle/ a_i \in P(Z_{48}), 1 \leq i \leq 6 \right\}$

be the MOD subset matrix collection $\{P, \times_n\}$, $\{P, +\}$, $\{P, \times\}$ and $\{P, \cup\}$ and $\{P, \cap\}$ be the MOD matrix subset semigroups. P_o, $P_{\times}^{+}$, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\cup}^{\times}$, $P_{\cap}^{\times}$, $P_{\cup}^{\times_n}$, $P_{\cap}^{\times_n}$ and $P_{\times_n}^{\times}$ be the MOD matrix subset special type of topological spaces associated with these five semigroups.

i) Find ideals in these five MOD semigroups.

ii) Show $\{P, \times\}$ and $\{P, \times_n\}$ have zero divisors and idempotents.

iii) Can $\{P, \times\}$ and $\{P, \times_n\}$ have nilpotents?

iv) Can $\{P, \times\}$ and $\{P, \times_n\}$ have S-zero divisors?

v) Can $\{P, \times\}$ and $\{P, \times_n\}$ be S-semigroups?

vi) Prove $\{P, +\}$ is always a S-semigroup.

vii) Prove the MOD subset matrix special type of topological spaces has no strong ideals.

viii) Prove MOD subset matrix special type of topological spaces have strong topological subspaces and none of them are topologoical ideals.

ix) Prove MOD subset topological matrix spaces under usual product $\times$ has left zero divisors which are not right zero divisors.

x) Prove MOD subset topological matrix spaces under $\times$ has left ideals which are not right ideals and vice versa.

xi) Can MOD subset topological matrix spaces have

a) S-zero divisors under $\times$ or $\times_o$?

b) S-idempotents under $\times$ or $\times_o$?

c) S-ideals under $\times$ or $\times_n$?

d) S-topological subset matrix subspaces which are not S-ideals?

(By MOD S-topological matrix subspaces we mean under both the operations of the MOD special type of subset spaces the associated subsemigroups must be S-subsemigroups).

xii) Obtain any other special features enjoyed by these 10 distinct MOD special type of subset matrix topological spaces.

75. Let $S_1 = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle/ a_i \in P(C(Z_{11})), 1 \leq i \leq 6\right\}$ be the MOD matrix subset finite complex number collection $\{S_1, \times_n\}$, $\{S_1, \cup\}$, $\{S_1, \cap\}$ and $\{S_1, +\}$ are the four MOD subset matrix semigroups and related with these MOD subset matrix semigroups we have 6 MOD subset matrix special topological spaces.

i) Study questions (i) to (x) of problem (72) for this S_1

ii) Compare this S_1 with W of problem (73)

iii) What is the role played by the finite complex number subset collection?

76. Let $M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle/ a_i \in P(C(Z_{24})); 1 \leq i \leq 9\right\}$ be the MOD subset matrix finite complex number collection

i) Study questions (i) to (xii) of problem (74) for this M.

ii) Distinguish this M from P of problem 74.

77. Let $T = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix} \middle/ a_i \in P(C(Z_{16})); 1 \leq i \leq 8\right\}$ be the MOD subset matrix finite complex number collection.

i) Study questions (i) to (x) of problem (71) for this T.

ii) Distinguish between S of problem 74 and that of T.

78. Let N = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} / a_i \in\right.$

$P(\langle Z_{12} \cup \rangle)$; $1 \leq i \leq 12\}$ be the collection of MOD subset matrix neutrosophic collection.

i) Study questions (i) to (x) of problem (72) for this N.

79. Let O = $\left\{\begin{bmatrix} a_1 & a_6 \\ a_2 & a_7 \\ a_3 & a_8 \\ a_4 & a_9 \\ a_5 & a_{10} \end{bmatrix} / a_i \in P(\langle Z_{19} \cup I\rangle), 1 \leq i \leq 10\right\}$ be

the collection of all MOD subset matrix.

i) Study questions (i) to (x) of problem 72 for this O.

ii) Compare N and S problems 78 and 75 respectively with O.

80. Let B = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} / a_i \in P(\langle Z_{480} \cup I\rangle);\right.$

$1 \leq i \leq 25\}$ be the MOD subset matrix neutrosophic collection.

i) Study questions (xii) to (xi) of problem 74 for this B.

ii) Compare B with problem 76 of M.

81. Let P = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} / a_i \in P(\langle Z_{19} \cup h\rangle), 1 \leq i \leq 6\right\}$ be the MOD subset matrix collection dual like number collection.

i) Study questions (i) to (x) of problem (72) for this A.

ii) Compare this A with O of problem (79).

82. Let D = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} / a_i \in P(\langle Z_{40} \cup h),\right.$

$1 \leq i \leq 16\}$ be the MOD subset matrix special dual like number collection.

i) Study questions (i) to (xii) problem (74) for this D.

ii) Obtain all special features associated with D.

83. Let $E = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle/ a_i \in P(\langle Z_{45} \cup g\rangle); 1 \leq i \leq 12\right\}$ be

the MOD subset matrix dual number collection

i) Study questions (i) to (x) of problem (72) for this E.

ii) Prove E has more number of zero divisors and nilpotents.

iii) Derive all special features associated with E.

84. Let $F = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle/ a_i \in\right.$

$P(\langle Z_{140} \cup g\rangle)$, $1 \leq i \leq 25\}$ be the MOD square matrix dual number subset collection.

i) Study questions (i) to (xii) of problem (74) for this F.

ii) Compare this F with D in problem 82.

85. Let $G = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} \middle/ a_i \in P(\langle Z_{48} \cup k\rangle);\right.$

$1 \leq i \leq 10\}$ be the MOD subset matrix of special quasi dual number collection.

i) Study questions (i) to (x) of problem (72) for this G.

ii) Enlist all the special features enjoyed by this G.

86. Let H = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{bmatrix} \right. / a_i \in$

$P(\langle Z_{43} \cup k\rangle)$, $1 \leq i \leq 36\}$ be the MOD special quasi dual number matrix subset collection.

i) Study questions (i) to (xii) of problem 74 for this H.

ii) Compare this H with F of problem 84.

87. Let S = $\{P(Z_{12}^I) / Z_{12}^I = \{a + I_t^{12}$ where $a \in Z_{12}$, $t \in Z_{12}$ is a zero or a zero divisor or a nilpotent or an idempotent$\}$ be the MOD natural neutrosophic subset collection. $\{S, +\}$, $\{S, \cup\}$, $\{S, \cap\}$ and $\{S, \times\}$ be the MOD natural neutrosophic subset semigroup. S_o, $S_\times^+$, $S_\cup^+$, $S_\cap^+$, $S_\cup^\times$ and $S_\cap^\times$ be the MOD natural neutrosophic subset special type of topological spaces.

i) Study questions (i) to (v) of problem (1) for this S.

ii) Distinguish S from R $=\{P(Z_{12})\}$ where Z_{12} is replaced by Z_{12}^I.

iii) Does the collection of MOD natural neutrosophic elements of Z_{12}^{I} enjoy any other special property?

iv) What can be the probable applications of MOD natural neutrosophic special type of topological spaces?

v) Are these MOD natural neutrosophic special type of topological spaces discrete? Justify your claim.

vi) Show the MOD natural neutrosophic special type of topological subset spaces has strong subspaces but has no strong ideals.

vii) Show MOD natural neutrosophic special type of topological subset zero divisors, idempotents and nilpotent.

viii) Obtain all other special features associated with these MOD natural neutrosophic subset special type of topological spaces.

88. Let $S_1 = \{P(Z_{43}^{I})\}$ be the collection of MOD natural neutrosophic subsets collection.

i) Study questions (i) to (viii) of problem (87) for this S_1

ii) Compare S of problem 87 with this S_1.

89. Let $B = \{P(\langle Z_{18} \cup I\rangle_I = \{a + bI + I_t^I \,/\, a, b \in Z_{18}$, $t \in \langle Z_{18} \cup I\rangle$ where t is an indeterminate or an idempotent or nilpotent or a zero divisor, $I^2 = I\}$ be the collection of all MOD natural neutrosophic-neutrosophic subsets. $\{B, +\}$, $\{B, \times\}$, $\{B, \cup\}$ and $\{B, \cap\}$ be the MOD natural neutrosophic semigroups. Let B_o, $B_{\times}^{+}$, $B_{\cup}^{+}$, $B_{\cap}^{+}$, $B_{\cup}^{\times}$ and $B_{\cap}^{+}$ be the MOD natural neutrosophic-neutrosophic special type of subset topological spaces built using $\{B, +\}$, $\{B, \times\}$, $\{B, \cup\}$ and $\{B, \cap\}$.

i) Study questions (i) to (viii) of problem (87) for this B.

ii) Compare B with S of problem 87.

90. Let D = $\{P(\langle Z_{47} \cup I\rangle_I)\}$ be the collection of MOD natural neutrosophic-neutrosophic subset special type of topological spaces, D_o, $D_{\times}^{+}$, $D_{\cup}^{+}$, $D_{\cap}^{+}$, $D_{\cup}^{\times}$ and $D_{\cap}^{\times}$ related with the MOD natural neutrosophic-neutrosophic subset semigroups, $\{D, \cup\}$, $\{D, \cap\}$, $\{D, +\}$ and $\{D, \times\}$.

i) Study questions (i) to (viii) of problem (87) for this D.

ii) Compare S_1 of problem 88 with D.

91. Let E = $\{P(C^I (Z_{48})) = \{$collection of all subsets from $C^I(Z_{48}) = \{a + bi_F + I_t^c$ / a, b $\in Z_{48}$, i_F^2 ; t $\in$ C(Z_{48}), is a zero divisor or nilpotent or an idempotent$\}$ be collection of all MOD natural neutrosophic finite complex number subets. E_o, $E_{\times}^{+}$, $E_{\cup}^{+}$, $E_{\cap}^{+}$, $E_{\cup}^{\times}$ and $E_{\cap}^{+}$ be the MOD natural neutrosophic finite complex number subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (87) for this E.

ii) Compare this E with D of problem 90.

92. Let F = $\{P(C^I(Z_{13}))\}$ be the MOD natural neutrosophic finite complex number subsets F_o, $F_{\times}^{+}$, $F_{\cup}^{+}$, $F_{\cap}^{+}$, $F_{\cup}^{\times}$ and $F_{\cap}^{+}$ be the MOD natural neutrosophic finite complex number subset special type of topological spaces.

i) Study questions (i) to (viii) of problem 87 for this F.

ii) Compare this F with E of problem 91.

93. Let H = {P($\langle Z_{24} \cup g\rangle_I$) = {collection of all subsets from MOD natural neutrosophic dual number $\langle Z_{24} \cup g\rangle_I$ = {a + bg + I_t^g / a, b ∈ Z_{24}, $g^2 = 0$, t ∈ $\langle Z_{24} \cup g\rangle$; where t is a nilpotent or idempotent or a zero divisor} be the MOD natural neutrosophic finite complex number subsets. H_o, $H_\cup^+$, $H_\cap^+$, $H_\times^\times$, $H_\cup^\times$ and $H_\cap^\times$ be the MOD natural neutrosophic finite complex number subset special topological spaces associated with H.

i) Study questions (i) to (viii) of problem 87 for this H.

ii) Compare this H with E of problem 91.

iii) Give all the special features associated with H.

94. Let I = {P($\langle Z_{47} \cup g\rangle_I$)} be the MOD natural neutrosophic dual number subsets collections, I_o, $I_\times^+$, $I_\cap^+$, $I_\cup^+$, $I_\cup^\times$ and $I_\cap^+$ be the MOD natural neutrosophic dual number subset special type of topological spaces.

i) Study questions (i) to (viii) of problem 87 for this I.

ii) Compare this I with H of problem 93.

95. Let J = {P($\langle Z_{12} \cup h\rangle_I$)} = {collection of all MOD natural neutrsophic special dual like number subsets $\langle Z_{12} \cup h\rangle_I$ = {a + bh + I_t^h / a, b ∈ Z_{12}, $h^2 = h$, t ∈ $\langle Z_{12} \cup h\rangle$ is a zero divisor or nilpotent or an idempotent of $\langle Z_{12} \cup h\rangle$} be the MOD natural neutrosophic special dual like number subsets collection. J_o, $J_\times^+$, $J_\cup^+$, $J_\cap^+$, $J_\cup^\times$ and $J_\cap^+$ be the MOD natural neutrosophic special dual like number subset special topological spaces.

i) Study questions (i) to (viii) of problem (87) for this J.

ii) Compare I of problem 94 with this J.

iii) Obtain all special features associated with J.

96. Let K = $\{P(\langle Z_{48} \cup k\rangle_I)\}$ = {collection of all subsets from $\langle Z_{48} \cup k\rangle_I$, I = $\{a + bk + I_t^k$ / a, b $\in Z_{48}$, $k^2 = 47k$, t $\in \langle Z_{48} \cup h\rangle$ where t is an idempotent or nilpotent or a zero divisor}} be the MOD natural neutrosophic special quasi dual number subset collection. K_o, $K_\cup^+$, $K_\cap^+$, $K_\times^+$, $K_\cup^\times$ and $K_\cap^\times$ be the MOD natural neutrosophic special quasi dual number subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (87) for this K.

ii) Compare J of problem 95 with this K.

iii) Obtain all special features associated with this K.

97. Let L = $\{P(\langle Z_{23} \cup k\rangle_I)\}$ = {collection of all natural neutrosophic special quasi dual number subsets from $\{\langle Z_{23} \cup k\rangle_I = \{a + bk + I_t^k$ / a, b $\in Z_3$, $K^2 = 22k$, t $\in \langle Z_3 \cup k\rangle$ is a zero divisor or idempotents or nilpotents} MOD natural neutrosophic special quasi dual number subset collections. L_o, $L_\times^+$, $L_\cup^+$, $L_\cap^+$, $L_\cup^\times$ and $L_\cap^\times$ be the MOD natural neutrosophic special quasi dual number subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (87) for this L.

ii) Obtain all special features associated with this L.

98. Let N = {P(M)} = {collection of all matrix subsets of

M with entries from Z_{14}, where M = $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$ / $a_i \in Z_{14}$}; $1 \leq i \leq$ 5}} be the MOD subset matrix collection.

i) Study questions (i) to (viii) of problem 89 where {N, +}, {N, $\times_n$}, {N, $\cup$} and {N, $\cap$} are the associated MOD matrix subset semigroup.

ii) Prove these related topological has MOD subset matrix topological nilpotents, MOD subset matrix topological zero divisors and idempotents.

iii) Derive all special features associated with N.

99. Let P = {P(R)} = {collection of all subset matrices from matrices from

$$R = \left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} \middle/ a_i \in C^I(Z_{12});\right.$$

$1 \leq i \leq 8$}} be the MOD natural neutrosophic finite complex number subset matrix collection. R_o, $R^{+}_{\times_n}$, $R^{+}_{\cup}$, $R^{+}_{\cap}$, $R^{\times_n}_{\cup}$ and $R^{\times_n}_{\cap}$ be the MOD natural neutrosophic finite complex number subset matrix special type of topological spaces associated with the MOD natural neutrosophic finite complex number subset matrix semigroups {R, $\times_n$}, {R, $\cup$}, {R, $\cap$} and {R, +}.

i) Study questions (i) to (viii) of problem (89) of R.

ii) Study the special feature enjoyed by R_o, $R^{+}_{\times_n}$ and so on.

iii) Compare N of problem 98 with this T.

100. Let T = {P(W)} = {Collection of all subsets from

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle/ a_i \in \langle Z_{11} \cup g \rangle_I; \right.$$

$1 \leq i \leq 16$}} be the MOD natural neutrosophic dual number subset matrix collections, T_o, $T^{+}_{\times}$, $T^{+}_{\times_n}$, $T^{+}_{\cup}$, $T^{+}_{\cap}$, $T^{\times}_{\cup}$, $T^{\times}_{\cap}$, $T^{\times_n}_{\cup}$, $T^{\times}_{\times_n}$ and $T^{\times_n}_{\cap}$ be the MOD natural neutrosophic dual number matrix subset special type of topological spaces associated with {T, ∪}, {T, ∩}, {T, +}, {T, $\times_n$} and {T, ×}.

i) Study questions (i) to (viii) of problem (89) for this T.

ii) Compare this T with R of problem (99).

101. Let S = {P(V)} = {collection of all subset matrices from

$$V = \left\{ \begin{bmatrix} a_1 & a_5 & a_9 \\ a_2 & a_6 & a_{10} \\ a_3 & a_7 & a_{11} \\ a_4 & a_8 & a_{12} \end{bmatrix} \middle/ a_i \in \langle Z_{48} \cup k \rangle_I,\ 1 \leq i \leq 12 \right\}\}$$

be the MOD natural neutrosophic special quasi dual number subset matrix collection.

i) Study questions (i) to (viii) of problem (89) for this V.

ii) Compare this V with T of problem (100).

102. Let

$$B = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} / a_i \in P(\langle Z_{10} \cup I\rangle);$$

$1 \leq i \leq 10\}$ be the MOD neutrosophic matrices with entries from subsets of $\langle Z_{10} \cup I\rangle$; collection. $\{B, +\}$, $\{B, \cup\}$, $\{B, \cap\}$ and $\{B, \times_n\}$ be the MOD neutrosohic subset entries matrix subsemigroups. B_o, $B^+_\cup$, $B^+_\cap$, $B^+_\times$, $B^\times_\cup$ and $B^\times_\cap$ be the 6 distinct MOD neutrosophic subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (89) for this B.

103. Let

$$D = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} / a_i \in P(\langle Z_{12} \cup g\rangle_I);\ 1 \leq i \leq 7\}$$

be the MOD natural neutrosophic dual number matrices with subset entries.

i) Study questions (i) to (viii) of problem (89) for this D.

ii) Compare this D with B of problem (102).

iii) Enumerate all the special features associated with this D.

104. Let

$$E = \{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} / a_i \in P(\langle Z_{48} \cup k \rangle_I);\ 1 \leq i \leq 10\}$$

be the MOD natural neutrosophic special quasi dual number subset matrix collection. E_o, $E_{\times}^{+}$, $E_{\cup}^{+}$, $E_{\cap}^{+}$, $E_{\cup}^{\times}$ and $E_{\cap}^{+}$ be the MOD natural neutrosophic special quasi dual number subset matrix special type of topological space associated with {E, +}, {E, ×}, {E, ∪} and {E, ∩}.

i) Study questions (i) to (viii) of problem (89) for this E.

ii) Compare this E with D of problem (103).

105. Let

$$M = \{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} / a_i \in P(C^I\langle Z_{40}\rangle)$$

= {collection of all subsets from $C^I(Z_{40})$ = {$a + b\, i_F + I_t^c$ / a, b ∈ Z_{40}, $i_F^2 = 39$, t ∈ $C(Z_{40})$ where t is a zero divisor or idempotent or nilpotent of $C(Z_{40})$; $1 \leq i \leq 25$} be the MOD natural neutrosophic finite complex number subset matrix collection. M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$, $M_{\cap}^{\times}$, $M_{\times_n}^{+}$, $M_{\times}^{\times_n}$, $M_{\cup}^{\times_n}$ and $M_{\cap}^{\times_n}$ be the MOD natural neutrosophic finite complex number subset matrix special type of topological spaces associated with the MOD natural neutrosophic finite complex number subset – matrix semigroup (M, +},{M, ∪}, {M, ∩}, {M, $\times_n$} and {M, ×}.

i) Prove $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{\times}$ and $M_{\times_n}^{\times}$ are MOD natural neutrosophic finite complex number subset matrix special type of toplogical spaces which are non commutative.

ii) Show these four spaces $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{\times}$ and $M_{\times_n}^{\times}$ have (a) Right MOD subset matrix topological zero divisors which are left zero divisors and vice versa.

iii) Find all MOD subset right ideals of these four spaces which are not left ideals and vice versa.

106. Let

$$G = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(Z_{48}^{I})\}$$

= {collection of polynomials with coefficients from the subsets of Z_{48}^{I} = {a + I_t^{46}/ a ∈ Z_{48} and t ∈ Z_{48} is such that t is a nilpotent or an idempotent or a zero divisor of Z_{48}}} be the MOD natural neutrosophic subset coefficient polynomials. {G, +}, {G, ∪}, {G, ∩} and {G, ×} are MOD natural neutrosophic subset polynomial semigroups. G_o, $G_{\times}^{+}$, $G_{\cup}^{+}$, $G_{\cap}^{+}$, $G_{\cup}^{\times}$ and $G_{\cap}^{\times}$ are the MOD natural neutrosophic subset coefficient polynomial special type of topological spaces associated with semigroups.

i) Find all MOD zero divisor subset polynomials of the topological spaces $G_{\times}^{+}$, $G_{\cup}^{\times}$ and $G_{\cap}^{\times}$. (a) If $x \in G_{\times}^{+}$ is a MOD topological zero divisor then the same x in $G_{\cup}^{\times}$ and $G_{\cap}^{\times}$ are also MOD subset topological zero divisor.

ii) Find a strong MOD neutrosophic subset coefficient polynomial special type of topological subspace.

iii) Prove there does not exist MOD neutrosophic subset coefficient polynomial special type of topological ideals.

iv) Show if $W \subseteq G_o$ is a MOD subset coefficient polynomial subspace of G_o then $W \subseteq G_{\times}^{+}$ is not a subspace $G_{\times}^{+}$ or $G_{\cup}^{+}$ or $G_{\cap}^{+}$ and $G_{\cup}^{\times}$ or $G_{\cap}^{\times}$

v) Obtain any other special feature associated with these spaces.

107. Let

$$H = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(Z_{12} \cup g\rangle)\}$$

be the MOD dual number subset coefficient polynomial collection. {H, +}, {H, ∪}, {H, ∩} and {H, ×} be the MOD dual number subset coefficient polynomial semigroups. H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ be the related MOD dual number subset coefficient polynomial special type of topological spaces associated with these MOD semigroups.

i) Study questions (i) to (v) of problem (106) for this H.

ii) Compare this H with G of problem 105.

108. Let

$$I = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(C^I(Z_{27}))\}$$

be the MOD natural neutrosophic finite complex number subset coefficient polynomials. I_o, $I_\times^+$, $I_\cup^+$, $I_\cap^+$, $I_\cup^\times$ and $I_\cap^\times$ be the MOD natural neutrosophic finite complex number subset coefficient polynomial special type of topological spaces associated with I.

i) Study questions (i) to (v) of problem (106) for this I.

ii) Compare with this I the H of problem 106.

iii) Compare with this I G of problem 105.

109. Let J = P(S[x]) = {collection of all subsets from

$$S[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle Z_{18} \cup I\rangle\}\}$$

be the MOD neutrosophic subsets of polynomial. Let {J, ×}, {J, +}, {J, ∪} and {J, ∩} be the MOD neutrosophic subset polynomial semigroups. Let J_o, $J_\times^+$, $J_\cup^+$, $J_\cap^+$, $J_\cup^\times$ and $J_\cap^\times$ be the MOD natural neutrosophic subsets in polynomial special type of topological spaces.

i) Find all strong MOD subset special type of topological spaces of all the six spaces.

ii) Prove in general if Y is a MOD subspace of any one of the topological spaces then in general need not be the MOD subspaces of other MOD topological spaces.

iii) Obtain MOD nilpotents and zero divisors of these topological spaces.

iv) Enumerate any special property associated with J.

v) Can J have MOD idempotents, justify your claim?

110. Let L = {P(M[x])} = {collection of all polynomial subsets from

$M[x] = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in \langle Z_{48} \cup k\rangle = Pa + bk \,/\, a, b \in Z_{48}$, $k^2 = 47k\}\}\}$ be the MOD special quasi dual number polynomial subsets from P(M[x]).

i) Study questions (i) to (v) of problem (108) for this L

ii) Compare this L with J in problem (108).

111. Let

$$P = \{\sum_{i=0}^{9} a_i x^i \,/\, a_i \in P(\langle Z_{48} \cup I\rangle),\ x^{10} = 1\}$$

be the MOD neutrosophic subset coefficient finite degree polynomial {P, +}, {P, +}, {P, ∪} and {P, ∩} be the MOD neutrosophic coefficient finite degree polynomial semigroups. P_o, $P^{+}_{\times}$, $P^{+}_{\cup}$, $P^{+}_{\cap}$, $P^{\times}_{\cup}$ and $P^{\times}_{\cap}$ be the MOD neutrosophic coefficient polynomial special type of topological spaces associated with the four semigroups.

i) Prove all these topological spaces are of finite order.

ii) Find all MOD subset polynomial coefficient strong spaces.

iii) Can these MOD topological spaces associated with P have strong ideals? Justify.

iv) Find all MOD topological zero divisors, nilpotents and idemoptents (if any) in $P_{\cup}^{\times}$, $P_{\cap}^{\times}$, and $P_{+}^{\times}$.

112. Let R = $\{\sum_{i=0}^{18} a_i x^i$ / $a_i \in P(\langle Z_{12} \cup g\rangle)$ (or $C^I(Z_{12})$); $x^{19} = 1\}$ be the MOD natural neutrosophic dual number (finite complex number) subset coefficient polynomials of finite order. R_o, $R_{\cup}^{+}$, $R_{\cap}^{+}$, $R_{\times}^{+}$, $R_{\cup}^{\times}$ and $R_{\cap}^{\times}$ be the MOD subset natural neutrosophic polynomial special type of topological spaces.

i) Study questions (i) to (iv) of problem (110) for this R.

ii) Enumerate all the special features associated with this R.

113. Let V = $P(M[x]_{10})$ = {collection of all polynomial subsets of finite order from $M[x]_{10} = \{\sum_{i=0}^{10} a_i x^i$ / $a_i \in \langle Z_{15} \cup h\rangle_I$; $x^{11} = 1\}\}$ be the MOD special dual like number coefficient polynomial subsets V_o, $V_{\times}^{+}$, $V_{\cup}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be the MOD special dual like number coefficient polynomial subsets special type of topological spaces.

i) Prove $o(V) < \infty$.

ii) Find all strong MOD special dual like number coefficient polynomial special type of topological subspaces.

iii) Prove V has no MOD subset polynomial special type of topological ideals.

iv) Find all MOD subset topological zero divisor and nilpotents if any.

v) Find all special features associated with this V.

114. Let W = P(N[x]$_5$) = {collection of all polynomial subsets from $N[x]_5 = \{\sum_{i=0}^{5} a_i x^i \,/\, x^6 = 1,\ a_i \in \langle Z_{25} \cup g\rangle_I$ (or $C^I(Z_{14})$ or $\langle Z_{10} \cup h\rangle_I$ or $\langle Z_{16} \cup g\rangle)\}\}$ be the MOD natural neutrosophic polynomial subsets of finite order. W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ be the MOD natural neutrosophic polynomial subset special type of topological spaces.

Study questions (i) to (v) of problem (113) for this W.

Chapter Three

MOD SUBSET SPECIAL TYPE OF INTERVAL TOPOLOGICAL SPACES

In this chapter we proceed onto define mod special type of interval topological spaces using $[0, n)$, ${}^{I}[0, n)$, $\langle [0, n) \cup g \rangle$, $C([0, n))$; $C^{I}([0, n))$, $\langle [0, n) \cup g \rangle_I$, $\langle [0, n) \cup h \rangle$, $\langle [0, n) \cup h \rangle_I$, $\langle [0, n) \cup k \rangle$ and $\langle [0, n) \cup k \rangle_I$.

We will illustrate this situation by some examples and obtain all special features associated with them.

Example 3.1. Let S = {P([0, 4)) be the power set of [0, 4). S is called mod interval subsets. Define on S the operation $+$, $\cup$, $\cap$ and $\times$.

Clearly $\{S, +\}$ is the mod interval subset semigroup under $+$.

$\{S, \cup\}$ be the mod interval subset semigroup under $\cup$.

$\{S, \cap\}$ be the mod interval subset semigroup under $\cap$.

$\{S, \times\}$ be the mod interval subset semigroup under $\times$.

We will illustrate this situation by some working.

Let A = {3.2, 1.5, 0.3, 1.2} and B ={1.5, 0.3, 0.4, 0.01}$\in$ S.

$A \cup B$ ={3.2, 1.5, 0.3, 1.2, 0.4, 0.01} I

$A \cap B$ = {1.5, 0.3) II

$A \times B$ ={0.8, 2.25, 0.45, 1.80, 0.96, 0.09, 0.36, 1.28, 0.6, 0.12, 0.48, 0.032, 0.015, 0.003, 0.012} III

A + B = {0.7, 3, 1.8, 2.7, 3.5, 1.8, 0.6, 1.5, 3.6, 1.9, 0.7, 1.6, 3.21, 1.51, 0.31, 1.21} IV

Clearly I, II, III and IV are distinct. So all the mod interval subset semigroups are distinct.

Thus S_o = {S, $\cup$, $\cap$}, $S^{+}_{\times}$ = {S, +, $\times$}, $S^{\times}_{\cup}$ = {S, $\times$, $\cup$}, $S^{\times}_{\cap}$ = {S, $\times$, $\cap$}, $S^{+}_{\cup}$ = {S,+, $\cup$} and $S^{+}_{\cap}$ = {S, +, $\cap$} be the six distinct mod subset interval special type of topological spaces.

All these spaces enjoy special properties. S_o, $S^{+}_{\cup}$, $S^{+}_{\cap}$, $S^{\times}_{\cup}$ and $S^{\times}_{\cap}$ has mod interval subsets which are topological idempotents.

However S_o, $S^{+}_{\cup}$ and $S^{+}_{\cap}$ does not contain zero divisor, but the other three mod topological spaces contain zero divisors; which are defined as mod interval subsets special type topological space zero divisors.

Let A ={0, 2} $\in$ S.

$A \times A$ ={0} is the mod interval subset special type of topological nilpotent subset of S.

Apart from this we are not in a position to get mod interval subset special topological space zero divisors.

However only the set B $=\{0, 1\} \in$ S is such that $B \times B = \{0, 1\}$; which we term as mod interval trivial special topological idempotent of $S_{\cup}^{+}$, $S_{\cap}^{+}$ and $S_{+}^{\times} = S_{\times}^{+}$.

Example 3.2. Let T $=\{P(\langle[0, 6) \cup g\rangle)\}$ = {collection of all subsets from $\langle[0, 6) \cup g\rangle = \{a + bg \,/ a, b \in [0, 6), g^2 = 0\}$} be the mod interval subset dual numbers.

$\{T, +\}$, $\{T, \times\}$, $\{T, \cup\}$and $\{T, \cap\}$ be the mod interval dual number semigroups.

$T_o =$

$\{T, \cup, \cap\}$, $T_{\times}^{+} = \{T, +, \times\}$, $T_{\cup}^{+} = \{T, +, \cup\}$, $T_{\cap}^{+} = \{T, +, \cap\}$, $T_{\cup}^{\times} = \{T, \cup, \times\}$ and $T_{\cap}^{\times} = \{T, \cap, \times\}$ be the six mod interval dual number subset special type of topological spaces.

$T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and $T_{+}^{\times} = T_{\times}^{+}$ have infinite number of mod interval subset special topological space nilpotents and zero divisors.

Let $A = \{0.3g, 4g, 2.35g, 1.78g\}$

$B = \{0.003g, 1.0098g, g, 1.4g, 5.00789g\} \in T.$

Clearly $A \times B = \{0\}$ and $A \times A = \{0\}$ and $B \times B = \{0\}$.

Infact the reader is left with the task of finding mod interval subset dual number special type of topological subspaces. One can find easily ideals but they are not strong.

Further if $P \subseteq T_o$ is an ideal P need not be an ideal of $T_{\times}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$, $T_{\cup}^{+}$ and $T_{\cap}^{+}$.

Similarly if $R \subseteq T_{\times}^{+}$ is an ideal, R need not be an ideal of $T_{\cup}^{+}$ or $T_{\cap}^{+}$ or $T_{\cup}^{\times}$ or $T_{\cap}^{\times}$ or T_o.

So obtaining mod interval dual number subset special type of topological strong ideals is not possible. Only we can have strong subspaces.

Example 3.3. Let R = {P(C[0, 6))} be the power set of C([0, 6)) = $\{a + bi_F / a, b \in [0, 6); i_F^2 = 5\}$. R will be known as mod interval finite complex number subsets.

Define on R, +, ×, ∪ and ∩ so that {R, +}, {R, ∪}, {R, ∩} and {R, ×} are mod interval finite complex number subset semigroups.

All the four semigroups are distinct. $R_o = \{R, \cup, \cap\}$, $R_{\cup}^{+} = \{R, +, \cup\}$, $R_{\times}^{+} = \{R, +, \times\}$, $R_{\cap}^{+} = \{R, +, \cap\}$, $R_{\cup}^{\times} = \{R, \times, \cup\}$ and $R_{\cap}^{\times} = \{R, \times, \cap\}$ are the mod finite complex number interval subset special type of topological spaces all of which are of infinite cardinality and are distinct.

Finding subspaces, mod topological zero divisors, mod topological nilpotents and mod topological ideals are left as an exercise to the reader.

Example 3.4. Let T = {P(⟨[0, 7) ∪ g⟩)} be the mod interval finite dual number subset collection. {T, +}, {T, ∪}, {T, ∩} and {T, ×} be the mod interval finite dual number subset semigroups, T_o, $T_{\times}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$, $T_{\cup}^{+}$ and $T_{\cap}^{+}$ be the mod interval finite dual number subset special type of topological spaces.

All of them are distinct and is of infinite cardinality. Further the mod topological interval spaces $T_{\times}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{+}$ contain infinite number of zero divisors and nilpotents.

This has mod strong subspaces of both finite and infinite order which are not mod topological ideals.

Consider B = {P(Z_{17})}; B_o, $B_{\times}^{+}$, $B_{\cup}^{\times}$, $B_{\cap}^{\times}$, $B_{\cup}^{+}$ and $B_{\cap}^{+}$ are mod subset subspaces; so B is a strong mod subset topological subspaces of T.

D = {P(⟨$Z_{17} \cup g$⟩)} is again a strong mod topological subset subspace of T. Both B and D are of finite order and they are not ideals of T.

E′ ={P([0, 17)g)} $\subseteq$ T is again a mod subset strong topological subspace of T which is not an ideal of T.

Similarly F = {P([0, 17))} $\subseteq$ T is a mod subset strong topological subspace of E which is not an ideal of T. Thus both E′ and F are of infinite cardinality.

Further we once again just recall a mod subset topological subspace $L_o \subseteq T_o$ to be an ideal if for all subsets I of L_o and for all subsets W of T_o.

$W \cap I \in L_o$ and $W \cup I \in L_o$.

Likewise for $L_{\times}^{+} \subseteq T_{\times}^{+}$ is an ideal if for all I of $L_{\times}^{+}$ and W of $T_{\times}^{+}$

$W + I \in L_{\times}^{+}$ and $W \times I \in L_{\times}^{+}$.

So finding mod subset interval topological ideals happens to be a very difficult problem.

Similar definition in case of $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$.

When $I \subset T$ is an ideal for all the 6 topological spaces we call I to be mod strong subset interval topological ideal of T_o, $T_{\times}^{+}$, $T_{\cup}^{+}$, …, $T_{\cup}^{\times}$.

Study in this direction is challenging for there is no mod subset interval strong special type of topological ideals.

However there are mod subset interval strong special type of topological subspaces associated with T.

Example 3.5. Let A = $\{P({}^{I}[0, 6))\}$ = {collection of all subsets from ${}^{I}[0, 6\ ) = \{a + I_t^{[0,6)}$ where t is a nilpotent or zero divisor or idempotent from Z_6; $a \in [0, 6)\}\}$ be the mod natural neutrosophic interval subset collection.

$\{A, +\}$, $\{A, \cup\}$, $\{A, \cap\}$ and $\{A, \times\}$ be the mod natural neutrosophic modulo integer interval subset semigroups. All the four semigroups are distinct.

Further all these semigroups are of infinite order having both finite and infinite order subsemigroups. But all ideals A are of infinite order $\{A, \times\}$ can have zero divisors and nilpotents $\{A, +\}$ can have idempotents. Every element in $\{A, \cup\}$ and $\{A, \cap\}$ are idempotents.

We can have in $\{A, \cap\}$ ideals of finite order.

$A_o = \{A, \cap, \cup\}$, $A_{\times}^{+} = \{A, +, \times\}$,

$A_{\cup}^{+} = \{A, +, \cup\}$, $A_{\cap}^{+} = \{A, +, \cap\}$,

$A_{\cup}^{\times} = \{A, \times, \cup\}$ and $A_{\cap}^{\times} = \{A, \times, \cap\}$ be the mod subset interval natural neutrosophic modulo integer special type of topological spaces. All these spaces are of infinite order and are distinct.

