DETERMINATION OF COINCIDENCE SUMMING
CORRECTION FACTORS FOR 22Na POINT SOURCE

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Abstract. The coincidence summing effect plays an important role in HPGe spectrometry, especially at low source-detector distances, due to a large solid angle; therefore, the calculation of correction factors is necessary. The aim of the research described in this paper was to compare values of correction factors for a 22Na point source obtained using the GESPECOR software package (Monte-Carlo method) and experimentally obtained values. Measurements were performed using a semiconductor HPGe spectrometer and the point source axially positioned at nine different distances from the detector end-cap. For the purpose of determining correction factors, a system of equations was formed, which, besides nuclear data as the input parameters, uses the experimentally obtained values of the total count in the entire spectrum, as well as the counts in the full energy peaks. The system of equations was solved for each particular case and correction factors were determined. By comparing the results obtained using the experimental and Monte-Carlo method, it was found that the correction factors for the 22Na point source have discrepancies less than 3%. The significance of these discrepancies was also verified from a statistical point of view using a Student's t-test.

Key words: Coincidence summing, HPGe spectrometry, Monte-Carlo method

1. INTRODUCTION

The coincidence summing effect occurs when two or more gamma photons are emitted in a cascade during the decay of radionuclides in a time interval that is significantly smaller than the resolving time of the detector. Therefore, as a result of this effect, the sum peak could be recorded in the spectrum. The coincidence summing effect leads to the loss of counts under the full energy peaks, which results in a count increase under the sum peak. Then, a correction of the efficiency for individual full energy peaks must be done. The coincidence summing effect depends on the radionuclide decay scheme and also depends on the source-detector solid angle. For close geometries, this effect is dominant, while it decreases with an increase of source-to-detector distance [1]. The determination of coincidence summing correction factors in gamma spectrometry was the subject of several publications [...] this problem gained a new dimension by introducing the Monte-Carlo method [6, 7, 8]. For the purpose of this research, a 22Na point source was axially positioned at nine different distances from the detector end-cap. Based on these measurements, experimental values for efficiencies were obtained and used for the calculation of the coincidence summing correction factors. This paper presents a comparison between the correction factors obtained with the Monte-Carlo simulation, using the GESPECOR v4.2. software package (Germanium Spectrometry Correction Factors) and the experimentally obtained values.

2. DETERMINATION OF CORRECTION FACTORS IN THE CASE OF 22Na

The 22Na radionuclide is one of the most important positron emitters, mainly because of its relatively long half-life of (2.6029 ± 0.0008) years [9], with a simple decay scheme (Fig. 1). 22Na decays predominantly by 90.3% β^+ and 9.64% EC to the 1274.537 keV level (I_γ = 99.94%) of 22Ne, and a very small fraction (0.055%) disintegrates to the ground state of 22Ne [9]. Radionuclides emitting β^+ particles should have an adequate treatment in gamma-ray spectrometry. When a positron is emitted, it interacts with a surrounding electron and in the process of annihilation two gamma [...] keV (I_{511}=180.7%) [9], [...] ill be created. Thus, the peak of 511 keV or 1022 keV will be formed, as well as sum-peaks on 1785.537 keV and 2296.537 keV. Since the source was positioned above the detector at nine different distances and photons from the annihilation process are emitted under the angle of 180°, only one photon with the energy of 511 keV is deposited in crystal, so the peak on 1022 keV could not be formed.
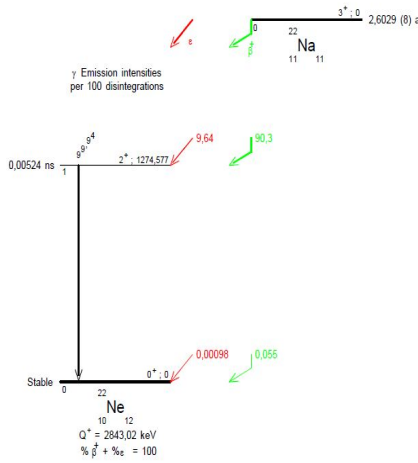
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Figure 1. Decay scheme of ^{22}Na [9]

