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Neutrosophic Extended Triplet Group Action and Burnside's Lemma

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Abstract: The aim of this article is mainly to discuss the neutrosophic extended triplet (NET) group actions and Burnside's lemma of NET group. We introduce NET orbits, stabilizers, conjugates and NET group action. Then, we give and proof the Orbit stabilizer formula for NET group by utilizing the notion of NET set theory. Moreover, some results related to NET group action, and Burnside's lemma are obtained.

Keywords: NET group action; NET orbit; NET stabilizer; NET conjugate; Burnside's lemma; NET fixed points; The fundamental theorem about NET group action.

1. Introduction

Galois is well known as the first researcher associating group theory and field theory, along the theory particularly called Galois theory. The concept of groupoid gives a more flexible and powerful approach to the concept of symmetry (see [1]). Symmetry groups come out in the review of combinatorics outline and algebraic number theory, along with physics and chemistry. For instance, Burnside's lemma can be utilized to compute combinatorial objects related along symmetry groups. A group action is a precise method of solving the technique wither the elements of a group meet transformations of any space in a method such protects the structure of a certain space. Just as there is a natural similarity among the set of a group elements and the set of space transformations, a group can be explained as acting on the space in a canonical way. A familiar method of defining no-canonical groups is to express a homomorphism f from a group G to the group of symmetries (an object is invariant to some of different transformations; including reflection, rotation) of a set X. The action of an element $g \in G$ on a point $x \in X$ is supposed to be similar to the action of its image $f(g) \in Sym(X)$ on the point x. The stabilizers of the action are the vertex groups, and the orbits of the action are the elements, of the action groupoid. Some other facts about group theory can be revealed in [2-5].

Neutrosophy is a new branch of philosophy, presented by Florentic Smarandache [6] in 1980, which studies the interactions with different ideational spectra in our everyday life. A NET is an object of the structure $(x, e^{neut(x)}, e^{anti(x)})$, for $x \in N$, was firstly presented by Florentine Smarandache [7-9] in 2016. In this theory, the extended neutral and the extended opposites can similar or non-identical from the classical unitary element and inverse element respectively. The NETs are depend on real triads: (friend, neutral, enemy), (pro, neutral, against), (accept, pending, reject), and in general $(x, neut(x), anti(x))$ as in neutrosophy is a conclusion of Hegel's dialectics that is depend on x and $anti(x)$. This theory acknowledges every concept or idea x together along its opposite $anti(x)$ and along their spectrum of neutralities $neut(x)$ among them. Neutrosophy is the foundation of neutrosophic logic, neutrosophic set, neutrosophic probability, and neutrosophic statistics that are utilized or applied in engineering (like software and information fusion), medicine, military, airspace, cybernetics, and physics. Kandasamy and Smarandache [10] introduced many new neutrosophic notions in graphs and applied it to the case of neutrosophic cognitive and relational maps. The same researchers [11] were introduced the concept of neutrosophic algebraic structures for groups, loops, semi groups and groupoids and also their $N-$ algebraic structures in 2006. Smarandache and Mumtaz Ali [12] proposed neutrosophic triplets and by utilizing these they defined NTG and the application areas of NTGs. They also define NT field [13] and NT in physics [14]. Smarandache investigated physical structures of hybrid NT ring [15]. Zhang et al [16] examined the Notion of cancellable NTG and group coincide in 2017. Şahın and Kargın [17], [18] firstly introduced new structures called NT normed space and NT inner product respectively. Smarandache et al [19] studied new algebraic structure called NT G-module which is constructed on NTGs and NT vector spaces. The above set theories have been applied to many different areas including real decision making problems [20-44]. Furthermore, Abdel Basset et al applied this theory to decision making approach for selecting supply chain sustainability metrics [48], an approach of TOPSIS technique [49, 51], iot-based enterprises [50, 52], calculation of the green supply chain management [53] and neutrosophic ANP and VIKOR method for achieving sustainable supplier selection [54].

The paper deals with action of a NET set on NETGs and Burnside's lemma. We provide basic definitions, notations, facts, and examples about NETs which play a significant role to define and build new algebraic structures. Then, the concept of NET orbits, stabilizers, fixed points and conjugates are given and their difference between the classical structures are briefly discussed. Finally, some results related to NET group actions and Burnside's lemma are obtained.

2. Preliminaries

Since some properties of NETs are used in this work, it is important to have a keen knowledge of NETs. We will point out some few NETs and concepts of NET group, NT normal subgroup, and NT cosets according to what needed in this work.

Definition 2.1 [12, 14] A NT has a form $(a,\ neut(a),\ anti(a))$, for $(a, neut(a), anti(a)) \in N$, accordingly $neut(a)$ and $anti(a) \in N$ are neutral and opposite of a, that is different from the unitary element, thus: $a * neut(a) = neut(a) * a = a$ and $a * anti(a) = anti(a) * a = neut(a)$ respectively. In general, a may have one or more than one neut's and one or more than one anti's.

Definition 2.2 [8, 14] A NET is a NT, defined as definition 1, but where the neutral of a (symbolized by $e^{neut(a)}$ and called "extended neutral") is equal to the classical unitary element. As a consequence, the "extended opposite" of a, symbolized by $e^{anti(a)}$ is also same to the classical inverse element. Thus, a NET has a form $(a, e^{neut(a)}, e^{anti(a)})$, for $a \in N$, where $e^{neut(a)}$ and $e^{anti(a)}$ in N are the extended neutral and negation of a respectively, thus :

$$a * e^{neut(a)} = e^{neut(a)} * a = a,$$

which can be the same or non-identical from the classical unitary element if any and

$$a * e^{anti(a)} = e^{anti(a)} * a = e^{neut(a)}.$$

Generally, for each a ∊ N there are one or more $e^{neut(a)}$'s and $e^{anti(a)}$'s.

Definition 2.3 [12, 14] Suppose $(N,*)$ is a NT set. Subsequently $(N,*)$ is called a NTG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for and $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N$, one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in N$.

(2) $(N,*)$ is associative, i.e. for any

one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) * (c, neut(c), anti(c)) \in N.$

Theorem 2.4 [46] Let $(N, *)$ be a commutative NET relating to $*$ and $(a, neut(a), anti(a)), (b, neut(b), anti(b)) \in N$;

(i) $neut(a) * neut(b) = neut(a * b);$

(ii) $anti(a) * anti(b) = anti(a * b);$

Definition 2.5 [8, 14] Assume $(N, *)$ is a NET strong set. Subsequently $(N, *)$ is called a NETG, if the axioms given below are holds.