Studying compactness, connectedness etc; happens to be challenging problem.

Let X = $\{3 + I_0^6, 2 + I_3^6, I_2^6 + I_4^6, 4, 1, 0\}$ and Y = $\{3 + I_0^6, 1, 0, 4 + I_3^6 + I_4^6, I_2^6\} \in A$.

We find, $X \cup Y = \{3 + I_0^6, 2 + I_3^6, I_2^6 + I_4^6, 4, 1, 0, I_2^6, 4 + I_3^6 + I_4^6\}$ I

$X \cap Y = \{0, 1, 3 + I_0^6\}$ II

$X + Y = \{I_0^6, 5 + I_0^6 + I_3^6, 3 + I_0^6 + I_2^6 + I_4^6, 1 + I_0^6, 4 + I_0^6, 3 + I_0^6, 3 + I_3^6, 1 + I_2^6 + I_4^6, 5, 2, 1, 1 + I_0^6 + I_3^6 + I_4^6, I_3^6 + I_4^6, 4 + I_2^6 + I_4^6 + I_3^6, 2 + I_3^6 + I_4^6, 5 + I_3^6 + I_4^6, 4 + I_3^6 + I_4^6, 3 + I_0^6 + I_2^6, 2 + I_3^6 + I_2^6, I_2^6 + I_4^6, 4 + I_2^6, 1 + I_2^6, I_2^6\}$ III

$X \times Y = \{3 + I_0^6, 2 + I_3^6, I_2^6 + I_4^6, 4, 1, 0\} \times \{0, 1, 3 + I_0^6, 4 + I_3^6 + I_4^6, I_2^6\} = \{0, I_0^6, I_3^6, I_2^6 + I_4^6, 3 + I_0^6, 2 + I_3^6, I_2^6 + I_4^6, 4, 1, 3 + I_0^6, I_3^6 + I_0^6, I_2^6 + I_4^6 + I_0^6, I_3^6 + I_4^6 + I_0^6, 2 + I_3^6 + I_4^6 + I_0^6, I_2^6 + I_4^6 + I_0^6, 4 + I_3^6 + I_4^6, I_2^6 + I_0^6, I_2^6 + I_0^6, I_4^6 + I_2^6\}$ IV

We have used natural neutrosophic zero dominated product and not the usual zero dominated product. The four equations are distinct hence the six topologies are also distinct.

Finding subspaces ideals etc are left as exercise to the reader.

A has mod natural neutrosophic special topological idempotents subsets with respect to + and × also.

Example 3.6. Let B = {P(⟨[0, 9) ∪⟩$_I$)} = {collection of all natural neutrosophic interval dual number subset collection from ⟨[0, 9) ∪ g⟩$_I$ ={a + bg + I_t^g / a, b ∈ [0, 9), $g^2 = 0$, t ∈ ⟨Z_9 ∪ g⟩ is a zero divisor or idempotent or nilpotent} be the mod natural neutrosophic interval dual number subset collection. {B, ×}, {B, ∪}, {B, ∩} and {B, +} be the four distinct mod natural neutrosophic interval dual number subset semigroups and B_o, $B_\cup^+$, $B_\cap^+$, $B_\times^+$, $B_\cup^\times$ and $B_\cap^\times$ be the mod natural neutrosophic interval dual number subset special type of topological spaces associated with these mod subset semigroups.

This has infinite number of mod natural neutrosophic dual number interval subset special type of topological zero divisors and nilpotents under the × operation.

However there are only a finite collection of mod interval topological space idempotent subsets under ×.

Study in this direction is a matter of routine so left as an exercise to the reader.

These mod spaces has both of finite order as well as of infinite order, mod topological subspaces.

Example 3.7. Let B = {P(⟨[0, 28) ∪ h⟩$_I$)} = {collection of all mod natural neutrosophic special dual like number interval subsets from ⟨[0, 28) ∪ h⟩$_I$ = {a + bh + I_t^h / a, b ∈ [0, 28), $h^2 = h$; t ∈ ⟨Z_{28} ∪ h⟩ is such that t is a zero divisor or nilpotent or an idempotent}. {B, +}, {B, ∪}, {B, ∩} and {B, ×} be the mod natural neutrosophic interval special dual like number subset semigroup.

Let B_o, $B_\times^+$, $B_\cup^+$, $B_\cap^+$, $B_\cup^\times$ and $B_\cap^\times$ be the mod subset interval natural neutrosophic special type of topological spaces built using these semigroups on B.

All properties associated with B can be obtained using the fact $h^2 = h$; this task is left as an exercise to the reader.

Example 3.8. Let C = {P(C^I([0, 4))} be the collection of all mod natural neutrosophic finite complex number subset collection. {C, +}, {C, ×}, {C, ∪} and {C, ∩} are the mod natural neutrosophic finite complex number subset semigroup.

C_o, $C_\times^+$, $C_\cup^+$, $C_\cap^+$, $C_\cup^\times$ and $C_\cap^\times$ are the six distinct mod special type of topological spaces associated with C.

All properties associated with these mod semigroups and mod topological spaces is left as an exercise to the reader.

Example 3.9. Let D = {P(⟨[0, 24) ∪ I⟩$_I$) = {collection of all subsets from the mod natural neutrosophic neutrosophic interval ⟨[0,24) ∪ I⟩$_I$ = {a + bI + I_t^I / a, b ∈ [0, 24), I^2 = I, t ∈ ⟨Z_{24} ∪ I⟩}} be the mod natural neutrosophic neutrosophic interval subset collection, {D, +}, {D, ×}, {D, ∪} and {D, ∩} be the mod natural neutrosophic neutrosophic subset interval semigroups and D_o, $D_\times^+$, $D_\cup^+$, $D_\cap^+$, $D_\cup^\times$ and $D_\cap^\times$ are the mod natural neutrosophic neutrosophic subset interval special type of topological spaces associated with D.

Study of strong subspaces of finite and infinite order is interesting.

However all these spaces have both finite and infinite order strong subspaces but no strong ideals.

In view of all these we give the following theorem.

Theorem 3.1. *Let S = P([0, n)) (or P(^{I}[0, n)) or P(⟨[0, n) ∪ I⟩) or P(⟨[0, n) ∪I⟩$_I$) or P(C([0, n))) or P(C^I([0, n))) or P(⟨[0, n) ∪ g⟩) or P(⟨[0, n) ∪ g⟩$_I$), P(⟨[0, n)∪h⟩) or P(⟨[0, n)∪h⟩$_I$)) or P(⟨[0, n)∪k⟩)) or P(⟨[0, n)∪k⟩$_I$)) be the mod interval subset* *collections (or mod natural neutrosophic subset collections, appropriately associated with these sets).*

i) *S is of infinite order.*

ii) *{S, +}, {S, ×}, {S, ∪} and {S, ∩} are mod interval subset semigroups of infinite order.*

iii) *These semigroups contain strong subsemigroups of finite and infinite order.*

iv) *These semigroup do not contain strong ideals.*

v) *These semigroups contain mod subset interval zero divisors, idempotents and nilpotents depending on the sets chosen and n chosen.*

vi) *All mod subset interval special type of topological spaces* S_o, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$, $S_{\times}^{+}$ *and* $S_{\cap}^{\times}$ *are of infinite order and are distinct.*

vii) *These mod subset special type of topological spaces has strong mod subset interval subspaces of both finite and infinite order.*

viii) *S has no mod interval subset topological ideals.*

ix) *All mod topological interval subset ideals of S are of infinite order.*

x) $S_{\times}^{+}$, $S_{\cup}^{\times}$ *and* $S_{\cap}^{\times}$ *has mod subset interval topological zero divisors, idempotents and nilpotents depending of the subsets collection used and on n.*

xi) $S_{\times}^{+}$, $S_{\cup}^{\times}$, $S_{\cap}^{\times}$; …, S_o *have idempotents with respect +, ×, ∪ and ∩ (+ and × depend on the set).*

The proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe mod interval subset matrix special type of topological spaces and mod interval matrix subset special type of topological spaces and the mod natural neutrosophic analogues by examples.

Example 3.10. Let W = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle/ a_i \in P([0, 42)); 1 \leq i \leq 5 \right\}$ be the mod interval subset matrix collection. {W, +}, {W, $\cup$}, {W, $\cap$} and {W, $\times_n$} be the mod interval subset matrix semigroups.

W_o, $W^{+}_{\times_n}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times_n}_{\cup}$ and $W^{\times_n}_{\cap}$ be the mod interval subset matrix special type of topological spaces.

Finding mod strong topological subspaces of both finite and infinite order is a matter of routine so left as an exercise to the reader.

$W^{\times_n}_{\cup}$, $W^{\times_n}_{\cap}$ and $W^{+}_{\times_n}$ has infinite number of mod interval subset matrix topological zero divisors,

$W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times_n}_{\cup}$, $W^{\times_n}_{\cap}$ and W_o has idempotents with respect to $\cup$ and $\cap$. $W^{\times_n}_{\cap}$, $W^{\times_n}_{\cup}$ and $W^{+}_{\times_n}$ has idempotents with respect to $\times_n$.

W has both finite and infinite order strong mod interval subset matrix special type of topological subspaces, however strong ideals of mod interval subset matrix special type of topological space does not exist.

Example 3.11. Let V = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right.$ / $a_i \in$ P(⟨[0, 26) ∪ g⟩); $1 \leq i \leq 12$, $g^2 = 0$} be the mod interval dual number subset matrix collection. {V, $\times_n$}, {V, ∪}, {V, ∩} and {V, +} be the mod interval dual number subset matrix semigroups.

V_o, $V^+_{\times_n}$, $V^+_{\cup}$, $V^+_{\cap}$, $V^{\times_n}_{\cup}$ and $V^{\times_n}_{\cap}$ be mod interval dual number subset matrix special type of topological spaces. $V^+_{\times_n}$, $V^{\times_n}_{\cup}$ and $V^{\times_n}_{\cap}$ has infinite topological zero divisors. V_o, $V^+_{\cup}$ and $V^+_{\cap}$ has every subset matrix to be an idempotent under ∪ or (and) ∩.

V has strong mod dual number interval subset matrix special type of topological subspaces of both finite and infinite order, however has no strong mod dual number interval subset matrix special type of topological ideals.

Example 3.12. Let Y = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right.$ / $a_i \in$ P(C[0, 48)); $1 \leq i \leq 9$} be mod interval finite complex number subset matrix collection, {Y, +}, {Y, $\times_n$}, {Y, ×},{Y, ∩} and {Y, ∪} be the mod interval finite complex number subset matrix semigroups.

Let Y_o, $Y^+_{\times}$, $Y^+_{\times_n}$, $Y^+_{\cup}$, $Y^+_{\cap}$, $Y^{\times}_{\cup}$, $Y^{\times}_{\cap}$, $Y^{\times_n}_{\cup}$, $Y^{\times_n}_{\cap}$ and $Y^{\times}_{\times_n}$ be the mod interval finite complex number subset matrix special type of topological spaces associated with these five mod semigroups.

We see $Y^+_{\times}$, $Y^{\cup}_{\times}$, $Y^{\cap}_{\times}$ and $Y^{\times}_{\times_n}$ are mod non commutative interval finite complex number subset matrix special topological space.

The reader is left with the task of finding mod interval finite complex number subset matrix topological right ideals which is not left ideals and vice versa.

These spaces, $Y_{\times}^{+}$, $Y_{\times}^{\cup}$, $Y_{\times}^{\cap}$ and $Y_{\times}^{\times_n}$ can have mod interval finite complex number subset matrix special type of topological right zero divisors which are not left zero divisors and vice versa.

The study in this direction is a matter of routine so left as exercise to the reader.

Example 3.13. Let Z ={$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix}$ / $a_i \in P(^I[0, 256))$; $1 \leq i \leq 8$} be the mod interval natural neutrosophic matrix with subset entries collection. {Z, +}, {Z, $\cup$}, {Z, $\cap$} and {Z, $\times_n$} be the mod interval natural neutrosophic subset matrix semigroups.

{Z, +} has idempotents. Further in {Z, $\cup$} and {Z, $\cap$} every element is an idempotent. {Z, $\times_n$} also has nilpotents and idempotents.

We see the mod natural neutrosophic subset matrix special type of topological spaces has mod natural neutrosophic subset matrix special type of topological idempotents, nilpotents and zero divisors.

Next we proceed onto describe mod interval natural neutrosophic finite complex number matrix with subset entries by some examples.

Example 3.14. Let S = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \right.$ / $a_i \in P(C^I([0, 43)); 1 \leq i \leq 10\}$ be the mod interval natural neutrosophic finite complex number matrices with subset entries collection. {S, +}, {S, $\times_n$}, {S, $\cup$} and {S, $\cap$} are mod interval natural neutrosophic finite complex number matrices with subset entries semigroup.

$$A = \begin{bmatrix} \{0.3i_F, 1, 2i_F + 3\} & \{40, 10 + i_F\} \\ \{4 + i_F, 2, 0\} & \{1, 3i_F, 10\} \\ \{20, 10i_F, 1\} & \{1, 0, 5i_F\} \\ \{4i_F, 0, 2 + 8i_F\} & \{2, 5\} \\ \{i_F, 30i_F\} & \{0.1, 0.8\} \end{bmatrix}$$

and $$B = \begin{bmatrix} \{0.3i_F, 1, 2\} & \{0, 1\} \\ \{5, 2, 9i_F\} & \{0, 1, 3i_F\} \\ \{10i_F, 10\} & \{20, 20i_F\} \\ \{0, 5i_F, 20\} & \{30, 20i_F, 10\} \\ \{1, 5i_F\} & \{0.1, 0.5\} \end{bmatrix} \in S.$$

$$A \cup B = \begin{bmatrix} \{0.3i_F, 1, 2i_F + 3, 2\} & \{40, 10 + i_F, 0, 1\} \\ \{0, 2, 5, 9i_F, 4 + i_F\} & \{1, 10, 0, 3i_F\} \\ \{10i_F, 10, 1, 20\} & \{1, 0, 5i_F, 20, 20i_F\} \\ \{0, 4i_F, 20, 5i_F, 2 + 8i_F\} & \{2, 5, 10, 30, 20i_F\} \\ \{i_F, 30i_F, 5i_F, 1\} & \{0.1, 0.5, 0.8\} \end{bmatrix} \qquad \text{I}$$

$$A \cap B = \begin{bmatrix} \{0.3i_F, 1\} & \{\phi\} \\ \{2\} & \{1, 3i_F\} \\ \{10i_F\} & \{\phi\} \\ \{0\} & \{\phi\} \\ \{\phi\} & \{0.1\} \end{bmatrix} \qquad \text{II}$$

$$A \times B = \begin{bmatrix} \begin{matrix}\{0.3i_F, 1, 2i_F + 3, 2, \\ 3.78, 0.9i_F + 25.2, \\ 0.6i_F, 4i_F + 6\}\end{matrix} & \{0, 40, 10 + i_F\} \\ \begin{matrix}\{0, 10, 20 + 5i_F, 4, \\ 8 + 2i_F, 18i_F, 36i_F + 34\}\end{matrix} & \begin{matrix}\{0, 1, 3i_F, 10, \\ 30i_F, 34\}\end{matrix} \\ \{10, 10i_F, 29, 28, 28i_F, 14i_F\} & \{20, 20i_F, 0, 14i_F 29\} \\ \begin{matrix}\{0, 37i_F, 40 + 31i_F, 24, \\ 10i_F + 39\}\end{matrix} & \\ \{i_F, 30i_F, 38, 22\} & \{0.01, 0.08, 0.05, 0.4\} \end{bmatrix}$$

Finally we find A + B;

A + B =

$$\begin{bmatrix} \begin{matrix}\{1 + 0.3i_F, 2, 2i_F + 4, \\ 2 + 0.3i_F, 3, 5 + 2i_F, \\ 0.6i_F, 1 + 0.3i_F, 3 + 2.3i_F\}\end{matrix} & \begin{matrix}\{40, 10 + i_F, \\ 41, 11 + i_F\}\end{matrix} \\ \begin{matrix}\{5, 2, 9i_F, 7, 4, 2 + 9i_F \\ 9 + i_F, 6 + i_F, 4 + 10i_F\}\end{matrix} & \begin{matrix}\{1, 3i_F, 10, 2, 11, \\ 1 + 3i_F, 6i_F, 10 + 3i_F\}\end{matrix} \\ \begin{matrix}\{30, 10 + 10i_F, 11, \\ 20 + 10i_F, 20i_F.1 + 10i_F\}\end{matrix} & \begin{matrix}\{20, 20i_F, 21, 1 + 20i_F \\ 20 + 5i_F, 25i_F\}\end{matrix} \\ \begin{matrix}\{4i_F 0, 2 + 8i_F, 9i_F, 5i_F, \\ 2 + 13i_F, 20 + 4i_F, 20, 22 + 8i_F\}\end{matrix} & \begin{matrix}\{32, 35, 2 + 20i_F, 5 + 20i_F \\ 12, 15\}\end{matrix} \\ \{1 + i_F, 1 + 30i_F, 6i_F, 35i_F\} & \{0.2, 0.9, 0.6, 1.3\} \end{bmatrix} \qquad \text{IV}$$

Thus all the four mod interval subset matrix entries semigroups are different.

Hence the related 6 mod natural neutrosophic interval subset entries matrix special type of topological spaces are also distinct.

These spaces has mod topological idempotents with result $\cup$, $\cap$ and $+$.

However finding mod topological idempotents happens to be a difficult task.

Example 3.15. Let M = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle/ a_i \in P(\langle [0, 24) \cup g\rangle_I); 1 \leq i \leq 4\right\}$ be the mod interval natural neutrosophic dual number matrices with subset entries collection {M, +}, {M, $\cup$}, {M, $\cap$}, {M, $\times_n$} and {M, $\times$} be the mod interval natural neutrosophic dual number matrices with subset semigroups.

M_o, $M_{\times}^{+}$, $M_{\times_n}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\times_n}^{\times}$, $M_{\cup}^{\times}$, $M_{\cap}^{\times}$, $M_{\cup}^{\times_n}$ and $M_{\cap}^{\times_n}$ be the mod interval natural neutrosophic dual number matrix subset special type of topological spaces.

We will show how the two products $\times$ and $\times_n$ are different.

Let $A = \begin{bmatrix} \{g, 5g2, 1\} & \{8g+12, 0\} \\ \{0, 4g+6, 3\} & \{12g, 6g+6\} \end{bmatrix}$ and

$B = \begin{bmatrix} \{6+12g, 6g, 3, 1\} & \{12g, 6g+6\} \\ \{0, 3, g+1\} & \{12g, 8g\} \end{bmatrix}$ belong to M.

$$A \times B = \begin{bmatrix} \{1, g, 5g+2, 3, 3g, 15g+6, \\ 0, 6g, 12g, 12+6g\} + \\ \{0, 12, 20g+12\} & \{8g, 0, 8, 16+16g\} \\ & +\{0\} \\ \{0, 18+12g, 9, 3, 18g, \\ 12, 12g, 12g+18 \\ 4g+6\} + \{0, 12g, \\ 18g+18, 12g, 6+12g\} & \{0, 8g\} + \{0\} \end{bmatrix}$$

$$= \begin{bmatrix} \{1, g, 5g+2, 3, 3g, \\ 15g+6, 0, 6g, 12g, 12+6g, \\ 13, 12+g, 5g+14, 15, 3g+12, \\ 18+15g, 12+12g, 20g+13, & \{8g, 0, 8 \\ 21g+12, 25g+14, 20g+15, & 16+16g\} \\ 23g+12, 11+18, 2g+12, \\ 8g+12, 2g\} \\ \{0, 18+12g, 9, 3, 18g, \\ 12, 12g, 12g+18, \\ 4g+6, 18, 12g+9, \\ 3+12g, 6g, 12+12g \\ 18, 16g+6, 6+12g, & \{0, 8g\} \\ 15+12g, 9+12g, \\ 6+6g, 18+12g, 6 \\ 12+16g\} \end{bmatrix}$$

Clearly $A \times_n B \neq A \times B$. It is left for the reader to verify.

Thus M has mod topological zero divisors, nilpotents under $\times_n$ and $\times$.

Also M has mod topological right zero divisors which are not left zero divisors and vice versa under the product ×.

So the mod topological spaces $M_{\cup}^{\times}$, $M_{\times}^{+}$, $M_{\times_n}^{\times}$ and $M_{\cap}^{\times}$ are non commutative and {M, ×} is a mod non commutative semigroup which contribute to it.

Example 3.16. Let N = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle/ a_i \in P(\langle [0,20) \cup h\rangle_I;\ 1 \leq i \leq 5\right\}$ be the mod interval natural neutrosophic special dual like number matrix with entries as subsets collection. {N, +}, {N, $\times_n$}, {N, ∪} and {N, ∩} be the mod interval natural neutrosophic special dual like number matrix subsets semigroups.

N_o, $N_{\times_n}^{\times}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times_n}$ and $N_{\cap}^{\times_n}$ be the mod interval natural neutrosophic matrix subsets special type of topological spaces.

N has mod topological idempotents under all the four operations.

Example 3.17. Let P = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle/ a_i \in P(\langle [0,43) \cup k\rangle_I;\ 1 \leq i \leq 12\right\}$ be the mod interval natural neutrosophic matrix subsets special quasi dual number collection {P, $\times_n$}, {P, ∪}, {P, ∩} and {P, +} be the mod interval natural neutrosophic special quasi dual number matrix subset semigroups.

P_o, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\times_n}^{+}$, $P_{\cup}^{\times_n}$ and $P_{\cap}^{\times_n}$ be the mod interval natural neutrosophic special quasi dual number matrix subset special type of topological spaces.

Finding nontrivial mod topological idempotents, nilpotents are zero divisors are left as an exercise to the reader.

Example 3.18. Let Q = $\left\{\begin{bmatrix} a_1 & a_6 \\ a_2 & a_7 \\ a_3 & a_8 \\ a_4 & a_9 \\ a_5 & a_{10} \end{bmatrix} \middle/ a_i \in P(\langle[0,48 \cup I\rangle_I); 1 \leq i \leq 10\right\}$ be the mod interval natural neutrosophic-neutrosophic subset matrix collection.

{Q, +}, {Q, $\times_n$}, {Q, $\cup$} and {Q, $\cap$} be the mod interval natural neutrosophic-neutrosophic subset matrix semigroup and Q_o, $Q_{\times_n}^{+}$, $Q_{\cup}^{+}$, $Q_{\cap}^{+}$, $Q_{\cap}^{\times_n}$ and $Q_{\cup}^{\times_n}$ be the mod interval natural neutrosophic-neutrosophic subset matrix topological spaces.

Studying their properties is left as an exercise to the reader.

Next we proceed onto prove the following result.

Theorem 3.2. *Let M ={A =$(a_{ij})_{m \times n}$ / $a_{ij} \in$ P([0, p)) (or $P(^I[0, p))$ or P(C[0, t)) or $P(C^I[0, t))$ or P(⟨[0, s) $\cup$ I⟩) or P(⟨[0, s)$\cup$I⟩$_I$) or P(⟨[0, a) $\cup$ g⟩) or P(⟨[0, a) $\cup$ g⟩$_I$) or P(⟨[0, b) $\cup$ h⟩) or P(⟨[0, b) $\cup$ h⟩$_I$) or P(⟨[0, c) $\cup$ k⟩) or P(⟨[0, c) $\cup$ k⟩$_I$) where p, t, s, a, b and c $\in Z^+$), $1 \leq i \leq m$, $1 \leq j \leq n$; $2 \leq m, n < \infty$} be the mod interval subset entries matrix collection (mod interval natural neutrosophic subset entries matrix collection and so on).*

i) *{M, +}, {M, ∪}, {M, $\times_n$} and {M, ∩} are the mod interval matrix with subset entries semigroups.*

ii) *M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ are mod interval subset entries matrix special type of topological spaces.*

iii) *There are mod interval subset matrix topological zero divisors and nilpotents for appropriate values of the interval.*

iv) *M has mod interval subset matrix strong topological spaces.*

v) *M has no mod interval subset topological strong ideals.*

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe using mod interval matrix subset various topological spaces by examples.

Example 3.19. Let A ={P(M) where M = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \right.$ / $a_i \in [0, 42)$; $1 \leq i \leq 5$}} be the mod interval matrix subset collection {A, ∪}, {A, ∩}, {A, +} and {A, $\times_n$} are mod interval matrix subsets semigroup.

A_o, $A^{+}_{\times_n}$, $A^{+}_{\cup}$, $A^{+}_{\cap}$, $A^{\times_n}_{\cup}$ and $A^{\times_n}_{\cap}$ are mod interval matrix subset special type of topological spaces.

Let $x = \{\begin{bmatrix} 2 \\ 20 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \\ 10 \end{bmatrix}, \begin{bmatrix} 0 \\ 10 \\ 1 \\ 0 \\ 8 \end{bmatrix}\}$ and $y = \{\begin{bmatrix} 3 \\ 0 \\ 1 \\ 4 \\ 30 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \\ 10 \end{bmatrix}\} \in A$.

$$x \cap y = \{\begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \\ 10 \end{bmatrix}\} \qquad \text{I}$$

$$x \cup y = \{\begin{bmatrix} 3 \\ 20 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \\ 10 \end{bmatrix}, \begin{bmatrix} 0 \\ 10 \\ 1 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 4 \\ 30 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 4 \\ 2 \\ 0 \end{bmatrix}\} \qquad \text{II}$$

$$x + y = \{\begin{bmatrix} 6 \\ 20 \\ 1 \\ 5 \\ 34 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \\ 9 \\ 40 \end{bmatrix}, \begin{bmatrix} 3 \\ 10 \\ 2 \\ 4 \\ 38 \end{bmatrix}, \begin{bmatrix} 8 \\ 20 \\ 4 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 6 \\ 7 \\ 10 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ 5 \\ 2 \\ 8 \end{bmatrix},$$

$$\begin{bmatrix} 3 \\ 21 \\ 2 \\ 6 \\ 14 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 4 \\ 10 \\ 20 \end{bmatrix}, \begin{bmatrix} 0 \\ 11 \\ 3 \\ 5 \\ 18 \end{bmatrix}\} \qquad \text{III}$$

$$x \times_n y = \{\begin{bmatrix} 9 \\ 0 \\ 0 \\ 4 \\ 36 \end{bmatrix}, \begin{bmatrix} 15 \\ 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 20 \\ 0 \\ 5 \\ 40 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 20 \\ 16 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 8 \\ 10 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ 25 \\ 16 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 30 \end{bmatrix},$$

$$\begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 10 \\ 2 \\ 0 \\ 38 \end{bmatrix}\} \qquad \text{IV}$$

We see all the four operations are distinct so the mod interval topological spaces are also distinct M is of infinite order M has mod strong subspaces of both finite and infinite order.

However all ideals are of infinite order and none of them are strong ideals.

Example 3.20: Let N = P (D) = {collection of all subsets from $D = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} / a_i \in {}^{I}[0, 10); 0 \le i \le 12\}\}$ be the mod interval natural neutrosophic matrix subset collection. {N, +}, {N, $\cup$}, {N, $\cap$} and {N, $\times_n$} be the mod interval natural neutrosophic matrix subset semigroups.

N_o, $N_{\times_n}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times_n}$ and $N_{\cap}^{\times_n}$ be the mod interval natural neutrosophic matrix special type of topological spaces.

This has mod strong subspaces and has no mod strong ideals.

$N_{\cup}^{\times_n}$, $N_{\cap}^{\times_n}$ and $N_{\times_n}^{+}$ have mod topological zero divisors and idempotents. However $N_{\cup}^{\times_n}$, $N_{\cap}^{\times_n}$ and $N_{\times_n}^{+}$ has no nontrivial nilpotents.

Example 3.21. Let V = {P(W) where

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right. \Bigg/ a_i \in C([0, 24));\ i_F^2 = 23,\ 1 \leq i \leq$$

16} be the mod interval finite complex number matrix subset collection.

{V, +}, {V, $\times_n$}, {V, $\times$}, {V, $\cap$} and {V, $\cup$} be the mod interval finite complex number matrix subset semigroups.

Clearly {V, $\times$} is the only non-commutative structure V_o, $V_{\times_n}^{+}$, $V_{\times}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{+}$, $V_{\cup}^{\times_n}$, $V_{\times}^{\times_n}$, $V_{\cap}^{\times_n}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be the mod interval finite complex number matrix subset special type of topological spaces associated with V.

Clearly $V_{\cup}^{\times}$, $V_{\cap}^{\times}$, $V_{+}^{\times}$ and $V_{\times_n}^{\times}$ are all non-commutative mod interval finite complex number matrix subset topological spaces associated with V.

All properties associated with these mod interval topological spaces is considered as a matter of routine so leftas exercise to the reader.

Example 3.22. Let M ={P(T) where T = { $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix}$ / $a_i \in$ $C^I([0, 12))$; $1 \leq i \leq 18$} be the mod interval natural neutrosophic finite complex number matrix subset collection {T, +}, {T, ∪}, {T, ∩}and {T, $\times_n$} be the mod interval natural neutrosophic finite complex number subset semigroups.

T_o, $T^{+}_{\times_n}$, $T^{+}_{\cup}$, $T^{+}_{\cap}$, $T^{\times}_{\cup}$ and $T^{\times}_{\cap}$ be the mod interval natural neutrosophic finite complex number matrix subset special type of topological spaces associated with these four semigroups.

Study of properties associated with T is a matter of routine so left as exercise to the reader.

Example 3.23. Let B = {P(L) / L = { $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}$ / $a_i \in \langle [0, 24) \cup I \rangle$; $1 \leq i \leq 6$}} be the mod interval neutrosophic matrix subset collection.

{B, +}, {B, ∪}, {B, ∩} and {B, $\times_n$} are the mod interval neutrosophic matrix subset semigroups.

B_o, $B^{+}_{\times_n}$, $B^{+}_{\cup}$, $B^{+}_{\cap}$, $B^{\times_n}_{\cup}$ and $B^{\times_n}_{\cap}$ are the mod interval neutrosophic matrix subset special type of topological spaces associated with the semigroup.

$$D = \{P(S) / S = \{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{bmatrix} / a_i \in [0, 24)I; 1 \leq i \leq 6\}\}$$ be the

mod interval neutrosophic matrix subset collection $D \subseteq B$ is a mod interval neutrosophic strong topological subspaces of B however D is not a strong ideal of B.

Getting other properties related with B is a matter of routine so left as an exercise to the reader.

Example 3.24. $C = \{P(N) / N = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} / a_i \in$ $\langle [0, 6 \cup I \rangle_I; 1 \leq i \leq 16\}\}$ be the mod interval natural neutrosophic-neutrosophic matrix subset collection.

$\{C, +\}$, $\{C, \cup\}$, $\{C, \cap\}$, $\{C, \times_n\}$ and $\{C, \times\}$ be the mod interval natural neutrosophic-neutrosophic matrix subset semigroup.

Let C_o, $C_{\times}^{+}$, $C_{\times_n}^{+}$, $C_{\cup}^{+}$, $C_{\cap}^{+}$, $C_{\cup}^{\times}$, $C_{\cap}^{\times}$, $C_{\times}^{\times_n}$, $C_{\cup}^{\times_n}$ and $C_{\cap}^{\times_n}$ be the mod natural neutrosophic-neutrosophic matrix subset special type of topological spaces.

Clearly $C_{\times}^{+}$, $C_{\times}^{\times_n}$, $C_{\cup}^{\times}$ and $C_{\cap}^{\times}$ are mod interval natural neutrosophic-neutrosophic subset matrix non-commutative special type of topological spaces.

Find mod strong subspaces and show C has no mod strong ideals.

Example 3.25. Let $F = \{P(Z) / Z =$

$\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{pmatrix}$ / $a_i \in \langle[0,12) \cup g\rangle$; $1 \leq i \leq 12\}$ be the mod interval dual number matrix subset collection.

{F, +}, {F, $\cup$}, {F, $\cap$} and {F, $\times_n$} be the mod interval dual number matrix subset semigroup.

F_o, $F_{\cup}^{+}$, $F_{\cap}^{+}$, $F_{\times_n}^{+}$, $F_{\cup}^{\times_n}$ and $F_{\cap}^{\times_n}$ be the mod interval dual number matrix subset special type of topological spaces.

These spaces $F_{\cup}^{\times_n}$, $F_{\cap}^{\times_n}$ and $F_{+}^{\times_n}$ have mod interval dual number topological nilpotents and zero divisors in infinite order.

Further F has mod interval dual number subset matrix topological subspaces P which are such that P $\times_n$ P = $\{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\}$.

This is the special feature associated mainly with mod interval dual number matrix subset special type of topological spaces.

Example 3.26. Let B ={P(Y) / Y = $\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$ / $a_i \in \langle[0, 12) \cup g\rangle_I$; $1 \leq i \leq 5\}\}$ be the mod interval natural neutrosophic dual number matrix subset collection. {Y, +}, {Y, $\cup$}, {Y, $\cap$} and {Y, $\times_n$} be the mod interval natural neutrosophic dual number matrix subset semigroups.

Y_o, $Y^{+}_{\times_n}$, $Y^{+}_{\cup}$, $Y^{+}_{\cap}$, $Y^{\times_n}_{\cup}$ and $Y^{\times_n}_{\cap}$ be the mod interval natural neutrosophic dual number matrix subset special type of topological spaces associated with these semigroups.

Y_o, $Y^{+}_{\times_n}$, $Y^{+}_{\cup}$, $Y^{+}_{\cap}$, $Y^{\times_n}_{\cup}$ and $Y^{\times_n}_{\cap}$ be the mod interval natural neutrosophic dual number matrix subset special topological spaces associated with these four semigroups.

Y has mod interval natural neutrosophic dual number matrix subset strong topological subspaces and some of them are such that then square is zero.

We have the following result.

Theorem 3.3. *Let $S = \{P(B) / B = \{M = (m_{ij}) / m_{ij} \in \langle[0, n) \cup g\rangle$ (or $\langle[0, n) \cup g\rangle_I$); $1 \leq i \leq t$, $1 \leq j \leq s\}\}$ be the mod interval dual number (or mod interval natural neutrosophic dual number) matrix subsets collection. $\{S, +\}$, $\{S, \times_n\}$, $\{S, \times\}$ (when $t = s$), $\{S, \cup\}$ and $\{S, \cap\}$ be the mod interval dual number (or mod interval natural neutrosophic dual number) matrix subset semigroups. S_o, $S^{+}_{\times_n}$, $S^{+}_{\cup}$, $S^{+}_{\cap}$,* (or/and $S^{+}_{\times}$, $S^{\times}_{\cup}$, $S^{\times}_{\cap}$, $S^{\times}_{\times_n}$) $S^{\times_n}_{\cup}$ and $S^{\times_n}_{\cap}$ *be the mod interval natural neutrosophic dual number special type of topological spaces according as $s = t$ or $s \neq t$.*

i) $o(S) = \infty$.

ii) *S has finite order mod interval matrix subset topological spaces which are strong.*

iii) *S has strong mod interval matrix subset topological spaces which are of infinite order.*

iv) *S has mod interval matrix subset topological spaces has zero divisors, nilpotents and idempotents for appropriate n of [0, n).*

v) *All mod interval matrix subset topological ideals of S are of infinite order.*

vi) S has no mod strong topological ideals.

vii) S has mod topological subspaces M of both finite and infinite order such that $M \times_n M = \{(0)\}$, $M \times M = \{(0)\}$.

Proof is direct and hence left as an exercise to the reader.

The seventh property is only a special feature enjoyed by mod interval dual numbers and mod natural neutrosophic interval dual numbers.

Example 3.27. Let $W = \{P(Z) / \{Z = \{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} / a_i \in \langle [0, 40) \cup h\rangle;\ 1 \leq i \leq 10\}\}$ be the mod interval special dual like number matrix subset collection.

$\{W, +\}$, $\{W, \cup\}$, $\{W, \cap\}$ and $\{W, \times_n\}$ be the mod interval special dual like number matrix subset semigroups W_o, $W^{+}_{\times_n}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times_n}_{\cup}$ and $W^{\times_n}_{\cap}$ be the mod interval special dual like number matrix subset special type of topological spaces using the mod semigroups.

W has mod interval matrix subset special dual like number strong special type of topological subspaces of both finite and infinite order.

We have mod interval special dual like number matrix subset topological zero divisors, idempotents and nilpotents.

Example 3.28. Let F = {P(T) / T ={collection of all matrices $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ / $a_i \in \langle[0, 48) \cup h\rangle_I$; $1 \leq i \leq 9$}} be the mod interval natural neutrosophic special dual like number matrix subset collection {F, +}, {F, $\cup$}, {F, $\cap$} and {F, $\times_n$} be the mod interval natural neutrosophic special dual like number matrix subset semigroups.

