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SUPPORTING MATHEMATICAL AND DIGITAL COMPETENCES USEFUL FOR STEM EDUCATION

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Abstract: *Acronym STEM (Science, Technology, Engineering, and Mathematics) has become very frequently word among many stakeholders in the school policy. Mathematical and computational thinking are important for STEM Education. There are common thinking skills, but computational thinking focuses more on automation. Mathematical thinking focuses more on proofing. We present n our contribution the theoretical requirements that are needed for students in mathematical and digital competences. Practical examples represent, how it is possible to develop mentioned competences in educational practice.*

Keywords: STEM Education, mathematical and informatical thinking, mathematical and digital competencies, competitions Bebras and Mathematical Olympiad, language and technology.

INTRODUCTION

The acronym STEM (Science, Technology, Engineering, and Mathematics) is the term, which is very frequently used among many stakeholders in the school policy. According El Nagdi et al. (2018) many real-world problems are complex and need multidisciplinary approach. Tackling such problems requires not just the ability to use design thinking or inquiry, but also the ability to choose the best approach or combination of approaches that capitalizes on the strengths of each way of thinking. From this perspective, STEM encompasses the content, skills, and ways of thinking of each of the disciplines, but it also includes an understanding of the interactions between the disciplines and the ways they support and complement each other. It is important for each STEM discipline in education to build notions with better understanding by each learner.

Every STEM subject uses mathematical knowledge and thinking. For this reason, it is important to support development of mathematical competences. Recommendation (2006) defines mathematical competence as the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts). For this goal it is important to use real-world problems with practical applications.

It is important for necessary knowledge in mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers. An individual should have the skills to apply basic mathematical principles and processes in everyday contexts at home and work, and to follow and assess chains of arguments. An individual should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids. A positive attitude in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity.

Digital competence involves and obtains the confident and critical use of Information Society Technology (IST) for work, leisure and communication. It is underpinned by basic skills in Information and Communication Technologies (ICT): the use of computers to retrieve, assess, store, produce, present and exchange information, and to communicate and participate in collaborative networks via the Internet.

Digital competence needs a sound understanding and knowledge of the nature, role and opportunities of IST in everyday contexts: in personal and social life as well as at work. This includes main computer applications such as word processing, spreadsheets, databases, information storage and management, and an understanding of the opportunities and potential risks of the Internet and communication via electronic media (e-mail, network tools) for work, leisure, information sharing and collaborative networking, learning and research.

The main goal of the article is to present the theoretical requirements that are required of students in mathematical and digital competences. The second goal is to present practical solutions that will allow the education of these requirements set out in the theoretical part. The examples are selected so that they build competences and skills commonly referred to as computational thinking. To this end, the authors presented specific exercises that can be used in school practice to achieve the theoretical assumptions made by the requirements.

Skills needed for digital competence include the ability to search, collect and process information and use it in a critical and systematic way, assessing relevance and distinguishing the real from the virtual while recognising the links. Individuals should have skills to use tools to produce, present and understand complex information and the ability to access, search and use internet-based services. Individuals should also be able use IST to support critical thinking, creativity, and innovation.

Use of IST requires a critical and reflective attitude towards available information and a responsible use of the interactive media. An interest in engaging in communities and networks for cultural, social and/or professional purposes also supports this competence.

1. MATHEMATICAL COMPETENCES

The State Educational Curriculum in mathematics for lower secondary level (see SEC (2014)) expects that pupils:

- obtain the ability to use mathematics in their future life,
- develop their logical and critical thinking,
- learn to argue, communicate, and collaborate in a group by problem solving,
- can recognize mathematics as part of human culture and an important tool for social progress,
- can read with understanding compact texts containing numbers, dependencies and relationships and discontinuous texts containing tables, graphs, and diagrams,
- use understandable and mastered procedures and algorithms to solve problems, they can also to mathematize the real situation and interpret the obtained results,
- search, retrieve and process obtained information from adequately processed resources, including work with textbooks and another texts,
- develop skills that are related to the learning process, to teaching activities and to rational and independent learning.

