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A Thermo-Gas-Dynamic Model for the Bifurcation Analysis of Refrigerant-Lubricated Gas Foil Bearing Rotor Systems

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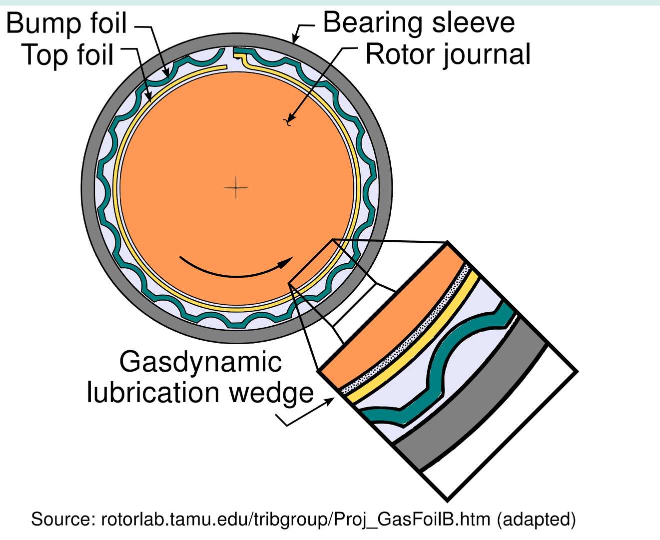
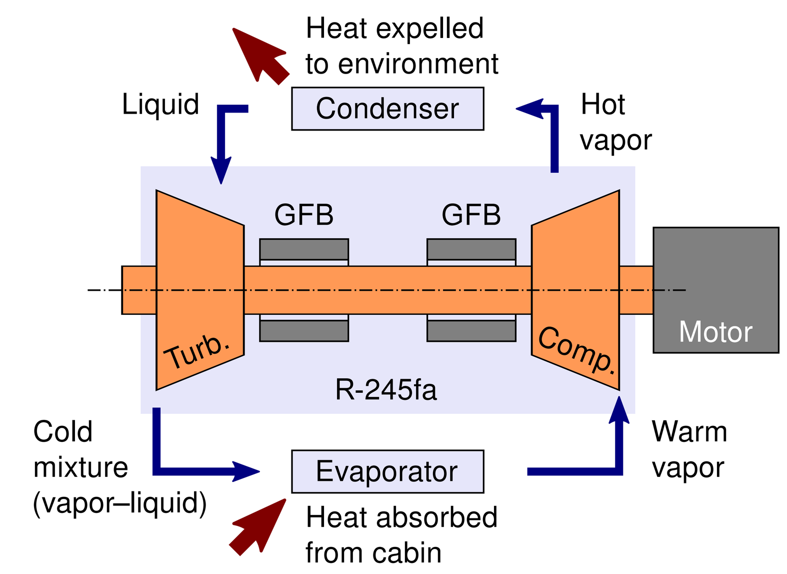
Self-Acting Gas Foil Bearings (GFBs)



- High-speed rotor supported by gasdynamic lubrication wedge
- Oil-free machinery offers high energy efficiency and low wear

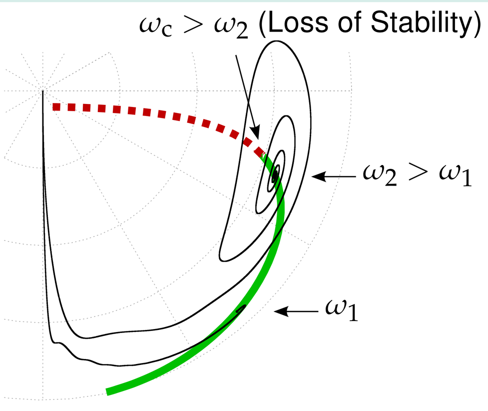
Application: Vapor-Compression Refrigeration

