Early Mathematical Learning: Easy as 1, 2, 3?

By

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Abstract

In Northern Ireland, more than 1 in 6 children do not achieve the expected standard in numeracy by the end of primary school (Northern Ireland Audit Office, 2013). However, early mathematical achievement is predictive of later educational achievement, employment and future life chances (Williams, 2003). Therefore, understanding what influences the learning and development of early mathematical skills should be of utmost importance for governments, policy makers, educational practitioners and researchers. This research aimed to explore the individual differences and potential factors that contribute to early mathematical achievement.

This thesis adopted a qualitative (Chapter 3), mixed methods (Chapter 4 and 5) and quantitative (Chapter 6) approach to understand the factors that may influence a child's mathematical learning and development. At school-entry children vary in their numeracy skills suggesting that the home environment may influence a child's learning and development. Semi-structured interviews with parents, of pre-school aged children, were used to investigate child interactions and specific parental views and experiences in relation to mathematical practices at home. Thematic analysis was used to explore behaviour relevant to the home numeracy environment. Based on these views a questionnaire that measures the different aspects of the home numeracy environment was developed and validated to reach three main levels of psychometric soundness; construct, content and criterion validity. This new home numeracy environment measure was then used in a longitudinal study which aimed to understand how children develop mathematical skills over time. A latent transition analysis was used to describe children's precise *learner profiles* and *learning pathways* during this transition from pre-school to school education and the key predictors of children's *pathway* membership over time were identified. Findings demonstrate that there is no one factor solely driving mathematical development over time but a range of factors that should be considered by educational practitioners and researchers to further children's mathematical development.

Abbreviations

ACPT-P	Auditory Continuous Performance Test-Pre-school
AIC	Akaike's Information Criterion
ANS	Approximate Number System
AOR	Adjusted Odds Ratio
APA	American Psychological Association
BAS–II	British Ability Scale
BBC	British Broadcasting Corporation
BIC	Bayesian Information Criterion
BPVS–III	British Picture Vocabulary Scale
BSID–II	Bayley Scales of Infant Development
BTEC	Business and Technology Education Council
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CPT	Continuous Performance Test
DEL	Department for Employment and Learning
EFA	Exploratory Factor Analysis
ENS	Exact Numerical Skills
EPPSE	Effective Provision of Pre-School, Primary and Secondary Education
FIML	Full Information Maximum Likelihood
FSM	Free School Meals
GCSE	General Certificate of Secondary Education
HEQ	Higher Education Qualification
HLE	Home Literacy Environment
HNE	Home Numeracy Environment
HOME	Home Observation for Measurement in the Environment
IES	Inverse Efficiency Score
KABC	Kaufman Assessment Battery for Children
KMO	Kaiser-Meyer-Olkin
LLC	Limited Liability Company
LCA	Latent Class Analysis
LTA	Latent Transition Analysis
LPA	Latent Profile Analysis
LPTA	Latent Profile Transition Analysis
MDI	Mental Development Index
ML	Maximum Likelihood

MLR	Robust Maximum Likelihood
NS-SEC	National Statistics Socio-economic Classification
OECD	Organisation of Economic Co-operation and Development
OfCom	Office of Communications
Ofsted	Office for Standards in Education, Children's Services and Skills
ONS	Office for National Statistics
OR	Odds Ratio
PE	Proportion of Errors
PHMQ	Pre-school Home Maths Questionnaire
PIAAC	Programme for the International Assessment of Adult Competencies
PPCT	Process-Person-Context-Time model
PVECIQ	Parents' Views and Experiences with Pediatric Cochlear Implant Questionnaire
REC	Research Ethics Committee
RMSEA	Root Mean Square Error of Approximation
RT	Response Time
SD	Standard Deviation
SDT	Social Development Theory
SES	Socio-economic Status
SFON	Spontaneous Focusing on Numerosity
SOC2010	Standard Occupational Classification 2010
SPSS	Statistical Package for the Social Sciences
SRMR	Standardised Root Mean Square Residual
STEM	Science, Technology, Engineering and Mathematics
STM	Short-term Memory
TEMA	Test of Early Mathematics Ability
Time 1	T1
Time 2	T2
Time 3	T3
TLI	Tucker–Lewis Index
UK	United Kingdom

Chapter 1: Overview

Overview

This chapter will provide a general introduction to the current literature and research problems pertinent to this thesis. It includes a brief background into the overall importance of numeracy and literacy skills in the workplace, school readiness policies and contextualises children's numeracy competencies. This chapter conveys the significance of the current research through a statement of what the purpose and current problems are along with the aims of the research and concludes with an outline of the structure of the thesis.

1.1 Introduction

1.1.1 Contextualising numeracy and literacy skills in the workplace.

Since the turn of the century, highly educated workers job prospects have grown dramatically whereas medium and low educated workers job prospects have declined (Programme for the International Assessment of Adult Competencies: (PIAAC) Organisation of Economic Co-operation and Development (OECD), 2016). As the workplace evolves to meet economic demands, some jobs disappear, and others change. Clear contributing factors to modern job performance is sound basic numeracy and literacy skills. These skills are vital to economic success and play a central role in allowing blue and white-collar workers to adapt and advance professionally (Windisch, 2016). Furthermore, strong numeracy and literacy skills are also associated with successful entrepreneurship and a lowered risk of poverty (José-Luis, 2015; World Bank, 2012).

However, in the United Kingdom (survey taken only in England and Northern Ireland) adults aged 16 to 65 score below average in literacy proficiency, when compared to 23 Organisation of Economic Co-operation and Development (OECD) membership countries (OECD, 2013). Moreover, 16 to 65-year olds in the United Kingdom (survey taken only in England and Northern Ireland) perform poorer in numeracy than in literacy assessments (OECD, 2013). This is of particular concern, as individuals with weak numeracy and literacy proficiencies are more likely to be in low-paid jobs or unemployment, tend to have poorer health, be less involved at a civic engagement level (i.e. communities working together) and are less likely to improve their skills through adult education and training (OECD, 2013). The needs of adult learners are diverse, and as adults it is harder to sustain learners' motivation to train and develop new skills (Windisch, 2016). Therefore, it is of utmost importance that educational professionals and governments make sure every child is provided with optimal educational experiences as tackling numeracy and literacy weaknesses are challenging with adults (Windisch, 2016).

1.1.2 School readiness policies.

Children begin pre-school with varying levels of school readiness with those children who enter pre-school with foundational mathematics and reading skills more likely to succeed in school than those who do not (Duncan et al., 2007). The term *school readiness* features in many guides and reviews of education (Ofsted, 2014) and is used to describe how prepared children are to succeed in school. *Success in school* in this instance means academic excellence (Garbarino, 1976). School readiness assessments started as specific cognitive ability measures for special educational provisions within mainstream education. Recently however, more inclusive educational policies mean that school readiness assessments focus on the adjustments a school may make to meet all children's developmental needs. However, the measures used (i.e. specific cognitive ability measures) to assess school readiness provide an incomplete picture of children's school readiness as they do not assess family support and therefore do not consider evidence of children's social skills or the support that they receive at home (Hughes, White, Foley & Devine, 2018). Thus, the characteristics of a child who is *school ready* are not agreed on nationally. Moreover, there is no clear guidance of what age children should be *school ready* (Ofsted, 2014).

Various sources state that children should reach school-readiness milestones on entry into primary one, whereas others suggest that these should be achieved by the start of pre-school (Allen, 2011; Field, 2010; Ofsted, 2014). The Allen Report (2011) encourages the promotion of early intervention programmes to make children school ready by five years-old. In contrast, the Field Report (2010) debates the importance of pre-school and how the home background establishes a child's school readiness. Therefore, school readiness is more complex and there are many dimensions that need to be considered to obtain a well-rounded picture of a child who is school ready. Furthermore, the Tickell Review (2011) discusses what contributes to school readiness and proposes a greater emphasis of the parent and/or carers as key roles in their children's learning. Defining the key characteristics of school readiness is essential for ensuring that children can be well prepared for starting school (Ofsted, 2014).

1.1.3 Contextualising children's numeracy competencies.

Researchers and practitioners in many child development fields have a growing consensus that children's experiences during the first five years of life influences many aspects of development and that these early childhood experiences can have considerable, long-lasting effects throughout childhood, adolescence and adulthood (Hoff, 2003; World Bank, 2015). All children do not begin school with the same level of mathematical competence. On average, children from lower socio-economic status (SES) families are entering pre-school approximately one year behind in mathematical knowledge than children from higher SES families (Starkey, Klein, & Wakeley, 2004). These differences are present at three years of age (Starkey & Klein, 2008) and persist regardless of children's participation in school readiness programs, such as Head Start (Pigott & Israel, 2005; Starkey et al., 2004).

The skills and abilities that children have developed at entry to pre-school predict early academic achievement and educational attainment (Alexander, Entwisle, Blyth & McAdoo, 1988; Entwisle, Alexander, & Olson, 2005). Throughout the past few decades the home environment has been identified as being a contributing factor in a child's educational and cognitive development (Casey, Bradley, Nelson & Whaley, 1988; Wachs, 1989). It is well documented that individual differences in numeracy skills, as well as literacy skills, are evident at school-entry which precedes formal instruction, this suggests that the early environment that children experience influences their learning (Duncan et al., 2007). Yet, other research suggests that the origins of some early mathematical skills may be innate (e.g. Feigenson, 2011; Izard, Sann, Spelke, & Streri, 2009; McCrink & Wynn, 2004; Xu & Spelke, 2000). Nevertheless, school-entry mathematics skills were found to be more important for later mathematics, reading and science achievement than school-entry reading skills (Claessens & Engel, 2013). Thus, identifying the predictors of foundational mathematical achievement benefits the success in more than one core curriculum subject, and in turn raises school academic achievement.

In addition to social factors (i.e. SES and the home environment) as predictors of foundational mathematical achievement there are a range of mathematical abilities with domain-general and domain-specific explanations that are potential predictors (e.g. Aunio & Räsänen, 2016; Blair & Razza, 2007; Duncan et al., 2007; Gilmore et al., 2013; Gray, Rogers, Martinussen & Tannock, 2015; Krajewski & Schneider, 2009a; Jordan, Kaplan, Ramineni & Locuniak, 2009; Lyons, Price, Vaessen, Blomert & Ansari, 2014; Muldoon, Towse, Simms, Perra & Menzies, 2013). This thesis will explore the relationship between social factors, domain-general and domain-specific factors and their overall association with mathematical outcomes.

1.2 Statement of problems

In Northern Ireland, more than 1 in 6 children do not achieve the expected standard in numeracy by the end of primary school (Northern Ireland Audit Office, 2013). However, early mathematical achievement is predictive of later educational achievement, employment and future life chances (Williams, 2003). As such, it is essential to develop our understanding of the factors that contribute to early mathematical achievement.

On average, children from low-income families perform considerably poorer in mathematics than their peers raised in middle or upper-income homes, thus leading to a vicious cycle of poverty (Jordan, Kaplan, Locuniak & Ramineni, 2007; Jordan, Kaplan, Nabors Oláh & Locuniak, 2006; National Mathematics Advisory Panel, 2008). A study by Woolfson and colleagues (2013) used an instrument (the Scottish Early Development Instrument: SEDI, originally developed in Canada) to evaluate five key developmental domains; (1) language and cognitive development, (2) communication and general knowledge, (3) physical health and wellbeing, (4) social competence and (5) emotional maturity. Low SES children were 2 to 3 times more likely than high SES children to score low in at least one developmental domain. Nevertheless, 17% of high SES children were 'developmentally vulnerable', indicating that those in need cannot be identified by SES alone.

Another factor that could contribute to early disadvantages for children is the quality of the home learning environment. The quality of the home learning environment is frequently defined by the availability of educational resources, for example books, reading, playing with numbers, counting and board games (Anders et al., 2012; Cankaya & LeFevre, 2016; Gunn, Simmons, & Kameenui, 1995; Melhuish et al., 2008; Skwarchuk et al., 2014). Melhuish and colleagues (2013) investigated the long-term effects of different pre-school provision on child development (3 to 11-year-olds in Northern Ireland) and found that children from homes with the lowest frequency of home learning environment activities were almost 3 times less likely to attain Level 5 in mathematics at the end of Key Stage 2, than children from homes with a higher frequency of home learning environment activities. Furthermore, children who experienced high-quality pre-schools were 3.4 times more likely to attain Level 5 in mathematics than children without pre-school experience. Thus, the frequency of home learning environment activities and the quality of pre-school learning environments can diminish or benefit individual success later in life (Sénéchal & LeFevre, 2002).

Recently, the methods that assess the quality of the home numeracy environment has changed. Parents' self-reports of the frequency of number activities occurring in the home has become the most common method (i.e. Kleemans, Peeters, Segers & Verhoeven, 2012; LeFevre et al., 2009, 2010b; Melhuish et al., 2008; Skwarchuk, 2009; Skwarchuk, Sowinski, & LeFevre, 2014; Anders et al., 2012). There is a vast amount of literature that examines the role of the home literacy environment (i.e. parents helping their children to read words and the frequency of shared reading; Skwarchuk et al., 2014) in comparison to the home numeracy environment (i.e. parents helping their children to count; Burgess, Hecht & Lonigan, 2002; Frijters, Barron & Brunello, 2000; Kirby & Hogan, 2008; LeFevre et al., 2009; Sénéchal &

LeFevre, 2002; Sénéchal, LeFevre, Thomas & Daley, 1998). As such, previously researchers have drawn questions from home literacy environment (HLE) questionnaires to create home numeracy environment (HNE) questionnaires, with other home numeracy measures based on variations of the Home Observation for Measurement in the Environment (HOME) inventory (Caldwell & Bradley, 1984; Anders et al., 2012). The reasoning behind this has been somewhat unclear, leading researchers to assume that since the early home environment (i.e. during pre-school years) has been connected to children's literacy skills it is theoretically reasonable to predict that the early home environment will impact children's numeracy skills (Blevins-Knabe, 2016; LeFevre et al., 2009; 2010b; Lukie et al., 2014). Ideally the development of theory or measurements should be both deductive and inductive (Williamson, Karp, Dalphin & Gray, 1982). As far as the author is aware no research has used an inductive approach such as developing questionnaire items based on interviews with parents of 3 to 4-year olds for a home numeracy environment questionnaire measure.

1.3 Purpose of research

One of the overarching purposes of this thesis is to develop and validate a HNE questionnaire that is relevant for use in Northern Ireland. This study aims to develop a home numeracy environment scale using semi-structured interviews with parents as existing scales are either; (a) not culturally appropriate, (b) very brief or (c) outdated (e.g. LeFevre et al., 2009; Melhuish et al., 2008).

The second purpose of this thesis was to use longitudinal methods, in particular using a person-oriented approaches to statistical analysis, to describe children's precise *learner profiles* and *learning pathways* of mathematic specific skills during the transition from pre-school to school educational settings. Many mathematical cognition research questions require methods that take a person-centred approach, yet this is rarely achieved. Much of previous research uses a variable-oriented approach to statistical analysis. Bergman and Magnusson (1997; Bergman, Magnusson & El-Khouri, 2003) proposed a distinction between variable-oriented and person-oriented approaches to statistical analysis of empirical data (Collins & Lanza, 2010, 2013). In variable-oriented approaches the emphasis is on identifying relations between variables, and it is assumed that these relations apply across all people. In contrast, in person-oriented approaches the emphasis is on the individual, looking at subtypes of individuals that exhibit similar patterns of individual characteristics (Bergman & Magnusson, 1997). Developmental scientists have argued that the use of longitudinal studies is essential for understanding causes of developmental change (Morrison & Ornstein, 1996; Magnusson & Cairns, 1996; Magnusson & Stattin, 2006; Grammer, Coffman, Ornstein & Morrison, 2013). Therefore, this thesis will explore person-oriented approaches to statistical analysis using longitudinal methods. Furthermore, this thesis aims to identify key predictors of mathematical achievement tracking the relationship between social factors, (i.e. the HNE and SES), general cognitive skills, (i.e. working memory), language ability and children's mathematical outcomes.

1.4 Aims

The overall aims of this thesis were to address the limitations of previous research and current gaps in existing literature by:

1. Investigating the dominant and common views and experiences relevant to the home numeracy environment (HNE) using an exploratory approach in the form of semi-structured interviews.
2. Creating a HNE questionnaire measure using both deductive (i.e. theory-driven items) and inductive (i.e. using semi-structured interviews to produce new items) approaches to scale development.
3. Discussing every stage of the scale development and validation process to increase the psychometric soundness of the HNE measure. The HNE questionnaire was evaluated across five psychometric properties; (1) construct validity, (2) factor structure, (3) scale score reliability, (4) content validity, and (5) criterion validity.
4. Tracking children's basic numerical skill development from pre-school to school. A latent transition analysis will be used to describe children's precise *learner profiles* and *learning pathways* during this transition.
5. Identifying the key predictors of children's *pathway* membership over time. This study considers a variety of demographic characteristics (i.e. gender, age, SES and parents' highest educational qualification), as well as predictors associated with multiple components of the home environment (i.e. the home numeracy environment measures), domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary). Therefore, this study will incorporate potential predictors of pathway membership to extend knowledge on children's development of mathematical skills in early childhood.

1.5 Chapter overviews

This thesis comprises seven chapters. The current chapter has generically outlined the problems, purpose and gaps in recent literature, and aims of the thesis. Chapter 2 is an in-depth literature review that explores the influential studies that have given recent research a firm foundation on which to build on (e.g. Bronfenbrenner's bioecological systems theory (1994); Bronfenbrenner & Morris, 1998). The literature review seeks to outline current models and theories with regards to the factors that may influence a child's learning and development in mathematics, outlining the gaps in literature this thesis seeks to address.

Chapter 3 has formed the basis of a peer-reviewed qualitative paper on parents' views and experiences of the HNE (Cahoon, Cassidy & Simms, 2017). The main focus of this study was to investigate the dominant and common views and experiences relevant to the HNE using an exploratory approach in the form of semi-structured interviews. The findings are organised thematically to increase the understanding of how parents of pre-schoolers perceive how they teach children about numbers and under what circumstances numeracy occurs in the home. Chapter 3 is the bases of the inductive approach to the scale development process discussed in Chapter 4. Based on the information gathered from pre-schoolers' parents during the semi-structured interviews (Chapter 3), items were developed to create the initial HNE measure in Chapter 4.

Chapters 4 and 5 will guide the reader through the various stages of scale development and validation of the HNE measure. Chapter 4 presents phase one, known as the scale development process. This process comprises four developmental stages of a questionnaire measure including; (stage 1) item generation, (stage 2) questionnaire administration, (stage 3) initial item reduction and (stage 4) an exploratory factor analysis. These four development stages are the processes that lead to construct validity. Chapter 5 subsequently presents phase two, the scale validation process, covering both (stage 5) content and (stage 6) criterion validity. Therefore, between Chapter 4 and 5 three levels of psychometric soundness in scale development will be focused on; construct, content and criterion of the Pre-school Home Maths Questionnaire (PHMQ).

Chapter 6 reports a longitudinal study tracking children's development of basic mathematical skills during the transitional phase from pre-school to primary school. Firstly, in Chapter 6 a confirmatory factor analysis will be completed to allow the researcher to gain further evidence of the construct validity of the new PHMQ measure (Hinkin, 1998). Secondly, this chapter discusses children's precise *learner profiles* and *learning pathways* over time. Potential predictors of pathway membership are also discussed in detail. Sections of the PHMQ are used as predictors to understand if the HNE is associated with pathway membership. Chapter 7 is the discussion, in which the key results of Chapters 3, 4, 5 and 6 are discussed in detail in relation to the wider existing literature.

Chapter 2: Background Literature

Overview

The purpose of this chapter is to present a review of influential studies that have given recent research a firm foundation on which to build on. This literature review seeks to outline current models and theories with regards to the factors that may influence a child's learning and development in mathematics, outlining the gaps in literature this thesis seeks to address.

2.1 Introduction

For decades, policy makers, educational practitioners and researchers have been interested in the factors that influence mathematics achievement because weak numeracy and underperformance in the subject has stark consequences with low mathematics performers more likely to be in low-paid jobs or unemployment and tend to have poorer health (Murray, 2013; OECD, 2013).

2.1.1 Nature verses nurture.

Scientists have long considered the interaction between nature and nurture in child development. What abilities are developed naturally and what abilities are learned? Some research suggests that the origins of some early mathematical skills may be intrinsic (Baillargeon & Carey, 2012; Dehaene, 2001; Feigenson, 2011; Feigenson, Libertus & Halberda, 2013; Geary, Berch & Koepke, 2015; Izard et al., 2009; McCrink & Wynn, 2004; Starkey, Spelke & Gelman, 1990; Xu & Spelke, 2000). A classic study by Starkey, Spelke and Gelman (1990) found that 6 to 8-month-old infants could detect the number of distinct entities in a sequence of sounds or a visible scene. Arguably these findings provide evidence that infants detect changes in quantity, suggesting that the emergence of the earliest numerical abilities does not depend upon the development of language or complex activities with number (Starkey, Spelke & Gelman, 1990). However, there is some research that has refuted this evidence. Mix, Levine and Huttenlocher (1997) found that when infants were shown visual displays, they had no significant preference for either the equivalent (i.e. auditory sequences numerically equivalent to the visual display) or non-equivalent (auditory sequences not numerically equivalent to the visual display) visual display once the rate and duration of the auditory sequences were varied randomly. Yet, there is a growing amount of research in support of infants recognising changes to numerical arrays, comparing numbers of items across auditory and visual sensory modalities, and adding and subtracting approximate quantities before formal education (e.g. Feigenson, 2011; Izard et al., 2009; McCrink & Wynn, 2004; Xu & Spelke, 2000).

Despite this reservation, it is essential that children become fluent in mathematics by building on and developing basic numerical processing abilities. *Basic numerical processing abilities* refers to basic number processing, such as counting. Higher mathematical abilities such as calculations usually develops after a person masters basic number processing. Vygotsky (1978a) stated that infants are born with the basic abilities for intellectual development and subsequently, through interactions, these develop into more sophisticated mental processes. Vygotsky's Social Development Theory (SDT, Vygotsky, 1978b) states three major principles to learning; (1) social interaction, (2) the more knowledgeable other and, (3) the zone of proximal development. Vygotsky's theory contradicted Piaget's view that learning follows development as Vygotsky claimed social learning precedes development. Vygotsky states;

> *"Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (inter-psychological) and then inside the child (intra-psychological)." - Vygotsky, 1978b, pg. 57.*

This social interaction is the basis of Vygotsky's SDT, however to gain an understanding of cognitive development Vygotsky asserts the more knowledgeable other and the zone of proximal development as the main principles. These two fundamental aspects refer to a more competent person (i.e. parent, teacher, peer or even computer) providing a higher ability or understanding than the learner (more knowledgeable other) for the learner to become independently proficient (zone of proximal development) (Vygotsky, 1978b). Thus, creating a learning context that allows children to play an active role in their learning may be crucial for development. Another theory that undertakes the perspective that children take active participation in their own learning, like Vygotsky's SDT, is Bronfenbrenner's bioecological systems theory (Bronfenbrenner, 1979).

2.1.2 Bronfenbrenner's bioecological systems theory.

According to Bronfenbrenner's ecological systems theory (1994), also known as the bioecological systems theory (Bronfenbrenner & Morris, 1998), humans create the environments that shape the course of development (Bronfenbrenner, 2005). Bronfenbrenner (2005) stated:

> *"The recognition that developmental processes are profoundly affected by events and conditions in the larger environment accords major importance to public policies and practices that influence the nature of the environment and, as a result, have significant effects, often unintended, on the development of children growing up in families, classrooms, and other settings". – Bronfenbrenner, 2005.*

This emphasises that policy makers should be aware of how policies, and the way in which they are implemented, can affect human development. Whilst, developmental researchers should focus attention on the indirect effects of public policies on developmental processes (Bronfenbrenner, 2005). This multifaceted system suggests that a person's development occurs in the midst of a complex environment (Ayoola et al., 2017). Bronfenbrenner’s conceptualisation of a child’s development are five multilevel nested systems known as; (1) microsystems, (2) mesosystems, (3) exosystems, (4) macrosystems, and (5) chronosystems (Bronfenbrenner, 1977, 1979, 1994, 2005) which provides a coherent understanding of the complex environment in which every child exists.

The microsystem (1) focuses on the individual and their immediate environment. It is “the complex of relations between the developing person and environment in an immediate setting containing the person” (Bronfenbrenner, 1977, pg. 515). Thus, the microsystem comprises the home environment, pre-school or school, peer group or the community environment of which the child exists. The mesosystem (2) encompasses the interaction of the different microsystems for instance, the interaction between family (the home environment) and friends (peer group) or between family and school. The exosystem (3) refers to interactions between various settings that do not directly involve the developing individual but may have an influence on that person's behaviour and development. For instance, the workplace of the child's parent; if the parent takes home work related stress the parent may provide lower quality child care (Ayoola et al., 2017; Bronfenbrenner, 2005). The macrosystem (4) comprises characteristics of a given culture, belief systems, public policy and economic conditions, as well as others, under which families live (Bronfenbrenner, 1976). Therefore, the macrosystem influences the nature of interaction within all other levels of the ecology of human development (Bronfenbrenner, 2005, 1994). Finally, the chronosystem (5) adds the dimension of time to demonstrate the influence of both change and transition in the child’s environment.

Drawing on Bronfenbrenner’s bioecological systems theory (1979) is particularly important as it facilitates exploration of the individual, social, systemic and cultural explanations and how these are inter-connected in explaining a child’s mathematical development. Bronfenbrenner's bioecological systems theory was updated recently to include four components and was deemed the Process-Person-Context-Time (PPCT) model (Bronfenbrenner, 2005). *Proximal processes* (1) (the microsystem) are hypothesised as the primary drive of development (Bronfenbrenner, 2001; 2005), these are interactions between the person and their environment. The *person* (2) is an individual’s characteristics such as biological, cognitive, behavioural, and emotional characteristics. Bronfenbrenner and Morris (2006) distinguish three types of characteristics as the most influential to a person’s proximal processes;

demand, resource and force. *Demand* characteristics refer to qualities of an individual which invite or discourage reactions from the social environment (e.g. age, gender or physical appearance). These characteristics can disrupt or foster the initial interactions the individual has with their environment (Bronfenbrenner & Morris, 2006; Tudge, Mokrova, Hatfield & Karnik, 2009). In contrast, *resources* are not immediately apparent. Resources include both material resources (e.g. caring parents, educational opportunities and housing etc.) and past experiences (e.g. mental, emotional and knowledge etc.) required for proximal processes to occur (Bronfenbrenner & Morris, 1998; Tudge et al., 2009). Finally, *force* characteristics refer to a person's temperament, motivation, and persistence. For example, Bronfenbrenner suggested that two children may have equal resource characteristics, but their learning trajectories may differ dramatically due to variations in their motivation to succeed and persistence on tasks (Bronfenbrenner & Morris, 2006; Tudge et al., 2009).

Context (3) refers to a person's development being influenced by both immediate and distant environments (Bronfenbrenner & Morris, 2006). With Bronfenbrenner (2005) elaborating and reforming the bioecological systems theory into the PPCT model, *context* is now only one of the four components within the full PPCT model. This is in contrast to the bioecological systems theory in which the context (or environments) made up the majority of the theory (i.e. the microsystem, mesosystem, exosystem and macrosystem). Within the PPCT model *time* (4) was distinguished as three types of time; microtime, mesotime and macrotime. Microtime is what happens during an interaction while mesotime is these microtime interactions occurring over longer periods of time. Macrotime (i.e. the chronosystem in the bioecological systems theory) is where these "processes are likely to vary according to the specific historical events that are occurring" (Tudge et al., 2009, pg. 201).

Vygotsky and Bronfenbrenner's theories claim that the child and the wider context play an important and crucial role in learning and development. However, their work rarely investigated how combinations of these components are associated with children's outcomes over time (Dennis, 2010; Rubin, Burgess, Dwyer & Hastings, 2003). To gain a greater understanding of a child's mathematical development, it is necessary to investigate the components of the PPCT model and test their associations to comprehend how and what influences mathematical success.

2.1.3 Individual differences and potential factors that influence mathematical outcomes.

Globally children are either labelled good or poor at mathematics skills (Dowker, 2005) however, poor performance in one mathematic skill can occur somewhat independently of poor performance in another mathematic skill (Dowker, 2005; Holmes & Dowker, 2013). Much research has incorporated the idea of early numerical development comprising different mathematics component skills (Bisanz, Sherman, Rasmussen, Ho & Campbell, 2005; Dowker, 2008), cognitive skills (LeFevre et al., 2010a; Krajewski & Schneider, 2009b) and other factors such as the home numeracy environment, language ability and SES (Belsky et al., 2007; De Smedt & Boets, 2010; Göbel & Snowling, 2010; Melhuish et al., 2008; Skwarchuk et al., 2014; Starkey, Klein, & Wakeley, 2004). Research that addresses individual differences and some of the potential factors (e.g. the frequency of home learning environment activities, domain-general components i.e. executive functions and domain specific components i.e. early numerical competencies) that may influence a child's development of mathematics skills will now be discussed.

2.1.4 The home learning environment.

The home environment can support learning and development (Manolitsis, Georgiou & Tziraki, 2013; Pomerantz, Moorman & Litwack, 2007). The home learning environment is a significant predictor of reading and mathematics achievement (Anders et al, 2012; Melhuish et al., 2008) but also can influence children's social and behavioural development (Sylva, Melhuish, Sammons, Siraj-Blatchford & Taggart, 2004; 2008). Studies that explore the nature of the home learning environment have found wide variations between families. For instance, the quality of the home learning environment is associated with the availability of educational resources, such as books and board games (Anders et al, 2012; Cankaya & LeFevre, 2016; Gunn et al., 1995; Skwarchuk et al., 2014). Research shows that families with more economic strain and low parental education, especially mothers with low levels of education, are moderately associated with a low-quality home learning environment (Dearing et al., 2012; Melhuish et al., 2008; Totsika & Sylva, 2004). Additionally, the nature and frequency of parent involvement in joint activities, such as reading to the child (an activity utilised in the HLE) or counting (an activity involved in the HNE), affects the quality of the home learning environment and in turn effects a child's cognitive and mathematical development (Belsky et al., 2007; Melhuish et al., 2008; Skwarchuk et al., 2014). Thus, these home environment characteristics should be considered when exploring children's mathematical development.

Much of the home learning environment research has been based on Eccles (1993) expectancy-value model (e.g. Huntsinger, Jose, Larson, Balsink Krieg & Shaligram, 2000;

Huntsinger, Jose, Liaw, & Ching, 1997). Eccles theoretical model combines five interrelated areas that may influence a child's developmental outcomes; (1) parent, family, and neighbourhood characteristics (e.g. education level of parents', ethnicity, socioeconomic status), (2) the child's characteristics (e.g. gender, birth order), (3) general attitudes and beliefs of parents' (e.g. academic attitudes, child-rearing beliefs), (4) parents' child-specific beliefs (e.g. expectations of child's achievements, views of their child's capabilities in different domains) and (5) parents' practices (e.g. teaching strategies, time use with child, encouragement to participate in activities). Previous literature demonstrates that the quality of the home learning environment can be differentiated into three major components; (a) the structural characteristics of a family (Krajewski & Schneider, 2009a; Huntsinger et al., 1997, 2000), (b) educational attitudes and expectations of parents (LeFevre, Polyzoi, Skwarchuk, Fast & Sowinski, 2010b), and (c) parent-child interactions, measured either in relation to the domain of literacy or numeracy (Sénéchal & LeFevre, 2002; LeFevre et al., 2009; Huntsinger et al., 1997; Huntsinger et al., 2000) or irrespective of domain (Melhuish et al., 2008; Anders et al., 2012). Overall, all studies conclude that the quality of the home learning environment is important, and that quality is linked to a child's social and academic outcomes (Bakermans-Kranenburg, Van IJzendoorn, & Bradley, 2005; Melhuish et al., 2008). However, it is apparent that little is known about the role of numeracy activities, in comparison to literacy activities, in promoting early childhood learning at home and its dependence on family background (LeFevre et al., 2009).

2.1.4.1 Home literacy environment.

There is a vast amount of literature that examines the role of the home literacy environment (HLE) in comparison to the home numeracy environment (HNE; Sénéchal & LeFevre, 2002; Burgess et al., 2002; Frijters et al., 2000; Kirby & Hogan, 2008; Sénéchal, LeFevre, Thomas & Daley, 1998). Research demonstrates that home literacy activities can boost children's literacy and language skills (Sénéchal & LeFevre, 2002). Some evidence exists that suggests that the HLE is a better predictor of children's numeracy than the HNE (Anders et al., 2012). However, one explanation for this evidence could be that the measure used, in this case the Kaufman Assessment Battery for Children (KABC; Melchers & Preuss, 2003), that requires not only numeracy but also language skills (Abedi & Lord, 2001; Anders et al., 2012), much like other mathematical tests. However, school-entry mathematical skills were found to be important in predicting later mathematical, reading and science achievement above school-entry reading skills (Claessens & Engel, 2013). Further, those who entered preschool with high levels of mathematical skills had faster growth in maths competences (Aunola, Leskinen, Lerkkanen & Nurmi, 2004). Thus, improving early mathematical achievement benefits the success in more than one core curriculum subject, and in turn raises school academic achievement. Therefore, it is essential to understand how early mathematics skills develop due to its wider impact on academic achievement more generally.

2.1.4.2 Measures of the home numeracy environment.

Arguably the most common methods of measuring the home numeracy environment are questionnaire measures that assess the frequency of number activities occurring with and without parents in the home (Blevins-Knabe & Musun-Miller, 1996; Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; DeFlorio & Beliakoff, 2015; Kleemans, Peeters, Segers & Verhoeven, 2012; LeFevre et al., 2009, 2010b; Melhuish et al., 2008; Missall, Hojnoski, Caskie & Repasky, 2015; Skwarchuk, 2009; Skwarchuk et al., 2014; Skwarchuk & LeFevre, 2015; Pan, Gauvain, Liu & Cheng, 2006; Ramani, Rowe, Eason & Leech, 2015) and examining the frequency of numeracy behaviours through observations of parents (usually mothers) and child dyads (Vandermaas-Peeler, Ferretti, & Loving, 2012; Vandermaas-Peeler, Nelson, & Bumpass, 2007; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009; Vandermaas-Peeler & Pittard, 2014) including, in some cases, triads comprising the parent, target child and sibling/s (Benigno & Ellis, 2004). However, utilising questionnaire measures and observations methods to assess the home numeracy environment are somewhat new. Further it has been argued that questionnaire measures and observations methods may not be assessing the same concept (Missall, Hojnoski, & Moreano, 2017). Studies on the effects of the home environment on children’s number skills originally focused on language development (Durkin, Shire, Riem, Crowther & Rutter, 1986), case studies (Young-Loveridge, 1989), or on social aspects of the home, for instance social class (Saxe et al., 1987) or culture differences (Huntsinger et al., 1997).

For instance, Durkin et al. (1986) focused on language development, examining the spontaneous reference to number words and counting between mothers and children (aged 9–36 months) in a longitudinal study. It was concluded that it was unclear how much influence mothers number word contributions had on children as in some cases contributions from mothers offered conflicting information and that more research was necessary to establish the importance of mother-child interaction on numeracy development. Young-Loveridge (1989) used a case study approach and found that home numeracy activities of six children correlated with pre-school children’s performance on number tasks. It emerged that home numeracy activities were more important in determining development of number concepts than SES, as measured by father's occupation or mother's education. However, although Durkin et al. (1986) and Young-Loveridge (1989) provided an account of the home numeracy environment their small sample sizes make it difficult to generalise these findings. On the contrary, Saxe et al. (1987) used a larger sample (78 middle- and working- class mothers and pre-school children) that examined the influence of SES on the home environment and numerical understanding. Saxe et al. (1987) discovered that middle-class mothers recounted engaging in more complex numeracy activities than working-class mothers, as well as having higher educational achievement goals for their children. Further, middle-class children demonstrated greater competence on complex numerical tasks than their working-class peers.

These three different studies, the case study, interview and/or observations (Durkin et al., 1986; Young-Loveridge, 1989; Saxe et al., 1987), found links between home numeracy activities and mathematical outcomes. Nevertheless, these studies have many limitations and much of the research cannot be generalised. Yet, research has continued to discover this common trend between the frequency of number-related activities and children's performance on number tasks. However, the measures used to understand children's experiences in the home has changed recently.

2.1.4.3 Questionnaire based measures.

2.1.4.3.1 Recent changes within the home numeracy environment.

The home numeracy environment is continuously changing for instance, the recent increase in technology usage in the home (OfCom, 2013, 2016). OfCom (2016) state that there are two devices in the home that continue to be used by children: television sets (92% for 3-4-year olds and 96% for 5-7s) and tablets (55% for 3-4s and 67% for 5-7s). Thus, technology advances have potentially expanded the reach of numeracy learning in the home. Yet, questions about educational technology are rarely used in home numeracy environment questionnaire measures. It is only recently that these types of questions have been added and even at this it is usually one question for example, how often did you and your child engage in the following activities? "Uses maths software" (Huntsinger, Jose, & Luo, 2016) and "Playing counting games using child computer or arithmetic software" (Kleemans et al., 2012). More research is needed to understand the broad array of educational technology that may be watched and/or played on tablets (Cahoon, Cassidy & Simms, 2017), as well as other number-related activities that may have emerged in recent years. Thus, there is a necessity for an improved, more fine-tuned psychometric measure that assesses current numeracy interactions and activities that children are exposed to.

2.1.4.3.2 Structure of home numeracy environment measures.

The home numeracy environment has been measured as a unidimensional construct (Blevins-Knabe et al., 1996; Kleemans et al., 2012) where all activities occurring in the home environment related to mathematics have been measured. Kleemans et al. (2012) found that parent–child numeracy activities and parents' numeracy expectations are unique predictors of early numeracy skills, after controlling for linguistic and cognitive child factors. This emphasises the importance of home numeracy experiences on early numeracy skills. This unidimensional approach provides a general overview of the influence of the home numeracy environment however, it does not give specific information or understanding of what types of activities and environments enhance early numeracy skills. Subsequent advanced constructs have been proposed. These constructs involve two separate types of activities; informal and formal numeracy activities (Skwarchuk et al., 2014; LeFevre et al., 2009), or mathematics and spatial activities (Dearing et al., 2012). Another multidimensional model (Hart, Ganley & Purpura, 2016) has recently involved three separate factors: the direct numeracy environment, the indirect numeracy environment, and the spatial environment. The implications of these multidimensional models will be discussed in detail.

Even without the dichotomisation of numeracy activities (e.g. formal and informal numeracy activities), questionnaire measures have yielded inconsistencies between home activities and children's number skills (for example, Blevin-Knabe et al., 1996; DeFlorio & Beliakoff, 2015; Missall, Hojnoski, Caskie & Repasky, 2015). Some studies have found unique and positive associations between the home numeracy environment and mathematics skills in 4 to 7-year olds (Kleemans et al., 2012; Dearing et al., 2012; Manolitsis et al., 2013; Niklas & Schneider, 2014). Whereas, Missall et al. (2015) in a study involving children aged 3 to 5-years found no relation between the home numeracy environment and a range of numeracy skills. Yet, DeFlorio and Beliakoff (2015) found significant associations between the frequency and range of home mathematics activities and mathematics skills, but these associations were reduced to non-significance after accounting for SES and parents' expectations for their children's mathematical learning. However, previous literature has demonstrated that SES did not predict early mathematical ability (Krajewski & Schneider, 2009a). Thus, it seems more important to explore parents' expectations, and the frequency of home numeracy experiences, rather than only assessing a family's SES to understand early mathematical development. Hence, more research is essential to understand the mechanisms that promote early mathematics skills in the home.

2.1.4.4 Inconsistencies in home environment questionnaires.

2.1.4.4.1 Dichotomisation of numeracy activities.

Most studies involving questionnaire measures have resulted in inconsistencies between home activities and children's number skills. Sénéchal and LeFevre (2002) proposed a home literacy model with two separate pathways, *formal* and *informal*, that linked children's experiences to their acquisition of early literacy skills. The *formal* literacy experiences pathway was assessed through frequency of parent involvement in literacy activities (e.g. reading and writing words) whereas the *informal* literacy pathway was investigated through children's exposure to shared reading with parents (developed by Sénéchal, LeFevre, Hudson & Lawson, 1996). Skwarchuk, Sowinski and LeFevre (2014) proposed a theoretical model of the home numeracy environment, inspired by a home literacy model (Sénéchal & LeFevre, 2002). Suggesting that participating in *formal* practices (assessed through frequency of parent involvement in numeracy activities) would support the development of symbolic mathematics knowledge. Symbolic number refers to cultural symbols attributed to quantities (e.g. Arabic digits i.e. 1 or number words i.e. one). While *informal* mathematics exposure (measured through a number games checklist) would promote non-symbolic mathematics skills. Non-symbolic (e.g. dot estimation) intuitions of numerosity is relied upon to quickly approximate the numerosity of sets of objects without resorting to counting (Dehaene, 1997). Skwarchuk et al. (2014) found that *formal* home numeracy practices accounted for unique variance in children's symbolic number knowledge whereas *informal* exposure to games with numerical content predicted children's non-symbolic arithmetic performance, thus supporting their hypothesis.

However, this hypothesised conceptual model of the home numeracy environment (Skwarchuk et al., 2014) has rarely been replicated. For example, Huntsinger et al. (2016) used a questionnaire measuring the frequency of numeracy activities in the home and found that participating in formal mathematics activities predicted both formal (learned through explicit instruction using rules, principles, and procedures e.g. calculations both addition and subtraction) and informal (acquired outside of formal schooling e.g. concepts of relative magnitude) mathematics knowledge, whereas engaging in informal activities predicted neither. In contrast, LeFevre et al. (2009) discovered significant associations only between parent's reports of informal activities and symbolic mathematics knowledge. Furthermore, LeFevre et al. (2010b) found no relations between informal home numeracy practices, as measured through a questionnaire, and children's symbolic number knowledge for Greek or Canadian pre-school aged children. This thesis will seek to gain insight into the *formal* and *informal* experiences of parent-child interactions in the home numeracy environment by interviewing parents who have pre-school aged children. Furthermore, this thesis will address how these interactions may affect the development of children's mathematical learning in the home through developing and validating a new home numeracy environment questionnaire that moves beyond the scope of previous questionnaires (i.e. by using an inductive approach) and creating a measure of informal mathematics exposure, measured through a new number games checklist relevant to the United Kingdom.

Dearing et al. (2012) also dichotomised home activities and investigated how two different types of activities, mathematics and spatial activities, occurring in the home related to mathematical development in a female population (N=127; mean age = 6.72). By sampling a female only population Dearing et al. (2012) increased the statistical power for identifying individual differences among females. Gender differences are smaller for mathematics than spatial problems, with males generally performing better in spatial tasks than females (Dearing et al, 2012; Halpern et al., 2007) Thus, Dearing et al. (2012) aimed to address females' early numerical and spatial reasoning skills within the types of spatial environments provided at home. Mathematics activities were related to developing numerical skills, for example "counts down using numbers (10, 9, 8, 7, . . .)" or "memorizes maths facts (such as 2 + 2)". Whereas, spatial activities were likely to develop spatial skills, such as "playing with puzzles", "draws maps", and "builds with construction toys (such as building blocks)". These spatial activities were suggested as a foundation to geometry and measurement skills (Dearing et al., 2012; Levine, Ratliff, Huttenlocher & Cannon, 2012; Hart et al., 2016). Dearing et al. (2012) found that mathematics activities were closely related to females' arithmetic skills. However, although family SES predicted engagement in spatial activities, these activities were not related to females spatial reasoning skills.

2.1.4.4.2 Multidimensional model numeracy activities.

A multidimensional model involving three separate factors (Hart et al., 2016) moves beyond the assumption of only two environments existing in the home. Hart et al. (2016) broke down the home environment into three specific factors; the direct numeracy environment, the indirect numeracy environment, and the spatial environment. Findings demonstrated that parents who reported undertaking more general home mathematics activities (defined as a combination score of all three environments) reported having children with higher mathematical skills, whereas parents who indicated doing more spatial activities reported having children with lower math skills. However, in longitudinal studies, early spatial skills predict long-term mathematics performance (Krajewski & Ennemoser, 2009; Wolfgang, Stannard, & Jones, 2001) and spatial skills have been found to be important for later success in science, technology, engineering and mathematics (STEM) fields (Wai, Lubinski, & Benbow, 2009). Thus, it does seem important to explore spatial skills further in questionnaire measures. The aforementioned studies indicate the potential impact of the home numeracy environment on learning and the inconsistencies in the research.

2.1.5 Domain-general components.

Many parents engage their children in numerical and literacy activities to prepare their children for school (Duncan et al., 2007; Sénéchal & LeFevre, 2002). However, many parents do not know that they can prepare their children for school by supporting the development of executive function skills (Hutchison & Phillips, 2018). Executive functioning can be used to describe cognitive processes including a variety of behaviours such as planning, self-regulation, problem-solving, strategy use, and goal directed behaviour (Lee, Romine, Wolfe, & Wong, 2002; Miller & Cohen, 2001). It has been proposed that executive function processes are related to how successful an individual is when performing complex tasks (Miyake et al., 2000), in academic achievement (St. Clair-Thompson & Gathercole, 2006), and success in life (Garavan, Ross, & Stein, 1999).

Performing mathematics tasks is a complex process that requires the manipulation of many cognitive factors (Cargnelutti, Tomasetto & Passolunghi, 2017) as mathematics involves mastering a sequence of problem-solving to reach a goal (Best, Miller & Naglieri, 2011; Clements, Sarama & Germeroth, 2016; McClelland, Cameron, Wanless & Murray, 2007). The link between processes of executive functions and academic achievement is well documented for older students (Bielaczyc, Pirolli, & Brown, 1995; Zimmerman, 2002). However, the development of executive functions in early years and how this contributes to academic skills has only recently gained momentum in research (Clements et al., 2016).

Executive function skills emerge early and continue to develop through-out the life span (Schmitt, Geldhof, Purpura, Duncan & McClelland, 2017). Between ages two to five there are many structural changes in the prefrontal cortex which allow for the enhancement of executive function skills (Zelazo & Müller, 2002, 2011). The pre-school to school transition is important for the development of executive functions (Schmitt et al., 2017). Children must adapt to a more structured educational situations in school that requires greater executive function skills compared to less structured environments experienced in pre-schools (Schmitt et al., 2017). Executive functions, mathematics and literacy skills seem to develop during the same period (National Mathematics Advisory Panel, 2008; National Early Literacy Panel, 2008). However, some researchers argue that executive functions are the foundation for academic achievement as children must be able to hold their attention and avoid distractions, as well as actively remember and sustain on challenging tasks (Blair & Raver, 2015; McClelland et al., 2007; Schmitt et al., 2017).

Furthermore, studies suggest that the predictive relation between executive functions (such as self-regulation; Blair, Ursache, Greenberg & Vernon-Feagans, 2015) and mathematics seem stronger than the association between executive functions and literacy in children, both in pre-school and early school years (Blair & Razza, 2007; Blair et al., 2015; Schmitt, Pratt & McCleeland, 2014; Schmitt et al., 2017). Superior executive functions may be vital for the development of mathematics skills, for instance cardinality (i.e. the number of items in a set) or calculations that involve changing attention and inhibiting previously learned rules (Schmitt et al., 2017). One of the fundamental problems with measuring executive functioning is the *task impurity problem* (Rabbitt, 1997, 2004). Executive functions tasks nearly always implicate other non-executive cognitive abilities such as verbal ability, processing speed or visual–spatial ability. Thus, executive tasks are complex and identifying which executive functions predict mathematical achievement can be challenging.

2.1.6 Domain-specific components.

Besides domain-general components, throughout the last decade, research has expanded to explain important associations between basic numerical processing abilities and the development of school level mathematics skills (De Smedt, Noël, Gilmore & Ansari, 2013; Price & Wilkey, 2017). Complex mathematics skills rely on the mastery and integration of a range of basic numerical processing abilities, facts and concepts, whether these are innate and/or acquired (Butterworth, 1999; Dehaene, 1997; Geary, 2013; Lyons et al., 2014). There are many different basic numerical processing abilities that have been found to be good predictors of later mathematics performance, such as counting skills, basic arithmetical skills, approximate number skills, numeral ordering, number line estimation and numerical language (Aunio & Räsänen, 2016; Aunola, Leskinen, Lerkkanen & Nurmi, 2004; Gilmore et al., 2013; Libertus, Feigenson & Halberda, 2011; Lyons et al., 2014; Krajewski & Schneider, 2009a; Muldoon et al., 2013; van der Sluis, de Jong & van der Leij, 2007; Geary, 2004; Passolunghi, Vercelloni, & Schadee, 2007). However, it is vital to identify the basic skills that are most predictive of mathematical success during the early school years.

'Number sense', which is the ability to non-verbally, non-symbolically represent numbers, is deemed to be a precursor to formal understanding of mathematics (Ansari, 2008; Dehaene, 1997, 2001). Both non-symbolic and symbolic number representations are associated with 'number sense' (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Verguts & Fias, 2004) and play an important role in the achievement of higher mathematical abilities (De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009; Piazza et al., 2010). Foundational non-symbolic numerical skills can be referred to as the approximate number system (ANS, Halberda, Mazzocco, & Feigenson, 2008). ANS is active across the lifespan, from infancy to adulthood (Droit-Volet, Clement & Fayol, 2008; Halberda & Feigenson, 2008; Halberda, Wilmer, Naiman & Germine, 2012; Izard, Dehaene-Lambertz & Dehaene, 2008; Izard et al., 2009; Libertus Feigenson & Halberda, 2011, 2013; Libertus, Odic & Halberda, 2012). Previous evidence shows a positive association between ANS (non-verbal comparison task) and mathematical achievement in children (Libertus et al., 2011). However, it has been unclear whether the link depends on formal mathematics instruction (Libertus et al., 2011). Those who have not yet required formal mathematical learning, such as infants (Izard et al., 2009) and children aged 5 (Barth, Beckman & Spelke, 2008), have been shown to have an innate basic number processing ability, which suggests early abstract numerical representations. However, recent findings indicate that the ANS does not mature until adolescence (Libertus et al., 2011). Additionally, ANS representations are imperfect estimates and become gradually inaccurate with increasing magnitude (Libertus et al., 2011; De Smedt, Noël, Gilmore & Ansari, 2013). For infants and young children imperfect estimates are more apparent, with acuity of ANS representations sharpening throughout childhood and becoming refined within adulthood (Halberda & Feigenson, 2008). However, it remains undetermined as to when these imperfect estimates of ANS integrate with mathematics abilities.

Despite some researchers finding evidence that supports the role of ANS in older children and adults (Fazio, Bailey, Thompson & Siegler, 2014; Lourenco, Bonny, Fernandez & Rao, 2012; Paulsen, Woldorff & Brannon, 2010; Lyons & Beilock, 2011), many researchers argue that symbolic and exact numerical skills (ENS) are superior to non-symbolic ANS skills (Bartelet, Vaessen, Blomert & Ansari, 2014; Holloway & Ansari, 2009; Sasanguie, De Smedt, Defever & Reynvoet, 2012; Toll, Van Viersen, Kroesbergen & Van Luit, 2015). While other research suggests that ANS refines into the ENS over development with the acquisition of symbolic numerical knowledge (Buckley & Gillman, 1974; Castronovo & Göbel, 2012; Siegler & Booth, 2004; Ashcraft & Moore, 2012). The prolonged nature of non-symbolic and symbolic system (or ANS and ENS development) has implications for understanding the interplay between individual differences and 'number sense'. Therefore, it is crucial to use longitudinal studies to unravel the learning trajectories of early children's numerical development, as well as identify the basic skills that are most predictive of early mathematics success.