2.1. Theoretical approach to experimental determination of correction factors

For the purpose of calculating correction factors, a system of equations was formed:

$$N_1 = Ap_1\varepsilon_1(1 - p_2\varepsilon_{t2}), \tag{1}$$

$$N_2 = Ap_2\varepsilon_2(1 - p_1\varepsilon_{t1}), \tag{2}$$

$$N_{12} = Ap_1\varepsilon_1 p_2\varepsilon_2, \tag{3}$$

$$T = A(p_1\varepsilon_{t1} + p_2\varepsilon_{t2} - p_1 p_2\varepsilon_{t1}\varepsilon_{t2}), \tag{4}$$

where:
N_1, N_2 – net count rates under the full energy peaks corresponding to energies E_1 (511 keV) and E_2 (1274.537 keV), respectively,
N_{12} – net count rate under the sum-peak corresponding to the energy of 1785.537 keV,
T – total count rate in the entire spectrum,
A – source activity,
p_1, p_2 – photon emission intensities corresponding to energies E_1 and E_2, respectively,
ε_1, ε_2 – full energy peak efficiencies corresponding to energies E_1 and E_2, respectively, and,
ε_{t1}, ε_{t2} – total efficiencies which correspond to energies E_1 and E_2, respectively.

Due to a considerable statistical uncertainty of the net count rate N_{12}, equation (3) makes the solution of the system of equations unstable. Thus, the system of equations with four unknowns was formed from equations (1), (2), and (4). The system of equations was solved by introducing an approximation in which total efficiencies were presented as a function of full energy peak efficiencies:

$$\frac{\varepsilon_{t1}}{\varepsilon_1} = \sqrt{k}\,E_1 + 1, \tag{5}$$

and

$$\frac{\varepsilon_{t2}}{\varepsilon_2} = \sqrt{k}\,E_2 + 1, \tag{6}$$

where k is a parameter which enables solving the system of equations. Introducing of this parameter was based on reducing of unknowns in system of equations. To solve this system of equations, the Mathematica 5.0 software package (Wolfram Research, Inc., USA) was used [10]. The values of corrected efficiencies $\varepsilon_i^{'}$ were obtained directly by solving the system of equations, while uncorrected values for efficiencies were calculated using the formula:

$$\varepsilon_i = \frac{N_i}{A\,p_i}, \ i = 1, 2. \tag{7}$$

Correction factors were obtained as a ratio between these two values:

$$K_i = \frac{\varepsilon_i^{'}}{\varepsilon_i}, \ i = 1, 2. \tag{8}$$

2.2. Numerical approach to determining correction factors using GESPECOR

GESPECOR is one of the most important software packages based on the Monte-Carlo simulation method, which gives the possibility for direct calculation of correction factors [11]. The Monte-Carlo method is based on a simulation of passing a large number of photons through specific medium (sample, detector, or some other specific layer). For the purpose of correction-factor calculation, several input parameters in the simulation were defined: parameters for ^{22}Na, related to detector and point source geometry, the material from which they were made, and the type of detector (HPGe). The input number of photons was set to 10^6. The correction factors were calculated for energies of 511 keV and 1274.537 keV.

3. EXPERIMENTAL SETTINGS/CONDITIONS AND RESULTS

For gamma-ray detection, the semiconductor HPGe spectrometer, p-type, GEM30-70, Ortec, with auxiliary electronics was used. The detector has a relative efficiency of 35.5% and a resolution of 1.66 keV at 1332 keV (^{60}Co). The detector is placed inside a lead shield whose thickness is 10 cm.

The radioactive point source/standard ^{22}Na is purchased from the Czech Metrology Institute (the activity of this source on the reference date was (5786 ± 46) Bq) and the source was placed and measured on the central detector axis at nine different distances from the detector end-cap, as shown in Table 1. A spectrum analysis was performed using the Genie2000 software, Canberra. The measurement time was between 65000 s and 250000 s, depending on the point source position. The values of calculated correction factors in case of ^{22}Na, for energies of