Mathematics education in lower secondary level participates in developing pupils' ability to use ICT resources to search, process, store and present information. The use of appropriate software should facilitate some difficult calculations or procedures and also should focus on the base of the solved problem.

Computer aided mathematics education should lead to building a relationship between mathematics and reality, gaining experience in mathematical real-world situations and creating mathematical models. Open source software such GeoGebra helps to create many suitable models during the teaching hour.

2. INFORMATICAL COMPETENCES

The concept of IT (Information Technology) competence is based on two perspectives, which are particularly important in the context of considerations regarding new technologies: Jan van Dijk's model (Van Dijk, 2012) of access to new media and the information-knowledge-wisdom model of Neil Postman (Postman, 2011). The Van Dijk model includes four levels of access to new media:

- motivational access
- physical access,
- skills access,
- usage access.

The basic issue that decides about using new technologies is motivation (motivational access) - it depends on the decision to buy a computer and network connection and to acquire the skills necessary to use the appropriate applications. The next issue is physical access to computers and the Internet at home, work, school or other place (material access), and it also benefits from them, as access does not have to mean use (especially with no motivation). As a third level, van Dijk distinguishes competence access, as the use of computers and the Internet requires appropriate competence in the use of software, searching for information on the network, assessment of its credibility and suitability as well as the ability to process it and use it for its own purposes.

One of the documents that presents the scope of digital competences is the recommendation of the European Commission, which indicates in the document "European approach to media literacy in the digital environment" (Pérez-Tornero, Paredes, Baena, 2010) as the priority of the following issues of media and information education:

- ease of use of all existing media, from newspapers to virtual communities;
- active use of media through, interactive TV, using search engines or participating in virtual communities;
- critical approach to media regarding their quality and content;
- creative media use;
- understanding of the media economy and the difference between pluralism and the media market;
- awareness of issues related to copyright that are necessary for the "culture of legality".

Thus information literacy, it is defined as a set of knowledge and skills determining effective use of information resources, from the moment of recognizing the information gap, by defining information needs, selecting information sources, finding desirable information in them and critically evaluating them, to using them in his work and presenting to the recipients in the right form.

The importance of information competences is unquestionable, as they are - apart from preparing for the process of lifelong learning - one of the key conditions for the citizen's participation in the information society and decide on its development. These competences acquire particular importance especially in the perspective of the development of new information and communication technologies and the avalanche development of electronic information sources (Savolainen 2002).

The widespread use of such concepts as information noise, disinformation or information overload, indicates that information not only surrounds every human being, but can also be hemmed in. And this in turn means that every person should have a team of knowledge skills, allowing for efficient use of information resources. These skills can be grouped in several areas (Dąbrowska, Drzewiecki, Górecka 2012).

The first of the highlighted areas is in its essence most closely related to the problems of information sources and concerns such issues as knowledge of these sources and the ability to choose the ones that will be most suitable for the implementation of a given task. This is, therefore, a general "orientation" in the available sources of information and the ability to choose those that, in terms of credibility, ease of access, and adjustment to the age of users will be the most appropriate for the implementation of a given task.

The second area concerns the process of using selected sources, and thus direct search of information. This area includes issues related to the ability to build appropriate strategies and the selection of appropriate information retrieval techniques, and therefore such skills as, for example, narrowing down search results, using logical operators, etc.

The third area concerns the critical approach. This set of issues includes both critical assessment of sources and the information itself as well as information verification and evaluation processes.

The fourth area contains elements of work using information and information management. This area includes issues such as organizing and classifying information for personal use, recording and improving the results of work, as well as presenting it in a form that best suits the needs of recipients.

3. PYTHON AS A TOOL FOR EDUCATION OF PROGRAMMING

According to the Computing Our Future report (Computing Our Future, 2015), programming has already been introduced into education programmes in most European Union countries. The leading countries in this area include Great Britain and Denmark. The teaching of programming in schools is supported by educational programs run by non-governmental organizations and commercial entities. It looks a bit different in the United States, where the most programs devoted to learning programming work. There, it is primarily a domain of informal, or commercial, education (in the form of paid courses) and charity (educational programs of non-governmental organizations).