Knops, Nuerk and Göbel (2017) discuss 18 articles that investigated how domain-general components interact with numerical processes and concluded that domain-specific numerical variables predicted arithmetic performance above and beyond domain-general variables. Nevertheless, domain-general components warrant examination as young children who begin pre-school with superior executive function skills have an advantage in terms of mathematics performance than their weaker executive function peers, that persists into the secondary school (Clements et al., 2016). Thus, sole focus on domain-specific numerical variables to explain the development of mathematical skills is not adequate (Knops et al., 2017) as numerical processing is a complex skill involving several interrelated mechanisms (Kaufmann et al., 2013). The following reviews a theoretical model of numerical processing that suggests that there are distinct systems for numerical processing.

2.1.8 Pathways to early mathematics model.

The pathways to early mathematics model proposed by LeFevre et al. (2010a; based on the triple-code model of number processing Dehaene, 1992; Dehaene, Piazza, Pinel & Cohen 2003; Dehaene & Cohen, 1995) proposed that three precursors, verbal, visuo-spatial short-term memory (STM) and quantitative skills, contribute to different aspects of numerical competence varying depending on each numerical task's demands. In a two-year longitudinal study LeFevre et al. (2010a) a model was tested with pre-school (aged 4 years 5 months to 5:8) and kindergarten children (aged 5 years 4 months to 6:6). It was discovered that the pathways from the early mathematics model contributed independently to early numeracy skills. Children's verbal skills made a unique contribution to a number naming task (verbal number task) but not to non-verbal arithmetic (non-verbal number task). Whereas, children's subitising latency (quantitative skill), pattern-matching process on small sets and counting-based processes on larger sets, made unique contributions to children's performance on non-verbal arithmetic (non-verbal number task) but not on the number naming task (verbal number task). Additionally, visuo-spatial attention skills made independent unique contributions to both types of early number tasks.

These pathways from the early mathematics model related differently to performance on a variety of mathematical outcomes two years later. There were four standardised subtests of mathematical knowledge; the *Numeration* subtest that assessed children's knowledge of the numerical order, *Measurement* subtest that assessed children's ability to compare quantities and *Geometry* subtest that assessed children's processing and understanding of spatial arrays (from the KeyMaths Test-Revised; Connolly, 2000) and the *Calculation* subtest that assessed computation skills (from the Woodcock-Johnson Test of Achievement-Revised; Woodcock & Johnson, 1989). Furthermore, two computerised research-based measures of mathematical knowledge were assessed; the number line task and magnitude comparison task. Children's verbal skills made significant contributions to all standardised and research-based mathematical measures two years later. However, quantitative skills and visuo-spatial STM skills only predicted specific outcome measures. The quantitative pathway made significant contributions to both research-based measures but only predicted performance on the Numeration and Calculation subtests of the standardised mathematical attainment measures. The visuo-spatial STM skills' pathway made significant contributions to all standardised mathematical attainment tests and to their number line performance but failed to predict children's performance on the symbolic comparison task.

The pathways to early mathematics model (LeFevre et al., 2010a) provided a comprehensive understanding of the three precursors and their pathways to early mathematics. This was interpreted as evidence that different mathematical cognitive demands contribute to different early number competences. The focus in research has generally been on what mathematical cognitive precursors lead to mathematical success or in what order of development do these mathematical cognitive precursors occur to lead to stronger mathematical abilities, these studies are known as variable-centred approaches. However, many mathematical cognition research questions require methods that take a person-centred approach that emphasises the individual, yet this is rarely achieved.

2.1.9 Variable-centred versus person-centred approaches.

Most research, including the pathways to early mathematics model proposed by LeFevre et al. (2010a) invoke a variable-centred approaches. Bergman and Magnusson (1997; Bergman, Magnusson & El-Khouri, 2003) proposed a distinction between variable-oriented and person-centred approaches to statistical analysis of empirical data (Collins & Lanza, 2010, 2013). In variable-centred approaches, such as regression analysis, factor analysis, and structural equation modelling, the emphasis is on identifying relations between variables that can be applied to all learners in the same way. These methods limit the ability to deal with heterogeneity within and between individuals (Hickendorff, Edelsbrunner, Schneider, Trezise & McMullen, 2017). In contrast, person-centred approaches, such as cluster analysis, latent profile analysis and latent transition analysis, the emphasis is on the individual. Bergman and Magnusson (1997) stated; "operationally, this focus often involves studying individuals on the basis of their patterns of individual characteristics that are relevant for the problem under consideration" (p. 293). Thus, a person-centred approach is studying individuals while look for subtypes of individuals that exhibit similar patterns of individual characteristics. Research on developmental trajectories of mathematical outcomes indicate that the time between pre-school and school-entry, when evidence of basic numerical processing abilities and executive functions begins to develop, may be the optimal time to examine individual characteristics between different mathematical profiles and pathways to mathematics outcomes (Bergman & Magnusson, 1997; LeFevre et al., 2010a; Schmitt et al., 2017).

2.1.10 Rationale.

This thesis will use the Process-Person-Context-Time (PPCT) model (Bronfenbrenner, 2005) to examine the associations between proximal processes, the person, the context and changes over time. *Proximal processes* will be assessed by interviewing parents about their interactions with their child and experiences relevant to the home numeracy environment (Chapter 3). As such, these *proximal processes* will then be assessed through a frequency of numeracy activities section of the Pre-school Home Maths Questionnaire (PHMQ). As mentioned previously (in Chapter 1), the development and validation of the PHMQ will be discussed in Chapter 4 and then utilised within the longitudinal study in Chapter 5. Ideally the development of measurements should be both deductive and inductive (Williamson et al., 1982) which has not been the case with former home numeracy environment questionnaires. This thesis will seek to gain insight into the *formal* and *informal* experiences of parent-child interactions in the home numeracy environment and then how these parent-child interactions effect the development of children's mathematical learning. Furthermore, the frequency of number activities scales that are available have rarely been validated beyond construct validity (e.g. LeFevre et al., 2009). Schoenfeldt (1984, p.78) stated that "the construction of the measuring devices is perhaps the most important segment of any study". Thus, this thesis will build on three levels of psychometric soundness; construct, content and criterion validity of the PHMQ.

Person characteristics will be measured through a child's demographic characteristics (i.e. child's gender), cognitive skills (i.e. working memory) and mathematical specific skills (i.e. cardinality). The researcher also included demographics of the primary parent (i.e. the parent that spends the most time with the child) such as age, race, parenting educational beliefs, etc. as *person* characteristics of the child with whom the developing person of interest (the child) was interacting. The *context* will be investigated through two of the multilevel nested systems; the microsystem and macrosystem. The microsystem will be explored through assessing the home environment. The macrosystem will be measured through economic conditions (i.e. socio-economic status) and material resources (i.e. checklists from the PHMQ this will be discussed in more detail in Chapter 4 and 5). Although different mesosystems were assessed, results are limited as teachers were given a questionnaire about what types of mathematics activities were completed in the classroom. However, this focuses more on the teacher within their workplace context, than the interaction between the classroom and home life. Thus, the researcher is not able to state that the interaction of the different microsystems was evaluated. *Time* (the chronosystem) was measured as a longitudinal study was carried out, and therefore the researcher could examine the interrelated impact of each proximal process, person, and context over time (Bronfenbrenner, 1994; Tudge et al., 2009). This broad view of early mathematical development expands research by adopting a more holistic view of the relationship between multiple factors and children's mathematical development.

2.1.11 Aims.

The overall aims of this thesis were to address the limitations of previous research and current gaps in existing literature by:

1. Investigating the dominant and common views and experiences relevant to the home numeracy environment (HNE) using an exploratory approach in the form of semi-structured interviews.
2. Creating a HNE questionnaire measure using both deductive (i.e. theory-driven items) and inductive (i.e. using semi-structured interviews to produce new items) approaches to scale development.
3. Discussing every stage of the scale development and validation process to increase the psychometric soundness of the HNE measure. The HNE questionnaire was evaluated across five psychometric properties; (1) construct validity, (2) factor structure, (3) scale score reliability, (4) content validity, and (5) criterion validity.
4. Tracking children's basic numerical skill development from pre-school to school. A latent transition analysis will be used to describe children's precise *learner profiles* and *learning pathways* during this transition.
5. Identifying the key predictors of children's *pathway* membership over time. This study considers a variety of demographic characteristics (i.e. gender, age, SES and parents' highest educational qualification), as well as predictors associated with multiple components of the home environment (i.e. the home numeracy environment measures), domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary). Therefore, this study will incorporate potential predictors of pathway membership to extend knowledge on children's development of mathematical skills in early childhood.

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Chapter 3: Parents' Views and Experiences of the Informal and Formal Home Numeracy Environment

Overview

Chapter 3 has formed the basis of a peer-reviewed qualitative paper on parents' views and experiences of the home numeracy environment (HNE, Cahoon, Cassidy & Simms, 2017). The main focus of this study was to investigate the dominant and common views and experiences relevant to the HNE using an exploratory approach in the form of semi-structured interviews. The findings are organised thematically to increase the understanding of how parents of pre-schoolers perceive how they teach children about numbers and under what circumstances numeracy occurs in the home. Chapter 3 is the bases of the inductive approach to the scale development process discussed in Chapter 4.

3.1 Introduction

3.1.1 Background.

Numeracy skills are important for virtually every activity at home and beyond (Niklas & Schnelder, 2014). Recently there has been an increasing emphasis on the importance of numeracy skills in the workplace (Hoyles, Noss, Kent & Bakker, 2010; Noss, 1997). Consequently, proficiency in a range of numeracy and mathematical skills is important, not only for the individual but also for the national economy (Clark-Wilson, Sutherland & Oldknow, 2011; Norris, 2012). Early mathematical achievement predicts children's growth in mathematics and, as such, later educational achievement, employment and future life chances (Duncan et al., 2007; Williams, 2003). Given the significance of mathematical competence, it is essential to obtain a strong foundation in mathematics from a young age. However, there is a lack of research focusing on learning outside of the school context, thus overlooking the potential importance of early numerical experiences and how they might affect growth in numeracy skills (Butterworth, 2005; High, 2008).

The home environment may affect a child's learning and development as it is evident at school-entry that children vary in their literacy and numeracy skills (Segers, Kleemans & Verhoeven, 2015; Skwarchuk, Sowinski & LeFevre, 2014). Specifically, variations in the quality of the home learning environment have been observed to contribute to differences in children's cognitive (e.g. measured by the Mental Development Index (MDI), an index in the Bayley Scales of Infant Development (BSID–II); Bayley, 1993; Lugo-Gill & Tamis-LeMonda, 2008) and social emotional development (Duncan, Brooks-Gunn & Klebanov, 1994). Previous literature demonstrates that the quality of the home learning environment can be differentiated into three major components; (a) the structural characteristics of a family (Krajewski & Schneider, 2009a; Huntsinger et al., 1997; Huntsinger et al., 2000) such as family composition and SES, (b) educational attitudes and expectations of parents (LeFevre et al., 2010b), and (c) parent-child interactions, measured either in relation to the domain of literacy (i.e. the HLE) or numeracy (i.e. the HNE; Sénéchal & LeFevre, 2002; LeFevre et al., 2009; Huntsinger et al., 1997; Huntsinger et al., 2000) or irrespective of domain (Melhuish et al., 2008; Anders et al., 2012).

Conversely, there are limitations to the components (i.e. in this next example parent-child interactions) measured previously. For instance, Niklas and Schneider (2017) found that the home learning environment predicted early abilities and also competencies at the end of primary school, even after controlling for former academic achievement, and child and family characteristics. However, their measure of the home learning environment only included one question about numeracy activities. Thus, any detailed conclusions about the quality or content of home numeracy activities that are beneficial for future learning cannot be made from this study. Nevertheless, overall, many studies conclude that the quality of the home learning environment is important, and that quality is linked to a child's social and academic outcomes (Bakermans-Kranenburg et al., 2005; Melhuish et al., 2008). However, it is apparent that little is known about the role of numeracy activities, in comparison to literacy activities, in promoting early childhood learning at home and its dependence on family background (LeFevre et al., 2009).

The influence of the home literacy environment on the growth of early linguistic competencies has been well researched (Aikens & Barbarin 2008; Evans, Shaw, & Bell, 2000; Scarborough & Dobrich 1994). Yet research on the home numeracy environment and its impact on the acquisition of mathematical skills is in its infancy (Skwarchuk et al., 2014; Niklas & Schneider, 2014). Some studies have examined parents' reports of the home numeracy environment. LeFevre, Clarke and Stringer (2002) reported that the frequency that parents interacted with their child by directly teaching early numeracy skills (e.g. simple addition) was positively associated with children's school-based mathematical achievement. In contrast, other studies have found that parent's reports of engaging in home numeracy activities was not significantly correlated with children's numeracy skills (Blevins-Knabe, Austin, Musun, Eddy & Jones, 2000 (Study 3); Blevins-Knabe & Musun-Miller, 1996; Missall et al., 2015).

A potential reason for these mixed findings across studies is that there is no consensus in the definition that encompasses the everyday routine and practices occurring in the home numeracy environment. LeFevre et al. (2009) investigated parents' reports of numeracy activities and defined two types: *direct* activities, which involved explicitly, and intentionally teaching about numbers or arithmetic to develop children's mathematical skills (e.g. counting objects) and *indirect* activities, which involved numbers in real-world tasks (e.g. playing board games with dice) that include 'hidden' mathematical instructions that occur incidentally. LeFevre et al. (2009) found that children's mathematical skills were related to the frequency with which parents reported engaging their children in indirect numeracy activities. Additionally, the terms *formal* and *informal* mathematics have been used across studies (Anderson, 1998; Barwell, 2016; Skwarchuk et al., 2014; Song & Ginsburg, 1987). Anderson (1998) used the terms formal and informal to refer to 'partnership' styles between teachers and parents that were formed either through informal methods, such as parent-teacher conversations or newsletters or through more formal methods, such as written reports and parent-teacher interviews. In contrast, Song and Ginsburg (1987) used the term *informal* to refer to how children acquired numeracy skills through spontaneous interactions with their environment, imitations of adults, and watching TV, and used the term *formal* to refer to written work in school. Moreover, Skwarchuk et al. (2014) developed a clear distinction between formal and informal activities, mapping onto the previously mentioned direct and indirect activities, respectively (LeFevre et al., 2009). Thus, cross-study comparisons are difficult due to the different definitions used, contributing to the lack of understanding about what kind of parent involvement brings about positive academic effects or how pedagogically-focused parents may have to be to influence their child's mathematical development (Aubrey, Bottle & Godfrey, 2003).

A key way that parents interact with their children in the home environment is though game-playing. Tudge (1990) noted that through these activities' children master basic mathematical skills by observing more competent players who demonstrate higher-level skills. The point when the less competent person becomes independently proficient is known as the 'zone of proximal development' (for more information on Vygotsky's Social Development Theory refer to 2.1.1 Nature verses nurture; Vygotsky, 1978b). Continual modification of tasks enables the child to learn as the more competent person provides the appropriate level of challenge, known as 'Scaffolding' (Berk & Winsler, 1995). Mediation techniques can be used by parents to facilitate their children's acquisition of numerical skills, such as asking questions, prompting children, requesting explanations, providing answers, and offering information on strategies (Anderson, 1997; Bjorklund, Hubertz & Reubens, 2004; Kritzer, 2011).

Nevertheless, interventions targeting children's numeracy learning at home are lacking (Niklas, Cohrssen & Taylor, 2016; Starkey & Klein, 2000). Further, there is a lack of consistency in opinion on how to successfully intervene to improve the home numeracy environment to benefit early learning. Some propose that intensive interventions are important (Sheldon & Epstein, 2005; Starkey & Klein, 2000) while others state that even non-intensive interventions can be effective, concluding that even with small budgets interventions should be undertaken (Niklas et al., 2016). However, more information is needed to distinguish *what* number-related experiences these interventions should focus on. A potential target may be parent-child interactions. For example, Bjorklund et al., (2004) examined the relationship between parental guidance and children's numeracy behaviour in a game context (e.g. chutes and ladders) and mathematics context (e.g. arithmetic problems) and found that parents provided varying levels of support and appropriately adjusted their behaviours to meet their child's abilities. However, parents' instructions (e.g. prompting or using cognitive directives, such as demonstrating a strategy) did not always lead to their children effectively using the identical strategy that the parent had displayed (e.g. single-item counting, adding from one, adding from larger addends) in both contexts. This demonstrates that the influence of parent guidance is contingent on both children's abilities and the context in which numeracy is presented (Benigno & Ellis, 2004; Niklas et al., 2016). In addition to children interacting with their parents and caregivers at home, interactions with others, such as siblings, have been observed to play an important role in learning numerical concepts (Clements, 2004; Howe et al., 2015; Howe, Ross, & Recchia, 2011) and therefore may also be a target for interventions.

3.1.2 Rationale.

The aforementioned studies (Bjorklund et al., 2004; Benigno & Ellis, 2004) imposed tasks on parents and children and subsequently monitored their behaviour. In contrast, the current study was exploratory and aimed to gain opinions from parents on their everyday routine activities and understand the way in which parents encourage the development of early numeracy skills in the home. The main focus of this study was to investigate the dominant and common views and experiences relevant to the home numeracy environment using an exploratory approach in the form of semi-structured interviews. This enabled increasing understanding of how parents perceive how they teach children about numbers and under what circumstances numeracy occurs in the home. In this study, when defining the home numeracy environment all number-related activities that occurred at home are included, as well as those occurring in the family car and garden. Activities that occurred with both parents and/or siblings were also included.

3.2 Method

3.2.1 Participants.

This study was an inductive, qualitative design based on recorded interviews with eight parents recruited from a local public leisure facility, on the basis of purposive sampling. Parents view classes in a viewing area; an employee of the leisure facility made an announcement to parents about the study. The researcher attended subsequent classes and approached parents to be recruited into the study. Inclusion criteria for participation was that the person was a parent or guardian of at least one child aged between 3 to 4 years, and that they were the primary care-giver. Three of the participating parents were fathers and five were mothers. All parents had at least one child aged between 37 months and 59 months (M_{age} = 47.5 months). In Northern Ireland pre-school playgroup accepts children between 2-5 years, whereas nurseries and pre-schools are for 3-4-year olds in the year before children begin full time formal education. The children (56% female) experienced a variety of different childcare settings when not at home with parent(s); nursery school (n = 3), pre-school playgroup (n = 3), private day care (n = 2), or attended school (n = 1). The highest level of parent education was doctoral level (n = 1), undergraduate degree (n = 3), higher secondary school (i.e. was awarded final educational qualification at eighteen years-old; n = 1), or lower secondary school (i.e. was awarded final educational qualification at sixteen years-old; n = 3). Parent's highest level of mathematical education was as follows: doctoral level (n = 1), undergraduate degree (n = 2), higher secondary school (n = 1), and lower secondary school (n = 4). Mathematical education was defined as including any mathematical or statistical training.

3.2.2 Procedure.

The research procedures were reviewed and approved by School of Psychology Research Ethics Committee (REC) before the study commenced. Parents were provided with an in-depth participant information sheet that they were requested to read, this informed the parents of the requirements of the study, data protection and their right to withdraw from the study at any time. Parents were made aware that they could request to stop the interview at any time they wished without any negative repercussions. If they wished to participate in the study, they completed a consent form that they returned to the researcher before commencing the interview. Parents complete a demographic questionnaire and took part in an interview at their convenience. Interviews were semi-structured and a topic guide, consisting of five open-ended questions (see Appendix 3.1 for questions), was created to enable a detailed exploration of their home numeracy environment using a responsive approach.

The questions contained in the topic guide were developed from previous research. As an initial question parents were asked about their child's interest in mathematics, which was based on Fisher, Dobbs-Oates, Doctoroff and Arnold (2012) study that indicated a relationship between high levels of interest and strong mathematical skills. A question on what types of numerical activities occurred in the home, was based on previous home numeracy environment scales, which assess the frequency of different activities (Kleemans, Peeters, Segers & Verhoeven, 2012; LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski, 2014; Melhuish et al., 2008). A question to explore both the circumstances in which number-related activities occurred and the opportunities parents created in the home for their child to learn numeracy was developed from a study that investigated how collaborative parent–child interactions and children's interests affected exposure to home numeracy activities (Lukie et al., 2014). One question was derived from Vandermaas-Peeler, Ferretti and Loving (2012), which was an observational study that investigated the specific processes parents used to encourage and support their child in learning numbers. Previous findings have indicated that parents believed that literacy activities were more vital than numeracy activities (Early et al. 2010; Blevins-Knabe et al., 2000) therefore a question was also asked about the frequency and structure of mathematical activities in comparison to reading in the home. The topic guide questions were used flexibly in order to generate statements from parents that provide insight into behaviour relevant to the home numeracy environment, how parents might teach their children numeracy skills, and under what circumstances. The researcher interviewed each of the 8 participants individually. The individual interview sessions took approximately 45 minutes each.

3.2.3 Data analysis.

Data saturation was found after six parents were interviewed, another two interviews were completed to confirm the saturation; this is consistent with other studies (Isman, Mahmoud Warsame, Johansson, Fried & Berggren, 2013; Isman, Ekéus, & Berggren, 2013). Data saturation is achieved when further coding is not achievable, thus the ability to obtain additional new information has been reached and enough information has been collected to replicate the study (Fusch & Ness, 2015). Interviews were audio recorded and transcribed verbatim. To identify the dominant and common themes in participants' responses, thematic analysis was conducted on the interviews. The six stages of the thematic analysis process used in the current study are as follows: (1) familiarising with the data by reading and re-reading the transcripts and writing down initial ideas, (2) generating initial codes systematically across the entire data set matching data relevant to each code, (3) searching for themes by matching codes into potential themes and assembling all data relevant to each potential theme, (4) reviewing themes and generating a thematic 'map' which involves two levels of reviewing and refining themes – reviewing the coded data extracts to ensure strong evidence exists to support the theme (Level 1) as well as the entire data set (Level 2), (5) develop clear definitions and naming each theme, (6) producing the report by selecting and analysing quotations that represent the themes, research question and literature (for a detailed description of the process see Braun & Clarke, 2006). In keeping with previous literature (e.g. Walton & French, 2016), identifying information about the participants was excluded when presenting extracts from the transcripts. Each interview transcript was examined using the aforementioned six analytic steps and it was determined that 18 codes were present in the participant's responses (see Appendix 3.2 for the 18 codes, definitions and the linked codes and themes). An inter-rater reliability measure was applied with a second coder to enhance coding credibility for 25% of the interview transcripts by calculating Cohen's kappa (Cohen, 1960). Cohen's kappa values for all codes were 1.00, except for one code with a value of 0.64. Cohen's kappa values over 0.6 indicate statistically acceptable levels of agreement (Hruschka et al., 2004).

3.3 Results

The findings are organised into six themes: (1) numeracy environment structure, (2) frequency of number-related experiences, (3) levels of number knowledge, (4) technology attitudes (5) parent-child interactions and (6) social interaction.

3.3.1 Theme 1. Numeracy environment structure.

Most participants indicated a lack of structure when teaching numeracy. Some participants discussed how they helped their child learn numeracy in an informal way:

> *"The likes of his dinner, he would have smiley faces and I would ask, "How many smiley faces are on the plate?" and he would start to count them. The same with the bath too, he knows he has five ducks so if you only give him four, "there's only four ducks, I need five ducks"" – Grace – Son (3)*

> *"We do it (mathematics) simply by asking; "How many sausages do you want?", or "How many pieces do you want your toast cut up into?"" – Emily – Son (4) and Daughter (3)*

This suggests that numeracy is taught through everyday activities, indicating that number-related activities are generally unplanned in the home, thus parents report that the home numeracy environment is unstructured. However, even though these activities are spontaneous, they provide an alternative learning opportunity for their child to acquire number knowledge. The findings demonstrated that there are two reasons for the existence of this unstructured/informal numeracy environment. First, children's interest in numbers drives the frequency of the activities:

> *"She's not the slightest bit interested (in addition) … so I generally sneak maths in" – Sarah – Daughter (4)*

Therefore, numeracy must go undetected if a child is not interested and so the frequency of numeracy activities is low. Second, the planning, awareness and time involved in preparing a structured/formal environment for number-related activities may influence the frequency of numeracy-related activities occurring in the home. One participant noted:

> *"It's a bit like parenting you often think "Oh it'll come naturally" and "Oh well I automatically teach my children about everything" maths, English and things, but you need to think. You need to almost have the plans in place… but it's difficult to remember to highlight maths" – Peter – Son (3)*

Initially this participant suggested that teaching should be instinctive but admitted that it is difficult to spontaneously formulate plans in order to teach numeracy. From these findings, a structured/formal environment was defined as parents explicitly planning number-related activities, parent's awareness of their opportunity to teach numeracy, organising strategies for their child to learn and develop number skills and setting aside time for teaching numeracy. Whereas an unstructured/informal environment was defined as parents having a spontaneous approach when referring to numeracy.

3.3.2 Theme 2. Frequency of number-related experiences.

To understand how frequently number-related activities took place, participants were asked to compare the frequency and structure of number-related activities to reading activities. Most participants stated that reading was a structured daily activity that they dedicated specific time to, whereas all participants specified that number-related experiences would be unstructured and did not occur at a prescribed time. This suggested that reading occurred more often than number-related experiences. In spite of this, participants realised through the course of the interview that number-related experiences could occur more frequently than reading:

> *"Maths is slightly more than reading because you don't have to have anything in front of you to do maths, we can just ask him 'what's five plus five?' but for reading we have to have a book and to be sitting on the sofa. Reading is four or five days a week, for about 15 minutes every day but maths probably would be slightly more, we would ask him something about maths everyday" – Jack – Son (4)*

> *"I'd say daily. It's just part of life, but I suppose we should be more conscious of the fact that we are doing it (number-related activities)" – Peter – Son (3)*

This theme suggests that parents are not necessarily cognisant that they are educating their child about numbers on a daily basis, and thus report doing number-based activities less frequently than reading activities. Therefore, reports from parents may not be a true reflection of the frequency of these activities. Furthermore, most participants stated that some of the books they read to their children involved numbers. The following extract gives an example of the types of books one parent would read to their child:

> *"Gruffalo, Tiger came to Tea, Hungry Caterpillar… that's good for the numbers actually, Hungry Caterpillar, because you go through the- "on Monday he ate one orange, and on Tuesday he ate two strawberries"[1] so he can count those out" – Peter – Son (3)*

This statement illustrates that some children might be accessing some structured number learning through reading books that involve numbers or shapes, albeit only occasionally. In regard to direct and indirect numeracy activities as defined by LeFevre et al. (2009) participants mentioned direct activities (such as, "counting" "blocks", "numbers off license plates", "food" or "stairs") that focus on number learning at a higher frequency than indirect activities (such as, "card games" or "money") in which the development of numeracy skills are likely to be incidental.

3.3.3 Theme 3. Levels of number knowledge.

The majority of participants mentioned that using rhymes was helpful to familiarise their children with counting words. One participant noted:

> *"We would sing songs, one starts of 'Chook, chook, chook. Good morning, Mrs. Hen"' It teaches them to add up to ten. So, you've got six speckled hens, two brown and two yellow and then it eventually adds up to ten and the last line is 'there's ten little chicks'. It has been helpful because she was able to count up to ten before she went to nursery and she knows that if you've got nine sweeties and mummy gives you one more, you've got ten sweeties" – Sophie – Daughter (4)*

However, this was the only example where the parent was confident that her child knew and understood the meaning of number words through the practising of rhymes. The majority of participants were hesitant to say that their child understood the meaning of the number words. One participant stated:

> *"He learnt pretty early to count and then over the last while he started to get confused with the likes of 7 and 11, so he would go from 6, 11, 12, 13. So I think he is still using a rhythm rather than understanding the amounts above say 4 or 5" – Christopher – Son (3)*

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[1] Carle (1974)

This emphasises that parents recognise that their young children do not have a full understanding of numbers words and their meanings. This also suggests that counting rhymes, although useful in increasing familiarity with number words, may not be sufficient to develop children's understanding of these words.

3.3.4 Theme 4. Views of technology.

It was apparent that television programmes and computer-based applications were being used extensively in the home. Despite the extensive use, all parents expressed that they struggled to limit the duration of technology usage. It was apparent that parents were more relaxed with the rules they enforced if the television programme or computer-based application was used as a tool for children to acquire knowledge:

> *"I would try to limit the games, the platform games like 'Crossy Road'. I would be much more relaxed if it was the maths game or something that he might learn from as opposed to trying to get a chicken across a hundred roads" – Peter – Son (3)*

Parents indicated that technology aided them beyond what they felt they could accomplish independently with their child:

> *"Technology maths games are useful, it's something that I couldn't do myself" – Jude – Daughter (4)*

Also, participants suggested that technology could engage a child and direct their attention towards learning:

> *"Any kinds of visual aids are helpful, especially if you have a child who maybe doesn't have the ability to focus" – Sarah – Daughter (4)*

> *"For a while because we have five children we would actually, against all the rules, plonk him and his sister in front of the computer in the mornings sometimes just to bring a bit of sanity to the house and we would put on Number Jacks or something like that so Number Jacks would probably be the big one in terms of maths" – Emily – Son (4) and Daughter (3)*

This illustrates that children find these technologies absorbing generally, but they also display interest in them when the activity contains numerical information. Although parents may use technology to occupy their children whilst they carry out household activities, parents are sensitive to the quality of the content of their children's viewing and interaction.

3.3.5 Theme 5. Parent-child interactions.

Parents initiated number-related activities spontaneously, yet the activities they led were complex:

> *"We put cars in front of him and count them and teach doubles on his fingers. We would also give him scenarios with his cars, with numbers less than five. So, four plus three, that kind of thing" – Jack – Son (4)*

> *"When playing with her Jenga blocks, I get her to build me a tower with five blocks"" – Sophie – Daughter (4)*

In contrast, children initiate number-related activities in the form of generally simplistic counting activities:

> *"He would bring it up himself, if we get to the bottom of the stairs he would say, "Count mummy"" – Grace – Son (3)*

> *"He runs out to the hopscotch and starts trying to count to 10" – Christopher – Son (3)*

This could suggest that children associate certain objects with counting. Even if children initiated the number-related activity, parents described that they were likely to control and direct the activity. Every parent reported actively helping their child learn numeracy skills with most offering guided instructions when their child made an error. However, the way in which parents aided their children ranged by activity. The two activities in which parents reported adjusting their behaviour when their child made an error were after their child had missed a number when counting and when teaching basic arithmetic, such as addition or subtraction. Parents reported three types of interactions which they felt aided their child's understanding; providing the correct answer, explaining different scenarios in order to reach the correct answer, and encouraging their child to repeat the activity.

In regard to missing numbers while counting, parents reported always providing the answer and then encouraging their child to count again:

> *"He would skip a number, so you have to correct him and then he would do it right" – Grace – Son (3)*

"I would count from 5 and say 5, 6 and then make sure I'm emphasising 7, 8, 9, 10, then making sure he knows where 11 fits in again" – Christopher – Son (3)

Parents over-emphasised and stressed the numbers that their child had missed when counting. Parents reported using repetition to make sure that their child would remember the information for future activities.

Parent-child interactions while working on arithmetic were more complex and depended on the child's abilities or interest in the number-related activity. If a child provided an incorrect answer to an arithmetic problem the majority of the participants reported that they explained the scenario in a way that aided understanding and then encouraged their child to repeat the task:

"If I said to him "What's one and one?", and he said "ten", then I would say, "Really? So, one and one is ten?" (Demonstrated on fingers), and then he would count it himself so I'm not saying, "No that's wrong, this is the answer", but kind of asking are you sure about that? Getting him to think about it" – Emily – Son (4) and Daughter (3)

There are two important issues to note regarding parent-child interactions. First, parents described that they usually explained a numerical problem by visually demonstrating the sets of numbers involved:

"If he physically saw things and there was some subtraction and items were removed he would count out the answer". – Jack – Son (4)

Parents discussed using concrete manipulatives to demonstrate how to count, add, subtract or group sets of objects, which they believed could improve their child's understanding of numeracy concepts. Second, in addition to explaining, parents reported using encouragement and providing reassurance to inspire their child's confidence when attempting to answer numeracy problems:

"I would keep saying, "You try it again", "try it again", "you can do it", and help her. Make sure that she understands what's right and what's wrong because there are definite answers with maths, you are either right or wrong." – Jude – Daughter (4)

Parents realised that positive encouragement boosted their child's self-esteem. Moreover, participants noted that by making the number-related activities fun they hoped their child would be more likely to learn:

> *"I would make it fun for him to count and when he can count, he thinks he is great especially when he gets it right" – Grace – Son (3)*

> *"It's good to learn but it needs to be fun and if it's fun I think they will learn from it" – Emily – Son (4) and Daughter (3)*

Overall, parents had the desire to create an enjoyable home numeracy environment in order to keep their child's attention, boost self-esteem and facilitate learning.

3.3.6 Theme 6. Social interaction.

As well as parental interaction, parents reported children interacting with siblings when doing number-related activities. From the eight parent interviews, five of the target children had siblings. These five parents expressed that when number-related activities were occurring, their target child sometimes interacted with siblings, and this was regarded as positive. Triad interactions (i.e. parent, target child, and older sibling/s) through homework were particularly highlighted. Overall parents believed that their child was picking up information that was being taught to older siblings, even if the numeracy was more advanced than what would be expected from their younger child:

> *"The focus is on the older children, that they get to see about maths and about volume (while baking) but it's like everything younger children benefit from that and although he is only three and is not being told this is a hundred grams of sugar, the hope is that he picks that up along the way with his siblings" – Peter – Son (3)*

> *"We would do his homework (older sibling, age 6 years-old) with her (younger sibling, age 4 years-old) she does pick it up like coins and money" – Jude – Daughter (4)*

Doing homework together as a family facilitated opportunities for the parent to ask their younger children questions about numeracy, as the younger children wanted to be included in their older siblings' activities. Parents reported that their younger child was more likely to concentrate on the question asked of them, and have an interest in answering, if their older sibling was involved. However, one parent did describe that although the child was "consistently listening to older siblings talking about doing sums" that "he couldn't really understand the sums”. This parent interpreted his child’s interest in activities as displaying an enthusiasm for numbers even if the child did not fully understand numeracy concepts.

3.4 Discussion

The diversity of the six themes identified through this study illustrates how the home numeracy environment may be influenced by parents' views and experiences of numeracy-related activities, reported behaviours of their child and children's interactions with others. Theme one, *numeracy environment structure*, illustrates the types of environments that parents create for their children to learn numeracy in the home. Theme two, *frequency of number-related experiences*, suggests that parents are not always cognisant when educating their child about numbers and in fact numeracy-related experiences could be occurring more frequently than reading activities. Theme three, *levels of number knowledge*, reveals that more meaningful explanations may be necessary from parents in order for their child to understand number words and their meanings. Theme four, *technology attitudes*, demonstrates that technology is being used extensively in the home and parents are concerned by the content of their children's viewing and interaction. Theme five, *parent-child interactions*, emphasises that parents usually aided their child's understanding through three types of interactions which were adjusted for each type of numerical problem a child got incorrect. Finally, theme six, *social interaction*, suggests that triad numerical interactions are occurring in the home and parents believe younger siblings are learning numeracy skills from older siblings. Themes one to three, five and six support previous research in the area of home numeracy environment, Theme four is an emerging area for future research which, to date, has not been sufficiently studied.

The findings have provided a comparative definition for the terms informal and formal. In the home environment, number-related activities were mainly spontaneous and taught through everyday tasks, this aligns with Song and Ginsburg (1987) use of the term *informal* learning, where children acquired numeracy through spontaneous interactions with the environment. It is evident that parents can create a formal mathematical environment at home (Song & Ginsburg's (1987) use of the term *formal* learning, which referred only to written work in school). The four components to creating a formal numeracy environment may be explicit planning, parental awareness, organised strategies, and setting aside time for numeracy, yet this was not often achieved (theme one). To elaborate on the terms formal and informal, in this study formal referred to creating a structured environment (e.g. parents explicitly plan number-related activities and are aware of their opportunity to teach numeracy), and informal referred to having an unstructured home numeracy environment (e.g. a spontaneous activity such as, counting out food). The evidence suggested that the home numeracy environment is largely unstructured, thus the home numeracy environment is mainly an informal learning environment. Direct activities, as defined by LeFevre et al. (2009), were mentioned at a higher frequency than indirect activities in the current study. However, the numeracy activities that occur in the home can be contingent on the environment, the situation that the parent creates, and how pedagogically-focused parents are with their children (Aubrey et al., 2003; Benigno & Ellis, 2004; Berk & Winsler, 1995).

Further, it was evident that parents may not always be cognisant when undertaking numerical activities with their child in the home. This finding is consistent with previous literature, which cites that parents are not always aware of the mathematical potential of children's early, informal experiences (Anderson, 1998). In a previous study Vandermaas-Peeler et al. (2012) randomly assigned parent-child dyads to a numeracy awareness group where numeracy instructions where provided to incorporate into the games and a comparison group provided with no numeracy instructions. Parents who were made aware provided guidance at approximately twice the rate of parents in the comparison group. It was concluded that parental awareness could enhance children's exposure to numeracy content and enrich socio-cultural interactions related to numeracy. Thus, the information that was gathered strengthens the possibility of targeting and intervene in the family context to make parents aware of the potential of home activities. Future research could investigate the efficacy of interventions that raise parents' awareness of *activities* (i.e. *direct* and *indirect activities*) and promote positive interactions in the *environments* (i.e. *unstructured/informal* and *structured/formal environments*) in order to assess the impact on children's learning. During the interviews that formed the current study, parents came to the realisation that number-related activities occurred every day, and that there were more frequent opportunities to teach numeracy than reading (theme two). It is important to note here that neither mathematics or literacy should to be done over the other but that they both should have a place in the home learning environment. The parent reports in this study are consistent with previous studies in which mothers were found to incorporate numbers into their young children's daily routines by counting food, learning numbers, or reading numbers off license plates (Aubrey et al., 2003; Kritzer, 2011).

An additional key finding of this study was the identification that emerging technologies are utilised for home numeracy activities (theme four). Thus, technology advances have potentially expanded the reach of numeracy learning in the home. A recent OfCom report (2013) stated that at home approximately one quarter (28%) of children aged 3-4 use tablet computers, the increasing accessibility of these types of technologies have potential to modify the types of numerical content young children may be exposed to in the home environment. Findings also showed that parents expressed a struggle to limit the duration of technology usage but that if a child could acquire knowledge from the activity a parent was more likely *not* to enforce a time limit. This discovery is similar to the findings of Mayo and Siraj (2015) who mentioned parents' struggle to enforce and maintain the duration of technology usage and limiting the duration was dependent on the type of technology the child was using. Future research should investigate how often, and to what extent, electronic devices are integrated into pedagogic planning in the home and its impact on young children's learning.

All parents reported supporting their child to learn numeracy however, these findings should be treated with caution due to the potential impact of social desirability on responses. Nevertheless, the main types of interactions mentioned were providing the correct answer when their child made an error, explaining different scenarios in order for their child to reach the correct answer, and encouragement to repeat the activity (theme five). These interactions align with those identified in an observational study (Bjorklund et al., 2004) and interviews completed with parents and children by the Effective Provision of Pre-School, Primary and Secondary Education (EPPSE; ages 3-16) research project (Mayo & Siraj, 2015). Mayo and Siraj (2015) found that if parents felt they were unable to provide help they simply provided answers without explanations or they made sure their child received help from a sibling or other outlet. This current study confirms that parent guidance is contingent on both children's abilities and the context in which numeracy is presented (Bjorklund et al., 2004). In addition, parent guidance may also be dependent on parent's abilities and, as found in this study, some parents may rely on technology for support (Mayo & Siraj, 2015).

Children's self-initiated activities do not seem to be enough to learn number word meanings, procedures of practice, and any associated numerical knowledge (Fuson, 1988; Nunes & Bryant, 1996). Thus, situational guidance (Berk & Winsler, 1995) is required in order to ensure children grasp conceptual understanding of number words. Parents discussed that their children interacted with siblings when number-related activities were occurring (theme six), consistent with previous literature (Howe et al., 2015; Howe et al., 2011). In this study triad interactions mainly occurred through homework. Parents believed that children were processing information being taught to older siblings, even if the numeracy was more advanced than what would be expected from their younger child. However, Benigno and Ellis (2004) found that the presence of a sibling meant parents were less likely to utilise some interactions as teaching opportunities. Thus, there may be a need for parents to be aware and to adapt different strategies for both their younger and older children for effective learning. Nevertheless, numerical interactions between siblings are occurring in the home, yet more research may be necessary to understand if children learn from their older siblings when parents are not available to guide the learning experience.

3.4.1. Limitations.

It is important to note that purposive sampling was used in this study as is typically used in qualitative research (Etikan, Musa & Alkassim, 2016). Due to the non-random selection of participants, caution should be taken when generalising these findings. In addition, as little is known about the everyday experiences involved in the home numeracy environment this study was exploratory and thus tackling new ideas has prepared the groundwork for further research (Singh, 2007). However, exploratory research can be open to bias, but the study uses rigorous qualitative research methods and the resulting thematic analysis had strong inter-rater reliability.

3.4.2 Conclusion.

Numeracy experiences in the home and parent's involvement in their child's early learning is important for later success (Duncan et al., 2007; Bjorklund et al., 2004). Given the dearth of research that explores the home numeracy environment via interviews with parents, the findings of this study offer a unique contribution to literature on the behaviour of parents, and early number-related activities that occur in the home. Literature demonstrates equivocal definitions, rendering is difficult to determine what defines an effective home numeracy environment that facilitates development in mathematics. This is further complicated by the lack of agreement on what parental involvement and interactions matter most. However, a common theme in the varying definitions through-out literature is that every learning experience in the home are shared learning experiences for children, whether this is between parents or siblings. It is evident that steps are needed to make parents aware of their informal teaching of numeracy in the home to develop a more effective home numeracy environment for children to learn.

For clarification, from these findings a *structured/formal environment* was defined as parents explicitly planning number-related activities, parent's awareness of their opportunity to teach numeracy, organising strategies for their child to learn and develop number skills and setting aside time for teaching numeracy. Whereas, an *unstructured/informal environment* was defined as parents having a spontaneous approach when referring to numeracy. However, this is in regard to the *environment* under which number-related activities occur as opposed to the *activities* that happen within the environment. As defined by LeFevre et al. (2009) *direct* activities involved explicitly and intentionally teaching about numbers or arithmetic to develop children's mathematical skills (e.g. counting objects) and *indirect* activities involved numbers in real-world tasks (e.g. playing board games with dice) that include 'hidden' mathematical instructions that occur incidentally.

Consequently, these *environment* and *activity* definitions can be seen to contradict each other due to the findings from this study. For instance, the numerical environment discussed in this study was more likely to be an *unstructured/informal environment* however, *direct* activities were mentioned at a higher frequency. Therefore, even though parents have a spontaneous approach (i.e. definition of an unstructured/informal environment) they are explicitly and intentionally teaching about numbers or arithmetic to develop children's mathematical skills (i.e. definition of a direct activities) which is similar, in part, to the definition of a *structured/formal environment* (i.e. parent's awareness of their opportunity to teach numeracy). Therefore, although the word 'explicitly and intentionally' are used within the definition used by LeFevre et al. (2009) to describe *direct* activities this study suggests that the parent may actually be unaware of 'explicitly and intentionally' teaching about numbers or arithmetic to develop children's mathematical skills in an *unstructured/informal environment* (see section 3.3.1 Theme 1. Numeracy environment structure for finding). Therefore, for clarification proposes the researcher would suggest that when discussing the *environment* under which number-related activities occur the terms *structured/formal* and *unstructured/informal environment* be utilised. Whereas, when discussing the *activities* that happen within the environment the terms *direct* and *indirect* activities be used.

Chapter 4: The Scale Development Process of a Pre-school Home Maths Questionnaire (PHMQ)

Overview

Chapter 4 focuses on various stages of scale development using a well-established framework to reduce the likelihood of measurement problems (Price & Mueller, 1986). This study utilised multiple item development methods to tap into the construct known as the home numeracy environment (HNE) by using the process described by Hinkin (1998). This chapter presents phase one, known as *the scale development process*. This process comprises four development stages of questionnaire development explained by Hinkin (1998) including; (stage 1) item generation, (stage 2) questionnaire administration, (stage 3) initial item reduction and (stage 4) an exploratory factor analysis. The focus of phase one is on construct validity, which combines theory and psychometric measurement (Kerlinger, 1986). This chapter builds on the findings from the previous chapter.

There are three types of validity that need to be addressed to ensure psychometric soundness of a measure; construct, content and criterion validity. Chapter 4 will address construct validity, known as phase one *the scale development process* and Chapter 5 will present content and criterion validity, known as phase 2 *the scale validation process.*

4.1 Introduction

4.1.1 Phase one: The scale development process.

It is essential that the measure being created adequately represents the construct under examination, in this case the home numeracy environment (HNE), and it is of utmost importance that the measurement instrument reaches psychometric soundness (Schoenfeldt, 1984). The term psychometric soundness is a reference to a test's reliability and validity (Hinkin, 1998). Each stage of the scale development process will be discussed to increase the psychometric soundness of the HNE measure. This chapter will address construct validity, known as phase one *the scale development process.* Figure 1 illustrates the stages of phase one of *the scale development process*. Phase one comprises four stages, based on Hinkin (1998) tutorial on the development of measures. The four stages include; (stage 1) item generation, (stage 2) questionnaire administration, (stage 3) initial item reduction and (stage 4) an exploratory factor analysis. Phase one presents construct validity, which addresses two further psychometric properties, factor structure (i.e. dimensionality) and scale score reliability.

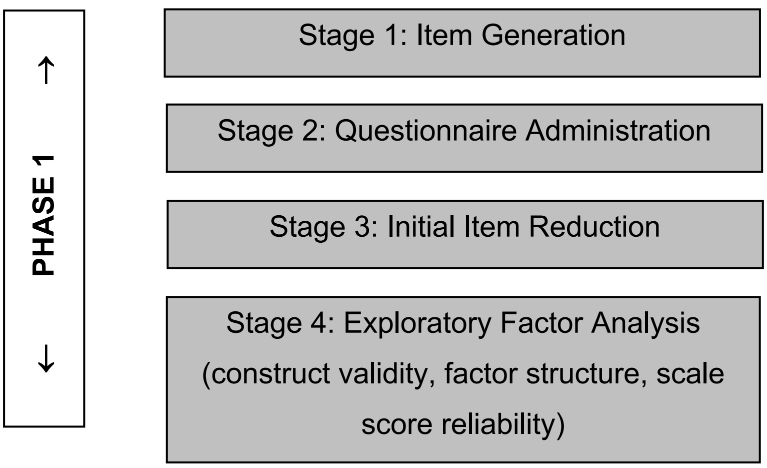


Figure 1. Visual representation of phase one of scale development

4.1.1.1 Stage 1: Item generation.

Once a theoretical foundation for the potential measure is developed, the first stage to scale development is (1) item generation; the creation of items that assess the construct. The current study utilised multiple item development methods to attempt to measure the HNE construct. The fundamental goal was to demonstrate content validity by sampling systematically all content that is potentially relevant to the target construct (Clark & Watson, 1995). However, domain sampling theory suggests that it is not plausible to measure a complete domain, but it is vital to draw potential items in order to sufficiently represent the construct under examination (Ghiselli, Campbell & Zedeck, 1981; Hinkin, 1998). The development of theory or measurements should ideally be both deductive and inductive (Williamson, Karp, Dalphin & Gray, 1982), therefore the current study used both approaches. Deductive scale development suggests that theory provides enough information to generate the initial set of items (Hinkin, 1998). Whereas, inductive theory builds from interviews or case studies, producing new theory from data or in this case measurement items (Eisenhardt & Graebner, 2007).

As far as the author is aware, no research has developed questionnaire items based on interviews with parents of 3 to 4-year olds in order to develop a home *numeracy* environment questionnaire measure. Although Melhuish et al. (2008) did use interviews with parents to develop a home *learning* environment index. Questions regarding the frequency that children engaged in 14 activities were included in the interview. The 14 activities covered social (e.g. play with friends at home, and elsewhere), routine (e.g. regular bedtime), literacy (e.g. going to the library), numeracy (e.g. playing with numbers) and spatial (e.g. painting and drawing) activities and during the interviews the participants answers were coded on a 7-point Likert scale for each activity. Later a selection of these 14 activities were used based on a multilevel model to construct a 7-item home learning environment index. Melhuish et al. (2008) describe this interview process as semi-structured interviews with most questions being pre-coded, but if an interview process has pre-coded outcomes then the interview is better described as a structured interview (Fox, 2009). This type of structured interviewing procedure is not exploratory and has predetermined outcomes which restricts conclusions (Fox, 2009) and does not allow for home numeracy activities to be discovered.

The semi-structured interviews in the previous chapter (Chapter 3) were exploratory and aimed to gain opinions from parents on their everyday routine activities and understand the way in which parents encourage the development of early numeracy skills in the home. The six findings were; (1) numeracy environment structure, (2) frequency of number-related experiences, (3) levels of number knowledge, (4) views of technology, (5) parent-child interactions and (6) social interaction. The themes illustrate how the home numeracy environment may be influenced by parents' attitudes and expectations, and parent-child interactions (Cahoon et al., 2017). The previously discussed interviews and consequent six themes will be used to generate items for a United Kingdom (UK) based home numeracy environment questionnaire. Melhuish et al. (2008) is the only home learning environment questionnaire generated specifically for the UK thus other existing home numeracy environment scales may not be culturally appropriate.

Furthermore, many general home environment questionnaires are very brief with only a few items regarding numeracy. For instance, Melhuish et al. (2008) in their home learning environment index only included two items on numeracy activities (e.g. the frequency of playing with letters/numbers and numbers/shapes). Likewise, some home numeracy environment questionnaires are also very brief for example, Kleemans et al. (2012) only had four parent–child numeracy activities items. A small number of home numeracy items may not be representative of the everyday routines and practices that occur in the home making well-rounded conclusions difficult. Overall, existing questionnaires are either not culturally appropriate or very brief. In contrast to previous studies, this study will consider technology-based educational experiences and social interactions with parents and siblings. Thus, this study moves beyond the limits of previous research.

4.1.1.2 Stage 2: Questionnaire administration.

Once the items have been generated the second stage of scale development is (2) questionnaire administration. At this stage the items should be administered to a sample representative of the actual population of interest (Hinkin, 1998), in this case parents with children aged 3 to 4 years old. The aim is to examine how well items confirmed expectations concerning the psychometric properties of the new measure (Hinkin, 1998).

4.1.1.3 Stage 3: Initial item reduction.

After data collection a questionnaire should undergo (3) initial item reduction which is the third stage of scale development. Initial item reduction refers to questions that are removed due to lack of variance. A potential explanation for lack of variability in responses to questions can be due to many factors. For example, the "halo effect", which is the tendency for one impression to shape or influence all other judgements (Fitzpatrick, 1991; Wilson, Hewitt, Matthews, Richards & Shepperd, 2006). Furthermore, it is important that the scale used (e.g.

rank order or rating items) for the scaled items produces necessary variance for subsequent statistical analyses (Stone, 1978). These aspects will be examined during the third stage of scale development.