F_o, $F^{+}_{\times_n}$, $F^{+}_{\times}$, $F^{+}_{\cup}$, $F^{+}_{\cap}$, $F^{\times_n}_{\cup}$, $F^{\times_n}_{\cap}$, $F^{\times_n}_{\times}$, $F^{\times}_{\cup}$ and $F^{\times}_{\cap}$ be the mod interval natural neutrosophic special like number matrix subset special type of topological spaces G = {P(H) / H = $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ / $a_i \in Z_{48}$, $1 \leq i \leq 9$} be the mod interval special dual like number subset matrix special type of topological subspaces which is strong and is of finite order.

Replace H in G by K where entries are from $Z_{48}H$.

Then this K is also a mod strong topological subspace and so on.

Example 3.29. Let L ={P(W) /

W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \end{bmatrix}\right.$ / $a_i \in \langle[0, 24) \cup k\rangle$; $1 \leq i \leq 14$}} be the mod interval special quasi dual like matrix subset collection.

It is matter of routine to define mod interval semigroups and mod interval subset special topological spaces and study the properties related with them.

Example 3.30. Let Z ={P(M) / M = $\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ / $a_i \in \langle[0, 4) \cup k\rangle_I$; $1 \leq i \leq 4\}\}$ be the mod interval natural neutrosophic special quasi dual number matrix subset collection.

{Z,+}, {Z, $\times_n$}, {Z, $\cup$} and {Z, $\cap$} be the mod interval natural neutrosophic special quasi dual number matrix subset semigroups Z_o, $Z_{\cup}^{+}$, $Z_{\cap}^{+}$, $Z_{\times_n}^{+}$, $Z_{\cup}^{\times_n}$ and $Z_{\cap}^{\times_n}$ be the mod natural neutrosophic interval special quasi dual number matrix subset topological space.

$$Y = \{P(N) / N = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} / a_i \in [0, 4)k;\ 1 \leq i \leq 4\} \subseteq Z$$ be

the mod natural neutrosophic interval special quasi dual number special type of matrix subset topological subspaces Y is a strong mod interval topological subspace of infinite order which is not a strong topological ideal but not even a mod interval topological ideal.

Study in this direction is left as an exercise to the reader.

Now we proceed onto describe mod interval subset special type of polynomial topological spaces by examples.

Example 3.31. Let W = M[x] = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P([0, 10))\}$ be the mod interval polynomial with subset coefficients collection {M[x], $\cup$}, {M[x], $\cap$}, {M[x], $\times$} and {M[x], +} be the mod interval polynomial subset coefficient semigroup.

Let $p(x) = \{3, 2.5, 0.1\}x^3 + \{0.7, 0.2\}x + \{0.5, 2, 0.1\}$ and $q(x) = \{0.1, 2.5, 6, 3\}x^3 + \{2, 1\}x + \{2, 0.1, 7\} \in M[x]$

$p(x) + q(x) = \{3.1, 2.6, 0.2, 5.5, 5, 2.6, 9, 8.5, 6.1, 6, 3.1\}x^3 + \{2.7, 2.2, 1.7, 1.2\}x + \{2.5, 4, 2.1, 0.6, 0.2, 7.5, 9, 7.1\}$ I

$p(x) \cap q(x) = \{3, 0.1, 2.5\}x^3 + \{2, 0.1\}$ II

$p(x) \cup q(x) = \{3, 2.5, 0.1, 6\}x^3 + \{0.7, 0.2, 2, 1\}x$
$+ \{0.1, 2, 0.5, 7\}$ III

$p(x) \times q(x) = \{0.3, 0.25, 0.01, 7.5, 5, 6.25, 0.25, 8, 0, 0.6, 9\}x^6 + \{0.07, 0.02, 1.75, 0.5, 4.2, 1.2, 2.1, 0.6\}x^4 + \{0.05, 0.2, 0.01, 1.25, 5, 0.25, 3, 2, 0.6, 1.5, 6, 0.3\}x^3 + \{6, 5, 0.2, 3, 2.5, 0.3\}x^4 + \{1.4, 0.4, 0.7, 0.2\}x^2 + \{1, 4, 0.2, 0.5, 2, 0.1\}x + \{1, 4, 0.2, 0.05, 0.01, 3.5, 4, 0.7\} + \{1.4, 0.4, 0.07, 0.02, 4.9\}x$

$= \{0.3, 0.25, 0.01, 7.5, 5, 6.25, 0.25, 8, 0.6, 9\}x^6 + \{6.07, 6.02, 7.75, 6.5, 0.2, 7.2, 8.1, 6.6, 5.07, 5.02, 6.75, 5.5, 9.2, 6.2, 7.1, 5.6, 0.27, 0.22, 1.95, 0.7, 4.4, 1.4, 2.3, 0.8, 3.07, 3.02, 4.75, 3.5, 7.2, 4.2, 5.1, 3.6, 2.57, 2.52, 4.25, 3, 6.7, 3.7, 4.6, 3.1, 0.37, 0.32, 2.05, 0.8, 4.5, 1.5, 2.4, 0.9\}x^4 + \{0.05, 0.2, 0.01, 1.25, 5, 0.25, 3, 2, 0.6, 1.5, 6, 0.3\}x^3 + \{1.4, 0.4, 0.7, 0.2\}x^2 + \{2.4, 5.4, 1.6, 1.9, 3.4, 1.5, 1.4, 4.4, 0.6, 0.9, 2.4, 0.5, 1.07, 4.07, 0.27, 0.57, 2.07, 0.17, 1.02, 4.02, 0.22, 0.52, 2.02, 0.12, 5.9, 8.9, 5.1, 5.4, 6.9, 5\}x + \{1, 4, 0.2, 0.05, 0.01, 3.5, 4, 0.7\}$ IV

All the four mod interval subset polynomial coefficient semigroups are distinct.

Hence W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ are the six distinct mod interval subset coefficient polynomial special type of topological spaces are distinct.

Finding strong mod topological spaces, mod topological ideals and mod topological nilpotents and zero divisors happens to be matter of routine.

Example 3.32. Let S = W[x] = $\{\sum_{i=0}^{\infty} a_i x^i / a_I \in P\{^I[0, 48))\}$ be mod interval natural neutrosophic subset coefficient polynomial collection.

{S, +}, {S, $\times_o$}, {S, $\times$}, {S, $\cup$} and {S, $\cap$} be the mod interval natural neutrosophic subset coefficient polynomial semigroups.

Let S_o, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\times}^{+}$, $S_{\times_0}^{+}$, $S_{\cap}^{\times}$, $S_{\cup}^{\times}$, $S_{\times_0}^{\times}$, $S_{\cup}^{\times_0}$ and $S_{\cap}^{\times_0}$ be the mod natural neutrosophic interval subset coefficient polynomial special type of topological spaces associated with the semigroups.

Let p(x) = $\{I_4^{48} + 15, I_{16}^{48}, 3, 0.331 + I_6^{48}\}x^7 + \{I_2^{48}, I_{16}^{48} + 0.33, 0.1112, I_{12}^{48}\}x^4 + \{I_{12}^{48} + I_8^{48} + 6.33, I_0^{48} + I_{32}^{48} + I_6^{48} + 0.75, 16.72, 0.3792\} \in S$.

Interested reader can prove the two operation $\times_0$ and $\times$ are distinct on S. The mod interval natural neutrosophic subset coefficient topological zeros with respect to $\times$ and $\times_0$ are different.

There are mod strong subspaces which are not strong ideals.

For instance L = V[x] = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P([0, 48)\} \subseteq S$ is a mod strong interval subset coefficient polynomial topological space but is not a mod strong interval subset ideal.

However L is an ideal of S_o alone.

Example 3.33. Let $R = P[x] = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in C([0, 20));$ $i_F^2 = 19\}$ be the mod interval finite complex number subset coefficient polynomial collection $\{R, +\}$, $\{R, \times\}$, $\{R, \cup\}$ and $\{R, \cap\}$ be the mod interval finite complex number subset coefficient polynomial semigroups and R_o, $R_\cup^+$, $R_\cap^+$, $R_\times^+$, $R_\cup^\times$ and $R_\cap^\times$ the related mod interval finite complex number coefficient topological spaces.

All properties can be derived related to R. All ideal of R are of infinite order and R has no mod interval strong topological ideals.

Example 3.34. Let $T = \{B[x]\} = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(C^I[0, 12))\}$ the mod interval natural neutrosophic finite complex number subset coefficient polynomial collection, $\{T, \times\}$, $\{T, \times_o\}$, $\{T, +\}$, $\{T, \cup\}$ and $\{T, \cap\}$ be the mod interval natural neutrosophic finite complex number subset coefficient polynomial semigroups.

Let T_o, $T_\times^+$, $T_{\times_n}^\times$, $T_{\times_n}^+$, $T_\cup^+$, $T_\cap^+$, $T_\cup^\times$, $T_\cap^\times$, $T_\cup^{\times_n}$ and $T_\cap^{\times_n}$ be the mod interval natural neutrosophic finite complex number subset coefficient polynomial special type of topological spaces associated with these 5 semigroups.

All properties can be derived with appropriate modification.

Example 3.35. Let $E = \{D[x]\} = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(\langle [0, 20) \cup I \rangle\}$ be the mod interval neutrosophic subset coefficient polynomial collection. E_o, $E^+_\cup$, $E^+_\cap$, $E^+_{\times_0}$, $E^+_\times$, $E^\times_\cup$, $E^\times_\cap$, $E^\times_{\times_0}$, $E^{\times_0}_\cup$ and $E^{\times_0}_\cap$ be the mod interval special type of topological spaces.

$W = B[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P([0, 20)I)\}$ be the mod interval neutrosophic subset coefficient polynomial strong special type of topological subspace.

Clearly W is not a strong mod topological ideal.

Example 3.36. Let $H = \{V[x]\} = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle[0, 15) \cup I\rangle_I)\}$ be the mod interval natural neutrosophic-neutrosophic subset coefficient polynomial collection.

As usual H_o, $H^+_{\times_0}$, $H^+_\times$, $H^+_\cup$, $H^+_\cap$, $H^\times_{\times_0}$, $H^\times_\cup$, $H^\times_\cap$, $H^{\times_0}_\cup$ and $H^{\times_0}_\cap$ be the mod interval natural neutrosophic-neutrosophic subset coefficient polynomial special type of topological spaces. H_0, $H^+_\cup$, $H^+_\cap$ has mod finite order topological subspaces.

Under $H^\times_{\times_0}$, $H^+_\times$, $H^\times_\cup$, $H^\times_\cap$, $H^{\times_0}_\cup$ and $H^{\times_0}_\cap$ all subspaces are only of infinite order.

Example 3.37. Let $J = \{W[x]\} = \sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle[0, 24) \cup g\rangle_I)\}$ be the mod interval natural neutrosophic dual number subset coefficient polynomials collection $\{J, +\}$, $\{J, \times_o\}$, $\{J, \times\}$, $\{J, \cup\}$ and $\{J, \cap\}$ be the mod interval natural neutrosophic dual number subset coefficient polynomial semigroups.

Associated with these 5 semigroups we have 10 distinct mod interval natural neutrosophic dual number subset coefficient polynomial special type topological spaces.

The main feature enjoyed by these spaces is we have mod interval natural neutrosophic dual number subset coefficient polynomials special type of subspaces V such that V $\times$ V = {0} and V can be both of finite or of infinite order.

There are strong mod interval subset special type of topological spaces however they are not mod strong topological ideals.

Example 3.38. Let M ={T[x]} = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(\langle[0, 12) \cup g\rangle)\}$ be the mod interval dual number subset coefficient polynomial collections. {M, +}, {M, $\times$}, {M, $\cup$} and {M, $\cap$} be the mod interval dual number subset coefficient polynomial semigroups.

Associated with these mod semigroups we have M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$, the mod interval dual number subset coefficient polynomial special type of topological spaces.

$$W_1 = \{L[x]\} = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(Z_{12})\},$$

$$W_2 = \{P[x]\} = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(Z_{12}g)\} \text{ and}$$

$W_3 = \{R[x]\} = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle Z_{12} \cup g\rangle)\}$ be the mod interval dual number subset coefficients polynomial topological subspaces.

All these are strong mod topological subspaces.

None of them are ideals. We see $W_2 \times W_2 = \{0\}$.

Example 3.39. Let K = {L[x]} = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, 10) \cup h\rangle), h^2 = h\}$ be the mod interval special dual like number subset coefficient polynomial collection.

{K, +}, {K, $\cup$}, {K, $\cap$} and {K, $\times$} be the mod interval special dual like number subset coefficient polynomial semigroups.

K_o, $K_{\cup}^{+}$, $K_{\cap}^{+}$, $K_{\times}^{+}$, $K_{\cap}^{\times}$ and $K_{\cup}^{\times}$ be the mod interval special dual like number subset coefficient polynomial special type of topological spaces associated with these semigroups.

T = {P[x]} = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in [0, 10)h\} \subseteq K$ is the mod interval subset coefficient polynomial strong topological subspace of K but is not a strong topological ideal.

Only $T_{\cap}^{\times}$ is an ideal if we demand for every p(x) $\in$ T and for every q(x) $\in$ K; p(x) $\times$ q(x) $\in$ $T_{\cap}^{\times}$ and p(x) $\cap$ q(x) $\in$ $T_{\cap}^{\times}$. However p(x) $\cup$ q(x) $\notin$ T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$.

p(x) + q(x) $\notin$ T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$ and $T_{\times}^{+}$. But if we restrain it is sufficient if under only one of the operation it should belong to T then except $T_{\cup}^{+}$ all will be ideals for $\cap$ or $\times$ the set T is an ideal, however p(x) $\cup$ q(x) and p(x) + q(x) do not belong to T for any p(x) $\in$ T and q(x) $\in$ K. Hence the claim.

This has mod topological zero divisors but mod topological idempotents does not exist. Also this has mod topological nilpotents as [0, 10) has no nilpotents.

Example 3.40. Let L = {M[x]} = $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in$ P (⟨[0, 23) ∪ h⟩$_I$} be the mod interval natural neutrosophic special dual like number subset coefficient polynomial collection {L, +}, {L, ∪}, {L, ∩}, {L, $\times_o$} and {L, ×} be the mod interval natural neutrosophic special dual like number subset coefficient polynomial semigroups.

L_o, $L^{+}_{\times_0}$, L^{+}_{x}, $L^{+}_{\cap}$, $L^{+}_{\cup}$, $L^{\times}_{\cup}$, $L^{\times}_{\cap}$, $L^{\times}_{x_0}$, $L^{\times_0}_{\cup}$ and $L^{\times_0}_{\cap}$ be the mod interval natural neutrosophic special dual like number subset coefficient polynomial special type of topological spaces associated with the five mod interval semigroups.

The study of properties related with L happens to be a matter of routine so left as an exercise to the reader.

Example 3.41. Let M = {R[x]} = {$\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in$ P ⟨[0, 24) ∪ k⟩; k^2 = 23k} be the mod interval special quasi dual number subset coefficient polynomial collection. {M, +}, {M, ×}, {M, ∪} and {M, ∩} be the mod interval subset coefficient semigroups.

M_o, M^{+}_{x}, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times}_{\cup}$ and $M^{\times}_{\cap}$ be the mod interval special quasi dual number subset coefficient polynomial special type of topological spaces.

M has mod topological zero divisors, only mod topological trivial idempotents and mod topological nilpotents as 24 is such that it contains zero divisors and nilpotents.

Example 3.42. Let N = {S[x]} = {$\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in$ P (⟨[0, 7) ∪ k⟩$_I$)} be the mod interval natural neutrosophic special quasi dual number subset coefficient polynomial collection.

$\{N, +\}$, $\{N, \times_o\}$, $\{N, \times\}$, $\{N, \cup\}$ and $\{N, \cap\}$ be the mod interval natural neutrosophic special quasi dual number subset coefficient polynomial semigroups.

N_o, $N_{\times}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\times_0}^{+}$, $N_{\cup}^{\times}$, $N_{\cap}^{\times}$, $N_{\times_0}^{\times}$, $N_{\cup}^{\times_0}$ and $N_{\cap}^{\times_0}$ be the mod interval neutral neutrosophic special quasi dual number special type of topological space of subset coefficients polynomial.

Study of the related properties is a matter of routine so left as an exercise to the reader.

Next we proceed onto describe by examples mod interval polynomial subset special type of topological structures.

Example 3.43. Let W = P(S[x]) = {collection of all subsets from $S[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in [0, 12)\}\}$ be the mod interval polynomial subset collection, $\{W, +\}$, $\{W, \cup\}$, $\{W, \cap\}$ and $\{W, \times\}$ be the mod interval polynomial subset semigroups.

Let $A = \{0.5x^8 + 10.5,\ 5x^3 + 0.01x + 6.01,\ 2.5x^2 + 0.005\}$ and $B = \{2.5x^2 + 0.005,\ 7x^2 + 5x + 0.05,\ 10x^2 + 0.06\} \in W$.

$A \cup B = \{0.5x^8 + 10.5,\ 5x^3 + 0.01\,x + 6.01,\ 2.5x^2 + 0.005,\ 7x^2 + 5x + 0.05,\ 10x^2 + 0.06\}$ I

$A \cap B = \{2.5x^2 + 0.005\}$ II

$A + B = \{0.5x^8 + 2.5x^2 + 10.505,\ 5x^3 + 0.01x + 2.5x^2 + 0.01x + 6.01,\ 5x^2 + 0.01,\ 0.5x^3 + 7x^2 + 5x + 10.55,\ 5x^3 + 7x^2 + 5.01x + 6.06,\ 0.5x^8 + 10x^2 + 10.56,\ 9.5x^2 + 5x + 0.055,\ 12.5x^2 + 0.065 + 5x^3 + 10x^2 + 0.01x + 0.06\}$ III

$A \times B = \{1.25x^{10} + 26.25x^2 + 0.0025x^8 + 0.0525,\ 12.5x^5 + 0.025x^3 + 3.025x^2 + 0.025x^3 + 0.00005x + 3.005,\ 6.25x^4 + 0.0125x^2 + 0.0125x^2 + 0.000025,\ 3.5\,x^{10} + 1.5x^2 + 2.5x^9 + 52.5\,x + 0.025x^8 + 0.525,\ 35x^5 + 0.07x^3 + 6.07x^2 + 25x^4 + 0.05x^2 + 6.05x + 0.25x^3 + 0.0005x + 0.3005,\ 3.5x^4 + 0.035x^2 + 0.5x^3 + 0.025x + 0.125x^2 + 0.00025x^{10} + 9x^2 + 0.03x^8 + 0.630,\ 2x^5 + 0x^3 + 0.1x^2 + 0.3x^3 + 0.0006x + 0.3606,\ x^2 + 0.05x^2 + 0.150x^2 + 0.00030\}$ IV

Clearly all the equations I, II, III and IV are distinct hence the mod interval polynomial subset topological spaces associated these four mod semigroups are distinct.

W_o, W_x^+, $W_\cup^+$, $W_\cap^+$, $W_\cup^\times$ and $W_\cap^\times$ has mod interval polynomial subset strong topological spaces. No nontrivial idempotents with respect to $\times$. However under $\cup$ and $\cap$ every element of W is an idempotent.

Example 3.44. Let N = {P(W[x])} = {collection of all subsets from $W[x] = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in {}^I[0, 4)\}\}$ be the mod interval natural neutrosophic polynomial subsets collection.

$\{N, +\}$, $\{N, \times\}$, $\{N, \times_0\}$, $\{N, \cup\}$ and $\{N, \cap\}$ be the mod interval natural neutrosophic polynomial subsets semigroup.

We see N_o, $N_{x_0}^+$, $N_\times^+$, $N_\cup^+$, $N_\cap^+$, $N_{\times_0}^\times$, $N_\cup^\times$, $N_\cap^\times$, $N_\cup^{\times_0}$ and $N_\cap^{\times_0}$ are the mod interval natural neutrosophic polynomial subset special type of topological spaces associated with these 5 semigroups.

Properties related with them can be desired with appropriate changes.

Example 3.45. Let V = P(W[x]) = {collection of all subsets from W[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in$ C ([0, 40))}} be the mod interval finite complex number polynomial subset collection, {V, ×}, {V, ∩}, {V, +}and {V, ∪} be the mod interval finite complex number polynomial subsets semigroup.

V_o, $V_\times^+$, $V_\cup^+$, $V_\cap^+$, $V_\cup^\times$ and $V_\cap^\times$ be their respective mod interval finite complex number polynomial subset special type of topological spaces.

All related properties of V can be desired a routine way with appropriate changes.

Example 3.46. Let Z = {P(V[x])} = {collection of all subsets from V[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in C^I$ ([0, 7))} be the mod interval natural neutrosophic finite complex modulo integer polynomial subset collection.

{Z, +}, {Z, $\times_o$}, {Z, ×}, {Z, ∪} and {Z, ∩} be the mod interval natural neutrosophic finite complex number polynomial subset semigroups.

Z_o, $Z_\times^+$, $Z_{\times_0}^+$, $Z_\cup^+$, $Z_\cap^+$, $Z_\cup^\times$, $Z_\cap^\times$, $Z_{\times_0}^\times$, $Z_\cup^{\times_0}$ and $Z_\cap^{\times_0}$ be the mod interval natural neutrosophic finite complex number polynomial subset special type of topological spaces associated with these 5 semigroups.

It is left as an exercise for the reader to derive all the related properties.

Example 3.47. Let Y = {P(T[x])} = {collection of all subsets from T[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 9) \cup I\rangle\}\}$ be the mod interval neutrosophic polynomial subset collection.

We see {Y, +}, {Y, ∪}, {Y, ∩} and {Y, ×} be the mod interval neutrosophic polynomial subset semigroups.

Y_o, $Y_\times^+$, $Y_\cup^+$, $Y_\cap^+, Y_\cup^\times$ and $Y_\cap^\times$ be the mod interval neutrosophic subset polynomial special type of topological spaces associated with the semigroups.

P_1 = P(R[x]) = {collection of all subsets from R[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in [0, 9)\}\}$,

P_2 = P(R[x]) = {collection of all subsets from $R_1[x]$ = $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in [0, 9)I\}\} \subseteq$ Y are both mod interval neutrosophic polynomial subset strong topological subspaces that are not strong topological ideals.

However $R_1[x]$ is an ideal for some of the topological spaces like $T_\cap^\times$.

Study in this direction is innovative and interesting.

Example 3.48. Let R = {P(L[x])} = {collection of all subsets from L[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 21) \cup I\rangle_I\}\}$ be the mod natural neutrosophic-neutrosophic polynomial subset collection. {R, +}, {R, ∪}, {R, ∩}, {R, ×} and {R, $\times_n$} be the mod interval natural neutrosophic-neutrosophic polynomial subset semigroup.

R_o, $R^{+}_{\times_0}$, $R^{+}_{\times}$, $R^{+}_{\cup}$, $R^{+}_{\cap}$, $R^{\times_0}_{\cup}$, $R^{\times_0}_{\cap}$, $R^{\times}_{\cup}$, $R^{\times}_{\cap}$ and $R^{\times_0}_{\times}$ be the mod interval natural neutrosophic-neutrosophic polynomial subset special type of topological spaces.

All properties of these mod special spaces can be derived with appropriate modifications.

Example 3.49. Let M = {P(B[x])} = {collection all subsets from $B[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle[0, 24) \cup g\rangle\}\}$ be the mod interval dual number subset polynomial collection.

{M, +}, {M, ×}, {M, ∪} and {M, ∩} be the mod interval dual number polynomial subset semigroups.

M_o, $M^{+}_{\times}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times}_{\cup}$ and $M^{\times}_{\cap}$ be the mod interval dual number subset polynomial special type of topological spaces. This mod topological space has nilpotents and zero divisors.

Infact M has mod topological subspaces say S such that $S \times S = \{0\}$.

Example 3.50. Let B = {P(D[x])} = {collection of all subsets from $D[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle[0, 13) \cup g\rangle_I\}\}$ be the mod interval natural neutrosophic dual number polynomial subset collection.

{B, +}, {B, ∪}, {B, ∩}, {B, ×} and {B, $\times_0$} be the mod interval natural neutrosophic dual number polynomial subset semigroup. B_o, $B^{+}_{\times_0}$, $B^{+}_{\times}$, $B^{+}_{\cup}$, $B^{+}_{\cap}$, $B^{\times}_{\times_0}$, $B^{\times}_{\cap}$, $B^{\times}_{\cup}$, $B^{\times_0}_{\cup}$ and $B^{\times_0}_{\cap}$ be the mod interval natural neutrosophic dual number polynomial subset special type of topological spaces associated with this five mod interval semigroups.

Only these spaces have both mod interval natural neutrosophic zero divisors and nilpotents, mixed zero divisors and nilpotents and mod interval usual zero divisors and nilpotent.

This mod dual number spaces behaves differently and distinct from other so study of these happens to be both innovative and interesting.

Example 3.51. Let S = {P(R[x]) = {collection of all subsets from R[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 12) \cup h\rangle\}\}$ be the mod interval special dual like number polynomial subset collection {S, +}, {S, ×}, {S,∪} and {S, ∩} be the mod interval special dual like number polynomial subset semigroups.

Let S_o, $S_{\cup}^{+}$, $S_{\cap}^{+}, S_{\times}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ be the mod interval special dual like number polynomial subset special type of topological spaces.

T = {P(M[x])} = {collection of all subsets from M[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in [0, 12)h\}\}$ be the mod interval special dual like number subset special type of strong topological subspaces.

Clearly T is not a mod strong ideal of S.

Study in this direction is also innovative and left as an exercise to the reader.

Example 3.52. Let W = {P(T[x]) = {collection of all subsets from T[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 17) \cup h\rangle_I\}\}$ be the mod interval natural neutrosophic special quasi dual like number polynomial subsets collection, {W, +}, {W, ∪}, {W, ∩}, {W, $\times_0$} and

{W,x} be the mod interval natural neutrosophic special dual like number polynomial subset semigroups.

W_o, $W^{+}_{\times}$, $W^{+}_{\times_0}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times}_{\cup}$, $W^{\times}_{\cap}$, $W^{\times_0}_{\times}$, $W^{\times_0}_{\cup}$ and $W^{\times_0}_{\cap}$ be the mod interval natural neutrosophic special dual like number polynomial subset special type of topological spaces built using these 5 mod interval semigroups.

This W has strong mod interval topological subspaces but has no mod interval topological ideals.

Study in this direction is interesting and this work is left as an exercise to the reader.

Example 3.53. Let M = {P(C[x])} = {collection of all subsets from C[x] = $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 42) \cup k\rangle$}} be the mod interval special quasi dual number polynomial subsets collection {M, +}, {M, ×}, {M, ∪} and {M, ∩} be the mod interval special quasi dual number polynomial subset semigroups.

M_o, $M^{+}_{\times}$ $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times}_{\cup}$ and $M^{\times}_{\cap}$ be the mod interval special quasi dual number polynomial subset special type of topological spaces associated with these semigroups.

N = {P(V[x])} = {collection of all subsets from V[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in [0, 42)k\}\}$ is a mod interval special quasi dual number polynomial subset strong special type of topological subspace which is not a mod topological ideal.

Study in this direction is innovative as $k^2 = 41k$ and is left as an exercise to the reader.

Example 3.54. Let F = {P(G[x])} = {collection of all subsets from G[x] = { $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in \langle[0, 12) \cup k\rangle_I$}} be the mod interval natural neutrosophic polynomial subset collection. {F, +}, {F, ×}, {F, ∪}, {F, ∩} and {F, $\times_o$} be the mod interval natural neutrosophic polynomial subset semigroup.

Let F_o, $F_{\cup}^{+}$, $F_{\cap}^{+}$, $F_{\times}^{+}$, $F_{\times_0}^{+}$, $F_{\cup}^{\times}$, $F_{\cap}^{\times}$, $F_{\times_0}^{\times}$, $F_{\cup}^{\times_0}$ and $F_{\cap}^{\times_0}$ be the mod interval natural neutrosophic polynomial subset special type of topological spaces associated with these semigroups. F has mod topological natural neutrosophic zero divisors and nilpotents and mod topological usual zero divisors.

Next we proceed onto briefly describe the mod interval finite degree polynomial subsets and subset coefficient polynomials in the following by examples.

Example 3.55. Let S = P($M[x]_9$) = {collection of all subsets from $M[x]_9$ = { $\sum_{i=0}^{9} a_i x^i$ / $a_i \in [0, 6)$, $x^{10} = 1$}} be the mod interval finite degree polynomial subsets collection.

{S, +}, {S, ×}, {S, ∪} and {S, ∩} be the mod interval finite degree polynomial subset semigroups. S_o, $S_{\times}^{+}$, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ be the mod interval finite degree polynomial subset special type of topological spaces.

Clearly o(S) = ∞.

Let A = {$0.1x^6 + 0.3$, $2.01x^2 + 0.03x + 1.1$} and B = {$0.1x^6 + 0.3$, $0.5x^5 + 0.6x^3 + 2x + 0.1$, $0.7 + 0.10x$} ∈ S.

Clearly A ∩ B = {$0.1x^6 + 0.3$} I

$A \cup B = \{0.1x^6 + 0.3, 2.01x^2 + 0.03x + 1.1, 0.5x^5 + 0.6x^3 + 2x + 0.1, 0.7 + 0.10x\}$ II

$A + B = \{0.2x^6 + 0.6, 0.1x^6 + 2.01x^2 + 0.03x + 1.4, 0.1x^6 + 0.5x^5 + 0.6x^3 + 2x + 0.4, 0.5x^5 + 0.6 x^3 + 2.03x + 1.2, 0.1x^6 + 0.10x + 1, 2.01x^2 + 0.13x + 1.8$ III

$A \times B = \{0.01x^{12} + 0.06x^6 + 0.09, 0.201x^8 + 0.003x^7 + 0.11x^6 + 0.603x^2 + 0.009x + 0.33, 0.05x^{11} + 0.15x^5 + 0.06x^9 + 0.18x^3 + 0.2x^7 + 0.6x + 0.01 x^6, 0.03, 1.005x^7 + 1.206x^5 + 4.02x^3 + 0.201x^2 + 0.015x^6 + 0.018 x^4 + 0.66x^3 + 0.55 x^5 + 0.06x^2 + 2.2x + 0.003x + 0.11, 0.07x^6 + 0.21 + 0.2x^7 + 0.03x, 0.147 x^3 + 0.003 x^2 + 0.11x\}$ IV

Clearly the equations I, II, III and IV are distinct forcing all the mod interval finite polynomial special type of topological spaces to be distinct.

Let $A = \{3x^2 + 3, 3x^8 + 3x^2 + 3\}$ and $B = \{2x^2 + 4, 2x^3 + 4x + 2\} \in S$. Clearly $A \times B = \{0\}$. Thus S has mod topological zero divisors but has no mod topological nontrivial idempotents.

Further S has mod interval finite degree polynomial subset strong special type of topological subspaces but has no mod interval topological strong ideals.

All ideals are also of infinite order as $o(S) = \infty$.

Example 3.56. Let $H = \{P(W[x]_9)\}$ = {collection of all subsets from $W[x] = \{\sum_{i=0}^{9} a_i x^i \,/\, a_i \in {}^{I}[0, 16), x^{10} = 1\}\}$ be the mod interval natural neutrosophic finite degree polynomial subsets collection. $\{H, +\}$, $\{H, \cup\}$, $\{H, \cap\}$, $\{H, \times_0\}$ and $\{H, \times\}$ be the mod interval natural neutrosophic finite degree polynomial subset semigroup.

H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\times_0}^{+}$, $H_{\times_0}^{\times}$, $H_{\cup}^{\times}$, $H_{\cap}^{\times}$, $H_{\cup}^{\times_0}$ and $H_{\cap}^{\times_0}$ be the mod interval natural neutrosophic finite degree polynomial special type of topological spaces associated H. o(H) = ∞.

Example 3.57. Let D = {P(E[x]$_{12}$)} = {collection of all subsets from E[x]$_{12}$ = $\{\sum_{i=0}^{12} a_i x^i$ / $a_i \in \langle[0, 5)\cup I\rangle_I$; $x^{13} = 1\}\}$ be the mod interval natural neutrosophic-neutrosophic polynomial subset collection.

{D, +}, {D, $\times_0$}, {D, ×}, {D, ∪} and {D, ∩} be the mod interval natural neutrosophic-neutrosophic polynomial subset semigroups.

D_o, $D_{\times_0}^{+}$, $D_{\times}^{+}$, $D_{\cup}^{+}$, $D_{\cap}^{+}$, $D_{\cup}^{\times}$, $D_{\times_0}^{\times}$, $D_{\cap}^{\times}$, $D_{\cup}^{\times_0}$ and $D_{\cap}^{\times_0}$ be the mod interval natural neutrosophic-neutrosophic polynomial subset special type of topological spaces related with the 5 mod interval semigroups.

This has finite order mod interval natural neutrosophic-neutrosophic polynomial subset strong topological subspaces.

Example 3.58. Let M ={P(F[x]$_{10}$)} = {collection of all subsets from F[x]$_{10}$ = $\{\sum_{i=0}^{10} a_i x^i$ / $a_i \in C^I([0, 12))$, $x^{11} = 1\}\}$ be the mod interval natural neutrosophic finite complex number finite degree polynomial subset collection. {M, +}, {M, ×}, {M, $\times_0$}, {M, ∪} and {M, ∩} be the mod interval natural neutrosophic finite complex number finite degree polynomial semigroup.

All the 10 mod interval natural neutrosophic finite complex number finite degree polynomial special topological spaces are distinct.

Further their related properties can be derived with appropriate modifications.

Example 3.59. Let G = P(L[x]$_6$) = {collection of all subsets from $L[x]_6 = \{\sum_{i=0}^{6} a_i x^i$ / $a_i \in \langle[0, 16) \cup g\rangle$, $x^7 = 1\}\}$ be the mod interval dual number finite degree polynomial subsets. G_o, $G_{\times}^{+}$, $G_{\cup}^{+}$, $G_{\cap}^{+}$, $G_{\cup}^{\times}$ and $G_{\cap}^{\times}$ be the mod interval dual number finite degree polynomial subset special type of topological spaces.

All properties can be derived with appropriate modifications.

In view of this we have the following theorem.

Theorem 3.4. *Let G = P(S[x]$_n$) = {Collection of all subsets from S[x] = {$\sum_{i=0}^{n} a_i x^i$ / $a_i \in$ [0,m) (or ^{I}[0, m) or C([0, m)) or C^I([0,m)), or ⟨[0, m) ∪ I⟩ or ⟨[0, m) ∪ I⟩$_I$ or ⟨[0, m) ∪ g⟩$_I$ or ⟨[0, m) ∪ g⟩ or ⟨[0, m) ∪ h⟩ or ⟨[0, m) ∪ h⟩$_I$ or ⟨[0, m) ∪ k⟩ or ⟨[0, m) ∪ k⟩$_I$); $n < \infty$, $2 \leq m < \infty$}} be the mod interval finite degree polynomial subset collection.*

i) *{G, +}, {G, ×}, {G, $\times_0$}, {G, ∪} and {G, ∩} be the five mod interval finite degree polynomial subset semigroup where all the operations are relevant.*

ii) *G_o, $G_{\cup}^{+}$, $G_{\cap}^{+}$, $G_{\times}^{+}$, $G_{\times_0}^{+}$, $G_{\cup}^{\times}$, $G_{\cap}^{\times}$, $G_{\cup}^{\times_0}$, $G_{\times_0}^{\times}$* and *$G_{\cap}^{\times_0}$ are the mod interval finite degree polynomial subset special type of topological spaces.*

iii) *G has mod interval finite degree polynomial subset special type of strong topological subspaces.*

iv) *G has mod interval finite degree polynomial subset special type of topological ideals and not strong ideals.*

v) *G has mod interval finite degree polynomial subset special type of topological zero divisors or nilpotents for appropriate values of m.*

vi) *v is true when $\langle[0, m) \cup g\rangle$ or $\langle[0, m) \cup g\rangle_I$ are taken.*

vii) *G has no mod interval topological idempotents.*

Proof of these can be given with appropriate modifications so left as an exercise to the reader.

Example 3.60. Let H = { $\sum_{i=0}^{9} a_i x^i$ / $a_i \in$ P ([0, 24)), x^{10} = 1} be the mod interval subset coefficient polynomial collection, H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ be the mod interval special topological spaces o(H) = ∞.

All related properties can be studied for H with appropriate modification.