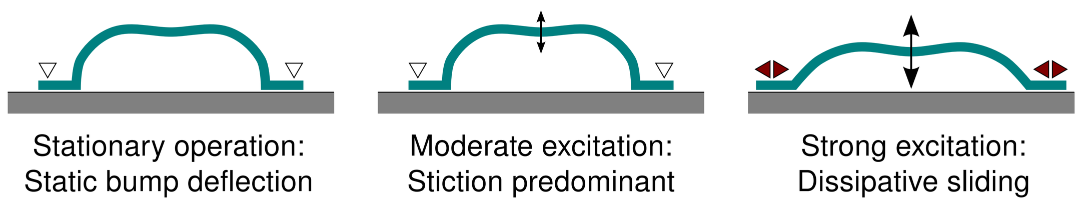


- System optimized by using refrigerant as lubricating fluid

Challenge: Self-Excited Vibrations

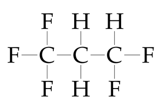
- Stationary operating points tend to become unstable at elevated rotational speeds
- Occurrence of self-excited rotor vibrations with large amplitudes (fluid whirl)
- Vibrations calmed down by deliberately introduced friction



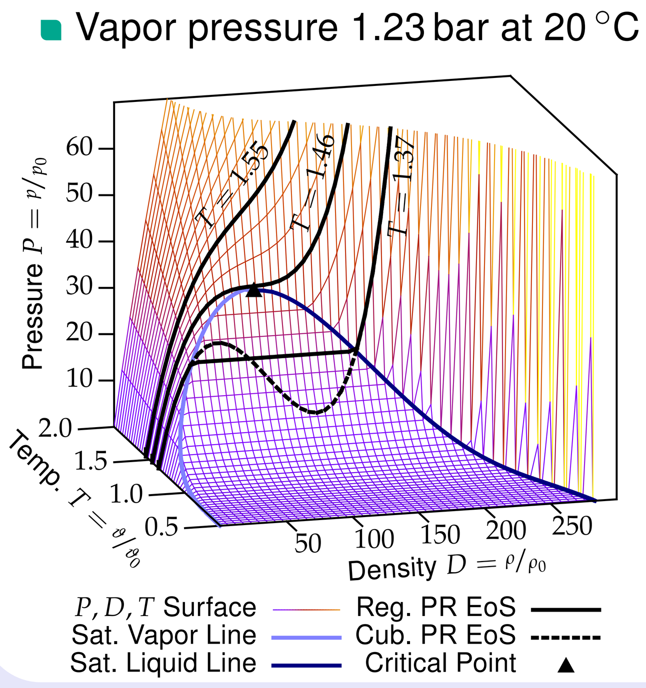


Fluid Model for Non-Ideal Gases

- Refrigerant R-245fa (1,1,1,3,3-pentafluoropropane)



- Vapor pressure 1.23 bar at 20 °C



- Cubic Peng–Robinson equation of state (PR EoS)

$$P_{\mathrm{PR}}(D,T) = \frac{1}{p_0}\left[\frac{R_\mathrm{m}\vartheta_0 T}{\left(\frac{M_\mathrm{m}}{\rho_0 D}\right) - b} - \frac{a(\vartheta_0 T)}{\left(\frac{M_\mathrm{m}}{\rho_0 D}\right)^2 + 2b\left(\frac{M_\mathrm{m}}{\rho_0 D}\right) - b^2}\right]$$

- Equilibrium vapor pressure (coexistence curve) by fitting simplified Clausius–Clapeyron solution $P_{\mathrm{sat}}(T) = \exp\left(C_0 - C_1 T^{-1} - C_2 \ln T\right)$
- Regularization of PR EoS by algebraic solution of cubic equation $P_{\mathrm{PR}}(D,T) = P_{\mathrm{sat}}(T)$ with roots $D_\mathrm{v}(T) < D_\mathrm{m}(T) < D_\mathrm{l}(T)$
- Mass fraction of liquid

$$W_\mathrm{l}(D,T) = \frac{D - D_\mathrm{v}(T)}{D_\mathrm{l}(T) - D_\mathrm{v}(T)}$$