(1) $(N, *)$ is well-defined, i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N,$ one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in N.$

(2) $(N, *)$ is associative, i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b)), (c, neut(c), anti(c)) \in N$, one has

$$(a, neut(a), anti(a)) * \big((b, neut(b), anti(b)) * (c, neut(c), anti(c))\big)$$
$$= \big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big) * (c, neut(c), anti(c)).$$

Definition 2.6 [47] Assume that $(N_1, *)$ and $(N_2, \circ)$ are two NETG's. A mapping $f : N_1 \to N_2$ is called a neutro-homomorphism if:

(1) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N_1$, we have

$$f\big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big)$$
$$= f\big((a, neut(a), anti(a))\big) * f\big((b, neut(b), anti(b))\big)$$

(2) If $(a, neut(a), anti(a))$ is a NET from N_1, Then $f(neut(a)) = neut(f(a))$ and $f(anti(a)) = anti(f(a)).$

Definition 2.7 [45] Assume that $(N_1, *)$ is a NETG and H is a subset of N_1. H is called a NET subgroup of N if itself forms a NETG under $*$. On other hand it means :

(1) $e^{neut(a)}$ lies in H.

(2) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in H,$
$(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in H.$

(3) If $(a, neut(a), anti(a)) \in H,$ then $e^{anti(a)} \in H.$

Definition 2.8 [45] A NET subgroup H of a NETG N is called a NT normal subgroup of N if $(a, neut(a), anti(a))H = H(a, neut(a), anti(a)), \forall (a, neut(a), anti(a)) \in N$ and we represent it as $H (\quad N.$

3. NET Group Action

A NETG action is a representation of the elements of a NETG as a symmetries of a NET set. It is a precise method of solving the technique in which the elements of a NETG meet transformations of any space in a method that maintains the structure of that space. Just as a group action plays an important role in the classical group theory, NETG action enacts identical role in the theory of NETG theory.

Definition 3.1 An action of N on X (left NETG action) is a map $N \times X \to X$ denoted

$$\big((n, neut(n), anti(n)), (x, neut(x), anti(x))\big) \to (n, neut(n), anti(n))(x, neut(x), anti(x))$$

as shown $$1(x, neut(x), anti(x)) = (x, neut(x), anti(x))$$

and
$$(n, neut(n), anti(n))\big((h, neut(h), anti(h))(x, neut(x), anti(x))\big)$$
$$= \big((n, neut(n), anti(n))(h, neut(h), anti(h))\big)(x, neut(x), anti(x))$$

for all $(x, neut(x), anti(x))$ in X and $(n, neut(n), anti(n)), (h, neut(h), anti(h))$ in N. Given a NET action of N on X, we call X a $N-$set. A $N-$map between $N-$sets X and Y is a map $f : X \to Y$ of NET sets that respects the $N-$action, meaning that,

$f\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right)=(n,neut(n),anti(n))\,f\left((x,neut(x),anti(x))\right)$ for all $(x,neut(x),anti(x))$ in X and $(n,neut(n),anti(n))$ in $N.$ To give a NET action of N on X is equivalent to giving a NETG neutro-homomorphism from N to the NETG of bijections of $X.$
Note that a NETG action is not the same thing as a binary structure, we combine two elements of X to get a third element of X (we combine two apples and get an apple). In a NETG action, we combine an element of N with an element of X to get an element of X (we combine an apple and an orange and get another orange).
It is critical to note that $(n,neut(n),anti(n))\cdot\left((h,neut(h),anti(h))\cdot(x,neut(x),anti(x))\right)$ has two actions of N on elements of $X.$ under other conditions

$$\left((n,neut(n),anti(n))(h,neut(h),anti(h))\right)\cdot(x,neut(x),anti(x))$$

has one multiplication in the NETG $\left((n,neut(n),anti(n))(h,neut(h),anti(h))\right)$ and then one action of an element of N on $X.$

Example 3.2 For a NET subgroup $H\subset N,$ consider the left NT coset space $N/H=\{(a,neut(a),anti(a))H:(a,neut(a),anti(a))\in N\}.$ (We do not care wether or not $H\lhd N,$ as we are just thinking about N/H as a set.) Let N act on N/H by left multiplication. That is for $(n,neut(n),anti(n))\in N$ and a left NT coset $(a,neut(a),anti(a))H$ ($(a,neut(a),anti(a))\in N$), set

$$(n,neut(n),anti(n))\cdot(a,neut(a),anti(a))H=(n,neut(n),anti(n))(a,neut(a),anti(a))H$$
$$=\left\{\begin{matrix}(n,neut(n),anti(n))(y,neut(y),anti(y)): \\ (y,neut(y),anti(y))\in(a,neut(a),anti(a))H\end{matrix}\right\}.$$

This is an action of N on N/H, since $1_N(a,neut(a),anti(a))H=(a,neut(a),anti(a))H$ and

$$(n_1,neut(n_1),anti(n_1))\cdot\left((n_2,neut(n_2),anti(n_2))\cdot(a,neut(a),anti(a))H\right)$$
$$=(n_1,neut(n_1),anti(n_1))\cdot\left((n_2,neut(n_2),anti(n_2))(a,neut(a),anti(a))H\right)$$

$$=(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))(a,neut(a),anti(a))H$$
$$=\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)(a,neut(a),anti(a))H.$$

Note: NET Groups Acting Independently by Multiplication

All NETG acts independently like so, NET set $N=N$ and $X=N.$ Then for $(n,neut(n),anti(n))\in N$ and $(n,neut(n),anti(n))\in X=N,$ we define
$(n,neut(n),anti(n))\cdot\left((n,neut(n),anti(n))\right)$
$=(n,neut(n),anti(n))\left((n,neut(n),anti(n))\right)\in X=N.$

Example 3.3 Each NETG N acts independently $\left(X=N\right)$ by left multiplication functions. In other words, we set $\pi_{(n,neut(n),anti(n))}:N\rightarrow N$ by

$$\pi_{(n,neut(n),anti(n))}\left((h,neut(h),anti(h))\right)=(n,neut(n),anti(n))(h,neut(h),anti(h))$$

for all $(n,neut(n),anti(n))\in N$ and $(h,neut(h),anti(h))\in H.$ Subsequently, the axioms for being a NETG action are $1_N(h,neut(h),anti(h))=(h,neut(h),anti(h))$ for all $(h,neut(h),anti(h))\in N$ and

$$(n_1,neut(n_1),anti(n_1))\left((n_2,neut(n_2),anti(n_2))(h,neut(h),anti(h)\right)$$
$$=\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)(h,neut(h),anti(h))$$

for all $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2)),(h,neut(h),anti(h))\in N,$ which are both true whereby 1_N is a neutrality and multiplication in N is associative.

The notation for the NET effect of N is $\pi(n,neut(n),anti(n))$ or

$$\pi_{(n,neut(n),anti(n))}\big((x,neut(x),anti(x))\big)$$

simply as $(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))$ or

$$(n,neut(n),anti(n))(x,neut(x),anti(x)).$$

In this explanation, the conditions for the left NETG action take the succeeding shape:

i. for all $(x,neut(x),anti(x))\in X, 1_N(x,neut(x),anti(x))=(x,neut(x),anti(x))$.

ii. for every $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N$ an $(x,neut(x),anti(x))\in X$,

$$\begin{aligned}&(n_1,neut(n_1),anti(n_1))\cdot\big((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))\big)\\&=\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)\cdot(x,neut(x),anti(x)).\end{aligned}$$

Theorem 3.4 Let a NETG action N act on the NET set $X.$ If $(x,neut(x),anti(x))\in X$, $(n,neut(n),anti(n))\in N$, and

$$(y,neut(y),anti(y))=(n,neut(n),anti(n))(x,neut(x),anti(x)),$$

then $(x,neut(x),anti(x))=(n,neut(n),anti(n))^{-1}\cdot(y,neut(y),anti(y))$.