4.1.1.4 Stage 4: Exploratory factor analysis

The fourth stage of scale development is using an (4) exploratory factor analysis to refine new scales within a questionnaire. There are a number of different scaling techniques however, Likert-type scales are the most frequently used in questionnaires (Cook, Hepworth & Warr, 1981; Hinkin, 1998) and are the most suitable for use in factor analysis. Therefore, a factor analysis will be utilised to decrease a set of observed variables into a smaller set of observed variables.

4.1.2 Rationale.

The main aim is to understand the primary environment for a child aged 3 to 4-years and in turn explain how the early home environment influences young children's development of early mathematical skills (Blevins-Knabe, 2016; DeFlorio & Beliakoff, 2015). The foundation for the creation of HNE measures has been that since the early home environment (i.e. during pre-school years) has been connected to children's literacy skills it is theoretically reasonable to predict that the early home environment will impact children's numeracy skills (Blevins-Knabe, 2016; LeFevre et al., 2009; 2010b; Lukie et al., 2014). Accordingly, researchers have drawn questions from home literacy environment questionnaires to create home numeracy environment questionnaires. Alternatively, other home numeracy questionnaire measures are based on variations of the Home Observation for Measurement in the Environment (HOME) inventory (Caldwell & Bradley, 1984) for example, Anders et al. (2012). However, as the development of theory or measurements should ideally be both deductive and inductive (Williamson et al., 1982) the current study used both approaches.

The purpose of this study is to address the issues with current home numeracy environment scales. For instance, some researchers who have created home numeracy environment scales have not provided adequate information about item generation and refinement, scale dimensionality, scale score reliability, or validity (e.g. Kleemans et al., 2012; LeFevre et al., 2009; Melhuish et al., 2008). By creating a measure that addresses these issues the researcher aims to develop a home numeracy environment scale that is strong psychometrically. This study aims to develop and refine a parent-focused questionnaire designed to measure the frequency and quality of mathematical experiences of pre-school children in their homes. The main aim of this investigation is to provide the framework used for the development of a home numeracy measure in accordance with established psychometric principles.

4.2 Method

Phase 1: The Scale Development Process

4.2.1 Stage 1: Item generation.

4.2.1.1 Inductive approach to scale development.

The first item generation method was to develop items based on the interviews (Chapter 3) to produce new measurement items. As theoretical definitions of the HNE construct vary from study to study and no consensus definition has been established of what encompasses the everyday routine and practices occurring in the HNE (Cahoon et al., 2017; LeFevre et al., 2009; Skwarchuk et al., 2014), more information was necessary to develop the HNE items. Interviews have been recommended as a scale development method in this scenario (Butler, 1991; Linehan, Comtois, Brown, Heard & Wagner, 2006; Kipnis, Schmidt & Wilkinson, 1980; Wolf, Putnam, James & Stiles, 1978). Based on the information gathered from pre-schoolers' parents during the semi-structured interviews (Cahoon et al., 2017) (Chapter 3) 44 items were developed to create the initial Pre-school Home Maths Questionnaire (PHMQ).

4.2.1.2 Deductive approach to scale development.

The second item generation method was to develop a base set of items that assessed the HNE drawn from previous HNE measures (e.g. LeFevre, et al., 2009; Lukie et al., 2014; Kleemans et al., 2012; Melhuish et al., 2008) and previous parent-child interaction research such as, observational research involving parent guidance and support (e.g. Vandermaas-Peeler, Boomgarden, Finn & Pittard, 2012; Bjorklund, Hubertz & Reubens, 2004). All items were cross-referenced between those mentioned from the interviews (e.g. a numeracy activity such as counting objects) and items from other HNE measures or cited in previous parent-child interaction research. Twenty-five items from previous research were added to the questions established from the inductive approach, as they were mentioned in both previous research and the interviews. For example, parents mentioned in the interviews that they felt their child (aged 3 to 4) was learning number-related concepts from their siblings. Further, Benigno et al. (2004) in an observation study found that interactions were occurring between parent target-child and sibling. It was therefore deemed important that the question, "Do you feel that your child has learnt skills from their siblings?" was added as it was mentioned in both previous research and the interviews. An advantage of using a deductive approach in addition to an inductive approach to scale development is that it helps to guarantee content validity in the final scales (Hinkin, 1998).

4.2.1.3 Summary of items.

Together, the inductive items (N = 44) and deductive items (N = 25) totalled to 69 items that were categorised into eight home environment relevant dimensions based on the questions characteristics for example, literacy questions were gathered into a literacy home environment dimension. These categories were; (1) two parent expectation questions (e.g. “What is the highest educational level and mathematical achievement the parent/guardian would want the target child to complete?”), (2) four literacy questions (e.g. the frequency of child engagement in reading) and (3) three counting ability questions (e.g. “How high can your child count up to?”). These questions are known as benchmark questions; questions that give context to results by allowing comparison between participant responses.

A further two categories were; (4) five parent-child teaching methods questions (e.g. “What are the specific interactions the parent/guardian would do to encourage and support the target child to learn numeracy?”) and (5) 13 target child-sibling interactions questions (e.g. “What numerical activities siblings are most likely to do together?”). These two categories involving interactions with parents and interactions with siblings were named as interaction questions. For the main questions in these two dimensions parents were asked to arrange answers in rank order (four ranking options for parent-child teaching methods and eleven ranking options for target child-sibling interactions).

Another category was the frequency of numeracy activities questions; (6) 38 questions, which were generated from the interviews. The 38 items were placed in a random order to control for order bias. Parents were asked, "In the past month, how often did you and your child engage in the following?”. Response options were on a 5-point Likert scale as follows: activity did not occur, few times a month, about once a month, few times a week, and almost daily. The last two categories were; (7) three questions on parent’s view of their child’s understanding of numeracy (e.g. “do you believe that your child understands the meaning of number words up to 5?”), and (8) one support question (e.g. “do you believe it is important for caregivers to support numeracy learning in the home?”) were added to the PHMQ. The questionnaire also comprised 29 demographic questions, such as participant’s relationship to the target child, SES (classified by using The National Statistics Socio-economic Classification (NS-SEC), Rose & Pevalin, 2010), employment of other adults living in household, birth order of target child etc. In the initial PHMQ measure there were 69 items (e.g. items generated from inductive and deductive approach), excluding demographic questions. Overall, there was a total of 98 questions. A detailed breakdown of the items and how they were generated is presented in Appendix 4.2, Table 2. The original questionnaire is presented in Appendix 4.1.

4.2.2 Stage 2: Questionnaire administration.

4.2.2.1 Participants.

To acquire an equal spread of participants (e.g. across SES) through data collection, the proportion of free school meals (FSM) per school was calculated for 67 schools across Northern Ireland, using Department of Education (2014) statistics. FSM is increasingly used as a proxy for SES variables in UK educational research (Hobbs & Vignoles, 2007). However, FSM is not a perfect proxy as families different extensively and thus cannot truly be compared based on FSM alone. For instance, 8% of non-FSM children are in workless families, while 43% of FSM children are in families with two part-time workers or one or more full-time workers. Thus, the bias produced by using only FSM as a SES variables is context-specific. Furthermore, FSM is an imperfect proxy of mothers' and partners' education and social class (Hobbs et al., 2007). Therefore, to avoid imperfect proxy bias (i.e. a proxy that correlates with the key variable but cannot be understood in isolation) parents were asked in the PHMQ to complete 8 questions from NS-SEC (2010), which allowed the researcher to derive SES using the Standard Occupational Classification (SOC2010). SES descriptives will be discussed in more detail in the results.

The FSM statistics were divided into three proportions to distinguish schools that had low (4-18%), medium (19-58%) and high (59-85%) FSM eligibility. The average FSM eligibility was 37.7%. It was anticipated that an equal spread of pre-schools would be contacted from the three FSM eligibility categories. However, there was a low participation rate from the pre-schools in the medium FSM eligibility category, so more pre-schools were contacted from this category. A total of 26 pre-schools were contacted and invited to take part in the study. All pre-schools responded and 11 responded positively.

Clark and Watson (1995) recommend that approximately 200-300 respondents be assessed at the stage of construct validity. In a scale development study LeFevre et al. (2009) recruited 258 children, from this, 146 parents returned a questionnaire giving a participation rate of 57%. Hence for this study the recruitment of approximately 300 purposively sampled participants was necessary. However, from the 309 PHMQ that were distributed to the parents of the children in the 11 pre-schools only 87 questionnaires were returned, giving a participation rate of 28%. The proportion of PHMQ returned from each of the low, medium and high FSM categories were 30%, 42.5% and 27.5%, respectively. Due to the low participation rate four play centres across Northern Ireland were contacted and invited to take part in the study. A play centre is a soft obstacle play area for children up to the age of 8 at which parents/guardians supervise play. Thus, it was deemed an ideal area to target parents with children aged between 3 to 4 years old. Three play centres responded, and two play centres agreed to be involved. From the two play centres 88 questionnaires were collected, giving a grand total of 175 completed HNE questionnaires. The criteria for participation across all studies that involved the PHMQ was that the parent/guardian was related to the target child aged between 3 to 4 years-old. Three questionnaires were omitted as these participants were child-minders and therefore did not meet the inclusion criteria for participation in the study.

A total of 172 parents/guardians were involved, all of whom provided informed consent before participating in the study. Participants were asked to specify their relationship to the target child: 148 mothers, 18 fathers, 3 grandparents, 2 foster parents and 1 adoptive parent completed the questionnaire. 157 (91.3%) of participants specified that they were the primary carers for the target child (spend most of the time with the target child). The target child (52.3% female) that the parents/guardians were asked questions on were aged between 36 months to 60 months (M_{age} = 46.2 months). Tables 1 to 5 report all the demographic data for the study.

4.2.2.2 Materials.

A brief cover letter and participant information sheet was distributed to relevant parents/guardians, these materials contained information on the purpose of the study, what was required of the parent/guardian, what will happen with the information once collected and an explanation on how to withdraw from the study. Participant consent forms were also distributed. The parent/guardian signed a consent form stating that they wished to be involved in the study. The parent/guardian also ticked six boxes confirming that they agreed to take part, that they cared for a child between the ages of 3 to 4 years old and read and understood the information sheet. Further, they agreed to answer demographic questions on their family, understood that their participation is voluntary and that they were free to withdraw without giving a reason, and finally that the researchers would hold all information collected securely and would not share the information with any other party. For the pre-school parents/guardians, a reminder flyer was distributed by the school to those who did not returned the questionnaires after approximately two weeks. The questionnaires were returned to teachers for collection in sealed envelopes to maintain confidentiality.

4.2.2.3 Procedure.

Before the questionnaire was administrated to parents/guardians the questionnaire was piloted in a student population (n = 10) to confirm the length of time it took to complete the questionnaire and to make sure the presentation was easy to read and understand. The questionnaire took approximately 10-15 minutes to complete and adjustments were made to the questionnaire to make sure participants would understand the terminology.

The researcher collected the data by convenience sampling. The participant read the cover letter, information sheet and signed the consent form, then completed the PHMQ. The participants that completed the PHMQ in the play centres did the questionnaire on the day they agreed to the study and they did not take them home. An incentive to complete the questionnaire was given to all participating parents in the play centres, which was a £5.00 Amazon voucher.

4.2.2.4 Ethical considerations.

The research procedures were reviewed and approved by School of Psychology Research Ethics Committee (REC) before the study commenced. Personal information was stored in a secure location. All paper-based data (e.g. PHMQ) was placed in a locked filing cabinet. Electronic data (e.g. SPSS files) were anonymous and stored on a password-protected laptop. Parents/guardians provided a signed consent form indicating that they agreed to take part in the study. The questionnaires were returned to teachers for collection in sealed envelopes to maintain confidentiality.

4.2.2.5 Data analysis.

Before analysis began the data was entered into SPSS Version 23 and checked by second researcher via a double entry method. The second researcher was intensively trained before inputting the data and data was verified to get a 100% match in cases, which means there were no mistakes; increasing the validity of the data. Double entering data is substantially more effective than other data checking methods, such as visually checking (Barchard & Verenikina, 2013). Preliminary analyses were conducted, and the demographic composition of the participants was determined using frequencies, means and standard deviations. An exploratory factor analysis was used to investigate variable relationships for complex concepts (e.g. the frequencies of mathematical activities) allowing the researcher to investigate concepts that are not easily measured directly by collapsing large numbers of variables into a few interpretable underlying factors.

4.2.2.6 Sample size for exploratory factor analysis.

There is little agreement between researchers on what the appropriate sample size is to conduct a factor analysis (Floyd & Widaman, 1995) with some recommending absolute number of cases (N), while others state that the subject-to-variable ratio (p) is more important (Arrindell & van der Ende, 1985; Hutcheson & Sofroniou, 1999; MacCallum, Widaman, Zhang & Hong, 1999). In this study, 38 variables would be used for the exploratory factor analysis

(EFA). Thus, it was important to aim for approximately 150-200 participants to reach a value of between 1:4-1:5 subject-to-variable ratio. This would then be consistent with previous research which suggests that a ratio of 1:3-1:6 subject-to-variable is acceptable (Arrindell & van der Ende, 1985; Cattell, 1978). A subject-to-variable ratio of 1:4.5 was achieved with 172 participants.

4.3 Results

Overview

The method section has covered both (stage 1) item generation and (stage 2) questionnaire administration, the first two stages of the four stages to the scale development process explained by Hinkin (1998). The results section will now discuss (stage 3) initial item reduction and (stage 4) an exploratory factor analysis. However, before these sections the demographic composition of participants will be discussed.

4.3.1 Demographic composition of participants.

4.3.1.1 Sex-age distribution.

Eighteen males completed the questionnaire with a mean age of 37.2 years (SD = 7.5 years). 144 females took part and had a mean age of 35.0 years (SD = 6.4 years). The overall mean age of participants was 35.3 years (SD = 6.5 years). The mean age of the target child (that the parents/guardians were asked questions on) was 3.33 years (SD = 0.5 years). Male target children had a higher mean age (M_{age} = 3.37 years, SD = 0.6 years) compared to females (M_{age} = 3.29 years, SD = 0.5 years), although there was no significant difference.

Table 1. *Descriptive statistics of age for participant and target child*

		Mean	Std. Deviation	Minimum	Maximum
Age of participant (N = 162)	Age (yr.)	35.26	6.51	23	65
Sex – Age	Male (N = 18)	37.17	7.45	28	61
	Female (N = 144)	35.02	6.37	23	65
Age of target child (N = 172)	Age (yr.)	3.33	0.51	3	4
Sex – Age	Male (N = 82)	3.37	0.56	3	4
	Female (N = 90)	3.29	0.46	3	4

4.3.1.2 Socio-economic classification.

Table 2 summarises the job categories of the participants. The job categories used to derive SES from the NS-SEC (2010) were 14 functional and three residual operational categories. The functional categories represent a range of specific employment statuses and labour market positions. In contrast, the residual categories can be grouped together as 'not classified' jobs (e.g. full-time students) (NS-SEC, 2010). Most participants had jobs in the *lower professional and higher technical occupations* (26.2%) which includes jobs such as, a nurse, mortgage specialist and primary school teacher. This was followed by both *semi-routine occupations* (14.5%) that involved jobs such as, dental nurse, receptionist and clinical support worker. Fourteen percent of respondents had *intermediate occupations* (14.0%) such as, clerical officer, civil servant and sales assistant. *Higher managerial and administrative occupations* (0.6%), *employers in small organisations* (1.2%), *lower technical occupations* (1.7%) and *routine occupations* (1.7%) were all categories with the least number of participants these included jobs such as human resources manager, foster carer, chef and waitress respectively. The data illustrates that participants were from a wide range of SES backgrounds.

Table 2. *Socio-economic classification*

Operational categories	Participants *n* (%) (n = 172)
Employers in large establishments	0 (0)
Higher managerial and administrative occupations	1 (0.6)
Higher professional occupations	18 (10.5)
Lower professional and higher technical occupations	45 (26.2)
Lower managerial and administrative occupations	12 (7.0)
Higher supervisory occupations	11 (6.4)
Intermediate occupations	24 (14.0)
Employers in small organisations	2 (1.2)
Own account workers	4 (2.3)
Lower supervisory occupations	13 (7.6)
Lower technical occupations	3 (1.7)
Semi-routine occupations	25 (14.5)
Routine occupations	3 (1.7)
Never worked and long-term unemployed	7 (4.1)
Full-time students	2 (1.2)
Occupations not stated or inadequately described	2 (1.2)
Not classifiable for other reasons	0 (0)

4.3.1.3 Education levels.

Table 3 summarises participants highest educational and mathematical qualifications. There was a wide spread of highest educational level achieved by the participants in the study with most participants reaching degree level (29.7%) and only 2.3% of participants having no qualifications. Most participants had reached at least a GCSEs / O level / Irish Junior Certificate in mathematics (65.1%) with only 5.2% of participants having no mathematical qualification.

Table 3. *Educational attainment*

	Education level	Participants *n* (%) (n = 171)
Highest Educational Qualification	GCSEs / O level / Irish Junior Certificate	48 (27.9)
	A levels / BTEC / Irish Leaving Certificate	34 (19.8)
	Degree	51 (29.7)
	Masters	20 (11.6)
	PhD	2 (1.2)
	No qualifications	4 (2.3)
	Other	12 (7.0)
Highest Mathematical Qualification	GCSEs / O level / Irish Junior Certificate	112 (65.1)
	A levels / BTEC / Irish Leaving Certificate	24 (14.0)
	Degree	14 (8.1)
	Masters	5 (2.9)
	No qualification	9 (5.2)
	Other	7 (4.1)

4.3.1.4 Further demographics.

Table 4 illustrates the marital status, ethnicity and first language spoken by the participants. The majority of participants were married (62.8%). Other common marital statuses were single (14%) and cohabiting (16.9%). The majority of participants were White/Caucasian (94.2%). Other ethnic backgrounds were Chinese and Mixed race (both 1.7%). The first language spoken by participants was mostly English (95.9%). Other first languages spoken were Chinese and Polish (both 1.2%), and Cantonese and Hungarian (both 0.6%).

Table 4. *Further demographic characteristics of the participants*

		Participants *n* (%) (n = 171)
Marital Status	Single	24 (14)
	Married	108 (62.8)
	Cohabitating	29 (16.9)
	Divorced	4 (2.3)
	Separated	5 (2.9)
	Widowed	1 (0.6)
Ethnicity	White/Caucasian	162 (94.2)
	Chinese	3 (1.7)
	Mixed	3 (1.7)
	Other	2 (1.2)
	Black or African American	1 (0.6)
First language spoken	English	165 (95.9)
	Chinese	2 (1.2)
	Polish	2 (1.2)
	Cantonese	1 (0.6)
	Hungarian	1 (0.6)

4.3.1.5 Other adults in household.

Table 5 summarises the relationship of other adults living in the same household as the target child. The majority of adults living in the same household were fathers (66.3%). 11.6% of adults living in the same household were mothers and 16.3% of participants lived alone.

Table 5. *Other adult living at home relationship to target child*

Relationship to target child	Participants *n* (%) (n = 171)
Mother	20 (11.6)
Stepmother	1 (0.6)
Father	114 (66.3)
Stepfather	3 (1.7)
Grandparent	5 (2.9)
Live alone	28 (16.3)

4.3.2 Stage 3: Initial item reduction.

4.3.2.1 Summary of items.

The removal or adjustment of items/questions will be discussed in reference to the eight-home environment relevant dimensions mentioned and broken down in the Method section, 4.2.1.3 Summary of items. The eight-home environment relevant dimensions that made up the PHMQ are; (1) parent expectation questions, (2) literacy questions, (3) counting ability questions. These questions are known as benchmark questions as they give context to results by allowing comparison between participant responses. (4) Parent-child teaching methods questions and (5) target child-sibling interactions questions. These two categories, involving target child interactions with parents and siblings, were named as interaction questions. The (6) frequency of numeracy activities questions, were used within the exploratory factor analysis. Finally, (7) questions on parent's view of their child's understanding of numeracy and (8) a support question.

4.3.2.2 Benchmark questions.

All benchmark questions were retained for the final questionnaire. These benchmark questions give context to results by allowing comparison between participant responses, such as parent expectation (2 questions), literacy (4 questions) and counting (3 questions).

4.3.2.3 Interaction questions.

The parent-child teaching methods questions (5 questions) were kept due to good variation in results however, the target child-sibling questions (originally 13 questions) were reduced. Out of the total number of target children involved in the study (N = 172) 85.5% of children had siblings (N = 147). When parents were asked if they felt their child was learning number skills from their siblings 50% (N = 86) stated yes and 16.9% (N = 29) stated no, with a total of 33.1% stating that this did not apply; this category applied to both only children and those siblings who were understood to be too young to learn from (e.g. infants), this question was kept. Another question from the target child-sibling interactions dimension was dropped due to the lack of variation in results. The question was, "What would your participating child be more likely to do when engaged in a mathematical based activity with siblings?". 57.6% (N = 99) of participants answered that their child would *take part in the activity*, with only 12.2% (N = 21) of participants stating that their child would *observe the activity*. Criteria for lack of variability for this question was >50%. A potential explanation for lack of variability in responses to this question is the "halo effect". For this question parents may want it to appear that their child takes part in an activity even if it is too advanced for their child to take part in. This finding was discovered in the interviews in chapter 3 and confirms this point, see 3.3.6 Theme 6. Social interaction for more detail.

The 11 ranking options for target child-sibling interactions were reduced to 7 ranking options. The threshold for cut off was any rank option that scored over 20% in the least likely categories. Therefore, the four least likely activities to occur between the target child and siblings were removed (see Appendix 4.2, Table 2 for overview). These activities were; observing older siblings' homework (23.2%), taking part in older siblings' homework (34.8%), mathematics applications on technology device (e.g. Playing Number Jacks on iPhone) (27.3%) and play board games or card games together (e.g. "jack change it") (24.5%). Furthermore, another reason for reducing rank order options was that participants found it too difficult to rank order 11 options. However, this was piloted with a group of undergraduate students (N = 10) who also found it difficult to rank order the 7 options. Therefore, this question was changed to match the 5-point Likert scale of the frequency of numeracy activities questions. This change is discussed further in section 5.2.1.1 New scaling for target child-sibling interaction.

4.3.2.4 Other home numeracy dimensions.

Due to lack of variation in responses four questions were removed from the questionnaire; children's understanding of numeracy items (3 questions) and importance of support question (1 question). The questions were "Do you believe that your child understands the meaning of number words up to 5?" to which 77.3% (N = 133) answered yes, "Do you believe that your child understands the meaning of odd and even?" to which 91.9% (N = 153) answered no, "Do you believe that your child understands the meaning of more and less? (e.g. one pile of clothes bigger than another set of clothes)" to which 92.4% (N = 159) answered yes and finally "Do you believe it is important for caregivers to support numeracy learning in the home?" to which 98.8% (N = 170) answered yes. Criteria for lack of variability for these questions was the same as mentioned before >50%, and they are classic "halo effect" questions (Fitzpatrick, 1991; Wilson, Hewitt, Matthews, Richards & Shepperd, 2006) (see Appendix 4.2, Table 2 for overview).

4.3.3 Stage 4: Exploratory factor analysis.

An exploratory factor analysis was conducted on the responses to the 38 frequency of numeracy activities items. The 38 items were analysed using a principle components analysis with oblique rotation (direct oblimin). Thus, reducing the number of variables and determining activities grouped together. The suitability of data for factor analysis was assessed. The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis (KMO = .80), and all KMO values for individual items were greater than .59. Five factors had eigenvalues over Kaiser's criterion of 1 and in combination explained 53.14% of the common variance. The scree plot (Figure 2) showed inflexions that would justify retaining 5 factors. The factors were labelled as follows; (1) parent - child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting which comprised 28 items from the frequency of home numeracy activities component of the questionnaire. Ten items did not load onto any factor and therefore these were removed from further analysis. Table 6 shows the factor loadings after rotation. Cronbach's alpha for the total scale was .89. Cronbach's alpha for the total subscales based on these factors were acceptable, ranging from .76 for the *counting* factor to .81 for both the *parent - child interactions* and *computer maths games* factors, thus display good internal reliability.

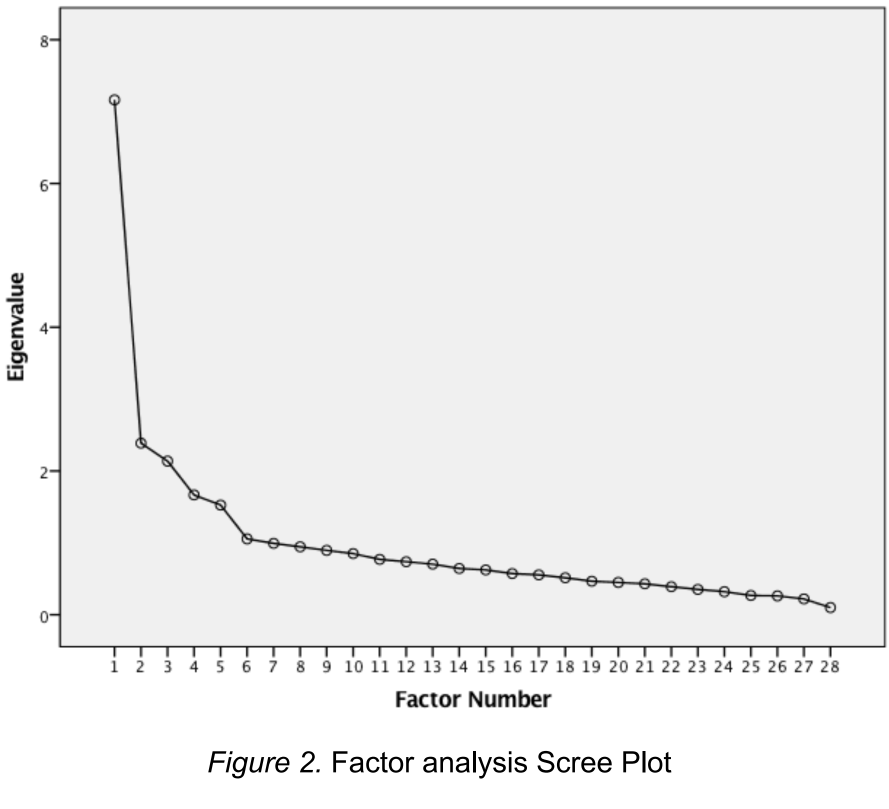


Figure 2. Factor analysis Scree Plot

Table 6. *Summary of exploratory factor analysis results for home numeracy environment questionnaire*

	Rotated Factor Loadings				
Items	Parent – child interactions	Computer maths games	TV programmes	Shape	Counting
Identifying names of written numbers	**.65**	-.01	.11	.08	.03
Write numbers	**.59**	.05	.06	.03	.14
Teaching about measurements (e.g. baking or height)	**.54**	-.04	-.03	.02	-.18
Time terminology (e.g. big hand, little hand)	**.50**	.09	-.05	-.03	-.07
Asking shape related questions (e.g. “how many sides does a circle have?”)	**.49**	.07	-.09	.20	-.12
Scenarios number games (e.g. "if I have two toy cars and I take one away, how many cars do I have?")	**.49**	.10	.04	-.09	-.25
Teaching about money (e.g. playing shop or buying sweeties)	**.43**	.12	-.10	.02	-.28
Sticker books	**.38**	-.02	.18	.14	-.08
Maths related websites (e.g. coolmaths.com)	-.02	**.71**	.00	-.05	.01
Racing games (e.g. the faster they complete sums, the faster the boat moves)	-.17	**.67**	.03	.01	-.02
Size/matching apps (e.g. “put the big skirt on the small girl”)	-.03	**.65**	.04	.07	-.01
Maths applications (e.g. Number Jacks)	.19	**.63**	.10	-.05	-.01
Add and subtraction games	.20	**.60**	.09	-.04	.01
Filling in the gap number games (e.g. what is next in the sequence?)	.16	**.51**	-.06	.07	-.01
Watching number related TV shows (e.g. Number Jacks or Numtums)	.13	.00	**.89**	-.07	-.03
Rhyming TV shows involving numbers (e.g. Number Jacks)	.03	.11	**.85**	.02	-.04
Watch educational programs (e.g. Dora the Explorer)	-.15	.13	**.38**	.14	-.19
Sorting shapes	-.03	.06	-.03	**.62**	-.19
Sorting objects by size	-.05	.04	-.04	**.61**	-.34
Creating patterns with objects (e.g. arranging blocks into shapes)	.10	.12	-.12	**.61**	.02
Playing with building blocks	-.04	-.04	.12	**.58**	.15

Play with jigsaws	.09	-.12	.08	**.54**	.04
Pairing/matching games	.07	.09	-.03	**.44**	-.13
Counting out food, dinner plates, knives and forks	-.04	.01	.11	.09	**-.61**
Counting	.07	-.09	.05	-.07	**-.59**
Counting objects (e.g. ducks in bath, blocks, new toys, books)	.04	.05	.06	.15	**-.55**
Counting on fingers/hands	.15	.01	.02	.01	**-.55**
Comparing sets of objects (e.g. brother has more than mum)	.09	.20	-.03	.05	**-.52**
Eigenvalues	7.16	2.39	2.14	1.67	1.53
% of variance	25.58	8.52	7.63	5.95	5.45
α	.81	.81	.79	.78	.76

Note: Factor loadings over .40 appear in bold. N=172

Note: Oblique rotation allows for correlation hence, the counting factor having a negative factor loading shows that it is negatively correlated with the other four factors.

4.3.3.1 Data reduction summary.

From the initial 69 items, 19 items were removed for different reasons mentioned previously (See Appendix 4.2, Table 2 for breakdown). In sum, 14 deductive items and 5 inductive items were removed thus a total of 50 items were retained. Instead of eight-home environment relevant dimensions mentioned previously there are now six-home environment relevant dimensions with the removal of the questions on parent's view of their child's understanding of numeracy and the support question.

4.4 Discussion

4.4.1 Phase one: The scale development process.

This chapter presented phase one, *the scale development process,* comprising four stages, based on Hinkin (1998) tutorial on the development of measures. The four stages include; (1) item generation, (2) questionnaire administration, (3) initial item reduction and (4) an exploratory factor analysis.

4.4.1.1 Stage 1: Item generation

Kleemans et al. (2012) and Lukie et al. (2014) home numeracy measure were based on LeFevre et al. (2009). Preceding this, LeFevre et al. (2009) reported that the list of home activities came from a variety of sources. However, these sources are not stated and therefore it can only be assumed that these items were generated through a deductive process. Hinkin (1998) suggests that a theoretical foundation (deductive process) provides enough information to generate the initial set of items. However, Williamson et al. (1982) state that ideally to develop a measure both a deductive and inductive process should be utilised. This study builds items from previous interviews (chapter 3; Cahoon et al., 2017) producing new theory from data (Eisenhardt et al., 2007) as well as developing items through previous literature (LeFevre et al., 2009; Vandermaas-Peeler, Ferretti & Loving, 2012; Melhuish et al., 2008; Lukie et al., 2014; Kleemans et al., 2008; Benigno et al., 2004), see Appendix 4.2 for breakdown of items. Therefore, in contrast to previous work on children's home numeracy environment (e.g., Kleemans et al., 2012; Lukie et al., 2014; LeFevre et al., 2009) this study assesses a range of home mathematics activities through both deductive and inductive processes. As far as the author is aware, this is the first study that uses both processes to develop a home numeracy environment questionnaire.

The five subscales found within the frequency of numeracy activities scale were; (1) parent - child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting. These five subscales demonstrate a comprehensive breakdown of numeracy related activities occurring in the home. The majority of activities covered in previous self-report measures are counting related activities (Blevins-Knabe, 2016) for instance, counting objects in a group (Skwarchuk, 2009; Missall et al., 2015). Furthermore, shape related activities have also been covered in previous self-report home numeracy environment measures for example, naming shapes (Missall et al., 2015). The activities within the (1) *parent - child interactions* subscale involves activities that require the assistance of a parent to complete the activity as a child would not be able to do these activities independently which is similar to those activities within LeFevre et al. (2009) home numeracy measure (e.g. printing numbers). These activities are easy to observe in the early years (Blevins-Knabe, 2016). Although not the exact items from previous scales, these types of activities have been widely covered within the three subscales from the frequency of numeracy-activities scale within the PHMQ, (5) *counting* (e.g. counting out food, dinner plates, knives and forks), (4) *shape* (e.g. sorting objects by size) and (1) *parent - child interactions* (e.g. write numbers).

Items about the use of educational technology are rarely used in home numeracy environment questionnaire measures and if they are this is usually only one item (Huntsinger, et al., 2016; Kleemans et al., 2012). More research is needed to understand the broad array of educational technology that may be watched and/or played on tablets (chapter 3; Cahoon, Cassidy & Simms, 2017). Therefore, in contrast to previous home numeracy scales (Huntsinger, et al., 2016; Kleemans et al., 2012; LeFevre et al., 2009) activities that involved educational technology were added to the frequency of numeracy activities scale. These types of activities were widely covered within two factors from the frequency of numeracy-activities scale within the PHMQ; (2) *computer maths games*, and (3) *TV programmes.*

Ofcom (2016) has stated that there are two devices in the home that continue to be used by children: television sets (92% for 3-4s and 96% for 5-7s) and tablets (55% for 3-4s and 67% for 5-7s). This is up since Ofcom's 2013 report were in the home approximately one quarter (28%) of children aged 3 – 4 use tablets. Ofcom (2016) state that a large number of children are accessing websites that provide educational support, such as 'MyMaths Ltd' which is ranked in the top 40 most visited sites, 'BBC Learning' within British Broadcasting Corporation (BBC) sites receiving 276,000 visitors, and 'Coolmath.com LLC' (stands for limited liability company) within the Mode Media Property receiving 463,000 visitors. Therefore, the addition of the two subscales *(2) computer maths games* and (3) *TV programmes* expands the scope of previous home numeracy environment measures by expanding the number of items related educational technology, which include items such as, 'Maths related websites (e.g. coolmaths.com)'. Overall, the PHMQ is an inclusive measure of the home numeracy environment.

4.4.1.2 Stage 3: Initial item reduction

Overall, there were eight home environment relevant dimensions with the PHMQ; (1) parent expectation questions, (2) literacy questions, (3) counting ability questions, (4) parent-child teaching methods questions, (5) target child-sibling interactions questions, (6) frequency of numeracy activities questions, (7) questions on parent's view of their child's understanding of numeracy and (8) a support question. Through the initial item deduction six-home environment relevant dimensions were retained. Questions on parent's view of their child's understanding of numeracy (7) and the support question (8) were removed due to the "halo effect", which is the tendency for one impression to shape or influence all other judgements (Fitzpatrick, 1991; Wilson et al., 2006). From the initial 69 items within the PHMQ, 19 items were removed for different reasons mentioned previously (See Appendix 4.2, Table 2 for breakdown). In sum, 14 deductive items and 5 inductive items were removed thus a total of 50 items were retained. The scaling of some questions were changed (see 4.3.2.3 Interaction questions for more detail) these questions will be piloted and discussed in the next chapter (see 5.2.1 Pilot for more detail).

4.4.1.3 Stage 4: Exploratory factor analysis

The development of the PHMQ revealed that five subscales can be identified in the frequency of numeracy activities scale with high levels of reliability (α = .76 to .81). The levels of reliability, as assessed by internal consistency, are high providing strong item covariance thus the sampling domain has been captured sufficiently (Churchill, 1979). Hinkin et al. (1998) proposes that α =.70 ought to serve as an absolute minimum for newly developed measures. Suggesting that the internal consistency reliability should be considerably higher than .70. Therefore, by following the method discussed by Hinkin et al (1998), the new frequency of numeracy activities scale can be considered as having good internal consistency.

This high level of reliability is consistent with other studies in which a factor analysis was used to refine the home numeracy environment measure. For instance, LeFevre et al. (2009) reported a reliability between .71 and .84 for their numeracy-related activities measure comprising of four factors; (1) number skills, (2) games, (3) applications and (4) number books. Kleemans et al. (2012) established two factors in their home numeracy questionnaire, (1) parent-child numeracy activities and (2) parents' numeracy expectations, with a reliability of .76 and .83, respectively. Further, Lukie et al. (2014) established a four-factor model, (1) exploratory cognitive play, (2) active play, (3) crafts, and (4) screen time, within their child-interest scale with a reliability ranging between .60 to .79.

LeFevre et al. (2009) classified that those activities reported in the (1) number skills and (4) number books subscales reflected *direct* teaching activities and the (2) games and (3) application factors reflected *indirect* experiences. Within this study there was no clear evidence for the five subscales, (1) parent – child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting reflecting *direct* teaching activities or *indirect* experiences (for further information see section 6.3.3 Confirmatory factor analysis).

4.4.2 Conclusion.

The main aim of this chapter was to provide the framework used for the development of a home numeracy measure in accordance with established psychometric principles. Phase one, *the scale development process,* presented construct validity, which addressed two further psychometric properties. Firstly, *factor structure* evident through the five-factor structure of the frequency of numeracy activities scale found through the exploratory factor analysis and *scale score reliability* demonstrated through the high levels of reliability (α = .76 to .81). Overall, the PHMQ is an inclusive measure of the home numeracy environment and the frequency of numeracy activities scale reaches three levels of psychometric properties; (1) construct validity, (2) factor structure, (3) scale score reliability.

Chapter 5: The Scale Validation Process of a Pre-school Home Maths Questionnaire (PHMQ)

Overview

Chapter 4 presented phase one, *the scale development process,* comprising four stages of questionnaire development explained by Hinkin (1998). Phase one addressed construct validity, which incorporated two further psychometric properties, factor structure (i.e. dimensionality) and scale score reliability. Chapter 5 presents phase two, *the scale validation process.* Phase two involves two stages, (stage 5) content and (stage 6) criterion validity using the process explained by Nunes, Pretzlik and Ilicak (2005). Therefore, the Pre-school Home Maths Questionnaire (PHMQ) in particular the frequency of numeracy activities scale was evaluated across five psychometric properties; construct validity, factor structure, scale score reliability, content validity and criterion validity. This chapter builds on the findings from the previous chapter.

5.1 Introduction

5.1.1 Phase two: The scale validation process.

Phase one addressed construct validity, which incorporated two further psychometric properties, factor structure (i.e. dimensionality) and scale score reliability. However, further reliability validations are fundamental for the development of quality measures (Hinkin, 1998; Schmitt & Klimoski, 1991). Thus, phase two, *the scale validation process* involves two stages, (stage 5) content and (stage 6) criterion validity using the process explained by Nunes, Pretzlik and Ilicak (2005). Figure 3 illustrates the stages of phase two of *the scale validation process*. Each stage of the scale validation process is discussed to increase the psychometric soundness of the HNE measure.

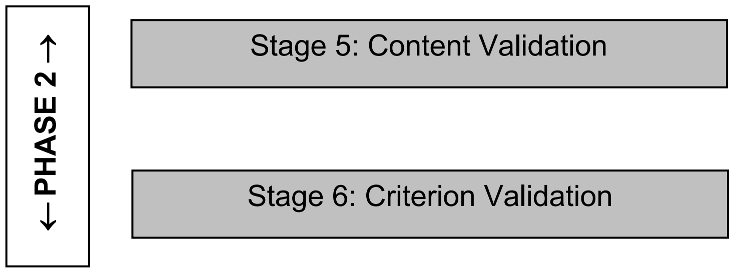


Figure 3. Visual representation of phase two of scale validation

5.1.1.1 Stage 5: Content validity.

Content validity, stage 5, considers whether appropriate questions have been asked in the measure (Nunes et al., 2005). Hence, the PHMQ, in particular the frequency of numeracy activities scale (one section from the PHMQ), needs to reflect whether the parents are asked questions that show a sufficient and comprehensive description of their views and experiences (Nunes et al., 2005). Content validity compares the themes identified in the questionnaire with those emerging in the interviews (Nunes et al., 2005). This content validity method was utilised by Nunes et al. (2005) which was based on psychological measurement theories on the construction of scales (e.g. Guilford, 1954). Nunes et al. (2005) analysed the reliability and validity of a questionnaire which was an assessment of pediatric cochlear implantation originally designed by Archbold, Lutman, Gregory, O'Neil and Nikolpoulos (2002). The questionnaire was named "Parents' views and experiences with pediatric cochlear implant questionnaire" (PVECIQ) and recently changed to ''Parent outcome profile from pediatric cochlear implantation''. Parents responded to the questionnaire and to an interview with their children or teenagers (aged between 5 to 16 years) had the pediatric cochlear implant for at least 3 years.

Parents' responses from the interviews were placed into content categories based on the ten themes of the PVECIQ. A theme within the PVECIQ that will now be used as an example to explain content validity is *Communication*. During the interviews parents mentioned when asked ''what are your best moments?'' that *communication at a distance* was one of the best moments. Parents were satisfied when their child could chat on the phone with friends, extended family or when contacting their children by phone when they (the parent) were away from home. This communication type was covered within the PVECIQ by the question, "We can now chat even when s/he cannot see my face" (question 27, theme *communication*). This is the method that will be used in this chapter to establish content validity.

5.1.1.2 Stage 6: Criterion validity.

Criterion validity investigates contrast cases of parents with very high or very low scores on each of the themes within a questionnaire and compares the contrasting cases to the interview responses (Nunes et al., 2005). This was the method used by Nunes et al. (2005) with the ten themes within the PVECIQ. An example from Nunes et al. (2005) use of criterion validity on one theme, *communication*, is as follows. A participant who had a very low score in *communication* in the questionnaire was described in the interview as understanding sign language better than English and found it difficult to communicate with new people. In contrast, a different participant who received a high score in *communication* in the questionnaire was described in the interview as finding it easy to communicate with both adults and children and participated well in discussions. Therefore, contrasting cases of parents with very high or very low scores on the sub-scales on the frequency of number activities scale will be identified, much like Nunes et al. (2005).

It is anticipated that interviews responses will vary considerably between parents with extreme cases (i.e. high and low scores) on the frequency of numeracy activities scale (a section from the PHMQ involved in the factor analysis). Guilford (1954) recommended that scale items should be contrasted by developing indicators for the different points on a scale. It is important that scale extremes (i.e. high and low) should be labelled sufficiently to support appropriate self-reports. This is more easily achieved when the scale measures are well-defined traits (Nunes et al., 2005). Parent's views and experiences within the home environment on their child's day-to-day activities is still an undetermined concept as researchers are still exploring the dimensions essential to explaining this experience and the meaning of different views (chapter 3; Cahoon et al., 2017). However, it should be possible to identify the dimensions of children's number-related experiences by comparing parent's descriptions of very high and very low frequencies of numerical activities in the home. The description of both extremes (i.e. high and low scores) provides rich information for the development of the PHMQ, in particular the frequency of numeracy activities scale, through the identification of home numeracy activities and the anchoring points in these (Nunes et al., 2005). Anchoring points are found by comparing the extreme cases. Consequently, if no anchoring points can be found and parents who score very high do not differ in their views and experiences within the interviews from those who obtain very low scores the PHMQ cannot be validated through criterion validity (Nunes et al., 2005).

5.1.2 Rationale.

This study aims to validate a parent-focused questionnaire designed to measure the frequency and quality of mathematical experiences of pre-school children in their homes. The main aim of this investigation is to carry out an independent assessment of the measure to develop a valid measure of early numeracy experiences occurring in the home addressing two stages of psychometric soundness, (5) content and (6) criterion validity.

5.2 Method

Overview.

Before the validation process the newly reduced version of the PHMQ was piloted to confirm the length of time it took to complete the questionnaire, to ensure any changes made to the presentation of questions were easy to read and understand. The questionnaire for the pilot included some demographic questions, target child-sibling interaction questions and the new number game checklist (discussed in the following section, 5.2.1.2 Informal home numeracy practice). The validation process involved participants completing both an interview and a slightly longer PHMQ (including the frequency of numeracy activities dimension). Before the validation processes (stage 5) content and (stage 6) criterion validity are discussed, the method and results for the piloted questionnaire will be examined.

5.2.1 Pilot.

5.2.1.1 New scaling for target child-sibling interaction.

Originally the target child-sibling interactions questions (e.g. “What numerical activities siblings are most likely to do together?”) were 11 ranking options and were reduced to 7 ranking options as it was found that 11 ranking options was too difficult for participants to fill in. Findings from existing research is mixed when related to what type of question and answer approaches have greater predictive validity; ranking or rating. Krosnick (1999) originally stated that ranking questions had greater predictive validity. However, in recent research, rating questions had significantly higher average correlations than rank order questions leading to greater validity with rating order questions. Therefore, there is a strong justification for changing this question to a rating style question due to greater validity (Krosnick, Thomas, & Shaeffer, 2003; Maio, Roese, Seligman & Katz, 1996). Thus, it was necessary to pilot this section of the research instrument (Baker, 1994) to ensure participants understands the question in the same way. The results from this ranking to rating scale change will be discussed in the following results section, (see 5.3.2.1 New scaling for target child-sibling interaction).

5.2.1.2 Informal home numeracy practices (number games exposure checklist).

As a measure of formal home numeracy experiences, Skwarchuk and colleagues (2014) asked parents the frequencies of parent-initiated activities. According to previous literature the frequency of numeracy activities section in the PHMQ could be used as a measure of direct and indirect home numeracy experiences (LeFevre et al., 2009). Skwarchuk and colleagues (2014) created a measure to assess the informal numeracy experiences by developing a number games title checklist (performance on a checklist with real and fake game titles). This framework was adapted from Sénéchal and LeFevre (2002) study that used parent's knowledge of storybook titles as a measure of informal home literacy practices. The number games title checklist by Skwarchuk and colleagues (2014) was created for a Canadian sample, thus a new number games checklist was developed as a measure of *informal home numeracy practices* (number games exposure checklist) so that the games were relevant to the United Kingdom.

To develop the board game checklist the researcher visited three local retail establishments, two of which specialised only in children's toys (i.e. Smyths, Toys R Us and Argos). Information was gathered both in store and online about commercially available board games suitable for children aged 3 to 6 years. To compile the list the researcher used selection criteria to allow parents a chance to have knowledge of the games, these are as follows. In Sénéchal et al. (1996) *book title checklist* fairy tale games (i.e. those games that involved fairy tale characters from movies or television) for which a movie or television version existed were eliminated due to possible over familiarisation. To allow for the games to be readily available to parents only those game titles that were available in two of the three retail establishments were selected. Lastly to ensure that the games were accessible to all parents regardless of income level only games that were under £15 were selected. Games were categorised according to whether they included numerical components (counting, adding and recognising numbers). In contrast to Skwarchuk et al. (2014) that included 25 titles (10 numerical games, 10 non-numerical games, and 5 plausible but non-existent games), this board game checklist consisted of 30 game titles; 10 numerical; 10 non-numerical and 10 plausible but non-existing games. The number of plausible but non-existent games was increased to 10 as this was equal to that of the numerical and non-numerical game. Refer to Table 7 for a summary of board game checklist and breakdown of selection criteria, the games are placed in alphabetic order.

Table 7. *Summary of board game checklist and breakdown of selection criteria*

Numerical, non-numerical or non-existing	Board games	Skwarchuk et al. 2014	Available at: Smyth's toy store	Cost (£)	Age range (*yr.*)	Available at: Toy's 'r' Us	Cost (£)	Age range (*yr.*)	Available at: Argos	Cost (£)	Age range (*yr.*)
Numerical	Battleships	No	Yes	6.99	3+	Yes	5.99	7+	Yes	15.99	7+
Non-existing	Beach Shelter	No									
Non-numerical	Buckaroo	No	Yes	15.99	4+	Yes	17.99	4+	Yes	14.99	4+
Numerical	Build A Beetle	No	Yes	6.99	4+				Yes	7.99	4+
Non-numerical	Chasin' Cheeky	No	Yes	9.99	3+	Yes	10.99	3+	No		
Non-existing	Croc Doctor	No									
Non-numerical	Crocodile Dentist	Yes	Yes	13.49	4+	Yes	15.99	4+	Yes	13.49	4+
Non-existing	Doctor Pop-up	No									
Non-existing	Dog Tales	No									
Numerical	Doh Nutters Game	No	Yes	14.99	4+	Yes	14.99	4+	Yes	14.99	4+
Numerical	Dominoes	Yes	Yes	6.99	4+	Yes	7.99	4+	Yes	8.99	4+
Non-numerical	Elefun	Yes	Yes	24.99	3+	Yes	11.99	3+	Yes	14.99	3+
Non-existing	Exasperation	Yes									
Numerical	Frustration	No	Yes	10.99	6+	Yes	11.99	5+	Yes	11.99	6+
Non-numerical	Guess who?	No	Yes	11.99	5+	Yes	14.99	6+	Yes	14.99	5+
Non-existing	Head to toe	No									
Numerical	Hungry Hungry Hippo	Yes	Yes	11.99	4+	Yes	14.99	4+	Yes	14.99	4+
Non-numerical	Kerplunk	No	Yes	12.99	5+	No			Yes	14.99	5+
Numerical	Ludo	No	Yes	9.99	5+	Yes	9.99	5+	Yes	4.99	3+
Non-existing	Mailman	No									
Non-existing	Mashup	No									
Numerical	Monopoly Junior	Yes	Yes	10.99	5+	Yes	9.74	5+	Yes	11.95	5+
Non-numerical	Operation	No	Yes	9.99	6+	Yes	17.99	4+	Yes	17.99	6+
Non-existing	Pepper Pigs	No									
Non-numerical	Pie Face	No	Yes	14.99	5+	Yes	15.99	5+	Yes	19.99	5+
Non-numerical	Pop-up Pirate	No	Yes	9.99	4+	Yes	14.99	4+	No		
Non-numerical	Shark Chase	No	No			Yes	19.99	5+	Yes	12.99	5+
Numerical	Snakes and Ladders	Yes	Yes	4.99	3+	Yes	9.99	5+	Yes	4.99	4+
Non-existing	Spider Web Master	No									

Numerical	The Mashin Max Game	No	Yes	14.99	4+	Yes	19.99	4+	Yes	14.99	4+
Below omitted											
Numerical	Cluedo Junior	No	No			No			Yes		
Numerical	Catch Me If You Can	No	Yes	4.99	4+	No			No		
Non-numerical	Scramble	No	Yes	12.99	5+	No			No		
Non-numerical	Rat attack	No	Yes	5.99	4+	No			No		
Non-numerical	Screwball Scramble	No	No			Yes	15.99	5+	No		
Non-numerical	Wet head	No	Yes	19.99	6+	Yes	19.99	4+	No		
Non-numerical	Twister	No	Yes	11.99	6+	Yes	11.99	6+	Yes	14.99	6+
Non-numerical	Greedy Granny	No	Yes	19.99	5+	No			No		
Non-numerical	Pig Goes Pop	No	Yes	18.99	4+	Yes	19.99	4+	Yes	18.99	4+

Note: All prices checked June 2016.

The newly created number games checklist was cross-referenced with Skwarchuk et al. (2014) number game exposure checklist. Four numerical, 2 non-numerical and 1 plausible but non-existing games were taken from Skwarchuk et al. (2014) checklist as they also reached the selection criteria used in this study. The items taken from Skwarchuk et al. (2014) can be seen in table 7. As in previous home numeracy research (Skwarchuk et al., 2014), parents were asked to indicate their familiarity with children's game titles. Parents were asked not to guess or stop to verify any game titles online or in a catalogue and informing parents that the lists included non-existing games minimised guessing. To calculate the number game checklist score, the total of correctly marked number games was corrected for guessing in the same way as Skwarchuk et al. (2014) study was corrected (e.g. if 7 number games and 1 non-existing games were selected, this was scored as (7-1/10) x 100 = 60%). Therefore, overall the PHMQ was made up of seven-home environment relevant dimensions with the addition of the *informal home numeracy practices* (number game exposure) section to the PHMQ.