- Fluid film thickness

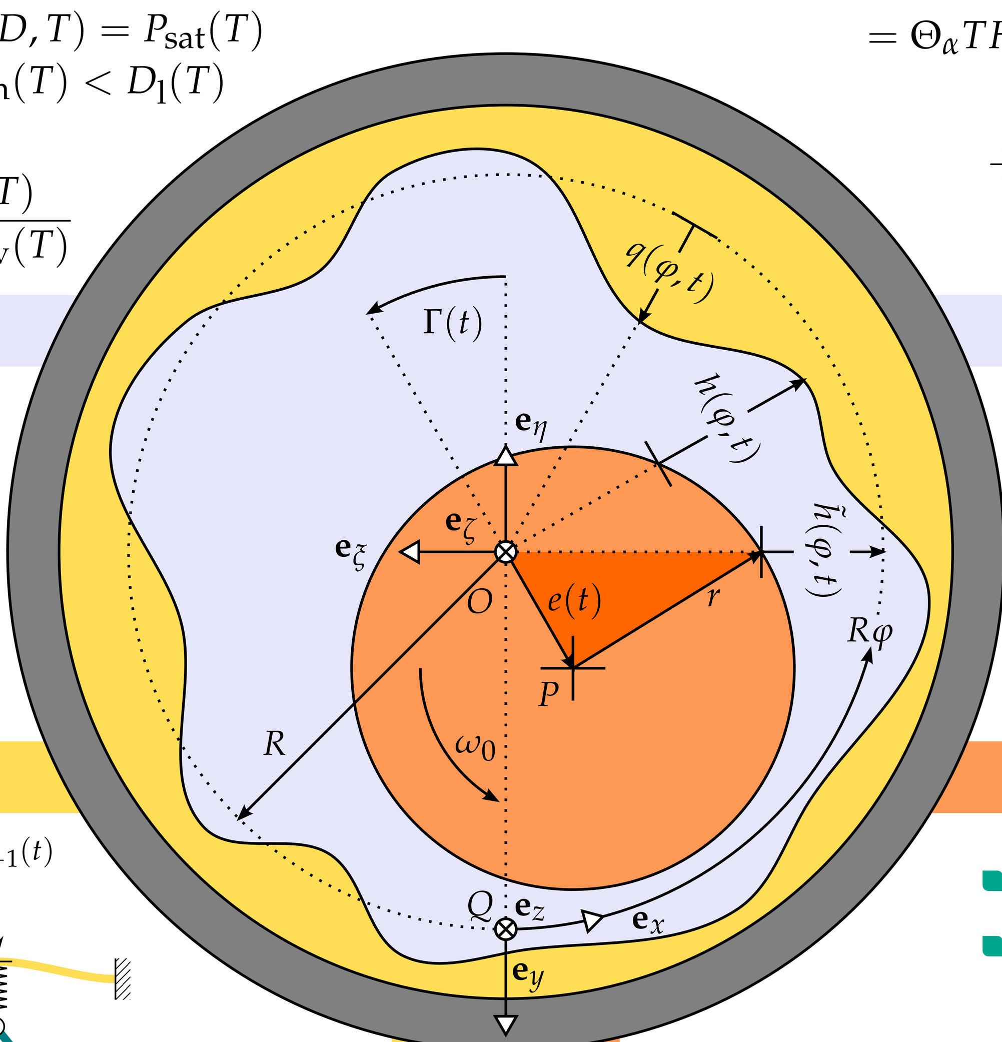
$$H(\varphi,\tau) = \underbrace{1}_{\text{Clearance}} \underbrace{- Q(\varphi,\tau)}_{\text{Structure deformation}} \underbrace{- \varepsilon(\tau)\cos\left[\varphi - \gamma(\tau)\right]}_{\text{Rotor journal displacement}}$$