If $(x,neut(x),anti(x))\neq(x',neut(x'),anti(x'))$ then

$$(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))\neq(n,neut(n),anti(n))\cdot(x',neut(x'),anti(x')).$$

Proof : From $(y,neut(y),anti(y))=(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))$ we get

$$\begin{aligned}&(n,neut(n),anti(n))^{-1}\cdot(y,neut(y),anti(y))\\&=(n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\&=\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)(x,neut(x),anti(x))\\&=1_N(x,neut(x),anti(x))=(x,neut(x),anti(x)).\end{aligned}$$

To show $(x,neut(x),anti(x))\neq(x',neut(x'),anti(x'))\Rightarrow$

$$(n,neut(n),anti(n))(x,neut(x),anti(x))\neq(n,neut(n),anti(n))(x',neut(x'),anti(x')),$$

we show the contrapositive : if

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(n,neut(n),anti(n))(x',neut(x'),anti(x'))$$

then applying $(n,neut(n),anti(n))^{-1}$ to both sides gives

$$\begin{aligned}&(n,neut(n),anti(n))^{-1}\cdot\big((n,neut(n),anti(n))\cdot(x,neut(x),anti(x))\big)\\&=(n,neut(n),anti(n))^{-1}\cdot\big((n,neut(n),anti(n))\cdot(x',neut(x'),anti(x'))\big)\end{aligned}$$

so

$$\begin{aligned}&\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)\cdot(x,neut(x),anti(x))\\&=\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)\cdot(x',neut(x'),anti(x'))\end{aligned}$$

so

$$(x, neut(x), anti(x)) = (x', neut(x'), anti(x')).$$

On the other hand to imagine action of a NETG on a NET set is such it's a definite neutro-homomorphism. On hand are the facts.

Theorem 3.5 Actions of the NETG N on the NET set X are identical NETG neutro-homeomorphisms from $N \to Sym(X)$, the NETG of permutations of X.

Proof: Assume we've an action of N on the NET set X. We observe $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$ as a function of $(x, neut(x), anti(x))$ (with $(n, neut(n), anti(n))$ fixed). That is, for each $(n, neut(n), anti(n)) \in N$ we have a function $\pi_{(n,neut(n),anti(n))}: X \to X$ by

$$\pi_{(n,neut(n),anti(n))}\big((x, neut(x), anti(x))\big) = (n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)).$$

The axiom $1_N \cdot (x, neut(x), anti(x)) = (x, neut(x), anti(x))$ says π_1 is the neutrality function on X.

The axiom

$$(n_1, neut(n_1), anti(n_1))\big((n_2, neut(n_2), anti(n_2)) \cdot (x, neut(x), anti(x)\big)$$

$$= \big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big) \cdot (x, neut(x), anti(x))$$

says

$$\pi_{(n_1, neut(n_1), anti(n_1))} \circ \pi_{(n_2, neut(n_2), anti(n_2))}$$
$$= \pi_{(n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))},$$

so structure of functions on X match multiplication in N. Additionally, $\pi_{(n,neut(n),anti(n))}$ is an invertible function whereby $\pi_{(n_1, neut(n_1), anti(n_1))^{-1}}$ is an anti-neutral: the composite of $\pi_{(n_1, neut(n_1), anti(n_1))}$ and $\pi_{(n_1, neut(n_1), anti(n_1))^{-1}}$ is π_1, which is the neutral function on X. Therefore, $\pi_{(n_1, neut(n_1), anti(n_1))} \in Sym(X)$ and $(n, neut(n), anti(n)) \to \pi_{(n_1, neut(n_1), anti(n_1))}$ is a neutro-homomorphism $N \to Sym(X)$.

Contrariwise, assume we've a homomorphism $f : N \to Sym(X)$. For every $(n, neut(n), anti(n))$, we have a permutation $f\big((n, neut(n), anti(n))\big)$ on X, and

$$f\big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big)$$

$$= f\big((n_1, neut(n_1), anti(n_1))\big) \circ f\big((n_2, neut(n_2), anti(n_2))\big).$$

Setting $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$
$$= f\big((n, neut(n), anti(n))\big)\big((x, neut(x), anti(x))\big)$$

introduces a NETG action of N on X, whereby the neutro-homomorphism properties of f submits the defining properties of a NETG action. From this view point, the NET set of $(n, neut(n), anti(n)) \in N$ that act trivially

$$\big((n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))\big) = (x, neut(x), anti(x))$$

for all $(x, neut(x), anti(x)) \in X$ is straightforwardly the neutrosophic kernel of the neutro-homomorphism $N \to Sym(X)$ related to the action. Consequently the above mentioned $(n, neut(n), anti(n))$ such act trivially on X are assumed to lie in the neutrosophic kernel of the action.

Example 3.6 To build N act independently by conjugation, take $X = N$ and let

$$(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$$
$$=(n,neut(n),anti(n))(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1}.$$

Here, $(n, neut(n), anti(n)) \in N$ and $(x, neut(x), anti(x)) \in N$. Since

$$1_N \cdot (x, neut(x), anti(x)) = 1_N (x, neut(x), anti(x)) 1_N{}^{-1} = (x, neut(x), anti(x))$$

and

$$(n_1, neut(n_1), anti(n_1)) \cdot \big((n_2,neut(n_2),anti(n_2)) \cdot (x,neut(x),anti(x))\big)$$
$$= (n_1, neut(n_1), anti(n_1)) \cdot$$
$$\Big((n_2,neut(n_2),anti(n_2)) \cdot (x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\Big)$$
$$=(n_1,neut(n_1),anti(n_1))$$
$$\Big((n_2,neut(n_2),anti(n_2)) \cdot (x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\Big)$$
$$(n_1,neut(n_1),anti(n_1))^{-1}$$
$$=\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(x,neut(x),anti(x))$$
$$\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)^{-1}$$
$$= \big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big) \cdot (x, neut(x), anti(x)),$$

neutrosophic conjugation is a NET action.

Definition 3.7 Assume such N is a NETG and X is a NET set. A right NETG action of N on X is a rule for merging elements $(n, neut(n), anti(n)) \in N$ and elements $(x, neut(x), anti(x)) \in X$, symbolized by $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$,
$(n, neut(n), anti(n)) \cdot \big((x, neut(x), anti(x))\big) \in X$ for all $(x, neut(x), anti(x)) \in X$ and $(n, neut(n), anti(n)) \in N$. We also need the succeeding conditions.

I. $(x,neut(x),anti(x))1_N=(x,neut(x),anti(x))$ for all $(x, neut(x), anti(x)) \in X$.

II. $$\big((x,neut(x),anti(x))\cdot(n_2,neut(n_2),anti(n_2))\big)\cdot(n_1,neut(n_1),anti(n_1))$$
$$=(x,neut(x),anti(x))\big((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\big)$$

for all $(x,neut(x),anti(x))\in X$ and $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N.$

Remark 3.8 Left NETG actions are not very distinct from right NETG actions. The only distinction exists in condition (ii).