5.2.1.3 Participants.

Consistent with previous research 20 to 30 participants would be necessary when piloting the new aspects of the PHMQ (Hill, 1998; Isaac & Michael, 1995). Of the 20-30 participants, approximately 6-10 participants were necessary to complete the interview and the slightly longer PHMQ for the content and criterion validity processes.

5.2.1.4 Procedure.

The research procedures were reviewed and approved by School of Psychology Research Ethics Committee (REC) before the study commenced. The pilot questionnaire, including target child-sibling interaction questions and the new number game checklist took approximately 5 minutes to complete.

5.2.1.5 Data analysis.

The PHMQ data from the pilot questionnaire (and extended questionnaire, used alongside the interview) were entered by the researcher into SPSS Version 23 and then re-entered by an additional researcher. The placement student was trained before inputting the data and data was verified to get a 100% match in cases.

5.3 Results

Overview

The findings from the piloted questionnaire will now be examined. However, before the results from the pilot the demographic composition of participants will be discussed in comparison to the previous chapter. The method and results for the validation processes, (stage 5) content and (stage 6) criterion validity will then be presented.

5.3.1 Demographic composition of participants.

Twenty-four mothers, 4 fathers and 2 grandparents took park in the pilot of the PHMQ. Five males completed the questionnaire with a mean age of 47 years (SD = 18.9 years). 25 females took part and had a mean age of 33.6 years (SD = 5 years). The mean age of the target child (that the parents/guardians were asked questions on) was 3.5 years (SD = 0.4 years). Male target children had a higher mean age (M_{age} = 3.5 years, SD = 0.4 years) compared to females (46.7% female, M_{age} = 3.5 years, SD = 0.5 years), although there was no significant difference.

Table 8. *Descriptive statistics of age for participant and target child for pilot*

		Mean	Std. Deviation	Minimum	Maximum
Age of participant (N = 30)	Age (yr.)	33.59	10.85	20	73
Sex – Age	Male (N = 5)	47.00	18.87	28	73
	Female (N = 22)	33.59	5.04	20	39
Age of target child (N = 30)	Age (yr.)	3.49	0.41	3	4
Sex – Age	Male (N = 16)	3.52	0.39	3	4
	Female (N = 14)	3.45	0.45	3	4

This is a similar demographic to those who took part in the questionnaire administration for phase one, *the scale development process*. For instance, mostly females took part in both studies (N = 144 females, N = 18 males in phase one; N = 25 females, N = 5 males in pilot) with a similar mean age (M_{age} = 35.0 years, SD = 6.4 years in phase one; M_{age} =33.6 years, SD = 5 years in pilot). Further, the mean age of the target child were similar (M_{age} = 3.33 years, SD = 0.5 years in phase one; M_{age} = 3.5 years, SD = 0.4 years in pilot), with male target children having a higher mean age in both studies (M_{age} = 3.37 years, SD = 0.6 years in phase one; M_{age} = 3.5 years, SD = 0.4 years) compared to females (M_{age} = 3.29 years, SD = 0.5 years in phase one; M_{age} = 3.5 years, SD = 0.5 years in pilot).

5.3.1.1 Participant breakdown for pilot.

Participants (M_{age} = 33.6 years) were asked what their relationship to the target child was; 24 mothers, 4 fathers and 2 grandparents completed the questionnaire. Twenty-seven (90%) participants specified that they were the primary carers for the target child. The target children (46.7% female), that the parents/guardians were asked questions on, were aged between 36 months to 53 months (M_{age} = 42.4 months).

5.3.2 Pilot.

5.3.2.1 New scaling for target child-sibling interaction.

Out of the 30 parents/guardians that took part in the pilot 27 (90%) stated that their target child had a sibling. The pilot data demonstrated that the change to the psychological measurement from rank order to rating scales for the target child-sibling interaction questions had a wide variation in responses therefore this dimension of the PHMQ was retained.

5.3.2.2 Informal home numeracy practices.

The total of non-existing games was scored out of 10. The median number of non-existing games selected was 0. Thus, parents did not guess when filling out the number games checklist. The total of correctly checked number games was scored out of 10. Results ranged from 0-8 with a mean score of 4.8 showing good variability in results, therefore this number game checklist was retained for further analysis.

5.4 Method

Phase 2: The Scale Validation Process

5.4.1 Stage 5: Content validity.

5.4.1.1 Participants.

The criteria for participation was that the parent/guardian was related to the target child aged between 3 to 4, and for the interviews to proceed the participant needed to be the primary carers for the target child. The questionnaire was piloted with 30 participants, of which 8 participants agreed to take part in the extended questionnaire and the interview.

5.4.1.2 Materials.

A brief cover letter and participant information sheet were used containing information on the purpose of the study, what was required, what will happen with the information collected and explanation on how to withdraw from the study. Participant consent forms were also distributed. Participants returned a signed consent form confirming that they agreed to take part, that they cared for a child between the ages of 3 to 4 years old, read and understood the information sheet, agree to answer some demographic questions on their family, understand that their participation is voluntary and that they were free to withdraw without giving a reason, and finally that the researchers would hold all information collected securely and would not share the information with any other party. If the parent/guardian wished to be involved in the extended questionnaire and interview the participant was given a different consent form with the addition that they agreed to take part in a recorded interview.

The schedule for the semi-structured interview used in this study began with a general introduction to the study, including the aims of the research and a reminder that the child in question was their 3 to 4 year old. To open the discussion, questions that would give the interviewer an understanding of the child's home environment were discussed. This was deemed essential for allowing the conversation to flow smoothly. The questions covered in the interview schedule were all covered in the PHMQ and were placed in a different order to the questions covered in the PHMQ. Questions were counterbalanced to control for order effects in this repeated measure design. Under each dimension of the PHMQ the interviewer had questions to prompt parents if the issue was not spontaneously addressed. At the end of the interview, the parent/guardian was asked if their child had expressed an interest in any other subject to understand if anything was missed in the PHMQ, and if there was anything that may have been missed with regards to how they support their child to learn about numbers in the home, these questions adequately opened further issues that might not have been covered by the previous questions and allowed parents/guardians to share their views. Interviews were audio-recorded on an Olympus DM-450 digital voice recorder.

5.4.1.3 Procedures.

The manager of the play centres who had taken part in the previous questionnaire administration study was contacted via phone to request permission to collect further data on their premises. Verbal permission was granted and when the researcher visited the establishment an information sheet was given to the manager to read which examined the additional interviews that would be taking place. A consent form was then signed by the manager. Participants were recruited via convenience sampling and approached face-to-face. The researcher asked the potential participants if they would like to complete a questionnaire, or if they would be interested in taking part in an extended version of the questionnaire and an additional interview. The slightly longer PHMQ (including the frequency of numeracy activities dimension) was used alongside the interview took approximately 10 minutes to complete. Half the parents were administered the questionnaire before the interview and half of the parents were given the questionnaire after the interview. The individual interviews lasted approximately 40 minutes and were scheduled at a time that was convenient for the parent/guardian.

5.4.1.4 Ethical considerations.

The research procedures were reviewed and approved by School of Psychology Research Ethics Committee (REC) before the study commenced. The interview, the audio file from the digital voice recorder was transferred from the digital recorder to the researcher’s private laptop. The audio files were stored in a password protected folder on a laptop so that transcription could be achieved more frequently. Once transcribed, hard copies of the interview transcripts were kept in the same location as the completed questionnaires in a locked cabinet. Consent forms were kept in a different location, in a locked cabinet, to the interview transcripts and questionnaires.

5.4.1.5 Data analysis.

The subscales in the questionnaire; frequency of numeracy activities, containing the five subscales labelled as (1) parent – child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting, will be used to assess the content and criterion validity. Further, other relevant dimensions from the PHMQ such as the frequency of reading compared to numeracy, target child-sibling interaction, structure of the home numeracy environment and parent-child teaching methods, will be discussed to assess the content validity of the PHMQ and assess why these areas were included in the PHMQ.

5.4.1.5.1 Content validity.

Content validity reflects whether relevant questions were asked in the measure (Nunes et al., 2005). This was assessed by comparison with parents' responses to the PHMQ and an interview. The interviews were used to find any issues seen as significant to the parents but not covered in the PHMQ, or vice versa. The parents' responses were coded using NVivo (Version 11) into content categories based on the five-factor model (e.g. the subscales from the frequency of numeracy activities; (1) parent-child interaction, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting). Responses were analysed to investigate the breadth of coverage of the PHMQ.

5.4.1.5.2 Criterion validity.

Contrasting cases were identified by obtaining the total scores for the five *frequency of numeracy activities* subscales and were calculated for each participant. Scores ranged from 0 to 4, based on the 5-point Likert scale as follows; activity did not occur (0), few times a month (1), about once a month (2), few times a week (3), and almost daily (4). Respondents whose scores were either high or low were identified for each subscale. High scores where therefore closer to 4 and low scores were closer to 0. The parents' interview transcripts were then searched for comments relevant to the subscales.

5.5 Results

Overview

This results section will discuss phase two findings of (5) content and (6) criterion validity of *the scale validation process*.

5.5.1 Participants

5.5.1.1 Participant breakdown for interview.

The 8 participants (M_{age} = 37.8 years) that agreed to take part in the extended questionnaire and the interview were; 6 mothers, 1 father and 1 grandparent; all 8 participants stated that they were the primary carer to the target child. The target child (50% female) was aged between 36 months to 49 months (M_{age} = 42.8 months).

5.5.2 Stage 5: Content validity.

Content validity was evaluated by comparison with parents' responses in the PHMQ to the information gathered in an interview following a method used by Nunes et al. (2005). There was an agreement between parents' views in the interview and those assessed by the PHMQ. Issues surrounding the five frequency of numeracy activities subscales were mentioned in the eight interviews and used to assess the content validity. The definitions and sample comments illustrating each subscale dimension are summarised in Table 9.

Table 9. *Subscale dimensions and sample commentary from interviews*

Subscale dimension with definitions	Frequency items	Examples from interviews
1. Parent –child interaction Any number-based interaction between the primary parent/guardian and their child in the home. Activities were a parent is necessary for the child to learn from the activity.	1. Write numbers 2. Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?") 3. Teaching about measurements (e.g. baking, height) 4. Sticker books 5. Identifying names of written numbers 6. Teaching about money (e.g. informal – playing shop or formal – buying sweeties) 7. Time terminology (e.g. big hand, little hand) 8. Asking shape related questions (e.g. "how many sides does a circle have?")	"They'll (target child and older sibling) play together with Play Doh but there is usually a bit of a dispute if you leave them together alone. It's better if adults play with him than any of his peer group. He is still at the solidity play, well a bit of parallel play, but he's not moved onto co-operating" – Participant 1 "If we are baking I would try and get her to count the bun cases" – Participant 2 "She loves jigsaws. It's always supervised with mummy, and me going "You find another piece of Ariel's tail for me" but she loves it" – Participant 6
2. Computer maths games Any computer - based activities (such as, tablet or smartphone usage) that occur in the home, specifically games that involve number, shape or problem solving.	1. Maths applications (e.g. Number Jacks) 2. Maths related websites (e.g. coolmaths.com) 3. Racing games (e.g. faster they complete sums the faster the boat moves) 4. Size/matching apps (e.g. "put the big skirt on the small girl") 5. Add and subtraction games 6. Filling in the gap number games (e.g. what is next in the sequence?)	"On the iPad, he does the shadow into the shape, the racing games, and the one with the balloons on the number train" – Participant 1 "There's a Cbeebies app and the games on that are all educational" – Participant 2 "This EduKitchen app is good. So, there's a recycling bin and they pick up all the rubbish. They would have fruit and

		then wrappers to work out which ones go in the recycling bin so it is quite educational" – Participant 4
3. TV programmes Any educational TV programmes watched in the home involving rhymes and/or numbers.	1. Watching number related TV shows (e.g. Number Jacks or Numtums) 2. Rhyming TV shows involving numbers (e.g. Number Jacks) 3. Watch educational programmes (e.g. Dora the Explorer)	"TV can be a great motivator. You can say to them if we finish this then we'll put on Peppa Pig. It's great because they'll complete it before they go and watch TV" – Participant 1 "He prefers cartoons but he does watch things like Mr Tumble and Gigglebiz. and there is Kerwhizz too. It's a game show with aliens and ask number, shape or what's missing questions" – Participant 4 "I'd rather them watch the Numtums (than non-educational TV), I think it's quite good" – Participant 5
4. Shape Any shape, pattern or sorting based activity in the home.	1. Sorting shapes 2. Play with jigsaws 3. Sorting objects by size 4. Pairing/matching games 5. Playing with building blocks 6. Creating patterns with objects (e.g. arranging blocks into shapes)	He does the game with the wooden shapes, where you fit them into the holes and he loves matching cards like animal dominos where you match all the cows together" – Participant 4 "She's good at jigsaws. She knows to do the straight edge, she'll work from the corner. She has an 8 piece, 12 piece, 18 piece and a 24-piece jigsaw. She can do the 24 piece, she might need help. The smaller ones she can do on her own but the larger ones she'll need a bit of help to get started" – Participant 7

		"We would do puzzles together, jigsaws, and you can see his progression with more pieces now" - Participant 8
5. Counting Activities that involve the counting or comparing of objects in the home.	1. Counting 2. Counting on fingers/hands 3. Comparing sets of objects (e.g. brother has more than mum) 4. Counting out food, dinner plates, knives and forks 5. Counting objects (e.g. ducks in bath, blocks, new toys, books)	"She will count on her own without me prompting her. She's very particular, almost an OCD level where everything has to be exact, she's very exact when she comes to counting" – Participant 3 "He looks forward to his bedtime stories. In fairness, he gets to pick stories and now and again we'd say well you've been good so pick out 4 and he would go out and pick out 4 books. He picks out 2 books normally" – Participant 5 "She sits and count away to herself whenever she is playing, but she can only reliably count to 10 and then it becomes 33 and 54 and random numbers" – Participant 6

5.5.2.1 Initial observations.

From table 9 it is apparent that each topic mentioned in the *frequency of numeracy activities* scale was covered in the interview thus, all items were retained in the *frequency of numeracy activities* scale. Data saturation was found in the eight interviews which is consistent with other studies (Isman, Ekéus & Berggren, 2013; Isman, Mahmoud Warsame, Johansson, Fried & Berggren, 2013). Data saturation is achieved when further coding is not achievable, thus the ability to obtain additional new information has been reached (Fusch & Ness, 2015).

5.5.2.1.1 Parent – child interaction subscale.

The *parent – child interaction* subscale involves any number-based interaction between the primary parent/guardian and their child in the home. Activities were a parent is necessary for the child to learn from the activity. This theme and subscale was confirmed as all parents/guardians mentioned interactions between their child and themselves when working on a complex task. Although some examples have been mentioned in table 9 here is another example;

> *"Lately I have been doing [scenarios] like, "I've got 3 fingers on this hand and I've got 4 on this hand, together that equals?" and numbers are very kind of showy stuff rather than in your head" – Participant 1*

This quote demonstrates how a parent broke down an addition task in order to facilitate learning therefore, this theme and subscale was confirmed.

5.5.2.1.2 Computer maths games subscale.

The *computer maths games* subscale involves any computer - based activities (such as, tablet or smartphone usage) that occur in the home, specifically games that involve number, shape or problem solving. Many computer - based activities were mentioned in the interviews and some examples of these can be seen in table 9, therefore this theme was confirmed.

5.5.2.1.3 TV programmes subscale.

The *TV programmes* subscale was defined as any educational TV programmes watched in the home involving rhymes and/or numbers. Again, there are some examples of the types of TV programmes the children are watching in table 9. A wide range of TV programmes are being watched from educational (e.g. Mr Tumble, Gigglebiz, Kerwhizz, NumTums, Number Jacks and Cbeebies) to non-educational (e.g. Disney movies, My Little Pony and Paw Patrol). Here is another example of a TV programme involving number;

> *"Number Jacks is actually really good. They had a train and whenever the train went through the tunnel numbers were disappear, above the bridge is the number 1 so they are subtracting by 1" – Participant 4*

Therefore, the theme *TV programmes* was confirmed through the content validity process as children were watching number related TV shows and educational programmes.

5.5.2.1.4 Shape subscale.

The *shape* subscale was described as any shape, pattern or sorting based activity in the home. Every parent/guardian mentioned that they did some sort of shape-based activities including "puzzles", "dominos", "matching cards", "Jenga blocks" and so on. This confirms that shape-based activities are occurring in the home.

5.5.2.1.5 Counting subscale.

The *counting* subscale was termed as activities that involve the counting or comparing of objects in the home. Counting activities were mentioned by every parent/guardian in a variety of different ways for instance, "counting the legs of animals in a book", "counting on fingers", "counting blocks by shape" and so on. Therefore, counting is achieved in a variety of different ways in the home confirming that counting is an important aspect in daily life for a child aged 3 to 4.

5.5.2.2 New issue.

An extra issue was identified in the interviews, which suggested the need for increasing the breadth of the *frequency of numeracy activities* scale. YouTube was mentioned by half the participants. Children watched a range of videos including educational videos. Here are some examples of the types of videos children watched;

> *"She likes watching a couple of YouTube videos. She loves the videos where people open, they are called blind bags, the likes of My Little Pony or Paw Patrol. It's almost like a kinder eggs surprise thing and it will have one of the characters in them. She counts the characters sometimes" – Participant 3*

> *"Oh Number Jacks. He has only started to watch Number Jacks on YouTube. He likes that. It's quite good" – Participant 4*

> *"They usually watch YouTube videos. People have made up YouTube videos using the characters from Frozen or Paw Patrol or whatever and changing their colours or do the finger family" – Participant 6*

Due to the range of videos being watched which included number, the item 'Maths related YouTube videos (e.g. NumTums)' was added to the frequency of numeracy activities scale.

5.5.2.3 Other themes from the interviews.

There were other themes that developed during the analysis of the interviews. These themes were the same themes that were found in the first interviews (chapter 3; Cahoon et al., 2017) and thus are already included in the PHMQ. The themes were; the home numeracy environment structure, frequency of number-related experiences compared to reading, parent-child teaching methods and target child-sibling interaction. In chapter 3 the themes that cover this content are; (theme one) numeracy environment structure (theme two) frequency of number-related experiences, (theme five) parent-child interactions and (theme six) social interaction, respectively. These themes from the current interviews will also be discussed to confirm the content validity of the other PHMQ dimensions.

5.5.2.3.1 Structure of the home numeracy environment.

This theme is different in that it does not reflect one specific dimension of the PHMQ, moreover, it reflects two PHMQ dimensions and the balance between structured/formal and unstructured/informal numeracy environments.

Most participants discussed how they played and taught number-related activities in an unstructured/informal way that was generally unplanned;

> *"He seems to be excelling at maths. He loves the counting and will do it himself now and he is only 3. I can hear him when he is on his own counting out figures, counting out Peppa Pig and separating things… even his Shreddies and Cheerio this morning for breakfast he counted those. So, we can nearly be counting all day without realising you're doing it, with nearly everything" – Participant 1*

> *"When we are walking places it's easier to count things like how many red cars are there, so when you are out numeracy would be easier" – Participant 2*

> *"I haven't gone out of my way to get numeracy games it's just everyday objects" – Participant 5*

The reason for the unplanned nature of numeracy-related activities was related to children's interests, this finding is consistent with the first round of interviews (chapter 3; Cahoon et al., 2017). However, not unlike children's interests influencing the activities they want to be involved in (chapter 3; Cahoon et al., 2017) parent's interests may also influence the frequency of number-based activities in the home;

> *"I'm not strong at maths and I used to imagine that it was hurting my brain, so I suppose I lean towards literacy, and that's what I would concentrate on with the kids. My mum really likes maths, so she'll ask more things regarding numbers and sums. If Ella wants to know anything about numbers, I want her to learn so I will participant, absolutely" – Participant 6*

This discovery that parent's interests may also influence the frequency of number-based activities in the home is a new addition to what could influence the structure of the home numeracy environment. This, however, does not influence the content of the PHMQ.

5.5.2.3.2 Frequency of number-related experiences compared to reading.

There was a literacy dimension to the PHMQ which includes 4 questions such as, "How often do you and your child engage in reading?" answered on a 5 point-Likert scale, "Do any of the books you read to the participating child involve numbers?", "If so how many?" and "Would you do maths activities more or less than reading?". These questions were included as benchmark questions after the first round of interviews (chapter 3; Cahoon et al., 2017). The information found in the current interviews will now be discussed to validate the inclusion of these benchmark questions.

Participants stated that reading was a "more structured", usually daily activity whereas number-related activities were "spontaneous". This suggested that reading occurred more often than number-related experiences. However, like the first interviews (chapter 3; Cahoon et al., 2017) participants realised that number-related experiences could occur more frequently than reading as number-related experiences did not occur at a prescribed time like reading;

> *"Reading would be every day, but when you are out and about it's easier to count things" – Participant 2*

> *"The reading is obviously book based. Numeracy is more spontaneous. We try and link numeracy into things she's done like cooking, count the number of toys or*

counting the legs of animals in a book or the number of animals on a page. It's less structured than the reading side of things but we do it often" – Participant 3

"Reading would be more structured. Numbers would be the sort of thing that creeps up on a day to day activity… Reading is structured, were the numbers is spontaneous" – Participant 4

Further, most participants stated that numbers were involved during reading;

"If you're reading a book and if there is a picture, she'll say "Look mummy there's three dogs" or she'll count them "One, two, three" from the picture. There probably is more numeracy than literacy at the minute just because she is quite young" – Participant 2

"Sometimes we would find things in books, sometimes I will say "find…" he is really into pirate so "find five swords in the picture". In that instance I suppose there is that element of counting when he is searching for things, that's quite frequent actually" – Participant 4

"She'll count, she has a pirate book. When you get it right you push the button and it makes a little noise. The first page is 'Count which arrow has four diamonds' and there's one arrow with 3 dots and an arrow with 4 diamonds. She counts the one with the dots 1, 2, 3 and then she will count the one with the 4 diamonds. They are very close together and she can still go 1, 2, 3, 4 and then she will press the button when she gets it right" – Participant 7

This suggests that children are accessing number learning through books. Thus, there is some structured numeracy-related activities occurring in the home. The above quotes illustrate that the 4 questions in the literacy dimension of the PHMQ are necessary questions that gain a broader understanding of the home numeracy environment.

5.5.2.3.3 Parent-child teaching methods

The different types of interactions that parents use to aid their child's learning and understanding of numbers was also a theme (chapter 3; Cahoon et al.,2017) and a section was added to the PHMQ on the types of interactions parents use with their child. The parent-child teaching methods dimensions in the PHMQ contains 5 questions. One question is, "Who is more likely to bring up numeracy activities?" and the other "What are the specific interactions the parent/guardian would do to encourage and support the target child to learn numeracy?", with 4 rank order options (e.g. question and encourage your child without explanation, prompt, explain and work through the problem together, provide answer and move on and adjust your behaviour). See appendices 4.1 or 5.1 (the original and final PHMQ) for full parent-child teaching methods question. Each rank order option was mentioned in the current interviews;

> *"Lately it has been, "I've got 3 fingers on this hand and I've got 4 fingers on this hand, together that equals?". Very showy stuff rather than in your head" – Participant 5*

> *"Ella has these four dollies and I would say "Now Ella you have four dollies you could give two of those you Rob". It's working with items and visualising numbers, but practically as well" - Participant 6*

> *"She loves counting, she's really good at counting, she would count up to 20 and then I would try to do "One and add another one, what does that make?" (moved objects to demonstrate) but she's not really getting it yet, she is too little" – Participant 7*

The evidence found in the current interviews confirm the content of the PHMQ.

5.5.2.3.4 Target child-sibling interaction.

All participants who took part in the interview stated that the target child had siblings. However, two participants mentioned that the sibling was younger than the target child, accordingly six participant's responses were examined. The questions in the target child-sibling interaction dimension of the PHMQ included the 7-ranking numeracy-related activities options and the question, "Do you feel that your child has learnt number skills from their siblings?".

There were a wide variety of activities occurring between parent, target child, and older sibling/s, (i.e. triad interactions). Comparable to the first interviews (chapter 3; Cahoon et al., 2017), parents believed that their child was learning information that was being taught to older siblings or from what they overheard their older siblings doing;

> *"Her older brother is interested in maths. I would say she is maybe following his lead. She has an IKEA kitchen in the living room and I hear him counting sometimes. Then when he is at school I can hear her counting things just because that's what he does" – Participant 2*

> *"Most of the numeracy between the two of them would be about sharing. How much Rob has compared to how much she has and how to make it the same" - Participant 6*

> *"Amy is his older half-sister. They interact well considering the age gap. Amy would be very good, she would be a lot better than me, at going through things like colour. I would say she has taught Jake colours and she would go through the days of the week with him too" – Participant 8*

The questions that are included in the PHMQ under the target child-sibling interaction dimension can be confirmed as appropriate questions based on the evidence from in the interviews.

5.5.2.4 Summary of Content Validity.

Content validity was evaluated by comparing parents' responses in the PHMQ to an interview following a method used by Nunes et al. (2005). There was agreement between parents' views in the interview and those assessed by the PHMQ. Only two new issues arose from the current interviews that were not mentioned in the previous interviews (chapter 3; Cahoon et al., 2017). The first was that parent's interest in mathematics may influence the frequency of numeracy activities occurring in the home, however this does not influence the content with the PHMQ. The second new issue that arose was due to the range of videos being watched which included number, the item 'Maths related YouTube videos (e.g. NumTums)' was added to the frequency of numeracy activities scale. Overall, the analysis of interviews confirmed the significance of the dimensions included in the PHMQ.

5.5.3 Stage 6: Criterion validity.

Criterion validity examines the convergence between two measures, in this case the PHMQ and the interview findings. To carry out the comparison between results, participants with very low or very high scores in each subscale were selected. When parents were asked, "In the past month, how often did you and your child engage in the following?" response options were on a 5-point Likert scale as follows: activity did not occur (0), few times a month (1), about once a month (2), few times a week (3), and almost daily (4). Therefore, scored ranged from 0 to 4 with 0 defined as low frequency of activities and 4 defined as high frequency of activities. Following the method used by Nunes et al. (2005) the interview findings were classified by themes and then analysed to identify contrasting cases. The contrasting cases in the interviews were then compared against the participants' responses to extreme points in the subscales. For each of the contrasting cases a summary is presented.

5.5.3.1 Parent – child interaction.

The subscale named *parent – child interactions* was defined as any number-based interaction between the primary parent/guardian and their child in the home.

Participant 5 scored the lowest on the *parent – child interactions* subscale (M = .63). Additionally, Participant 5 was the only parent who stated that their child was likely to bring up numeracy activities more than the caregiver (under the parent-child teaching methods questions). This suggested that the parent perhaps does not interact with their child as often as the other parents involved in this study. During the interview with Participant 5, they described few instances when number-related activities would be played together. The frequency of number-related activities was low;

> *"He would help me bake now and again, not too often because of all the mess that comes with it but now and again he would help me cook and measure out ingredients" – Participant 5*

Furthermore Participant 5 suggested that emphasis on number-related activities was low in the home;

> *"I suppose our main focus would be colours rather than numbers" – Participant 5*

Also, number-related activities using everyday objects were not considered without prompting;

> *"It's not something that I have thought about (asking number-related questions while reading) but he got a homework book back, and there was a question in it about "what age do you think the girl is?" and he had to count the balloons. It wouldn't be something that I would have thought of" – Participant 5*

In contrast, Participant 1 who received a highest score (M = 2.75), reported playing many number-based activities spontaneously with everyday objects;

> *"We play with Play-Doh, rolling it up in balls, squashing it and counting it. This brings up counting and the shapes" – Participant 1*

> *"We'll count the animals" (in a book they own at home) – Participant 1*

> *"I do like him to help me tidy up, so should it be "Can you put one block back in the box?"" – Participant 1*

Participant 3 who also scored high on the same subscale (M = 2.38) and perceived that it was best to interact and teach their child when "she's got the attention span for it" stating that it "depends on the time of the day";

> *"Obviously in the evening she's a bit tired and it's more fun rather than learning, and in the early afternoon when she's finished nursery we'll try and re-enforce what she has learnt that day whether it be the alphabet or numbers; any kind of homework" – Participant 3*

Therefore, it can be concluded that the subscale, *Parent - child interactions* subscale does work in discriminating contrasting cases.

5.5.3.2 Computer maths games.

The subscale named *computer maths games* was defined as computer-based activities that occur in the home, specifically, games that involved number, shape or problem solving. Most participants allowed their child to use technology whether it was a tablet or smartphone.

Participant 3 who scored low on the *computer maths games* subscale (M = 1.00) describes some of the games his child played on tablets and smartphones;

"She has jigsaws and puzzles on her tablet. Her favourite one is probably the picture were one bit is missing and you slide the bits and pieces around to get the full picture. She is better at that than I am" – Participant 3

"She has a Peppa Pig game. It's an electronic board where Peppa Pig asks, "Press the letter P" or "Press the number 5". She plays that occasionally" – Participant 3

Participant 3 limited the duration of time on the tablet and smartphone;

"We try and limit her duration on the smartphone and tablet. At one point, we thought that she was very dependent on using the smartphone and the tablet. It was a case of you go and do that while we go and do stuff, but we caught on to that quite early" – Participant 3

"In this day and age there is more portable media and I worry how that would affect her learning. I think over use of the game will affect her imagination, that creativity, that's why we limit it to maybe an hour a day at the very most" – Participant 3

Participant 4 who received the lowest score on the *computer maths games* subscale (M = .83) explained that only educational games could be used on the tablet, however even these games were only "occasionally" allowed to be played;

"I prefer the games to be educational… The Edukitchen app is really good and the Cbeebies app is good too, because it makes him think. Furchester hotel as well. There is a problem he has to solve in each room and there are three ways he can solve the problem" – Participant 4

In contrast, Participant 8, who scored the highest on the *computer maths games* subscale (M = 2.00), stated that she allowed the tablet to be used "every day" and had downloaded applications on her tablet for learning;

"There is a Cbeebies app that I downloaded and it's for learning. He does colour in and counting activities on it" – Participant 8

Participant 8 described the different activities that her son did on the tablet and the duration of the activities;

"I would probably get half an hour's peace out of the Cbeebies app, whereas when he watches Batman on YouTube I would get an hour" – Participant 8

There was seemingly no time limit when it came to technology usage for Participant 8, therefore this could explain the higher score on the *computer maths games* subscale. It is noteworthy however, that although Participant 8 set no time limits on tablet usage, Participant 8 did have one rule and that was no tablet “after 6 o’clock”.

5.5.3.3 TV programmes.

Every participant scored a higher frequency in the watching *TV programmes* subscale than the *computer maths games* subscale. The *TV programmes* subscale was defined as any educational TV programmes watched in the home involving rhymes and/or numbers.

Participant 6 scored the lowest on the *TV programmes* subscale (M = 1.00). Participant 6 mentioned TV programmes that were not geared towards specific learning outcomes;

> *“She likes a bit of My Little Pony but mostly Paw Patrol. Oh and Disney films, she loves Rapunzel and she loves Frozen” – Participant 6*

On the other hand, Participant 7 who scored high on the *TV programmes* subscale (M = 3.00) described more TV programmes that contained educational content;

> *“She loves PJ masks, Peppa Pig, Lazy town, Numtums and Octonauts” – Participant 7*

Participant 5 also scored high on this subscale (M = 3.00) and mentioned a lot of educational games;

> *“The Cbeebies TV shows do have numbers because there’s Numtums and Squiggle It too… I’d rather them watch Numtums and stuff like that. I think it’s good for learning” – Participant 5*

The types of TV programmes being watched may influence the frequency and perhaps be one reason for the contrasting cases. This would be expected as the *TV programmes* subscale only involves questions about educational programmes (e.g. Watching number related TV shows, for example Numtums), Rhyming TV shows involving numbers (e.g. Number Jacks) and Watch educational programs (e.g. Dora the Explorer)). Therefore, those children who are mostly watching non-educational TV programmes would score low on the *TV programmes* subscale. It is important to note that a child’s interest plays a factor in the TV programmes they want to watch (as mentioned in Chapter 3), and this could influence high and low frequencies on this subscale. Nevertheless, the subscale seems to identify contrasting cases well.

5.5.3.4 Shape.

The subscale named *shape* is defined as any shape, pattern or sorting based activity in the home (e.g. jigsaws, playing with building blocks, pairing and/or matching games).

According to parental description, Participant 6 who scored the lowest on the *shape* subscale (M = 1.33), stated that she would not organise shape-related activities;

> *"I don't do that many structured activities. If they wanted to do painting or building blocks or do a jigsaw I would sit with them" – Participant 6*

Participant 5 who also scored low on the *shape* subscale (M = 1.67) suggested that activities had changed since the birth of their second child and thus shape-related activities are not often done;

> *"He used to play jigsaws quite often before Rachel was born. He used to be very focused he would have sat and done a jigsaw and I actually thought he was quite smart at one point because he was doing the bigger jigsaws, bigger wooden ones that have 48 pieces" – Participant 5*

In contrast, Participant 2 who scored high on the *shape* subscale (M = 3.00) described her daughter's favourite games;

> *"She loves Jenga. Jenga's her new favourite game. Sometimes we build houses with the Jenga block but she does quite like playing Jenga, pushing the blocks out. She's actually quite good at it" – Participant 2*

> *"She loves puzzles. She loves jigsaws. She has lots of jigsaws" – Participant 2*

This suggests that perhaps it is the child's interest that influences the frequency of shape-related activities occurring. It may also depend on how much time, effort or planning a parent/guardian puts into an activity. Participant 1 scored the highest on the 'shape' subscale (M = 4.00) and mentioned numerous activities were shapes would be brought up in a spontaneous manner through everyday activities;

> *"He's great at matching, we match beads and bags of pegs. Also in my sewing box we'll sort buttons into big, medium, small piles" – Participant 1*

> *"To keep him engaged if you change the visual object, he thinks it's something new… he'll identify and sort out by colour and then he'll count" – Participant 1*

Due to the wide variation in the scores on this subscale, it is determined that the subscale discriminates contrasting cases well.

5.5.3.5 Counting.

The *counting* subscale is defined as activities that involve the counting or comparing of objects in the home, for example, the counting of fingers, counting of food related items and so on.

Participant 5 who scored the lowest on the *counting* subscale (M = 1.60) suggested in the interview, as stated above, that their "main focus would be colours rather than number" but that the rote learning of numbers would be brought up whereas calculations would not;

> *"We would count the stairs and he would be counting along with me, but we've always been 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It's just rote learning at the moment not sums" – Participant 5*

Participant 7 who scored second lowest (M = 2.40) on the *counting* subscale suggested the same as Participant 5;

> *"We count every day whether it be steps or how many things are in front of her because I do want her to start learning but it's probably a bit early to do like subtraction with her or anything like adding" – Participant 7*

For both Participants 5 and 7 counting was an activity that was mostly brought up by the parent. Whereas Participants 2 and 3, who scored high on the *counting* subscale (both M = 3.60) stated that mathematics was brought up naturally by their child;

> *"She likes counting. If we are out somewhere she will count flowers, or she will count dogs, like "They have two dogs."" – Participant 2*

> *"She would count things spontaneously, be it if she's jumping up and down or the number of cows in a field, we live near a field, she'll count the cows' without me telling her too" – Participant 3*

These statements demonstrate that counting may be covered more often in the home if both the parent and the child are likely to bring up counting. Further Participant 2 specified that the reason for her daughter bring up numbers spontaneously may be because of her older son;

> *"She doesn't get as much time on her own as he did (older son) but she picks up a lot of things from him, so a lot of her spontaneous counting out objects is because he does it and she is copying him" – Participant 2*

Hence, if there are more people in the household to bring up numbers, perhaps there are more opportunities for counting experiences. Participant 2 also had high expectations for number learning;

> *"We do more numbers (than reading) at this stage just because she is so young, and I wouldn't be expecting her to be able to read, yet I would expect her to be able to count and recognise numbers" – Participant 2*

Participant 3, as mentioned previously, aimed to "re-enforce" what their daughter had "learnt that day" in nursery. Therefore, counting may be bought up more frequently if the parent has high expectations for their child to learn about numbers. In conclusion, the subscale *counting* discriminates contrasting cases.

5.5.3.6 Summary of Criterion Validity.

To summarise, the examination of criterion validity by analysing contrasting cases indicates that the subscales detect differences between the extreme high and low scores. Thus, there are clear differences between the views of parents as assessed through the interviews.

5.6 Discussion

5.6.1 Phase two: The scale validation process.

This chapter presented phase two, *the scale validation process,* comprising of two stages, based on Nunes et al. (2005) validation methods. The two stages are; (stage 5) content and (stage 6) criterion validity. These two stages of validation are the stages that follow the first four stages from the *scale development process* (chapter 4).

5.6.1.1 Stage 5: Content validity

The goal of developing items for the PHMQ measure was to result in a questionnaire that illustrates the theoretical domain of interest, in this case the home numeracy environment, in turn demonstrating content validity (Hinkin et al., 1998). After phase one, in which items were generated and refined through the use of exploratory factor analysis, the items were subject to a further assessment of content validity following the procedure used by Nunes et al. (2005) in phase two. The analysis of the PHMQ content validity demonstrates that the themes included in the PHMQ are raised by parents in the interviews. One new item spontaneously raised by the parents was that their children watched a range of videos on YouTube, including educational videos. YouTube is predominantly utilised, with 37% of 3 to 4-year-olds and 54% of 5 to 7-year-olds, using the YouTube app or website (Ofcom, 2016). As confirmed by the interviews with parents, younger children mostly use YouTube to consume traditional, 'TV-like' content (Ofcom, 2016). Therefore, the item 'Maths related YouTube videos (e.g. NumTums)' was added to one of the subscales of the frequency of numeracy activities scale (see section 6.3.3 Confirmatory factor analysis for the frequency of numeracy activities, for the analysis on which subscale this item was added to).

5.6.1.2 Stage 6: Criterion Validity.

The examination of criterion validity by means of analysing contrasting cases from the frequency of numeracy activities scale and the interviews demonstrate that the scale detects differences between extreme cases well. There were clear differences between the views and experiences of parents with low and high scores across all five subscales: (1) parent - child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting.

5.6.2 Conclusion.

By following the procedures used by Hinkin (1998) and Nunes et al. (2005) the new PHMQ measure demonstrates construct, content, criterion validity and satisfies APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998), which was the ultimate objective of this scale development and validation process[2]. Further evidence for this was that other studies that have used this process have resulted in the creation of other measures that appear to be psychometrically sound (Hinkin & Schriesheim, 1989; Kumar & Beyerlein, 1991).

These scale development and validation analyses provide confidence that the new PHMQ measure, in particular the *frequency of numeracy activities* scale, possess reliability and validity and would be suitable for use in future research (Hinkin, 1998). Some of the HNE scales discussed in chapter 4 (see 4.1 Introduction for this discussion) did not provide adequate information about item generation and refinement, scale dimensionality, scale score reliability, or validity (e.g. Kleemans et al., 2012; LeFevre et al., 2009; Melhuish et al., 2008). In previous literature a major weakness to studying the HNE is the lack of information describing the psychometric integrity of scales used to measure the construct of the HNE. The current study provides these details and thus appears to be psychometrically sound (Hinkin & Schriesheim, 1989; MacKenzie, Podsakoff & Fetter, 1991). The PHMQ covers a vast array of HNE areas thus, it is concluded that the PHMQ can be used to describe the HNE that a parent creates for their child to learn numeracy. The PHMQ can allow researchers to obtain data in a quantifiable way quickly in order to understand how parents contribute to their child learning numeracy related concepts and skills.

[2] A confirmatory factor analysis will be completed in the next chapter to allow the researcher to gain further evidence of the construct validity of the new PHMQ measure (Hinkin, 1998).

Chapter 6: How Children Develop Numerical Skills Over Time

Overview

This chapter reports a longitudinal study tracking children's development of basic mathematical skills during the transitional phase from pre-school to primary school. First, this chapter reports the results of a confirmatory factor analysis to gain further evidence of the construct validity of the frequency of numeracy activities scale within the PHMQ measure (Hinkin, 1998). Second, this chapter discusses children's number skills *learner profiles* and *learning pathways* over time. Potential predictors of pathway membership are also discussed in detail. Certain sections of the PHMQ measure are used as predictors to understand if the home numeracy environment influences pathway membership.

6.1 Introduction

The origins of some early mathematical skills may be intrinsic (Baillargeon & Carey, 2012; Dehaene, 2001; Feigenson, 2011; Izard, Sann, Spelke, & Streri, 2009; McCrink & Wynn, 2004; Starkey, Spelke & Gelman, 1990; Xu & Spelke, 2000). Yet there is also strong evidence that individual experiences are important for the development of children's mathematical skills (Geary, Berch & Koepke, 2015). Children vary substantially in their level of number knowledge prior to school-entry (Manolitsis, Georgiou & Tziraki, 2013; Zill & West, 2001). Understanding why some children start school more prepared to learn mathematics than their peers is critical, as early mathematics skills are among the strongest predictors of later academic achievement (Duncan et al., 2007). Furthermore, mathematical skills are vital for college entry (Sadler & Tai, 2007) as well as achievement in STEM degrees (Wolniak, 2016). Most research suggests that proficiency in mathematics is not unitary but actually comprises different component mathematics skills (e.g. ordering and cardinality; Bisanz, Sherman, Rasmussen, Ho & Campbell 2005; Cowan et al., 2011; Dowker, 2005, 2008; Jordan, Mulhern, & Wylie, 2009), cognitive skills (LeFevre et al., 2010a; Krajewski & Schneider, 2009b) and other individual differences such as SES, the HNE and language development (Belsky et al., 2007; De Smedt & Boets, 2010; Göbel & Snowling, 2010; Melhuish et al., 2008; Skwarchuk et al., 2014; Starkey, Klein, & Wakeley, 2004). These component mathematics skills, cognitive skills and individual differences that may influence mathematical outcomes will now be discussed.

6.1.1 Individual differences.

6.1.1.1 Socio-economic status.

There are many different factors that contribute to SES (i.e. occupational measures, parental income, mother's educational levels etc.) with some relating strongly with academic achievement. In fact, there is evidence that parental education is the best predictor, with maternal education being most predictive in the early years (Sammons et al., 2004; Mercy & Steelman, 1982). However, in a meta-analysis by Sirin (2005) a lower mean correlation between SES and school achievement (mean correlation of r=.299) was found compared to a meta-analysis completed by White in 1982 (mean correlation of r=.343). Thus, results showed a slight decrease in the average correlation between SES and school achievement (Sirin, 2005). A potential explanation may be that the approach to research on SES and school achievement has changed recently which may account for the difference in relationship over time. In earlier research SES was contextualised as static throughout the lifespan whereas recent research emphasises a developmental approach between SES and school achievement (Sirin, 2005). In this study two proxies of SES will be considered as factors that could influence mathematical development. SES will be assessed by occupational measures, as this is the most widely used measure of SES, and by parental education, as there is evidence that suggests this is the best predictor of children's academic achievement (Galobardes, Shaw, Lawlor, Lynch & Smith, 2006; Sammons et al., 2004; Mercy & Steelman, 1982).

6.1.1.2 The home numeracy environment.

Questionnaire measures that assess the home numeracy environment have yielded inconsistencies between home activities and children's number skills (for example, Blevin-Knabe et al., 1996; DeFlorio & Beliakoff, 2015; Missall, Hojnoski, Caskie & Repasky, 2015). Some studies have found unique and positive associations between the home numeracy environment and mathematical skills (Kleemans et al., 2012; Dearing et al., 2012; Manolitsis et al., 2013; Niklas & Schneider, 2014). Whereas, Missall et al. (2015) found no relation between the home numeracy environment and a range of numeracy skills. Yet, DeFlorio and Beliakoff (2015) found significant associations between the frequency and range of home mathematics activities and mathematical skills, but these associations were reduced to non-significance after accounting for SES and parents' expectations for their children's mathematical learning. However, previous literature has demonstrated that SES did not predict early mathematical ability (Krajewski & Schneider, 2009a). Thus, it seems more important to explore the frequency of home numeracy experiences, rather than only assessing a family's SES to understand early mathematical development. Hence, more research is essential to understand the mechanisms that promote early mathematical skills in the home.

6.1.1.3 Language development.

The home learning environment has been measured either in relation to literacy or numeracy (e.g. Huntsinger et al., 1997; Huntsinger et al., 2000; LeFevre et al., 2009; Sénéchal & LeFevre, 2002). Some evidence exists that suggests the home literacy environment is a better predictor of children's numeracy than the home numeracy environment (Anders et al., 2012). However, one explanation for this finding could be that the outcome measure used (in Anders et al., 2012 case the Kaufman Assessment Battery for Children (KABC; Melchers & Preuss, 2003)) requires not only numeracy but also language skills (Abedi & Lord, 2001; Anders et al., 2012), much like other mathematical tests. Previous research has reported that individuals with reading or language problems perform poorly on arithmetic tasks (e.g. multiplication and fact retrieval) compared to those without these problems (De Smedt & Boets, 2010; Göbel & Snowing, 2010; Miles, Haslam & Wheeler, 2001; Moll, Snowling, Göbel & Hulme, 2015; Simmons & Singleton, 2006). Some research even claims that mathematics is language-dependent skill (Ascher & D'Ambrosio, 1994). Thus, language ability will be considered as a factor that could influence children's mathematical development.

6.1.2 Component skills.

6.1.2.1 Domain-general components.

Executive functions are defined as the procedure "responsible for the monitoring and regulation of cognitive processes during the performance of complex cognitive tasks" (van der Sluis, de Jong & van der Leij, 2007, pg. 1). However, universally there is no general consensus on the definition that completely encompasses the *components* of executive functions. For instance, Eslinger (1996) found 33 definitions of executive functions. Some researchers suggest that executive functions are best theorised as individual components (i.e. shifting, inhibition, updating, working memory, fluency) that are loosely related. In fact, some researchers only include inhibition (i.e. the ability to withholding a dominant response) and working memory as components of executive functions (e.g. Miyake, Friedman, Emerson, Witzki & Howerter, 2000). Others claim that executive functions share a mutual executive *attention* component (Blair, 2006; Duncan, Emslie, Williams, Johnson, & Freer 1996; Shallice & Burgess, 1993). Thus, the measurement of executive functions is a complex issue. However, there are some common features (e.g. components such as inhibition, working memory, planning etc.) in most definitions of executive function and substantial evidence that executive functioning plays an important role in learning during childhood (Sergeant, Geurts & Oosterlaan, 2002).

Most longitudinal evidence between cognitive and mathematical skills comes from school-age children and adults. However, recent evidence indicates this association is present before school-entry (e.g. Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2017; Bull, Espy & Wiebe, 2008; Lauer & Lourenco, 2016). Cognitive skills, such as working memory, include numerous components/processes that function in a synchronised manner in order to temporarily store and manipulate information (Baddeley, 2003; Cowan, 2008; Ericsson & Kintsch, 1995; Miyake & Shah, 1999; Oberauer, 2005). These components/processes include separate verbal (i.e. phonological loop concerned with verbal and acoustic information) and visuospatial subsystems (i.e. visuospatial sketchpad providing an interface between visual and spatial information) each of which has a limited capacity. Evidence suggests that executive functions, such as verbal and visual working memory, in younger children are best explained as a unitary construct (Hughes, Ensor, Wilson & Graham, 2009; Schmitt et al., 2017; Wiebe, Espy & Charak, 2008) that increasingly become differentiated as children get older (Huizinga, Dolan, & van der Molen, 2006; Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003). Therefore, due to the young age group in the current study (i.e. children in pre-school) a similar theoretical approach is maintained by focusing empirically on the unity of working memory by to only measuring children’s verbal working memory (as opposed to both verbal and visual).

Research shows that signs of attention-related skills (i.e. self-regulation and task persistence) can be identified as early as age 2 years 5 months but continue to develop and reach relative stability between 6 and 8 years old (Olson, Sameroff, Kerr, Lopez & Wellman, 2005; Posner & Rothbart, 2000). While research relating attention with later academic achievement are uncommon, there is consistent evidence that sustained attention and participation in classroom activities predicts achievement during pre-school and early school years (Alexander, Entwisle & Dauber, 1993; Raver, Smith-Donald, Hayes & Jones, 2005). Furthermore, sustained attention skills have been are associated with later academic achievement, independent of cognitive (McClelland, Morrison, & Holmes, 2000; Yen, Konold, & McDermott, 2004) and reading and vocabulary skills (Howse, Lange, Farran, & Boyles, 2003). Therefore, it is important to understand the impact that these executive functions (i.e. verbal working memory and sustained attention) have on children’s mathematical development. Besides executive function skills, throughout the last decade, research has expanded to explain important associations between basic numerical processing abilities and the development of school level mathematics skills (De Smedt, Noël, Gilmore & Ansari, 2013; Price & Wilkey, 2017). Thus, understanding the development of executive functions, as well as basic numerical processing abilities, is of fundamental importance to further our understanding of children’s learning and development.

6.1.2.2 Domain-specific components.

Previous research has indicated that early number knowledge is key to later mathematical development (Duncan et al., 2007; Göbel, Watson, Lervåg, & Hulme, 2014; National Research Council, 2009; Nguyen et al., 2016; Watts, Duncan, Siegler, & Davis-Kean, 2014) and there are many basic numerical processing abilities that are important in a child’s learning and development of mathematics skills (Lyons, Price, Vaessen, Blomert & Ansari, 2014). For instance, cardinal principle knowledge, number naming, dot comparison, and numeral ordering tasks etc. (e.g. Aulet & Lourenco, 2018; Batchelor, Keeble & Gilmore, 2015; Cordes & Gelman, 2005; De Smedt, Noël, Gilmore & Ansari, 2013; Lyons et al., 2014; Nanu, McMullen, Munck, Hannula-Sormunen & Pipari Study Group, 2018; Methe, Hintze & Floyd, 2008). However, some basic numerical processing abilities may be more important in the development of *early* mathematics skills during pre-school. For instance, children typically learn counting principles such as, cardinality and numeral ordering knowledge around 3 years of age (Bermejo, 1996; Gelman & Gallistel, 1978).

Different mathematics skills develop simultaneously, although they influence and reinforce one another across development (Baroody & Ginsburg, 1986; Rittle-Johnson, Siegler, & Alibali, 2001; Sarnecka & Carey, 2008). Dowker (2008) revealed how some pre-schoolers were unable to do basic counting but understood how counting could be used to determine cardinality. Meanwhile other children were capable of basic counting but lacked cardinal knowledge. This indicates that there might not be a single pathway in mathematical development. Some evidence suggests that no skill is a required prerequisite for another skill (Coles & Sinclair, 2018; Dowker, 2008; Gray & Reeve, 2016; Holmes & Dowker, 2013) as pre-schoolers may perform poorly on foundation skills (e.g. counting) and yet may succeed on seemingly more complex tasks (e.g. cardinality). Some researchers suggest that children's *experience* of counting helps the development of knowledge of *principles* (Briars & Siegler, 1984; Fuson, 1988). However, other researchers propose a mutual development theory wherein principles and experiences develop together and reinforce each other during the course of development (Baroody & Ginsburg, 1986; Rittle-Johnson & Siegler, 1998). Thus, many researchers differ in their understanding of the relationship between children's counting skills and knowledge of counting principles (Dowker, 2008).