- Reynolds equation

$$\frac{\partial}{\partial\tau}(DH) + \underbrace{\frac{\Lambda}{2}\frac{\partial}{\partial\varphi}(DH)}_{\text{Couette flow}} = \underbrace{\frac{\partial}{\partial\varphi}\left(\frac{DH^3}{2V}\frac{\partial P}{\partial\varphi}\right) + \kappa^2\frac{\partial}{\partial Z}\left(\frac{DH^3}{2V}\frac{\partial P}{\partial Z}\right)}_{\text{Poiseuille flow}}$$

- Energy equation

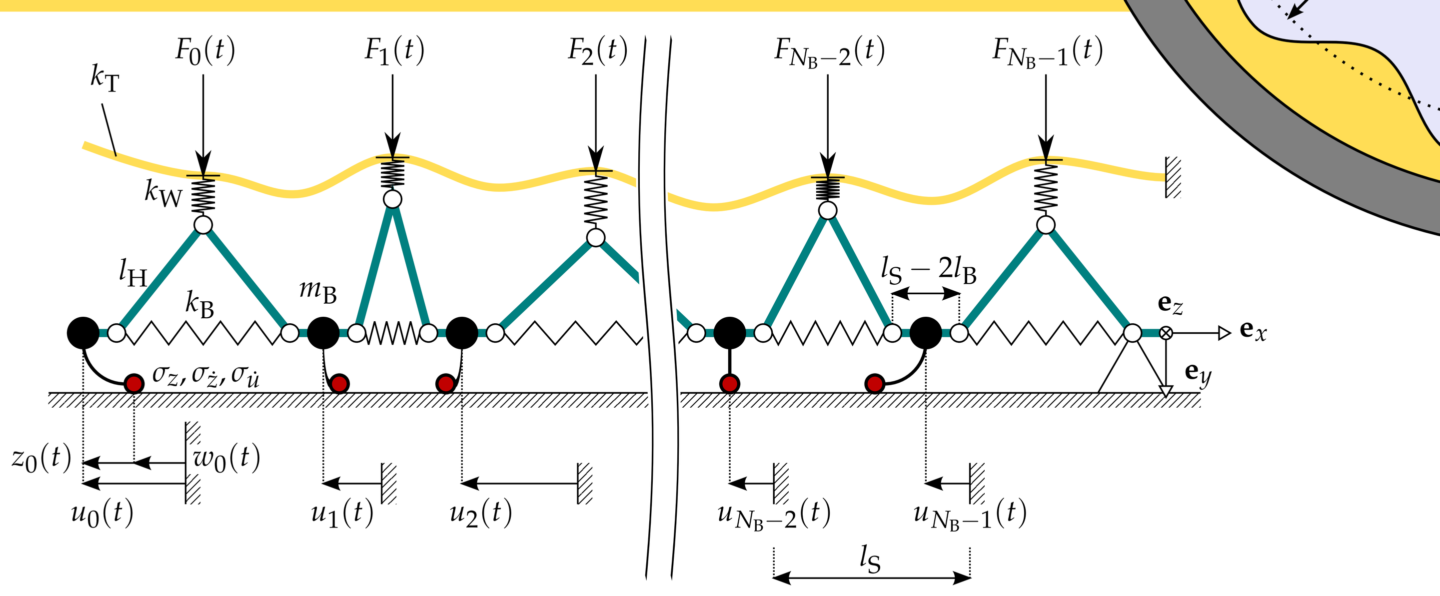
$$\Theta_{c_p} DH^2\left[\frac{\partial T}{\partial\tau} + \frac{\partial T}{\partial\varphi}\left(\frac{\Lambda}{2} - \frac{H^2\partial P}{2V\partial\varphi}\right) + \kappa^2\frac{\partial T}{\partial Z}\left(-\frac{H^2\partial P}{2V\partial Z}\right)\right]$$
$$= \Theta_\alpha TH^2\left[\frac{\partial P}{\partial\tau} + \frac{\partial P}{\partial\varphi}\left(\frac{\Lambda}{2} - \frac{H^2\partial P}{2V\partial\varphi}\right) + \kappa^2\frac{\partial P}{\partial Z}\left(-\frac{H^2\partial P}{2V\partial Z}\right)\right]$$
$$+ \underbrace{2V\left[\frac{\Lambda^2}{12} + \frac{H^4}{4V^2}\left(\frac{\partial P}{\partial\varphi}\right)^2 + \kappa^2\frac{H^4}{4V^2}\left(\frac{\partial P}{\partial Z}\right)^2\right]}_{\text{Viscous dissipation}} + \underbrace{2\Theta_k H(T_\mathrm{a} - T)}_{\text{Heat transfer}}$$