- For left NETG actions, implementing $(n_2,neut(n_2),anti(n_2))$ to an element and then applying $(n_1,neut(n_1),anti(n_1))$ to the result is the same as applying

$$(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\in N.$$

- For right NETG actions applying $(n_2,neut(n_2),anti(n_2))$ and then $(n_1,neut(n_1),anti(n_1))$ is the same as applying $(n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\in N.$

Let us see the example of a right NETG action (beyond the Rubik's cube example, which as we wrote things is a right NETG action). Also it is easy to do matrices multiplying vectors from the right.

Example 3.9 (A NETG acting on a NET set of NT cosets). Assume such N is a NETG and H is a NET subgroup. Examine the NET set $X=\{Ha/(a,neut(a),anti(a))\in N\}$ of right NT cosets of $H.$ subsequently N acts on X by right multiplication, That is, we describe

$$\big(H(a,neut(a),anti(a))\big)\cdot(n,neut(n),anti(n))$$
$$=H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)$$

for $(n,neut(n),anti(n))\in N$ and $H(a,neut(a),anti(a))\in X.$ First let's chect that this is well defined, hence assume such $H(a,neut(a),anti(a))=H(a',neut(a'),anti(a')),$ then $(a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\in H.$ Now, we have to prove that

for any $(n,neut(n),anti(n))\in N.$ But $(a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\in H$ so that

$$(a',neut(a'),anti(a'))(n,neut(n),anti(n))$$
$$=\big((a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\big)\begin{pmatrix}(a,neut(a),\\ anti(a))(n,neut(n),anti(n))\end{pmatrix}$$
$$\in H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)$$
$$H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)=H\big((a',neut(a'),anti(a'))(n,neut(n),anti(n))\big)$$

so that

$$(a',neut(a'),anti(a'))(n,neut(n),anti(n))\in H\begin{pmatrix}(a,neut(a),anti(a))(n,neut(n),\\ anti(n))\end{pmatrix}.$$

But certainly $H\big((a',neut(a'),anti(a'))(n,neut(n),anti(n))\big)$ also contains

$$1_N\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right)=(a',neut(a'),anti(a'))(n,neut(n),anti(n)).$$

Thus the two cosets $H\left((a,neut(a),anti(a))(n,neut(n),anti(n))\right)$ and

$H\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right)$ have the elements

$(a',neut(a'),anti(a'))(n,neut(n),anti(n))$ in common. This proves that

$$H\left((a,neut(a),anti(a))(n,neut(n),anti(n))\right)=H\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right)$$

since NT cosets are either same or separate.

Now we've proved that this is well defined, we have to show it is also an action. Definitely axiom (i) is holds since

$$\left(H(a,neut(a),anti(a))\right)\cdot 1_N=H\left((a,neut(a),anti(a))1_N\right)=H(a,neut(a),anti(a)).$$

Lastly, we have to show axiom (ii). Assume such

$$(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N.$$ Then

$$\begin{aligned}
&\left(\left(H(a,neut(a),anti(a))\right)\cdot(n_2,neut(n_2),anti(n_2))\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=\left(H\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=H\left(\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)(n_1,neut(n_1),anti(n_1))\\
&=H\left((a,neut(a),anti(a))\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)\right)\\
&=\left(H(a,neut(a),anti(a))\right)\cdot\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)
\end{aligned}$$

which proves (ii) and ends the proof. Of course, N also acts on the set of left NT cosets of H by multiplication on the left.

Definition 3.10 A NETG action of N on X is called NET faithful if distinct elements of N act on X in dis-similar methods: when $(n_1,neut(n_1),anti(n_1))\neq(n_2,neut(n_2),anti(n_2))$ in N, there is an $(x,neut(x),anti(x))\in X$ such that

$$(n_1,neut(n_1),anti(n_1))\cdot(x,neut(x),anti(x))\neq(n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x)).$$

Note that when we say $(n_1,neut(n_1),anti(n_1))$ and $(n_2,neut(n_2),anti(n_2))$ act distinctly, we signify they act distinctly somewhere, not all place. This is consistent with what it signifies to say two functions are disjoint. They take distinct values somewhere, not all place.

Example 3.11 The action of N independently by left multiplication is faithful: distinct elements send 1_N to distinct places.

Example 3.12 When H is a NET subgroup of N and N acts on N/H left multiplication $(n_1, neut(n_1), anti(n_1))$ and $(n_2, neut(n_2), anti(n_2))$ in N act in the similar method on N/H exactly when

$$(n_1, neut(n_1), anti(n_1))(n, neut(n), anti(n))H = (n_2, neut(n_2), anti(n_2))(n, neut(n), anti(n))H$$

for all $(n, neut(n), anti(n)) \in N$, which means

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1)) \in \bigcap_{(n,neut(n),anti(n))}$$
$$\in N(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}.$$

So the left multiplication action of N on N/H is NET faithful in the case that the NET subgroups $(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}$ (as $(n, neut(n), anti(n))$ varies) have trivial intersection.

Viewing NETG actions as neutro-homeomorphisms, a NET faithful action of N on X is an injective neutro-homomorphism $N \to Sym(X)$. Non faithful actions are not injective as NETG neutro-homeomorphisms, and many important homeomorphisms are not injective.

Remark 3.13 What we've been calling a NETG action could be a left and right NETG action. The difference among left and right actions is how a product $(n, neut(n), anti(n))(n', neut(n'), anti(n'))$ acts: in a left action $(n', neut(n'), anti(n'))$ acts first and $(n, neut(n), anti(n))$ acts second, while in a right action $(n, neut(n), anti(n))$ acts first and $(n', neut(n'), anti(n'))$ acts second.

We can introduce the NET conjugate of $(h, neut(h), anti(h))$ by $(n, neut(n), anti(n))$ as

$$(n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))$$

Instead $\quad (n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))^{-1},$

and this convention fits well with the right NET conjugation action but not left action : setting

$$(h, neut(h), anti(h))^{(n,neut(n),anti(n))} = (n, neut(n), anti(n))^{-1}(h, neut(h), anti(h))(n, neut(n), anti(n))$$

we have $(h, neut(h), anti(h))^{1_N} = (h, neut(h), anti(h))$ and

$$\left((h, neut(h), anti(h))^{(n_1,neut(n_1),anti(n_1))}\right)^{(n_2,neut(n_2),anti(n_2))}$$
$$= (h, neut(h), anti(h))^{(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))}.$$

The distinction among left and right actions of a NETG is mostly unreal, whereby subsetituting $(n, neut(n), anti(n))$ with $(n, neut(n), anti(n))^{-1}$ in the NETG changes left actions into right actions and contrarily since inversion backwards the order of multiplication in N. So for us "NETG action" means "left NETG action".