6.1.2.2.1 Domain-specific components to be used in this study.

The term *Arabic digit* represent number names (e.g. 5 represents the number name "five"; Liebeck, 1990; Wright, Martland, and Stafford 2006). The term *digit recognition* refers to the child's ability to state the name *Arabic digit* (Wright et al. 2006). Over time, children learn to recognise digits and use this knowledge for calculating or mathematical problems (Cook 1996; Munn, 1994; Tolchinsky, 2003). Thus, due to the foundational nature of digit recognition children in this study will be asked to identify written Arabic digits.

Symbolic number refers to cultural symbols attributed to quantities (e.g. Arabic digits i.e. 1 or number words i.e. one). While, non-symbolic (e.g. dot estimation) intuitions of numerosity is relied upon to quickly approximate the numerosity of sets of objects without resorting to counting or the use of number symbols (Dehaene, 1997; for further information on non-symbolic and symbolic skills see section 2.1.6 Domain-specific components). A recent prominent theory suggests that Arabic digits are associated with or 'mapped onto' the innate approximate number system (ANS) over the course of learning, also known as the 'mapping hypothesis' (Dehaene, 2007; Piazza, 2011; for a review see Leibovich & Ansari, 2016). In other words, when developed children map symbolic representations onto their pre-existing non-symbolic representations. Therefore, the non-symbolic system may assist in the acquisition of numbers (Gallistel & Gelman, 1992). However, as suggested by Batchelor, Keeble and Gilmore (2015) the symbolic system may be learnt independently of the non-symbolic system. Subsequently, after the symbolic system has developed the 'mapping' between non-symbolic and symbolic representations might occur (e.g. Le Corre & Carey, 2007).

The development of non-symbolic and symbolic number processing has typically been tested through magnitude comparison tasks (e.g. De Smedt, Verschaffel & Ghesquière, 2009; Halberda, Mazzocco, & Feigenson, 2008; Holloway & Ansari, 2009; Inglis, Attridge, Batchelor & Gilmore, 2011). In magnitude comparison tasks participants are shown two numerosities (i.e. dot arrays or Arabic digits/number words) and asked to select the more numerous. Researchers have proposed that performance on non-symbolic magnitude comparison tasks is associated with mathematics achievement. Nevertheless, the evidence to support this hypothesis is mixed (see De Smedt et al., 2013). Some researchers have found a significant relationship between non-symbolic dot comparison performance and mathematics skills in children (Halberda, Mazzocco, & Feigenson, 2008; Inglis, Attridge, Batchelor & Gilmore, 2011; Libertus, Feigenson & Halberda, 2013; Mundy & Gilmore, 2009) and in adults (Halberda, Ly, Wilmer, Naiman & Germine, 2012; Libertus, Odic & Halberda, 2012; Lourenco, Bonny, Fernandez & Rao, 2012; Lyons & Beilock, 2011), while others find no significant relationship between non-symbolic dot comparison performance and mathematics skills in children's (Holloway & Ansari, 2009; Mundy & Gilmore, 2009; Sasanguie, Göbel, Moll, Smets & Reynvoet, 2013; Sasanguie, De Smedt, Defever & Reynvoet, 2012) and in adults (Batchelor & Gilmore, 2011; Castronovo & Göbel, 2012; Inglis, Attridge; Price, Palmer, Battista & Ansari, 2012).

Most associations between numerical magnitude processing skills (i.e. both symbolic and non-symbolic magnitude comparison tasks) and broader mathematical competence (e.g. arithmetic) have shown positive correlations but vary substantially in their strength (Schneider et al., 2016). However, in a meta-analysis (Schneider et al., 2016) the effect size was significantly higher for the symbolic than for the non-symbolic magnitude comparison task and decreased very slightly with age. Thus, symbolic had higher associations with broader mathematical competence than non-symbolic numerical magnitude processing. Batchelor, Keeble and Gilmore (2015) introduced a cross-notation comparison task to be used with pre-school aged children to allow for the direct assessment of mapping between magnitude representations (e.g. dot arrays versus verbal number words), thus a version of this task will be used to assess children's mapping magnitude representations skills.

Another task that assesses cardinal principle knowledge (i.e. the last count word indicates the number of items in a set) will be used in this study. Some researchers suggest that principles guide children to learn to count (Gelman & Gallistel, 1978; Gelman, 1997). For instance, Gelman and Gallistel (1978) generated five principles that govern and define counting: (1) *the one-to-one principle* when each item is assigned one and only one unique count word, (2) *the stable-order principle* when the counting words are consistently used in a stable order and (3) *the cardinal principle*. These first three principles are known as the how-to-count principles. (4) *The abstraction principle* deals with the characterisation of what is countable and (5) *the order-irrelevance principle* states that the order of enumeration is irrelevant and does not affect the resulting number within the set. However, (3) *the cardinal principle* is the only principle that describes conceptual knowledge and what a child *must have* as opposed to the other four principles which are procedural and describe what a child *must do* (Gilmore, Göbel & Inglis, 2018). Therefore, cardinal principle knowledge was deemed an important numerical measure to be utilised within this pre-school aged study, when children are learning to count.

Children apply cardinal principle knowledge to numbers on the Give-N task. In Give-N tasks children are asked to produce sets of items (e.g., Condry & Spelke, 2008; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Lee, 2009; Wynn, 1990, 1992). As stated previously in Chapter 2, cardinality is the number of items in a set. A child demonstrates cardinal principle knowledge when counting a set of objects and the last word uttered correctly, in the correct order, expresses the number of items in the set. Thus, cardinal principle knowledge is perhaps necessary to give number words their meanings (Sarnecka & Wright, 2013). Cardinality has been shown to be important for pre-school children's mathematics skills development (e.g. Batchelor, Keeble & Gilmore, 2015; Bermejo, 1996; Lyons, Price, Vaessen, Blomert & Ansari, 2014; Mussolin, Nys & Leybaert, 2012; Wagner & Johnson, 2011). Interestingly, some researchers have found a positive association between performance on the non-symbolic comparison task and performance on Give-N tasks (Mussolin, Nys & Leybaert, 2012; Wagner & Johnson, 2011). The Give-N task will be used to assess cardinality as other measures of cardinality are thought to provide a weaker measure (Batchelor, Keeble & Gilmore, 2015). For instance, the "How many?" task (Huntley-Fenner & Cannon, 2000; Slaughter, Kamppi, & Paynter, 2006) does not demonstrate that children understand cardinality as children may learn to state the final number in a counting sequence in response to the question, "How many?" (Batchelor, Keeble & Gilmore, 2015).

Spontaneous focusing on numerosity (SFON) is a recently-developed construct which captures an individual's self-initiated or non-guided focusing on the numerical aspects of their environment (Hannula & Lehtinen, 2005; Batchelor, Inglis & Gilmore, 2015). Pre-school children's (aged 4-5-year olds) SFON performance has been found to be positively associate with their symbolic numerical processing and performance on a standardised test of arithmetic (Batchelor et al., 2015). Further, children who had predominant SFON tendencies developed faster in their cardinality recognition skills longitudinally (from 3-4-years old) than children with non- predominant SFON tendencies (Hannula, 2005). Therefore, children's development in cardinality recognition skills may be related to their SFON tendency. Due to this pervious research a measure of SFON will be included within this study.

Order processing has only recently been considered as an important predictor of numerical abilities in young children (Kaufmann, Vogel, Starke, Kremser & Schocke, 2009) and has been seen as a crucial building block of the development of numerical representations (Sury & Rubinsten, 2012). A cross-sectional study by Lyons et al. (2014) across Grades 1–6 (i.e. children 7 to 12 years old) examined the unique relations between eight basic numerical skills (e.g. numeral ordering, numeral comparison, dot comparison, counting etc.) and early arithmetic ability through a multiple regression model. One of the main findings was that the relative importance of symbolic number ability appears to shift from cardinal to ordinal processing. Furthermore, children's Grade 6 (M_{age}= 12.18 yrs.) numerical-ordering performance was a better predictor of mental arithmetic performance than the seven other numerical tasks and remained so even after controlling reading ability and nonverbal intelligence (i.e. non-numerical factors).

There is a lack of longitudinal studies, however, that have researched the link between numerical ordinal processing and calculation abilities (Attout, Noël & Majerus, 2014; Lyons & Beilock, 2011; Rubinsten & Sury, 2011). Although, in a longitudinal study tracking pre-school children through 2nd grade of primary school (mean age of 68 months at Time 1; Attout et al., 2014) a strong link between numerical ordinal abilities and calculation abilities was only found at a cross-sectional level but not at a longitudinal level. Whereas, numerical magnitude abilities at Time 1 predicted calculation abilities 1 and 2 years later (Attout et al., 2014). The results indicated that there is no causal association between numerical ordinal processing abilities and later calculation abilities. However, ordinal processing and calculation abilities were associated via different pathways than magnitude-processing abilities. Therefore, it is important to assess cardinality, ordinal processing, mapping magnitude representations and basic counting abilities in young children's development of mathematics skills to understand individual differences in mathematics skills development.

6.1.3 Rationale.

Traditional analytical variable-centered approaches (e.g. regression analysis, factor analysis, and structural equation modelling) assume that the relation between variables can be applied to all learners in the same way. These methods limit the ability to deal with heterogeneity within and between individuals (Hickendorff et al., 2017) as they show average trends at the group level but do not deal with the nuance of individual variation. These methods are therefore not adequate to explore contextual variations that may affect learning and development (Lindblom-Ylänne, Parpala & Postareff, 2015). The current study aimed to address the empirical gap by taking a person-centred approach as opposed to a variable-centred approach. A person-centred, longitudinal research design was utilised to discover how pre-school children with a range of basic numerical skill profiles make the transition from pre-school to school learning. Thus, going beyond previous research by identifying children's basic numerical skill pathways as opposed to only analysing the average child experience (i.e. a variable-centered approaches; Lindblom-Ylänne et al., 2015). Many previous studies have used cross-sectional approaches (e.g. Batchelor, Keeble & Gilmore, 2015; Lyons et al., 2014) but have not considered learner profiles and how children's learning pathways change over time, this study addresses this gap. Studies that have targeted the developmental trajectories in children's mathematical skills (Aunola et al. 2004; Chong & Siegel 2008; Jordan et al. 2006, 2007; Morgan, Fargas & Wu 2009) have shown that children who enter pre-school with low performance in basic number skills stay behind their peers throughout later school years. Therefore, it is important to understand children's mathematics skill profiles and what effects pathway membership using longitudinal research methods.

6.1.4 Aims.

The current study aimed to examine the person-centered development of participants as learners across their transition from pre-school to primary school over the course of 8 months. This study aims to address limitations of previous studies and current gaps in existing literature by tracking children's basic numerical skill development from pre-school to school. A latent transition analysis will be used to describe children's precise *learner profiles* and *learning pathways* during this transition. Furthermore, identifying the key predictors of children's *pathway* membership over time. This study considers a variety of demographic characteristics (i.e. gender, age, SES and parents' highest educational qualification), as well as predictors associated with multiple components of the home environment (for clarity, the only sections of the PHMQ that will be used as predictors of children's pathway membership are the *frequency of numeracy activities* scale and the *informal home numeracy practices* (number games exposure checklist) sections), domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary). Therefore, this study will incorporate potential predictors of pathway membership to provide knowledge on children's development of mathematical skills in early childhood.

6.2 Method

6.2.1 Participants.

A total of 10 primary schools that had pre-school provision were contacted and invited to take part in the current study. Eight of the primary schools had taken part in previous research (Chapter 4). Each primary school responded and seven stated that they would like to be involved in the project, giving a potential recruitment pool of approximately 341 children and parents. Parents of the pre-school children were recruited to take part in the Pre-school Home Maths Questionnaire (PHMQ) at Time 1 and the children were recruited to take part in game like tasks at 3-time points tracking their transition between pre-school and primary school over the course of 8 months. Time 1 started in February 2017. Between Time 1 (T1) and Time 2 (T2) there was a 3-month gap with T2 starting in May 2017. Between T2 and Time 3 (T3) there was a 5-month gap with T3 starting in October 2017. 152 parents agreed to complete the PHMQ and also consented for their child to take part in the study.

To control for a potential 30% loss rate from T1 to T3 the aim was to recruit 140 participants through the seven pre-schools at T1. This is consistent with previous longitudinal studies were data collection started in pre-school for instance, 28% loss rate over 5-year period (Senechal & LeFevre, 2002) and 29% over 1.5 years (Lonigan, Burgess & Anthony, 2000). 152 children were recruited at T1. There was a 22.4% loss rate (total of 34 participants) from T1 to T3. The reasons for non-participation across time points are shown in Table 10. Between T1 and T3 22 of the 152 children moved from a pre-school to a different primary school. This dispersal of children meant that 14 new schools were contacted before T3. 11 of these new schools agreed to take part in the study.

Table 10. *Reasons for non-participation across time points*

Reason for non-participation	T1 Participants *n* (%)	T2 Participants *n* (%)	T3 Participants *n* (%)
Learning disability e.g. autism	3 (2.0)	1 (0.7)	/
Refuse to consent	5 (3.3)	/	/
Selective mutism	2 (1.3)	/	/
Absent e.g. holiday/sick/hospital	4 (2.6)	/	1 (0.7)
Incorrect age i.e. in nursery for 2 years	9 (5.9)	/	/
Unable to locate participant	/	/	1 (0.7)
No reply from new T3 schools	/	/	3 (2.0)
Unable to make suitable testing appointment	/	/	5 (3.3)
Total non-participation	23 (15.1)	1 (0.7)	10 (6.6)
Total at end of T1, T2, T3	129	128	118

Therefore, from the 152 dyads recruited at T1, a total of 136 parents completed the PHMQ and 118 children completed the game like tasks at T3. This meant that 104 parent-child dyads provided complete data throughout the full study. From the total sample, 124 mothers and 12 fathers completed the questionnaire. 127 (83.6%) participants specified that they were the primary carers for the target child (i.e. spend most of the time with the target child). The 152-target child (52.6% female) that the parents/guardians were asked questions on were aged between 38 months to 54 months at time point 1 (M_{age} = 47.8 months). A full break down of the participants' demographic information is presented in the results.

Consistent with recruitment in the previous chapter proportions of FSM were classified as: low (4-18%), medium (19-58%) and high (59-85%) FSM Eligibility, thus the same classification approach was used across studies. These values were calculated using openly available data from the Department of Education (2014) statistics on 67 schools across Northern Ireland that had a pre-school. A total of 136 (89%) of parents returned the PHMQ. The proportion of PHMQ returned from each of the FSM Eligibility categories were 32%, 50% and 18%, respectively. However, to ensure an equal spread of participants and to avoid imperfect proxy bias (i.e. a proxy that correlates with the key variable but cannot be understood in isolation) parents completed 8 questions from The National Statistics Socio-economic Classification (NS-SEC 2010), which allowed the researcher to derive socio-economic status (SES) using the Standard Occupational Classification (2010) (SOC2010). SES descriptives will be discussed in more detail in the results section.

6.2.2 Materials.

6.2.2.1 Recruitment materials.

Ten primary schools with connecting pre-schools were approached by post, which included a brief cover letter, invitation letter, permission form and copy of the PHMQ. The invitation letter contained information on the aims of the study, what was required from parents, their child, the pre-school and primary 1 teacher, and what will happen with the information once collected. The head teacher was required to sign and return a permission form before recruitment could commence. If there was no response from the head teacher, the primary school was contacted by telephone to follow up the written contact. Permission from the head teachers to distribute parent information sheets and consent forms was gained between November and December 2016.

After receiving consent from the head teacher, all parents/guardians of the children attending the pre-school were sent a cover letter, parent information sheet and consent form via the pre-school teacher. The parent information sheet included information on the purpose of the study, what was required from the parent/guardian and their child, what will happen with the information once collected and an explanation on how to withdraw from the study if they wished to do so. If parents wished to be involved in the study, they ticked boxes confirming that they agreed to their child taking part in this study, to completing in the PHMQ, that they had read and understood the information sheet, and understood that their participation was voluntary and that they were free to withdraw without giving a reason. Parents also agreed that the researchers would hold all information collected securely and would not share the information with any other party, to their child being audio recorded during one of the games and to state that their child does not have any diagnosed learning disabilities, attentional or neurological disorders. Permission from the parents/guardians was gained in January 2017. After the consent form was returned, the parents/guardians were sent the PHMQ via the pre-school teacher.

Only children whose parents/guardians returned a signed consent form participated in the study. The children whose parents/guardians gave consent were also asked for their assent before the researcher began to administer the tasks. At time point 1 the pre-school head teacher was asked to complete a questionnaire, which asks what they are teaching the children in their class. At T3 the primary 1 teacher was asked to complete the same questionnaire.

6.2.3 Procedure.

6.2.3.1 Overall procedure for parents.

The research procedures were reviewed and approved by School of Psychology Research Ethics Committee (REC) before the study commenced. The parent of the target child was asked to complete the PHMQ at T1. Parents who complete and return the PHMQ were entered into a prize draw for a £50.00 Amazon voucher. The PHMQ was returned to the pre-school teacher and collected by the researcher during T1. Parents/guardians provided a signed consent form stating that they wish to take part in the study. Consent forms were kept in a different location, in a locked cabinet, to the interview transcripts and questionnaires. The children whose parents/guardians gave consent were also asked for their assent before the researcher began the tasks.

6.2.3.2 Overall procedure for teachers.

As there were 7 pre-school schools involved there may be a difference in the schools teaching and therefore learning outcomes for the children. In order to control for this the pre-school teacher and the primary 1 teacher was asked to complete a brief questionnaire. The teacher was provided with a teacher information sheet and was required to sign and return a consent form. The questionnaire requested information on teaching content and curriculum to further understand any differences between schools. This information was not analysed further as there was no difference between children's learning outcomes between schools e.g. no group of children from one school achieved higher than another.

6.2.3.3 Overall procedure for children.

This longitudinal examination assessed how pre-school children developed mathematical skills over time with their transition from pre-school to primary school. The researcher made two visits to the pre-schools and one visit to the primary schools when the children were in primary one, resulting in 3-time points. Children completed a series of game-like tasks with the researcher on a one-to-one basis in a school corridor or public space; the environment was familiar to the children. At T1 and T3 children completed a maximum of three 20-minute sessions consisting of tasks that measured 1) mathematical achievement, 2) vocabulary skills, 3) basis mathematical skills, such as, cardinal principle knowledge, digit recognition, numerical ordering, spontaneous focusing on numerosity, and mapping magnitude representations, as well as 4) general cognitive tasks, specifically, verbal working memory and sustained attention. At T2 children completed two 20-minute sessions; they were not assessed on their mathematical achievement and vocabulary skills at this time point. Children were given a sticker after completing the games with the researcher.

6.2.3.4 Task materials and procedure.

The child participants completed 9 different tasks. Table 11 summarises each task and the time point(s) when they were administered.

Table 11. *Breakdown of tasks that children completed*

	Task name	Type of measure	Construct that the task measures	Times
Outcome measure	British Ability Scale (BASII) – Early Number Concepts	Standardised	Mathematical achievement	T1 & T3
Receptive vocabulary	British Picture Vocabulary Scale (BPVS)	Standardised	Receptive vocabulary	T1 & T3
Domain specific – Maths skills	Give-N	Experimental	Cardinal Principle Knowledge	T1, T2, T3
	Digit Recognition	Experimental	Digit Recognition	T1, T2, T3
	Numeral Ordering	Experimental	Ordering ability	T1, T2, T3
	Spontaneous Focus on Numbers (SFON) – Picture task	Experimental	SFON	T1, T2, T3
	Cross-Notation Comparison	Experimental	Mapping Magnitude Representations	T1, T2, T3
Domain general – Cognitive skills	Auditory Continuous Performance Test – Pre-school (ACPT-P)	Experimental	Sustained Attention	T1, T2, T3
	Animal Recall	Experimental	Working Memory – Verbal task	T1, T2, T3

6.2.3.5 Outcome measure.

6.2.3.5.1 British ability scale – Early number concepts.

The British Ability Scale (BAS-II) Early Number Concepts (Elliot, Smith, & McCulloch, 1996) was administered as an outcome measure of mathematical achievement. For this task, the child answered questions about size, number and other numerical concepts. The task stimuli include 10 green plastic squares and an easel used to present a series of pictures (Diagnostic Scales Stimulus Booklet 1). There were 30 questions in total with a maximum score of 35. The assessment was stopped when the child made 5 consecutive errors. There are three suggested starting points based on the child's age. At T1 the researcher started at item 1 but at T3 the researcher started at item 4. If the child got fewer than 3 correct within that decision point the researcher went back to previous starting point, if applicable. The researcher coded the child's answers with a score of 1 if correct and a score of 0 if incorrect. Item 3 was an exception to this and was scored between 0-3. The researcher was to provide only neutral encouragement to the child during the task, except for the designated teaching items (items 4 and 5). For these items, the researcher provided specific feedback e.g. "yes that's correct" but if the child had not answered correctly or had not understood the question, they gave the correct response. The child's correct responses are totalled to give a raw score (i.e. number of correct answers). The BAS was normed on 1480 children aged 3–8 years 11 months (Elliott, Smith & McCulloch, 1996).

6.2.3.6 Receptive vocabulary measure.

6.2.3.6.1 British picture vocabulary scale.

The British Picture Vocabulary Scale – Third Edition (BPVS-III; Dunn, Dunn, Styles & Sewell, 2009) is a standardised non-reading assessment of receptive vocabulary. For this task, the researcher stated a word that covers a range of subjects, such as verbs, animals, emotions, toys and attributes and the child responded by selecting a picture from four options that best illustrated the word's meaning. The task stimulus is an easel used to present a series of pictures (the BPVS3 Testbook). Following the administrator script praise was given for both correct and incorrect responses in order to motivate child to do their best. Comments such as "Good! You are doing well" were given but not over stated. If pupils asked if they got a question correct the researcher said, "That was a good answer". The researcher began from the Start Set, according to age. The Basal Set is the set where no more than one error is made. If more than one error is made, the researcher found the Basal Set by testing backwards through preceding sets until no more than one error is made in a set. After the Basal Set was found the researcher tested forward by set the child made 8 or more errors within a set of 12, this is known as the Ceiling Set. The number of errors in each set is calculated and the total number of errors made is subtracted from the Basal Set through to the Ceiling Set to gain a raw score. The correct responses were totalled to give a raw score for each participant. There are 14 Sets giving a maximum score of 168. The BPVS-III was normed on 3278 children with and without disabilities aged 3 to 16 years (Dunn & Dunn, 2009). The BPVS-III has an internal reliability of r = 0.91 and criterion validity with the Wechsler Intelligence Scale for Children (2005) of r = 0.76 (Dunn & Dunn, 2009; Hannant, 2018).

6.2.3.7 Domain specific.

6.2.3.7.1 Give-N task.

Th Give-N task was adapted from Wynn (1990, 1992) and was presented in line with previous studies (e.g. Batchelor, Keeble & Gilmore, 2015; Condry & Spelke, 2008). The Give-N task asked children to generate sets of a given number from a given set of counters. Materials included a colourful puppet called *Fluffy* (32 cm high), a yellow plastic plate (25 cm in diameter), and a set of 18 plastic counters (each 1.5 cm in diameter). There were five different colours for the five different numbers asked (e.g. 18 white, 18 green counters etc.). The five numbers used in this task were 3, 4, 6, 11 and 15. The numbers 3 and 4 were considered to be the low number trials and the numbers 6, 11 and 15 were considered to be the high number trials. The researcher began the task by placing the puppet on the table and saying, "This is Fluffy. The way we play this game is: "I will tell you what to put on the plate, and you put it there and slide it over to Fluffy, like this". The researcher demonstrated this by placing a counter on the plate and sliding the plate across the table to Fluffy. Figure 4 demonstrates the experimental set-up of the Give-N task.

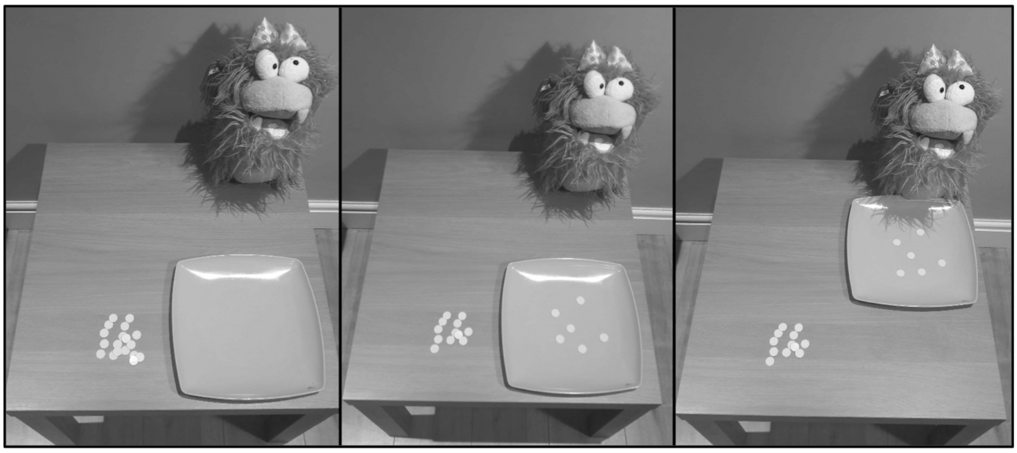


Figure 4. Showing the experimental set-up of the Give-N task
Step 1. Illustrates the set up at the start of the task. Step 2. Demonstrates the counters being placed on the plate by the child. Step 3. Shows how to researcher knows the child is finished placing the counters on the plate, with the plate slide across and sitting in front of the puppet.

The researcher then placed the plate in front of the child with 18 coloured counters beside the plate and asked, "Can you give three counters to Fluffy?". After the child put one or more counters on the plate and slid the plate over to Fluffy, the researcher asked a question repeating the original number word (e.g., "Is that three?"). If the child said "Yes" then the researcher said, "Thank you!" and placed the item(s) back in an envelope. If the child said "No" then the researcher restated the original request. This was the script for the low number trials (3 and 4). The follow-up questions for the high number trials encourage the child to count.

The follow-up question was the same for both low and high number trials (e.g., “Is that six?”). However, for the high number trials if the child said “yes,” the researcher said, “Can you count and make sure it’s six?”. If the child counted and ended with a number other than six, the researcher said, “Can you fix it so it’s six?” If the child answered “no” to the original follow-up question, the researcher said, “Can you count and fix it so it’s six?”. The child’s final response (after counting and fixing) was recorded. The single digit numbers were in block one and the double-digit numbers were in block two. The researcher stopped the test when a child made a mistake (i.e. gave the wrong number of counters) with six or more numbers in block one. The numbers were randomised within each block (e.g. 4, 6, 3 (block 1) 15, 11 (block 2) or 6, 3, 4 (block 1), 11, 15 (block 2)). Each number was requested up to three times and for each correct response (e.g. 4 counters was requested, and the child gave Fluffy 4 counters) children were awarded one point. A proportion score out of 15 was calculated for each participant.

6.2.3.7.2 Digit recognition task.

Number symbols were presented on individual sheets contained in a folder and the researcher sat opposite the child. The digits were presented in Arial, size 200, landscape. The researcher said: “We are going to play another game, I want you to tell me what the number is” and then opened the folder and stated, “What number is this?”. The instruction could be repeated once, if needed. There were 2 blocks with 6 numbers in each. The first block contained single digit numbers (i.e. 3, 2, 5, 8, 7, 9) and the second block contained numbers between 10 to 20 (i.e. 12, 14, 11, 16, 20, 18). The researcher stopped the test when a child made a mistake with four or more numbers in one block. For each correct response children were awarded one point. A proportion score out of 12 was calculated for each participant.

6.2.3.7.3 Numerical ordering task.

Children were presented with three number cards (12.5cm x 9 cm) presented horizontally as Arabic numerals. The Arabic numerals were presented in Calibri, size 140, portrait. For all trials numbers were presented with *consecutive numbers* (e.g. 8, 7, 9), *gaps of 2* and *gaps of 3* numbers counterbalanced. There were 18 one-digit trials that were presented from left to right by the experimenter. The distance between the placed number cards (i.e. 2cm) stayed the same throughout the trials. There were 6 consecutive number trials (e.g. 8, 7, 9), 6 trials with gaps of 2 (e.g. 5, 9, 7), and 6 trials with gaps of 3 (e.g. 1, 7, 4). The researcher sat beside the child and read the numbers aloud as they put the number cards on the table. Once all three numbers were presented the researcher asked the child, “Can you put the numbers in the right order from the smallest number to the biggest number?”. As the researcher stated this information the researcher demonstrated where the small number should be on the left-hand side and where the biggest number should be on the right-hand side. After the child moved the number cards into the order they perceived to be correct or the child looked like they had finished the researcher asked, “Are the numbers in the right order from the smallest number to the biggest number?”. If the child answered “Yes” the researcher said, “Good job” and moved onto the next trial. If the child answered “No” the researcher said, “Can you fix the number cards, so they are in the right order from the smallest number to the biggest number?”, the child was given time to do this. After the second try if the order was still incorrect but the child believed it as correct the researcher replied, “Good job” and the next trial was presented.

Before the 18 test trials commenced 4 practice trials were given. During the practice trials if the order of the numbers were placed in the incorrect order by the participant after the second attempt the researcher stated, “Good try, but if we put the numbers like this [arrange into order] then they are smallest to biggest”. Only during the 4 practice trials were the cards moved into the correct order by the researcher to demonstrate to the child how they were to move them. No time limit was put in place for children to move the number cards. If child got 6 or more trials wrong the task was stopped. For each correct response children were awarded one point, giving a maximum score of 18. The number cards were in the same order for all children. A proportion score was calculated for each participant.

6.2.3.7.4 Spontaneous focus on numbers task.

The materials used in this spontaneous focus on numbers (SFON) task were three cartoon pictures (25.0 cm 17.5 cm) each in A4 clear punched poly pockets placed in a folder and an Olympus DM-450 digital voice recorder. The three cartoon pictures are the same pictures used by Batchelor and colleagues (2015), see Figure 5 for the pictures. The three cartoon pictures contained numerous small arrays (of objects, people or animals) that could be counted, for example, “three chicks” (Picture 1), “two children” (Picture 2), “four flowers” (Picture 3). The pictures were presented in the same order for each child.


Picture 1


Picture 2


Picture 3

Figure 5. The pictures used in the first, second and third trial of the SFON picture task

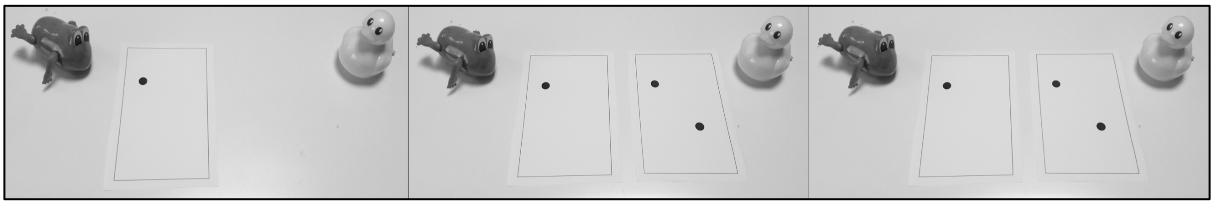
Following the guidelines used by Batchelor, Inglis & Gilmore (2015) the researcher sat opposite the child, introducing the task by stating, “This game is all about pictures. I’m going to show you a picture, but I’m not going to see the picture. Only you get to see the picture. This means I need your help to tell me what’s in the picture”. The SFON task was recorded but the researcher wrote down any words that were not clearly spoken by the child (i.e. if the child mumbled) for clarity when transcribing. The researchers request was repeated if the child was hesitant to speak by saying, “Can you tell me what you can see?”. The researcher prompted the child to speak louder if the child spoke too quietly. No time limit was put in place for children to respond. The researcher waited for 3 seconds after the child appeared to have finished and then asked, “Is that everything?”. The next trial was introduced when the child was ready to move on. Children received a score of 0 or 1 contingent on whether they spontaneously focused on numerosity or not for each of the three SFON trials, thus the maximum score for this task was 3 (Batchelor et al., 2015). As the SFON task assesses spontaneous focusing on numerosity in the environment rather than a child’s ability to accurately count stimuli the numbers mentioned did not need to be the exact number represented in the picture for example, three chicks. The children only had to mention a number to gain a score of 1 for that picture.

6.2.3.7.5 Cross-notation comparison task.

Following Batchelor, Keeble and Gilmore (2015), children were presented with two characters, either a duck or a frog, and instructed to choose between the two characters on who had the most balls (the ‘balls’ were dot arrays) after presenting the child with two numerosities. Each child was given 12 trials. If a child scored more than 6 in the first 12 trials that child was given a further 6 trials, giving a total of 18 trials. The trials were counter-balanced thus, for half the trials the first numerosity was presented by the researcher as a verbal number word. For the other half the first numerosity was presented non-symbolically as dot arrays on a laminated card (18cm x 12cm). To aid understanding of the verbal number word trials, the child was presented with a picture of a box on a card and the researchers stated, “The [Character] has hidden their balls in a box. [Character] has [*n*] balls.” As the researcher presented the non-symbolic card they said: “The [Character] has this many balls.” The researcher then asked, “Who has the most balls?”. Both of the presented cards (i.e. the picture of a box and the dot array) stayed on the table next to each of the characters (i.e. the duck or the frog) until a response was given by the child. Children responded by pointing to and/or naming the character. Children were not allowed to count the dot arrays to answer this task. If a child made any visible counting acts (i.e. counting into themselves or on their hands) the researcher reminded the child not to count by saying, “For this game it’s important that you don’t count”. The child’s response was marked down as incorrect if there was a visible sign of counting (score of 0).

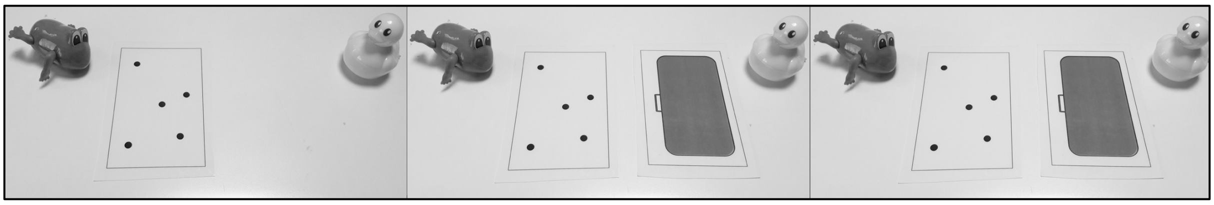
Before beginning the experimental trials two practice trials were administered that were non-symbolic (dot array versus dot array) thus familiarising the child with the instructions of the game. Children were asked to compare the following sets: 2 dots vs. 1 dot and 4 dots vs. 2 dots. Trials 1 and 2 were presented by the researcher consecutively as they involved numbers within the subitising range. Trial 3 onwards were presented to each child in a random order. The numerosities used ranged between one and fifteen. Four trials involved at least one numerosity within the subitising range (i.e. 1–3), and the other 14 trials included two numerosities outside the subitising range (i.e. 4–15). This procedure was used for task consistency and is the same as that used by Batchelor et al. (2015). Figure 6 (1) is a practice trial example of a non-symbolic (dot comparison) and Figure 6 (2) is an example of a cross-notation comparison trial.

(1)



“The frog has this many balls.” “The duck has this many balls.” “Who has the most balls?”.

(2)



“The frog has this many balls.” “The duck has hidden their balls in a box. The duck has four balls.” “Who has the most balls?”.

Figure 6. Set-up of the cross-notation comparison task

(1) is an example of the practice trial which was a non-symbolic (dot comparison) and (2) is an example of a cross-notation comparison trial.

Table 12. *Cross-notation Comparison Trials*

Trial no.	Distance	Dot Array	Verbal Number Word	Presented first
1	Small	1 The frog has this many…	3 The duck has three…	Verbal number word
2	Small	3	2	Dot Array
3	Large	1	5	Dot Array
4	Large	6	2	Verbal number word
5	Small	5	4	Dot Array
6	Small	7	5	Verbal number word
7	Large	4	8	Dot Array
8	Large	5	10	Verbal number word
9	Small	6	9	Dot Array
10	Small	7	10	Verbal number word
11	Large	9	4	Dot Array
12	Large	10	6	Verbal number word
13	Small	11	12	Dot Array
14	Small	13	15	Verbal number word
15	Small	14	11	Dot Array
16	Large	9	13	Verbal number word
17	Large	15	10	Dot Array
18	Large	12	7	Verbal number word

The numerical distance between the presented stimuli varied. For half of the trials there was a small distance between the numbers (i.e. a distance of 1-3) and for the other half there was a large numerical distance (i.e. a distance of 4 or 5). The verbal number word, dot array and the characters were the larger quality an equal number of times, thus the task was counterbalanced (see Table 12). Half way through the trials the characters were swapped, for instance, the frog starts on the left for the first 9 trials and then moves to the right for the final 9 trials. All the dots on the cards were identical (i.e. 0.8cm in diameter). For each correct response children were awarded one point, giving a maximum score of 18. A proportion score was calculated for each participant.

6.2.3.8 Domain general.

6.2.3.8.1 Verbal-animal recall task.

Working memory was measured using an adapted version of an animal recall task (McCormack, Simms, McGourty & Beckers, 2013). A total of 22 animals were used as stimuli, all animals were recognisable by children aged 3-5 years-old. The researcher sat beside the child placing the task folder in front of the child and introduces the task by reading out the instructions. Children initially received a series of practice trials that were repeated if necessary. The practice trials were designed to ensure that children understood the task instructions. Specifically, that the participants knew that they needed to name the colours of the smiley faces that appeared on the cards in between animal pictures and then recall the animals in the correct order once the researcher asks, “What animal(s) did you see?”. The test trials consisted of four sets of one-animal trials, with subsequent levels involving four sets of two-animal, three-animal, four-animal trials and five-animal trials, thus five levels of increasing difficulty were used (see Table 13 for the order in which the animals were presented). If all the animals in one of the four trials in a level were recalled, even in the incorrect order, children moved to the next level. The task was terminated when the child failed to recall all the animals in any of the four trials at a level. No animal was repeated within a level and no animal was repeated more than 3 times throughout the whole task. For each correct response, which was classified as an animal recalled in the correct position, children were awarded one point and the accuracy score was used in the analysis, the maximum score was 60.

Table 13. *Showing the order, the animals were presented*

Trial number	1^{st} animal presented	2^{nd} animal presented	3^{rd} animal presented	4^{th} animal presented	5^{th} animal presented
Practice					
1	Elephant				
2	Sheep				
3	Fish	Rabbit			
4	Lion	Dog			
Trials					
1	Tiger				
2	Cat				
3	Crocodile				
4	Penguin				
5	Mouse	Bear			
6	Bird	Spider			
7	Zebra	Horse			
8	Hippo	Pig			
9	Cow	Frog	Monkey		
10	Sheep	Snake	Rabbit		
11	Lion	Fish	Dog		
12	Horse	Tiger	Mouse		
13	Elephant	Cat	Crocodile	Hippo	
14	Bear	Penguin	Spider	Horse	
15	Snake	Pig	Frog	Zebra	
16	Monkey	Bird	Fish	Lion	
17	Cow	Hippo	Zebra	Penguin	Mouse
18	Cat	Rabbit	Elephant	Monkey	Sheep
19	Pig	Crocodile	Fish	Spider	Bear
20	Frog	Horse	Bird	Dog	Tiger

6.2.3.8.2 Auditory sustained attention.

Familiarisation phase.

The participants were seated in front of a laptop computer (MacBook Air 13-inch) and heard both a dog barking and cat meowing at different intervals. Participants were required to press the spacebar button for the target sound (dog barking) and to inhibit their response by abstaining from pushing the button for the non-target (cat meowing). The test procedure commenced with a familiarisation phase whereby the researcher instructed and explained the protocol to the participant. The researcher sat beside the child and had the task open in PsychoPy2 with the 'Start' screen shown and introduced the task by saying "We are going to play a game on the laptop. I want you to listen very carefully for the sound. There will be two sounds. The first is a cat meowing. The cat meow sounds like this". The researcher clicked a wireless optical mouse (Logitech M185) that triggered the cat meowing and then stated, "OK? Whenever you hear the cat meow, don't do anything. Let it go by. Ok listen". The researcher clicked the wireless mouse triggering the cat meow and continued, "The second sound is a dog barking, like this". The researcher clicked the wireless mouse triggering the dog bark and continued, "Place your hands on two green pads on the laptop (researcher pointed to the two green pads on laptop). Whenever you hear the dog bark, hit this black button (researcher pointed to black button - the middle of the space bar) as fast as you can, then place your hands back on the green pads. Here, you try (researcher encourages child to press the space bar with two fingers). "Are you ready? Try this". The researcher clicked the wireless mouse triggering the dog bark and said, "Now hit it" (researcher pointed to black button with two fingers). At this point the child should press the black spacebar and the researcher stated, "Good! Now put your hands back on the green pads". After the child pressed the black spacebar a new screen with the word 'Again?' appeared. If the child behaviour indicated that they lost interest or were distracted the researcher stated, "I'd like you to keep playing" which was repeated only once. If black spacebar was pressed multiple times the researcher stated, "Only hit the button once". If the child did not respond, the familiarisation instructions were repeated once. If familiarisation instructions were not repeated the researcher clicked to the 'Practice' screen.

Practice phase.

The familiarisation phase was followed by a practice phase, which aimed to confirm that the participant comprehended the task instructions and was capable of fulfilling its demands. The child had to successfully complete two practice trials (one including the target and the other the non-target) to continue onto the test phase. The 'Practice' screen was shown, and the researcher introduced the practice trial by saying, "Now you get to practice this game on your own. Remember, place your hands on the green pads on the laptop. Listen for the dog bark. Hit the black button only when you hear the dog bark. Don't do anything for the cat meowing. If you make a mistake, keep going. Ready? Go." The researcher clicked the wireless mouse triggering the start of the practice trial. The child was presented four stimuli; two targets and two non-targets. If the child responded correctly to the target stimuli, the examiner said, "That was great", and the test phase was given. If the child did not respond correctly, the practice phase was repeated. If the child was unable to complete the initial practice trials, they were granted two additional attempts to successfully complete the practice phase. It is important to note, however, that all participants were able to successfully complete the practice trials and no participant was excluded based on this criterion.

Test phase.

The practice phase was followed by a test phase. The 'Trial' screen was shown (the word *trial* was used instead of *test* so to not cause any anxiety for the child), and the researcher introduced the test phase by saying, "Now we get to play the whole game. Remember, place your hands on the green pads on the laptop. Listen for the dog bark. Whenever you hear the dog bark, hit the black button as fast as you can. Ready? Go." The researcher clicked the wireless mouse triggering the start of the test phase. The researcher remained seated beside the child. The presentation of the auditory stimuli was randomised but divided into four sections so that every 11 trials a small white box appeared in the right-hand side for 10 ms. If the child looked away from the laptop and attended to the examiner or placed their hands elsewhere, they were encouraged as follows, "Hands up here, listen for the dog". This encouragement was given every 11 trials, but only if necessary. This encouragement was only included at T1 and did not significantly impact on the total time spent administering this task. The specific ratio of targets to non-targets during the first and third set were 6 cats to 5 dogs and the specific ratio of targets to non-targets during the second and fourth set were 5 cats to 6 dogs which were randomised in presentation order. The accuracy scores, total omissions (failing to respond to the target), total commissions (false alarms) and response rate were calculated. The first three scores mentioned had a maximum score of 22 and the response rate ranged from 0-4 seconds. The scoring technique used in the analysis is described in the results section.

Further information on the auditory sustained attention task.

An experimental measure of auditory sustained attention was presented on a MacBook Air (13-inch) using PsychoPy2 (version 1.84.2). The Auditory Continuous Performance Test-Preschool (ACPT-P) developed by Mahone, Pillion and Hiemenz (2001) provided the basis for the development of an adapted computerised Continuous Performance Test (CPT) used in this study. In contrast to the original version that used a dog bark and a church bell, the current auditory task incorporated the sounds of a dog (target) and cat (non-target) much like Guy, Rogers and Cornish (2013). The auditory task incorporated a similar design and administration. The auditory task included 44 trials, each consisting of 22 non-target or distractors and 22 targets.

In the auditory task, two familiar environmental sounds (dog barking and cat meowing) were edited to 690 ms each, with an interval of 3000 ms and an additional 1000 ms blank screen transition to next stimulus, in line with Guy, Rogers and Cornish (2013) stimuli. Each child had 3690 ms response time. However, after a piloting this task the timings were changed to give the children a longer time to respond. The stimuli were presented for 1 second, the interval for 4 seconds and blank screen for 1 second. This presentation set-up therefore allowed each participant a 4 second window to respond.

6.2.4 Ethical considerations.

The one-to-one sessions took place in an environment that was familiar to the child. Children completed the game-like tasks on a one-to-one basis with researchers who had considerable experience in assessing young children. The audio files from the digital voice recorder (i.e. SFON task) were transferred to the researcher's private laptop. The audio files were stored in a password-protected folder on a laptop. Once transcribed, hard copies of the interview transcripts were kept in the same location as the completed questionnaires in a locked cabinet.

6.2.5 Data Analyses.

All data collected was entered and verified using SPSS Version 23 by the researcher and then 10% of the data was re-entered by an additional researcher. The additional researcher was trained before inputting the data and data was verified to get a 100% match in cases, which means there were no data entry mistakes; increasing the validity of the data. Descriptive statistics of demographic characteristics were calculated using SPSS, these will be discussed in detail. A confirmatory factor analysis, latent profile analyses and latent transition analyses were all completed in Mplus Version 1.5 (Muthén & Muthén, 1998-2017).

6.2.5.1 Confirmatory factor analysis.

6.2.5.1.2 Assessment of the factor structure of the frequency of numeracy activities.

A confirmatory factor analysis (CFA) was necessary to assess the factor structure of the frequency of numeracy activities scale in the PHMQ. Mplus Version 1.5 (Muthén & Muthén, 1998-2017) was used to explore the factor structure instead of SPSS as Mplus allows the researcher to place each item in the factor suggested by the exploratory factor analysis to test if the model fits. A CFA was utilised on the five subscales identified in Chapter 4 through the exploratory factor analysis (i.e. the parent-child interaction, computer maths games, TV programmes, shape and counting)

The evaluation of model fit for the CFA is based on goodness-of-fit indices (Geiser, 2012). The chi-square value, in combination with several fit indices were assessed to make a joint evaluation of the model (Wiggins, Netuveli, Hyde, Higgs & Blane, 2008; Geiser, 2012; Dimitrov, 2010). The chi-square value provides evidence for model fit when the value is not significant. However, the chi-square value has previously been reported to yield a false tendency to reject the model fit with large samples and a false tendency to support the model fit occurs with small samples (Dimitrov, 2010). Thus, Hu and Bentler (1999) recommended a joint evaluation of several fit indices to assess model fit (Dimitrov, 2010). The fit indices are the comparative fit index (CFI; Bentler, 1990), Tucker–Lewis Index (TLI; Tucker & Lewis, 1973), root mean square error of approximation (RMSEA, Steiger, 1990) and standardised root mean square residual (SRMR). In addition to three information criteria; Akaike's information criterion (AIC; Akaike, 1987), the Bayesian information criterion (BIC; Schwartz, 1978) and the sample size - adjusted BIC.

6.2.5.2 Latent transition analysis measurement model (step one).

In the current study, the latent analyses were completed with Mplus Version 1.5 (Muthén & Muthén, 1998-2017) utilising the maximum likelihood robust (MLR) estimator. This MLR estimator offers standard errors, robust to violations of the assumption of normality (Fryer, 2017). These frequently arise when utilising ordinal measures (Fryer, 2017), which are used in the current study. Prior to conducting the Latent Transition Analysis (LTA), a series of Latent Profile Analysis (LPA) were conducted (Lazarfeld & Henry, 1968; Muthén, 2001). Thus, two types of models will be discussed: (1) LPA a term used to describe a model with continuous cluster indicators (Fryer, 2017), as opposed to Latent Class Analysis (LCA) which is typically used for categorical indicators. Both these types of models are used to trace back the heterogeneity in a group to a number of underlying homogeneous subgroups at a specific measurement point (Hickendorff et al., 2017). Thus, the profiles that are formed obtain as much *similarity within* a profile while at the same time as much *difference between* the profiles as possible (Lanza & Cooper, 2016). The (2) LTA is the longitudinal extension of these models where the transitional component reflects changes in learners' profile membership over time, demonstrating potential non-linear learning pathways (Hickendorff et al., 2017). LTA will estimate where children start (T1, Spring term of pre-school) giving their initial group profile and then provide the same information after the children's transition from pre-school to primary school education (T3, Autumn term of Primary one). LTA plots the transitions of children between these profiles by providing probability estimates of both profile memberships and pathway transitions (Fryer, 2017). Therefore, through LTA the researcher will be able to describe children's precise *learner profiles* and *learning pathways* during the transition from pre-school to school education (Fryer, 2017). The indicators in the current study are continuous variables therefore a Latent Profile Transition Analysis (LPTA) will be used, also referred to as LTA is this study.

The reason for conducting preliminary LPA tests is due to the limited amount of literature that identifies learner profiles and learning pathways in this age group. Thus, there are no existing profiles reported in previous literature that could be tested within the present data set. Further, using a simpler cross-sectional model (i.e. LPA) than the more complex longitudinal (i.e. LTA or LPTA) allows the researcher to investigate each time point separately. LPA confirms the extent that the extracted latent profiles can be replicated at each cross-sectional time point (Kam, Morin, Meyer & Topolnytsky, 2016). Also, confirming that those LPA profiles extracted would converge with profiles extracted using full information maximum likelihood (FIML; used to handle missing data) on the full longitudinal data set (Kam et al., 2016). A challenge with LPA and LTA models is determining the number of profiles in the data. The criteria for making the decision on how many profiles there are depends on the theoretical and practical meaning to the extracted profiles (Marsh, Lüdtke, Trautwein & Morin, 2009; Muthén, 2003) and the statistical adequacy of the solution.

In the current study, a number of statistical tests and indices were used to help in the decision process of how many profiles should be extracted (McLachlan & Peel, 2000; Kam et al., 2016; Fryer, 2017). Seven fit indices were employed: three likelihood ratio tests and four information criterion indices. The three likelihood ratio tests, Vuong-Lo-Mendell-Rubin likelihood ratio test (Vuong, 1989), Lo-Mendell-Rubin likelihood ratio test (Lo, Mendell & Rubin, 2001) and Bootstrap Likelihood Ratio Test (McLachlan & Peel, 2000), all provide an assessment of whether there is a statistically significant improvement for one more profile being added to the model (Fryer et al., 2017; Nylund, Asparouhov & Muthén, 2007). Therefore, these likelihood ratios are not available for model one (models with only one profile). The three information criterion indices are selection criterion were lower values reveal the preferred model; Akaike's information criterion (AIC; Akaike, 1987), the Bayesian information criterion (BIC; Schwartz, 1978) and the sample size - adjusted BIC. As a post hoc evaluation of group separation, an entropy criterion should be investigated. Entropy criterion is a summary measure for the quality of the classification in the model (Geiser, 2012). Entropy values closer to 1 indicate good classification accuracy of a population into subgroups (Geiser, 2012; Fryer, 2017). However, all four information criteria have their disadvantages, nevertheless the BIC is usually the most useful selection criterion guide for latent models (Fryer, 2017; Kam et al., 2016; Nylund-Gibson, Grimm, Quirk & Furlong, 2014).