Fluid Dynamics $P(\varphi,Z,\tau)$ ↓ **Structure Dynamics** $Q(\varphi,\tau)$, $Q'(\varphi,\tau)$ ↑

Rotor Dynamics $\varepsilon(\tau)$, $\varepsilon'(\tau)$, $\gamma(\tau)$, $\gamma'(\tau)$ ↑ **Fluid Dynamics** $P(\varphi,Z,\tau)$ ↓

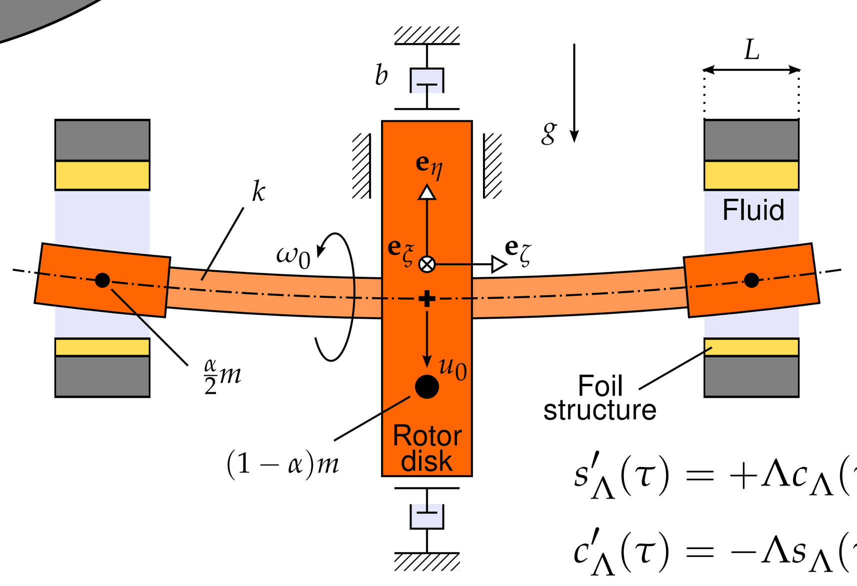
Foil Structure Friction Model



- Triangular spring–mass–rod arrangement with superposed elastic beam model
- Elasto-plastic bristle friction $Z_n'(\tau) = U_n'(\tau) - \alpha_n(\tau)\beta_n(\tau)\left|U_n'(\tau)\right|\frac{\sigma_Z Z_n(\tau)}{\mu_n(\tau) f_{\perp,n}(\tau)}$

Jeffcott–Laval Rotor Model

- Elastic horizontal rotor symmetrically mounted on two GFBs
- Small proportion $\alpha/2$ of total mass shifted to each journal



- Constant vertical load and unbalanced disk
- Rotational speed (bearing number) $\Lambda = \frac{6\mu_0\omega_0}{p_0}\left(\frac{R}{C}\right)^2$
- Unbalance transformation