Definition 3.14 Let a NETG N act on NET set X. For each $(x, neut(x), anti(x)) \in X$, its orbit is

$$Orb_{(x,neut(x),anti(x))} = \{(n,neut(n),anti(n))(x,neut(x),anti(x)):(n,neut(n),anti(n))\in N\} \subset X$$

and its stabilizer is

$$Stab_{(x,neut(x),anti(x))} = \{(n,neut(n),anti(n))\in N:(n,neut(n),anti(n))(x,neut(x),anti(x))\} \subset N.$$

(The stabilizer of NET $(x, neut(x), anti(x))$ is symbolized by $N_{(x,neut(x),anti(x))}$, where N is NETG.) We call $(x, neut(x), anti(x))$ a NET fixed point for the action when

$$(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)) = (x, neut(x), anti(x))$$

for every $(n, neut(n), anti(n)) \in N$, that is, when

$$Orb_{(x,neut(x),anti(x))}=\{(x,neut(x),anti(x))\}$$

(or equivalently, when $Stab_{(x,neut(x),anti(x))}=N$). The orbit of NETs of a point is a geometric notion: it is the NET set of places where the points can be moved by the NETG action. Under other conditions, the stabilizer of a NET of a point is an algebraic notion: it is the NET set of NETG elements that fix the point. Mostly we'll denote the elements of X as points and we'll denote the size of a NET orbit as its length.

Definition 3.15 Let N be a NETG, $(n, neut(n), anti(n)) \in N$, and let H be a NET subgroup of N.

$$\begin{aligned}&(a, neut(a), anti(a))H(a, neut(a), anti(a))^{-1}\\ &= \left\{\begin{matrix}(a, neut(a), anti(a))(h, neut(h), anti(h))(a, neut(a), anti(a))^{-1}: \\ (h, neut(h), anti(h)) \in H\end{matrix}\right\}\end{aligned}$$

is called a NET conjugate of H and the NET center of N is

$$Z_N=\left\{\begin{matrix}(a,neut(a),anti(a))\in N:(a,neut(a),anti(a))(n,neut(n),anti(n)) \\ =(n,neut(n),anti(n))(a,neut(a),anti(a)):\forall(n,neut(n),anti(n))\in N\end{matrix}\right\}.$$

Remark 3.16 When we imagine about a NET set as a geometric object, it is useful to describe to its elements as points. For instance, when we imagine about N / H as a NET set on which N acts, it is helpful to imagine about the NT cosets of H, which are the elements N / H, as the points in N / H. simultaneously, though, a NT coset is a NET subset of N.

All of our applications of NETG actions to group theory will flow from the similarities among NET orbits, stabilizers, and fixed points, which we now build explicit in our the following fundamental examples of NETG actions.

Example 3.17 When a NETG N acts independently by conjugation,

a) the NET orbit of $(a, neut(a), anti(a))$ is

$$Orb(a,neut(a),anti(a)) = \left\{ \begin{array}{l} (n,neut(n),anti(n))(a,neut(a),anti(a)) \\ (n,neut(n),anti(n))^{-1} : (n,neut(n),anti(n)) \in N \end{array} \right\},$$

which is the conjugacy class of $(a, neut(a), anti(a))$,

b) $$Stab(a,neut(a),anti(a)) = \left\{ \begin{array}{l} (n,neut(n),anti(n)) : (n,neut(n),anti(n)) \\ (a,neut(a),anti(a))(n,neut(n),anti(n))^{-1} \\ = (a,neut(a),anti(a)) \end{array} \right\}$$

c) $$Z(a,neut(a),anti(a)) = \left\{ \begin{array}{l} (n,neut(n),anti(n)) \\ :(n,neut(n),anti(n))(a,neut(a),anti(a)) \\ = (a,neut(a),anti(a))(n,neut(n),anti(n)) \end{array} \right\}$$

is the NET centralizer of $(a, neut(a), anti(a))$.

d) $(a, neut(a), anti(a))$ is a NET fixed point when it commutes with all elements of N, and thus the NET fixed points of conjugation form the NET center of N, and thus the NET fixed points of NET conjugation form the center of N.

Example 3.18 When H acts on N by conjugation,

i. the orbit of $(a, neut(a), anti(a))$ is

$$Orb(a,neut(a),anti(a)) = \left\{ \begin{array}{l} (h,neut(h),anti(h))(a,neut(a),anti(a)) \\ (h,neut(h),anti(h))^{-1} : (h,neut(h),anti(h)) \in H \end{array} \right\},$$

which has no special name (elements of N that are $H-$conjugate to $(a, neut(a), anti(a))$),

ii. $$\begin{aligned} Stab(a,neut(a),anti(a)) &= \{(h,neut(h),anti(h)) : \\ &(h, neut(h), anti(h))(a, neut(a), anti(a))(h, neut(h), anti(h))^{-1} \\ &= (h, neut(h), anti(h)) \} \\ &= \{(h, neut(h), anti(h)) : (h, neut(h), anti(h))(a, neut(a), anti(a)) \\ &= (a, neut(a), anti(a))(h, neut(h), anti(h)) \} \end{aligned}$$

is the elements of H commuting with $(a, neut(a), anti(a))$ (this is $H \cap Z((a, neut(a), anti(a)))$ is the NET centralizer of $(a, neut(a), anti(a))$ in N).

iii. $(a, neut(a), anti(a))$ is a NET fixed point when it commutes with all elements of H, so the NET fixed points of $H -$ conjugation on N shape the NET centralizer of H in N.

Theorem 3.19 the Fundamental Theorem about NETG Action

Let a NETG N act on a NET set X.

a. Different NET orbits of the action are disjoint and form a portion of X.

b. For each $(x, neut(x), anti(x)) \in X$, $Stab_{(x, neut(x), anti(x))}$ is a NET subgroup of N and

$$Stab_{(n, neut(n), anti(n))(x, neut(x), anti(x))} = (n, neut(n), anti(n))$$
$$Stab_{(x, neut(x), anti(x))} Stab_{(n, neut(n), anti(n))} (n, neut(n), anti(n))^{-1}$$

for all $(n, neut(n), anti(n)) \in N$.

c. For each $(x, neut(x), anti(x)) \in X$, there is a bijections

$$Orb_{(x, neut(x), anti(x))} \to N / Stab_{(x, neut(x), anti(x))} \text{ by}$$

$$(n, neut(n), anti(n))(x, neut(x), anti(x))$$
$$\to (n, neut(n), anti(n)) Stab_{(x, neut(x), anti(x))}.$$

More concretely, $\begin{aligned}&(n, neut(n), anti(n))(x, neut(x), anti(x))\\ &= (n', neut(n'), anti(n'))(x, neut(x), anti(x))\end{aligned}$

in the case that $(n, neut(n), anti(n))$ and $(n', neut(n'), anti(n'))$ lie in the similar NET coset of $Stab_{(x, neut(x), anti(x))}$, and different NT left cosets of $Stab_{(x, neut(x), anti(x))}$ correspond to different points in $Orb_{(x, neut(x), anti(x))}$. In particular, if $(x, neut(x), anti(x))$ and $(y, neut(y), anti(y))$ are in the same NET orbit then

$$\left\{\begin{aligned}&(n, neut(n), anti(n)) \in N : (n, neut(n), anti(n))(x, neut(x), anti(x))\\ &= (y, neut(y), anti(y))\end{aligned}\right\}$$

is a NT left coset of $Stab_{(x, neut(x), anti(x))}$, and

$$\left|Orb(x,neut(x),anti(x))\right|=\left[N:Stab(x,neut(x),anti(x))\right].$$

Parts b and c Show the role of conjugate NET subgroups and neutrosophic triplet cosets of a NET subgroup when working with NETG actions. The formula in part c that relates the length of a NET orbit to the index in N of a NET stabilizer for a point in the NET orbit, is named the NET orbit-stabilizer formula.