Example 3.61. Let K = { $\sum_{i=0}^{20} a_i x^i$ / $a_i \in P(\langle[0, 120) \cup g\rangle_I)$, x^{21} = 1}, g^2 = 0} be the mod interval subset coefficient polynomial collection. Let K_o, $K_{\times}^{+}$, $K_{\times_0}^{+}$, $K_{\cup}^{+}$, $K_{\cap}^{+}$, $K_{\times}^{\times_0}$, $K_{\cup}^{\times_0}$, $K_{\cap}^{\times_0}$, $K_{\cup}^{\times}$ and $K_{\cap}^{\times}$ be the mod interval subset coefficient polynomial of finite degree special topological space K has mod interval subset strong topological subspaces which are not topological ideals. K has infinite number of zero divisors and nilpotents.

Example 3.62. Let Q = { $\sum_{i=0}^{5} a_i x^i$ / $a_i \in P(\langle[0, 112) \cup k\rangle_I)$, x^6 = 1} the mod interval natural neutrosophic special quasi dual number subset coefficient polynomial collection. Q_o, $Q_{\times}^{+}$, $Q_{\times_0}^{+}$, …, $Q_{\cup}^{\times_0}$ be the 10 mod interval topological spaces.

All related properties can be derived with conditions $x^6 = 1$ and $k^2 = 11k$.

The theorem can be derived for these mod interval topological spaces with subset coefficient polynomials of finite degree with appropriate changes in that theorem.

We suggest the following problems for the interested reader.

PROBLEMS

1. Let W = P([0, n)) be the mod interval subset collection. {W, +}, {W, $\cap$}, {W, $\cup$} and {W, $\times$} be the mod interval subset semigroups.

   i) Find idempotents if any in {W, +} and {W, $\times$}.

   ii) Prove every subset of W is an idempotent in case of {W, $\cup$} and {W, $\cap$}.

   iii) Find all zero divisors in {W, $\times$}.

   iv) Define the six mod interval subset type I topological spaces using the four mod semigroups and prove they are distinct.

   v) Prove these mod interval subset topological spaces have mod interval special type of topological zero divisors, idempotents and nilpotents depending on n of [0, n).

   vi) Find all mod interval strong subset topological spaces of both finite and infinite order.

   vii) Prove W has no mod interval special type of strong subset topological ideals.

viii) Obtain any other special feature associated with W.

2. Let V = {P([0, 43))} be the mod interval subset collection. {V, +}, {V, ×}, {V, ∩} and {V, ∪} be the mod interval subset semigroups.

   Study questions (i) to (viii) of problem (1) for this V.

3. Let X = {P([0, 24))} be the mod interval subset collection of [0, 24). {X, ∪}, {X, +}, {X, ×}, {X, ∩} be the mod interval subset semigroups.

   Study questions (i) to (viii) of problem (1) for this X.

4. Let Y = {P([0, 256))} be the mod interval subset collection of [0, 256).

   i) Study questions (i) to (viii) of problem (1) for this Y.

   ii) Compare Y with X of problem (3) and V of problem (2).

5. Let A ={P(⟨[0, 5) ∪ g⟩)} be the mod interval subset dual number collection. {A, +}, {A, ∪}, {A, ∩} and {A, ×} be the mod interval subset dual number semigroups.

   i) Study questions (i) to (viii) of problem (1) for this A.

   ii) Compare A of this problem with V of problem (2).

6. Let B = {P(⟨[0,24) ∪ g⟩)} be the mod interval subset dual number collection. {B, +}, {B, ∪}, {B, ×} and {B, ∩} be the mod interval subset dual number semigroups.

i) Study questions (i) to (viii) of problem (1) for this B.

ii) Compare this B with problem (5) of A.

iii) Prove both A and B have infinite number of zero divisors.

7. Let C = $\{P(\langle[0, 243) \cup g\rangle)\}$ be the mod interval subset dual number subsets $\{C, +\}$, $\{C, \times\}$, $\{C, \cup\}$ and $\{C, \cap\}$ be the mod interval subset dual number semigroups.

i) Study questions (i) to (viii) of problem (1) for this C.

ii) Compare this (C) with A and B of problems (5) and (6) respectively.

8. Let D = $\{P(\langle[0, 43) \cup I\rangle)\}$ be the mod interval neutrosophic subset collection. $\{D, \cup\}$, $\{D, \cap\}$, $\{D, \times\}$ and $\{D, +\}$ be the mod interval neutrosophic subset semigroups.

i) Study questions (i) to (viii) of problem (1) for this D.

ii) Compare this D with C of problem (7).

9. Let E = $\{P(\langle[0, 6) \cup I\rangle)\}$ be the mod interval neutrosophic subset collection.

i) Study questions (i) to (viii) of problem (1) for this E.

ii) Compare this E with D of problem 8.

iii) Obtain all the special features associated with this E.

10. Let F = $\{P(\langle[0, 24) \cup h\rangle)\}$ be the mod interval special dual like number subset collection. $\{F, +\}$, $\{F, \times\}$, $\{F,$

$\cup$} and {F, $\times$} be the mod interval subset special dual like number semigroups.

i) Study questions (i) to (viii) of problem (1) for this F.

ii) Compare this F with E and D of problems (9) and (8) respectively.

iii) Study all the special features associated with this F.

11. Let G = {P($\langle$[0, 53) $\cup$ h$\rangle$)} be the mod special dual like number interval subset collection {G, +}, {G, $\cup$}, {G, $\cap$} and {G, $\times$} be the mod special dual like number interval subset collection.

i) Study questions (i) to (viii) of problem (1) for this G.

ii) Compare this G with F of problem (10).

12. Let H = {P(C[0, 20))} = collection of all mod interval finite complex number subset collection. {H, +}, {H, $\cup$}, {H, $\cap$} and {H, $\times$} be the mod subset interval finite complex number semigroups.

i) Study questions (i) to (viii) of problem (1) for this H.

ii) Compare this H with G of problem 11.

13. Let J = {P(C[0, 59))} be the mod interval finite complex number subset collection. {J, +}, {J, $\cup$}, {J, $\cap$} and {J, $\times$} be the mod interval finite complex subset semigroups.

i) Study questions (i) to (viii) of problem (1) for this J.

ii) Compare this J with H of problem (12).

14. Let K = $\{P(\langle[0, 48) \cup k\rangle)\}$ be the mod interval special quasi dual number subset collection.

    i) Study questions (i) to (viii) of problem (1) for this K.

    ii) Compare this K with J of problem 13.

15. Let L = $\{P(\langle[0, 47) \cup k\rangle)\}$ be the mod subset interval special quasi dual number collection $\{L, +\}$, $\{L, \cup\}$, $\{L, \cap\}$ and $\{L, \times\}$ be the mod subset interval special quasi dual number semigroups.

    i) Study questions (i) to (viii) of problem (1) for this L.

    ii) Compare this L with K of problem 14.

    iii) Enlist all the special features associated with L.

16. Let S = $P(\langle[0, 48)\rangle)$ = collection of all subsets from mod interval natural neutrosophic numbers. $\{S, +\}$, $\{S, \times\}$, $\{S, \cup\}$ and $\{S, \cap\}$ be the mod natural neutrosophic interval subset semigroups.

    i) Study questions (i) to (viii) of problem (1) for this S.

    ii) Prove $\{S, +\}$ has idempotents.

    iii) Prove $\{S, \times\}$ has both interval zero dominant zero divisors under $\times_0$ and has mod natural neutrosophic zero dominated zero divisors.

17. Let T = $\{P(\langle^{I}[0, 49)\rangle)\}$ be the mod interval natural neutrosophic subset collection. $\{T, +\}$, $\{T, \times\}$, $\{T, \cup\}$ and $\{T, \cap\}$ be the mod interval natural neutrosophic subset semigroups.

    i) Study questions (i) to (viii) of problem (1) for this T.

ii) Study questions (ii) and (iii) of problem (16) for this T.

iii) Compare this T with S of problem (16).

iv) Enumerate all special features associated with this T.

18. Let M = $\{P(C^I([0, 4))\}$ be the mod natural neutrosophic interval finite complex number subsets. $\{M, +\}$, $\{M, \times\}$, $\{M, \cup\}$ and $\{M, \cap\}$ be the mod interval natural neutrosophic finite complex number subset semigroup.

i) Study questions (i) to (viii) of problem (1) for this M.

ii) Study questions (ii) and (iii) of problem (16) for this M.

iii) Compare M with T of problem (17).

19. Let N = $\{C^I([0, 12))\}$ be the mod interval natural neutrosophic finite complex number subset collection. $\{N, \times\}$, $\{N, +\}$, $\{N, \cup\}$ and $\{N, \cap\}$ be the mod interval natural neutrosophic finite complex number subset semigroups.

i) Study questions (i) to (viii) of problem (1) for this N.

ii) Study questions (ii) and (iii) of problem (16) for this N.

iii) Compare this N with M of problem (18).

iv) Enumerate all special features associated with this N.

20. Let O = $\{P(\langle[0, 28) \cup I\rangle_I)\}$ be the collection of all mod interval natural neutrosophic-neutrosophic subset collection. $\{O, \times\}$, $\{O, \cup\}$, $\{O, \cap\}$ and $\{O, +\}$ be the mod interval natural neutrosophic-neutrosophic subset

semigroups. $O_o, O_{\times}^{+}, O_{\cup}^{+}, O_{\cap}^{+}, O_{\cup}^{\times}$ and $O_{\cap}^{\times}$ be the mod natural neutrosophic-neutrosophic subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (1) for this O.

ii) Compare this O with N of problem 19.

iii) Compare O with M of problem 18.

21. Let P = $\{P(\langle[0, 17) \cup I\rangle_I)$ be the mod interval natural neutrosophic-neutrosophic subset collection. $\{P, \times\}$, $\{P, \cup\}$, $\{P, \cap\}$ and $\{P, \times\}$ be the mod interval natural neutrosophic-neutrosophic subset semigroup.

P_o, $P_{\times}^{+}$, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\cup}^{\times}$ and $P_{\cap}^{\times}$ be the mod interval natural neutrosophic-neutrosophic subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (1) for this P.

ii) Compare this P with O of problem 20.

iii) Compare this P with M of problem 18.

22. Let Q $=\{P(\langle[0, 24) \cup g\rangle_I)\}$ be the mod natural neutrosophic dual number interval subset collection $\{Q, \times\}$, $\{Q, \cup\}$, $\{Q, \cap\}$ and $\{Q, +\}$ be the mod natural neutrosophic dual number interval subset semigroups. Q_o, $Q_{\times}^{+}$, $Q_{\cup}^{+}$, $Q_{\cap}^{+}$, $Q_{\cup}^{\times}$ and $Q_{\cap}^{\times}$ be the mod natural neutrosophic dual number interval subset special type of topological spaces.

i) Study questions (i) to (viii) of problem (1) for this Q.

ii) Enumerate all special features associated with this Q.

23. Let R = $\{P(\langle[0, 47) \cup g\rangle_I)\}$ be the mod natural neutrosophic interval dual number collection of subsets. $\{R, +\}$, $\{R, \cup\}$, $\{R, \cap\}$ and $\{R, \times\}$ be the mod natural neutrosophic interval dual number subset. R_o, $R_\times^+$, $R_\cup^+$, $R_\cap^+$, $R_\cup^\times$ and $R_\cap^\times$ be the mod natural neutrosophic interval dual number special type of topological spaces

    i) Study questions (i) to (viii) of problem (1) for this R.

    ii) Compare this R with Q of problem (22).

    iii) Obtain all special features associated with R.

    iv) Prove R has infinite number of zero divisors and nilpotents.

24. Let S = $\{P(\langle[0, 14) \cup h\rangle_I)\}$ be the collection of all mod natural neutrosophic special dual like number subset collection. $\{S, +\}$, $\{S, \cup\}$, $\{S, \cap\}$ and $\{S, \times\}$ be the mod interval natural neutrosophic special dual like number subset semigroups.

    S_o, $S_\cup^+$, $S_\cap^+$, $S_\times^+$, $S_\cup^\times$ and $S_\cap^\times$ be the mod interval natural neutrosophic special dual like number subset special type of topological spaces.

    i) Study (i) to (viii) of problem (1) for this S.

    ii) Enumerate all special features associated with S.

    iv) Compare this S with R of problem (23).

25. Let T = $\{P(\langle[0, 29) \cup h\rangle_I)\}$ be the collection of all mod interval natural neutrosophic special dual like number subset collection. $\{T, +\}$, $\{T, \times\}$, $\{T, \cup\}$ and $\{T, \cap\}$ be the mod interval natural neutrosophic special dual like number subset semigroups. T_o, $T_\times^+$, $T_\cup^+$, $T_\cap^+$, $T_\cup^\times$ and $T_\cap^\times$ be the mod interval natural neutrosophic special

dual like number subset topological spaces associated with these semigroups.

i) Study (i) to (viii) of problem (1) for this T.

ii) Enumerate all special features enjoyed by this T.

iii) Compare this T with S of problem (24).

26. Let V = {P(⟨[0, 15) ∪ k⟩$_I$)} be the mod interval natural neutrosophic special quasi dual number subset collection. {V, +}, {V, ×}, {V, ∪} and {V, ∩} be the mod interval natural neutrosophic special quasi dual number subset semigroups and V_o, $V_{\times}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{+}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be special type of topological spaces associated with these semigroups.

i) Study questions (i) to (viii) of problem (1) for this V.

ii) Obtain all special features associated with V.

iii) Compare this V with T of problem (25).

27. Let W = {P(⟨[0, 17) ∪ k⟩$_I$ be the mod interval natural neutrosophic special quasi dual number subset collection {W, +}, {W, ∩}, {W, ∪} and {W, ×} be the mod subset interval special quasi dual number semigroups. W_o, $W_{\times}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{\times}$ and $W_{\cup}^{\times}$ be the mod subset interval special quasi dual number interval special type of topological spaces associated with these semigroups.

i) Study questions (i) to (viii) of problem (1) for this W.

ii) Compare this W with V of problem (26).

28. Let M = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \right.$ / $a_i \in P([0, 15))$; $1 \leq i \leq 5\}$ be the mod interval subset matrix collection.

i) Prove {M, $\cup$}, {M, $\times_n$}, {M, $\cap$} and {M, +} are mod interval subset matrix semigroups of infinite order.

ii) Prove all the 6 mod interval subset matrix special type of topological spaces using M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ are distinct.

iii) Are these six spaces discrete?

v) Can these six spaces be compact?

vi) Prove M has mod interval subset matrix strong special type of topological subspaces?

vii) Can M have strong mod interval subset matrix topological ideal?

viii) Find all mod interal subset topological zero divisors and nilpotents of M.

ix) Can M have mod interval subset topological idempotents?

x) Mention all the classical properties of this topological space satisfied by this M.

29. Let N = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right.$ / $a_i \in P([0, 23))$; $1 \leq i \leq 16\}$ be the mod interval subset entries matrix collection. {N, +}, {N, $\cup$}, {N, $\cap$}, {N, $\times_n$} and {N, $\times$} be the mod

interval subset matrix semigroups. N_o, $N_{\times}^{+}$, $N_{\times_n}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times}$, $N_{\cap}^{\times}$, $N_{\times_n}^{\times}$, $N_{\cup}^{\times_n}$ and $N_{\cap}^{\times_n}$ be the mod interval subset matrix special type of topological spaces associated with these 5 semigroups.

i) Study questions (i) to (x) of problem (28) for this N.

ii) Prove $N_{\times}^{+}$, $N_{\cup}^{\times}$, $N_{\times_n}^{\times}$ and $N_{\cap}^{\times}$ are mod interval non-commutative subset matrix topological spaces.

iii) Prove these four spaces have mod interval topological right zero divisors which are not mod interval topological left zero divisors and vice versa.

iv) Prove all these four spaces has mod interval topological subset right ideals which are not left ideals and vice versa.

30. Let V = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} / a_i \in \{P(^I[0, 48))\}; 1 \leq i \leq 10\right\}$ be the mod natural neutrosophic subset interval matrix collection. $\{V, +\}$, $\{V, \times\}$, $\{V, \cap\}$ and $\{V, \cup\}$ be the mod natural neutrosophic interval subset matrix semigroups. V_o, $V_{\times_n}^{+}$, $V_{\cup}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{\times_n}$ and $V_{\cap}^{\times_n}$ be the mod natural neutrosophic subset interval matrix special type of topological spaces associated with these semigroups.

i) Study questions (i) to (x) of problems (28) for this V.

ii) Obtain all special feature associated with this V.

31. Let $W = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ / $a_i \in P(^I[0, 23))$; $1 \leq i \leq 9\}$ be the mod natural neutrosophic subset interval matrix collection. $\{W, +\}$, $\{W, \times_n\}$, $\{W, \times\}$, $\{W, \cap\}$ and $\{W, \cup\}$ be the mod subset natural neutrosophic interval semigroups.

    i) Study questions (i) to (x) of problem (28) for this W.

    ii) Study questions (ii) to (iv) of problem (29) for this W.

32. Let $Y = \{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix}$ / $a_i \in P(\langle[0, 7) \cup g\rangle_I)$; $1 \leq i \leq 8\}$ be the mod interval natural neutrosophic subset dual number matrices. $\{Y, \cup\}$, $\{Y, \cap\}$, $\{Y, +\}$ and $\{Y, \times_n\}$ be the mod natural neutrosophic interval dual number subset matrix semigroups and Y_o, $Y^{+}_{\times_n}$, $Y^{+}_{\cup}$, $Y^{+}_{\cap}$, $Y^{\times}_{\cup}$ and $Y^{\times}_{\cap}$ be the mod natural neutrosophic dual number interval subset matrix special type of topological spaces.

    i) Study questions (i) to (x) of problem (28) for this Y.

    ii) Prove this Y has more number of zero divisors and nilpotents.

33. Let $Z = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}$ / $a_i \in P(\langle[0, 480) \cup g\rangle_I)$; $1 \leq i \leq 12\}$ be the mod natural neutrosophic dual number interval subset matrix collection. $\{Z, +\}$, $\{Z, \times_n\}$, $\{Z, \cup\}$ and $\{Z, \cap\}$ be the mod natural

neutrosophic subset matrix interval dual number semigroups and Z_o, $Z^{+}_{\times_n}$, $Z^{+}_{\cup}$, $Z^{+}_{\cap}$, $Z^{\times_n}_{\cup}$ and $Z^{\times_n}_{\cap}$ be the mod natural neutrosophic dual number subset interval matrix special type of topological spaces.

i) Study questions (i) to (x) of problem (28) for this Z.

ii) Compare this Z with Y of problem (32).

34. Let $A = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle/ a_i \in P([0, 24) \cup I\rangle_I), 1 \leq i \leq 6 \right\}$ be the mod natural neutrosophic-neutrosophic interval subset matrix collection {A, +}, {A, ∪}, {A, ∩} and {A, $\times_n$} be the mod interval natural neutrosophic-neutrospohic subset matrix semigroups.

i) Study questions (i) to (x) of problem (28) for this A.

35. Let $B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle/ a_i \in P(\langle[0, 47) \cup I\rangle_I); \right.$ $1 \leq i \leq 16\}$ to be mod natural neutrosophic interval subset matrix collection {B, +}, {B, ×}, {B, $\times_n$}, {B, ∪} and {B, ∩} be the mod natural neutrosophic interval subset matrix. B_o, $B^{+}_{\cup}$, $B^{+}_{\cap}$, $B^{+}_{\times_n}$, $B^{+}_{\times}$, $B^{\times}_{\cup}$, $B^{\times}_{\cap}$, $B^{\times_n}_{\times}$, $B^{\times_n}_{\cup}$ and $B^{\times_n}_{\cap}$ be the mod natural neutrosophic neutrosophic subset interval matrix special type of topological spaces built using these mod semigroups.

i) Study questions (i) to (x) of problem (28) for this B.

ii) Compare B with A of problem (34).

iii) Obtain all special features enjoyed by B.

36. Let C = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right.$ / $a_i \in P(C([0, 20)))$; $1 \leq i \leq 12\}$ be the mod interval finite complex number subset matrix collection. {C, +}, {C, $\times_n$}, {C, $\cup$} and {C, $\cap$} be the mod interval finite complex number subset matrix semigroups. C_o, $C^+_{\times_n}$, $C^+_{\cup}$, $C^+_{\cap}$, $C^{\times_n}_{\cup}$ and $C^{\times_n}_{\cap}$ be the mod interval finite complex number subset matrix special type of topological spaces.

i) Study questions (i) to (x) of problem (28) for this C.

ii) Enumerate all special features enjoyed by this C.

37. Let D = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right.$ / $a_i \in P(C([0, 43)))$; $1 \leq i \leq 9\}$ be the mod interval finite complex number subset matrix collection. {D, +}, {D, $\cup$}, {D, $\cap$}, {D, $\times_n$} and {D, $\times$} be the mod interval finite complex number subset matrix semigroups. D_o, $D^+_{\times_n}$, $D^+_{\times}$, $D^+_{\cup}$, $D^+_{\cap}$, $D^{\times}_{\cup}$, $D^{\times}_{\cap}$, $D^{\times_n}_{\times}$, $D^{\times_n}_{\cup}$ and $D^{\times_n}_{\cap}$ be the mod interval finite complex number subset special type topological spaces associated with the mod semigroups.

i) Study questions (i) to (x) of problem (28) for this D.

ii) Compare this D with (C) of problem (35).

38. Let F = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}\right.$ / $a_i \in P(\langle[0, 20) \cup k\rangle)$; $1 \leq i \leq 6\}$ be the mod interval special quasi dual number subset matrix collection. {F, +}, {F, ×}, {F, ∪} and {F, ∩} be the mod interval subset special quasi dual number matrix semigroup. F_o, $F^{+}_{\times_n}$, $F^{+}_{\cup}$, $F^{+}_{\cap}$, $F^{\times_n}_{\cup}$ and $F^{\times_n}_{\cap}$ be the mod interval special quasi dual number subset matrix special topological space.

i) Study questions (i) to (x) of problem (28) for this F.

ii) Enumerate all special feature enjoyed by F.

39. Let G = $\left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix}\right.$ / $a_i \in P(\langle[0,12) \cup g\rangle)$, $1 \leq i \leq 10\}$ be the mod interval dual number subset matrix collection.

Study questions (i) to (x) of problem (28) for this G.

40. Let H = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix}\right.$ / $a_i \in P(\langle[0, 14) \cup h\rangle_I)$; $1 \leq i \leq 8\}$ be the mod natural neutrosophic special dual like number interval subset matrix collection. {H, +}, {H, $\times_n$}, {H, ∪} and {H, ∩} be the mod natural neutrosophic dual number special dual like number interval matrix subset semigroups.

i) Study questions (i) to (x) of problem (28) for this H.

ii) Enumerate all special features associated with this H.

41. Let J = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \right.$ / $a_i \in P(\langle[0, 18) \cup k\rangle_I)$; $1 \leq i \leq 15\}$ be the mod natural neutrosophic interval special quasi dual number subset matrix collection. {J, +}, {J, ∪}, {J, ∩} and {J, $\times_n$} be the mod natural neutrosophic interval special quasi dual number subset matrix semigroups. J_o, $J^{+}_{\times_n}$, $J^{+}_{\cup}$, $J^{+}_{\cap}$, $J^{\times_n}_{\cup}$ and $J^{\times_n}_{\cap}$ be the mod natural neutrosophic special quasi dual number inter subset matrix special type of topological spaces.

i) Study questions (i) to (x) of problem (28) for this J.

ii) Obtain all special features enjoyed by J.

42. Let K = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \right.$ / $a_i \in P(C^I[0, 28))$; $1 \leq i \leq 20\}$ be the mod natural neutrosophic finite complex number interval subset matrix collections.

i) Study questions (i) to (x) of problem (28) for this K.

ii) Find all special features enjoyed by this K.

43. Let M = P(R) = {collection of a matrix subset from M where $R = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle/ a_i \in [0, 17);\ 1 \leq i \leq 5 \right\}$} be the mod interval matrix subset collection. {M, +}, {M, $\times_n$}, {M, $\cup$} and {M, $\cap$} be the mod interval matrix subset semigroups. M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ be the mod interval matrix subset special type of topological spaces associated with these four mod semigroups.

i) Prove the four mod semigroups are distinct and are of infinite order.

ii) Prove the mod matrix subset topological spaces are of infinite order.

iii) Are these spaces discrete?

iv) Are these six mod interval subset matrix spaces connected?

v) Are these spaces compact?

vi) Can these mod interval matrix subset spaces have topological zero divisors?

vii) Can these mod interval matrix subset special type of topological spaces have topological nilpotents?

viii) Prove some of these mod interval subset special type of topological spaces have topological idempotents.

ix) Prove M has strong mod interval matrix subset special type of topological subspaces.

x) Prove M has no strong mod interval matrix subset topological ideals.

xi) Prove all ideals of M are of infinite order.

xii) Prove M has mod interval matrix subset topological subspaces of finite order.

44. Let T = P(B) = {collection of a matrix subset from $B = \{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ where $a_i \in {}^I[0, 42)$; $1 \leq i \leq 4\}$ be mod interval neutrosophic interval matrix subset collection. Let {T, +}, {T, $\times_n$}, {T, ×}, {T, ∩} and {T, ∪} be the mod natural neutrosophic interval matrix subset semigroup. T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\times}^{+}$, $T_{\times_n}^{+}$ $T_{\cup}^{\times}$, $T_{\cap}^{\times}$, $T_{\times_n}^{\times}$, $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ be the mod natural neutrosophic interval matrix subset special type of topological spaces associated with the 5 mod semigroups.

i) Prove $T_{\times}^{+}$, $T_{\times}^{\cup}$ $T_{\times}^{\cap}$ and $T_{\times}^{\times_n}$ are non commutative natural neutrosophic interval matrix subset special type of topological spaces.

ii) Study questions (i) to (xii) of problem (43) for this T.

iii) Compare this T with M of problem (43).

iv) Obtain all special features associated with T.

45. Let V = {P(L) = {collection of all matrix subsets from $L = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix}$ / $a_i \in C([0, 24))$; $1 \leq i \leq 10\}\}$ be mod interval finite complex number matrix subset collection. {V, $\times_n$}, {V, ∩}, {V, ∪} and {V, +} be the mod interval finite complex number matrix subset semigroups. V_o, $V_{\times_n}^{+}$, $V_{\cup}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{\times_n}$ and $V_{\cap}^{\times_n}$ be the mod interval finite complex number matrix subset special type of topological spaces related with the four mod semigroups.

i) Study questions (i) to (xii) of problem (43) for this V.

ii) Study questions (i), (iii) and (iv) of problem (44) for this V.

46. Let W ={P(B) = collection of all matrix subsets from $B = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} / a_i \in \langle[0, 12) \cup g\rangle; 1 \leq i \leq 9\}\}$ be the mod interval dual number matrix subset collection. Let {W, +}, {W, $\times_n$}, {W, $\times$}, {W, $\cup$}, {W, $\cap$} be the mod interval dual number matrix subset semigroups. W_o, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\times}^{+}$, $W_{\times_n}^{+}$, $W_{\cup}^{\times}$, $W_{\cap}^{\times}$, $W_{\cup}^{\times_n}$ and $W_{\times_n}^{\times}$ be the mod interval dual number matrix subset special type of topological spaces.

i) Study questions (i) to (xii) of problem (43) for this W.

ii) Compare this W with V of problem 45.

iii) Derive all the special features associated with this W.

47. Let Y = {P(D) = collection of all subsets from $D = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} / a_i \in \langle[0, 24) \cup I\rangle; I^2 = I; 1 \leq i \leq 5\}\}$ be the mod interval neutrosophic matrix subset collection. {Y, +}, {Y, $\cap$}, {Y, $\cup$} and {Y, $\times_n$} be the mod interval neutrosophic matrix subset semigroup. Y_o, $Y_{\times_n}^{+}$ $Y_{\cup}^{+}$, $Y_{\cap}^{+}$, $Y_{\cup}^{\times_n}$ and $Y_{\cap}^{\times_n}$ be the mod interval neutrosophic matrix subset special type of topological spaces using the four mod semigroups.

i) Study questions (i) to (xii) of problem (43) for this Y.

ii) Derive all the special features enjoyed by Y.

iii) Compare this Y with W of problem (46).

48. Let Z = {P(E) where

$$E = \{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{pmatrix}$$

where $a_i \in \langle [0, 20) \cap k \rangle$; $1 \leq i \leq 18\}$ be the mod interval special quasi dual number matrix subsets collection. {Z, +}, {Z, ∪}, {S, ∩} and {Z, $\times_n$} be the mod interval special quasi dual number matrix subset semigroups. Z_o, $Z_{\times}^{+}$, $Z_{\cup}^{+}$, $Z_{\cap}^{+}$, $Z_{\cup}^{\times}$ and $Z_{\cap}^{\times}$ be the mod interval special quasi dual number matrix subset special type of topological spaces associated with the semigroup.

Study questions (i) to (xii) of problem (43) for this Z.

49. Let P = {P(W) collection of all subsets from

$$W = \{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} / a_i \in {}^{I}[0, 43);\ 1 \leq i \leq 5\}\}$$ be the mod interval natural neutrosophic matrix subset collection.

i) Study questions (i) to (xii) of problem (43) for this P.

ii) Compare this P with Z of problem (48).

iii) Derive all special properties associated with P.

50. Let B = {P(S), collection of all matrix subsets from $S = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix}$ / $a_I \in C^I([0, 24))$; $1 \leq i \leq 15\}\}$ be the mod natural neutrosophic interval finite complex number matrix subset collection. {B, +}, {B, $\times_n$}, {B, $\cup$} and {B, $\cap$} be the mod natural neutrosophic interval finite complex number matrix subset semigroup. B_o, $B^+_{\times_n}$, $B^+_{\cup}$, $B^+_{\cap}$, $B^{\times_n}_{\cup}$ and $B^{\times_n}_{\cap}$ be the mod natural neutrosophic interval finite complex number matrix subset special type of topological spaces.

   i) Study questions (i) to (xii) of problem (43) for this B.

   ii) Compare this B with P of problem (49).

51. Let D = {P(L), collection of matrix subsets from $L = \{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix}$ / $a_i \in \{\langle [0, 40) \cup I \rangle_I\}$; $1 \leq i \leq 12\}\}$ be the mod interval natural neutrosophic neutrosophic matrix subset collection. Let {D, +},{D, $\times_n$}, {D, $\cup$} and {D, $\cap$} mod interval natural neutrosophic-neutrosophic matrix subset semigroups.

   D_o, $D^+_{\times_n}$, $D^+_{\cup}$, $D^+_{\cap}$, $D^{\times_n}_{\cup}$ and $D^{\times_n}_{\cap}$ be the mod interval natural neutrosophic-neutrosophic matrix subset topological spaces relative with the four mod semigroups.

   i) Study questions (i) to (xii) of problem (43) for this D.

ii) Compare this D with B of problem 50.

52. Let E = {P(M) = {collection of all subsets from $M = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{bmatrix} / a_i \in \{\langle 15 \cup g\rangle_I\}; 1 \leq i \leq 12\}$ be the mod interval natural neutrosophic dual number matrix subset collection. {M, +}, {M, $\times_n$}, {M, $\cup$}and {M, $\cap$} be the mod interval natural neutrosophic dual number matrix subset semigroup.

M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ be the mod interval natural neutrosophic dual number matrix subset special type of topological spaces.

i) Study questions (i) to (xii) of problem (43) for E.

ii) Compare this E with D of problem 51.

53. F = {P(S) = {collection of all matrix subsets from $S = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} / a_i \in \langle [0, 40) \cup h\rangle_I; 1 \leq i \leq 9\}\}$ be the mod interval natural neutrosophic special dual like number matrix subsets collection. {F, +}, {F, $\times_n$}, {F, $\times$}, {F, $\cup$} and {F, $\cap$} be the mod interval natural neutrosophic special dual like number matrix subset semigroups.

F_o, $F^{+}_{\times}$, $F^{+}_{\times_n}$, $F^{+}_{\cup}$, $F^{+}_{\cap}$, $F^{\times}_{\cup}$, $F^{\times}_{\cap}$, $F^{\times}_{\times_n}$, $F^{\times_n}_{\cup}$ and $F^{\times_n}_{\cap}$ be the mod interval natural neutrosophic special dual number matrix subset special types of topological spaces.

i) Study questions (i) to (xii) of problem (43) for this F.

ii) Compare this F with E of problem (52).

54. Let G = {P(M), where M = $\left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix}\right.$ / $a_i \in \langle [0, 12) \cup k\rangle_I$; $1 \leq i \leq 10\}\}$ be the mod interval natural neutrosophic special quasi dual number matrix subset collection.

{G, +}, {G, $\times_n$}, {G, $\cup$} and {G, $\cap$} be the mod interval natural neutrosophic special quasi dual number matrix subset semigroup, G_o, $G^{+}_{\times_n}$, $G^{+}_{\cup}$, $G^{+}_{\cap}$, $G^{\times_n}_{\cup}$ and $G^{\times_n}_{\cap}$ be the mod interval natural neutrosophic quasi dual number matrix subset special type of topological spaces.

i) Study questions (i) to (xii) of problem (43) for this G.

ii) Compare this G with F of problem (53).

55. Let H = {P(L[x]} = {collection of all subsets from L[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in [0, 24)\}\}$ be the mod interval polynomial subset collection.

i) $o(H) = \infty$.

ii) Prove {H, $\cup$}, {H, $\cap$}, {H, +} and {H, $\times$} be the mod interval polynomial subsetsemigroup.

iii) Prove only {H, $\cup$}, {H, $\cap$} and {H, +} can have finite order mod interval subsemigroups.

iv) Prove $\{H, \times\}$ cannot have mod subsemigroups of finite order.

v) Prove $\{H, \cup\}$ and $\{H, \cap\}$ can have mod interval idempotents.

vi) Prove H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ are 6 distinct mod topological spaces.

vii) How many mod interval strong topological subspaces of H are there?

viii) Prove H has no mod interval strong topological ideals.

ix) Find all mod interval topological zero divisors of H.

x) Does H contain mod interval topological nilpotents?

xi) Obtain any other special feature enjoyed by H.

56. Let I = $\{P(T[x])\}$ = {collection of all subsets from T[x] = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in {}^{I}[0, 35)\}$} be the mod interval polynomial subset collection.

Study questions (i) to (xi) of problem (55) for this I.

57. Let J = $\{P(S[x])\}$ = {collection of all subsets from S[x] = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in C([0, 12), i_F^2 = 11\}$} be the mod interval finite complex number coefficient polynomial subset collection.

Study questions (i) to (xi) of problem (55) for this J.

58. Let K = $\{P(R[x])\}$ = {collection of all subsets from R[x] = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in C^{I}([0, 29))\}$ be the mod interval

natural neutrosophic finite complex number coefficient polynomial subset collection.

Study questions (i) to (xi) of problem (55) for this K.

59. Let L = {P(V[x])} = {collection of all subsets from $V[x] = \{ \sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in (\langle [0, 48) \cup I \rangle, I^2 = I\}\}$ be the mod interval neutrosophic coefficient polynomial subset collection.

Study questions (i) to (xi) of problem (55) for this L.

60. Let N = {P(W[x])} = {collection of all subsets from $W[x] = \{ \sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in \langle [0, 17) \cup I \rangle_I \}\}$ be the mod interval natural neutrosophic-neutrosophic coefficient polynomial subsets collection.

Study questions (i) to (xi) of problem (55) for this N.

61. Let R = {P(Z[x])} = {collection of all subsets from Z[x] $= \{ \sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in \langle [0, 10) \cup g \rangle, g^2 = 0\}\}$ be the mod interval dual number coefficient polynomial subset collection.

Study questions (i) to (xi) of problem (55) for this R.

62. Let S = {P(Y[x])} = {collection of all subsets from Y[x] $= \{ \sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in \langle [0, 12) \cup g \rangle_I, g^2 = 0\}\}$be the mod interval natural neutrosophic dual number coefficient polynomial subsets collection.

i) Study questions (i) to (xi) of problem (55) for this S.

ii) Prove S has more mod interval zero divisors and nilpotents.

iii) Compare this S with R of problem 61.

63. Let V = {P(T[x])} = {collection of all subsets fron T[x] $= \{ \sum_{i=0}^{\infty} a_i x^i / a_i \in \langle [0, 47) \cup h\rangle, h^2 = h\}\}$ be the mod interval special dual like number coefficient polynomial subset collection.

i) Study questions (i) to (xi) of problem (55) for this V.

ii) Compare this V with S and R of problems (62) and (61) respectively.

64. Let W = {P(B[x])} = {collection of all subsets from $B[x] = \{ \sum_{i=0}^{\infty} a_i x^i / a_i \in \langle([0, 12) \cup h\rangle_I, h^2 = h\}\}$ be the mod interval natural neutrosophic special dual like number coefficient polynomial collection.

i) Study questions (i) to (xi) of problem (55) for this W.

ii) Compare this W with V of problem (64).