To avoid the problem of local maxima (Geiser, 2012, pg. 240; Uebersax, 2000), or chance selection of a suboptimal solution due to inaccurate parameter estimates, each latent profile model was conducted with 2000 random sets of start values to ensure that the best loglikelihood value was adequately replicated; a method similarly used by Kam et al. (2016). The default iterations were increased to 100 random starts and the 100 best solutions for final stage optimisation were retained (Hipp & Bauer, 2006; McLachlan & Peel, 2000). The use of an adequate number of random sets of starting values increases the chances that a latent model will find the optimal solution with the highest log likelihood value and guarantees that the true maximum will be found (Geiser, 2012). For the latent transition models, the number of random starts was increased to 5000 so that the best loglikelihood value was reliably replicated, as with a lower random start value convergence may not be likely, in particular with the larger number of model profiles (Kam et al., 2016).

The LPA and LTA had four indicators of mathematics specific skills (cardinal principle knowledge, digit recognition, numerical ordering and mapping magnitude representations) based on each separate time point and using the 128 to 118 participants who completed T1 (N= 128), T2 (N= 128) and T3 (N= 118) measures. To deal with the missing data (e.g. 10 participants dropping out at T3) full information maximum likelihood (FIML) will be used instead of imputing missing data. The model is estimated by FIML and thus all available information is used to estimate the model. When using a LCA, Wurpts and Geiser (2014) stated that conditions of N = 70 were not feasible under virtually any condition, therefore the sample (N=128 to 118) was deemed appropriate for analysis. Latent profile indicators do not need to be on the same metric (e.g. not z-scored or mean centred), as LPTA compares means of the same variables across classes (see Seltzer, Frank & Bryk, 1994, for detail on metrics).

This study considers a variety of demographic characteristics (i.e. gender, age, SES and HEQ), as well as predictors associated with multiple components of the home numeracy environment measured with parents (i.e. the frequency of numeracy activities scale and number games checklist), and child measures such as domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary). Firstly, a multinomial logistic regression was completed to understand the bivariate associations between pathways membership and the predictor variables individually. By considering the bivariate associations first the overall relationships are assessed between latent pathways and predictors without collinearity concerns and the differences between unadjusted and adjusted estimates are demonstrated (the unadjusted odds ratio is obtained by only studying the effect of one predictor variable, whereas when more than one predictor is considered an adjusted odds ratio is created which takes into account the effect due to all the additional variables included in the analysis). Then the adjusted associations are considered through a multivariate multinomial logistic regression analysis to understand the potential predictors of pathway membership. Blocks of predictors were entered into the model in a forward stepwise manner. This was a statistically driven model that explores unique predictors. Thus, predictors were selected for this final adjusted model if $p < 0.05$ for any association between pathway membership and the given predictor in the bivariate analyses. This will be discussed further in the results (see section 6.4.5.5.2 Adjusted associations between pathway membership and predictors).

6.3 Results

Overview.

In the following subsections, the subsequent analysis will be discussed (a) the demographic composition of participants, (b) how the score for each child measure was utilised for both domain general and domain specific skills, (c) the confirmatory factor analysis for the frequency of numeracy activities section from the PHMQ, (d) latent profile analysis and discussion on how the number of latent profiles was chosen, (e) latent transition analysis breakdown and multinomial logistic regression to understand the associations between pathways membership and the predictor variables.

6.3.1 Demographic composition of participants.

Due to the constraints of the LTA, only the demographics of those parents and children with data (n = 128) and the sex-age distribution of children across the three time points as 128 children at T1 and T2 and 118 at T3 will be discussed.

6.3.1.1 Sex-age distribution for parents.

Table 14 demonstrates that participants were 10 males (M_{age} = 36.3, SD = 6.6) and 98 females (M_{age} = 34.9, SD = 5.8). The overall mean age of the participants was 35 years-old (SD = 5.9, Range 21-46 years).

Table 14. *Descriptive statistics of age for participant*

		Mean	Std. Deviation	Minimum	Maximum
Age of parent (N = 128)	Age (yr.)	35.02	5.87	21	46
Sex – Age	Male (N = 10)	36.30	6.62	21	46
	Female (N = 98)	34.89	5.81	22	46

6.3.1.2 Sex-age distribution for children.

Table 15 shows the sex-age distribution of the children across the three time points. 128 children completed the tasks at T1 with a mean age of 4 years (SD = 3.3 months), and 70 (54%) were female. At T3 118 children completed the tasks with a mean age of 4 years 7 months (SD = 3.2 months), and 65 (55%) were female.

Table 15. *Descriptive statistics for discontinued children at each time point*

		Mean	Std. Deviation	Minimum	Maximum
T1 Age of children (N = 128)	Age (mo.)	48.56	3.34	43	54
Sex – Age	Male (N = 58)	48.75	3.58	43	54
	Female (N = 70)	48.40	3.13	43	54
T2 Age of children (N = 128)	Age (mo.)	51.78	3.38	46	58
Sex – Age	Male (N = 58)	51.91	3.64	46	58
	Female (N = 70)	51.67	3.17	46	58
T3 Age of children (N = 118)	Age (mo.)	56.55	3.24	51	63
Sex – Age	Male (N = 53)	56.68	3.47	51	63
	Female (N = 65)	56.45	3.07	51	62

6.3.1.3 Socio-economic classification.

Table 16 summarises parent job categories. The job categories used to derive SES from the National Statistics Socio-economic Classification (NS-SEC, 2010) were 14 functional and three residual operational categories. The functional categories represent a range of specific employment statuses and labour market positions, whereas the residual categories can be grouped together as ‘not classified’ jobs (e.g. full-time students, occupations not stated or inadequately described or not classifiable for other reasons) (NS-SEC, 2010).

Most participants had jobs in the *lower professional and higher technical occupations* category (18%). These categories include jobs such as, a nurse, mortgage specialist and primary school teacher. This was followed by the *intermediate occupations* category (15.6%) that includes jobs such as clerical officer, civil servant and sales assistant. Next was the *higher professional occupations* category (14.1%) that includes jobs such as architects, medical practitioners, higher education teaching professionals and programmers and software development professionals.

The categories *higher managerial and administrative occupations* (0.8%), *employers in small organisations* (1.6%), *higher supervisory occupations* (2.3%) and *own account workers* (3.9%) were all categories with the least number of participants, these included jobs such as production manager in mining and energy, estate manager, nursery assistant and hairdresser respectively. The data illustrates that participants were from a wide range of SES backgrounds.

Table 16. *Socio-economic classification*

No.	Operational categories	Participants *n* (%) (n = 128)
1	Employers in large establishments	0 (0)
2	Higher managerial and administrative occupations	1 (0.8)
3	Higher professional occupations	18 (14.1)
4	Lower professional and higher technical occupations	23 (18.0)
5	Lower managerial and administrative occupations	5 (3.9)
6	Higher supervisory occupations	3 (2.3)
7	Intermediate occupations	20 (15.6)
8	Employers in small organisations	2 (1.6)
9	Own account workers	5 (3.9)
10	Lower supervisory occupations	11 (8.6)
11	Lower technical occupations	0 (0)
12	Semi-routine occupations	10 (7.8)
13	Routine occupations	9 (7.0)
14	Never worked and long-term unemployed	5 (3.9)
15	Full-time students	1 (0.8)
16	Occupations not stated or inadequately described	1 (0.8)
17	Not classifiable for other reasons	0 (0)
#	Missing	14 (10.9)

None of the 17 operational categories from the NS-SEC (2010) are regarded as ordinal scales (e.g. high, middle or low SES), therefore for further analysis, SES data was converted into the three-class version described in NS-SEC (2010), this can be assumed to involve a form of hierarchy. In Table 16 the operational categories 1 to 6 fall into the *higher managerial, administrative and professional occupation* (n = 50, 39.1%), categories 7 to 9 fall into *intermediate occupations* (n = 27, 21.1%) and categories 10 to 13 fall into *routine and manual occupations* (n = 30, 23.4%). These three categories are referred to as high, middle and low SES, respectively, in further analyses. Overall, the majority of the participants are in the high SES category. Categories 14 to 17 are known as an unemployed category and are marked as missing in future analyses as suggested in the NS-SEC (2010).

6.3.1.4 Education levels.

Table 17 summarises parents' highest educational and mathematical qualifications. There was a wide spread in the highest educational level achieved by the participants in this study with a large proportion of participants reaching *degree* level (25.8%). According to the Office for National Statistics (ONS, 2017) in July to September 2017, 42% of the UK population aged 21 to 64 had achieved higher education qualifications, this included higher degrees such as Level 6 (bachelor's degree), Level 7 (master's degree) and Level 8 award (PhD or doctorate). In this study 45.4% of participants have achieved higher education qualifications, slightly higher than the UK average.

Table 17. *Educational attainment*

	Education level	Participants *n* (%) (n = 128)
Highest Educational Qualification	GCSEs / O level / Irish Junior Certificate	25 (19.5)
	A levels / BTEC / Irish Leaving Certificate	18 (14.1)
	Degree	33 (25.8)
	Masters	18 (14.1)
	PhD	7 (5.5)
	No qualifications	9 (7.0)
	Other	3 (2.3)
	Missing	15 (11.7)
Highest Mathematical Qualification	GCSEs / O level / Irish Junior Certificate	73 (57.0)
	A levels / BTEC / Irish Leaving Certificate	12 (9.4)
	Degree	6 (4.7)
	Masters	5 (3.9)
	PhD	5 (3.9)
	No qualification	10 (7.8)
	Other	2 (1.6)
	Missing	15 (11.7)

The second highest educational level achieved by the participants in this study was *GCSEs / O level / Irish Junior Certificate* level (19.5%). Next was both *A levels / BTEC / Irish Leaving Certificate* level and *masters* level (14.1%) and 7% of participants had *no qualifications*. These are similar the statistics published by the Office for National Statistics (ONS, 2017) which stated that 20% of the UK population aged 21 to 64 had achieved GCSE qualifications (equivalent to an A* to C), 21% had qualifications equivalent to an A level and 8% of the UK population had no qualifications. Most participants (57%) had a *GCSEs / O level / Irish Junior Certificate* in mathematics. The second highest mathematical qualification level achieved by the participants was both *A levels / BTEC / Irish Leaving Certificate* level and participants having no mathematical qualification (9.4%). Other participants had *degrees* (4.7%), *Masters* (3.9%) and *PhDs* (3.9%) in mathematics.

6.3.1.5 Further demographics.

Table 18 illustrates the marital status, ethnicity and first language spoken by the participants. The majority of participants were *married* (50.8%). Other common marital statuses were *single* (16.4%) and *cohabiting* (15.6%). The majority of participants were *White/Caucasian* (83.6%). Other ethnic backgrounds were *Asian* (2.3%), *Chinese* (1.6%) and *Black or African American* (0.8%). The first language spoken by participants was mostly *English* (82%). Other first languages spoken were *Chinese* (1.6%), *French*, *Hindi*, *Polish*, *Tamil*, *Bengali* and *Greek*, all making up 0.8% of the sample.

Table 18*. Descriptive statistics and frequencies of the participants*

		Participants *n* (%) (n = 128)
Marital Status	Single	21 (16.4)
	Married	65 (50.8)
	Cohabitating	20 (15.6)
	Divorced	2 (1.6)
	Separated	5 (3.9)
	Civil partnership	1 (0.8)
	Missing	14 (10.9)
Ethnicity	White/Caucasian	107 (83.6)
	Asian	3 (2.3)
	Chinese	2 (1.6)
	Black or African American	1 (0.8)
	Missing	14 (10.9)
First language spoken	English	105 (82.0)
	Chinese	2 (1.6)
	French	1 (0.8)
	Hindi	1 (0.8)
	Polish	1 (0.8)
	Tamil	1 (0.8)
	Bengali	1 (0.8)
	Greek	1 (0.8)
	Missing	15 (11.7)

6.3.1.6 Other adults in household.

Of the 128 parents 94 (73.4%) lived with some else in the household. Table 19 shows the relationship of the other adult living in the same household as the target child, the majority being fathers (55.5%). Table 20 displays the highest educational qualifications of the additional adult living in the child's household. There was a wide spread in the highest education qualifications attained by the sample with only 3.1% of other cohabitants having *no qualifications*. The majority of participants highest educational qualification was *GCSEs / O level / Irish Junior Certificate* (23.4%).

Table 19. *Other adult living at home relationship to target child*

	Participants *n* (%)
Mother	12 (9.4)
Father	71 (55.5)
Stepfather	4 (3.1)
Grandparent	4 (3.1)
Brother of parent	1 (0.8)

Table 20. *Educational achievement of other adult living in household*

	Education level	Participants *n* (%)
Highest Educational Qualification	GCSEs / O level / Irish Junior Certificate	30 (23.4)
	A levels / BTEC / Irish Leaving Certificate	11 (8.6)
	Degree	22 (17.2)
	Masters	15 (11.7)
	PhD	9 (7.0)
	No qualifications	4 (3.1)

6.3.2 Scoring for each child measure

6.3.2.1 Outcome and receptive vocabulary measure scores.

The British Ability Scale (BASII) Early Number Concepts, and receptive vocabulary variable, British Picture Vocabulary Scale (BPVS), were used as raw scores as opposed to age-equivalent scores or standardised scores as age will be used as a predictor variable in the latent transition analysis. Table 21 demonstrates the descriptive statistics (mean, standard deviation and significant change over time) for each measure across the three-time points. See Appendix 6.1. Table 3 for a Pearson zero-order correlation between the all longitudinal variables used with the children.

Table 21. *Descriptive statistics of each measure across time*

Measure	Scoring type		Mean	Std. Deviation	Sig. change	No. of children at floor	No. of children at ceiling
British Ability Scale (BASII) – Early Number Concepts	Raw score	Time 1*	16.51	5.79	<.01	0	0
		Time 2*	-	-	-	-	-
		Time 3**	21.90	4.67		0	0
British Picture Vocabulary Scale (BPVS)	Raw score	Time 1	57.29	16.92	<.01	0	0
		Time 2	-	-	-	-	-
		Time 3	69.58	14.22		0	0
Give-N	Proportion score	Time 1	.49	.35	<.01	20	11
		Time 2	.56	.35		24	14
		Time 3	.69	.26		4	12
Digit recognition	Proportion score	Time 1	.41	.31	<.01	19	8
		Time 2	.47	.31		11	13
		Time 3	.60	.31		1	24
Numeral ordering	Proportion score	Time 1	.34	.30	<.01	26	6
		Time 2	.40	.35		27	15
		Time 3	.60	.35		9	33
Spontaneous Focus on Numbers (SFON) – Picture task	Accuracy score	Time 1	.48	.87	<.05	90	8
		Time 2	.56	.86		81	6
		Time 3	.74	1.01		66	13
Cross-notation comparison	Proportion score	Time 1	.64	.21	<.01	1	7
		Time 2	.74	.21		1	23
		Time 3	.83	.17		0	35
Auditory Continuous Performance Test – Pre-school	Inverse efficiently score	Time 1	3.29	5.79	<.01	0	30
		Time 2	2.31	2.90		0	35
		Time 3	2.27	6.51		0	38
Verbal animal recall	Accuracy score	Time 1	4.04	2.91	<.01	16	0
		Time 2	5.28	3.49		4	0
		Time 3	6.21	3.41		6	0

Note: * N = 128 ** N = 118

6.3.2.2 Domain specific skills scores.

Proportion scores were calculated for the four domain specific skills, i.e. cardinal principle knowledge, digit recognition, numerical ordering and mapping magnitude representations, as proportion scores offer unidimensional scores and demonstrate convergent validity (Barchard & Russell, 2006; MacCann, Roberts, Matthews & Zeidner, 2004). For both the numerical ordering and cross-notation comparison tasks each child was given 12 trials, if a child scored more than 6 in the first 12 trials that child was administered a total of 18 trials. Three different proportion scores were calculated for both tasks; the proportion correct out of the maximum number of trials (out of 18), the proportion correct out of the number of trials the child actually did (either 12 or 18) and the proportion correct for the trials that all participants completed (out of 12). A correlation was carried out to discover which proportion score would be the most appropriate measure to use. Tables 22 and 23 demonstrate that all correlations between the three metrics mentioned above were over $r = 0.9$ for both the numerical ordering and cross-notation comparison tasks at every time point thus, there should be no difference to the overall outcomes of the analysis dependent on what metric was chosen. However, the proportion scores for the trials that all participants completed (out of 12) was deemed a more reasonable approach to marking this task than to assume that the children who were not administered the final 6 trials in both tasks due to cut off rules would have got all of the subsequent problems wrong. Therefore, the proportion score out of 12 was the chosen metric.

Table 22. *Pearson zero-order correlations between proportion score for the numerical ordering task*

	1. T1 numerical ordering proportion correct out of 18	2	3	4	5	6	7	8	9
2. T1 numerical ordering proportion correct out of 12 and 18	.983**								
3. T1 numerical ordering proportion correct out of 12	.975**	.987**							
4. T2 numerical ordering proportion correct out of 18	-	-	-						
5. T2 numerical ordering proportion correct out of 12 and 18	-	-	-	.989**					
6. T2 numerical ordering proportion correct out of 12	-	-	-	.983**	.992**				
7. T3 numerical ordering proportion correct out of 18	-	-	-	-	-	-			
8. T3 numerical ordering proportion correct out of 12 and 18	-	-	-	-	-	-	.995**		
9. T3 numerical ordering proportion correct out of 18	-	-	-	-	-	-	.986**	.988**	-

Note: * p < .05 ** p < .01 (two-tailed).

Table 23. *Pearson zero-order correlations between proportion score for the cross-notation comparison task*

	1. T1 Cross-notation comparison proportion correct out of 18	2	3	4	5	6	7	8	9
2. T1 Cross-notation comparison proportion correct out of 12 and 18	.973**								
3. T1 Cross-notation comparison proportion correct out of 12	.933**	.925**							
4. T2 Cross-notation comparison proportion correct out of 18	-	-	-						
5. T2 Cross-notation comparison proportion correct out of 12 and 18	-	-	-	.980**					
6. T2 Cross-notation comparison proportion correct out of 12	-	-	-	.948**	.952**				
7. T3 Cross-notation comparison proportion correct out of 18	-	-	-	-	-	-			
8. T3 Cross-notation comparison proportion correct out of 12 and 18	-	-	-	-	-	-	.983**		
9. T3 Cross-notation comparison proportion correct out of 18	-	-	-	-	-	-	.938**	.932**	-

Note: * p < .05 ** p < .01 (two-tailed).

Preliminary statistics revealed that SFON accuracy scores had a lack of variability. Children received a score of 0 or 1 contingent on whether they spontaneously focused on numerosity or not for each of the three SFON trials, thus the maximum score for this task was 3 (Batchelor et al., 2015). This is the same scoring format to that was used in Batchelor et al. (2015). In previous research (Batchelor et al., 2015) the task was used with children aged 4.5 to 5.6 years (n = 130), achieving a mean score of 1.16 for the SFON task. The current study involves children aged 4 to 4.7 years, slightly younger than that of Batchelor et al. (2015) therefore a mean of .48 (T1) to .74 (T3; see Table 21) over time could be the result of the task being too advanced for this current age group. Due to the floor effect, the SFON task was not included in any further analysis.

6.3.2.2.1 Combination score for domain specific skills.

An exploratory factor analysis was used on the four domain specific skills, cardinal principle knowledge, digit recognition, numerical ordering and mapping magnitude representations, as a factor analysis decreases a set of observed variables into a smaller set of observed variables (Hinkin et al., 1998) to create combined scores to be used in the latent transition analysis. The four measures were analysed in a principle components analysis with oblique rotation (direct oblimin) to determine whether the measures grouped together. The suitability of data for factor analysis was assessed. The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis as good (KMO = 0.75) (Hutcheson & Sofroniou, 1999). The one factor model had an eigenvalue over Kaiser’s criterion of 1 (eigenvalue = 2.54) explained 63.58% of the common variance. The factor was labelled, *combination score for mathematics skills*. Cronbach's alpha for the total measure was 0.81, thus displaying good internal reliability.

6.3.2.3 Domain general skills scores.

6.3.2.3.1 Verbal-animal recall task – working memory.

For the verbal-animal recall task the accuracy score was used in further analysis. The total accuracy score was used instead of a proportion correct score as there was a possibility of scoring a total of 60 due to the different levels within the task. However, no child scored above 20 at all three-time points meaning the if a proportion score was used the scores would be heavily skewed.

6.3.2.3.2 Auditory Continuous Performance Test – Sustained attention.

Townsend and Ashby (1978) proposed the inverse efficiency score (IES) to deal with the issue of how to combine speed and error. IES is calculated by taking the participants average correct response time (RT) of the condition (i.e. the average response time for the correct trials) divided by 1 take away the proportion of errors (PE), for example IES = RT / (1 – PE) (Bruyer & Brysbaert, 2011; Vandierendonck, 2017). Townsend and Ashby (1978, 1983) warned that the IES only works when there is a positive correlation between RT and PE (Bruyer & Brysbaert, 2011) therefore, this correlation in this data set was examined. The correlation was tested at T1 between RT and PE and was 0.36. Therefore, for the auditory sustained attention task this scoring process was used.

A factor analysis was used with the two domain general skills, verbal working memory and sustained attention, to understand if these two variables could be combined to generate a general cognitive score. However, the factor analysis suggested that they were separable constructs and further analyses utilised scores for these measures separately.

6.3.3 Confirmatory factor analysis for the frequency of numeracy activities.

In the overview sections of this Chapter (section 6.3 Results, Overview) five subsections of the results were outlined. The first two subsections (a) demographics and (b) scoring have already been discussed in detail. The third subsection will now be considered: (c) the confirmatory factor analysis for the frequency of numeracy activities section from the PHMQ.

To recap (section 6.2.5.1 Confirmatory factor analysis, 6.2.5.1.2 Assessment of the factor structure of the frequency of numeracy activities) a confirmatory factor analysis (CFA) was necessary to confirm the factor structure of the frequency of numeracy activities scale in the PHMQ. The researcher used Mplus Version 1.5 (Muthén & Muthén, 1998-2017) to explore the factor structure instead of SPSS as Mplus allows the researcher to place each item in the factor suggested by the exploratory factor analysis to test if the model fits. A CFA was utilised on the five subscales found in Chapter 4 through the exploratory factor analysis (i.e. the parent-child interaction, computer maths games, TV programmes, shape and counting).

6.3.3.1 Five-factor model.

In Mplus a CFA with robust maximum likelihood (MLR) was used because this has been widely used in CFA models when continuous observed variables slightly or moderately deviate from the normality and it is superior to Maximum Likelihood (ML; Li, 2016). Model one (Figure 7) proposes the five-factor model corresponding to the five subscales found in the previous chapter through the exploratory factor analysis (parent-child interaction, computer maths games, TV programmes, shape and counting).

6.3.3.1.1 Model fit

The selection of the most appropriate model was based upon goodness of fit statistics (see Table 24). The model had acceptable model fit indices reporting a CFI of 0.83 and a TLI of 0.81. Good fitting models are indicated by a CFI of > 0.95 (better model: > 0.97) and the same cut-off value for TLI applies (Geiser, 2012). A CFI > 0.90 is often regarded as an indicator of an adequate model fit (Hair et al., 2010; Coroiu et al., 2018; Awang, 2012) the same cut-off value for TLI applies (Forza & Filippini, 1998; Coroiu et al., 2018; Awang, 2012).

The CFI and the TLI are incremental fit indices that compare the fit of the target model to the fit of a baseline model (Geiser, 2012). In Mplus the baseline model, also known as the null independence model, assumes that the population covariance matrix of the observed variables is a diagonal matrix, in other words, it is assumed that there is no relationship between any of the variables (Geiser, 2012). As a consequence, it is possible that the null model is "too good", meaning that the average level of correlations in the current data is rather low. In this case, Kenny (2012) argued that CFI should not be computed if the RMSEA of the null model is less than 0.158 as the CFI obtained will be too small a value (Kenny & McCoach, 2003; Beldhuis, 2012). When investigating the RMSEA values the model demonstrated acceptable RMSEA values (< 0.08) (Awang, 2012), the RMSEA value was 0.07. Therefore, the five-factor model is a reasonable model.

The SRMR coefficient is a standardised measure for the evaluation of the model residuals, however SRMR is somewhat biased by sample size. Marsh, Hau and Wen (2004) state that the SRMR values for solutions based on small sample sizes are unacceptable (greater than 0.08), whereas those based on large sample sizes are acceptable. A value < 0.08 is generally considered a good fit (Hu & Bentler, 1999). Therefore, taking into consideration all fit criteria for assessing goodness of fit the model presents acceptable fit indices (CFI = 0.83, TLI = 0.81, RMSEA = 0.07, SRMR = 0.072), thus it seems reasonable that a five-factor model be deemed a suitable measurement model.

6.3.3.1.2 The additional 'Maths related YouTube videos (e.g. NumTums)' item.

As discussed previously (section 5.6.1.1 Stage 5: Content validity) an additional item was discovered through the process assessing content validity and added into the frequency of numeracy activities scale. This item was named 'Maths related YouTube videos (e.g. NumTums)'. As confirmed by the interviews with parents during content analysis, younger children mostly use YouTube to consume traditional, 'TV-like' content (Ofcom, 2016). Therefore, the item 'Maths related YouTube videos (e.g. NumTums)' was initially added to the *TV programmes* subscale of the frequency of numeracy activities scale.

However, on examination of the modification indices (i.e. restrictions that may be relaxed to obtain a significant improvement of the global model fit; Geiser 2012) it was apparent that the item, 'Maths related YouTube videos (e.g. NumTums)', should be placed within the *computer maths games* subscale which made for better model fit indices. The fit indices for the new item placed in the *TV programmes* subscale were CFI = 0.81, TLI = 0.79, RMSEA = 0.073, SRMR = 0.078. Whereas, the fit indices for new item placed in the *computer maths games* subscale were CFI = 0.82, TLI = 0.81, RMSEA = 0.070, SRMR = 0.072. As suggested by the modification indices and the model fit statistics the new item was placed in the *computer maths games* subscale. This was the only suggested modification indices, further evidence that the five-factor model is a suitable measurement model.

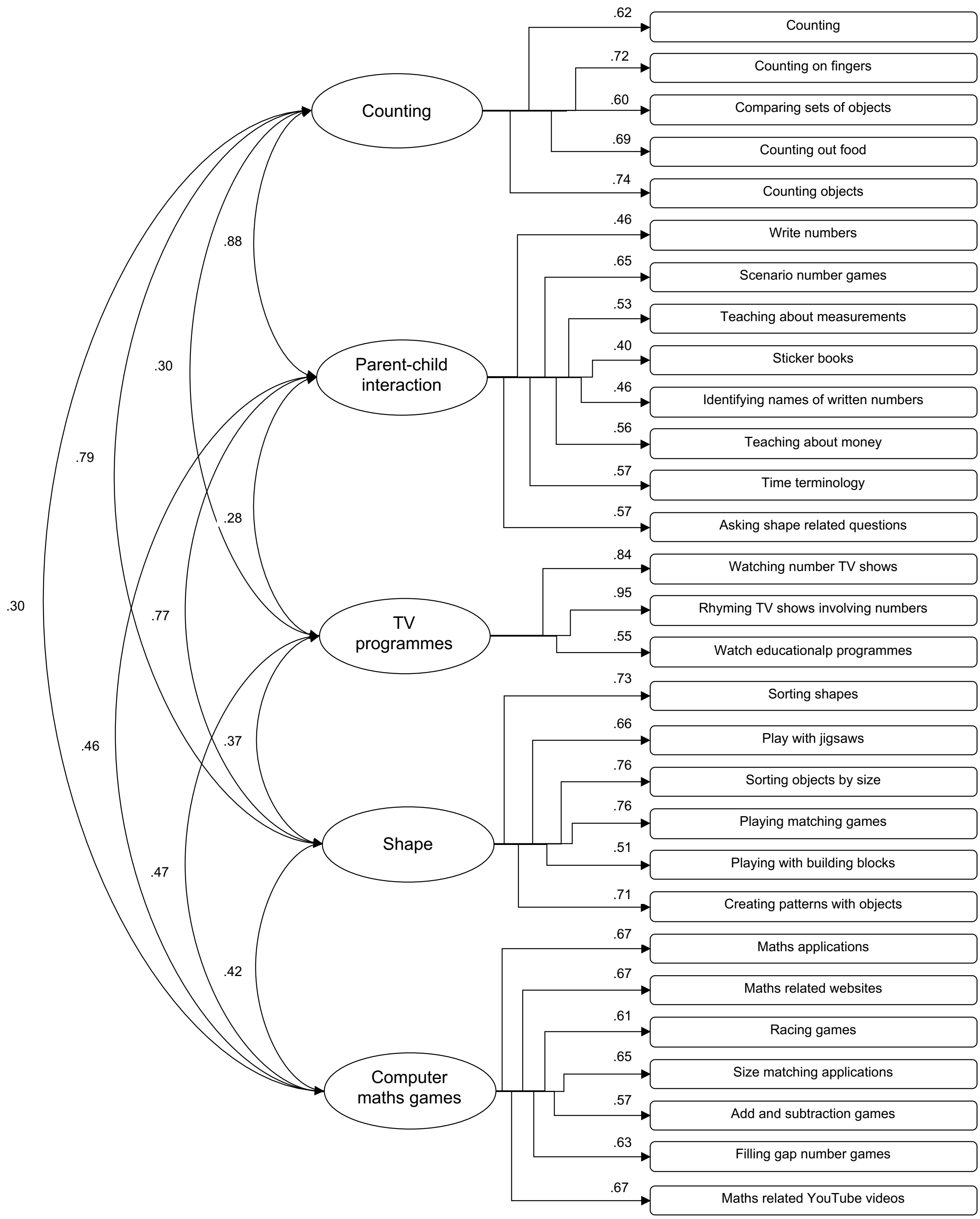


Figure 7. Five-factor model of the frequency of number activities scale in the PHMQ

Table 24. *Model fit statistics for the alternative models of frequency of numeracy activities*

Model no.	Model explained	χ^2(p)	df	CFI	TLI	RMSEA (90% CI) p	SRMR	AIC	BIC	Sample-Size Adjusted BIC
1	Five-factor model	610.552 (0.00)	367	0.83	0.81	0.070 (0.060 – 0.080) 0.00	0.072	11454.955	11737.482	11430.627

Note: N = 136; Estimator = MLR; n = 136; *χ2* **=** Chi-square Goodness of Fit statistic; *df* = degrees of freedom; p = Statistical significance; CFI = Comparative Fit Index; TLI = Tucker Lewis Index; RMSEA (90% CI) = Root-Mean-Square Error of Approximation with 90% confidence intervals; BIC = Bayesian Information Criterion; AIC = Akaike information criterion.

6.3.4 Latent profile analysis – cross-sectional profiles.

6.3.4.1 Preliminary analysis.

After the CFA, but before completing the latent profile analysis, it was deemed necessary to look at the correlations between the frequency of numeracy activities subscales (i.e. the parent-child interaction, computer maths games, TV programmes, shape and counting subscales), the scale as an overall score and the mathematics specific skills (i.e. cardinal principle knowledge (or Give-N), digit recognition, numerical ordering and mapping magnitude representations) used to create the math skills profiles in the latent profile analysis.

Table 25 shows that although the CFA demonstrated that the five-factor model was a suitable measurement model the sub-scales and scale (i.e. as an overall score) does not correlate with the mathematics specific skills. The only correlation was between the TV programmes subscale and the Give-N task ($r = -0.287$, $p < 0.01$). Due to the lack of correlation between mathematics specific skills and the frequency of numeracy activities subscales only an overall score of the frequency of numeracy activities scale will be used as a predictor of pathway membership.

Table 25. *Correlations between the frequency of numeracy activities subscale/scale and the mathematics specific skills*

	1. Counting (subscale)	2	3	4	5	6	7	8	9
2. Shape (subscale)	.658**								
3. Parent-child interaction (subscale)	.683**	.597**							
4. Computer maths games (subscale)	.257**	.305**	.362**						
5. TV programmes (subscale)	.306**	.352**	251**	.475**					
6. Give-N (Maths specific)	-.136	-.158	.022	.100	-.287**				
7. Digit Recognition (Maths specific)	-.084	-.038	.078	.074	-.145	.751**			
8. Ordering (Maths specific)	-.112	-.081	-.052	.073	-.004	.494**	.512**		
9. Mapping (Maths specific)	-.019	-.026	.094	.009	-.150	.494**	.453**	.340**	
10. Frequency of numeracy activities (scale)	.797**	.813**	.821**	.630**	.603**	-.110	-.019	-.050	-.012

Note: * p < .05 ** p < .01 (two-tailed).

6.3.4.2 Determining the number of latent profiles.

To determine the number of latent profiles the recommendations by Nylund et al. (2007) were followed. The model fit indices for 2 to 5-profiles solution at each time point are reported in Table 26. Table 26 demonstrates the reported fit for each model with constrained variance across profiles. Seven fit indices were employed; three likelihood ratio tests and four information criterion indices. The fit indices were used to help in the decision process of how many profiles should be extracted (McLachlan & Peel, 2000; Kam et al., 2016; Fryer, 2017). BIC, an information criterion index, supported a two-profile solution at T1 and a three-profile solution for both T2 and T3. Following previous research (Nylund-Gibson et al. 2014), the elbow of the BIC value, the last large decrease in the BIC value, was used as a guide (Fryer, 2017). In support of BIC, the three likelihood ratio tests, Vuong-Lo-Mendell-Rubin (Vuong, 1989), Lo-Mendell-Rubin (Lo, Mendell & Rubin, 2001) and Bootstrap Likelihood Ratio Test (McLachlan & Peel, 2000), were significance ($p < 0.001$) and entropy levelled out at a relatively high amount (i.e. $\Sigma = 0.96$ at T1 and $\Sigma = 0.93$ at T2 and T3) suggesting good separation for the profiles.

Table 26. *Latent profile analysis fit criterion for Time 1 to 3*

Fit criterion	Time 1					Time 2					Time 3				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Akaike information criterion (AIC)	198.76	-6.39	-42.01	-53.41	-63.65	228.08	19.03	-87.39	-105.20	-138.39	88.53	-114.19	-212.30	-226.31	-237.31
Bayesian information criterion (BIC)	221.57	30.68	9.33	12.19	16.21	250.90	56.10	-36.06	-39.61	-12.90	110.69	-78.17	-162.43	-162.58	-159.73
Sample size - adjusted BIC	196.27	-10.43	-47.60	-60.55	-72.34	225.59	14.99	-92.98	-112.35	-152.06	85.40	-119.27	-219.33	-235.29	-248.25
Entropy		0.96	0.89	0.88	0.88		0.93	0.93	0.90	0.95		0.91	0.93	0.91	0.91
Vuong-Lo-Mendell-Rubin		<0.001	0.28	0.08	0.53		<0.01	<0.001	0.06	0.37		<0.01	<0.001	0.15	0.55
Lo-Mendell-Rubin		<0.001	0.29	0.09	0.54		<0.01	<0.001	0.06	0.37		<0.01	<0.001	0.17	0.56
Parametric bootstrapped		<0.001	<0.001	<0.001	<0.001		<0.001	<0.001	<0.001	0.60		<0.001	<0.001	<0.001	<0.001

6.3.4.3 Characteristics of profiles.

6.3.4.3.1 Time 1.

At T1 the two-profile solution (Figure 8) represented one large group, known as the *high number skills group* (N = 73), and one moderately sized group, known as the *low number skills group* (N = 55). The *high number skills group* have scored high on cardinal principle knowledge (parameter estimate = 0.76), started to score high on digit recognition and thus started to develop ordering skills. However, the *high number skill group* are only scoring above chance in the symbolic and non-symbolic mapping task. The *low number skills group* score low on all of the number skill tasks at T1, scoring below chance on the mapping task.

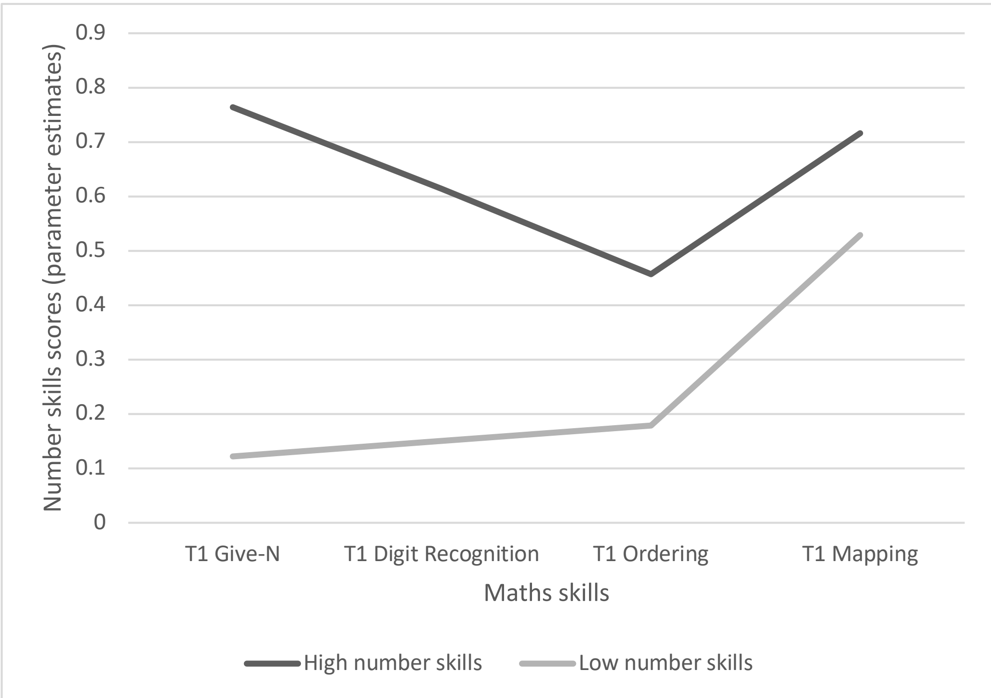


Figure 8. Number skills profiles at T1 (2 profiles)

6.3.4.3.2 Time 2.

At T2 the three-profile solution (Figure 9) represented three large profiles; *high number skills* (N = 41), *medium number skills* (N = 50) and *low number skills* (N = 37). The *high number skills* group scored high in all four number skill tasks. It can be assumed that the *medium number skills* group have a developing understanding of cardinal principle knowledge. The group have started to score higher on digit recognition however, score low on ordering skills. Further, the *medium number skill* group are only scoring above chance in the symbolic and non-symbolic mapping task. The *low number skills* group score low on all of the number skill tasks at T1, scoring below chance on the mapping task.

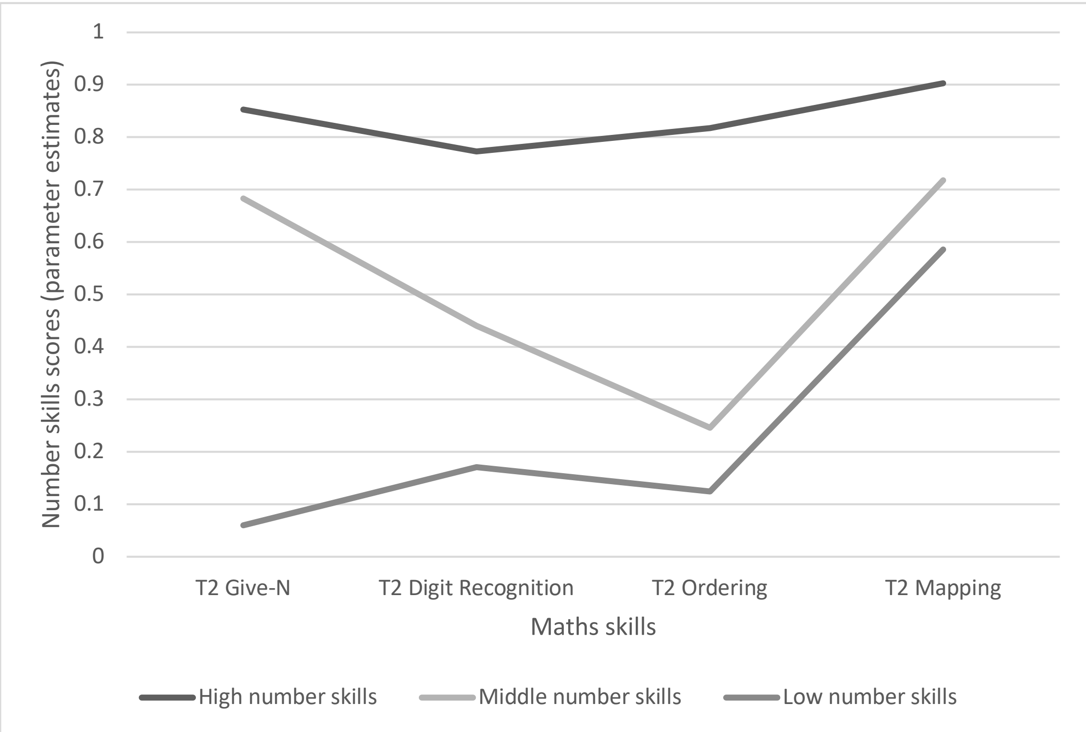


Figure 9. Number skills profiles at T2 (3 profiles)

6.3.4.3.3 Time 3.

At T3 the three-profile solution (Figure 10) represented one large group, *high number skills* (N = 59), one moderate-sized group, *medium number skills* (N = 44) and one small group, *low number skills* (N = 15). The *high number skills* group scored high in all four number skill tasks. The *medium number skills* group scored high on cardinal principle knowledge and symbolic and non-symbolic mapping task however their knowledge of digit recognition and ordering skills is still developing. The *low number skills* group scored low on all of the number skill tasks at T1, scoring at chance on the mapping task.

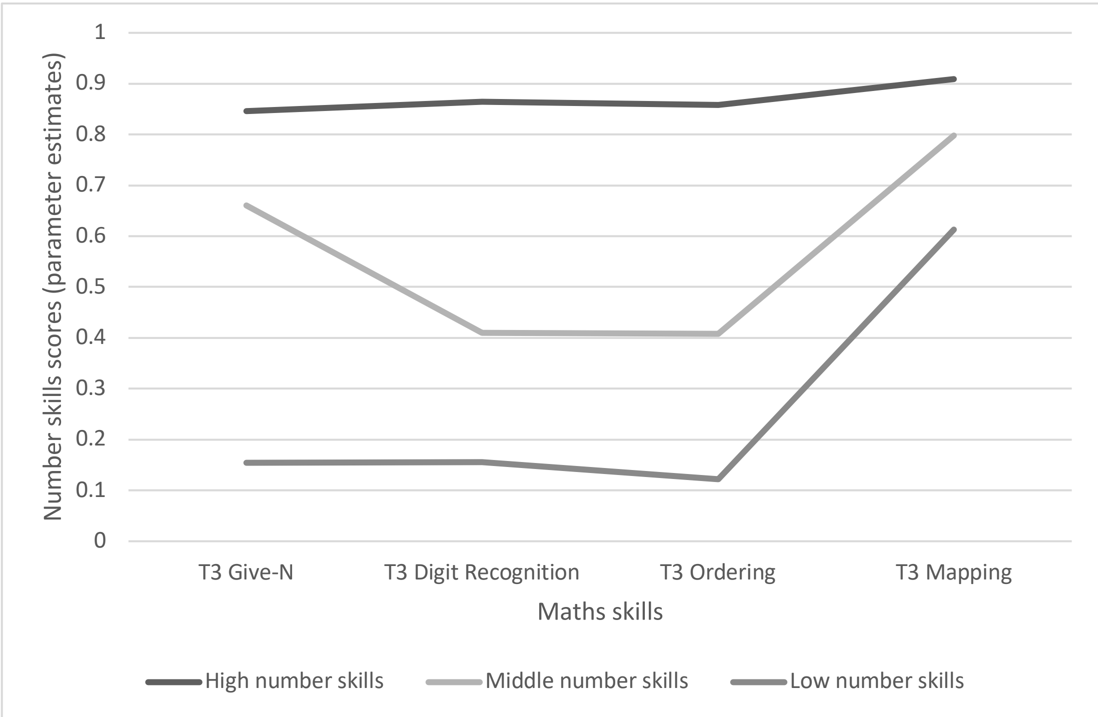


Figure 10. Number skills profiles at T3 (3 profiles)

6.3.4.3.4 Additional Time 1.

However, as suggested earlier the criteria for making the decision on how many profiles there are depends on the theoretical and practical meaning to the extracted profiles (Marsh, Lüdtke, Trautwein & Morin, 2009; Muthén, 2003), not just the statistical adequacy of the solution. As discussed at T1 the fit indices suggest that there were only 2-profiles. However, exploring only 2-profiles at T1 may be restrictive in a theoretical and practical way when it comes to the LTA and exploring learner pathways. Therefore, 3-profiles were investigated at T1 cross-sectionally.

At T1 the three-profile solution (Figure 11) represented one small group, *high number skills* (N = 31), one moderate-sized group, *medium number skills* (N = 45), and one large group, *low number skills* (N = 52). The *high number skills* group scored well in the number skill tasks, scoring above chance in the symbolic and non-symbolic mapping task. It can be assumed that the *medium number skills* group have an understanding of cardinal principle knowledge. The group have started to score higher on digit recognition however, score low on ordering skills. Further, the *medium number skill* group are scoring just above chance in the mapping task. The *low number skills* group score low on all of the number skill tasks at T1, scoring below chance on the symbolic and non-symbolic mapping task.

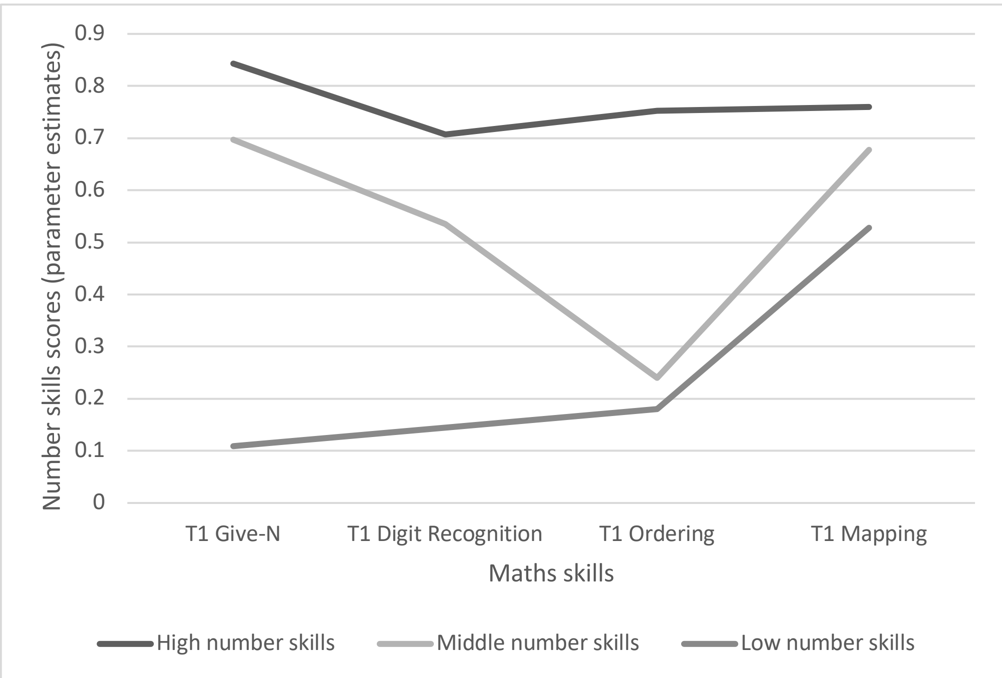


Figure 11. Number skills profiles at T1 (3 profiles)

Based on the cross-sectional LPA results, a latent transition analysis was conducted on the longitudinal data.

6.4.5 Latent transition analysis – longitudinal profiles.

6.4.5.1 Determining the model.

Similar to the LPA the model fit indices for 2 to 5-profiles solution were extracted and the best fitting model was evaluated (see Table 29). Based on the last moderately large decrease in the BIC value, that is the elbow of the BIC value (Fryer, 2017; discussed earlier under section 6.3.4.1 Determining the number of latent profiles), a three-profile model was supported. The three-profile model is also supported by the entropy value settled off at a relatively higher amount in a three-profile model (Σ = 0.951) suggesting good separation for the profiles (Nylund-Gibson et al., 2014; Fryer, 2017). The log-likelihood ratio tests are unavailable for all LPTA fit outputs.

Table 27. *Latent transition analysis fit statistics*

Fit criterion	Models			
	2	3	4	5
Akaike information criterion (AIC)	-297.481	-570.880	-660.820	-728.865
Bayesian information criterion (BIC)	-180.548	-394.055	-412.694	-398.030
Sample size - adjusted BIC	-310.212	-590.133	-687.836	-764.886
Entropy	0.962	0.951	0.939	0.957

6.4.5.2 Characteristics of profiles.

6.4.5.2.1 Labeling the profiles.

The three-profiles from the LPA were labeled according to the profile characteristics. Similar to the profiles in the LPA the profile characteristics represented high, medium and low number skills however the names of the profiles were changed. The high number skills group was changed to the *advanced number skills profile.* The *advanced number skills profile* was given this label as at T1 these children exhibit an understanding of cardinality and mapping skills. They have started to comprehend digit recognition skills and therefore, started to develop ordering skills (see Figure 12). These skills continue to develop over time for the children in the *advanced number skills profile*.

The medium number skills group was labeled as the *intermediate number skills profile*. These children have understood the concept of cardinal principle knowledge at T1 however, score at chance on the mapping skills, low on ordering skills and moderately on digit recognition skills. Over time these children continue to develop an understanding of cardinality and score above chance on the mapping skills task. These children only marginally continue to progress in their digit recognition and ordering skills. Therefore, it could be assumed that these children have started to grasp the concept and meaning of the count words, however they are unable to order numbers as they do not recognise digits fluently.

The low number skills group from the LPA was labelled as the *low number skills profile*, as these children do not develop an understanding of mathematical concepts over time. These children continue to score low on all number tasks and continue to score at chance on the mapping skills task.