Proof:

a) We show distinct NET orbits in a NETG action are not equal by showing that two NET orbits that overlap must coexist. Assume $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$ have a common element $(z,neut(z),anti(z))$.

$$(z,neut(z),anti(z))=(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))$$
$$(z,neut(z),anti(z))=(n_2,neut(n_2),anti(n_2))(y,neut(y),anti(y)).$$

We want to show $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$. It suffices to show $Orb(x,neut(x),anti(x))\subset Orb(y,neut(y),anti(y))$, since then we can switch the roles of $(x,neut(x),anti(x))$ and $(y,neut(y),anti(y))$ to obtain the converse insertion. For each point $(u,neut(u),anti(u))\in Orb(x,neut(x),anti(x))$, write

$$(u,neut(u),anti(u))=(n,neut(n),anti(n))(x,neut(x),anti(x))$$

for some $(n,neut(n),anti(n))\in N$. Since

$$\begin{aligned}
&(x,neut(x),anti(x))\\
&=(n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z)),(u,neut(u),anti(u))\\
&=(u,neut(u),anti(u))\left((n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z))\right)
\end{aligned}$$

$$\begin{aligned}
&=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\right)(z,neut(z),anti(z))\\
&=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\right)\begin{pmatrix}(n_2,neut(n_2),anti(n_2))\\(y,neut(y),anti(y))\end{pmatrix}\\
&=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}(n_2,neut(n_2),anti(n_2))\right)\\
&(y,neut(y),anti(y)),
\end{aligned}$$

which shows us that $(u,neut(u),anti(u))\in Orb(y,neut(y),anti(y))$. Therefore $Orb(x,neut(x),anti(x))\subset Orb(y,neut(y),anti(y))$. Every element of X is in some NET orbit (its own NET orbits), so the NET orbits partition X into disjoint NET subsets.

b) To see that $Stab(x,neut(x),anti(x))$ is a NET subgroup of N, we've $1_N\in Stab(x,neut(x),anti(x))$ since $1_N(x,neut(x),anti(x))=(x,neut(x),anti(x))$, and if $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in Stab(x,neut(x),anti(x))$, then

$$\begin{aligned}
&\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(x,neut(x),anti(x))\\
&=(n_1,neut(n_1),anti(n_1))\big((n_2,neut(n_2),anti(n_2))(x,neut(x),anti(x))\big)\\
&=(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))\\
&=(x,neut(x),anti(x)),
\end{aligned}$$

so $(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\in Stab(x,neut(x),anti(x))$. Thus $Stab(x,neut(x),anti(x))$ is closed under multiplication. Lastly,

$$(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))=(x,neut(x),anti(x))$$

$$\begin{aligned}
&\Rightarrow (n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\
&=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x))\\
&\Rightarrow (x,neut(x),anti(x))=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x)),
\end{aligned}$$

so $Stab(x,neut(x),anti(x))$ is closed under inversion. To prove

$$\begin{aligned}
&Stab(n,neut(n),anti(n))(x,neut(x),anti(x))\\
&=(n,neut(n),anti(n))Stab(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1},
\end{aligned}$$

for all $(x,neut(x),anti(x))\in X$ and $(n,neut(n),anti(n))\in N$, observe that

$$\begin{aligned}
&(h,neut(h),anti(h))\in Stab(n,neut(n),anti(n))(x,neut(x),anti(x))\\
&\Leftrightarrow (h,neut(h),anti(h))\cdot\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\
&=(n,neut(n),anti(n))(x,neut(x),anti(x))
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \big((h,neut(h),anti(h))(n,neut(n),anti(n))\big)(x,neut(x),anti(x))\\
&=(n,neut(n),anti(n))(x,neut(x),anti(x))
\end{aligned}$$

$$\Leftrightarrow (n,neut(n),anti(n))^{-1}\begin{pmatrix}((h,neut(h),anti(h))(n,neut(n),anti(n)))\\(x,neut(x),anti(x))\end{pmatrix}$$
$$=(n,neut(n),anti(n))^{-1}\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right)$$
$$\Leftrightarrow \left((n,neut(n),anti(n))^{-1}(h,neut(h),anti(h))(n,neut(n),anti(n))\right)$$
$$(x,neut(x),anti(x))=(x,neut(x),anti(x))$$
$$\Leftrightarrow (n,neut(n),anti(n))^{-1}(h,neut(h),anti(h))(n,neut(n),anti(n))$$
$$\in Stab_{(x,neut(x),anti(x))}$$
$$\Leftrightarrow (h,neut(h),anti(h))\in(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}$$
$$(n,neut(n),anti(n))^{-1},$$

so

$$Stab_{(x,neut(x),anti(x))(x,neut(x),anti(x))}$$
$$=(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1}}.$$

C) The condition

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(n',neut(n'),anti(n'))(x,neut(x),anti(x))$$

is equivalent to

$$(x,neut(x),anti(x))=\left((n,neut(n),anti(n))^{-1}(n',neut(n'),anti(n'))\right)(x,neut(x),anti(x)),$$

which means $(n,neut(n),anti(n))^{-1}(n',neut(n'),anti(n'))\in Stab_{(x,neut(x),anti(x))}$, or

$$(n',neut(n'),anti(n'))\in(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}.$$

Therefore $(n,neut(n),anti(n))$ and $(n',neut(n'),anti(n'))$ have the same effect on $(x,neut(x),anti(x))$ in the case that $(n,neut(n),anti(n))$ and $(n',neut(n'),anti(n'))$ lie in the similar NT coset of $Stab_{(x,neut(x),anti(x))}$. (Recall that for all NET subgroups H and

$$N,(n',neut(n'),anti(n'))\in(n,neut(n),anti(n))H$$
$$(n',neut(n'),anti(n'))H=(n,neut(n),anti(n))H.$$

Whereby $Orb_{(x,neut(x),anti(x))}$ consists of the points $(n,neut(n),anti(n))(x,neut(x),anti(x))$ for varying $(n,neut(n),anti(n))$, and we showed elements of N have the similar effect on $(x,neut(x),anti(x))$ if and only if they lie in the similar

NT left coset of $Stab_{(x,neut(x),anti(x))}$, we get a bijections between the points in the NET orbit of $(x, neut(x), anti(x))$ and the NT left cosets of $Stab_{(x,neut(x),anti(x))}$ by

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) \to (n, neut(n), anti(n))Stab_{(x,neut(x),anti(x))}.$$

Therefore the cardinality of the NET orbit of $(x, neut(x), anti(x))$, which is $\left|Orb_{(x,neut(x),anti(x))}\right|$ equals the cardinality of the NT left cosets of $Stab_{(x,neut(x),anti(x))}$ in N.