65. Let Z = {P(W[x])} = {collection of all subsets from $W[x] = \{ \sum_{i=0}^{\infty} a_i x^i / a_i \in \langle [0, 18) \cup k\rangle, k^2 = 17k\}\}$ be the mod interval special quasi dual number coefficient polynomial subsets collection.

i) Study questions (i) to (xi) of problem (55) for this Z.

ii) Compare this Z with W of problem (64).

66. Let D = {P(E[x])} = {collection of all subsets from E[x] = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle[0, 5) \cup k\rangle_I, k^2 = 4k\}\}$ be the mod interval natural neutrosophic special quasi dual number coefficient polynomial subset collection.

    i) Study questions (i) to (xi) of problem (55) for this D.

67. Let M = {P(F[x]$_{10}$)) = {collection of all subsets from F[x]$_{10}$ = $\{\sum_{i=0}^{10} a_i x^i / a_i \in [0, 12), x^{11} = 1\}\}$ be the mod interval coefficient polynomial subsets collection.

    i) o(M) = ∞.

    ii) Prove {M, +}, {M, ×}, {M, ∪} and {M, ∩} are mod internal semigroups of infinite order which contains subsemigroups of finite order.

    iii) Prove M_o, $M_{\times}^{+}$, ($M_{\times_0}^{+}$), $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$, $M_{\cap}^{\times}$, ($M_{\times_0}^{\times}$, $M_{\cup}^{\times_0}$, $M_{\cap}^{\times_0}$) are mod interval special type of polynomial subset topological spaces.

    iv) Prove M has mod interval strong topological subspaces but no strong topological ideals.

    v) Prove all topological ideals are of infinite order.

    vi) Prove for appropriate values of n in [0, n), M will have mod interval topological nilpotents and zero divisors.

    vii) Prove M has no nontrivial mod interval topological idempotents.

    viii) Prove M has mod interval strong topological subspaces of finite order.

    ix) Obtain any other special feature associated with M.

68. Let N = {P(B[x]$_{16}$)} = {collection of all subsets from $B[x]_{16} = \{ \sum_{i=0}^{\infty} a_i x^i$ / $a_i \in {}^I[0, 24)$, $x^{17} = 1\}\}$ be the mod interval natural neutrosophic coefficient finite degree polynomials subset collection.

    Study questions (i) to (ix) of problem (67) for this N.

69. Let P = {P(D[x]$_7$)} = {collection of all subsetsfrom $D[x]_7 = \{ \sum_{i=0}^{7} a_i x^i$ / $a_i \in \langle [0, 12) \cup g\rangle$; $g^2 = 0$, $x^8 = 1\}\}$ be the mod interval dual number coefficient of finite degree polynomial collection.

    Study questions (i) to (ix) of problem 67 for this P.

70. Let F = {P(G[x]$_6$)} = {collection of all subsets from $G[x]_6 = \{ \sum_{i=0}^{6} a_i x^i$ / $x^7 = 1$ $a_i \in \langle [0, 12) \cup g\rangle_I\}\}$ be the mod interval natural neutrosophic dual number coefficients polynomials of finite degree subsets collection.

    Study questions (i) to (ix) of problem (67) for this F.

71. Let H = {P(L[x]$_3$)} = {collection of all subsets from $L[x]_3 = \{ \sum_{i=0}^{3} a_i x^i$ / $a_i \in C^I([0, 12))$, $i_F^2 = 11$, $x^4 = 1\}\}$ be the mod interval natural neutrosophic finite complex number coefficient polynomials of finite degree subsets collection.

    Study questions (i) to (ix) of problem (67) for this H.

72. Let E = {P(T[x]$_8$)} = {collection of all subsets from $T[x]_8 = \{ \sum_{i=0}^{8} a_i x^i$ / $a_i \in \langle [0, 14) \cup I\rangle_I$, $x^9 = 1$, $I^2 = I\}\}$ be the mod interval natural neutrosophic neutrosophic

coefficient polynomials of finite degree subset collection.

Study questions (i) to (ix) of problem (67) for this E.

73. Let M = {P(T[x]$_9$)} ={collection of all subsets from T[x]$_9$ = { $\sum_{i=0}^{9} a_i x^i$ / $a_i \in \langle$ [0, 12) $\cup k\rangle_I$, $x^{10} = 1$, $k^2 = 11k$}} be the mod interval natural neutrosophic special quasi dual number coefficient polynomial of finite degree subsets.

i) Study questions (i) to (ix) of problem (67) for this M.

ii) Compare this M with E and H of problems (72) and (73) respectively.

74. Let T = { $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in$ P ([0, 17))} be the mod interval subset coefficient polynomial collection

i) o(T) = ∞.

ii) Prove {T, +}, {T, ×}, {T, ∩} and {T, ∪} are mod interval subset coefficient polynomial semigroups.

iii) In {T, ∩} and {T, ∪} every element is an mod idempotent.

iv) Find mod subsemigroups and ideals of these four mod semigroups.

v) Prove T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\times}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ are mod interval subset coefficient polynomial special type of topological spaces.

vi) Find all mod interval subset coefficient polynomial strong topological subspaces.

vii) Find all mod interval subset coefficient topological zero divisors and nilpotents.

viii) Prove T has no nontrivial mod interval subset coefficient topological idempotents in $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and $T_{+}^{\times}$.

ix) Prove T has no mod interval strong topological ideals.

75. Let S = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(^I[0, 42))\}$ be the mod interval natural neutrosophic subset coefficient polynomial collection.

i) Study questions (i) to (ix) of problem (74) for this T.

ii) Prove $\{S, \times_o\}$ and $\{S, \times\}$ are two distinct mod interval natural neutrosophic subset coefficient polynomial semigroups.

iii) $T_{\times_0}^{+}$, $T_{\times_0}^{\times}$, $T_{\cup}^{\times_0}$ and $T_{\cap}^{\times_0}$ are also different and distinct collection of mod interval natural neutrosophic subset coefficient polynomial topological spaces.

iv) Show in general a mod interval topological zero divisor is not a mod interval topological natural neutrosophic zero divisor and vice versa.

v) Prove S can have mixed mod interval topological zero divisors.

76. Let R = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(C[0, 12))\}$ be the mod interval finite complex number subset coefficient polynomial collection.

Study questions (i) to (ix) of problem (74) for this R.

77. Let W = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in C^I([0, 12))\}$ be the mod interval natural neutrosophic finite complex number subset coefficient polynomial collection.

    i) Study questions (i) to (ix) of problem (74) for this W.

    ii) Study questions (ii) to (v) of problem (75) for this W.

78. Let V = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, 43) \cup I\rangle)\}$ be the mod interval neutrosophic subset coefficient polynomial collection.

    Study questions (i) to (ix) of problem (74) for this V.

79. Let D = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, 14) \cup I\rangle_I)\}$ be the mod interval natural neutrosophic subset coefficient polynomial collection.

    i) Study questions (i) to (ix) of problem (74) for this D.

    ii) Study questions (ii) to (v) of problem (75) for this D.

80. Let E = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, 12) \cup g\rangle)\}$ be the mod interval dual number subset coefficient polynomial collection.

    i) Study questions (i) to (ix) of problem (74) for this E.

    ii) This E has more number of mod interval topological zero divisors and nilpotents.

81. Let H = { $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(\langle[0, 23) \cup g\rangle_I)$ be the mod interval natural neutrosophic dual number subset coefficient polynomial collection.

i) Study questions (i) to (ix) of problem (74) for this H.

ii) Study questions (ii) to (v) of problem (75) for this H.

iii) Prove this H has more number of mod interval topological zero divisors, nilpotents, natural neutrosophic zero divisors and nilpotents and mixed zero divisors and nilpotents.

82. Let G = { $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(\langle[0, 42) \cup h\rangle_I)$} be the mod interval natural neutrosophic special dual like number subset coefficient collection.

i) Study questions (i) to (ix) of problem (74) for this G.

ii) Study questions (ii) to (v) of problem (75) for this G.

iii) Compare this G with H of problem 81.

83. Let K = { $\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(\langle[0, 14) \cup k\rangle)$, $k^2 = 13k$}} be the mod interval special quasi dual number subset coefficient polynomial collection.

Study questions (i) to (ix) of problem (74) for this K.

84. Let L ={ $\sum_{i=0}^{12} a_i x^i$ / $a_i \in P([0, 12))$, $x^{13} = 1$} be the mod interval subset coefficient polynomial collection.

i) Study questions (i) to (ix) of problem (74) for this L.

ii) Derive all the special features enjoyed by L.

85. Let M ={ $\sum_{i=0}^{26} a_i x^i$ / $a_i \in P(^I[0, 17), x^{27} = 1\}$ be the mod interval natural neutrosophic subset coefficient finite degree polynomial collection.

i) Study questions (i) to (ix) of problem (74) for this M.

ii) Study questions (ii) to (v) of problem (85) for this M.

iii) Compare this M with L of problem 84.

86. Let R = { $\sum_{i=0}^{6} a_i x^i$ / $a_i \in P(C^I([0, 5))), x^7 = 1\}$ be the mod interval natural neutrosophic finite complex number subset coefficient polynomial collection.

i) Study questions (i) to (ix) of problem (74) for this R.

ii) Study questions (ii) to (v) of problem 85 for this R.

iii) Compare this R with M of problem 85.

Chapter Four

MOD SUBSET TOPOLOGICAL SPACES ON THE MOD PLANE $R_n(m)$ AND KAKUTANI'S THEOREM

The study of modulo values when we restrict to the first quadrant of the real plane $R_n(m) = [0, m) \times [0, m) = \{(a, b) / a, b \in [0, m)\}$ is defined as the MOD plane [20].

Algebraic operations are performed on $R_n(m)$ in [24].

The plane is represented by the following figure.

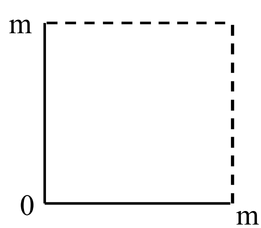

Figure 4.1

It is a half open square.

Let $P(R_n(m))$ = {collection of all subsets from $R_n(m)$}, we define the operations $\cup$ and $\cap$ on $P(R_n(m))$. $P(R_n(m))$ is an infinite collection with $R_n(m)$ as the greatest element under $\cup$ and ϕ as the least element under $\cap$. Infact $R_n(m)$ is an infinite distributive lattice.

Thus $\{R_n(m), \cup, \cap\}$ can be realized as a usual topological space of infinite order. One can also visualize $R_n(m)$ as a truncated topological space of the real plane. Here we call $R_n(m)$ as the MOD plane subset special type of topological space.

The advantage of building this type of MOD plane subset special type of topological space is that we can get infinite number of them in contrast with only one such real plane topological space.

The properties of $\cup$ or $\cap$ happens to result in a semilattice on $P(R_n(m))$. So as per need one can use these MOD plane subset topological spaces.

We will illustrate this situation by some examples.

Example 4.1. Let $R_n(5) = \{(a, b) / a, b \in [0, 5)\}$ be the MOD plane S = $P(R_n(5))$ = {collection of all subsets of $R_n(5)$}. $\{S, \cup, \cap\}$ is a MOD topological space of infinite order. Infact $\{S, \cup, \cap\} \subseteq \{P(R \times R), \cup, \cap\}$ is a subspace of $\{P(R\times R), \cup, \cap\}$.

Several open problems can be proposed at this stage for any $R_n(m)$; $2 \leq m < \infty$.

Likewise $\{(-\infty, \infty), \cup, \cap\}$ is a topological space so $\{[0, m), \cup, \cap\}$ is a subspace, here when we call or define MOD interval we assume in that intervals when some operation like + is performed $m - 1 + 1 = 0 \pmod n$ so we cannot say $m - 1 + 1$

is undefined in our spaces it takes the value 0 given by m – 1 + 1 ≡ 0 (mod m).

So we have to work in a different direction the Kakutani's fixed point theorem which is a challenging problem associated with MOD subset interval topological spaces.

We will illustrate this situation by some more examples.

Example 4.2. Let S ={P(R_n(12)) = {collection of all subsets from R_n(12) = {(a, b) / (a, b) ∈ [0, 12) × [0, 1)} be the collection of MOD plane subsets. {S, ∪, ∩} is the MOD plane subset special type of topological space.

Let A = {(3, 0.25), (0.7, 0.116), (11.035, 9.75), (1.1, 2.5), (0.113, 2.1), (4, 0.7), (5.3, 1)} and B = {(3, 0.25), (1.1, 2.5), (5.3, 1), (2, 3.55), (8.3, 7), (10, 10.35), (0.887, 0.12)} ∈ S.

Clearly A ∪ B ={(3, 0.25), (0.7, 0.116), (11.035, 9.75), (1.1, 2.5), (0.113, 2.1), (4, 0.7), (5.3, 1), (2, 3.55), (8.3, 7), (10, 10.35), (0.887, 0.12)} I

A ∩ B ={ (3, 0.25), (1.1, 2.5), (5.3, 1)} II

This is the way operations are performed on S and (S, ∪, ∩} is defined as a MOD subset plane topological space.

The following observations are pertinent, we have already in the earlier chapters defined MOD interval special type of topological spaces using the dual plane ⟨[0, m) ∪ g⟩, the neutrosophic plane ⟨[0, m) ∪ I⟩, the finite complex modulo integer plane C([0, m)), the MOD interval special quasi dual number plane ⟨[0, m) ∪ k⟩ and MOD interval special dual like number plane ⟨[0, m) ∪ h⟩.

We could not and have not done such study about MOD interval real plane $[0, m) \times [0, m) = R_n(m)$. This is study is mainly taken here to find the validity of Kakutani's theorem. For these planes are new so such study may not be existing in case of $\langle R \cup g \rangle = \{a + bg / a, b \in R, R \text{ reals}\}$ and so on. Only for C, the complex plane such study is complete in literature.

Now likewise the MOD interval special type of topological space built using $P([0, m))$; $2 \leq m < \infty$ is thoroughly studied in chapters II and III of this book.

Now the interval $[0, 1)$ does not satisfy the Kakutani's theorem for the space is not complete but we in our very definition of MOD interval $[0, m)$, assume $m - 1 + 1 \equiv 0 \pmod{m}$.

Hence we propose the following conjectures.

Conjecture 4.1. Let $P([0, m)) = T$ be the MOD interval subset collection. $\{T, \cup, \cap\}$ is a MOD interval subset special type of topological space in which $m - 1 + 1 \equiv 0 \pmod{m}$, can under this situation T satisfy the Kakutani's theorem?

Conjecture 4.2. Study conjecture 4.1 by replacing $[0, m)$ by each one of the following $\langle [0, m) \cup g \rangle$, $\langle [0, m) \cup h \rangle$, $\langle [0,m) \cup k \rangle$, $C([0\ m))$ and $\langle [0, m) \cup I \rangle$.

All these are MOD intervals or MOD planes closed under the binary operations $+$, $\times$, $\cup$ and $\cap$.

Conjecture 4.3. Let $\{S, +, \times\}$ where $S = P([0, m))$ be the MOD interval special type of subset topological space. Can $\{S, +, \times\}$ satisfy the Kakutani's theorem?

Conjecture 4.4. In conjecture 4.3 replace $[0, m)$ by each one of them $\langle [0, m) \cup g \rangle$, $\langle [0, m) \cup h \rangle$, $C([0, m))$, $\langle [0, m) \cup I \rangle$ and $\langle [0,$

m) $\cup$ k⟩ and analyse the validity or otherwise of the Kakutani's conjecture.

Conjecture 4.5. Let {P([0, m)), +, $\cup$}, {P([0, m), +, $\cap$}, {P([0, m), $\times$, $\cap$} and {P([0, m)), $\times$, $\cup$} be the four distinct MOD interval special type of subset topological spaces.

Can these four spaces satisfy Kakutani's theorem?

Conjecture 4.6. Can Kakutani's theorem be true for the following MOD interval subset special type of topological spaces:

i) {P(⟨[0, m) $\cup$ g⟩), + $\cup$}, {P(⟨[0, m) $\cup$ I⟩, +, $\cup$}, {P(⟨[0,m) $\cup$ h⟩), +, $\cup$}, {P(⟨[0,m) $\cup$ k⟩), +, $\cup$} and {P(C[0, m)), +, $\cup$}?

ii) Can Kakutani's theorem be true in case of the following MOD interval subset special type of topological spaces.

{P(⟨[0, m) $\cup$ g⟩, +, $\cap$}, {P(⟨[0, m) $\cup$ h⟩), +, $\cap$}, {P (⟨[0, m) $\cup$ k⟩), +, $\cap$}, {P(⟨[0, m) $\cup$ I⟩, +, $\cap$} and {P(C([0, m)), +, $\cap$}?

(Study if {+, $\cap$} is replaced by {$\times$, $\cap$} or {$\times$, $\cup$} on the above five spaces).

This type of study is new and innovative so only we have proposed these open conjectures and they happen to be difficult. Can we find fixed points as given by the Kakutani's theorem?

Clearly they are part of a plane or line. The plane or line is ideal and satisfies Kakutani's theorem.

We are not sure whether these properties are inherited by the MOD intervals and MOD planes.

Now we develop MOD plane subset special type of topological spaces using, $+$, $\times$, $\cup$ and $\cap$ operations.

To this end we give examples of MOD plane subset semigroups.

Example 4.3. Let S = $P(R_n(6))$ = {P([0, 6) × [0, 6)) = {collection of all subsets from $R_n(6)$ = [0, 6) × [0, 6)}} be the MOD plane subsets collections. {S, $\cup$} is a semigroup, in fact a semilattice.

Similarly {S, $\cap$} is a semigroup which is a semilattice. {S, +} is a MOD subset plane semigroup. {S, ×} is a MOD subset plane semigroup.

Let A = {(0.3, 5), (0.1,0.02), (0.12, 4.5), (0.001, 0), (1, 0.005)} and B = {(0.01, 2), (0.02, 0.05), (0.001, 0.005), (0.005, 0), (0, 0.3), (0.1,1.05)} $\in$ S = $P(R_n(6))$.

We find A × B ={(0.003, 4), (0.001, 0.04), (0.0012, 3), (0.00001, 0), (0.01, 0.01), (0.006, 0.25), (0.002, 0.001), (0.0024, 0.225), (0.00002, 0), (0.02, 0.00025), (0.0001, 0.025), (0.0001, 0.0001), (0.00012, 0.0225), (0.000001, 0), (0.001, 0.000025), (0.0015, 0), (0.0005, 0), (0.0006, 0), (0.000005, 0), (0.005, 0), (0, 1.5), (0, 0.006), (0, 1.25), (0,0), (0, 0.0015), (0.03, 5.25), (0.01, 0.0210), (0.012, 4.725), (0.0001, 0), (0.1, 0.05025)} I

A + B = {(0.31, 1), (0.11, 2.02), (0.13, 0.5), (0.011, 2), (1.01, 2.005), (0.32, 5.05), (0.12, 0.007), (0.14, 4.55), (0.021, 0.05), (1.02, 0.055), (0.301, 5.005), (0.101, 0.025), (0.121, 4.505), (0.002, 0.005), (1.005, 0.005), (0.305, 5), (0.125, 4.5), (0.105,0.0), (0.006, 0), (1.005, 0.005), (0.3, 5.3), (0.1, 0.32), (0.12, 4.8), (0.001, 0.3), (1, 0.305), (0.4, 0.05), (0.2, 1.07), (0.22, 5.55), (0.101, 1.05), (1.1, 1.055)} II

This is the way + and × operations are performed on $P(R_n(6))$.

We call these (S, +} and {S, ×} as MOD plane semigroups.

Hence we define {S, +, ×}, {S, ×, ∪}, {S, ×, ∩}, {S, +, ∪} and {S, +, ∩} for S = {P(R_n(m))}; $2 \leq m < \infty$ as the MOD plane subset special type of topological spaces.

We see all these six MOD plane subset topological spaces are distinct and enjoy different types of properties.

We will illustrate these situations by some examples.

Example 4.4. Let B = P(R_n(7)) = {collection of all subsets from the MOD plane $R_n(7)$ ={(a, b) / a, b ∈ [0, 7)}} be the MOD plane subsets collection.

{B, +}, {B, ×}, {B, ∪} and {B, ∩} are the four MOD plane subset semigroups. B_o = {B, ∪, ∩}, $B_{\times}^{+}$ = {B, +, ×}, $B_{\cup}^{+}$ = {B, ∪, +}, $B_{\cap}^{+}$ = {B, +, ∩}, $B_{\cup}^{\times}$ = {B, ∪, ×} and $B_{\cap}^{\times}$ = {B, ×, ∩} are the MOD plane subset special type of topological spaces associated with the four MOD plane subset semigroup.

Can we get Kakutani's theorem to be true in case of these MOD plane subset special type of topological spaces?

For these are just subset of the real plane, so B_o can infact be realized as the special type of proper subspace of this MOD plane subset topological space.

Study in this direction is not only innovative and interesting, these results are left as open conjectures.

That is validity of the Kakutani's conjectures in case of MOD plane subset special type of topological spaces is a challenging one.

Next we proceed onto describe MOD plane subset matrix special type of topological spaces and MOD plane matrix subset special type of topological spaces.

Here it is pertinent to keep on record that we can have 6 different types of MOD plane special topological spaces.

We will illustrate this situation by examples. Further authors wish to state by describing the operations by examples it makes easy for any researcher to understand the concept and that is vital.

Example 4.5. Let M = $\{(a_1, a_2, a_3, a_4) / a_i \in P([0, 12); 1 \leq i \leq 4\}$ be the MOD interval subset matrix collection. We will indicate how the operations are performed on M.

Let x = ({0.3, 10.1, 4}, {1.1, 0.5, 5}, {0.6, 0.2}, {4, 0.3, 1, 0.2}) and y = ({0, 3, 4, 0.1}, {0.7, 5}, {4, 0.6}, {1, 0.5, 0.7}) $\in$ M.

$x \cup y$ = ({0, 3, 10.1, 4, 0.1, 0.3}, {0.5, 5, 0.7, 1.1}, {4, 0.6, 0.2}, {4, 0.3, 1, 0.2, 0.5, 0.7}) I

$x \cap y$ = ({4}, {5}, {0.6}, {1}) II

x + y = ({0.3, 10.1, 4, 3.3, 1.1, 7, 4.3, 2.1, 8, 0.4, 10.2, 4.1}, {1.8, 1.2, 5.7, 6.1, 5.5, 10}, {4.6, 4.2, 1.2, 0.8}, {5, 4.5, 4.7, 1.3, 2, 1.2, 1.7, 0.8, 1, 1.5, 0.7, 0.9}) III

$x \times y$ = ({0, 0.9, 6.3, 1.2, 4.4, 4, 0.03, 1.01, 0.4}, {(0.77, 0.35, 3.5, 5.5, 2.5, 1}, {2.4, 0.8, 0.36, 0.12}, {4, 0.3, 1, 0.2, 2, 0.15, 0.5, 1, 2.8, 0.21, 0.7, 0.14}) IV

All the four equations are distinct so (M, +), {(M, $\cup$}, {M, $\cap$} and {M, $\times$} are four distinct MOD interval subset entries matrix semigroups of infinite order.

Thus M_o ={M, ∪, ∩}, $M_{\times}^{+}$ = {M, +, ×}, $M_{\cup}^{+}$ = {M, +, ∪}, $M_{\cap}^{+}$ = {M, +, ∩}, $M_{\cup}^{\times}$ = {M, ×, ∪} and $M_{\cap}^{\times}$ = {M, ×, ∩} are all six distinct MOD plane subset matrix entries special type of topological spaces.

This has MOD interval subset matrix special type of topological subspaces. Infact all these have MOD interval subset matrix special type of topological subspaces that are isomorphic with MOD plane subset special type of topological spaces like P_1 = {({a_1}, {0},{0}, {0}) /{a_1}∈ P[0, 12))} ≅ the MOD plane subset topological spaces got from P[0, 12) ≅ P_2 = {({0}, {a_2}, {0}, {0}) such that {a_2} ∈ P ([0, 12)} ≅ P_3 = {({0},{0}, {a_3}, {0}) /{a_3} ∈ P([0,12)} ≅ P_4 = {({0}, {0}, {0},{a_4}) / {a_4}∈ P[0,12)}.

Hence the claim.

Such study is done in the earlier chapters. Thus if Kakutani's theorem is true in the MOD interval subset matrix special type of topological spaces built using [0, n) then it is also true in case of some special type of subspaces of M.

Example 4.6. Let B = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle/ a_i \in P(Z_{19});\ 1 \leq i \leq 5 \right\}$ be the MOD plane subset matrix collection.

We will define ∪, ∩, $\times_n$ and + operation on B.

Let

$$x = \begin{bmatrix} \{(0,3), (0.1,0.5),(0.2,0),(1,0.1)\} \\ \{(0,0.1),(0.1,2),(2.1,0)\} \\ \{(4,3),(0,2),(0.01,0.02)\} \\ \{(0,0),(1,1),(0.003,0.004)\} \\ \{(1,0.01),(0.02,0.003)\} \end{bmatrix}$$

and

$$y = \begin{bmatrix} \{(0,0.1),(0.2,0),(1,5)\} \\ \{(0.1,2),(5,0.1)\} \\ \{(3,0),(2,0.5)\} \\ \{(1,0.8),(0.001,0.003)\} \\ \{(1.6,0),(2,0.5)\} \end{bmatrix}$$

be in B.

$$x \cup y = \begin{bmatrix} \{(0,3),(0.1,0.5),(0.2,0),(1,0.1),(1,5)\} \\ \{(0,0.1),(0.1,2),(2.1,0),(5,0.1)\} \\ \{(4,3),(0,2),(0.01,0.02),(3,0),(2,0.5)\} \\ \{(0,0),(1,1),(0.003,0.004),(1,0.8),(0.001,0.003)\} \\ \{(1,0.01),(0.02,0.003),(1.6,0),(2,0.5)\} \end{bmatrix} \quad \text{I}$$

Next we find $x \cap y$;

$$x \cap y = \begin{bmatrix} \{(0.2,0),(1,0.1)\} \\ \{(0.1,2)\} \\ \{\phi\} \\ \{\phi\} \\ \{\phi\} \end{bmatrix} \quad \text{II}$$

$$x + y = \begin{bmatrix} \{(1,3.1),(1.1,0.6),(1.2,0.1),(2,0.2),(0.2,3),(0.3,0.5)\} \\ \{(0.4,0),(1.2,0.1),(1.8),(1.1,5.5),(1.2,5),(2,5.1)\} \\ \{(0.1,2.1),(0.2,4),(2.2,2),(5,0.2),(5.1,2.1),(7.1,0.1)\} \\ \{(7,3),(3,2),(3.01,0.02),(6,3.5),(2,2.5),(2.01,0.52)\} \\ \{(1,0.8),(2,1.8),(1.003,0.804),(0.001,0.003), \\ (1.001,1.003),(0.004,0.023)\} \\ \{(2.6,0.01),(1.62,0.003),(3,0.51),(2.02,0.503)\} \end{bmatrix}$$

$$x \times_n y = \begin{bmatrix} \{(0,0.3),(0.1,0.05),(0.2,0),(1,0.01), \\ (0,0),(0.02,0),(0.04,0),(0.2,0), \\ (0.15),(0.1,2.5),(0.2,0),(10.5,0)\} \\ \{(0,0.2),(0.01,4),(0.21,0),(0,0.01), \\ (0.5,0.2),(10.5,0)\} \\ \{(12,0),(0,0),(0.03,0),(8,1.5), \\ (0,1),(0.02,0.01)\} \\ \{(0,0),(1,0.8),(0.003,0.0032), \\ (0.001,0.003),(0.000003,0.000012)\} \\ \{(1.6,0),(0.032,0),(2,0.005),(0.04,0.0015)\} \end{bmatrix} \quad \text{IV}$$

All the four equations are distinct, so all the four semigroups are distinct and hence all the six MOD plane subset matrix special type of topological spaces $M_o = \{M, \cup, \cap\}$, $M^{+}_{\times_n} = \{M, +, \times_n\}$, $M^{+}_{\cup} = \{M, +, \cup\}$, $M^{+}_{\cap} = \{M, +, \cap\}$, $M^{\times_n}_{\cup} = \{M, \cup, \times_n\}$ and $M^{\times_n}_{\cap} = \{M, \cap, \times_n\}$ are all distinct. These spaces have subspaces isomorphic to $S(R_n(19))$ as well as $S([0, 19))$.

However one is not in a position to say whether Kakutani's theorem is true in case of all these 6 MOD plane subset special type of topological spaces.

Example 4.7. Let W = $\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ / $a_i \in P(R_n(10))$; $1 \leq i \leq 4\}$

be the MOD plane subset matrix collection. (W, +), {W, $\times_n$}, {W, $\times$}, {W, $\cap$} and {W, $\cup$} are five distinct MOD plane subset matrix semigroups.

$$\text{Let } A = \begin{bmatrix} \{(0,0.1),(5,0.2) & \{(0,0),(5,4), \\ (1,0),(1,\ 0.1)\} & (0.4,2)\} \\ & \\ \{(0,7),(0.1,0.2) & \{(4,3),(0,0.5), \\ (1,1)\} & (0..2,0)\} \end{bmatrix}$$

$$B = \begin{bmatrix} \{(1,0.3),(0,4) & \{(0,0),(4,2) \\ (1,0.1)\} & (1,0.3)\} \\ & \\ \{(0,7),(4,0.2), & \{(0,2),(8,0), \\ (1,0),(0,5)\} & (0.1,0.5)\} \end{bmatrix}$$

A + B =

$$\begin{bmatrix} \begin{matrix}\{(1,0.4),(6,0.5), \\ (2,0.3),(2,0.4), \\ (0,4.1),(5,4.2), \\ (1,4),(1,4.1),(1,0.2), \\ (6,0.3),(2,0.1),(2,0.2)\}\end{matrix} & \begin{matrix}\{(0,0),(5,4),(4,2), \\ (1,0.3),(0.4,2),(4.4,4) \\ (9,6),(1.4,2.3),(6,4.3)\}\end{matrix} \\ & \\ \begin{matrix}\{(0,4),(0.1,7.2),(1,8), \\ (4,7.2),(4.1,0.4),(5,1.2), \\ (1,7),(1.1,0.2),(2,0),(0,2), \\ (0.1,5.2),(1,6)\}\end{matrix} & \begin{matrix}\{(4,5),(0,2.5),(0.2,2),(2,3), \\ (8,0.5),(8.2,0),(4.1,3.5), \\ (0.1,1),(0.3,0.5)\}\end{matrix} \end{bmatrix} \quad \text{I}$$

$$A \cup B = \begin{bmatrix} \{(0,0.1),(5,0.2), & \\ (1,0),(1,0.1),(1,0.3), & \{(0,0),(5,4),(4,2), \\ (0,4),(1,\ 0.1)\} & (0.4,2),(1,0.3)\} \\ & \\ \{(0,7),(0.1,0.2),(1,\ 1), & \{(4,3),(0,0.5),(0.2,0), \\ (4,0.2),(1,0),(0,5)\} & (0,2),(8,0),(0.1,0.5)\} \end{bmatrix} \quad \text{II}$$

$$A \cap B = \begin{bmatrix} \{(1,0.1)\} & \{(0,0)\} \\ \{(0,7)\} & \{\phi\} \end{bmatrix} \quad \text{III}$$

$$A \times_n B =$$

$$\begin{bmatrix} \{(0,0.03),(5,0.06),(1,0), & \\ (1,0.03),(0,0.4),(0,0.8), & \{(0,0),(0,8),(1.6,4) \\ (0,0),(0,0.4),(0,0.01), & (5,1.2),(0.4,0.6)\} \\ (5,0.02),(1,0.01)\} & \\ & \\ \{(0,9),(0,1.4),(0,7),(0,1.4) & \{(0,6),(0,1),(0,0), \\ (0.4,0.04),(0,0),(4,0.2), & (2,0),(1.6,0),(0.4,1.5), \\ (0.1,0),(1,0),(0,5),(0,1),(0,5)\} & (0,0.25),(0.02,0)\} \end{bmatrix} \quad \text{IV}$$

Next we find A × B and B × A,

$$
A \times B = \begin{bmatrix}
\begin{array}{l} \{0,0.1),(5,0.2),(1,0), \\ (1,0.1)\} \times \{(1,0.3),(0,4), \\ (1,0.1)\} + \{(0,0),(5,4), \\ (0.4,2)\} \times \{(0,7),(4,0.2), \\ (1,0),(0,5)\} \end{array} &
\begin{array}{l} \{(0,0.1),(5,0.2),(1,0), \\ (1,0.1)\} \times \{(0,0),(4,2), \\ (1,0.3)\} + \{(0,0),(5,4), \\ (0.2,0)\} \times \{(0,2),(8,0), \\ (0.1,0.5)\} \end{array} \\
\\
\begin{array}{l} \{(0,7),(0.1,0.2),(1,1)\} \times \\ \{(1,0.3),(0,4),(1,0.1)\} + \\ \{(4,3),(0,0.5),(0.2,0)\} \times \\ \{(0,7),(4,0.2),(1,0),(0,5)\} \end{array} &
\begin{array}{l} \{(0,7),(0.1,0.2),(1,1)\} \\ \times\{(0,0),(4,2),(1,0.3)\} + \\ \{(4,3),(0,0.5),(0.2,0)\} \\ \times\{(0,2),(8,0),(0.1,0.5)\} \end{array}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\begin{array}{l} \{(0,0.03.),(5,0.06),(1,0), \\ (1,0.03),(0,0.4),(0,0.8), \\ (0,0),(0,0.01),(5,0.02), \\ (1,0.01) + \{(0,0),(0,8), \\ (0,4),(0,0.8),(1.6,0.4), \\ (5,0)\} \end{array} &
\begin{array}{l} \{(0,0),(0,0.2),(0,0.4), \\ (4,0.2),(0,0.03),(5,0.06), \\ (1,0),(1,0.03)\} + \{(0,0),(0,8), \\ (1.6,0),(0.5,2),(0.02,0)\} \end{array} \\
\\
\begin{array}{l} \{(0,2.1),(0.1,0.06),(1,0.3), \\ (0.8),(0,0.8),(0,4),(0,0.7), \\ (0.1,0.02),(1,0.1)\} + \{(0,1), \\ (0,3.5),(0,0),(6,0.6), \\ (0,0.10),(0.8,0),(4,3), \\ (0.2,0),(0,5),(0,2.5)\} \end{array} &
\begin{array}{l} \{(0,0),(0,4),(0.4,0.4), \\ (4,2),(0,2.1),(0.1,0.06), \\ (1,0.3)\} + \{(0,6),(0,1), \\ (0,0),(2,0),(1.6,0), \\ (0.4,1.5),(0,0.25), \\ (0.02,0)\} \end{array}
\end{bmatrix}
$$

B × A =

$$\begin{bmatrix} \{(0,0.03),(5,0.06),(1,0), & \{(0,0),(0,0.2),(0,0.4), \\ (1,0.03),(0,0.4),(0,0.8), & (4,0.2),(0,0.03),(5,0.06), \\ (0,0),(0,0.01),(5,0.02), & (1,0),(1,0.03),(0,8), \\ (1,0.01),(0,8.03),(5,8.06), & (1.6,0),(0.5,2),(0.02,0), \\ (1,8),(1,8.03),(0,8.4), & (0,8.2),(0,8.4),(4,8.2), \\ (0,8.8),(0,8),(0,8.01), & (0,8.03),(5,8.06),(1,8), \\ (5,8.02),(1,8.01),(0,4), & (1,8.03),(0.5,2.2),(1.6,0.2), \\ (0,4.03),(5,4.06),(1,4), & (0.5,2.4),(1.6,0.4), \\ (1,4.03),(0,4.4),(0,4.8), & (4.5,2.2),(5.6,0.2), \\ (0,4.01),(5,4.02),(1,4.01), & (0.5,2.03),(1.6,0.03), \\ (0,0.83),(5,0.86),(1,0.8), & (5.5,2.06),(6.6,0.06), \\ (1,0.83),(0,1.2),(0,1.6), & (1.5,2),(2.6,0), \\ (0,0.8),(0,0.81),(5,0.82), & (1.5,2.3),(2.6,0.03), \\ (1,0.81),(1.6,0.43), & (1.02,0),(0.02,0.2), \\ (6,0.01),(6.6,0.46), & (1.02,0.03),(0.02,0.4), \\ (2.6,0.4),(2.6,0.43), & (4.02,0.2),(0.02,0.03), \\ (1.6,0.8),(1.6,1.2), & (5.02,0.06)\} \\ (1.6,0.4),(1.6,0.41), & \\ (6.6,0.42),(2.6,0.41), & \\ (5,0.03),(0,0.06),(6,0), & \\ (6,0.03),(5,0.4),(0,0.02), & \\ (5,0.8),(5,0),(5,0.01)\} & \end{bmatrix}$$

$$\left[\begin{array}{ll}
\{(0,3.1),(0.1,1.06), & \{(0,6),(0,1),(0,0), \\
(1,1.3),(0,9),(0,1.8), & (2,0),(1.6,0),(0.4,1.5), \\
(0,5),(0,1.7),(0.1,1.02), & (0,0.25),(0.02,0), \\
(1,1.1),(0,5.6),(0.1,3.56), & (0,4),(0.4,0.4), \\
(1,3.8),(0,1.5),(0,4.3), & (4,2),(0,2.1), \\
(0,7.5),(0,4.2),(0.9,0.02), & (0.1,0.06),(1,0.3), \\
(0.1,3.52),(1,3.6),(1.8,0.1), & (0.4,6.4),(4,8), \\
(0,2.1),(0.1,0.06),(1,0.3), & (0,8.2),(0.1,6.06), \\
(0,8),(0,0.8),(0,4),(0,0.7), & (1,6.3),(0,5),(0.4,1.4), \\
(0.1,0.02),(1,0.1),(6,2.7), & (4,3),(0,3.1),(0.1,1.06), \\
(6.1,0.66),(7,0.9),(6.8.6), & (1,1.3),(4.02,2),(2,4), \\
(6,1.4),(7,0.7),(6,4.6), & (2.4,0.4),(6,2),(2,2.1), \\
(6,1.3),(6.1,0.62),(1.8,0.3), & (0.12,0.06),(2.1,0.06), \\
(0,2.2),(0.1,0.16),(1,0.4), & (1.02,0.3),(3,0.3),(1.6,4), \\
(0,8.1),(0,0.9),(0,4.1), & (0.02,4),(0.42,0),(2,0.4), \\
(0,0.8),(0.1,0.12),(1,0.2), & (5.6,2),(1.6,2.1),(1.7,0.06), \\
(0.8,2.1),(0.9,0.06),(0.8,8), & (2.6,0.3),(0.1,0.31), \\
(0.8,4),(0.8,0.7),(4,5.1), & (0.4,5.5),(1,0.55), \\
(4.1,3.06),(5,3.3),(4,1),(4,3.8), & (0.8,1.9),(0.02,2.1), \\
(4,7),(4,3.7),(4.1,3.02),(5,3.1), & (4.4,3.5),(0.4,3.6), \\
(0.2,2.1),(0.3,0.06),(1.2,0.3), & (0.5,1.56),(1.4,1.8), \\
(0.2,8),(0.2,0.8),(0.2,4),(0.2,0.7), & (0,4.25),(0.4,0.65), \\
(0.3,0.02),(1.2,0.1),(0,7.1), & (4,2.25),(0,2.35)\} \\
(0.1,5.06),(1,5.3),(0,3),(0,5.8), & \\
(0,9),(0,5.7),(0.1,5.02), & \\
(1,5.1),(0,4.6),(0.1,2.56),(1,2.8), & \\
(0,0.5),(0,3.3),(0,6.5),(0,3.2), & \\
(0.1,2.57),(1,2.6)\} &
\end{array}\right]$$

This is the way × operation is performed on W.