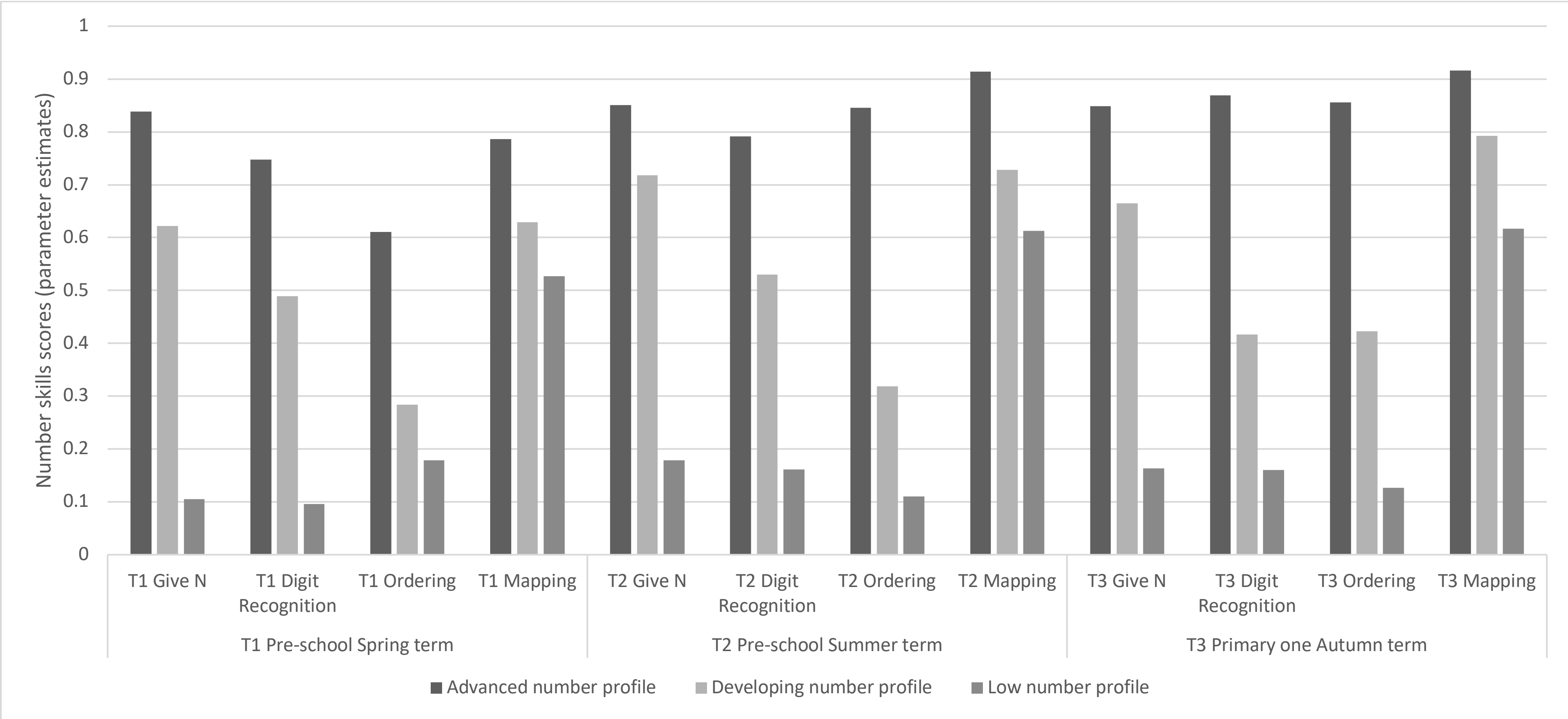


Figure 12. Profile characteristics over time

6.4.5.2.2 Size of profiles.

The size of the latent profiles in terms of profile membership (Table 28) change over time in plausible ways. At T1 and T2, most children were in the *low number skills profile* (38%) that low levels of expertise. The profile with the fewest children at T1 and T2 was the *advanced number profile* (30%) which demonstrated participants with high number skills. Most change occurred in knowledge profiles between T2 and T3 with fewer children in the *low number skills profile* (12%) and more children in the *advanced number profile* (52%). Table 28 shows the pathways of the 128 participants.

Table 28. *Assigned labels for the knowledge profiles with sample proportions at each time point*

Label of profile	No. of participants n (%)		
	T1	T2	T3
Advanced number skills profile	38 (30)	39 (30)	66 (52)
Intermediate number skills profile	41 (32)	41 (32)	46 (36)
Low number skills profile	49 (38)	48 (38)	16 (12)

6.4.5.3 Latent transition pathways.

Theoretically, as each child was in one of the three-profiles at each of the three-time points there is a possibility of there being 27 different transition pathways (i.e. $3^3 = 27$) within the latent transition model. However, the empirical results revealed that only 7 (i.e. 26%) of the 27 pathways had been taken by at least one child, while 20 of the theoretically possible pathways were not utilised by any children. Two pathways were travelled on by 1 participant each (<5% of participants). Previous literature suggests that pathways should not be analysed further if less than 5% of participants travelled a specific pathway (Schneider & Hardy, 2013). Therefore, these pathways will not be discussed in further detail. The remaining 5 pathways describe the development of 126 (98.4%) of the population. The pathways made by the 128 participants (100%) are shown in Table 29.

6.4.5.3.1 Consistent number skills pathways.

- Pathway 1 was named *consistently low number skills pathway* since children (12.5%) on this path are in the low number skills profile at all three time points.
- Pathway 3 was called *consistently intermediate number skills pathway* as children (11.7%) on this path are in the intermediate number skills profile at all three time points.
- Pathway 5 was termed *consistently advanced number skills pathway* since children (29.7%) on this path are in the advanced number skills profile at all three time points.

6.4.5.3.2 School-entry shifting pathways.

- Pathway 2 was called *low to intermediate number skills shifting pathway (at school-entry)* because children (24.2%) on this path started with low number skills at T1 and stayed there in T2 but then transitioned to the intermediate number skills profile between T2 and T3.
- Pathway 4 was termed *intermediate to advanced number skills shifting pathway (at school-entry)* as these children (20.3%) started in the intermediate number profile at T1 and stayed there in T2 but then transitioned to the advanced number profile between T2 and T3.

Table 29. *Pathways of conceptual change*

Pathway no.	Label of pathway	Number profiles			No. of participants n (%)
		T1	T2	T3	
Path 1	Consistently low number skills pathway	Low number skills	Low number skills	Low number skills	16 (12.5)
Path 2	Low to intermediate number skills shifting pathway (at school-entry)	Low number skills	Low number skills	Intermediate number skills	31 (24.2)
Path 3	Consistently intermediate number skills pathway	Intermediate number skills	Intermediate number skills	Intermediate number skills	15 (11.7)
Path 4	Intermediate to advanced number skills shifting pathway (at school-entry)	Intermediate number skills	Intermediate number skills	Advanced number skills	26 (20.3)
Path 5	Consistently advanced number skills pathway	Advanced number skills	Advanced number skills	Advanced number skills	38 (29.7)
Paths 6-7	Various pathways (less than 5% of participants)	Various profiles	Various profiles	Various profiles	2 (1.6)
Paths 8-27	Pathways with no assigned/identified participants				0 (0)

Overall the pathways demonstrate a general trend towards mathematical learning gains over time. Importantly, there was no pathway that demonstrated a decrease in number knowledge over time. For example, there was no pathway that had a child move from the advanced number to the intermediate number profile or the intermediate number to a low number profile. Figure 13 demonstrates children's transition pathways. Pathways 2 and 4 are the two pathways were children move up a profile. For instance, 57 (44.5%) children make a substantial change in their number skills development between T2 and T3, during the transition between pre-school and primary school. Whereas, 69 (53.9%) children remained in their profile over the three time points, those children in pathways 1, 3 and 5. It is important to note that although 69 children remained in their number profile over time, they are still improving in their performance on the mathematics specific skills tasks.

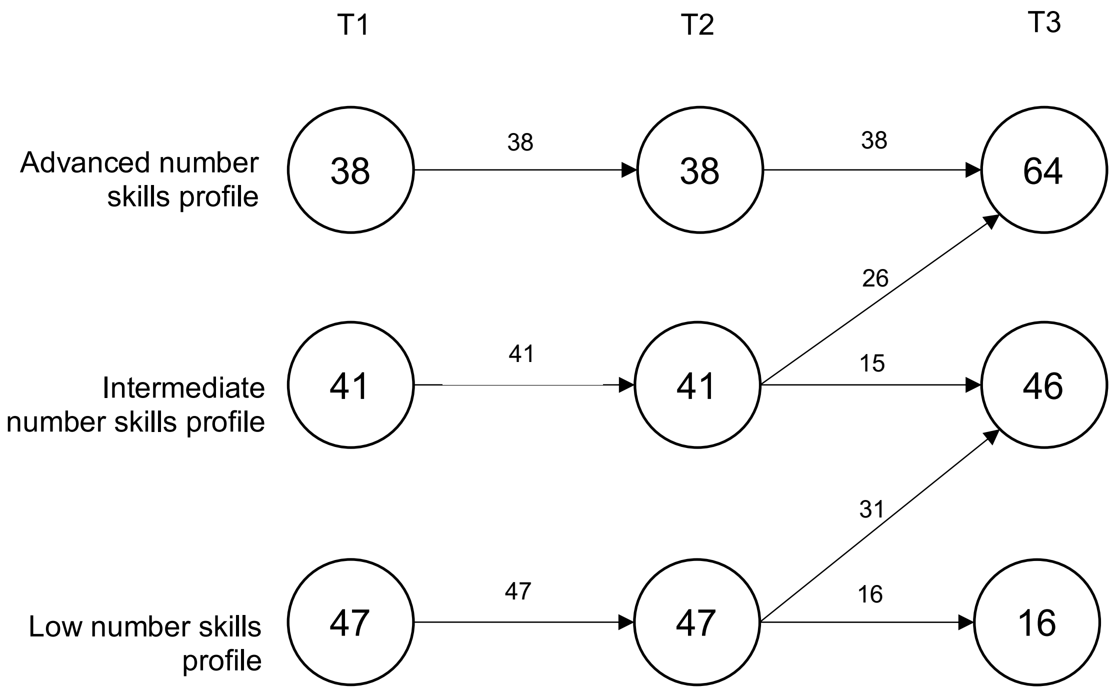


Figure 13. Diagram of the five transitional pathways

taken by 98.4% of the population over the three time points (T1, T2 and T3). The numbers represented in the diagram are the actual number of participants transitioning and not percentages.

6.4.5.4 Latent transition analysis – three-step method.

In order to understand the association between the final retained model and predictors it is important to ensure that the predictors do not impact on the structure of the model (for related discussions, see Marsh et al., 2009; Morin et al., 2011). Recent methodological research has provided a framework for avoiding this measurement parameter shift problem once predictors are added into the model, known as the three-step specification. The three-step method estimates the effects of auxiliary variables (i.e. covariates (or predictors) and distal outcome) in mixture models (Asparouhov & Muthén, 2013; Vermunt, 2010) by ensuring that the measurement of the latent profile variable (i.e. in this study the mathematics profile) is not affected by the inclusion of predictors by fixing the measurement parameters of the latent profile variable with predictors at values from the unconditional latent class model (i.e. the final retained model). The first step is explained previously (i.e. 6.2.5.2 Latent transition analysis measurement model (step one)) but as suggested it involves two additional steps. The second step is assigning individuals to latent profiles using the logits of classification probabilities (i.e. the average latent class probabilities for most likely latent class membership). Finally, the mixture model is estimated with measurement parameters that are fixed at values that account for the measurement error in the class assignment (see Asparouhov & Muthén, 2014; Nylund-Gibson, Grimm, Quirk & Furlong, 2014; Vermunt, 2010 for more information on three-step approaches). After the three-step method, predictors can be included in the model in the traditional manner and the predictors should not impact the model.

However, when completing the three-step method, before predictors were added, there was still movement in the profiles between time points (e.g. 1 child at T1 moved profiles and 2 children at T2 moved profiles). Although, this is minimal change in profiles the three-step method should have restricted this movement and it was deemed that the sample was too small to progress with the three-step method. In previous literature that utilised the three-step approach samples sizes were large for instance, a sample of 2172 children in Nylund-Gibson et al. (2014). Thus, although the recently developed three-step method has been demonstrated to be less biased, has lower mean squared error, and good confidence interval coverage, the conditions are limited. Asparouhov and Muthén (2014) considered only two sample sizes of 500 and 2,000. More research is necessary for the three-step method to be generalisable (Ryoo, Wang, Swearer, Hull & Shi, 2018), especially in the scenario in which sample sizes are relatively small.

An alternative solution was proposed based on classification probabilities (i.e. the average latent class probabilities for most likely latent class membership). From a statistical point of view, the number of participants in a pathway (or profile) at a time point is an estimated parameter of the latent transition model (i.e. a probability of a participant being in a pathway).

When the best model was identified (in this case the three-profile model) the model was run again with a “savedata” command which requests Mplus to create a new dataset with “cprobabilities”, also known as class (or classification) probabilities. This dataset was then explored to determine the class probabilities of each participant pathway membership. The majority of participants had high class probabilities (i.e. 80%+ class probabilities for 88% of the sample) in the pathways. Thus, it was deemed appropriate to complete a regression analysis on the LTA pathways by reading in these data into the next model (i.e. adding these class probabilities into the original data set). Allowing the researcher to have a *pathways* variable matched to each participants’ pathway membership.

6.4.6 Predictors of pathway membership.

A multinomial logistic regression analysis was used to identify predictors associated with pathway membership. Pathway 5, the *consistently advanced number skills pathway,* was used as the reference class as this was the highest scoring basic numerical skill pathway and had the largest number of participants (i.e. 28 (21.9%)). As explained earlier this study considers a variety of demographic characteristics (i.e. gender, age, SES and HEQ), as well as predictors associated with multiple components of the home numeracy environment measured with parents (i.e. the frequency of numeracy activities scale and number games checklist), and child measures such as domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary).

A multinomial logistic regression was completed to understand the bivariate associations between pathways membership and the predictor variables. By first considering the bivariate associations the overall relationships are assessed between latent pathways and predictors without collinearity concerns. Furthermore, the differences between the unadjusted and adjusted estimates can be explored. The unadjusted odds ratio is obtained by only studying the effect of one predictor variable, whereas when more than one predictor is considered an adjusted odds ratio is created that takes into account the effect due to all the additional variables included in the analysis. The adjusted associations were considered through a multivariate multinomial logistic regression analysis after the unadjusted multinomial logistic regression. Predictors were selected for this final adjusted model if $p < 0.05$ for any association between pathway membership and the given predictor in the bivariate analyses.

6.4.6.1 Bivariate association between pathway membership and predictors.

In the bivariate analyses (Table 31) compared to the *consistently advanced number skills pathway* (pathway 5), membership in the *consistently low number skills pathway* (pathway 1) and the *low to intermediate number skills shifting pathway (at school-entry)* (pathway 2) were significantly associated with age at T1 (OR = 0.75, $p < 0.001$; OR = 0.82, $p < .01$, respectively). The odds ratios demonstrate that as age increases children are less likely to be in pathways 1 and 2 than pathway 5. In short, for an additional month alive, the odds of being in pathway 1 is 25% lower than pathway 5 and the odds of being in pathway 2 is 18% lower than pathway 5.

Membership in pathways 1 and 2 (compared to 5) were significantly associated with receptive vocabulary (OR = 0.88, $p < 0.001$; OR = 0.92, $p < 0.001$, respectively) and working memory (OR = 0.63, $p < 0.01$; OR = 0.68, $p < 0.001$, respectively). The odds ratios demonstrate that as receptive vocabulary and working memory scores increased by one-unit children are less likely to be in pathway 1 and 2 than pathway 5. In short, for a one-unit increase in receptive vocabulary, the odds of being in pathway 1 is 12% lower than pathway 5 and the odds of being in pathway 2 is 8% lower than pathway 5. For a one-unit increase in working memory, the odds of being in pathway 1 is 37% lower than pathway 5 and the odds of being in pathway 2 is 32% lower than pathway 5.

There was no significant difference between the *consistently intermediate number skills pathway* (pathway 3) and pathway 5 with working memory but there was a significant different with receptive vocabulary (OR = 0.95, $p < 0.05$). The odds ratio shows that as receptive vocabulary scores increase children are less likely to be in pathway 3 than 5. For a one-unit increase in receptive vocabulary, the odds of being in pathway 3 is 5% lower than pathway 5. There was a higher percentage difference between pathway 1 and 5 that decreased through pathways 1 to 3 (12% (between pathway 1 and 5), 8% (between pathway 2 and 5), 5% (between pathway 3 and 5), respectively) in relation to working memory. This was confirmed by the mean score of receptive vocabulary increasing through the pathways (see Table 30). The means of both verbal working memory and receptive vocabulary are shown in Table 30.

There was a significant difference between the *intermediate to advanced number skills shifting pathway (at school-entry)* (pathway 4) and pathway 5 with working memory (OR = 0.76, $p < 0.01$) but no significant difference with receptive vocabulary. The odds ratio illustrates that as working memory scores increase children are less likely to be in pathway 4 than 5. For a one-unit increase in working memory, the odds of being in pathway 4 is 24% lower than pathway 5. Gender, sustained attention, the frequency of numeracy activities scale and number games checklist were not significant in the bivariate model.

Table 30. *Means of child measures according to pathways*

	Pathway 1	Pathway 2	Pathway 3	Pathway 4	Pathway 5
T1 Receptive vocabulary	41.56	50.68	57.07	60.27	68.39
T1 Working memory	2.63	2.97	4.33	3.58	5.76

Note: * p < .05 ** p < .01 (two-tailed).

Table 31. *Bivariate association between pathway membership and demographic characteristics, child and HNE measures*

	Pathway 1 – Consistently low number skills pathway		Pathway 2 - Low to intermediate number skills shifting pathway (at school-entry)		Pathway 3 – Consistently intermediate number skills pathway		Pathway 4 – Intermediate to advanced number skills shifting pathway (at school-entry)	
	OR	p Value	OR	p Value	OR	p Value	OR	p Value
Gender (Male)	0.583	0.371	0.547	0.220	2.333	0.244	0.428	0.103
Age at T1 (mo.)	**0.746**	< 0.001***	**0.823**	0.010**	0.856	0.113	0.949	0.553
SES								
High	Reference category							
Middle	0.000	0.075	1.979	0.272	1.220	0.790	0.964	0.954
Low	**8.091**	0.022*	2.063	0.259	0.642	0.630	0.706	0.623
HEQ								
GCSE	Reference category							
A-level	0.070	0.055	**0.055**	0.020*	0.107	0.092	**0.074**	0.038*
Degree	**0.022**	0.013*	**0.079**	0.029*	**0.045**	0.023*	0.153	0.105
Masters	0.000	0.170	**0.027**	0.004**	0.082	0.058	**0.067**	0.032*
PhD	0.081	0.120	**0.037**	0.030*	0.000	0.446	0.105	0.119
No qualification	0.183	0.213	**0.024**	0.019*	0.062	0.080	**0.043**	0.038*
T1 Receptive vocabulary	**0.876**	< 0.001***	**0.916**	< 0.001***	**0.946**	0.035*	0.961	0.056
T1 Working memory	**0.634**	0.004**	**0.681**	< 0.001***	0.848	0.074	**0.759**	0.002**
T1 Sustained attention	1.073	0.448	1.000	0.999	1.097	0.327	0.879	0.278
Frequency of numeracy activities	1.321	0.575	1.164	0.701	0.816	0.688	0.639	0.330
Number games checklist	0.894	0.403	1.225	0.110	1.185	0.250	1.156	0.295

Reference pathway: Pathway 5 the consistently advanced number skills pathway. OR, odds ratio. Bivariate association from a multinomial logistic regression.
Note: * p < .05, ** p < .01, *** p <.001 (two-tailed). Bold = p < .05. N = 126.

6.4.6.1.1 Results to be treated with caution.

For SES, high SES was used as a reference category and the only significant difference between pathways was between pathways 1 and 5 with low SES (OR = 8.09, $p < 0.05$). However, the odds ratio is large which could be due to the zero frequencies in the middle SES category (see Table 32 for the frequencies of demographic characteristics per pathway), thus this result should be treated with caution. Zero or small frequencies can be problematic as it can lead to coefficients that have unreasonably large standard errors (Field, 2013).

With regards to parent's higher education qualification (HEQ) GCSE (referred to as GCSE / O level / Irish Junior Certificate within the PHMQ) was used as the reference category. Compared to pathway 5, membership in the *consistently low number skills pathway* (pathway 1) and the *consistently intermediate number skills pathway* (pathway 3) were significantly associated with the HEQ degree (OR = 0.02, $p < 0.05$; OR = 0.05, $p < 0.05$, respectively). The odds ratios demonstrate that if parent's highest educational qualification is degree, children are less likely to be in the *consistent* pathways 1 and 3 than pathway 5. However, this result could be due to near zero frequencies in the degree category for pathways 1 and 3 (n = 1; n = 2, respectively) compared to pathway 5 (n = 11), therefore this result should be treated with caution.

Compared to pathway 5, membership in the *low to intermediate number skills shifting pathway (at school-entry)* (pathway 2) was significantly associated all the HEQ levels (A-level OR = .06, $p < .05$; degree OR = .08, $p < .05$; masters OR = .03, $p < .001$; PhD OR = .04, $p < .01$; no qualifications OR = .24, $p < .01$). Compared to pathway 5, membership in the *intermediate to advanced number skills shifting pathway (at school-entry)* (pathway 4) was significantly associated with A-level, Masters and no qualifications (OR = .07, $p < .05$; OR = .07, $p < .05$; OR = .04, $p < .05$, respectively).

Table 32. *Frequencies of demographic characteristics per pathway*

	Pathway 1	Pathway 2	Pathway 3	Pathway 4	Pathway 5
Age at T1	16	31	15	26	38
Male	8	16	3	15	14
Female	8	15	12	11	24
SES					
High	2	9	7	13	18
Middle	0	8	4	6	9
Low	7	8	2	4	8
HEQ					
GCSE	4	10	4	6	1
A-level	2	4	3	3	7
Degree	1	9	2	10	11
Masters	0	2	3	4	9
PhD	1	1	0	2	3
No qualification	3	1	1	1	4

Note: * p < .05 ** p < .01 (two-tailed).

6.4.6.2 Adjusted associations between pathway membership and predictors.

Blocks of predictors were entered into the model in a forward stepwise manner: 1) demographic characteristics (i.e. age, SES and HEQ) and 2) child measures (i.e. verbal working memory and receptive vocabulary). This is a statistically driven model that explores unique predictors. Thus, predictors were selected for this final adjusted model if $p < 0.05$ for any association between pathway membership and the given predictor in the bivariate analyses.

In the final adjusted model (Table 33) compared to the *consistently advanced number skills pathway* (pathway 5), membership in the *consistently low number skills pathway* (pathway 1) and the *low to intermediate number skills shifting pathway (at school-entry)* (pathway 2) were significantly associated with age at T1 (OR = 0.61, $p < 0.05$, OR = 0.81, $p < 0.05$, respectively). The odds ratios demonstrate that as age increases children are less likely to be in pathways 1 and 2 than pathway 5. In short, for an additional month alive, the odds of being in pathway 1 is 39% lower than pathway 5 and the odds of being in pathway 2 is 19% lower than pathway 5.

Membership in pathways 1, compared to 5, was significantly associated with receptive vocabulary (OR = 0.89, $p < 0.001$) but not working memory. The odds ratios demonstrate that as receptive vocabulary scores increase by one-unit children are less likely to be in pathway 1 than pathway 5. In short, for a one-unit increase in receptive vocabulary, the odds of being in pathway 1 is 11% lower than pathway 5. Membership in pathways 2 (compared to 5) was significantly associated with receptive vocabulary (OR = 0.93, $p < 0.01$) and working memory (OR = 0.76, $p < 0.05$). The odds ratios demonstrate that as receptive vocabulary and working memory scores increase by one-unit children are less likely to be in pathway 2 than pathway 5. For a one-unit increase in receptive vocabulary, the odds of being in pathway 2 is 7% lower than pathway 5 and for a one-unit increase in working memory, the odds of being in pathway 2 is 24% lower than pathway 5. There was no significant difference between the *consistently intermediate number skills pathway* (pathway 3) and pathway 5 with receptive vocabulary or working memory. There was a significant difference between the *intermediate to advanced number skills shifting pathway (at school-entry)* (pathway 4) and pathway 5 with working memory (OR = 0.79, $p < 0.01$) but no significant difference with receptive vocabulary. The odds ratio illustrates that as working memory scores increase children are less likely to be in pathway 4 than 5. For a one-unit increase in working memory, the odds of being in pathway 4 is 21% lower than pathway 5.

SES was not significant in the final adjusted model. With regards to parent's higher education qualification (HEQ) GCSE (referred to as GCSE / O level / Irish Junior Certificate within the PHMQ) was used as the reference category. Compared to pathway 5, membership in the *consistently low number skills pathway* (pathway 1) was significantly associated with the HEQ degree and no qualification (OR = 0.01, $p < 0.05$; OR = 0.02, $p < 0.05$, respectively). The odds ratios demonstrate that if parent's highest educational qualification is degree or parents have no qualification children are less likely to be in the *consistent* pathways 1 than pathway 5. Between the *low to intermediate number skills shifting pathway (at school-entry)* (pathway 2) and pathway 5 all HEQ levels were again significantly associated (A-level OR = 0.05, $p < 0.01$; degree OR = 0.05, $p < 0.01$; masters OR = 0.02, $p < 0.001$; PhD OR = 0.05, $p < 0.05$; no qualifications OR = 0.22, $p < 0.01$).

Compared to pathway 5, membership in the *consistently intermediate number skills pathway* (pathway 3) was significantly associated with the HEQ A-levels, degree and masters (OR = 0.06, $p < 0.05$; OR = 0.01, $p < 0.001$; OR = 0.03, $p < 0.01$, respectively). The odds ratios demonstrate that if parent's highest educational qualification is A-levels, degree and master's children are less likely to be in the *consistent* pathways 3 than pathway 5. Finally, compared to pathway 5, the *intermediate to advanced number skills shifting pathway (at school-entry)* (pathway 4) was significantly associated with the HEQ A-levels, degree, masters and no qualifications (OR = 0.05, $p < 0.05$; OR = 0.08, $p < 0.05$, OR = 0.04, $p < 0.05$; OR = 0.04, $p < 0.05$, respectively). However, based on the zero frequencies or near zero frequencies (mentioned previously in the bivariate results) among both SES and HEQ categories, results should be taken with caution.

Table 33. *Adjusted associations between pathway membership and demographics and child measures predictors*

	Pathway 1 – Consistently low number skills pathway		Pathway 2 - Low to intermediate number skills shifting pathway (at school-entry)		Pathway 3 – Consistently intermediate number skills pathway		Pathway 4 – Intermediate to advanced number skills shifting pathway (at school-entry)	
	AOR	p Value	AOR	p Value	AOR	p Value	AOR	p Value
Age at T1 (mo.)	**0.608**	0.019*	**0.807**	0.012*	0.812	0.093	0.939	0.495
SES								
High	Reference category							
Middle	0.000	0.632	1.346	0.682	0.514	0.442	0.747	0.678
Low	3.223	0.347	0.678	0.631	0.131	0.103	0.321	0.264
Higher Education Qualification								
GCSE	Reference category							
A-level	0.088	0.061	**0.045**	0.008**	**0.057**	0.034*	**0.053**	0.011*
Degree	**0.010**	0.011*	**0.051**	0.008**	**0.013**	< 0.001***	**0.079**	0.031*
Masters	0.000	0.306	**0.016**	< 0.001***	**0.026**	0.007**	**0.038**	0.010*
PhD	0.506	0.744	**0.045**	0.036*	0.000	0.319	0.056	0.053
No qualification	**0.022**	0.010*	**0.022**	0.007**	0.056	0.078	**0.040**	0.025*
T1 Receptive vocabulary	**0.889**	< 0.001***	**0.930**	0.005**	0.951	0.068	0.973	0.214
T1 Working memory	0.765	0.138	**0.756**	0.020*	0.912	0.340	**0.785**	0.010**

Reference pathway: Pathway 5 the consistently advanced number skills pathway. AOR, adjusted odds ratio. Adjusted associations from multivariate multinomial logistic regression. Note: * p < .05, ** p < .01, *** p <.001 (two-tailed). Bold = p < .05. N = 126.

6.4. Discussion

6.4.1 Confirmatory factor analysis.

A CFA was used to quantitatively assess the quality of the five-factor structure of the frequency of numeracy activities scale offering evidence of the construct validity of the scale (Hinkin, 1998). Taking into consideration all criteria for assessing goodness of fit the five-factor model (i.e. parent-child interaction, computer maths games, TV programmes, shape and counting) was deemed a suitable measurement model, confirming the findings of previous research (Chapters 4 and 5). As discussed previously (see section 4.4. Discussion, 4.4.1.1 Stage 1: Item generation) the five subscales demonstrate a comprehensive breakdown of numeracy related activities occurring in the home.

6.4.2 Learner profiles.

Latent profile and latent transition analyses were initially used to identify and describe children's precise *learner profiles*. In both analyses, the best model fit was a three-*learner profile* solution, representing high, medium and low number skills. The four indicators of mathematics specific skills (i.e. cardinal principle knowledge, digit recognition, numerical ordering and mapping magnitude representations) developed in a plausible way. The children in the *advanced number skills profile* represented high number skills by exhibiting cardinal principle knowledge and mapping magnitude representation skills. The children comprehended digit recognition skills and therefore, began to develop numerical ordering skills. These skills continue to develop over time from T1 to T3. The children in the *intermediate number skills profile* represented medium number skills and understood the concept of cardinal principle knowledge at T1 however, scored at chance on mapping skills, low on ordering skills and moderately on digit recognition skills. Over time these children continue to develop and understand cardinal principle knowledge and score above chance on mapping skills. These children only marginally continue to progress in their digit recognition and ordering skills. Therefore, it could be assumed that these children potentially understand the concept and meaning of number words however, they are unable to order numbers as they do not recognise digits fluently. The children in the *low number skills profile* scored low on all four indicators of mathematics specific skills over the three time points and scored at chance on mapping skills. This is consistent with previous research that proposes that mathematics knowledge begins to develop at a young age to varying degrees (Rittle-Johnson, Zippert & Boice, 2018).

As the children in the *low number skills profile* demonstrate low levels of expertise and do not show much variance across the four indicators of mathematics specific over time the development of the mathematics specific skills cannot be compared to other profiles. However, the development of the four mathematics specific skills that create the *advanced number skills* and *intermediate number skills profiles* can be compared longitudinally. Cardinal principle knowledge and mapping magnitude representations were the primary skills to develop in both profiles; over all three time points for the *intermediate number skills profile* and during pre-school for the *advanced number skills profile* (i.e. T1 and T2). At T3 the four mathematics specific skills in the *advanced number skills profile* were near ceiling. These two primary skills (i.e. cardinal principle knowledge and mapping magnitude representations) were followed by digit recognition and numerical ordering skills. This is consistent with previous research that suggests that mathematical cognition is not unitary but consists of many components that develop at different stages (Dowker, 2008).

The results from the current study suggest that children develop cardinality and mapping magnitude representations skills that involved no written Arabic digits before developing digit recognition skills and numerical ordering abilities that require the use of symbolic representations (i.e. Arabic digits). This is consistent with previous literature that suggests children may map newly learnt symbolic representations (i.e. Arabic digits) onto their pre-existing non-symbolic representations (i.e. dot arrays; Dehaene, 2007; Piazza, 2011; for a review see Leibovich & Ansari, 2016). Therefore, perhaps the non-symbolic system may help with the acquisition of numbers more broadly (Gallistel & Gelman, 1992). Although this was not a specific aim of the current study the statistical approach has enabled strong conclusions to be drawn on precursor skills for more complex mathematical development.

6.4.3 Learning pathways.

The latent transition analysis was also utilised to describe children's precise *learning pathways*. In total five plausible pathways were discovered. Three pathways showed children staying in their *learner profile* across all three time points and two pathways displayed a shift to the subsequent more advanced learner profile upon school-entry (i.e. shifting from a low to intermediate number profile). It is important to note that although 69 children (53.9%) remained in their learner profile over time their mathematical skills were still developing, i.e. scores on mathematics specific tasks increased over time. However, substantial developmental change was observed in the performance of the 57 children (44.5%) in the school-entry transition pathways. This was exemplified by these children transitioning between profiles.

Interestingly, no pathway skipped a profile (i.e. no child skipped from a low number skills profile to the advance number skills profile, missing the intermediate mathematics skill profile) but instead children systematically transitioned through each subsequent profile (i.e. low number skills to the intermediate number skills profile). The two transitions between profiles were observed at school-entry demonstrating that children make substantial learning gains once they enter school. During school children are prompted to make connections between new information and prior, or existing knowledge, by practising and recalling new information in different ways (e.g. revision or questions) leading to the consolidation of their learning material (Howard-Jones et al., 2018). Therefore, as the transitions between profiles occurs at school-entry perhaps this is due to children consolidating their mathematical skills and hence transitioning to the subsequent learner profile.

The sizes of the profiles also change over time in plausible ways as the proportion of children in the low number skills profile decreased from 38% at T1 to 12% at T3. Further, the proportion of children in the advanced number skills profile increased from 30% at T1 to 52% at T3. Overall, the *learning pathways* demonstrate a general trend towards mathematical learning gains over time. Importantly, there was no pathway that demonstrated a decrease in mathematical knowledge. This is similar to Schneider and Hardy (2013) who completed a LPTA and found that no pathway demonstrated a decrease in conceptual knowledge associated with the scientific topic of floating and sinking throughout an intervention study.

6.4.4 Predictors of pathway membership.

One of the main aims of this study was to understand what impacts on mathematical skill development (i.e. pathway membership) over time. This was important, as studies that have targeted the developmental trajectories in children's mathematical skills (e.g. Aunola et al. 2004; Chong and Siegel 2008; Jordan et al. 2006, 2007; Morgan, Fargas, and Wu 2009) have shown that children who enter pre-school with low performance in basic number skills stay behind their peers throughout later school years. Therefore, understanding the early mathematical development of different profiles of numeracy learners specifically what impacts on learning pathways through a person-centered approach was critical. This could allow for target interventions for those with low performance in basic number skills in the future.

In both the bivariate and multivariate analyses, younger children were more likely to be in the consistently low number skills pathway (pathway 1) and the low to intermediate number skills shifting pathway (at school-entry) (pathway 2) than the consistently advanced number skills pathway (pathway 5). These two pathways (i.e. pathways 1 and 2) involve children with the lowest mathematics skills (compared to pathways 3 to 5). Therefore, younger pre-school children are less likely to develop their number skills. Cardinality was the first basic math skill to develop in the intermediate number skills *profile* however, this skill had not yet developed in the low number skills *profile*. Previous research indicates that the first step to cardinal principle knowledge occurs when children learn the first few number words (Fuson, 1988; Gelman & Gallistel, 1978). Children typically learn the first few number words between 2 to 3 years of age, progressing over the next two years by gradually understanding larger number words (Carey, 2004; Le Corre & Carey, 2007; Spelke, 2017; Wynn, 1992). Therefore, these younger children (who were on average 3.56 (pathway 1) and 3.61 (pathway 2) years old) in the low number skills *pathways* have not developed cardinal principle knowledge, which would typically develop around 3 years of age (Bermejo, 1996).

In the bivariate analysis, those children with lower receptive vocabulary were more likely to be in the consistently low number skills pathway (pathways 1), the low to intermediate number skills shifting pathway (at school-entry) (pathway 2) and the enduring intermediate number pathway (pathway 3) than the consistently advanced number skills pathway (pathway 5). There was a high percentage difference in vocabulary scores (i.e. mean score differences) between pathway 1 and 5, this decreased through pathways 1 to 3 (12% (between pathway 1 and 5), 8% (between pathway 2 and 5), 5% (between pathway 3 and 5), respectively). This was evidenced by the mean score of receptive vocabulary decreasing across the pathways from the consistently advanced number skills to the consistently low number skills. In the adjusted multivariate model, only pathways 1 and 2 were significantly associated with receptive vocabulary compared to pathway 5. Again, there was a higher percentage difference between pathway 1 and 5 (11%) that decreased between pathway 2 and 5 (7%). In sum, this illustrates that as children score higher in receptive vocabulary, they are more likely to be in the consistently advanced number skills pathway than the lower number skills pathways (i.e. pathways 1 and 2). Findings are consistent with growingly popular ethnomathematical literature that indicates that language may impact the learning of mathematics (e.g. Ascher & D'Ambrosio, 1994; Kim, Ferrini-Mundy & Sfard, 2012).

In the adjusted multivariate model, children in the school-entry transition pathways (i.e. pathways 2 and 4) were significantly associated with working memory when compared to pathway 5. The odds ratio illustrated that as working memory scores increase children are less likely to be in the school-entry transition pathways than the consistently advanced number skills pathway suggesting that those in the advanced number skills pathway have higher working memory. This is consistent with previous literature that proposes that those with higher working memory capacity performed better on difficult mathematics problems (Osei-Boadi, 2016) and training studies that revealed significant improvements on school performance in mathematics on those receiving working memory training (Sánchez-Pérez et al., 2018).

Additionally, there was an association between working memory scores and the school-entry transition pathways (i.e. pathways 2 and 4), when compared to the advanced number skills pathway but this was not the case for those children in the consistent pathways (i.e. pathway 1 and 3). This indicates that working memory skills could enable the shift in mathematical learning pathways at school-entry. Working memory is one of the most fundamental executive functions (Barkley, 1997) that is present early in life, and shows rapid development throughout pre-school (e.g. Carlson, 2005; Davidson, Amso, Anderson & Diamond, 2006; Zelazo & Müller, 2002). As stated earlier most longitudinal evidence between cognitive skills and mathematics skills focus on school-age children and adults. Recent evidence indicates this association is present before school-entry, although research is limited (e.g. Verdine, Golinkoff, Hirsh-Pasek & Newcombe, 2017; Bull, Espy & Wiebe, 2008; Lauer & Lourenco, 2016). Bull and colleagues (2008) found a significant yet small contribution was made by verbal working memory (as measured by the digit backwards task) to mathematical skills at school-entry. Therefore, the current findings confirm previous research as there was a significant relationship between verbal working memory and the school-entry transition pathways. Moreover, the current study suggests that working memory skills enable the shift in mathematical learning pathways.

Sustained attention at T1 was not a significant predictor in the bivariate model. However, in pre-school education the development of sustained attention is not completely understood and somewhat scarce in the research literature (Guy et al., 2013). The current finding could be expected as attention-related skills are only developing during pre-school and do not reach relative stability until ages 6 and 8 (Olson, Sameroff, Kerr, Lopez, & Wellman, 2005; Posner & Rothbart, 2000). Furthermore, attention skills increase while children are engaged in academic studies (Duncan et al., 2007) therefore, as pre-school is not formal education this is perhaps further reason for sustained attention not being a predictor of any pathway membership as attention skills are not yet developed. Nevertheless, Continuous Performance Tests (CPT), the task used within this study, are predominantly utilised in school-aged children and adults and sparingly used with pre-schoolers due to task difficultly owing to task length, number of distractors and short inter-stimulus intervals (Guy et al., 2013; Mahone et al., 2001) thus more research may be necessary for improved CPTs that tap into pre-school children's sustained attention.

HEQ categories were more likely to be significantly associated with pathway membership than SES categories consistent with previous literature which suggests parental education is the best predictor of academic achievement (Sammons et al., 2004; Mercy & Steelman, 1982). However, the SES and HEQ results should be taken with caution due zero frequencies or *near to* zero frequencies among some SES and HEQ categories. It is recommended that in future research more data be collected to explore the causal effects of SES and HEQ on pathway membership.

The findings from this study indicated no statistically significant relations between parents reported frequency of numeracy activities with pathway membership at a bivariate level. Most studies involving questionnaire measures have resulted in inconsistencies between home activities and children's number skills (for more information on these inconsistencies see section 2.1.4.4 Inconsistencies in home environment questionnaires, 2.1.4.4.1 Dichotomisation of numeracy activities). An important reason for the discrepancy between results may involve the different ages of the children in these studies. Some research has found unique and positive associations between the home numeracy environment and mathematics skills with children between the ages of 4 years 10-months to 8 years olds (Kleemans et al., 2012 (M_{age} = 6.1 years, age range = 5 to 7 years); Dearing et al., 2012 (M_{age} = 6.72 years, SD = 0.34); Manolitsis et al., 2013 (M_{age} = 64.32 months, SD = 3.23, age range = 5 to 6 years); Niklas & Schneider, 2014 (M_{age} = 77 months, SD=4.5, 4 years 10 months to 8 years)). Whereas, Missall et al. (2015) in a study involving children aged 3 to 5 years 7-months (M_{age} = 53.6 months) found no relation between mathematics-related activities in the home and a range of numeracy skills. In the current study children were aged between 4 years and 4 years 7-month, similar to that of Missall et al. (2015). The measure of *informal home numeracy practices* (number games exposure checklist) was also not significant in the bivariate model. The measure previously developed by Skwarchuk et al. (2014) was correlated significantly with later mathematics skills. Further, informal exposure to numerical board games predicted children's non-symbolic arithmetic (Skwarchuk et al., 2014). However, children in Skwarchuk et al. (2014) study were aged 5 years 3 months to 6 years 5 months (Mage = 58 months). Hence, the younger age group within this current study could also explain the results. Most countries in Europe, and across the globe, have a school starting age of 6 (Ball, 1994; Bertram & Pascal, 2002; O'Donnell, 2004; Sharp, 2002; Woodhead, 1989; West & Varlaam, 1990). In the UK compulsory schooling starts at 5 years of age. In practice however, most children start Primary 1 at the beginning of the year in which they become five (Sharp, 2002). Hence, the children in this current UK sample are starting Primary 1 earlier when compared to other countries and this could also be a reason for the inconsistent findings. Furthermore, parents are engaging in home numeracy activities within the current study but perhaps children are too young for this engagement to make a difference to their learning trajectories.

6.4.5 Limitations.

The primary limitation of this study was that this framework cannot consider the new three-step specification method due to sample size limitations (i.e. small sample size; Vermunt, 2010; Asparouhov & Muthén, 2014). The three-step method deals with the measurement parameter shift problem by fixing the parameters estimated based on an unconditional model and in turn the predictors do not impact the nature of the model (Marsh et al., 2009; Morin et al., 2011; Ryoo, Wang, Swearer, Hull & Shi, 2018). Although an alternative method was utilised within this study that controlled for this movement, the recently developed three-step method has been demonstrated to be less biased, have lower mean squared error, and better confidence interval coverage than traditional methods. However, previous research has utilised the three-step approach with larger samples sizes (for a discussion on this see section 6.4.5.4 Latent transition analysis – three-step method). Thus, more research is necessary for the three-step method to be generalisable (Ryoo et al., 2018). Furthermore, increased sample size may be desirable to utilise this three-step method even though the majority of participants had high class probabilities in the pathways.

6.4.6 Conclusion.

The current study took a person-centered approach by examining different profiles and pathways of children's basic mathematics skills. This allowed for the exploration of what predicts mathematical skill development (i.e. pathway membership) during children's transition from pre-school to primary school over the course of 8 months. This study goes beyond previous research by addressing limitations imposed by only analysing the average child performance (i.e. a variable-centered approaches; Lindblom-Ylänne et al., 2015). Through use of a latent profile and latent transition analysis children's precise *learner profiles* and *learning pathways* were identified. Therefore, this study presents new findings as there are no existing mathematical *learner profiles* and *learning pathways* discovered in previous literature during this pre-school to school transition.

Another gap that this study has addressed is that previous research has used cross-sectional approaches (e.g. Batchelor, Keeble & Gilmore, 2015) and have therefore often failed to address what causes children's mathematical learning changes during this transition. Overall, the results indicate that many factors (i.e. age (mo.), receptive vocabulary and verbal working memory) contribute to mathematical skill development (i.e. pathway membership) over time and there is no one factor solely driving mathematical development. Findings highlighted the importance of language and working memory abilities on mathematical skills development over time. Therefore, to enhance children's mathematical development it is suggested that early years practitioners should focus on boosting language and working memory skills in pre-school, specifically used to assist younger children's mathematical skill development. Furthermore, by exploring the components through which young children develop foundational mathematical skills educational interventions could be used to target the development of language and working memory abilities in order to facilitate mathematical skills development.

Chapter 7: Discussion

Overview

This chapter is an overall discussion of the thesis, in which the key results of Chapters 3 through 6 are discussed. First, an overview of the aims is presented. A summary of each chapter is offered, followed by the implications, contributions and future recommendations based on the findings of these studies. Then the strengths and limitations of the studies and a final conclusion are presented.

7.1 Overview of the thesis aims

As adults' numeracy is part of the daily routine thus numerical literacy is a crucial skill for everyday life (Chiswick, Lee, & Miller, 2003; Gerardi, Goette, & Meier, 2013). The importance of understanding early numerical development is vital as there are "links between low… numeracy and crime, poor health choices, low educational attainment and unemployment" (Northern Ireland Audit Office, 2013). Therefore, it is of utmost importance that educational professionals and governments make sure every child is provided with optimal educational experiences as tackling numeracy weaknesses are challenging with adults (Windisch, 2016). Recent research suggests that children's experiences in the home or pre-school environments form the foundation for mathematical learning in primary school (LeFevre et al., 2009; Melhuish et al., 2013; Burchinal et al., 2008). Although, the home learning environment is of major importance for children's development over and above early institutional influences (Rossbach, 2005; Weinert, 2006). Therefore, this thesis aimed to develop a better understanding of the home numeracy environment and aimed to describe children's precise *learner profiles* and *learning pathways* of mathematic specific skills during the transition from pre-school to school educational settings. The overall aims of this thesis were to address limitations of previous studies and current gaps in existing literature by:

1. Investigating the dominant and common views and experiences relevant to the HNE using an exploratory approach in the form of semi-structured interviews.
2. Creating an HNE questionnaire measure using both deductive (i.e. theory driven items) and inductive (i.e. using semi-structured interviews to produce new items) approaches to scale development.
3. Discussing each stage of the scale development and validation process to increase the psychometric soundness of the HNE measure. The HNE questionnaire was evaluated across five psychometric properties; (1) construct validity, (2) factor structure, (3) scale score reliability, (4) content validity, and (5) criterion validity.

4. Tracking children's basic numerical skill development from pre-school to school. A latent transition analysis was used to describe children's precise *learner profiles* and *learning pathways* during this transition.
5. Identifying the key predictors of children's *pathway* membership over time. This study considers a variety of demographic characteristics (i.e. gender, age, SES and parents' highest educational qualification), as well as predictors associated with multiple components of the home environment (i.e. the frequency of numeracy activities scale and informal home numeracy practices (number games exposure checklist) from the PHMQ), domain-general skills (i.e. verbal working memory and sustained attention) and language (i.e. receptive vocabulary). Therefore, this study will incorporate potential predictors of pathway membership to extend knowledge on children's development of mathematical skills in early childhood.

7.2 Chapter 3

7.2.1 Research Summary.

Chapter 3 addressed the first aim of this thesis by investigating the dominant and common views and experiences relevant to the HNE using an exploratory approach. The diversity of the themes identified through the semi-structured interviews illustrated how the HNE may be influenced by parents' views and experiences of numeracy-related activities. Moreover, a child's own behaviours and children's interactions with others (e.g. siblings) may influence the learning environment. It was apparent that parents' attitudes differed with regard to the quality of parent–child interactions that occur during home activities similar to previous research such as Lukie et al. (2014). When discussing collaborative parent–child interactions Lukie et al. (2014) suggested that a parent may set out materials, but respect their child's autonomy by allowing independent play. While other parents may choose to interact collaboratively with their child on a task (e.g. printing letters/numbers or doing puzzles), this reflects the findings from *numeracy environment structure* (theme one) that illustrated the types of environments that parents create for their children to learn numeracy in the home. Although there were six themes discovered - the two of the most notable findings were the *numeracy environment structure* (theme one) which illustrated the types of environments that parents create for their children to learn numeracy in the home and *technology attitudes* (theme four) which demonstrated that technology is being used extensively in the home. The implications of themes one and four will be discussed in the next section (see 7.2.2 Study Implications).

7.2.2 Study Implications, Contributions and Future Recommendations.

From the *numeracy environment structure* (theme one) findings a *structured/formal environment* was defined as parents explicitly planning number-related activities, parent's awareness of their opportunity to teach numeracy, organising strategies for their child to learn and develop number skills and setting aside time for teaching numeracy. Whereas, an *unstructured/informal environment* was defined as parents having a spontaneous approach when referring to numeracy. However, this is in regard to the *environment* under which number-related activities occur as opposed to the *activities* that happen within the environment. As defined by LeFevre et al. (2009) *direct* activities involved explicitly and intentionally teaching about numbers or arithmetic to develop children's mathematical skills (e.g. counting objects) and *indirect* activities involved numbers in real-world tasks (e.g. playing board games with dice) that include 'hidden' mathematical instructions that occur incidentally.

Consequently, these *environment* and *activity* definitions can be seen to contradict each other due to the findings from this study. For instance, the numerical environment identified during this study was more likely to be an *unstructured/informal environment* however, *direct* activities were mentioned at a higher frequency than *indirect* activities. Therefore, even though parents have a spontaneous approach (i.e. definition of an unstructured/informal environment) they are explicitly and intentionally teaching about numbers or arithmetic to develop children's mathematical skills (i.e. definition of a direct activities by LeFevre et al., 2009) which is similar, in part, to the definition of a *structured/formal environment* (i.e. parent's awareness of their opportunity to teach numeracy). Therefore, although the words "explicitly and intentionally" are used within the definition by LeFevre et al. (2009) to describe *direct* activities this study suggests that the parent may actually be unaware and not cognisant of 'explicitly and intentionally' teaching about numbers or arithmetic to develop children's mathematical skills in an *unstructured/informal environment*.

This finding is important as past literature has yielded mixed findings across studies (see section 3.1.1 Background for mixed findings). One potential reason for the mixed findings is that there is no consensus in the definition that encompasses the everyday routine practices occurring in the home numeracy environment. Therefore, for clarification purposes this study would suggest that when discussing the *environment* under which number-related activities occur the terms *structured/formal* and *unstructured/informal environment* be utilised. Whereas, when discussing the *activities* that happen within the environment the terms *direct* and *indirect* activities be used. Furthermore, perhaps LeFevre et al.'s (2009) definition of *direct* activities should be adapted to encompass the finding that parents may not be cognisant of the effects of home numeracy activities on their child's learning, being that "explicitly and intentionally" may not be appropriate.

The *technology attitudes* (theme four) findings is an emerging area for future research which, to date, has not been sufficiently studied. The theme demonstrated that technology is being used extensively in the home and parents are concerned by the content of their children's viewing and interaction. Ofcom (2016) has stated that there are two devices in the home that continue to be used by children: television sets (92% for 3 to 4-year-olds and 96% for 5 to 7-year-olds) and tablets (55% for 3 to 4-year-olds and 67% for 5 to 7-year-olds). This is a significant increase from Ofcom's 2013 report in which one quarter (28%) of children aged 3 to 4 use tablets in the home. Future research should investigate how often, and to what extent, electronic devices are integrated into pedagogic planning in the home and its impact on young children's learning. The information that was gathered through the interviews could be used to target and intervene in the family context. Future research could investigate the efficacy of interventions that raise parents' awareness of *activities* (i.e. *direct* and *indirect activities*) and promote positive interactions in the *environments* (i.e. *unstructured/informal* and *structured/formal environments*) in order to assess the impact on children's learning. Furthermore, it would be beneficial to determine how parent–child interactions offer opportunities for early mathematical learning.

7.3 Chapter 4

7.3.1 Research Summary.

Chapter 4 focused on various stages of scale development using a well-established framework to reduce the likelihood of measurement problems (Price & Mueller, 1986). In turn Chapter 4 presents phase one, known as *the scale development process*. This process comprises four development stages of questionnaire development explained by Hinkin (1998) including; (stage 1) item generation, (stage 2) questionnaire administration, (stage 3) initial item reduction and (stage 4) an exploratory factor analysis. The focus of phase one was on construct validity, which combines theory and psychometric measurement (Kerlinger, 1986). Chapter 4 addressed the second aim and part of the third aim of this thesis. The second aim was to create an HNE questionnaire measure using both deductive (i.e. theory driven items) and inductive (i.e. using semi-structured interviews to produce new items) approaches to scale development. Chapter 4 evaluated three psychometric properties which were: (1) construct validity, (2) factor structure, (3) scale score reliability. In turn addressing part of the third aim of the thesis.