Remark 3.20 that the NET orbits of a NETG action are a partition results in a NETG theory: conjugacy classes are a partitioning of a NETG and the NT left cosets of a NET subgroup partition the NETG. The first result utilizes the action of a NETG independently by NET conjugation, having NET conjugacy classes as its NET orbits. The second result utilizes the right inverse multiplication action of the NET subgroup on the NETG.

Corollary 3.21 Let a finite NETG act on a NET set.

a) The length of every NET orbit divides the size of N.

b) Points in a common NET orbit have conjugate stabilizers, and in particular the size of the NET stabilizer is the similar for all points in a NET orbit.

Proof: **a)** The length of NET orbit is an index of a NET subgroup, so it divides $|N|$.

b) If $(x, neut(x), anti(x))$ and $(y, neut(y), anti(y))$ are in the same NET orbit, write

$$(y, neut(y), anti(y)) = (n, neut(n), anti(n))(x, neut(x), anti(x)).$$

Then,

$$Stab_{(y,neut(y),anti(y))} = Stab_{(n,neut(n),anti(n))(x,neut(x),anti(x))}$$
$$=(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}(n,neut(n),anti(n))^{-1},$$

so the NET stabilizers of $(x, neut(x), anti(x))$ and $(y, neut(y), anti(y))$ are conjugate NET subgroups.

A converse of part b is not generally true: points with NET conjugate stabilizers need not be in the same NET orbit. Even points with the same NET stabilizer need nor be in the same NET orbit. For example, if N acts on itself trivially then all points have NET stabilizer N and all orbits have size 1.

Corollary 3.22 Let a NETG N acts on a NET set X, where X is finite. Let the distinct NET orbits of X be symbolized by $(x_1,neut(x_1),anti(x_1)),...,(x_t,neut(x_t),anti(x_t))$. Then

$$|X| = \sum_{i=1}^{t}\left|Orb(x_i, neut(x_i), anti(x_i))\right| = \sum_{i=1}^{t}\left[N : Stab(x_i, neut(x_i), anti(x_i))\right].$$

Proof: The NET set X can be written as the union of its NET orbits, which are mutually disjoint. The NET orbit-stabilizer formula tells us how large each NET orbit is.

Example 3.23 As an application of the NET orbit-stabilizer formula we describe why $|HK| = {|H||K|}/{|H \cap K|}$ for NET subgroups H and K of a finite NETG $N.$ At this point

$$HK = \left\{ \begin{array}{l} (h, neut(h), anti(h)), (k, neut(k), anti(k)) : (h, neut(h), anti(h)) \in H, \\ (K, neut(K), anti(K)) \in K \end{array} \right\}$$

is the NET set of products, which usually is just a subset of $N.$ To count the size of $HK,$ let the direct product of NETG $H \times K$ act on the NET set HK like this :

$$\begin{aligned} &\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big) \cdot (x, neut(x), anti(x)) \\ &= (h, neut(h), anti(h))(x, neut(x), anti(x))(h, neut(h), anti(h))^{-1}, \end{aligned}$$

which gives us a NETG action (the NETG is $H \times K$ and the NET set is HK). There is only 1 NET orbit where by $1_N = 1_N 1_N \in HK$ and

$$(h, neut(h), anti(h)), (k, neut(k), anti(k)) = \Big((h,neut(h),anti(h)),(k,neut(k),anti(k))^{-1}\Big) \cdot 1_N.$$

So that the NET orbit-stabilizer formula shows us

$$|HK| = \frac{|H \times K|}{|Stab1_N|}$$

$$= \frac{|H||K|}{\left|\left\{ \begin{array}{l} \big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big) : (h,neut(h),anti(h)),(k,neut(k),anti(k)) \cdot 1_N \\ = 1_N \end{array} \right\}\right|}.$$

The condition $\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big) \cdot 1_N = 1_N$ means

$$(h,neut(h),anti(h))(k,neut(k),anti(k))^{-1} = 1_N, \text{ so}$$

$$Stab1_N = \Big\{\big((h,neut(h),anti(h))(h,neut(h),anti(h))\big) : (h,neut(h),anti(h)) \in H \cap K\Big\}.$$

So that $|Stab1_N| = |H \cap K|$ and $|HK| = {|H||K|}/{|H \cap K|}.$

Theorem 3.24 Burnside's Lemma

Let a finite NETG N act on a finite NET set X in relation to r NET orbits. Subsequently r is the average number of NET fixed points of the elements of the NETG.

$$r = \frac{1}{|N|} \sum_{(n,neut(n),anti(n)) \in N} \left| Fix_{(n,neut(n),anti(n))}(X) \right|,$$

where

$$Fix_{(n,neut(n),anti(n))}(X)=\begin{Bmatrix}(x,neut(x),anti(x))\in X:(n,neut(n),anti(n))\\ (x,neut(x),anti(x))=(x,neut(x),anti(x))\end{Bmatrix}$$

is the NET set of elements of X fixed by $(n, neut(n), anti(n))$.

Don't confuse the NET set $Fix_{(n,neut(n),anti(n))}(X)$ in relation to the NET fixed points of the action: $Fix_{(n,neut(n),anti(n))}(X)$ is only the points fixed by the elements $(n, neut(n), anti(n))$. The NET set of NET fixed points for the action of N is the intersection of the NET sets $Fix_{(n,neut(n),anti(n))}(X)$ as $(n, neut(n), anti(n))$ runs over the NETG.

Proof: we will count

$$\begin{Bmatrix}\left((n, neut(n), anti(n)), (x, neut(x), anti(x))\right) \in N\times X :\\ (n, neut(n), anti(n))(x, neut(x), anti(x)) = (x, neut(x), anti(x))\end{Bmatrix}$$

in two ways. By counting over $(n, neut(n), anti(n))$'s first we have to add up the number of $(x, neut(x), anti(x))'s$ with