Clearly it is left for the reader to verify A × B ≠ B × A in general.

It is easily verified {W, +}, {W, ∪}, {W, ∩}, {W, $\times_n$} and {W, ×} are the five distinct MOD plane subset matrix semigroups which will yield W_o = {W, ∪, ∩}, $W_{\cup}^{+}$ = {W, +, ∪}, $W_{\cap}^{+}$ = {W, +, ∩}, $W_{\times}^{+}$ = {W, +, ×}, $W_{\cup}^{\times}$ = {W, ×, ∪}, $W_{\cap}^{\times}$ = {W, ×, ∩}, $W_{\cup}^{\times_n}$ = {W, $\times_n$, ∪}, $W_{\cap}^{\times_n}$ = {W, $\times_n$, ∩}, $W_{\times}^{\times_n}$ = {W, $\times_n$, ×} and $W_{\times_n}^{+}$ = {W, +,$\times_n$} the 10 distinct MOD plane subset matrix special type of topological spaces.

It is left as open conjecture how many of the MOD plane subset matrix special type of topological spaces satisfy Kakutani's theorem?

It has become difficult to prove Kakutani's theorem in case of these MOD plane subset matrix special types of topological spaces.

Next we proceed onto describe by examples the MOD plane matrix subset topological spaces of special type.

Example 4.8. Let P(M) = B ={collection of all subsets from M

= {collection of all subsets from M = { $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ / $a_i \in R_n(6)$ ={(a, b)

/ a, b ∈ [0,6); 1≤ i ≤ 4} be the MOD plane matrix subset collection.

We show how operations +, ×, ∪ and ∩ are performed on B.

Let

$$A = \left\{ \begin{bmatrix} (0.3,1) \\ (5,0) \\ (0.05,1) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (1,1) \\ (2,3) \\ (0,0.1) \\ (0.5,0.2) \end{bmatrix}, \begin{bmatrix} (0,0) \\ (0.1,0.1) \\ (0.2,0.7) \\ (0.4,0) \end{bmatrix} \right\}$$

and

$$B_1 = \left\{ \begin{bmatrix} (0.3,1) \\ (5,0) \\ (0.05,1) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (1,1) \\ (0.1,0.2) \\ (0,0.1) \\ (1,5) \end{bmatrix} \right\}$$

be in B.

$$A + B_1 = \{ \begin{bmatrix} (0.6,2) \\ (4,0) \\ (1,2) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (1.3,2) \\ (1,3) \\ (0.05,1.1) \\ (0.5,0.2) \end{bmatrix}, \begin{bmatrix} (0.3,1) \\ (5.1,0.1) \\ (0.25,1.7) \\ (0.4,0) \end{bmatrix}, \begin{bmatrix} (1.3,2) \\ (5.1,0.2) \\ (0.05,1.1) \\ (1,5) \end{bmatrix},$$

$$\begin{bmatrix} (2,2) \\ (2.1,3.2) \\ (0,0.2) \\ (1.5,5.2) \end{bmatrix}, \begin{bmatrix} (1,1) \\ (0.2,0.3) \\ (0.2,0.8) \\ (1.4,5) \end{bmatrix} \} \qquad \text{I}$$

$$A \cap B_1 = \{ \begin{bmatrix} (0.3,1) \\ (5,0) \\ (0.05,1) \\ (0,0) \end{bmatrix} \} \qquad \text{II}$$

$$A \cup B_1 = \left\{\begin{bmatrix} (0.3,1) \\ (5,0) \\ (0.05,1) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (1,1) \\ (2,3) \\ (0,0.1) \\ (0.5,0.2) \end{bmatrix}, \begin{bmatrix} (0,0) \\ (0.1,0.1) \\ (0.2,0.7) \\ (0.4,0) \end{bmatrix}, \begin{bmatrix} (1,1) \\ (0.1,0.2) \\ (0,0.1) \\ (1,5) \end{bmatrix}\right\}$$

III

$$A \times_n B_1 = \{\begin{bmatrix} (0.09,1) \\ (1,0) \\ (0.0025,1) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (0.3,1) \\ (4,0) \\ (0,0.1) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (0,0) \\ (0.5,0) \\ (0.01,0.7) \\ (0,0) \end{bmatrix}, \begin{bmatrix} (0.3,1) \\ (0.5,0) \\ (0,0.1) \\ (0,0) \end{bmatrix},$$

$$\begin{bmatrix} (1,1) \\ (0.2,0.6) \\ (0,0.01) \\ (0.5,0.1) \end{bmatrix}, \begin{bmatrix} (0,0) \\ (0.01,0.02) \\ (0,0.07) \\ (0.4,0) \end{bmatrix}\} \qquad \text{IV}$$

Clearly I, II, III and IV are distinct . Hence $\{B, \cup\}$, $\{B, \cap\}$, $\{B, +\}$ and $\{B, \times_n\}$ are four distinct MOD plane matrix subset semigroup.

$B_o = \{B, \cup, \cap\}$, $B^{+}_{\times_n} = \{B, \times_n, +\}$, $B^{+}_{\cup} = \{B, \cup, +\}$, $B^{+}_{\cap} = \{B, \cap, +\}$, $B^{\times_n}_{\cup} = \{B, \cup, \times_n\}$ and $B^{\times_n}_{\cap} = \{B, \cap, \times_n\}$ are the six distinct MOD plane matrix subset special type of topological spaces.

Verifying Kakutani's theorem in case of these MOD plane matrix subset spaces also happens to be a challenging problem for the researchers so left as open conjectures.

Example 4.9. Let P = {P(T)} = {collection of all subsets from

$$T = \{\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{pmatrix} / a_i \in R_n(7);\ 1 \le i \le 6\}$$

be the MOD the MOD plane matrix subset collection. $\{P, +\}$, $\{P, \cap\}$, $\{P, \cup\}$ and $\{P, \times_n\}$ be the MOD plane matrix subset semigroups.

$P_o = \{P, \cup, \cap\}$, $P^+_{\times_n} = \{P, \times_n, +\}$, $P^+_{\cup} = \{P, +, \cup\}$, $P^+_{\cap} = \{P, \cup, +\}$, $P^{\times}_{\cup} = \{P, \times, \cup\}$ and $P^{\times}_{\cap} = \{P, \times, \cap\}$ be the MOD plane matrix subset topological spaces of special type.

$$\text{If } A = \{\begin{pmatrix} (0,1.1) & (1,0.5) & (0.01,0.2) \\ (4,0.3) & (0.5,0.8) & (3,4.2) \end{pmatrix},$$

$$\begin{pmatrix} (0,0) & (1,1) & (0.1,0.7) \\ (0.6,1) & (0,0.8) & (1.2,0.9) \end{pmatrix}, \begin{pmatrix} (0,2) & (0,5) & (0.9,1.2) \\ (1,0) & (1.6,0) & (0,0.3) \end{pmatrix}\}$$

$$B = \{\begin{pmatrix} (0,2) & (0,5) & (0.9,1.2) \\ (1,0) & (1.6,0) & (0,0.3) \end{pmatrix},$$

$$\begin{pmatrix} (1,1) & (0,0) & (0.1,0.8) \\ (0.2,0.6) & (0,6) & (0.9,0) \end{pmatrix}\} \in P.$$

$$\text{We see } A \cup B = \{\begin{pmatrix} (0.1,1) & (1,0.5) & (0.01,0.2) \\ (4,0.3) & (0.5,0.8) & (3,4.2) \end{pmatrix},$$

$$\begin{pmatrix} (0,0) & (1,1) & (0.1,0.7) \\ (0.6,1) & (0,0.8) & (1.2,0.9) \end{pmatrix}, \begin{pmatrix} (0,2) & (0,5) & (0.9,1.2) \\ (1,0) & (1.6,0) & (0,0.3) \end{pmatrix},$$

$$\begin{pmatrix} (1,1) & (0,0) & (0.1,0.8) \\ (0.2,0.6) & (0,6) & (0.9,0) \end{pmatrix}\} \qquad \text{I}$$

$$A \cap B = \{\begin{pmatrix} (0,2) & (0,5) & (0.9,1.2) \\ (1,0) & (1.6,0) & (0,0.3) \end{pmatrix}\} \qquad \text{II}$$

Next we proceed onto find

$$A + B = \{\begin{pmatrix} (0,3.1) & (1,5.5) & (0.91,1.4) \\ (5,0.3) & (2.1,0.8) & (3,4.5) \end{pmatrix},$$

$$\begin{pmatrix} (0,2) & (1,6) & (1,1.9) \\ (1.6,1) & (1.6,0.8) & (1.2,1.2) \end{pmatrix}, \begin{pmatrix} (0,4) & (0,3) & (1.8,2.4) \\ (2,0) & (3.2,0) & (0,0.6) \end{pmatrix},$$

$$\begin{pmatrix} (1,3) & (0,5) & (1,2) \\ (1.2,0.6) & (1.6,6) & (0.9,0.3) \end{pmatrix},$$

$$\begin{pmatrix} (1,2.1) & (1,0.5) & (0.11,1) \\ (4.2,0.9) & (0.5,6.8) & (3.9,4.2) \end{pmatrix},$$

$$\begin{pmatrix} (1,1) & (1,1) & (0.2,1.5) \\ (0.8,1.6) & (0,6.8) & (2.1,0.9) \end{pmatrix}\} \qquad \text{III}$$

We next find

$$A \times_n B = \{\begin{pmatrix} (0,2.2) & (0,2.5) & (0.099,0.24) \\ (4,0) & (0.8,0) & (0,1.26) \end{pmatrix},$$

$$\begin{pmatrix} (0,0) & (0,5) & (0.09,0.84) \\ (1.6,0) & (0,0) & (0,0.27) \end{pmatrix}, \begin{pmatrix} (0,4) & (0,4) & (0.81,1.44) \\ (1,0) & (2.56,0) & (0,0.09) \end{pmatrix}$$

$$\begin{pmatrix} (0,1.1) & (0,0) & (0.001,0.16) \\ (0.8,0.18) & (0,4.8) & (2.7,0) \end{pmatrix},$$

$$\begin{pmatrix} (0,0) & (0,0) & (0.01,0.56) \\ (0.12,0.6) & (0,4.8) & (1.08,0) \end{pmatrix}$$

$$\begin{pmatrix} (0,2) & (0,0) & (0.09,0.96) \\ (0.2,0) & (0,0) & (0,0) \end{pmatrix}\} \qquad \text{IV}$$

All the four MOD plane matrix subset semigroups are distinct as the four equations I, II, III and IV are distinct.

Hence all the six MOD plane matrix subset special type of topological spaces are distinct.

It is again left as a open conjecture whether the Kakutani's conjecture is true in case of these MOD plane matrix subset special type of topological spaces.

Thus if P(M) = B = {Collection of all subsets from M = {s × t matrices with entries from $R_n(m)$; $2 \leq m < \infty$, $2 \leq s$, $t < \infty$}} be the MOD plane matrix subset collection.

Will the six MOD plane matrix subset special type of topological spaces $B_o = \{B, \cup, \cap\}$, $B^{+}_{\times_n} = \{B, +, \times_n\}$, $B^{+}_{\cup} = \{B, +, \cup\}$, $B^{+}_{\cap} = \{B, +, \cap\}$, $B^{\times_n}_{\cup} = \{B, \times_n, \cup\}$ and $B^{\times_n}_{\cap} = \{B, \times_n, \cap\}$ satisfy the Kakutani's conjecture or which of the six MOD special type of topological spaces satisfy Kakutani's conjecture?

Next we proceed onto describe by examples MOD plane subset polynomial special type of topological spaces.

Example 4.10. Let V = S (P[x]) = {collection of all subsets from P[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in R_n(g)\}\}$ be the MOD plane subset polynomial collection.

We can define four distinct operations on V viz. +, ∪, ∩ and ×.

Let A = {(0, 0.3)x^3 + (1, 3)x + (0.01, 0.05), (8, 0.9)x^5 + (0.2, 0)x^2 + (0.1, 0.1), (1, 1)x^4 + (2, 0.7)x^2 + (0.3, 0.9)} and B = {(1, 0.5)x + (0, 0.05), (4, 2)x^3 + (0.1, 2)x + (0, 0.8)} ∈ V.

We see A ∩ B = {ϕ} I

A ∪ B = {(0, 0.3)x^3 + (1, 3)x + (0.01, 0.05), (8, 0.9)x^5 + (0.2, 0)x^2 + (0.1, 0.1), (1, 1)x^4 + (2, 0.7)x^2 + (0.3, 0.9), (1, 0.5)x + (0, 0.05), (4, 2)x^3 + (0.1, 2)x + (0, 0.8)} II

A + B = {(0, 0.3)x^3 +(2, 3.5)x + (0.01, 0.1), (8, 0.9)x^5 + (0.2, 0) x^2 + (1, 0.5)x + (0.1, 0.15), (1, 1)x^4 + (2, 0.7)x^2 + (1, 0.5)x + (0.3, 0.95), (4, 2.3)x^3 + (1.1, 5)x + (0.01, 0.85), (8, 0.9)x^5 + (4, 2)x^3 + (0.2, 0)x^2 + (0.1, 2)x + (0.1, 0.9), (1, 1)x^4 + (4, 2)x^3 + (2, 0.7)x^2 + (0.1, 2)x + (0.3, 1.7)} III

A × B ={(0, 0.15)x^4 + (1, 1.5)x^2 + (0.01, 0.025)x + (0, 0.015)x^3 + (0, 0.15)x + (0, 0.0025), (8, 0.45)x^6 + (0.2, 0)x^3 + (0.1, 0.05)x + (0, 0.045)x^5 + (0, 0)x^2 + (0, 0.005), (1, 0.5)x^5 + (2, 0.35)x^3 + (0.3, 0.45)x + (0, 0.05)x^4 + (0, 0.035)x^2 + (0, 0.045), (0, 0.6)x^6 + (4, 6)x^4 + (0.04, 0.1)x^3 + (0, 0.6)x^4 + (0.1, 6)x^2 +(0.001, 0.1)x + (0, 0.24)x^3 + (0, 2.4)x + (0, 0.040), (4, 1.8)x^8 + (0.8, 0)x^5 + (0.4, 0.2)x^3 + (0.8, 1.8)x^6 + (0.02, 0)x^3 + (0.01, 0.2)x +(0, 0.72) x^5 + (0, 0)x^2 + (0, 0.8), (4, 2)x^7 + (8, 1.4)x^5 + (1.2, 1.8)x^3 +(0.1, 2)x^5 + (0.2, 1.4)x^3 + (0.03, 1.8)x + (0, 0.8)x^4 + (0, 0.56)x^2 + (0, 0.72)} IV

All the four operations are distinct hence the MOD plane polynomial subset semigroups are distinct. Thus the six special type of topological spaces, V_o ={V, ∪, ∩}, $V_\times^+$ = {V, +, ×}, $V_\cup^+$ = {V, +, ∪}, {V, +, ∩} = $V_\cap^+$, $V_\cup^\times$ = {V, ×, ∪} and $V_\cap^\times$ = {V, ×, ∩} are the six distinct MOD plane polynomial subset special type of topological spaces of infinite order.

These contain infinite number of MOD plane subset topological zero divisors and nilpotents however does not contain MOD plane subset topological idempotents.

Of course V has MOD plane subset strong special type of topological subspaces, however no strong ideals.

Example 4.11. Let W = {P(S[x])} = {collection of all subsets from s[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in R_n(43)\}\}$ be the MOD plane polynomial subset collection.

We can as in the above example define the four operations +, ×, ∪ and ∩ and {W, +}, {W, ×}, {W, ∪} and {W, ∩} are four distinct MOD plane polynomial subset semigroups of infinite order.

Clearly W_o = {W, ∪, ∩}, $W_\times^+$ = {W, +, ×}, {W, +, ∩} = $W_\cup^+$, $W_\cup^\times$ = {W, ×, ∪} and $W_\cap^\times$ = {W, ×, ∩} are the MOD plane polynomial subset special type of topological spaces.

Is the Kakutani' theorem true in the case of the six MOD plane polynomial subset special type of topological spaces?

This sort is study is very new.

In view of this we propose the generalized form of the Kakutani's conjecture in case of MOD plane polynomial subset special type of topological spaces.

Let S = {P(M[x])} = {collection of all subsets from M[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in R_n(m)$; $2 \le m < \infty\}\}$ be the MOD plane polynomial subset collection. {S, +}, {S, ∪}, {S, ∩} and {S, ×} be the MOD plane polynomial subset semigroups.

S_o = {S, ∪, ∩}, $S_{\times}^{+}$ = {S, +, ×}, $S_{\cup}^{+}$ = {S, +, ∪}, $S_{\cap}^{+}$ = {S, +, ∩}, $S_{\cup}^{\times}$ = {S, ×, ∪} and $S_{\cap}^{\times}$ = {S, ×, ∩} be the MOD plane polynomial subset special type of topological spaces.

In which of the six MOD special type of topological polynomial subset spaces the Kakutani's theorem is true.

Next we proceed onto describe by MOD plane subset coefficient polynomial special type of topological spaces by examples.

Example 4.12. Let M = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(R_n(12)\}$ be the MOD plane subset coefficient polynomial collection. We can define +, ×, ∪ and ∩ operations on M.

(It is pertinent to keep on record that if p(x) the coefficient of x^2 term is present and in q(x), the coefficient x^2 term is not present just say for the sake of argument then in case of union operation '∪' we just put the {coefficient as that of x^2} ∪ {ϕ} and get the result however in case of '∩' operation we find {coefficient of x^2} ∩ {ϕ} = ϕ. This is the way ∪ and ∩ intersection operations are performed in M.)

We will illustrate this situation by an example from M.

Let $p_1(x)$ = {(3, 0.1), (2, 1), (0, 5), (4, 0)}x^3 + {(0.3, 0), (0, 2), (0, 0)}x +{(0.12, 0), (0.11, 0.004), (0.002, 0), (1, 1), (3.2, 0)} and $q_1(x)$ = {(1, 1), (2, 3) (0, 0.03)}x^2 +{(0.3, 0), (0, 2), (4, 0.003)}x +{(3, 5), (1, 1), (0.12, 0), (0, 0), (0, 8)} ∈ M.

We show how $p_1(x) \cup q_1(x)$ and $p_1(x) \cap q_1(x)$ are determined.

$p_1(x) \cap q_1(x)$ = ({(3, 0.1), (2, 1), (0, 5), (4, 0)} ∪ φ) x^3 + ({(1, 1), (2,3), (0, 03)} ∪ φ})x^2 +({(0.3, 0), (0, 2), (0,0)} ∪ {(0.3, 0), (0, 2), (4, 0.003)})x + ({(0.12, 0), (0.11, 0.004), (0.002,0), (1, 1), (3.2, 0)} ∪ {(3,5), (1,1), (0.12, 0), (0,0), (0,8)})
= {(3,0.1), (2,1), (0,5), (4,0)}x^3 + {(1,1), (2,3), (0, 0.03)}x^2 +{(0.3, 0), (0, 2), (0, 0), (4, 0.003)}x + {(0.12, 0), (0.11, 0.004), (0.002, 0), (1, 1), (3.2, 0), (3, 5), (0, 0), (0, 8)}.

We now find $p_1(x) \cap q_1(x)$ = ({(3, 0.1), (2, 1), (0, 5), (4, 0)} ∩ φ)x^3 + (({(1, 1), (2, 3), (0, 0.03)} ∩ φ)x^2 +({(0.3, 0), (0, 2), (0, 0)}) ∩ ({(0.3, 0), (0, 2), (4, 0.003)})x + ({(0.12, 0), (0.11, 0.004), (3.2, 0), (0.002, 0), (1, 11)} ∩ {(3, 5), (1, 1), (0.12, 0), (0, 0), (0, 8)} = {φ}x^3 +{φ}x^2 + {(0.3, 0), (0, 2)}x +{(1, 1), (0.12, 0)} = {(0.3, 0), (0, 2)}x + {(1, 1), (0.12, 0)}.

This is the way the ∩ and ∪ operations are performed on M).

Let p(x) ={(0, 0.2), (1, 0), (0.1, 0.5)}x^3 +{(1, 6), (0.6, 2)}x +{(0, 1), (0.3, 1)} and q(x) ={(0.1, 0), (0.1, 0.5), (0, 0.2)}x^3 +{(0.7, 2), (1, 6), (0, 1)}x +{(0, 1), (0.7, 0.2)} ∈ M.

p(x) + q(x) ={(0.1, 0.2), (1.1, 0), (0.2, 0.5), (0.2, 0.1), (0.1, 0.7), (1.1, 0.5), (0, 0.4), (1, 0.2)}x^3 + {(1.7, 8), (1.3, 4), (2,0), (1.6, 8), (1, 7), (0.6, 3)}x +{(0, 2), (0.3, 2), (0.7, 1.2), (1, 1.2)} I

p(x) ∪ q(x) = {(0, 0.2), (1, 0), (0.1, 0.5)}x^3 + {(1, 6), (0.6, 2)} x + {(0, 1), (0.3, 1)} ∪ {(0.1, 0), (0.1, 0.5), (0, 0.2)}x^3 + {(0.7, 2), (1, 6), (0, 1)}x +{(0, 1), (0.7, 0.2)} = ({(0, 0.2), (1, 0), (0.1, 0.5)} ∪ {(0.1, 0), (0.1, 0.5), (0, 0.02)})x^3 +({(1, 6), (0.6, 2)} ∪ {(0.7, 2), (1, 6), (0, 1)})x + ({(0, 1), (0.3, 1)} ∪ {(0, 1), (0.7, 0.2)}) = ({(0, 0.2) (1, 0), (0.1, 0.5), (0.1, 0))x^3 +{(1, 6), (0.6, 2), (0.7, 2)}x + {(0, 1), (0.3, 1), (0.7, 0.2)} II

p(x) ∩ q(x) = ({(0, 0.2), (1, 0), (0.1, 0.5)} ∩ {(0.1, 0), (0.1, 0.5), (0, 0.2)})x^3 + ({(1, 6), (0.6, 2)} ∩ {(0.7, 2), (1, 6)}) x + ({(0, 1), (0.3, 1)} ∩ {(0, 1), (0.7, 0.2)}) = {(0, 0.2), (0.1, 0.5)}x^3 +{(1, 6)}x + {(0, 1)} III

We now find p(x) × q(x) = ({(0, 0.2), (1, 0), (0.1, 0.5)} × {(0.1, 0), (0.1, 0.5), (0, 0.2)})x^6 + ({(0, 0.2), (1, 0), (0.1, 0.5)} × {(0.7, 2), (1, 6), (0, 1)})x^4 + ({(0, 0.2), (1,0), (0.1, 0.5)} × {(0, 1), (0.7, 0.2)})x^3 + ({(1, 6), (0.6, 2)} × {(0.1, 0), (0.1,0.5), (0, 0.2)})x^4 + ({(1, 6), (0.6, 2)}× {(0.7, 2), (1, 6), (0.1)})x^2 + ({(1, 6), (0.6, 2)} × {(0, 1), (0.7, 0.2)})x + ({((0, 1), (0.3, 1)} × {(0.1, 0), (0.1, 0.5), (0, 0.2)})x^3 +({(0, 1), (0.3, 1)}× {(0.7, 2), (1, 6), (0, 1)})x + {(0, 1), (0.3, 1)} × {(0, 1), (0.7, 0.2)} = {(0, 0), (0.1, 0), (0.01, 0), (0, 0.1), (0.01, 0.25), (0, 0.04)}x^6 + {(0, 0.4), (0.7, 0), (0.07, 0.1), (0, 1.2), (1, 0), (0.1, 3), (0, 0.2), (0, 0), (0, 0.5)}x^4 + {(0, 0.2), (0, 0), (0, 0.5), (0, 0.04), (0.7, 0), (0.07, 0.1)}x^3 + {(0.1,0), (0.1, 3), (0, 1.2), (0.06, 0), (0.06, 1), (0, 0.4)}x^4 +{(0.7, 0), (1, 0), (0, 6), (0.42, 4), (0.6, 0), (0, 2)}x^2 + {(0, 6), (0.7, 12), (0, 2), (0.42, 0.4)}x + {(0, 0), (0, 0.5), (0, 0.2), (0.03, 0), (0.03, 0.5), (0, 0.2)}x^3 + {(0, 2), (0, 6), (0, 1), (0.21, 2), (0.3, 6)}x + {(0, 1), (0, 0.2), (0.21, 0.2)} IV

All the four equations I, II, III and IV are distinct.

All the four values are distinct so {M, ∪}, {M, ∩}, {M, ×} and {M, ×} are the MOD plane subset coefficient polynomial semigroups.

Hence the six MOD plane subset coefficient polynomial special type of topological spaces M_o = {M, ∪, ∩}, $M_{\times}^{+}$ = {M,

$+, \times\}$, $M_{\cup}^{+} = \{M, +, \cup\}$, $M_{\cup}^{\times} = \{M, \times, \cup\}$, $M_{\cap}^{+} = \{M, +, \cap\}$ and $M_{\cap}^{\times} = \{M, \times, \cap\}$ are distinct.

It is a open conjecture to verify the validity of the Kakutani's theorem in case of this M.

Example 4.13. Let $S = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(R_n(11))\}$ be the MOD plane subset coefficient polynomial collection. $\{S, +\}$, $\{S, \cup\}$, $\{S, \cap\}$ and $\{S, \times\}$ be the MOD plane subset coefficient semigroup. $S_o = \{S, \cup, \cap\}$, $S_{\cup}^{+} = \{S, +, \cup\}$, $S_{\cap}^{+} = \{S, +, \cap\}$, $S_{\times}^{+} = \{S, +, \times\}$, $S_{\cup}^{\times} = \{S, \times, \cup\}$ and $S_{\cap}^{\times} = \{S, \times, \cap\}$ are the MOD plane subset coefficient polynomial special type of topological spaces all of which are distinct.

It is a challenging problem to study whether the Kakutani's theorem is true in case of all these six spaces or only some of them.

We propose the following conjecture.

Conjecture 4.7. Let $S = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(R_n(m)), 2 \leq m < \infty\}$ be the MOD plane subset coefficient polynomial collection.

Which of the six special type of topological spaces using MOD plane subset coefficient polynomials, $S_o = \{S, \cup, \cap\}$, $S_{\cup}^{+} = \{S, +, \times\}$, $S_{\cup}^{+} = \{S, +, \cup\}$, $S_{\cap}^{+} = \{S, +, \cap\}$, $S_{\cup}^{\times} = \{S, \times, \cup\}$ and $S_{\cap}^{\times} = \{S, \times, \cap\}$ satisfy Kakutani's theorem?

This is not only a challenging and difficult one.

Thus it is not easy to find a solution however can atleast try to prove Kakutani's theorem is not true in these cases.

Next we provide examples of MOD plane polynomial subsets of finite degree.

Example 4.14. Let S = P($M[x]_{12}$) = {collection of all subsets from $M[x]_{12}$ = $\{\sum_{i=0}^{13} a_i x^i$ / $a_i \in R_n(20)$; $x^{13} = 1\}\}$ be the MOD plane finite degree polynomial subset collection.

We can have four operation +, ∪, ∩ and × on S and are distinct. We just indicate the situation.

Let A ={$(0.7, 0)x^3 + (2, 0.01)x + (0, 3)$, $(1, 0.3)x^2 + (1, 0.001)$, $(1, 0.1)x + (0.2, 0)$} and B = {$(1,0.1)x + (0.2, 0)$, $(0.7, 2)x^4 + (0.3, 1)x^2 + (0.1, 0.2)$} ∈ S.

A ∪ B = {$(0.7, 0)x^3 + (2, 0.01)x + (0, 3)$, $(1, 0.3)x^2 + (1, 0.001)$, $(1, 0.1)x + (0.2, 0)$, $(0.7, 2)x^4 + (0.3, 1)x^2 + (0.1, 0.2)$} I

A ∩ B = {$(1, 0.1)x + (0.2, 0)$} II

A + B = {$(0.7, 0)x^3 +(3, 0.11)x + (0.2, 3)$, $(1, 0.3)x^2 + (1, 0.1)x + (1.2, 0.001)$, $(2, 0.2)x + (0.4, 0)$, $(0.7, 2)x^4 + (0.7, 0)x^3 + (0.3, 1)x^2 + (2, 0.01)x + (0.1, 3.2)$, $(0.7, 2)x^4 +(1.3, 1.3)x^2 + (1.1, 0.201)$, $(0.7,2)x^4 + (0.3, 1)x^2 + (1,0.1)x +(0.3, 0.2)$} III

A × B ={$(0.7, 0)x^4 + (2, 0.001)x^2 + (0, 0.3)x + (0.14, 0)x^3 + (0.4, 0)x$, $(1, 0.03)x^3 + (1,0.0001)x + (0.2, 0)x^2 + (0.2, 0.001)$, $(1, 0.01)x^2 + (0.4, 0)x + (0.04, 0)$, $(0.49,0)x^7 + (1.4, 0.02)x^5 + (0, 6)x^4 + (0.21, 0)x^5 + (0.6, 0.01)x^3 + (0, 3)x^2 + (0.07, 0)x^3 + (0.2,0.002)x + (0, 0.6)$, $(0.7, 0.6)x^6 + (0.7, 0.002)x^4 + (0.3, 0.3)x^4 + (0.3, 0.001)x^2 + (0.1, 0.06)x^2 + (0.1, 0.002)$, $(0.7, 0.2)x^5 + (0.14, 0)x^4 + (0.3, 0.1)x^3 + (0.06, 0)x^2 + (0.1, 0.02)x + (0.02, 0)$} IV

All the four operations are distinct hence the MOD plane finite degree polynomial subset semigroups {S, +}, {S, ∪}, {S,

$\cap\}$ and $\{S, \times\}$ are distinct. Hence the MOD plane special type of topological spaces $S_o = \{S, \cup, \cap\}$, $S_\times^+ = \{S, \times, +\}$, $S_\cup^+ = \{S, +, \cup\}$, $S_\cap^+ = \{S, \cap, +\}$, $S_\cup^\times = \{S, \times, \cup\}$ and $S_\cap^\times = \{S, \times, \cap\}$ are all distinct.

However one is not in a position to predict whether the Kakutani's theorem is true or not in these cases.

Example 4.15. Let $M = \{P(S[M]_{10})\}$ = {collection of all subsets from $S[M]_{10} = \{\sum_{i=0}^{10} a_i x^i \,/\, a_i \in R_n(7); x^{11} = 1\}\}$ be the MOD plane finite degree polynomial subset collection $\{M, +\}$, $(M, \cup\}$, $\{M, \cap\}$ and $\{M, \times\}$ are the MOD plane finite degree polynomial subset semigroups.

Let $M_o = \{M, \cup, \cap\}$, $\{M, +, \times\} = M_\times^+$, $M_\cup^+ = \{M, +, \cup\}$, $M_\cap^+ = \{M, +, \cap\}$, $M_\cup^\times = \{M, \times, \cup\}$ and $M_\cap^\times = \{M, \times, \cap\}$ be the MOD plane finite degree polynomial subset special type of topological spaces.

Verify Kakutani's theorem for these spaces.

In view of all these we suggest the following conjecture.

Conjecture 4.8. Let $W = \{P(S[x]_t)\}$ = {collection of all subsets from $S[x]_t = \{\sum_{i=0}^{t} a_i x^i \,/\, a_i \in R_n(m); x^{t+1} = 1; 2 \leq t < \infty, 2 \leq m < \infty\}$ be the MOD plane finite degree polynomial subset collection. $\{W, \cup\}$, $\{W, \cap\}$, $\{W, +\}$ and $\{W, \times\}$ be the MOD plane finite degree polynomial subset semigroup.

$W_o = \{W, \cup, \cap\}$, $W_\times^+ = \{W, \times, +\}$, $W_\cup^+ = \{W, +, \cup\}$, $W_\cap^+ = \{W, +, \cap\}$, $W_\cup^\times = \{W, \times, \cup\}$ and $W_\cap^\times = \{W, \times, \cap\}$ be the

MOD plane finite degree polynomial subset special type of topological spaces.

Verify the validity of Kakutani's theorem in case of these six spaces.

Next we proceed onto describe by examples the MOD plane subset coefficient finite degree polynomial special type of topological spaces.

Example 4.16. Let V = $S[x]_{19} = \{\sum_{i=0}^{19} a_i x^i$ / $a_i \in P(R_n(8))$; $x^{20} = 1\}$ be the collection of all MOD plane subset coefficient finite degree polynomial collection. On V we can define the four operations $\cup$, $\cap$, + and $\times$ and all the four operations are distinct.

Let p(x) = $\{(0, 0.2), (4, 0.6), (7, 1), (1, 1)\}x^3 + \{(6, 0.1), (0, 0), (0.3, 1)\}x^2 + \{(0.7, 0), (1, 1), (6, 0.6)\}x + \{(0, 3), (7, 0.5), (1, 0.1)\}$ and q(x) =$\{(2, 0.1), (1, 1), (0.4, 1), (1, 0)\}x^3 + \{(0.7, 0), (1, 1), (3, 0.3), (1, 2), (6, 0.4)\}x + \{0, 3), (7, 0.5), (1, 0.1), (2, 3), (0.001, 0.2)\} \in$ V.