Also, in Poland, the emphasis on programming from an early age has increased. From the 2017 school year, a new core curriculum was introduced, thanks to which: the school will create conditions for students to acquire knowledge and skills needed to solve problems using methods and techniques derived from computer science. In Polish schools the number of IT hours increased from 210 to 280 hours, because - as the head of the Ministry explains - we want the Polish school to respond to the needs of the 21st century. The change applies from the 4th grade to the completion of secondary school. The new core curriculum also includes classes I-III, where the emphasis has also been on IT education.

Python is an interpreted, high-level, general-purpose programming language. Created by Guido van Rossum and first released in 1991, Python's design philosophy emphasizes code readability with its notable use of significant whitespace. Its language constructs and object-oriented approach aim to help programmers write clear, logical code for small and large-scale projects. (Kuhlman 2009).

Python is dynamically typed and garbage-collected. It supports multiple programming paradigms, including procedural, object-oriented, and functional programming. Python is often described as a "batteries included" language due to its comprehensive standard library (Kuhlman, 2009).

Python, like other high-level languages, has many typical programming constructs, such as conditional statements, loops, variables and specific syntax characteristic of this language. For example, a parallel assignment (a, b = B% a, a), generators (yield), or written lists ([len (word) for woord in words]). In general, we deal with difficult problems in programming, which require a lot of mathematical knowledge. However, there are a few tasks, we will present one of them, which can be presented in an accessible form for the recipient with the benefit of his development of thinking, not just rewriting the code.

4. MATHEMATICAL AND INFORMATICAL THINKING IN EXAMPLES

Mathematical and digital competences need support during the educational process with suitable activities. One of them are competitions Mathematical Olympiad (2006) and Bebras (2019). They develop mathematical and informatical thinking. It is important in school education to develop thinking ability, not only knowledge of facts. Problem solving strategy is also important for these competitions.

To illustrate, in Bebras competition there was the following example:

Example 1. To arrange a dinner party Sara needs to talk to five friends: Alicia, Beat, Caroline, David and Emil. Sara can talk to Emil right away. However, to talk to her other friends, there are a few points to consider:

- Before she talks to David, she must first talk to Alicia.
- Before she talks to Beat, she must first talk to Emil.
- Before she talks to Caroline, she must first talk to Beat and David.
- Before she talks to Alicia, she must first talk to Beat and Emil.

In what order should Sara talk to all of her friends if she wants to talk to all of them?

Solution:

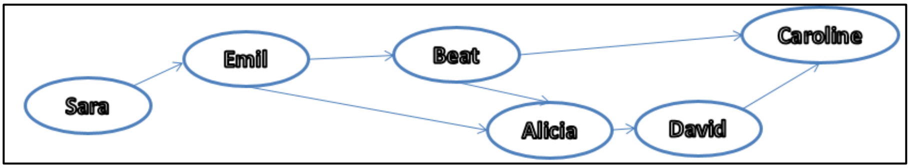

Figure 1. Solution of the Example 1

Source: Own work

The previous example used logical and combinatorial thinking. These kinds of thinking are possible to use in the following example from Mathematical Olympiad for 15 year old pupils in Slovakia:

Example 2. How many six-digits numbers exist, which have the product of digits the number 750?

Solution: The base for the solution is decomposition of the number 750 to prime numbers. It is 750 = 2. 3. 5. 5. 5.

It is possible from this decomposition to create six-digits numbers only in the case, that we use digits from this decomposition and the digit 1 at least once. It brings two possibilities:

- We have digits 1, 2, 3, 5, 5, 5. We obtain in this case (6!):(3!) = 6. 4. 5 = 120
- ○ possibilities.
- We can make from the digits 2 and 3 the digit 2.3 = 6. We use in this case
- ○ the digits 1, 1, 6, 5, 5, 5. We obtain (6!):(3! . 2!) = 6. 2. 5 = 60 possibilities.

Altogether we have 120 + 60 = 180 possibilities.