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) = (x, neut(x), anti(x)), \text{ so}$$

$$\left|\begin{Bmatrix}\left((n, neut(n), anti(n)), (x, neut(x), anti(x))\right) \in N\times X :\\ (n, neut(n), anti(n))(x, neut(x), anti(x)) = (x, neut(x), anti(x))\end{Bmatrix}\right|$$

$$=\sum_{(n,neut(n),anti(n))\in N}\left|Fix_{(n,neut(n),anti(n))}(X)\right|.$$

Next we count over the $(x, neut(x), anti(x))$'s and have to add up the number of $(n, neut(n), anti(n))$'s with $(n, neut(n), anti(n))(x, neut(x), anti(x)) = (x, neut(x), anti(x))$, i.e., with $(n, neut(n), anti(n)) \in Stab_{(x,neut(x),anti(x))}$:

$$\left|\begin{Bmatrix}\left((n, neut(n), anti(n)), (x, neut(x), anti(x))\right) \in N\times Y :\\ (n, neut(n), anti(n))(x, neut(x), anti(x)) = (x, neut(x), anti(x))\end{Bmatrix}\right|$$

$$=\sum_{(X,neut(X),anti(X))\in X}\left|Stab_{(x,neut(x),anti(x))}\right|.$$

Equating these two counts gives

$$= \sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$

$$= \sum_{(X,neut(X),anti(X))\in X} \left|Stab_{(x,neut(x),anti(x))}\right|.$$

By the NET orbit-stabilizer formula, $|N| \big/ \left|Stab_{(x,neut(x),anti(x))}\right| = \left|Orb_{(x,neut(x),anti(x))}\right|$, so

$$\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$

$$= \sum_{(X,neut(X),anti(X))\in X} \frac{|N|}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Divide by $|N|$:

$$\frac{1}{|N|}\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$

$$= \sum_{(x,neut(x),anti(x))\in X} \frac{1}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Let's examine the benefaction to the right side from points in a single NET orbit. If a NET orbit has n points in it, subsequently the sum over the points in that NET orbit is a sum of $\frac{1}{n}$ for n terms, and in other words equal to 1. Consequently the part of the sum over points in a NET orbit is 1, which makes the sum on the right side equal to the number of NET orbits, which is r.

Definition 3.25 Two actions of NETG N on a NET sets X and Y are called NET equivalent if there is a bijection $f : X \to Y$ as shown

$$f\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right) = (n,neut(n),anti(n))\, f\left((x,neut(x),anti(x))\right)$$

for all $(n,neut(n),anti(n)) \in N$ and $(x,neut(x),anti(x)) \in X$.

Actions of N on two NET sets are equivalent when N permutes elements in the similar method on the two NET sets following matching up the NET sets properly. When $f : X \to Y$ is a NET equivalence of NETG actions on X and Y,

$$(n,neut(n),anti(n))(x,neut(x),anti(x)) = (x,neut(x),anti(x))$$

if and only if

$$(n,neut(n),anti(n))\left(f\left((x,neut(x),anti(x))\right)\right) = f\left((x,neut(x),anti(x))\right),$$

so the NET stabilizer subgroups of $(x, neut(x), anti(x)) \in X$ and $f(x, neut(x), anti(x)) \in Y$ are the same.

Example 3.26 Let H and K be NET subgroup of N. The NETG N acts by left multiplication on N/H and N/K. If H and K are NET conjugate subgroups then these actions are equivalent: fix a representation $K=(n_0,neut(n_0),anti(n_0))H(n_0,neut(n_0),anti(n_0))^{-1}$ for some $(n_0,neut(n_0),anti(n_0))\in N$ and let $f : N/H \to N/K$ by

$$f\left((n, neut(n), anti(n))H\right) = (n, neut(n), anti(n))(n_0, neut(n_0), anti(n_0))^{-1}K.$$

This is well-defined (independent of the NT coset representatives for $(n, neut(n), anti(n))H$) since, for $(h, neut(h), anti(h)) \in H$,

$$\begin{aligned}
&f\left((n,neut(n),anti(n))h,neut(h),anti(h))H\right)\\
&=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}K\\
&=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}H(n_0,neut(n_0),anti(n_0))^{-1}\\
&=(n,neut(n),anti(n))H(n_0,neut(n_0),anti(n_0))^{-1}=(n,neut(n),anti(n))(n_0,neut(n_0),anti(n_0))^{-1}K.
\end{aligned}$$

There can be multiple equivalences between two equivalent NETG actions, just as there can be multiple neutro-isomorphisms between two isomorphic NETGs. If H and K are not NET conjugate then the actions have the same NET stabilizer subgroup, but the NET stabilizer subgroups of left NT cosets in N/H are NET conjugate to K, and none of the former and the latter are equal.

Theorem 3.27 An action of N that has one NET orbit is equivalent to the left multiplication action of N on some left NT coset space of N.

Proof : Assume that N acts on the NET set X in relation to one NET orbit. $Fix_{(x_0,neut(x_0),anti(x_0))} \in X$ and let $H = Stab_{(x_0,neut(x_0),anti(x_0))}$. We will Show the action of N on X is equivalent to the left multiplication action of N on N/H. Every $(x, neut(x), anti(x)) \in X$ has the form $(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0))$ for some $(n, neut(n), anti(n)) \in N$, and all elements in a left NT coset $(n, neut(n), anti(n))H$ have the same effect on $(x_0, neut(x_0), anti(x_0))$: for all $(h, neut(h), anti(h)) \in H$,

$$\begin{aligned}&\big((n,neut(n),anti(n))(h,neut(h),anti(h))\big)\big((x_0,neut(x_0),anti(x_0))\big)\\&=(n,neut(n),anti(n))\big((h,neut(h),anti(h))(x_0,neut(x_0),anti(x_0))\big).\end{aligned}$$

Let $f : N/H \to X$ by $f\big((n,neut(n),anti(n))H\big)=(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0))$.

This is well defined, as we just saw. Moreover,

$$\big((n, neut(n), anti(n)) \cdot (n', neut(n'), anti(n'))H\big) = (n, neut(n), anti(n))\, f\,\big((n', neut(n'), anti(n'))H\big)$$

since both sides equal

$$(n,neut(n),anti(n))(n',neut(n'),anti(n'))\big((n,neut(n),anti(n))\cdot(x_0,neut(x_0),anti(x_0))\big).$$

We will show f is a bijection. Since X has one NET orbit,

$$\begin{aligned}X &= \{(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0)) : (n,neut(n),anti(n))\in N\}\\&= \{ f\,\big((n, neut(n), anti(n))H\big) : (n, neut(n), anti(n)) \in N\},\end{aligned}$$

so f is onto. If $f\big((n_1,neut(n_1),anti(n_1))H\big)=f\big((n_2,neut(n_2),anti(n_2))H\big)$ then

$$(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0)) = (n_2, neut(n_2), anti(n_2))(x_0, neut(x_0), anti(x_0)),$$

so

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0)) = (x_0, neut(x_0), anti(x_0)).$$

Since $(x_0,neut(x_0),anti(x_0))$ has NET stabilizer H,

$$(n_2,neut(n_2),anti(n_2))^{-1}(n_1,neut(n_1),anti(n_1))\in H, \text{ so}$$

$$(n_1,neut(n_1),anti(n_1))H=(n_2,neut(n_2),anti(n_2))H.$$

Consequently f is one – to –one.
A special condition of this theorem tells that an action of N is equivalent to the left multiplication action of N independently in the case that the action has one NET orbit and the NET stabilizer subgroup are trivial.

5. Conclusion

The most important point of this research is first to define the NETs and subsequently use these NETs in order to describe the NETG action, NET orbits, stabilizers, and fixed point. We further introduced the Burnside's Lemma. Finally, we allow rise to a new field called NET Structures (namely, the neutrosophic extended triplet group action and Burnside's Lemma. Another researchers can work on the application of NETG action to NT vector spaces (representation of the NETG), number theory, analysis, geometry, and topological spaces.

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Conflicts of Interest

The authors declare no conflict of interest.

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