We fine p(x) + q(x), p(x) $\cap$ q(x), p(x) $\cup$ q(x) and p(x) $\times$ q(x).

p(x) + q(x) = $\{(2, 0.3), (6, 0.7), (1, 1.1), (3, 1.1), (1, 1.2), (5, 1.6), (0, 2), (2, 2), (0.4, 1.2), (4.4, 1.6), (7.4, 2), (1.4, 2), (1, 0.2), (5, 0.6), (0, 1), (2, 1)\}x^3 + \{(6, 0.1), (0, 0), (0.3, 1)\}x^2 + \{(1.4,0), (1.7,1), (6.7, 0.6), (2, 2), (7, 1.6), (3.7, 0.3), (4, 1.3), (1, 0.9), (1.7, 2), (2, 3), (7, 2.6), (6.7, 0.4), (7, 1.4), (4, 1)\}x + \{(0, 6), (7, 3.5), (1, 3.1), (7, 3.5), (6, 1), (0, 0.6), (1, 3.1), (0, 0.6), (2, 0.2), (2, 6), (1, 3.5), (3, 3.1), (7.001, 0.7), (1.001, 0.3), (0.001, 3.2)\}$ I

p(x) $\cup$ q(x) = $\{(0, 0.2), (4, 0.6), (7, 1), (1.1), (2.01), (0.4, 1), (1, 0)\}x^3 + \{(6, 0.1), (0, 0), (0.3, 1)\}x^2 + \{(0.7, 0), (1, 1), (6,$

0.6), (3, 0.3), (1, 2), (6, 0.4)}x + {0, 3), (7, 0.5), (1, 0.1), (2, 3), (0.001, 2)} II

p(x) $\cap$ q(x) = {(1,1)}x^3 + {ϕ}x^2 + {(0.7, 0), (1, 1)}x + {(0, 3), (7, 0.5), (1, 0.1)} III

p(x) $\times$ q(x) = {(0, 0.02), (1, 0.06), (6, 0.1), (2, 0.1), (0, 0.2), (4, 0.6), (7, 1), (1, 1), (0, 0.2), (1.6, 0.6), (2.8, 1), (0.4, 1), (0, 0), (4, 0), (7, 0), (1, 0)}x^6 + {(4, 0.01), (0,0), (0.6, 0.1), (6, 0.1), (0.3, 1), (2.4, 0.1), (0.12, 1), (6, 0), (0.3, 0)}x^5 +{(1.4, 0), (2, 0.1), (4, 0.06), (0.7, 0), (1, 1), (6, 0.6), (0.28, 0), (0.4, 1), (2.4, 0.6), (0.7, 0), (1, 0), (6, 0)}x^4 + {(0, 0.3), (6, 0.05), (2, 0.01), (0, 3), (7, 0.5), (1, 0.1), (2.8, 0.5), (0.4, 0.1), (0, 0), (7, 0.5), (1, 0)}x^3 + {(0, 0), (2.8, 0), (4.9, 0), (0.7, 0), (0, 0.2), (4, 0.6), (7, 1), (1 1), (0, 0.06), (4, 0.18), (5, 0.3), (3, 0.3), (0, 0.4), (4, 1.2), (7, 2), (1, 2), (0, 0.08), (0, 0.24), (2, 0.4), (6, 0.4)}x^4 + {(4.2, 0), (0, 0), (0.21, 0), (6, 0.1), (0.3, 1), (2, 0.03), (0.9, 0.3)}, (6, 0.2), (0.3, 2), (4, 0.04), (1.8, 0.4)}x^3 + {(0.49, 0), (0.7, 0), (4.2, 0), (0.7, 0), (1, 1), (6, 0.6), (2.1, 0), (3, 0.3), (2, 0.18), (1, 2), (6, 1.2), (6, 0.4), (4.2, 0), (4, 0.24)}x^2 + {(0, 0), (4.9, 0), (0.7, 0), (0, 3), (7, 0.5), (1, 0.1), (2.1, 0), (0, 0.9), (5, 0.15), (0, 6), (7, 1), (1, 0.2), (0, 0.12), (2, 0.2), (6, 0.04)}x + {(0, 0.6), (0, 1.8), (0, 3), (0, 0.1), (4, 0.3), (0, 1.8), (1, 0.5), (7, 0.5), (0, 0.02), (4, 0.06), (2, 3), (7, 0.1), (1, 0.1), (0, 0.6), (6, 3), (0,0.04), (0.004, 0.12), (0.007, 0.2), (0.001, 0.2)}x^3 +{(0, 0.3), (0, 0), (0, 3), (2, 0.05), (2.1, 0.5), (6, 0.01), (0.3, 0.1), (4, 0.3), (0.6, 3), (0.006, 0.02), (0.006, 0.12)}x^2 + {(0, 0), (0, 3), (0, 0.18), (4.9, 0), (7, 0.5), (2, 0.3), (0.7, 0), (1, 0.1), (6, 0.06), (6, 0), (2, 3), (4, 1.8), (0.007, 0), (0.001, 0.2), (0.006, 0.12)}x +{(0, 9), (0,1.5), (0, 0.3), (1, 0.25), (7, 0.05), (0, 0.3), (7, 0.05), (1, 0.01), (0, 1), (6, 1.5), (2, 0.3)} IV

We see all the four equations are distinct so the four MOD plane subset coefficient polynomial semigroups, are {V, +}, {V, $\cap$}, {V, $\cap$} and {V, $\times$} are distinct.

Thus the MOD plane subset coefficient polynomial special type of topological spaces;

$V_o = \{V, \cup, \cap\}$, $V_{\times}^{+} = \{V, +, \times\}$, $V_{\cup}^{+} = \{V, +, \cup\}$, $V_{\cap}^{+} = \{V, +, \cap\}$, $V_{\cup}^{\times} = \{V, \times, \cup\}$ and $V_{\cap}^{\times} = \{V, \times, \cap\}$ are all distinct.

Now the validity whether these six spaces satisfy Kakutani's theorem is challenging work. This is left as a open conjecture for the reader.

Example 4.17. Let $M = \{\sum_{i=0}^{9} a_i x^i / a_i \in (R_n(10)), x^{10} = 1\}$ be the MOD plane finite degree polynomial subset coefficient collection $\{M, \cup\}$, $\{M, \cap\}$, $\{M, +\}$ and $\{M, \times\}$ be the MOD plane finite degree polynomial subset coefficient semigroup.

$M_o = \{M, \cup, \cap\}$, $M_{\times}^{+} = \{M, +, \times\}$, $M_{\cup}^{\times} = \{M, \times, \cup\}$, $M_{\cap}^{\times} = \{M, \times, \cap\}$, $M_{\cup}^{+} = \{M, +, \cup\}$ and $M_{\cap}^{+} = \{M, +, \cap\}$ be the MOD plane finite degree polynomial with subset coefficient special type of topological spaces.

Study the validity of Kakutani's theorem in case of these six special types of topological spaces.

In view of this we propose the following conjecture.

Conjecture 4.9. Let

$$S = \{\sum_{i=0}^{t} a_i x^i \ / a_i \in P(R_n(m)), 2 \le t, m < \infty, x^{t+1} = 1\}$$

be the MOD plane finite degree polynomial with subset coefficients from $R_n(m)$; $S_o = \{S, \cup, \cap\}$, $S_{\times}^{+} = \{S, +, \times\}$, $S_{\cup}^{+} = \{S, +, \cup\}$, $S_{\cap}^{+} = \{S, +, \cap\}$, $S_{\cup}^{+} = \{S, \cup, \times\}$ and $S_{\cap}^{\times} = \{S, \cup, \times\}$ be the MOD plane finite degree polynomial subset coefficient special type of topological spaces.

Prove or disprove the validity of Kakutani's theorem in the case of the above special type of topological spaces.

However in this chapter we proceed onto discuss how far Kakutani's theorem can be studied in case of real neutrosophic space $\langle R \cup I\rangle = \{a + bI / a, b \in R, I^2 = I\}$, the real dual number plane $\langle R \cup g\rangle = \{a + bg / a, b \in R; g^2 = 0\}$, the real special dual like number space $\langle R \cup h\rangle = \{a + bh / a, b \in R, h^2 = h\}$ and the real special quasi dual number space $\langle R \cup k\rangle = \{a + bk / a, b \in R, k^2 = -k\}$.

All these four spaces are not like the real planes. These are planes which enjoy different kind of properties. However in all these planes if some one wishes to define a meaningful function or map f they must undoubtedly have at least a fixed point given by f(I) = I in case of real neutrosophic plane, f(g) = g is case of real dual number plane, f(h) = h in case of special dual like number plane and f(k) = k is case of real special quasi dual number plane.

Thus all these are partially real planes or to be more technical semireal planes. Subset special type of topological spaces have been defined using them [ ] now our contention is can we have the modified form of Kakutani's theorem or to be more specific will Kakutani's theorem be true on these spaces unconditionally.

Study in this direction is new for all these spaces demand a fixed point theorem for any nonempty subset S of

$$\underbrace{\langle R \cup I\rangle \times \ldots \times \langle R \cup I\rangle}_{\text{n times}}$$

any set valued function $\phi : S \to 2^S$ then will $I \in S$ be a fixed point of ϕ? (for any arbitrary subset S of $(\langle R \cup I\rangle)^n$.

Similar conjecture in case of $\langle R \cup g\rangle$, $\langle R \cup h\rangle$ and $\langle R \cup k\rangle$. However these are also planes how best is the Kakutani's fixed point theorem true in case of special type of topological spaces built using these neutrosophic plane $\langle R \cup I\rangle$, the dual number plane $\langle R \cup g\rangle$, the special dual like number plane $\langle R \cup h\rangle$ and the special quasi dual number plane $\langle R \cup k\rangle$.

Conjecture 4.10. Let R the reals in Kakutani's fixed point theorem be replaced by $\langle R \cup I\rangle$ or $\langle R \cup g\rangle$ or $\langle R \cup h\rangle$ or $\langle R \cup k\rangle$. Can Kakutani fixed point theorem hold in these four cases?

Conjecture 4.11. Let special subset topological spaces be constructed using $\langle R \cup I\rangle$ or $\langle R \cup k\rangle$ or $\langle R \cup h\rangle$ or $\langle R \cup g\rangle$; will Kakutani's theorem be true on these topological spaces?

In authors view the proof of the Kakutani's theorem in case of $\langle R \cup g\rangle$, $\langle R \cup I\rangle$, $\langle R \cup k\rangle$ and $\langle R \cup h\rangle$ happen to be a routine task with appropriate modifications by using only classical or usual topology on $\langle R \cup g\rangle$, $\langle R \cup I\rangle$, $\langle R \cup k\rangle$ and $\langle R \cup h\rangle$. However in case of special type of subset topological space the theorem happens to be difficult.

Next we proceed onto study the validity of Kakutani's fixed point theorem in case of the section of real plane $R_n(m)$; $2 \leq m < \infty$ where $R_n(m) = \{(a, b) / b, a \in [0, m)\}$. This is discussed earlier.

Now the notion of MOD plane topological spaces of special type is discussed for the validity of Kakutani's theorem.

Now in earlier chapter we have introduced MOD interval neutrosophic special type of topological spaces using $\langle [0, n) \cup I\rangle = \{a + bI / a, b \in [0, n)\}$, $2 \leq n < \infty$, MOD interval; dual number special type of topological spaces using $\langle [0, n) \cup g\rangle = \{a + bg / a, b \in [0, n), g^2 = 0\}$; $2 \leq n < \infty$ and MOD interval special dual like number special type of topological spaces using $\langle [0, n) \cup h\rangle = \{a + bh / a, b \in [0, n); h^2 = h\}$; $2 \leq n < \infty$, MOD interval quasi dual number special type of topological spaces using

$$\langle [0, n) \cup k\rangle = \{a + bk / a, b \in [0, n), k^2 = (n - 1) k\}.$$

This study is interesting and innovative for the MOD interval is a segment of the specific planes cut from the first quadrant of the respective planes.

Infact it is a misnomer we call it as MOD interval for the planes are sketched below for the reader to make a comparative study.

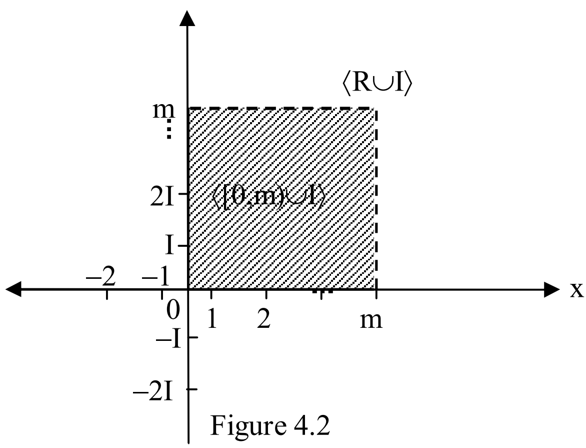

Figure 4.2

Shaded region is the MOD neutrosophic interval or the MOD neutrosophic plane.

$$\langle R \cup I\rangle = \{a + bI / a, b \in R, I^2 = I\}$$

$$\langle [0, m) \cup I\rangle = \{a + bI / a, b \in [0, m); 2 \leq m < \infty, I^2 = I\}$$

Thus $\langle [0, m) \cup I\rangle \subseteq \langle R \cup I\rangle$ and it exactly occupies part of the first quadrant of the neutrosophic plane $\langle R \cup I\rangle$.

Similarly

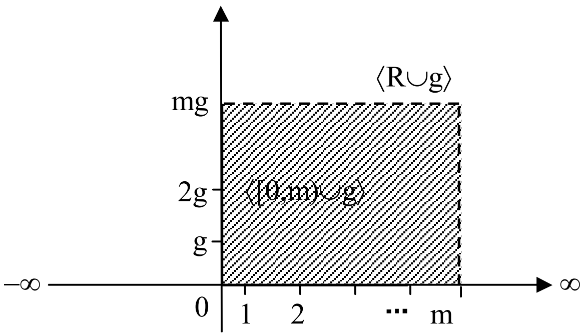

Figure 4.3

The dual number plane is $\langle R \cup g\rangle = \{a + bg / a, b \in R, g^2 = 0\}$ and the MOD dual number interval is $\langle [0, m) \cup g\rangle = \{a + bg / a, b \in [0, m), 2 \leq m < \infty, g^2 = 0\}$ is the MOD dual number plane shown by the shaded region.Clearly $\langle [0, m) \cup g\rangle \subseteq \langle R \cup g\rangle$, thus the MOD dual number plane is a subset (subplane) of $\langle R \cup g\rangle$ occupying some section of the first quadrant of $\langle R \cup g\rangle$.

Now consider the real special dual like number plane given by the following $\langle R \cup h\rangle = \{a + bh / a, b \in R, h^2 = h\}$

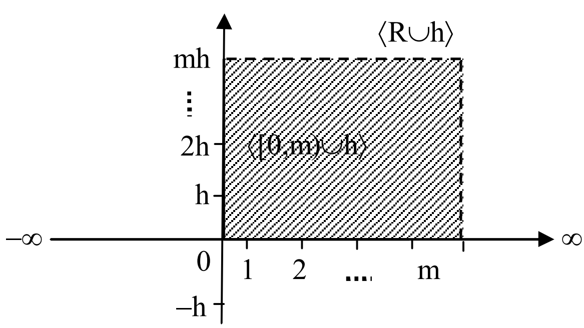

Figure 4.4

Now the MOD special dual like number interval / plane is $\langle[0, m) \cup h\rangle = \{a + bh / a, b \in [0, m), h^2 = h\}$ is shown by the shaded region.

Infact $\langle[0, m) \cup h\rangle \subseteq \langle R \cup h\rangle$ thus $\langle[0, m) \cup h\rangle$ is a subplane of $\langle R \cup h\rangle$.

Similarly $\langle R \cup k\rangle = \{a + bk /a, b \in R; k^2 = -k\}$ be the real special quasi dual plane. $\langle[0, m) \cup k\rangle =\{a + bk / a, b \in [0, m), k^2 = (m - 1)k\}$ be the MOD special quasi dual number plane.

The representation of them is as follows.

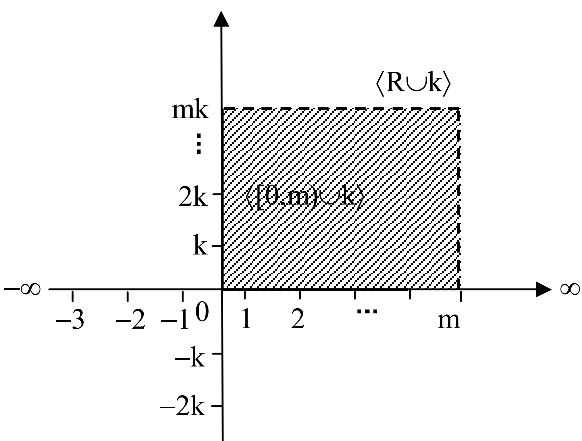

Figure 4.5

Here also it is seen the plane $\langle R \cup k\rangle$ contains the MOD special quasi dual number plane $\langle[0, m) \cup k\rangle$; that is shown by the shaded area in the figure. Thus $\langle[0, m) \cup k\rangle \subseteq \langle R \cup k\rangle$.

Finally we see $C([0, m)) = C(m) = \{a + bi_F / a,b \in [0, m), i_F^2 = (m - 1)\}$ is the MOD interval finite complex numbers. Clearly this also is a substructure but it is not extendable as in case of $i^2 = -1$ and $i_F^2 = (m - 1)$ with limitations infact all containment has limitations for $(m - 1)^2 = 1 \pmod m$ in all cases which is untrue in R.

Thus it is only as subset for none of the algebraic properties or analytic properties are true in this case. Yet with these limitations we have the following containment relation

$$C([0,n)) = C(n) \subseteq C = \{a + ib / i^2 = -1\ a, b \in R\}$$

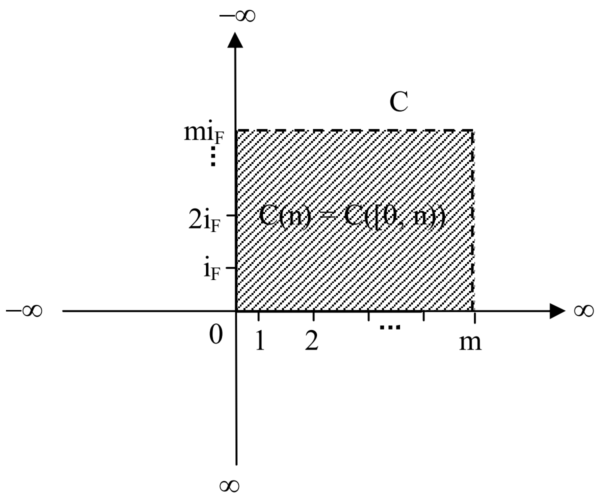

Figure 4.6

Thus all the four planes are only contained as respective subset. In case of complex plane and MOD finite complex number plane even this sort of containment is not true for we have to identify i with i_F that is $i \mapsto i_F$ then only such result is true a little deviant from other three planes for we see in case of real special quasi dual number plane, $k^2 = -k$ and in MOD special quasi dual number plane $\langle [0, m) \cup k \rangle$, $k^2 = (m-1)k$.

Now having seen the relative structure it is pertinent to keep on record there is lots of difference in their algebraic structures for the product on them can give zero divisors.

Thus with these deviations in mind only we build the topological structures on them.

We see the MOD interval topological spaces built using S = $\{\langle[0, n) \cup g\rangle\}$, H = $\{\langle[0, n) \cup h\rangle\}$, J = $\{\langle[0, n) \cup k\rangle\}$, L = $\{C([0,n))\}$ and M = $\{\langle[0, n) \cup I\rangle\}$ be the five MOD interval / planes and let A = P(S), B = P(H), C = P(J), D = P(L) and E = P(M) be the collection of all subsets of the five MOD interval planes respectively.

We can have the six types of MOD plane subset topological spaces built on each of them.

It is conjectured which of these MOD plane special type of topological spaces satisfy the Kakutani fixed point theorem?

So study in this direction is suggested as open conjectures as one is not in a position to know the properties of these MOD plane / interval spaces as well as planes built on $\langle R \cup I\rangle$, $\langle R \cup g\rangle$, $\langle R \cup h\rangle$ and $\langle R \cup k\rangle$.

Similarly study or adaptation of Kakutani's fixed point theorem in case of MOD finite complex number interval / planes using P(C[0, n)) happens to be a open conjecture.

Thus all the spaces whether in the first place satisfy the norms of a Kakutani's theorem is to be analysed. Further a function yielding convex graph is not feasible in these cases for even functions yielding simple smooth curves happen to become discontinuous in MOD planes, so study in this direction is difficult.

Next we study the properties of finite topological spaces built using MOD natural neutrosophic sets viz; Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup h\rangle_I$ and $\langle Z_n \cup k\rangle_I$.

We can find the MOD natural neutrosophic subset special type of topological spaces got from $P(Z_n^I)$, $P(C^I(Z_n))$, $P(\langle Z_n \cup I\rangle_I)$, $P(\langle Z_n \cup g\rangle_I$, $P(\langle Z_n \cup h\rangle_I)$ and $P(\langle Z_n \cup k\rangle_I)$. Such spaces have been introduced earlier. The following is conjecture is introduced.

Conjecture 4.12. Let $A = P(Z_n^I)$, $B = P(C^I(Z_n))$, $C = P((\langle Z_n \cup I\rangle_I))$ $D = P(\langle Z_n \cup g\rangle_I)$, $E = P(\langle Z_n \cup h\rangle_I)$ and $F = P(\langle Z_n \cup k\rangle_I)$ be the MOD natural neutrosophic collection of subsets.

Let the MOD natural neutrosophic subset special type of topological spaces be built on A, B, C, D, E and F.

Is Kakutani fixed point theorem true in these cases?

Next we see one example to illustrate how in these spaces one should always have certain elements to be fixed.

Example 4.18. Let $V = P(Z_4^I) = \{P(\{0, 1, 2, 3, I_0^4, I_2^4, I_2^4 + I_0^4, I_2^4 + 1, I_2^4 + 2, I_2^4 + 3, I_0^4 + 1, I_0^4 + 2, I_0^4 + 3, I_2^4 + I_0^4 + 1, I_2^4 + I_0^4 + 3, I_0^4 + I_2^4 + 2\})\}$ be the MOD natural neutrosophic subset collection.

Here we see I_0^4 cannot be mapped to any element but I_0^4, likewise I_2^4 should be mapped only onto I_2^4.

Now if $V_o = \{V, \cup, \cap\}$ is the MOD natural neutrosophic special type of subset topological space.

1. Is the Kakutani's fixed point theorem true in case of V_o?

2. Is the conditions for Kakutani's theorem true in case of V_o?

Study questions (1) and (2) in case of $V_{\times}^{+} = \{V, +, \times\}$, $V_{\cup}^{+} = \{V, +, \cup\}$, $V_{\cap}^{+} = \{V, +, \cap\}$, $V_{\cup}^{\times} = \{V, \times, \cup\}$ and $V_{\cap}^{\times} = \{V, \times, \cap\}$.

Now we can define on V max and min operation by taking face values. max $\{I_0^4, I_2^4\} = I_2^4$, max $\{I_0^4, a\} = a$ if we choose to keep in our study real value to be dominating so min $\{I_0^4, a\} = I_0^4$.

In a similar fashion if we want the MOD natural neutrosophic value dominating we put max $\{I_0^4, a\} = I_0^4$ and min $\{I_0^4, a\} = a$.

It is upto the reader to fix min $\{2, 3\} = 2$ and max$\{2, 3\} = 3$ using the face value ordering. So under these circumstances $\{V, \text{min}, \text{max}\}$ by face value ordering is a MOD natural neutrosophic subset special type of topological spaces.

Check the validity of the Kakutani's fixed point theorem for this V.

Can we use this to model Nash equilibrium in game theory where one can also say the predictions are not determinable that is an indeterminate or natural neutrosophic element in the prediction of the winner.

For in all cases of MOD natural neutrosophic elements we propose the following problem.

Can the fixed point exist even if other conditions of the Kakutani's theorem is not satisfied by the MOD topological spaces?

Example 4.19. Let V =$\{P(\langle Z_9 \cup g\rangle_I) = \{$collection of all subsets from $\langle Z_9 \cup g\rangle_I = \{a + bg + I_t^g \,/\, a, b \in Z_9, g^2 = 0, t \in Z_9$ is such that t is a nilpotent or a zero divisor or an idempotent in Z_9}} be the MOD natural neutrosophic dual number subsets collection. V_o, $V_{\times}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{+}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be the MOD natural neutrosophic dual number subsets special type of topological space.

Prove or disprove there is a fixed point for these spaces immaterial of the Kakutani other properties being true.

Example 4.20. Let T = {P($\langle Z_{48} \cup h\rangle_I$)} = {collection of all subsets from $\langle Z_{48} \cup h\rangle_I$ = {a + bh + I_t^h / a, b $\in Z_{48}$, h^2 = h, t $\in Z_{48}$ and t is nilpotent or zero-divisor or idempotent of Z_{48}}} be the MOD natural neutrosophic special dual like number subset collection. T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\times}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ be the MOD natural neutrosophic special dual like number subset special type of topological spaces.

The reader is left with the task to find fixed points and prove or disprove the space is connected or compact.

Example 4.21. Let F = {P($\langle Z_{21} \cup k\rangle_I$)} ={collection of all subsets from $\langle Z_{25} \cup k\rangle_I$ = {a + bk + I_t^k / a, b $\in Z_{21}$, k^2 = 20k, t is an idempotent or nilpotent or a zero divisor in Z_{21}} be the MOD natural neutrosophic special quasi dual number subset collection. F_o, $F_{\cup}^{+}$, $F_{\cap}^{+}$, $F_{\times}^{+}$, $F_{\cup}^{\times}$ and $F_{\cap}^{\times}$ be the MOD natural neutrosophic special quasi dual number subset special type of topological spaces.

Can this have Kakutani's fixed point? Prove or disprove these topological spaces are compact / connected.

Example 4.22. Let G = {P($C^I(Z_{24})$)} ={collection of all subsets from $C^I(Z_{24})$ ={a + bi_F + I_t^c / a, b $\in Z_{24}$, i_F^2 = 23, t $\in Z_{24}$ is an idempotent or a zero divisor or a nilpotent element in Z_{24}}} be the MOD natural neutrosophic finite complex number subset collection.

Let G_o, $G_{\times}^{+}$, $G_{\cup}^{+}$, $G_{\cap}^{\times}$, $G_{\cup}^{+}$ and $G_{\cap}^{\times}$ be MOD natural neutrosophic finite complex number subset special type of topological spaces.

Study the properties of Kakutani's theorem for these topological spaces built using G.

Example 4.23. Let H ={P($\langle Z_{40} \cup g\rangle_I$)} = {collection of all subsets from $\langle Z_{40} \cup g\rangle_I$ = {a + bg + I_t^g / a, b $\in Z_{40}$, $g^2 = 0$, t $\in Z_{40}$ is such that t is an idempotent or a zero divisor or a nilpotent}} be the MOD natural neutrosophic dual number subset collection. H_o, $H_{\times}^{+}$, $H_{\cup}^{+}$, $H_{\cap}^{+}$, $H_{\cup}^{\times}$ and $H_{\cap}^{\times}$ be the MOD natural neutrosophic dual number special type of topological spaces.

Verify Kakutani's theorem for these spaces.

Example 4.24. Let I ={P($\langle Z_{12} \cup h\rangle$)} = {collection of all subsets from $\langle Z_{12} \cup h\rangle_I$ ={a + bh + I_t^h / a, b $\in Z_{12}$, $h^2 = h$, t $\in Z_{12}$ is such that t is a nilpotent or an idempotent or a zero divisor}} be the MOD natural neutrosophic special dual like number subset collection. Study whether fixed point can be achieved for the spaces I_o, $I_{\times}^{+}$, $I_{\cup}^{+}$, $I_{\cap}^{+}$, $I_{\cup}^{\times}$ and $I_{\cap}^{\times}$.

Likewise we can study using MOD natural neutrosophic special quasi dual number subsets collection from P($\langle Z_n \cup k\rangle_I$) ={collection of all subsets from $\langle Z_n \cup k\rangle_I$ = {a + bk + I_t^k / a, b $\in Z_n$, $k^2 = (n - 1)k$, t $\in Z_n$ is such that t is a nilpotent or an idempotent or a zero divisor in Z_n.

Study the topological spaces built using D for Kakutani's theorem.

Likewise study for E = P($C^I(Z_n)$) ={collection of subsets from $C^I(Z_n)$ = {a + bi_F + I_t^c / a, b $\in Z_n$, $i_F^2 = (n - 1)$; t in Z_n is such that t is a nilpotent or a zero divisor or an idempotent.

All these study is a matter of routine. Further building matrices with subsets from $P(Z_n^I)$, $P(\langle Z_n \cup g\rangle_I)$, $P(\langle Z_n \cup h\rangle_I)$, $P(\langle Z_n \cup k\rangle_I$, $P(\langle Z_n \cup I\rangle_I$ and $P(C^I(Z_n))$ can be carried out and one can seek for the fixed point by modifying the Kakutani's theorem.

Finally one can also take collection of matrix subsets with entries from Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup h\rangle_I$, $\langle Z_n \cup k\rangle_I$ and $\langle Z_n \cup I\rangle_I$ and obtain fixed points whether other conditions of Kakutani's theorem is true or not.

Likewise one can get MOD natural neutrosophic number special type of topological spaces using subsets of $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(Z_n^I)\}$ or $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(C^I(z_n))\}$ or $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in (\langle [0, n) \cup g\rangle_I)\}$ or $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, n) \cup h\rangle_I)\}$ or $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in P(\langle [0, n) \cup k\rangle_I\}$.

Can we have fixed point ruling out the other properties of Kakutani's theorem?

Next we can study the existence of fixed point with or without the other properties of Kakutani's theorem for the following MOD natural neutrosophic finite degree polynomials with subset quotient special type of topological spaces built using $S = \{\sum_{i=0}^{m} a_i x^i / a_i \in P(Z_n^I), x^{m+1} = 1\}$ or $P(Z_n^I)$ replaced by $P(\langle Z_n \cup I\rangle_I)$ or $P(C^I(Z_n))$ or $P(\langle Z_n \cup g\rangle_I)$ or $P(\langle Z_n \cup h\rangle_I)$ or $P(\langle Z_n \cup k\rangle_I)$.

We can study similarly for the existence of fixed points or modified form of Kakutani's theorem in case of $(S[x]_m) =$

{collection of all subsets from $S[x]_m = \{\sum_{i=0}^{m} a_i x^i \,/\, x^{m+1} = 1, a_i \in$ $P(Z_n^I)\}$ = P the MOD natural neutrosophic special type of finite degree polynomial subsets topological spaces.

We also suggest the study of Z_n^I replaced by $C^I(Z_n)$ or $\langle Z_n \cup k\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup I\rangle_I$. For in all these spaces one can easily prove the existence of fixed points whether the other conditions of Kakutani's theorem is true or not.

Next we carry out the study on MOD interval natural neutrosophic number subsets which will be illustrated by some examples.

Example 4.25. Let S ={P(^{I}[0, 48))} = {collection of all subsets from ^{I}[0, 48) = {a + I_t^{48} / a ∈ [0, 48) and t is an idempotent or nilpotent or a zero divisor in Z_{48}} = collection of all MOD interval natural neutrosophic subsets.

S_o, $S_\times^+$, $S_\cup^+$, $S_\cap^+$, $S_\cup^\times$ and $S_\cap^\times$ be the MOD interval natural neutrosophic subset special type of topological spaces. Prove these have MOD natural neutrosophic fixed points even if the other properties of Kakutani's theorem is not true.

Example 4.26. Let R = {P(C^I([0, 24))} = {collection of all subsets from C^I([0, 24)) ={a + bi_F + I_t^c / a, b ∈ [0, 24), i_F^2 = 23, t ∈ Z_{24} is an idempotent or a nilpotent or an idempotent in Z_{24}} be the MOD interval natural neutrosophic finite complex number subsets collection.

R_o, $R_\cup^+$, $R_\cap^+$, $R_\times^+$, $R_\cup^\times$ and $R_\cap^\times$ be the MOD interval natural neutrosophic finite complex subsets special type of topological spaces.

Prove these topological spaces have fixed point.

Further prove or disprove these spaces satisfy the other properties of Kakutani's theorem. Similar study can be done using $P(\langle[0, n) \cup g\rangle_I)$ or $P(\langle[0, n) \cup h\rangle_I)$ or $P(\langle[0, n) \cup k\rangle_I)$ or $P(\langle[0, n) \cup I\rangle_I)$ as MOD interval neutrosophic subset collections their respective topological spaces can be analysed for fixed point as well as about the validity of Kakutani's theorem.

Example 4.27. Let B = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} / a_i \in P(C^I([0, 20)))\right.$ = {collection of all subsets from $C^I([0, 20)) = \{a + bi_F + I_t^c / a, b \in [0, 20), i_F^2 = 19, t \in Z_{20}$ is an idempotent or a nilpotent or a zero divisor? MOD interval natural neutrosophic finite complex number matrix with subset entries collection.

B_o, $B^+_{\times_n}$, $B^+_\cup$, $B^+_\cap$, $B^{\times_n}_\cup$ and $B^{\times_n}_\cap$ be the MOD interval finite complex number natural neutrosophic matrix with subset entries special type of topological spaces.

Prove these spaces have fixed point and further verify whether all other properties of Kakutani's theorem are true.

Study or prove the existence of fixed point when MOD interval natural neutrosophic spaces are constructed using subsets from $P(\langle[0, n) \cup g\rangle_I)$ or $P(([0, n) \cup h\rangle_I)$ or $P(\langle^I[0, n)\rangle)$ or $P(\langle[0, n) \cup k\rangle_I)$ or $P(\langle[0, n) \cup I\rangle_I)$. Further can for these spaces, all the properties of Kakutani's theorem hold good?

Example 4.28. Let A = P(M) = {collection of all subsets from

$$M = \{\begin{bmatrix} a_1 & a_2 & \dots & a_8 \\ a_9 & a_{10} & \dots & a_{16} \end{bmatrix} / a_i \in \langle [0, 49) \cup h \rangle_I$$

$=\{a + bh + I_t^h /a, b \in [0, 49), h^2 = h, t \in Z_{49}$ is a nilpotent or an idempotent or a zero divisor in Z_{49}, $1 \leq i \leq 16\}\}$ be the MOD interval natural neutrosophic special dual like number matrix subset collection. A_o, $A^{+}_{\times_n}$, $A^{+}_{\cup}$, $A^{+}_{\cap}$, $A^{\times_n}_{\cup}$ and $A^{\times_n}_{\cap}$ be the MOD interval natural neutrosophic special dual like number matrix subset special type of topological spaces.

Prove these spaces have fixed point but however test for the other properties of Kakutani's theorem.

Study the above example by replacing $\langle [0, 49) \cup I \rangle_I$ by ${}^I[0, 29)$ or $\langle [0, 24 \cup g \rangle_I$ or $\langle [0, 15) \cup k \rangle_I$ or $C^I([0, 40))$ for the fixed point of the respective MOD topological spaces and the validity of the Kakutani's theorem.

Example 4.29. Let $V = \{\sum_{i=0}^{m} a_i x^i / a_i \in P(\langle [0, 28) \cup g \rangle_I)\}$ be the MOD interval natural neutrosophic dual number subset coefficient polynomial collection. V_o, $V^{+}_{\times}$, $V^{+}_{\cup}$, $V^{+}_{\cap}$, $V^{\times}_{\cup}$ and $V^{\times}_{\cap}$ be MOD interval natural neutrosophic dual number subset coefficient polynomial special type of topological spaces.

Prove these spaces have fixed point. Test the validity of other properties of Kakutani's theorem satisfied by these spaces.

Study the above example when $P(\langle [0, 28) \cup g \rangle_I)$ is replaced by $P({}^I[0, 43))$ or $P(\langle [0, 142) \cup h \rangle_I)$ or $P(C^I([0, 48))$ or $P(\langle [0, 25) \cup k \rangle_I)$ or$P(\langle [0, 10) \cup I \rangle_I)$ and the related MOD interval natural neutrosophic subset coefficient polynomial topological spaces for fixed point and other properties of Kakutani's theorem.

Example 4.30. Let M = P(S[x]) = {collection of all subsets from $S[x] = \{\sum_{i=0}^{\infty} a_i x^i / a_i \in \langle[0, 11) \cup I\rangle_I\}$ be the MOD interval natural neutrosophic-neutrosophic polynomial subset collection. M_o, $M_{\times}^{+}$, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$ be the MOD interval natural neutrosophic-neutrosophic polynomial subset special type of topological spaces.

Prove all these spaces have fixed point and verify other properties of Kakutani's theorem for these spaces.