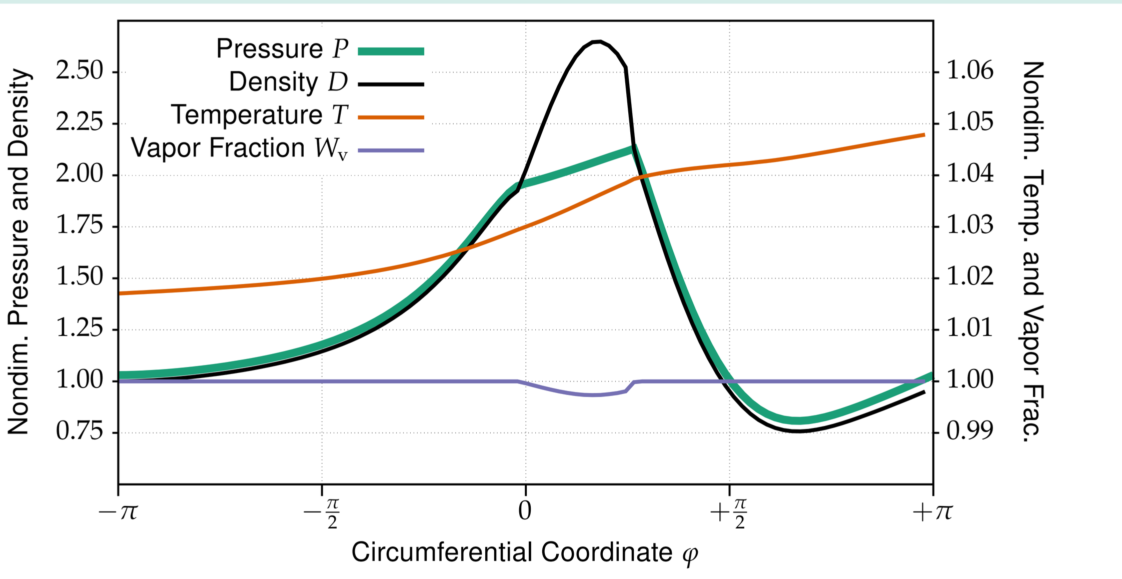
$$s_\Lambda'(\tau) = +\Lambda c_\Lambda(\tau) - s_\Lambda(\tau)\left[s_\Lambda(\tau)^2 + c_\Lambda(\tau)^2 - 1\right]$$
$$c_\Lambda'(\tau) = -\Lambda s_\Lambda(\tau) - c_\Lambda(\tau)\left[s_\Lambda(\tau)^2 + c_\Lambda(\tau)^2 - 1\right]$$

Computational Analysis

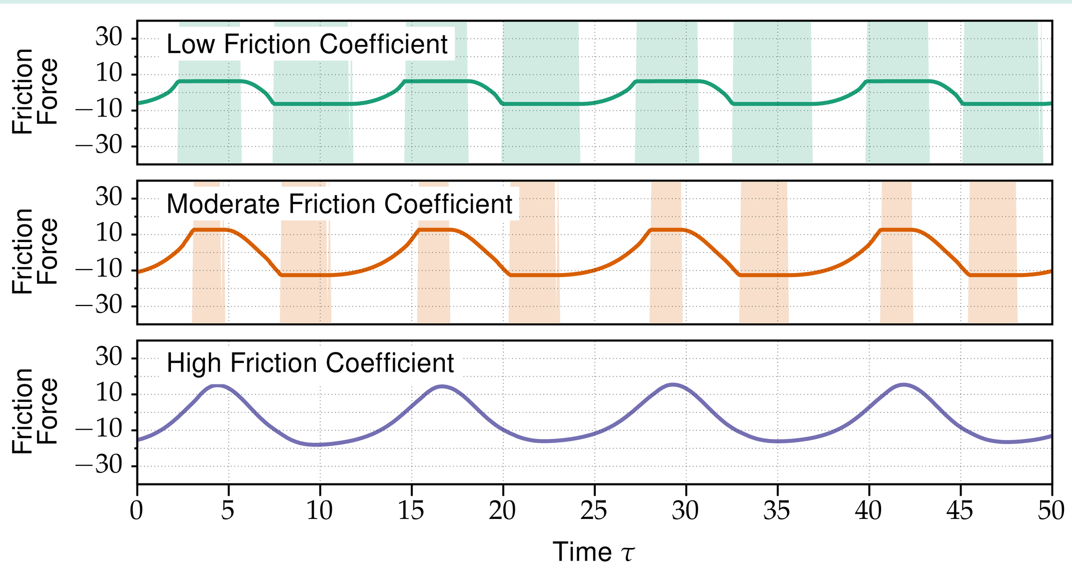
- Finite difference discretization on computational grid $N_\varphi \times N_Z = 469 \times 15$
- Simultaneous subproblem solution by means of collective state vector
- Nonlinear ODE system $\mathbf{s}'(\tau) = \mathbf{k}\{\mathbf{s}(\tau),\Lambda\}$ with $\mathbf{k}\colon \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$