The purpose of the following exercise prepared in Python is to show that each logical problem can be solved by means of stages and its simplification in the form of a sequence of code lines, which constitute an algorithmic part of the task. At each stage, the student may interrupt the work and ask additional questions. The greater the student's level of competence, the more stages he will solve himself.

In Polish education there is no recommendation as to which programming language to choose - a particular language is free in its choice, so the teacher can choose the programming language that best suits the requirements for hardware and software in its class.

Example 3: Write a function for (x, y), which will result in a number specifying the day, when a small beetle is at the top of a 10-meter pole. Assumptions for the task: beetle climbs by (x) centimetres in the day and falls by (y) centimetres at night. We assume that (x> y).

Example: The result for (300,100) is 5. The result for (4,2) is 499. The task comes from the LOGIA 13 competition, the first stage.

Necessary knowledge to solve: the student should know the basic mathematical operations (+, -, /, *), and complete division and sharing with the rest. The student should also know how to define a function in programming.

Solution: We will solve the task in stages with further difficulties. At the beginning, let's assume that the beetle climbs only and does not fall at night. It is for the value (y) that we must assume that: (y = 0). Then you have to assume that 1000 units is a multiple (x).

Then we will consider cases in which not necessarily 1000 is a multiple (x). Then we solve the problem, where (y) can take any value, that is, we have to subtract (x) from (y).

Consider the question when the beetle is at the top of a ten-centimetre pole if it climbs by (x) centimeters every day without falling off at night.

Assuming that 1000 is a multiple (x), it will be:

```
Def when (x):

Return 1000 // x
```

If we do not know if 1000 is a multiple (x), then the problem is a bit more difficult. We assume that we do not use other mathematical functions, e.g. rounding up. Auxiliary question, which we can present, when it will be at a height that allows one day to climb to the top. Then you have to add one day for the beetle to enter the pole.

```
Def when (x):

Return (1000-1) // x + 1
```

It is not difficult to notice that if the day falls by (x) and falls by (y) at night, he overcomes (x-y) daily. As in the first example, we assume that 1000 is a multiple (x-y).

Def when (x, y):

Return (1000 // x-y

Finally, we go to solving the right task: beetle in the day enters by (x), and at night it decreases by (y), and 1000 does not have to be a multiple (x-y).

Def when (x, y):

Return (1000-y-1) // (x-y) +1

We can find these materials in Python (2019). The above code can be tested on several examples. The task does not require advanced programming and mathematical skills. The task, however, is not easy. A standalone solution, however, is perfect for a novice programmer.

5. CONCLUSIONS

Mathematical and computational thinking are both essential in problem solving. There are common thinking skills, but informatical thinking focuses more on automation. Mathematical thinking focuses more on proofing. Pupils develop also their cognitive abilities and correctness to formulate properties of mathematic objects (see Kopáčová, Žilková, 2017). Using ICT with cognitive abilities of pupils obtained during the mathematics lessons is important in STEM Education. Interesting examples can be found in Koreňová, Fuchsová (2019). The greatest advantage of programming in Python is its simplicity based on the use of commands in it, which are literal meanings, e.g.: def, return (a + b), which in turn allow to: define, return values from adding. Another advantage is expressivity, which is 'expressed', it means the use of a small amount of code as opposed to other languages like java, JavaScript, C ++. The last significant advantage of the didactic point is the fact that in Python do not need to define minors as guilty languages. In Python, just a number or word is given. In other languages, you must assign this data to specific types, for example: "int", "string", "boolean".

These three advantages make solving logical, mathematical and algorithmic problems very intuitive, simple and short, which allows the student to focus on the problem, not to learn the details of a given programming language. It should be remembered that computational thinking has many features of any programming language, i.e. it can be in today's world of communication medium of the real and virtual world, and it can be a medium of cognition in general. Ludwig Wittgenstein truly gives this essence: "The limits of my language mean the limits of my world." (Martland, 1975). Today, this quote in the context of technology can be paraphrased: "The borders of our programming language of technology are the limits of our knowledge of the world through technology." Therefore, the development of competence in the field of computational thinking contributes to development: logical thinking, creativity in the search for solutions, heuristic thinking, algorithmic thinking, and innovation (see Papert, 1980).

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