In example 4.30 replace $\langle[0, 11) \cup I\rangle_I$ by ${}^{I}[0, 48)$, $\langle[0, 18) \cup g\rangle$, $\langle[0, 45) \cup h\rangle_I$, $\langle[0,12) \cup k\rangle_I$ and $C^I([0, 47))$ and study the respective MOD interval natural neutrosophic polynomial subsets special type of topological spaces for fixed point and other properties of Kakutani's theorem.

Example 4.31. Let W = $\{(S[x]_{10})$ = {collection of all subsets from $S[x]_{10} = \{\sum_{i=0}^{10} a_i x^i / x^{11} = 1, a_i \in \langle[0,12) \cup k\rangle_I\}\}$ be the MOD interval natural neutrosophic special quasi dual number finite degree polynomial subset collection. W_o, $W_{\times}^{+}$, $W_{\cup}^{+}$, $W_{\cap}^{+}$, $W_{\cup}^{\times}$ and $W_{\cap}^{\times}$ be the MOD interval natural neutrosophic special quasi dual number finite degree polynomial subset special type of topological spaces.

Study Kakutani's fixed point theorem for these spaces.

In W of example 4.31, replace $\langle[0, 12) \cup k\rangle_I$ by $C^I([0, 42))$, $\langle[0, 15) \cup g\rangle_I$, $\langle[0, 48) \cup h\rangle_I$, $\langle[0, 18) \cup I\rangle_I$ and study their respective MOD interval natural neutrosophic finite degree polynomial subset special type of topological spaces for Kakutani's theorem.

Example 4.32. Let $N = \{ \sum_{i=0}^{9} a_i x^i / a_i \in P(\langle [0, 27) \cup h\rangle_I); x^{10} = 1\}$ be the MOD neutrosophic special dual like number subset coefficient of finite degree polynomial subset collection. N_o, $N_{\times}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times}$ and $N_{\cap}^{\times}$ be the MOD interval natural neutrosophic special dual like number special type of topological spaces.

Study these spaces for the validity of the Kakutani's theorem.

In the above example replace $P(\langle [0, 27) \cup h\rangle_I$ by $P(^I[0, 48))$, $P(\langle [0, 48) \cup k\rangle_I)$, $P(\langle [0,7) \cup g\rangle_I)$, $P(\langle [0, 14) \cup I\rangle_I)$ and $P(C^I([0,40))$ and study their respective MOD interval natural neutrosophic finite degree polynomials with subset coefficients for the validity of the Kakutani's theorem.

We have suggested several problems for this chapter some of which are at research level.

Further the study of validity of Kakutani's theorem happens to be a difficult research for there are fixed points in MOD topological spaces with or without satisfying the other properties of Kakutani's theorem.

PROBLEMS

1. Obtain all the special properties enjoyed by MOD real planes $R_n(m) = \{[0, m) \times [0, m) = \{(a, b) / a, b \in [0, m), 2 \leq m < \infty\}\}$.

2. Prove $\{R_n(m), +\}$ is an abelian group of infinite order.

3. Prove $\{R_n(m), +\}$ can have both finite and infinite order subgroups.

4. Prove $\{R_n(m), \times\}$ is a MOD plane semigroup under the operation $\times$.

5. Prove $\{R_n(m), \times\}$ has zero divisors which are infinite in number.

6. Prove $\{R_n(m), \times\}$ can have idempotents if and only if Z_m has idempotents.

7. Can $\{R_n(m), \times\}$ have nilpotents? Justify your claim.

8. Prove $\{R_n(m), +, \times\}$ is a MOD plane ring. Study special features associated with it.

9. Let $S = \{R_n(20), +, \times\}$ be the MOD plane ring.

   i) Does S in general satisfy the distributive law?

   ii) Prove $o(S) = \infty$.

   iii) Find all MOD plane zero divisors of S.

   iv) Prove S has MOD plane idempotents.

   v) Prove S has MOD nilpotents.

   vi) Find all finite order subrings of S.

   vii) Prove all ideals of S are of infinite order.

   viii) Obtain any other special feature associated with S.

10. Let M = $\{R_n(19); +, \times\}$ be the MOD plane ring.

    i) Study questions (i) to (vii) of problem (9).

    ii) Distinguish this M from S of problem (9).

11. Let T = $P(R_n(12))$ = {collection of all subsets from $R_n(12)$} be the MOD plane subsets collection.

    i) Prove $\{T, +\}$, $\{T, \times\}$, $\{T, \cup\}$ and $\{T, \cap\}$ are MOD plane semigroups which are distinct.

    ii) Prove $\{T, \times\}$ is a semigroup of infinite order with infinite number of zero divisors.

    iii) Prove $T_o = \{T, \cup, \cap\}$, $T_\times^+ = \{T, +, \times\}$, $T_\cup^+ = \{T, +, \cup\}$, $T_\cap^+ = \{T, +, \cap\}$, $T_\cup^{\times_n} = \{T, \times_n, \cup\}$ and $T_\cap^{\times_n} = \{T, \times_n, \cap\}$ are six distinct MOD plane subset special type of topological spaces.

    iv) Which of these spaces are compact?

    v) Are these spaces connected?

    vi) Can these spaces satisfy the Kakutani's fixed point theorem?

    vii) Prove these MOD plane topological spaces can have subspaces of finite order.

    viii) Prove all MOD plane subset topological ideals are of infinite order.

    ix) Can $T_\times^+$ have MOD plane strong topological subspaces?

x) Prove T can never have MOD plane strong topological ideals?

xi) Prove T can never have MOD plane topological ideals?

xii) Can T have MOD plane topological ¾th strong ideals?

xiii) Can T have MOD plane topological ½ strong ideals?

xiv) Obtain any other special feature associated with T.

12. Let B = {P(R_n(18)} = {collection of all subsets from R_n(18)} be the MOD plane subset collection.

Study questions (i) to (xiii) of problem (11) for this B.

13. Let

$$N = \left\{\begin{pmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \end{pmatrix} \middle/ a_i \in P(R_n(10));\ 1 \leq i \leq 16\right\}$$

be the MOD plane matrix with subset entries from R_n(10).

Study questions (i) to (xiii) of problem (11) for this N with appropriate modifications.

14. Let

$$W = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle/ a_i \in P(R_n(24)),\ 1 \leq i \leq 4\right\}$$

be the MOD plane square matrix with entries from subsets of $P(R_n(24))$.

i) Study questions (i) to (xiii) of problem 11 for this W with appropriate changes.

ii) $\{W, \times\} = L$ be the MOD plane subset entries matrix semigroup.

L is a noncommutative semigroup.

iii) Prove $W_{+}^{\times}$, $W_{\cap}^{\times}$, $W_{\cup}^{\times}$ and $W_{\times_n}^{\times}$ are the four distinct MOD plane subset entries matrix special type of topological spaces which are noncommutative.

iv) Study relevant questions from (i) to (xii) of problem (11) for these four MOD plane subset entries matrix spaces given in (iii).

v) Obtain any other special feature associated with four MOD special types of topological spaces given in question (iii).

15. Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle/ a_i \in P(R_n(48));\ 1 \leq i \leq 16 \right\}$$

be the MOD plane subset entries matrix collection.

i) Study questions (i) to (xiii) of problem (11) for this M with appropriate changes.

ii) Study questions (ii) to (v) of problem (14) for this M.

iii) Obtain any other special feature enjoyed by this M.

16. Let B = P(M) = {collection of all subsets from

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \;/\; a_i \in R_n(24);\ 1 \leq i \leq 5 \right\}\}$$

be the MOD plane matrix subset collection.

i) Prove $\{B, +\}$, $\{B, \times_n\}$, $\{B, \cup\}$ and $\{B, \cap\}$ are MOD plane matrix subset semigroups.

ii) Prove all these four semigroups have MOD subsemigroups of both finite and infinite order.

iii) Which of the MOD semigroups has ideals of infinite order?

iv) Prove $\{B, \cap\}$ has ideals of finite order.

v) Can $\{B, \cup\}$ have ideals of finite or infinite order?

vi) Find all MOD plane zero divisors, MOD plane nilpotents and MOD plane idempotents in $\{B, \times_n\}$.

vii) Prove $\{B, +\}$ is only a MOD plane semigroup.

viii) What are the special features enjoyed by B_o = $\{B, \cup, \cap\}$, $B^{+}_{\times_n}$ = $\{B, +, \times_n\}$, $B^{+}_{\cup}$ = $\{B, +, \cup\}$, $B^{+}_{\cap}$ = $\{B, +, \cap\}$, $B^{\times_n}_{\cup}$ and $B^{\times_n}_{\cap}$ the six MOD plane matrix subset topological spaces. Enumerate all the special features associated with them.

ix) Can we say any of the MOD topological spaces satisfy the Kakutani's fixed point theorem?

x) Which of the MOD topological spaces are compact?

xi) Which of the MOD topological spaces are connected?

xii) Prove B has MOD plane matrix subset special type of topological subspaces which are strong.

xiii) Prove B cannot have MOD plane matrix subset special type of topological space ideals?

xiv) Prove the existence of MOD plane topological zero divisors in B.

xv) Obtain any other special feature enjoyed by B.

17. Let S = {P(T)} = {collection of all subsets from

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle/ a_i \in R(24); 1 \leq i \leq 9 \right\}$$

be the MOD plane matrix subset collection.

i) Study questions (i) to (xv) of problem (16) for this S.

ii) Prove {S, ×} is a MOD plane matrix subset semigroup under the usual product ×.

iii) Prove {S, ×} is non commutative and non associative MOD semigroup.

iv) Prove $S_{+}^{\times}$, $S_{\times_n}^{\times}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ are all MOD plane matrix subset special type of topological spaces which are non commutative.

v) Can any of the four MOD plane matrix subset special type of topological spaces satisfy the Kakutani's fixed point theorem?

vi) Does the four spaces mentioned in question (iii) of the problem (a) connected? (b) compact?

vii) Obtain all special features satisfied by these MOD plane non commutative special type of topological spaces.

18. Let W = P(V) = {collection of all subsets from

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \right. / a_i \in R_n(48);$$

$1 \leq i \leq 25$} by the MOD plane matrix subset collection.

i) Study questions (i) to (xv) of problem (16) for this W.

ii) Study questions (ii) to (vii) of problem (17) for this W.

19. Let V = {P(B)} = {collection of all matrix subsets from

$$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \Bigg/ a_i \in R_n(17),\ 1 \leq i \leq 21\}$$ be the

MOD plane matrix subset collection.

Study questions (i) to (xv) of problem (16) for this V.

20. Let T = S[x] = $\{\sum_{i=0}^{\infty} a_i x^i \;/\; a_i \in P(R_n(10))\}$ be the MOD plane subset coefficient polynomial collection.

i) Prove {T, +}, {T, ×}, {T, ∪} and {T, ∩} are MOD plane subset coefficient polynomial semigroups.

ii) Prove T_o, $T_{\times}^{+}$, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ are the four MOD plane subset coefficient polynomial special type of topological spaces.

iii) Do any one of them satisfy the Kakutani's fixed point theorem?

iv) Prove {T, +} and {T, ×} does not contain nontrivial MOD plane idempotents.

v) Are these MOD plane topological spaces (i) compact? (ii) connected?

vi) Find MOD plane subset coefficient polynomial special type of strong topological subspaces.

vii) Prove T cannot have MOD plane subset coefficient polynomial strong topological ideals.

viii) Does T contain MOD plane subset coefficient polynomial ¾ strong topological ideals?

ix) Does T contain MOD plane subset coefficient polynomial ½ strong topological ideals?

x) Characterize all MOD plane subset special type of topological zero divisors and nilpotents.

xi) Prove T cannot contain MOD plane subset special type of topological idempotents in $T_{\times}^{+}$.

xii) Obtain all special features associated with T.

21. Let S = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(R_n(17))\}$ be the MOD plane subset coefficient polynomial collection. Study questions (i) to (xii) of problem (20) for this S.

22. Let T = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in P(R_n(48))\}$ be the MOD plane subset coefficient polynomial collection.

Study questions (i) to (xii) of problem (20) for this T.

23. Let M = {P(S) where S = $\{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in R_n(40)\}$} be the MOD plane polynomial subset collection.

i) Study questions (i) to (xii) of problem (20) for this M.

ii) Enumerate all special features associated with M.

24. Let T = {P(N) where N = $\{\sum_{i=0}^{\infty} a_i x^i / a_i \in R_n(23)\}\}$ be the MOD plane polynomial subset collection.

i) Study questions (i) to (xii) of problem (20) for this T.

ii) Compare this T with M of problem (23).

25. Let S = $\{P(T[M]_{10})) = \{\sum_{i=0}^{10} a_i x^i / a_i \in R_n(24), x^{11} = 1\}\}$ be the MOD plane finite degree polynomial subset collection.

i) Study questions (i) to (xii) of problem (20) for this S.

ii) Distinguish this with problem in which the degree of the polynomial is infinite.

iii) Obtain any other special and striking features associated with this.

26. Let W = $\{P(M[x]_{23})\}$ = {collection of all subsets from $M[x]_{23} = \{\sum_{i=0}^{23} a_i x^i / a_i \in R_n(48); x^{24} = 1\}\}$ be the MOD plane finite degree polynomial subsets.

i) Study questions (i) to (xii) of problem (20) for this W

ii) Compare this W with S of problem (25).

27. Let $M = \{\sum_{i=0}^{8} a_i x^i / a_i \in P(R_n(20)); x^9 = 1\}$ be the MOD plane subset coefficient finite degree polynomials collection.

    i) Study questions (i) to (xii) of problem (20) for this M.

    ii) Compare this M with W of problem (26).

    iii) Obtain all special features enjoyed by this M.

28. Let $B = \{\sum_{i=0}^{12} a_i x^i / a_i \in P(R_n(12)); x^{13} = 1\}$ be the MOD plane subset coefficient finite degree polynomial collection.

    i) Study questions (i) to (xii) of problem (20) for this B.

    ii) Compare this B with M of problem (27).

29. Let $D = \{\sum_{i=0}^{5} a_i x^i / a_i \in P(R_n(9)); x^6 = 1\}$ be the MOD plane finite degree polynomial subset coefficient collection.

    i) Study questions (i) to (xii) of problem (20) for this D.

    ii) Compare this D with B of problem (28).

30. Let $S = \langle R \cup I \rangle = \{a + bI / a, b \in R, I^2 = I\}$ be the real neutrosophic plane. $V = P(S)$ ={collection of all subsets from S}; $\{V, +\}$, $\{V, \times\}$, $\{V, \cap\}$ and $\{V, \cup\}$ be the real

plane neutrosophic semigroups. V_o, $V_{\times}^{+}$, $V_{\cup}^{+}$, $V_{\cap}^{+}$, $V_{\cup}^{\times}$ and $V_{\cap}^{\times}$ be the real neutrosophic topological spaces of special type.

It is left as an open conjecture whether any of these six spaces satisfy the Kakutani's fixed point theorem.

31. Let $\langle R \cup g\rangle = \{a + bg / a, b \in R, g^2 = 0\}$ be the dual number plane. Let $B = P(\langle R \cup g\rangle)$ be the subsets collection of dual numbers.

Can any one of these dual number special type of topological spaces, B_o, $B_{\cup}^{+}$, $B_{\cap}^{+}$, $B_{\times}^{+}$, $B_{\cup}^{\times}$ and $B_{\cap}^{\times}$ satisfy the Kakutani's fixed point theorem.

32. Study problem (31) in which $\langle R \cup g\rangle$ is replaced by $\langle R \cup h\rangle = \{a + bh / a, b \in R, h^2 = h\}$.

33. Study problem (31) in which $\langle R \cup g\rangle$ is replaced by $\langle R \cup k\rangle = \{a + bk / a, b \in R, k^2 = -k\}$.

34. Study Kakutani fixed point theorem for the following MOD interval / plane special type of topological spaces, A, B, C, D and E where;

i) $A = P(\langle [0, n) \cup g\rangle) = \{$collection of subsets from the MOD interval dual numbers$\}$. A_o, $A_{\cup}^{+}$, $A_{\cup}^{+}$, $A_{\cup}^{\times}$, $A_{\cap}^{\times}$ and $_{\times}^{+}$ be the MOD interval / plane dual number special type of subset topological spaces.

ii) $B = P(\langle [0, n) \cup h\rangle = \{$collection of all subsets from the MOD interval / plane special dual like numbers$\}$. B_o, $B_{\times}^{+}$, $B_{\cup}^{+}$, $B_{\cap}^{+}$, $B_{\cup}^{\times}$ and $B_{\cap}^{\times}$ be the MOD interval / plane special dual like number subset special type of topological spaces.

iii) C ={P(⟨[0, n) ∪ k⟩)} = {collection of all subsets from ⟨[0, n) ∪ k⟩ ={a + bk / a, b ∈ [0, n), k^2 = (n – 1)k}} MOD interval / plane special quasi dual number subsets collection. C_o, $C_\times^+$, $C_\cup^+$, $C_\cap^+$, $C_\cup^\times$ and $C_\cap^\times$ be the MOD interval / plane special quasi dual number subset special type of topological spaces built on C.

iv) D ={P(⟨[0, n) ∪ I⟩)} ={collection of all subsets from the MOD interval / plane neutrosophic subsets from ⟨[0, n) ∪ I⟩ = {a + bI / a, b ∈ [0, n), I^2 = I}} be the MOD interval / plane natural neutrosophic subset collection be the MOD interval / plane natural neutrosophic subset special type of topological spaces.

v) E = {P(C([0, n)))} = {collection of all subsets from C([0, n)) = {a + bi_F / a, b ∈ [0, n), i_F^2 = n – 1 be the MOD plane / interval finite complex number collection, E_o, $E_\times^+$, $E_\cup^+$, $E_\cap^+$, $E_\cup^\times$ and $E_\cap^\times$ be the MOD plane / interval finite complex number subset special type of topological spaces. It is to be noted that this problem can be viewed as an open conjecture on MOD special type of topological spaces for the validity of the Kakutani's fixed point theorem.

35. Obtain all special features enjoyed by MOD natural neutrosophic subset semigroups.

36. Let S = P(Z_n^I) = {Collection of all subsets from Z_n^I = {a + I_t^n / a ∈ Z_n and t ∈ Z_n is a zero or a zero divisor or an idempotent or a nilpotent}} be the MOD natural neutrosophic subset collection.

i) Prove {S, +}, {S, ×}, {S, ∩} and {S, ∪} are MOD natural neutrosophic subset semigroups of finite order.

ii) Prove S_o, $S_{\times}^{+}$, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ are MOD natural neutrosophic subset special type of topological spaces of finite order.

iii) Find all MOD natural neutrosophic subset topological zero divisors, idempotents and nilpotents of the six spaces mentioned in (ii).

iv) Are these spaces connected?

v) Are these compact?

vi) Can the modified form Kakutani fixed point theorem be established for this S?

vii) Prove these have MOD natural neutrosophic strong subspaces.

viii) Can these spaces have MOD strong ideals?

ix) Obtain any other special feature associated with this S.

37. Let M = P (Z_9^I) = {collection of subsets from the MOD natural neutrosophic set Z_9^I} be the MOD natural neutrosophic subset collection.

Study questions (i) to (ix) of problem 36 for this M.

38. Let B = P(T) = {collection of all subsets from

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \Big/ a_i \in Z_{12}^I, 1 \le i \le 3 \right\}\}$$ be the MOD natural

neutrosophic matrix subsets. B_o, $B^{+}_{\times_n}$, $B^{+}_{\cup}$, $B^{+}_{\cap}$, $B^{\times_n}_{\cup}$ and $B^{\times_n}_{\cap}$ be the MOD natural neutrosophic matrix subset special type of topological spaces.

Can modified or special form of Kakutani's fixed point theorem be defined and made valid for these 6 spaces. mentioned above?

39. Let M = {collection of all subset matrices $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix}$ /

$a_i \in P(Z^I_{28})$, $1 \leq i \leq 10$} be the MOD natural neutrosophic matrix with entries as subsets collection. M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ be the MOD natural neutrosophic subset matrix special type of topological spaces. Study question (38) for this M.

40. Let

$$W = \{\begin{pmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \end{pmatrix} / a_i \in P(\langle Z_{10} \cup g\rangle_I;\ 1 \leq i \leq 18\}$$

be the MOD natural neutrosophic dual number subset matrix entries collection. Let W_o, $W^{+}_{\times_n}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times_n}_{\cup}$ and $W^{\times_n}_{\cap}$ be the MOD natural neutrosophic dual number subset matrix entries special type of topological spaces.

Study question (38) for these topological spaces constructed using W.

41. Let

$$M = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} / a_i \in P(\langle Z_{48} \cup I\rangle_I)\ 1 \leq i \leq 6\}$$

be the MOD natural neutrosophic neutrosophic subset matrix entries collection. M_o, $M^{+}_{\times_n}$, $M^{+}_{\cup}$, $M^{+}_{\cap}$, $M^{\times_n}_{\cup}$ and $M^{\times_n}_{\cap}$ be the MOD natural neutrosophic-neutrosophic matrix with subset entries special type of topological spaces.Study question (38) for these MOD special topological spaces constructed using M.

42. Let $W = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} / a_i \in P(\langle Z_{40} \cup g\rangle_I);\ 1 \leq i \leq 9\}$

be the MOD natural neutrosophic dual number subset entries matrix collection. W_o, $W^{+}_{\times}$, $W^{+}_{\times_n}$, $W^{+}_{\cup}$, $W^{+}_{\cap}$, $W^{\times}_{\cup}$, $W^{\times}_{\cap}$, $W^{\times_n}_{\cup}$, $W^{\times_n}_{\times}$ and $W^{\times_n}_{\cap}$ be MOD natural neutrosophic dual number subset entries matrix special type of topological spaces.

Study questions (38) for these 10 MOD topological spaces constructed using W.

43. Let $S = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}$ / $a_i \in P(C^I(Z_{12}))$, $1 \leq i \leq 16\}$ be the MOD natural neutrosophic finite complex number subset matrix. Let S_o, $S_{\times}^{+}$, $S_{\times_n}^{+}$, $S_{\times_n}^{\times}$, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$, $S_{\cap}^{\times}$, $S_{\cup}^{\times_n}$ and $S_{\cap}^{\times_n}$ be the MOD natural neutrosophic finite complex number subset matrix special type of topological spaces.

Study question (38) for these spaces.

44. Let $A = \{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P(Z_{12}^I)\}$ be the MOD natural neutrosophic subset coefficient polynomials collection. A_o, $A_{\times}^{+}$, $A_{\cap}^{+}$, $A_{\cup}^{+}$, $A_{\cup}^{\times}$, $A_{\cap}^{\times}$ and $A_{\cap}^{\times}$ be the MOD natural neutrosophic subset coefficient polynomial special type of topological spaces.

Study question (38) for these spaces.

45. Let $B = \{P(M[x])\}$ = {collection of all subsets from $M[x] = \{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in Z_{47}^I\}\}$ be the MOD natural neutrosophic polynomial subset collection. Let B_o, $B_{\times}^{+}$, $B_{\cup}^{+}$, $B_{\cap}^{+}$, $B_{\cup}^{\times}$ and $B_{\cap}^{\times}$ be the MOD natural neutrosophic polynomial subset special type of topological spaces.

Study question (38) for these spaces built using B.

46. Let C = {P(N[x]) = {collection of all subsets from N[x] $= \{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in Z_{42}^I$ }} be the MOD natural neutrosophic polynomial subsets collection. C_o, $C_\times^+$, $C_\cup^+$, $C_\cap^+$, $C_\cup^\times$ and $C_\cap^\times$ be the MOD natural neutrosophic polynomial subsets special type of topological spaces.

Study question (38) for these spaces built using C.

47. Let D ={P(M[x]$_{10}$)} = {collection of all subsets from $M[x]_{10} = \{\sum_{i=0}^{10} a_i x^i$ / $x^{11} = 1$, $a_i \in Z_{16}^I$ }} be the MOD natural neutrosophic finite degree polynomial subset collection. D_o, $D_\times^+$, $D_\cup^+$, $D_\cap^+$, $D_\cup^\times$ and $D_\cap^\times$ be the MOD natural neutrosophic finite degree polynomial subset special type of topological spaces.

Study question (38) for these spaces built using D.

48. Let S = $\{\sum_{i=0}^{8} a_i x^i$ / $a_i \in P(Z_{28}^I)$, $x^9 = 1\}$ be the MOD natural neutrosophic finite degree subset coefficient polynomial collection. S_o, $S_\times^+$, $S_\cup^+$, $S_\cap^+$, $S_\cup^\times$ and $S_\cap^\times$ be the MOD natural neutrosophic subset coefficient finite degree polynomial special type of topological spaces built using S.

Study question (38) for this S.

49. Let H = $\{\sum_{i=0}^{5} a_i x^i$ / $a_i \in P(Z_{19}^I)$, $x^6 = 1\}$ be the MOD natural neutrosophic subset coefficient finite degree polynomial collection. H_o, $H_\cup^+$, $H_\cap^+$, $H_\times^+$, $H_\cup^\times$ and $H_\cap^\times$ be the MOD natural neutrosophic subset coefficient

finite degree polynomial special type of topological spaces built using H.

Study question (38) for this H.

50. Let $K = \{\sum_{i=0}^{\infty} a_i x^i \ / \ a_i \in P(C^I(Z_{12}))\}$ be the MOD natural neutrosophic finite degree complex number subset coefficient polynomial collection. K_o, $K_{\times}^{+}$, $K_{\cup}^{+}$, $K_{\cap}^{+}$, $K_{\cup}^{\times}$ and $K_{\cap}^{\times}$ be the MOD natural neutrosophic finite complex number subset coefficient polynomial special type of topological spaces.

Study question (38) for this K.

51. Study for problem (50) when $P(C^I(Z_{12}))$ is replaced by $P(\langle Z_{17} \cup g\rangle_I)$, $P(\langle Z_{24} \cup h\rangle_I)$, $P(\langle Z_{42} \cup k\rangle_I)$ and $P(\langle Z_{10} \cup I\rangle_I)$ and analyse question (38) and distinguish the special features associated with them.

52. Let $L = \{P(M[x])\}$ ={collection of all subset polynomials from $M[x] = \{\sum_{i=0}^{\infty} a_i x^i \ / \ a_i \in \langle Z_{10} \cup I\rangle_I\}\}$ be the MOD natural neutrosophic-neutrosophic polynomial subset collection. L_o, $L_{\cup}^{+}$, $L_{\cap}^{+}$, $L_{\times}^{+}$, $L_{\cup}^{\times}$ and $L_{\cap}^{\times}$ be the MOD natural neutrosophic-neutrosophic polynomial subset special type of topologicals spaces.

Study questions (38) for this L.

53. Study L in problem (52) when $\langle Z_{10} \cup I\rangle_I$ is replaced by $\langle Z_{48} \cup g\rangle_I$, $\langle Z_{27} \cup h\rangle_I$, $\langle Z_{12} \cup k\rangle_I$, and $C^I(Z_{49})$, and analyse the question (38) for these spaces. Distinguish the special features enjoyed by these spaces.

54. Let $M = \{\sum_{i=0}^{8} a_i x^i$ / $a_i \in P(\langle Z_{19} \cup I\rangle_I)$; $x^9 = 1\}$ be the MOD natural neutrosophic-neutrosophic finite degree subset coefficient polynomial collection. M_o, $M_{\cup}^{+}$, $M_{\cap}^{+}$, $M_{\times}^{+}$, $M_{\cup}^{\times}$ and $M_{\cap}^{\times}$ be the MOD natural neutrosophic-neutrosophic finite degree polynomials with subset coefficient special type of topological spaces.

Study question (38) for this M.

55. Study question (54) if $\langle Z_{19} \cup I\rangle_I$ is replaced by $\langle Z_{18} \cup g\rangle_I$, $\langle Z_{23} \cup g\rangle_I$, $\langle Z_{48} \cup h\rangle_I$, $\langle Z_{119} \cup g\rangle_I$ and $C^I(Z_{24})$, the question (38) for the respective topological spaces.

56. Let P =$\{P(M[x]_{28})\}$ = {collection of all subsets from $M[x]_{28} = \{\sum_{i=0}^{28} a_i x^i$ / $x^{29} = 1$, $a_i \in C^I(Z_{28})\}\}$ be the MOD natural neutrosophic finite complex number finite degree polynomial subset collection from $M[x]_{28}$. P_o, $P_{\times}^{+}$, $P_{\cup}^{+}$, $P_{\cap}^{+}$, $P_{\cup}^{\times}$ and $P_{\cap}^{\times}$ be the MOD natural neutrosophic finite complex number finite degree polynomial subset special type of topological spaces.

Study question (38) for this P.

57. Replace in problem (56) $C^I(Z_{28})$ by $\langle Z_{43} \cup g\rangle_I$ $\langle Z_{24} \cup h\rangle_I$, $\langle Z_7 \cup k\rangle_I$ and $\langle Z_{12} \cup I\rangle_I$ and study problem (38) for these mod topological spaces.

58. Let M = $\{\{P({}^I[0, 48))$ = {collection of all subsets from ${}^I[0, 48) = \{a + I_t^{48}$ / $a \in [0, 48)$, t is a zero divisor or idempotent or nilpotent in $Z_{48}\}\}$ be the MOD natural neutrosophic subset collection. Let S_o, $S_{\times}^{+}$, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$ and $S_{\cap}^{\times}$ be the MOD natural neutrosophic subset special

type of topological space. Study problem (38) for all the spaces built using S.

59. Let T = {P(⟨[0, 20) ∪ g⟩$_I$)} = {collection of all subsets from ⟨[0, 20) ∪ g⟩$_I$ = {a + bg + I_t^{20} / a, b ∈ [0, 20), g^2 = 0' t is an idempotent or nilpotent or zero divisor in Z_{20}}} be the MOD natural neutrosophic dual number subset collection. T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and $T_{\times}^{+}$ be the MOD natural neutrosophic dual number subset special type of topological space using T.

Study question (38) for this T.

60. If in (59) when ⟨[0, 20) ∪ g⟩$_I$ is replaced by C^I([0,12), ⟨[0, 17) ∪ h⟩$_I$, ⟨[0, 11) ∪ k⟩$_I$ and ⟨[0,16) ∪ I⟩$_I$ for the respective MOD special interval topological spaces study question (38).

61. Let M = { $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix}$ / a_i ∈ P(^{I}[0, 24)); 1 ≤ i ≤ 12} be the MOD interval natural neutrosophic matrix with subset entries collection. M_o, $M_{\times_n}^{+}$, $M_{\cup}^{+}$ $M_{\cap}^{+}$, $M_{\cup}^{\times_n}$, and $M_{\cap}^{\times_n}$ be the MOD interval natural neutrosophic matrix with subset entries special type of topological spaces built using M.

Study question (38) for this M.

62. If P(^{I}[0, 24)) in problem 61 is replaced by P(⟨[0, 47) ∪ g⟩$_I$, P(⟨[0, 19) ∪ k⟩$_I$), P (⟨[0, 8) ∪I⟩$_I$) and P(⟨[0,9) ∪ h⟩$_I$) for their respective topological spaces. Study question (38).

63. Let W = {P(M)} = {collection of all subsets from

$$M = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}$$

where $a_i \in \langle[0, 8) \cup g\rangle_I$; $1 \leq i \leq 8\}\}$ be the MOD interval natural neutrosophic dual number matrix subset collection. W_o, $W_{\times_n}^{+}$, $W_{\cup}^{+}$ $W_{\cap}^{+}$, $W_{\cap}^{\times_n}$, and $W_{\cup}^{\times_n}$ be the MOD interval natural neutrosophic dual number matrix subset special type of topological spaces associated with W. Study question (38) for these MOD special type of topological spaces associated with W.

64. In problem 63 replace the space W by $\langle[0, 8) \cup g\rangle_I$ this by $C^I([0, 9))$, $\langle[0,28) \cup I\rangle_I$, $\langle[0,19) \cup h\rangle_I$, $\langle[0, 23) \cup k\rangle_I$. Study problem 38 for all these above mentioned spaces.

65. Let $N = \{\sum_{i=0}^{\infty} a_i x^i \; / \; a_i \in P(^I[0, n))\}$ be the MOD interval natural neutrosophic subset coefficient polynomial collection. N_o, $N_{\times}^{+}$, $N_{\cup}^{+}$, $N_{\cap}^{+}$, $N_{\cup}^{\times}$ and $N_{\cap}^{\times}$ be the MOD interval natural neutrosophic subset coefficient polynomial special type of topological spaces.

Study question (38) for this N.

66. Replace in problem 65, $P(^I[0, n))$ by $P(\langle[0, 18) \cup g\rangle_I)$, $P(\langle[0,196) \cup h\rangle_I$, $P(\langle[0, 48) \cup k\rangle_I)$, $P(\langle[0, 64) \cup I\rangle_I$ and study question (38) for all these MOD topological space.

i) Distinguish their similarities and dissimilarities.

ii) Compare them with each other.

67. Let V = P(S[x]) = {collection of all subsets from S[x] = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in {}^{I}[0, 47))\}\}$ be the MOD natural neutrosophic polynomial subset collection.

Study question (38) for this V.

68. In problem (67) replace ${}^{I}[0,47)$ by $\langle[0, 24) \cup g\rangle_I$, $\langle[0, 48) \cup h\rangle_I$, $\langle[0, 19) \cup k\rangle_I$, $\langle[0, 64) \cup I\rangle_I$ and $C^I([0, 27))$ and study the question (38) for these respective sets MOD special type of topological spaces.

69. Let Z = $\{\sum_{i=0}^{\infty} a_i x^i$ / $a_i \in P({}^{I}[0, 10))\}$ be the MOD interval natural neutrosophic subset coefficient polynomial collection. Z_o, $Z_{\cup}^{+}$, $Z_{\times}^{+}$, $Z_{\cap}^{+}$, $Z_{\cup}^{\times}$ and $Z_{\cap}^{\times}$ be the MOD interval natural neutrosophic subset coefficient polynomial special type of topological spaces.

Study question (38) for this Z.

70. Replace in problem 69, $P({}^{I}[0, 10))$ by $P(\langle[0, 27) \cup g\rangle_I)$, $P(\langle[0,48) \cup I\rangle_I)$, $P(\langle[0, 11) \cup h\rangle_I)$, $C^I([0, 40))$ and $P(\langle[0, 26) \cup k\rangle_I)$. Study the topological spaces associated with these sets the question (38).

71. Let Y = $\{\sum_{i=0}^{18} a_i x^i$ / $a_i \in P({}^{I}[0, 23))$; $x^{19} = 1\}$ be the MOD interval natural neutrosophic finite degree subset coefficient polynomial collection. Let Y_o, $Y_{\times}^{+}$, $Y_{\cup}^{+}$, $Y_{\cap}^{+}$, $Y_{\cup}^{\times}$ and $Y_{\cap}^{\times}$ be the MOD interval natural neutrosophic finite degree subset coefficient polynomial special type of topological spaces.

Study question (38) for the all types of sets of topological spaces built using Y.

72. Replace in problem 71, $P(^I[0, 23))$ in Y of problem (71) by $P(C^I[0,48))$, $P(\langle[0, 17) \cup I\rangle_I)$, $P(\langle[0, 81) \cup g\rangle_I)$, $P(\langle[0, 12) \cup k\rangle_I)$ and $P(\langle[0, 3) \cup h\rangle_I)$ and study for those MOD special topological spaces the problem 38.

73. Let $B = \{P(M[x]_{20})\}$ = {collection of all subsets from $M[x]_{20} = \{\sum_{i=0}^{20} a_i x^i \,/\, x^{21} = 1, a_i \in {}^I[0, 42)\}\}$ be the MOD interval natural neutrosophic finite degree polynomial subset collection. B_o, $B_\times^+$, $B_\cup^+$, $B_\cap^+$, $B_\cup^\times$ and $B_\cap^\times$ be the MOD interval natural neutrosophic finite degree polynomial special type of topological spaces.

Study question (38) for this B.

74. Replace B of problem (73) the set $^I[0, 42)$ by $C^I([0,5))$; $\langle[0, 9) \cup g\rangle_I$, $\langle[0, 12) \cup h\rangle$, $\langle[0, 121) \cup k\rangle_I$ and $\langle[0, 43) \cup I\rangle_I$ and for the related 5 distinct MOD interval topological spaces and study question (38).

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INDEX

N

R

S

T

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Dr.W.B.Vasantha Kandasamy is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 694 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 121st book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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Dr. Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu

In this book the authors describe several MOD subset special type of topological spaces. These spaces in many cases is forced to accept the existence of Kakutani's fixed point theorem however in some cases the other properties of the theorem may not in general be true. In fact the study and verification of Kakutani's theorem in case of these MOD subset special type of topological spaces constructed in this book happens to be a challenging problem.

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