511 keV and 1274.537 keV, obtained in the experiment (K_{1E} and K_{2E}) and obtained by the Monte-Carlo simulation (K_{1G} and K_{2G}) are numerically presented in Table 1 and graphically in Figures 2 and 3. In Table 1, the discrepancies between the mentioned correction factor values (ΔK_1 and ΔK_2) are included.
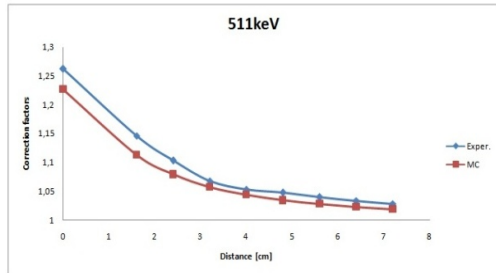

Figure 2. Correction factors obtained experimentally and with the Monte-Carlo method in case of 511 keV

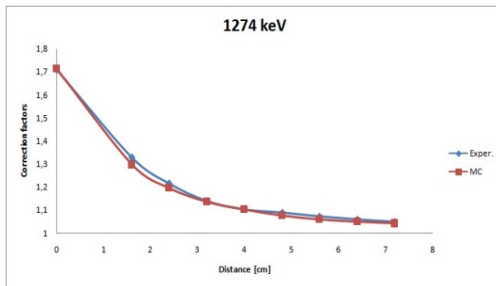Figure 3. Correction factors obtained experimentally and with the Monte-Carlo method in case of 1274 keV

4. DISCUSSION AND CONCLUSIONS

Correction factors obtained experimentally and with the Monte-Carlo simulation have the highest values in contact geometry, since in this position the solid angle has maximum value. Calculation of relative uncertainty for Monte-Carlo simulation, was obtained by variation of the following input parameters: crystal radius, crystal length, thickness of dead layer and distance from active face to entrance window. General law of uncertainty propagation was applied to obtain relative uncertainty. The relative uncertainty in the case of Monte-Carlo simulation for energies of 511 keV and 1274.537 keV amounts to 1% and 2.4%, respectively, whereas the relative measurement of uncertainty for experimentally obtained correction factors amounts to 1%.

From Table 1, it can be seen that the maximum discrepancy between the correction factor values is at a 1.6-cm distance from the detector end-cap and also in contact geometry at the energy value of 511 keV.

Based on the results given in Table 1, it was found that correction factor values differ up to 3%, from which it can be concluded that the correction factor values obtained by the Monte-Carlo method are in a good agreement with the experimentally obtained ones. The experimentally determined correction factors are generally greater than the correction factors obtained by the Monte-Carlo simulation, which is probably the result of inaccurately defined input simulation parameters that were related to the geometry of the detector. Therefore, by adjusting input parameters, these discrepancies would be even lower.

Table 1. Correction factor values for the coincidence summing effect in case of ^{22}Na, obtained experimentally and with the GESPECOR software

SOURCE-TO-DETECTOR DISTANCE	TCS FACTOR FOR 511 KEV		DIFFERENCE [%]	TCS FACTOR FOR 1274.5 KEV		DIFFERENCE [%]
D(CM)	EXPERIMENTAL K_{1E}	CALCULATION K_{1G}	ΔK_1	EXPERIMENTAL K_{2E}	CALCULATION K_{2G}	ΔK_2
0	1.262	1.227	2.85	1.708	1.712	-0.23
1.6	1.146	1.113	2.96	1.327	1.297	2.31
2.4	1.103	1.079	2.22	1.214	1.195	1.59
3.2	1.068	1.057	1.04	1.138	1.136	0.18
4.0	1.053	1.044	0.86	1.104	1.103	0.09
4.8	1.047	1.034	1.26	1.090	1.077	1.21
5.6	1.039	1.028	1.07	1.073	1.061	1.13
6.4	1.033	1.023	0.98	1.060	1.050	0.95
7.2	1.028	1.019	0.88	1.050	1.042	0.77

The significance of these differences was also verified from a statistical point of view with a Student's t-test of the difference between the arithmetic means of two small independent samples by setting the null hypothesis that there is no significant difference between the values of the correction factors obtained by the experimental and Monte-Carlo method.

The obtained t-test values of 0.50562 and 0.10231 for energies of 511 keV and 1274.537 keV are smaller than the t-test table critical value t=2.12 (for 16 degrees of freedom and a confidence level of 0.05), which accepts the null hypothesis and rejects the alternative.

This means that, from a statistical point of view, it can be concluded that the experimental and Monte-Carlo methods give comparable results for correction factors.

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