$$\mathbf{s}(\tau) = \left[\cdots D_{i,j}(\tau) \cdots T_{i,j}(\tau) \cdots \;\middle|\; \cdots U_n(\tau)\; U_n'(\tau)\; Z_n(\tau) \cdots \;\middle|\; X(\tau)\; X'(\tau) \cdots X_\mathrm{D}(\tau)\; X_\mathrm{D}'(\tau) \cdots\right]^\top \in \mathbb{R}^n$$

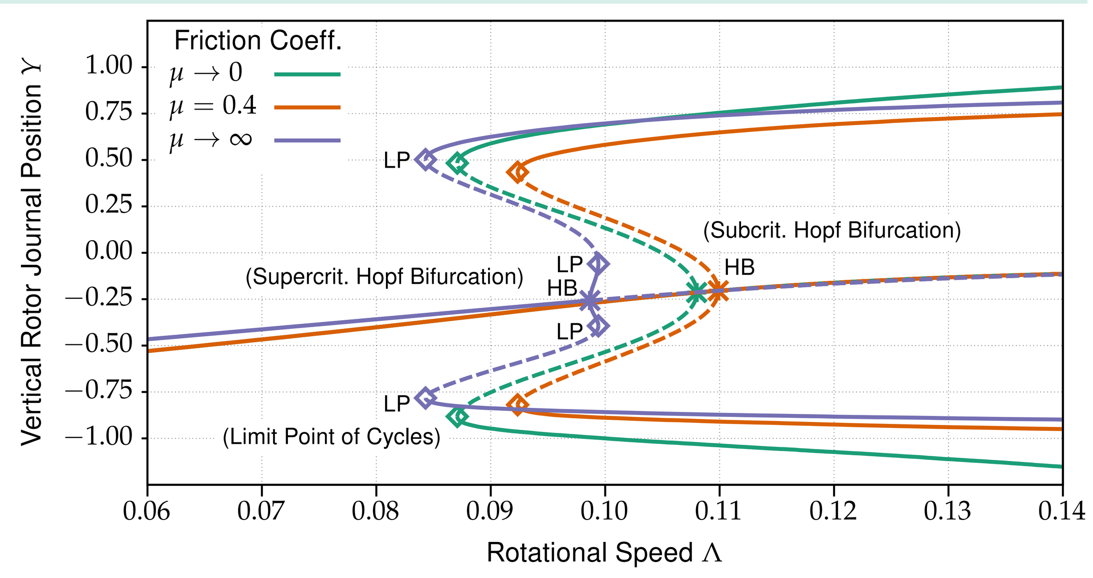
Results and Conclusions



1 Stable stationary operation under two-phase flow conditions



2 Detailed investigation of stick–slip transitions in foil contacts



3 Bifurcation diagram giving insights into coexisting solutions