As far as the author is aware, this was the first study that uses both an inductive and deductive approach to develop a home numeracy environment questionnaire. Initially 44 inductive and 25 deductive items totalling 69 items were categorised into eight home environment relevant dimensions based on the question's characteristics. The eight-home environment relevant dimensions that made up the PHMQ were; (1) parent expectation questions, (2) literacy questions, (3) counting ability questions. These questions are known as benchmark questions as they give context to results by allowing comparison between participant responses. (4) Parent-child teaching methods questions and (5) target child-sibling interactions questions. These two categories involving interactions with parents and interactions with siblings were named as interaction questions. The (6) frequency of numeracy activities questions, which were used within the exploratory factor analysis. Finally, (7) questions on parent's view of their child's understanding of numeracy and (8) a support question. From the initial 69 items, 19 items were removed after questionnaire administration for different reasons, for example, lack of variation in responses potentially explained by "halo effect" (i.e. the tendency for one impression to shape or influence all other judgements) or items not loading into any factor in the exploratory factor analysis. In sum, 5 inductive and 14 deductive items were removed thus a total of 50 items were retained. Instead of eight-home environment relevant dimensions mentioned previously in this study there are now six-home environment relevant dimensions with the removal of the questions on (7) parent's view of their child's understanding of numeracy and the (8) a support question (a detailed breakdown of the items and how they were generated is presented in Appendix 4.2, Table 2. The original questionnaire is presented in Appendix 4.1).

The five subscales found within the (6) frequency of numeracy activities scale were; (1) *parent - child interactions*, (2) *computer maths games*, (3) *TV programmes*, (4) *shape* and (5) *counting*. These five subscales demonstrate a comprehensive breakdown of numeracy related activities occurring in the home. Although not the exact items from previous scales, the types of activities covered within three subscales from the frequency of numeracy-activities scale, (5) *counting* (e.g. counting out food, dinner plates, knives and forks), (4) *shape* (e.g. sorting objects by size) and (1) *parent - child interactions* (e.g. write numbers) have been widely covered. However, items about the use of educational technology are rarely used in HNE questionnaires and if they are this is usually only one item (e.g. Huntsinger, et al., 2016;

Kleemans et al., 2012). More research is needed to understand the broad array of educational technology that may be watched and/or played on tablets (chapter 3; Cahoon, Cassidy & Simms, 2017). Therefore, in contrast to previous home numeracy scales (Huntsinger, et al., 2016; Kleemans et al., 2012; LeFevre et al., 2009) activities that involved educational technology were added to the frequency of numeracy activities scale. These types of activities were widely covered within two factors from the frequency of numeracy-activities scale within the PHMQ; (2) *computer maths games*, and (3) *TV programmes.*

7.3.2 Study Implications, Contributions and Future Recommendations.

Previous studies that develop HNE questionnaires have not provided sufficient information about item generation and refinement, scale dimensionality, scale score reliability, or validity (e.g. Kleemans et al., 2012; LeFevre et al., 2009; Melhuish et al., 2008). Thus, the purpose of this study was to address these issues by discussing item generation and refinement in detail and confirming that the (6) frequency of numeracy activities scale reached three levels of psychometric properties: (1) construct validity, (2) factor structure, (3) scale score reliability. The (2) *factor structure* was evidenced through the five-factor structure of the frequency of numeracy activities scale found through the exploratory factor analysis and (3) *scale score reliability* was demonstrated through the high levels of reliability (α = .76 to .81). In turn, this demonstrated that the frequency of numeracy activities scale has good (1) *construct validity* and can be considered as having good internal consistency, indicating a scale with strong psychometric properties.

The addition of the two subscales (2) *computer maths games* and (3) *TV programmes* expands the scope of previous HNE measures, that have lacked computer-based mathematics questions (e.g. Huntsinger et al., 2016; Kleemans et al., 2012; LeFevre et al., 2009), by expanding the number of items related to educational technology (e.g. ‘Maths related websites (e.g. coolmaths.com)’). Overall, the PHMQ is an inclusive measure of the HNE taking into consideration a wide variety of home environment relevant dimensions.

7.4 Chapter 5

7.4.1 Research Summary.

Chapter 5 addressed the second part of the third aim of this thesis, which was to validate the scale that was developed in Chapter 4. Chapter 5 addressed the final two psychometric properties which were: (4) content validity and (5) criterion validity. In turn Chapter 5 presents phase two, known as *the scale validation process*. Before the validation process the questionnaire was piloted as it included the new number game checklist (discussed in section 5.2.1.2 Informal home numeracy practice). Score on the number game checklist ranged from 0-8 (maximum score of 10) with a mean score of 4.8 showing good variability in results, therefore this number game checklist was retained for further analysis. These additional home environment relevant dimensions brought the total dimensions to 7, as follows: (1) parent expectation, (2) literacy, (3) counting ability, (4) parent-child teaching methods, (5) target child-sibling interactions, (6) frequency of numeracy activities questions and (7) informal home numeracy practice.

Content validity considered whether appropriate questions were asked in the PHMQ (Nunes et al., 2005). Hence, the PHMQ needed to reflect whether the parents were asked questions that show a sufficient and comprehensive description of their views and experiences (Nunes et al., 2005). The validation process involved participants completing both an interview and the PHMQ and comparing the home environment relevant dimensions with those emerging in the interviews (Nunes et al., 2005); the main focus was on the content validity of the frequency of numeracy activities. The content validity analysis demonstrated that the dimensions included in the PHMQ were raised and discussed by parents in the interviews. The parents in the interviews spontaneously raised only one new item, this referred to children watching a range of videos on YouTube, including educational videos. This is consistent with Ofcom (2016) statistics that reports that YouTube was predominantly used by children of a similar age to the participants in this study (i.e. 37% of 3 to 4-year-olds and 54% of 5 to 7-year-olds use the YouTube app or website). The item ‘Maths related YouTube videos (e.g. NumTums)’ was added to the frequency of numeracy activities scale (see section 6.3.3.1.2 The additional ‘Maths related YouTube videos (e.g. NumTums)’ item for more details on the placement of this item). There were other themes that developed during the analysis of the interviews. These themes replicated the findings in the first study interviews (Chapter 3; Cahoon et al., 2017) and thus were already included in the PHMQ. The themes were; the home numeracy environment structure, frequency of number-related experiences compared to reading, parent-child teaching methods and target child-sibling interaction. In Chapter 3 the themes that cover this content were; (theme one) numeracy environment structure (theme two) frequency of number-related experiences, (theme five) parent-child interactions and (theme six) social interaction, respectively.

Criterion validity investigated contrast cases of parents with very high or very low scores on each of the subscales within a frequency of numeracy activities scale and compares the contrasting cases to the interview responses (Nunes et al., 2005). There were clear differences between the views and experiences of parents with low and high scores across all five subscales: (1) parent - child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting.

7.4.2 Study Implications, Contributions and Future Recommendations.

The PHMQ measure demonstrates construct, content, criterion validity and satisfies APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998), which was the ultimate objective of this scale development and validation process. The PHMQ can allow researchers to obtain data in a quantifiable way quickly to further understand how parents contribute to their child learning numeracy related concepts and skills.

7.5 Chapter 6

7.5.1 Research Summary.

7.5.1.1 Confirmatory factor analysis.

An exploratory factor analysis alone does not consider the goodness of fit of the resulting factor structure (Long, 1983). Therefore, Chapter 6 begins with a confirmatory factor analysis that was conducted in Mplus; allowing each item to be placed in the factor suggested by the exploratory factor analysis to test if the model fits. This type of model is also known as multitrait model in which each item is restricted to load only on its appropriate factor (Hinkin, 1998). The five-factor model corresponded to the five subscales found in the previous chapter (Chapter 4) through the exploratory factor analysis. Based on model fit statistics and modification indices the five-factor model was deemed a suitable measurement model. The new item (i.e. ‘Maths related YouTube videos (e.g. NumTums)’) found during the content validity (stage 5) was placed within the *computer maths games* subscale, based on model fit statistics and modification indices. Overall, the five subscales demonstrated a comprehensive coverage of numeracy related activities occurring in the home.

7.5.1.2 Learner profiles and learning pathways.

The fourth aim of the thesis was to describe children’s precise *learner profiles* and *learning pathways*. Latent profile and latent transition analyses were initially used to identify and describe children’s precise *learner profiles*. In both analyses, the best model fit was a three-

learner profile solution, representing an *advanced number skills profile, intermediate number skills profile* and *low number skills profile* (i.e. high, medium and low number skills). Five plausible pathways were discovered during the latent transition analysis which were called: consistently low number skills pathway (pathway 1), low to intermediate number skills shifting pathway (at school-entry) (pathway 2), consistently intermediate number skills pathway (pathways 3), intermediate to advanced number skills shifting pathway (at school-entry) (pathway 4) and consistently advanced number skills (pathway 5). The three consistent pathways showed children staying in their *learner profile* across all three time points and the two transition pathways displayed a shift to the subsequent learner profile upon school-entry. The two transitions between profiles were observed at school-entry perhaps due to children consolidating their number skills at school-entry through practising and recalling new information in different ways (Howard-Jones et al., 2018).

7.5.1.3 Predictors of pathway membership.

The fifth aim of the thesis was to identify the key predictors of children's *pathway* membership over time. A multivariate multinomial logistic regression was completed to address the adjusted associations between pathway membership and demographics and child measure predictors. The findings were younger children were more likely to be in pathways 1 and 2 than pathway 5. These two pathways (i.e. pathways 1 and 2) represented lower number skills and thus it can be concluded that younger children are less likely to develop their number skills early on. Another finding was that as children score higher in receptive vocabulary, they are more likely to be in the consistently advanced number skills pathway (pathway 5) than the lower number skills pathways (i.e. pathways 1 and 2). This is consistent with growingly popular ethnomathematical literature that indicates that language may impact the learning of mathematics (e.g. Ascher & D'Ambrosio, 1994; Kim, Ferrini-Mundy & Sfard, 2012).

A particularly interesting result is in relation to working memory. In the adjusted multivariate model, children in the school-entry transition pathways (i.e. pathways 2 and 4) were significantly associated with working memory when compared to pathway 5. Therefore, those children in the consistently advanced number skills pathway have higher working memory than pathways 2 and 4. This is consistent with previous literature that proposes that those with higher working memory capacity performed better on difficult mathematics problems (Osei-Boadi, 2016). Additionally, however, pathways 2 and 4 are the school-entry transition pathways and these pathways were associated with working memory when compared to pathway 5 but the two consistent pathways (i.e. pathway 1 and 3) were not associated with working memory when compared to pathway 5. This demonstrates that working memory skills could enable the shift in mathematical learning pathways at school-entry. Potentially the most surprising finding is that there was no statistically significant relationship between parents reported frequency of numeracy activities with pathway membership at a bivariate level, however, this can be explained (see next section 7.5.2 Study Implications).

7.5.2 Study Implications, Contributions and Future Recommendations.

Overall, the results indicate that there is no one factor solely driving the longitudinal mathematical development. Findings highlighted the importance of language and working memory abilities on mathematical skills development over time. Therefore, to enhance children's mathematical development it is suggested that early years practitioners should focus on boosting language and working memory skills, in order to assist pre-school children's mathematical skill development. As previously stated, interventions targeting children's numeracy learning are lacking (Niklas et al., 2016; Starkey & Klein, 2000) and more information was needed to distinguish *what* number-related experiences these interventions should focus on. Based on the findings from this study future interventions in pre-schools, or early education programs, could focus on improving language and working memory skills as these may transfer to higher mathematics skills.

Although, it remains unclear what types of training interventions work (i.e. training in specific mathematics skills or working memory skills). Currently, there is little indication that the training of general cognitive functions (i.e. working memory or language) transfer to mathematical learning (Raghubar & Barnes, 2017). For instance, previous research (e.g. Cunningham & Sood, 2018) indicated that although working memory improved during a working memory training intervention programme, working memory also improved for the control group who did not receive training and there was no significant transfer to mathematical ability. Furthermore, it has been found that mathematical specific interventions were most effective for improving early numeracy (Raghubar & Barnes, 2017), thus is may be more important to focus on these individual component skills.

The frequency of numeracy activities scale was not significantly associated with pathway membership. Most studies involving questionnaire measures have resulted in inconsistencies between home activities and children's number skills (e.g. Kleemans et al., 2012; Missall et al., 2015). An important reason for the discrepancy between results may involve the different ages of the children in these studies. Some research has found unique and positive associations between the HNE and mathematics skills with 4 years 10-months to 8 years olds (Kleemans et al., 2012; Dearing et al., 2012; Manolitsis et al., 2013; Niklas & Schneider, 2014). Whereas, Missall et al. (2015) in a study involving children aged 3 to 5 years 7-months found no relation between mathematics-related activities in the home and a range of numeracy skills, but this does not negate the possibility that the relationship between HNE and achievement be observed later in development. In the current study children were aged between 4 years and 4 years 7-month, similar to that of Missall et al. (2015). In the future, it would be recommended that a fourth time point be added to understand if there would be an association between pathway membership and the frequency of numeracy activities scale when children are older. To conclude, although parents are engaging in home numeracy activities, perhaps children are too young at 4-years-old for an association to occur.

7.6 Strengths and Limitations

There are a number of general strengths and weaknesses within this thesis. A strength of the first study (Chapter 3) was that rigorous qualitative research methods were used, and the resulting thematic analysis had strong inter-rater reliability. The development of the PHMQ moved beyond the scope of previous questionnaires by using an inductive approach and creating a measure of informal mathematics exposure, measured through the new number games checklist relevant to the United Kingdom. Further, the steps of questionnaire development and validation (Chapters 4 and 5) that address construct, content and criterion validity were followed carefully using Hinkin (1998) and Nunes et al. (2005) studies as a guide. Previous frequency of number activities scales that are available have rarely been validated beyond construct validity (e.g. LeFevre et al., 2009). Schoenfeldt (1984, p.78) stated that “the construction of the measuring devices is perhaps the most important segment of any study”. Therefore, the Pre-school Home Maths Questionnaire (PHMQ), in particular the frequency of numeracy activities scale, was evaluated across five psychometric properties and therefore satisfies APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998). As with all questionnaire methods the PHMQ, is a self-report measure of the HNE and could be subject to social desirability bias. However, the PHMQ can allow researchers to obtain data in a quantifiable way quickly to further understand how parents contribute to their child’s learning. Overall, the PHMQ is an inclusive measure of the HNE and although only two sections of the PHMQ were utilised in the final longitudinal study in the future more sections could be explored to better understand the influences of children’s early mathematical development, however, this was beyond the scope of this thesis.

The final study (Chapter 6) advances understanding by addressing limitations imposed by only analysing the average child performance (i.e. a variable-centered approaches; Lindblom-Ylänne et al., 2015). Through use of a latent profile and latent transition analysis children's precise *learner profiles* and *learning pathways* were identified. Therefore, this study presents new findings as there are no existing publications that investigate mathematical *learner profiles* and *learning pathways* during this pre-school to school transition. 152 children were successfully recruited into the study; this is a relatively large sample size in the context of previously published longitudinal studies. However, in the context of latent profile and transition analyses the sample size is limited. Due to this limitation the results for the socio-economic proxies' predictors used within the study (i.e. SES and HEQ) are to be taken with caution due zero frequencies or *near to* zero frequencies among some SES and HEQ categories. It is recommended that in future research a larger sample should be collected to explore the causal effects of SES and HEQ on pathway membership. In addition, a larger sample size would also allow for the consideration of the new three-step specification method (Vermunt, 2010; Asparouhov & Muthén, 2014; previously discussed in section 6.4.5 Limitations). Although this limitation has been acknowledged an appropriate statistical alternative was used to deal with the measurement parameter shift problem and thus predictors did not impact the nature of the model (Marsh et al., 2009; Morin et al., 2011; Ryoo, Wang, Swearer, Hull & Shi, 2018; see section 6.4.5.4 Latent transition analysis – three-step method for more details).

7.7 Conclusion

As noted in Chapter 2 of this thesis (sections 2.1.2 Bronfenbrenner's Bioecological Systems Theory and 2.1.10 Rationale), the Process-Person-Context-Time (PPCT) model (Bronfenbrenner, 2005) was considered an appropriate theoretical framework for this study. *Proximal processes* were assessed through the semi-structured interviews and in turn the frequency of numeracy activities section of the PHMQ. *Person* characteristics were measured through some basic demographic characteristics (i.e. child's gender), cognitive skills (i.e. working memory) and mathematical specific skills (i.e. cardinality). The *context* was investigated through two of the multilevel nested systems; the microsystem and macrosystem. The microsystem was explored through assessing the home environment. The macrosystem was measured through other demographic characteristics such as economic conditions (i.e. SES) and material resources (i.e. checklists from the PHMQ). *Time* was also included within this study due to the longitudinal nature of the research project; therefore, the interrelated impact of each proximal process, person, and context over time was investigated

(Bronfenbrenner, 1994; Tudge et al., 2009). Therefore, this thesis allows for a holistic view of the relationship between multiple factors and children's early mathematical development.

This thesis provides invaluable information for understanding the developmental changes in numeracy learning and identifies *what* component skills contribute to early mathematical development during the transition from pre-school to school education. Overall the findings from this thesis (Chapters 3-5) show that the HNE is very broad. However, findings from the longitudinal study (Chapter 6) suggest children are perhaps too young (on average 4 years old) for this engagement to make a difference to their learning trajectories. Findings also highlight the importance of language and working memory abilities on mathematical development over time, particularly for younger pre-school children who were in lower number skills pathways. This has been shown to be important in previous research as children who enter pre-school with low performance in basic number skills stay behind their peers throughout later school years (Aunola et al., 2004; Chong & Siegel, 2008; Jordan et al., 2006, 2007; Morgan et al., 2009). In summation, the lack of evidence for effective training interventions indicates that more research is necessary to make differences to early mathematic development. However, this current research suggests that support tailored toward language and working memory development within early years education may be a promising avenue to focus research attention in order to generate effective interventions to improve children's mathematical development.

Appendices

Chapter 3

Appendix 3.1 Topic guide questions.

Topic guide questions:

1. Do you think your child is interested in maths? If so, why?
2. Would your child play number games? If so, what number games would be played?
3. Under what circumstances, would maths games be played?
4. What kind of interactions do you use to encourage and support your child to learning numbers?
5. Can you compare the frequency and structure of mathematical activities to reading at home?

Appendix 3.2 Codes generated using thematic analysis and definitions of codes.

Appendix Table 1. *Codes generated using thematic analysis and definitions of codes*

No. of code	Code	Definitions	Theme
1	Types of activities	Everyday number-related activities that occur in the home	Theme 1. Numeracy environment structure
2	Parent views	Parents views and experiences of numeracy-related activities	Theme 1. Numeracy environment structure
3	Numeracy environment structure	The types of environments that parents create for their children to learn numeracy in the home	Theme 1. Numeracy environment structure
4	Frequency of maths activities	The frequencies of numeracy-related experiences	Theme 2. Frequency of number-related experiences
5	Comparison of literacy-related and numeracy-related experiences	The frequencies of literacy-related experiences compared to number-related experiences	Theme 2. Frequency of number-related experiences
6	Numerical content in literacy	The frequencies of literacy-related experiences including potential overlap with numeracy-related experiences	Theme 2. Frequency of number-related experiences
7	Understanding numbers through rhythm	A parent's viewpoint of their child's understanding of number knowledge, including number words and their meanings	Theme 3. Levels of number knowledge
8	Views of technology	How views of technology may affect technology usage in the home	Theme 4. Views of technology
9	Technology limiting time	Limiting the duration of technology usage	Theme 4. Views of technology
10	Parent-child interactions	The types of interactions that occur between parent and child	Theme 5. Parent-child interactions
11	Adjusting behaviours	The way in which parents aided their child's learning, including demonstrating the numerical problem visually	Theme 5. Parent-child interactions
12	Provide answer	A parent providing the answer	Theme 5. Parent-child interactions
13	Explaining	A parent explaining a scenario to aid numerical understanding	Theme 5. Parent-child interactions
14	Encourage through questions	Parents encouragement and reassurance through seemingly though-provoking questions to enable child to answer numeracy problems	Theme 5. Parent-child interactions
15	Maths initiation and guidance	Who initiates and guides numeracy-related activities	Theme 5. Parent-child interactions
16	Parent lead	How a parent leads numeracy-related activities	Theme 5. Parent-child interactions
17	Child lead	How a child leads numeracy-related activities	Theme 5. Parent-child interactions
18	Social interactions	Triad numerical interaction occurring between parent, target child and older sibling/s in the home	Theme 6. Social interaction

Chapter 4

Appendix 4.1 Original PHMQ.

Instructions: Please complete the following questionnaire, answering all questions.
This questionnaire will take approximately 15 minutes to complete.
These questions are in relation to your child who is aged 3-4 years old
Please tick or circle the choice that best describes ***your family***.

ABOUT YOU

1. What age are you? ____

2. What is your relationship to the participating child:

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other:	

3. What is your current marital status?

(a) Single (never married)	
(b) Married	
(c) Cohabitating (not married)	
(d) Divorced	
(e) Separated	
(f) Widowed	

4. Are you the primary carer? (e.g. Spend most of the time with child)

(a) Yes	
(b) No	

5. What is your ethnic origin?

(a) Asian	
(b) Black or African American	
(c) White, Caucasian	
(d) Chinese	
(e) Mixed	
(f) Other:	

6. What is the first language you speak with your child?

(a) English	
(b) Irish	
(c) Spanish	
(d) French	
(e) Polish	
(f) Other:	

7. What is your highest educational qualification?

6. GCSEs / O level / Irish Junior Certificate	
7. A levels / BTEC / Irish Leaving Certificate	
8. Degree	
9. Masters	
10. PhD	
11. No qualifications	
12. Other:	

8. What is your highest level of mathematical achievement? (Including degrees that involve statistics)

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other:	

9. Are you currently employed? **If <u>currently</u> employed proceed to question 12.**

(a) Yes full-time	
(b) Yes part-time	
(c) No	

10. If no, have you previously been employed? **If <u>previously</u> employed proceed to question 12.**

(a) Yes	
(b) No	

11. If no, do you provide full-time child-care? **If <u>full-time carer</u>, please proceed to question 20.**

(a) Yes	
(b) No	

<u>Details of current/previous employment</u>

12. What is/was your main job title?

____

13. What activities do/did you mainly do in your job?

____

14. What does/did the firm/organisation you worked for mainly make or do? (e.g. Provide leisure services, retail industry, education)

____

15. Are/were you working as an employee or are/were you self-employed?

(a) Employee	
(b) Self-employed	

If Employee – Go to question 16

If Self-employed – Go to question 18

<u>Employee only</u>

16. In your job, do/did you have any <u>formal</u> responsibility for supervising the work of other employees?

(a) Yes	
(b) No	

17. How many people work/worked for the employer at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

<u>Self-employed only</u>

18. Are/were you working on your own or do/did you have employees?

(a) On own	
(b) With partner	
(c) No employees	
(d) Employees	

(e) Other:	

19. If you have/had employees, how many people do/did you employ at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

OTHER ADULTS LIVING IN HOUSEHOLD

20. Are there other adults living in your household?

(a) Yes		**If Yes – please continue**
(b) No		**If No – Go to question 24**

21. Is this adult currently employed?

(a) Yes full-time		**If No – Go to question 23**
(b) Yes part-time		
(c) No		

22. What is their occupation?

____

23. Person's relationship to child?

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other:	

ABOUT YOUR PARTICIPATING CHILD

These questions are in relation to your child who is aged 3-4 years old

24. When was your child born? ____/____/______ (Day/Month/Year)

25. Including the child in question, how many children do you have in total?

Total number of children: ____________________

26. What is the birth order of your participating child aged 3 – 4?

(a) First born (oldest)	
(b) Second born	
(c) Third born	
(d) Fourth born	
(e) Fifth born	
(f) Only child	
(g) Other:	

27. What is your participating child's gender?

(a) Male	
(b) Female	

28. How many languages can your participating child speak?

(a) One	
(b) Two	
(c) Other:	

29. What are these languages? ____

30. Ideally, how much education would you want your participating child to complete?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other:	

31. Ideally, what would you want your participating child's highest mathematical achievement to be?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other:	

LITERACY

32. In the past month, how often did you and your child engage in reading? ***Please circle***

activity did not occur | few times a month | about once a week | few times a week | almost daily

|____________|____________|____________|____________|

33. Do any of the books you read to the participating child involve numbers?

(a) Yes	
(b) No	

If Yes – How many? __________

34. Would you do maths activities more or less than reading?

(c) More	
(d) Less	
(e) Same	

NUMERACY

35. How high can your child currently count up to?

____

36. Did you ask your child to count to answer the above question?

(a) Yes	
(b) No	

37. How high do you think a child <u>at your child's age</u> should be able to count?

____

PARENT – CHILD INTERACTION

38. Who is more likely to bring up numeracy activities?

(a) You	
(b) Your child	
(c) Both	
(d) Other:	

39. Imagine you have asked your child a sum and they get the answer wrong, what are the specific things you say or do to encourage and support your child to learn maths?

Please order the following options in the order you would use each. ***1 - 'most likely' 4 - 'least likely'***

	Insert number below
(a) Question and encourage your child without explaination (e.g. "No that's not the right answer, what number do you think it would be?")	
(b) Prompt, explain and work through the problem together (e.g. Make sure he/she understand where they went wrong)	
(c) Provide answer and move on	
(d) Adjust your behaviour (e.g. demonstrate visually with objects/fingers)	

FREQUENCY OF HOUSEHOLD ACTIVITIES

40. In the past month, how often did you and your child engage in the following? *Please circle*

1. Counting

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Feeding objects (e.g. posting letters)

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Hopscotch

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Write numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?")

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

6. Counting on fingers/hands

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

7. Watching number related TV shows (e.g. Number Jacks or Numtums)

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

8. Teaching about measurements (e.g. baking, height)

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

9. Sticker books

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

10. Counting out turn taking (e.g. jumping to ten on trampoline)

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

11. Sorting shapes

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

12. Rhyming TV shows involving numbers (e.g. Number Jacks)

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

13. Using number cards (e.g. order the cards by number)

activity did not occur few times a month about once a week few times a week almost daily
|____________________|_________________|_________________|_________________|

14. Play with jigsaws

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

15. Rhyming storybooks (e.g. Dr Seuss)

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

16. Dot-to-dot number books

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

17. Watch educational programs (e.g. Dora the Explorer)

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

18. Sorting objects by size

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

19. Counting up stairs

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

20. Comparing sets of objects (e.g. brother has more than mum)

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

21. Pairing/matching games

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

22. Play card games (e.g. "jack change it")

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______|______|______|______|

23. Playing with building blocks

activity did not occur | few times a month | about once a week | few times a week | almost daily

24. Identifying names of written numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

25. Counting out food, dinner plates, knifes and forks

activity did not occur | few times a month | about once a week | few times a week | almost daily

26. Rhyming songs including counting (e.g. “1, 2, 3, 4, 5 once I caught a fish alive” or “ten green bottles”)

activity did not occur | few times a month | about once a week | few times a week | almost daily

27. Creating patterns with objects (e.g. arranging blocks into shapes)

activity did not occur | few times a month | about once a week | few times a week | almost daily

28. Being timed (e.g. hide and seek)

activity did not occur | few times a month | about once a week | few times a week | almost daily

29. Counting objects (e.g. ducks in bath, blocks, new toys, books)

activity did not occur | few times a month | about once a week | few times a week | almost daily

30. Teaching about money (e.g. informal – playing shop or formal – buying sweeties)

activity did not occur | few times a month | about once a week | few times a week | almost daily

31. Time terminology (e.g. big hand, little hand)

activity did not occur | few times a month | about once a week | few times a week | almost daily

32. Asking shape related questions (e.g. “how many sides does a circle have?”)

activity did not occur | few times a month | about once a week | few times a week | almost daily

TECHNOLOGY

41. The following questions are all relating to technology usage (computers, tablets, smart phones). If your child **does not** use technology, please go to question 43.

In the past month, how often did you and your child engage in the following? ***Please circle***

1. Maths applications (e.g. Number Jacks)

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Maths related websites (e.g. coolmaths.com)

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Racing games (e.g. faster they complete sums the faster the boat moves)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Size/matching apps (e.g. “put the big skirt on the small girl”)

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Add and subtraction games

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Filling in the gap number games (e.g. what is next in the sequence?)

activity did not occur | few times a month | about once a week | few times a week | almost daily

SIBLINGS

42. Do you feel that your child has learnt number skills from their siblings?

(a) Yes	
(b) No	
(c) Does not apply	

If this does not apply – Go to question 46

43. What would your participating child (aged 3 – 4) be more likely to do when engaged in a mathematical based activity with siblings?

(a) Observe the activity	
(b) Take part in the activity	

44. When your children are interacting mathematically, what types of activities are they most likely to do together?

Please order the following options in order. ***1 - 'most likely' 4 - 'least likely'***

	Insert number below
a) Counting objects together	
b) Arranging objects by size, shape or colour	
c) Observing older siblings homework	
d) Taking part in older siblings homework	
e) Maths applications on technology device (e.g. Playing Number Jacks on iPhone)	
f) Watching number related TV shows together (e.g. Number Jacks or Numtums)	
g) Sing rhyming songs together (e.g. "1, 2, 3, 4, 5 once I caught a fish alive")	
h) Reading books together that involve numbers (e.g. Hungry Caterpillar)	
i) Play board games or card games together (e.g. "jack change it")	
j) Timed games (e.g. hide and seek)	
k) Everyday activities that involve number (e.g. using money while shopping)	

UNDERSTANDING

45. Do you believe that your child understands the meaning of number words up to 5?

(a) Yes	
(b) No	

46. Do you believe that your child understands the meaning of odd and even?

(a) Yes	
(b) No	

47. Do you believe that your child understands the meaning of more and less? (e.g. one pile of clothes bigger than another set of clothes)

(a) Yes	
(b) No	

SUPPORT

48. Do you believe it is important for caregivers to support numeracy learning in the home?

(a) Yes	
(b) No	

Thank you for taking the time to fill in this questionnaire!

Appendix 4.2 Summary of items, how they were generated and initial item reduction criteria.

Appendix Table 2. *Summary of items, how they were generated and initial item reduction criteria*

Qu No. from the original PHMQ	Items with home numeracy dimension category breakdown	Stage 1: Item Generation; Inductive or Deductive approach *	Stage 1: Deductive items overlap with literature	Stage 3: Initial Item Reduction; Kept or Removed **	Stage 3: Initial Item Reduction; Reason for removal
	Parent expectations – Benchmark questions				
30	Ideally, how much education would you want your participating child to complete?	Inductive		Kept	/
31	Ideally, what would you want your participating child's highest mathematical achievement to be?	Inductive		Kept	/
	Literacy – Benchmark questions				
32	In the past month, how often did you and your child engage in reading?	Deductive	LeFevre et al., 2009	Kept	/
33	Do any of the books you read to the participating child involve numbers?	Inductive		Kept	/
33a	If Yes – How many?	Inductive		Kept	/
34	Would you do maths activities more or less than reading?	Inductive		Kept	/
	Numeracy – Benchmark questions				
35	How high can your child currently count up to?	Deductive	LeFevre et al., 2009	Kept	/
36	Did you ask your child to count to answer the above question?	Deductive	LeFevre et al., 2009	Kept	/
37	How high do you think a child at your child's age should be able to count?	Inductive		Kept	/
	Parent-child interaction – Interaction questions				
38	Who is more likely to bring up numeracy activities?	Inductive		Kept	/
39	What are the specific things you say or do to encourage and support your child to learn maths?			Kept	/
39a	Question and encourage your child without explanation	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
39b	Prompt, explain and work through the problem together	Deductive	Vandermaas-Peeler et al., 2012	Kept	/

39c	Provide answer and move on	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
39d	Adjust your behaviour	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
	Frequency of household activities				
40	In the past month, how often did you and your child engage in the following?				
1	Counting	Deductive	Melhuish et al., 2008	Kept	/
2	Feeding objects (e.g. posting letters)	Inductive		Removed	EFC
3	Hopscotch	Inductive		Removed	EFC
4	Write numbers	Deductive	LeFevre et al., 2009	Kept	/
5	Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?")	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
6	Counting on fingers/hands	Inductive		Kept	/
7	Watching number related TV shows (e.g. Number Jacks or Numtums)	Inductive		Kept	/
8	Teaching about measurements (e.g. baking, height)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
9	Sticker books	Inductive		Kept	/
10	Counting out turn taking (e.g. jumping to ten on trampoline)	Inductive		Removed	EFC
11	Sorting shapes	Deductive	LeFevre et al., 2009; Kleemans, Peeters, Segers & Verhoevena., 2012; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/

12	Rhyming TV shows involving numbers (e.g. Number Jacks)	Inductive		Kept	/
13	Using number cards (e.g. order the cards by number)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Removed	EFC
14	Play with jigsaws	Inductive		Kept	/
15	Rhyming storybooks (e.g. Dr Seuss)	Inductive		Removed	EFC
16	Dot-to-dot number books	Deductive	LeFevre et al., 2009	Removed	EFC
17	Watch educational programs (e.g. Dora the Explorer)	Deductive	LeFevre et al., 2009	Kept	/
18	Sorting objects by size	Deductive	LeFevre et al., 2009	Kept	/
19	Counting up stairs	Inductive		Removed	EFC
20	Comparing sets of objects (e.g. brother has more than mum)	Inductive		Kept	/
21	Pairing/matching games	Inductive		Kept	/
22	Play card games (e.g. “jack change it”)	Deductive	LeFevre et al., 2009	Removed	EFC
23	Playing with building blocks	Deductive	LeFevre et al., 2009	Kept	/
24	Identifying names of written numbers	Deductive	LeFevre et al., 2009	Kept	/
25	Counting out food, dinner plates, knifes and forks	Inductive		Kept	/
26	Rhyming songs including counting (e.g. “1, 2, 3, 4, 5 once I caught a fish alive” or “ten green bottles”)	Deductive	Kleemans, Peeters, Segers & Verhoevena., 2012; Melhuish et al., 2008	Removed	EFC
27	Creating patterns with objects (e.g. arranging blocks into shapes)	Inductive		Kept	/
28	Being timed (e.g. hide and seek)	Deductive	LeFevre et al., 2009	Removed	EFC
29	Counting objects (e.g. ducks in bath, blocks, new toys, books)	Deductive	LeFevre et al., 2009	Kept	/

30	Teaching about money (e.g. informal – playing shop or formal – buying sweeties)	Deductive	LeFevre et al., 2009	Kept	/
31	Time terminology (e.g. big hand, little hand)	Deductive	Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
32	Asking shape related questions (e.g. "how many sides does a circle have?")	Inductive		Kept	/
	Frequency of technology				
41	In the past month, how often did you and your child engage in the following?				
1	Maths applications (e.g. Number Jacks)	Inductive		Kept	/
2	Maths related websites (e.g. coolmaths.com)	Inductive		Kept	/
3	Racing games (e.g. faster they complete sums the faster the boat moves)	Inductive		Kept	/
4	Size/matching apps (e.g. "put the big skirt on the small girl")	Inductive		Kept	/
5	Add and subtraction games	Inductive		Kept	/
6	Filling in the gap number games (e.g. what is next in the sequence?)	Inductive		Kept	/
	Siblings – Interaction questions				
42	Do you feel that your child has learnt number skills from their siblings?	Deductive	Benigno et al. (2004)	Kept	/
43	What would your participating child (aged 3 – 4) be more likely to do when engaged in a mathematical based activity with siblings?	Inductive		Removed	Lack of variation in responses
44	When your children are interacting mathematically, what types of activities are they most likely to do together?				
44a	Counting objects together	Inductive		Kept	/
44b	Arranging objects by size, shape or colour	Inductive		Kept	/
44c	Observing older siblings homework	Inductive		Removed	Lack of variance; Least likely to occur in the home
44d	Taking part in older siblings homework	Inductive		Removed	Lack of variance; Least likely to occur in the home

44e	Maths applications on technology device (e.g. Playing Number Jacks on iPhone)	Inductive	Removed	Lack of variance; Least likely to occur in the home
44f	Watching number related TV shows together (e.g. Number Jacks or Numtums)	Inductive	Kept	/
44g	Sing rhyming songs together (e.g. “1, 2, 3, 4, 5 once I caught a fish alive”)	Inductive	Kept	/
44h	Reading books together that involve numbers (e.g. Hungry Caterpillar)	Inductive	Kept	/
44i	Play board games or card games together (e.g. “jack change it”)	Inductive	Removed	Lack of variance; Least likely to occur in the home
44j	Timed games (e.g. hide and seek)	Inductive	Kept	/
44k	Everyday activities that involve number (e.g. using money while shopping)	Inductive	Kept	/
	Understanding			
45	Do you believe that your child understands the meaning of number words up to 5?	Inductive	Removed	Lack of variation in responses
46	Do you believe that your child understands the meaning of odd and even?	Inductive	Removed	Lack of variation in responses
47	Do you believe that your child understands the meaning of more and less?	Inductive	Removed	Lack of variation in responses
	Support			
48	Do you believe it is important for caregivers to support numeracy learning in the home?	Inductive	Removed	Lack of variation in responses

Note: * Inductive items = 44 items; Deductive items = 25; Total items = 69. ** Inductive items removed = 14; Deductive items removed = 5; Total items after removal = 50.

Chapter 5

Appendix 5.1 Final PHMQ.

Instructions: Please complete the following questionnaire, answering all questions.
This questionnaire will take approximately 15 minutes to complete.
These questions are in relation to your child who is aged 3-4 years.
Please tick or circle the choice that best describes ***your family***.

ABOUT YOU

1. What age are you? ______________________________

2. What is your relationship to the participating child?

(a)	Mother	
(b)	Stepmother	
(c)	Father	
(d)	Stepfather	
(e)	Grandparent	
(f)	Foster parent	
(g)	Adoptive parent	
(h)	Other, please state:	

3. What is your current marital status?

(a)	Single (never married)	
(b)	Married	
(c)	Cohabitating (not married)	
(d)	Divorced	
(e)	Separated	
(f)	Widowed	

4. Are you the primary carer? (e.g. Spend most of the time with the child)

(a)	Yes	
(b)	No	

5. What is your ethnic origin?

(a)	Asian	
(b)	Black or African American	
(c)	White, Caucasian	
(d)	Chinese	
(e)	Mixed	
(f)	Other, please state:	

6. What is the first language you speak with your child?

(a) English	
(b) Irish	
(c) Spanish	
(d) French	
(e) Polish	
(f) Other, please state:	

7. What is your highest educational qualification?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

8. What is your highest level of mathematical achievement? (Including any degree that involves statistical training)

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

9. Are you currently employed? **If <u>currently</u> employed proceed to question 12.**

(a) Yes full-time	
(b) Yes part-time	
(c) No	

10. If no, have you previously been employed? **If <u>previously</u> employed proceed to question 12.**

(a) Yes	
(b) No	

11. If no, do you provide full-time child-care? **If <u>full-time carer</u>, please proceed to question 20.**

(a) Yes	
(b) No	

<u>Details of current/previous employment</u>

12. What is/was your main job title?

____

13. What activities do/did you mainly do in your job?

____

14. What does/did the firm/organisation you worked for mainly make or do? (e.g. Provide leisure services, retail industry, education)

____

15. Are/were you working as an employee or are/were you self-employed?

(a) Employee		**If Employee – Go to question 16**
(b) Self-employed		**If Self-employed – Go to question 18**

<u>Employee only</u>

16. In your job, do/did you have any <u>formal</u> responsibility for supervising the work of other employees?

(a) Yes	
(b) No	

17. How many people work/worked for the employer at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

Please continue to question 20

<u>Self-employed only</u>

18. Are/were you working on your own or do/did you have employees?

(a) On own	
(b) With partner	
(c) No employees	
(d) Employees	
(e) Other:	

19. If you have/had employees, how many people do/did you employ at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

<u>OTHER ADULTS LIVING IN HOUSEHOLD</u>

20. Are there other adults living in your household?

(a) Yes		**If Yes – please continue**
(b) No		**If No – Go to question 23**

21. Person’s relationship to child?

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other, please state:	

22. What is this adults highest educational qualification?

(a)	GCSEs / O level / Irish Junior Certificate	
(b)	A levels / BTEC / Irish Leaving Certificate	
(c)	Degree	
(d)	Masters	
(e)	PhD	
(f)	No qualifications	
(g)	Other, please state:	

ABOUT YOUR PARTICIPATING CHILD

These questions are in relation to your child who is aged 3-4 years.

23. When was your child born? ____/____/______ (Day/Month/Year)

24. Including the child in question, how many children do you have in total?

Total number of children: ____________________

25. What is the birth order of your participating child aged 3 – 4?

(a)	Only child	
(b)	First born (oldest)	
(c)	Second born	
(d)	Third born	
(e)	Fourth born	
(f)	Fifth born	
(g)	Other, please state:	

26. What is your participating child's gender?

(a)	Male	
(b)	Female	

27. How many languages can your participating child speak?

(a)	One	
(b)	Two	
(c)	Other, please state:	

28. What are these languages? ____

29. Ideally, how much education would you want your participating child to complete?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

30. Ideally, what would you want your participating child's highest mathematical achievement to be?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

LITERACY

31. In the past month, how often did you and your child engage in reading? ***Please circle***

activity did not occur | few times a month | about once a week | few times a week | almost daily

32. Do any of the books you read to the participating child involve numbers?

(a) Yes	
(b) No	

If Yes – How many? ___________ (give as number)

33. Would you do maths activities more or less than reading?

(a) More	
(b) Less	
(c) Same	

NUMERACY

34. How high can your child currently count up to?

____

35. Did you ask your child to count to answer the above question?

(a) Yes	
(b) No	

36. How high do you think a child <u>at your child's age</u> should be able to count?

____

37. Who is more likely to bring up numeracy activities?

(a) You	
(b) Your child	
(c) Both	
(d) Other:	

38. Imagine you have asked your child a sum and they get the answer wrong, what are the specific things you say or do to encourage and support your child to learn maths?

Please order the following options in the order you would use each. ***1 - 'most likely' to 4 - 'least likely'*** ***Please do not leave any blank***	*Insert number below*	*Example:*
(a) Question and encourage your child without explanation (e.g. "No that's not the right answer, what number do you think it would be?")		1
(b) Prompt, explain and work through the problem together (e.g. Make sure he/she understand where they went wrong)		2
(c) Provide answer and move on		3
(d) Adjust your behaviour (e.g. demonstrate visually with objects/fingers)		4

FREQUENCY OF HOUSEHOLD ACTIVITIES

39. In the past month, how often did you and your child engage in the following? *Please circle*

1. Counting

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Write numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Scenarios number games (e.g. “If I have two toy cars and I take one away, how many cars do I have?”)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Counting on fingers/hands

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Watching number related TV shows (e.g. Number Jacks or Numtums)

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Teaching about measurements (e.g. baking, height)

activity did not occur | few times a month | about once a week | few times a week | almost daily

7. Sticker books

activity did not occur | few times a month | about once a week | few times a week | almost daily

8. Sorting shapes

activity did not occur | few times a month | about once a week | few times a week | almost daily

9. Rhyming TV shows involving numbers (e.g. Number Jacks)

activity did not occur | few times a month | about once a week | few times a week | almost daily

10. Play with jigsaws

activity did not occur | few times a month | about once a week | few times a week | almost daily

11. Watch educational programs (e.g. Dora the Explorer)

activity did not occur | few times a month | about once a week | few times a week | almost daily

12. Sorting objects by size

activity did not occur | few times a month | about once a week | few times a week | almost daily

13. Comparing sets of objects (e.g. brother has more than mum)

activity did not occur | few times a month | about once a week | few times a week | almost daily

14. Pairing/matching games

activity did not occur | few times a month | about once a week | few times a week | almost daily

15. Playing with building blocks

activity did not occur | few times a month | about once a week | few times a week | almost daily

16. Identifying names of written numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

17. Counting out food, dinner plates, knifes and forks

activity did not occur | few times a month | about once a week | few times a week | almost daily

18. Creating patterns with objects (e.g. arranging blocks into shapes)

activity did not occur | few times a month | about once a week | few times a week | almost daily

19. Counting objects (e.g. ducks in bath, blocks, new toys, books)

activity did not occur | few times a month | about once a week | few times a week | almost daily

20. Teaching about money (e.g. informal – playing shop or formal – buying sweeties)

activity did not occur | few times a month | about once a week | few times a week | almost daily

21. Time terminology (e.g. big hand, little hand)

activity did not occur | few times a month | about once a week | few times a week | almost daily

22. Asking shape related questions (e.g. "how many sides does a circle have?")

activity did not occur | few times a month | about once a week | few times a week | almost daily

TECHNOLOGY

40. The following questions are all relating to technology usage (computers, tablets, smart phones). If your child **does not** use technology, please go to question 42.

In the past month, how often did your child engage in the following? ***Please circle***

1. Maths applications (e.g. Number Jacks)

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Maths related websites (e.g. coolmaths.com)

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Racing games (e.g. the faster they complete sums, the faster the boat moves)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Size/matching apps (e.g. "put the big skirt on the small girl")

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Add and subtraction games

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Filling in the gap number games (e.g. what is next in the sequence?)

activity did not occur | few times a month | about once a week | few times a week | almost daily

7. Maths related YouTube videos (e.g. NumTums)

activity did not occur	few times a month	about once a week	few times a week	almost daily

BOARD GAMES

41. Below you will see a list of games for nursery children. Some of these are popular children's games, and some are made up.

Please read the names and put a tick next to those games that you know to be real games. Do not guess, but only tick those you know.

It is extremely important that you answer without stopping to verify any games.

(a) Battleships	
(b) Beach Shelter	
(c) Buckaroo	
(d) Build A Beetle	
(e) Chasin' Cheeky	
(f) Croc Doctor	
(g) Crocodile Dentist	
(h) Doctor Pop-up	
(i) Dog Tales	
(j) Doh Nutters Game	
(k) Dominoes	
(l) Elefun	
(m) Exasperation	
(n) Frustration	
(o) Guess who?	
(p) Head to toe	
(q) Hungry Hungry Hippo	
(r) Kerplunk	
(s) Ludo	
(t) Mailman	
(u) Mashup	
(v) Monopoly Junior	
(w) Operation	
(x) Pepper Pigs	
(y) Pie Face	
(z) Pop-up Pirate	
(aa) Shark Chase	
(bb) Snakes and Ladders	
(cc) Spider Web Master	
(dd) The Mashin Max Game	

SIBLINGS

42. Do you feel that your child has learnt number skills from their siblings?

(a) Yes	
(b) No	
(c) Does not apply	

43. When your children are doing activities together that involve maths, what types of activities are they most likely to do together? Keeping this in mind, in the past month, how often have you and your child engage in the following? ***Please circle***

1. Counting objects together

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Arranging objects by size, shape or colour

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Watching number related TV shows together (e.g. Number Jacks or Numtums)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Sing rhyming songs together (e.g. "1, 2, 3, 4, 5 once I caught a fish alive")

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Reading books together that involve numbers (e.g. Hungry Caterpillar)

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Timed games (e.g. hide and seek)

activity did not occur | few times a month | about once a week | few times a week | almost daily

7. Everyday activities that involve number (e.g. using money while shopping)

activity did not occur | few times a month | about once a week | few times a week | almost daily

Thank you for taking the time to fill in this questionnaire!

Chapter 6

Appendix 6.1 Correlations between all longitudinal variables used with the children

Appendix Table 3. *Pearson zero-order correlations between all longitudinal variables used with the children*

	1. T1 BAS	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2. T3 BAS	.694**																							
3. T1 BPVS	.662**	.524**																						
4. T3 BPVS	.607**	.575**	.785**																					
5. T1 Cardinality	.773**	.646**	.609**	.602**																				
6. T2 Cardinality	.729**	.649**	.532**	.472**	.846**																			
7. T3 Cardinality	.656**	.765**	.477**	.483**	.623**	.704**																		
8. T1 DR	.634**	.544**	.430**	.384**	.751**	.720**	.631**																	
9. T2 DR	.642**	.537**	.430**	.391**	.751**	.722**	.668**	.872**																
10. T3 DR	.585**	.639**	.373**	.433**	.693**	.683**	.740**	.773**	.835**															
11. T1 Ordering	.480**	.448**	.393**	.330**	.494**	.399**	.380**	.512**	.493**	.383**														
12. T2 Ordering	.591**	.502**	.436**	.378**	.685**	.604**	.489**	.707**	.725**	.650**	.551**													
13. T3 Ordering	.634**	.668**	.503**	.528**	.665**	.640**	.694**	.688**	.661**	.737**	.471**	.658**												
14. T1 SFON	.160	.092	.113	.163	.220*	.200*	.081	.138	.161	.181*	.068	.095	.107											
15. T2 SFON	.148	.151	.067	.163	.168	.176*	.244**	.146	.197*	.231*	.133	.143	.191*	.181*										
16. T3 SFON	.162	.203*	.153	.235*	.273**	.204*	.182*	.209*	.175	.221*	.239**	.144	.241**	.213*	.307**									
17. T1 Mapping	.550**	.436**	.522**	.468**	.494**	.411**	.466**	.453**	.417**	.391**	.340**	.392**	.445**	.100	.118	.158								
18. T2 Mapping	.548**	.514**	.491**	.540**	.539**	.480**	.484**	.541**	.485**	.515**	.283**	.438**	.544**	.172	.139	.140	.609**							
19. T3 Mapping	.555**	.630**	.476**	.540**	.563**	.534**	.543**	.486**	.456**	.482**	.379**	.376**	.522**	.228*	.167	.227*	.530**	.615**						
20. T1 SA	-.210**	-.149	-.414**	-.386**	-.099	-.090	-.111	.032	.012	-.049	.057	-.040	-.146	-.050	-.103	-.049	-.317**	-.148	-.198*					
21. T2 SA	-288**	-.262**	-.345**	-.280**	-.309**	-.272**	-.193*	-.196*	-.212*	-.225*	-.205*	-.153	-.189*	-.074	-.138	-.071	-.291**	-.270**	-.372**	.294**				
22. T3 SA	-.301**	-.301**	-.276**	-.250**	-.196*	-.195*	-.232*	-.148	-.174	-.195*	-.147	-.139	-.212**	.055	-.081	-.005	-.204*	-.207*	-.347**	.141	.233*			
23. T1 WM	.493**	.350**	.425**	.383**	.454**	.408**	.269**	.339**	.312**	.302**	.393**	.425**	.293**	.011	.205*	.046	.426**	.373**	.241**	-.178*	-.294**	-.170		
24. T2 WM	.394**	.337**	.388**	.248**	.331**	.299**	.309**	.291**	.300**	.242**	.306**	.448**	.240**	.040	.131	.151	.306**	.336**	.288**	-.118	-.246**	-.180	.542**	
25. T3 WM	.420**	.443**	.412**	.392**	.401**	.330**	.368**	.317**	.361**	.343**	.260**	368**	.336**	.091	.157	.101	.388**	.397**	.284**	-.211*	-.264**	-.218*	.490**	.525**

Note: * p < .05 ** p < .01 (two-tailed). BAS = British Ability Scale, BPVS = British Picture Vocabulary Scale, DR = Digit Recognition, SFON = Spontaneous Focusing on Numerosity, SA = Sustained Attention, WM = Working Memory

References

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. Applied Measurement in Education, 14, 219–234.

Aikens, N. L., & Barbarin, O. (2008). Socioeconomic differences in reading trajectories: The contribution of family, neighbourhood, and school contexts. *Journal of Educational Psychology*, *100*(2), 235.

Akaike, H. (1987). Factor analysis and AIC. In *Selected Papers of Hirotugu Akaike* (pp. 371-386). Springer, New York, NY.

Alexander, K. L., Entwisle, D. R., & Dauber, S. L. (1993). First-grade classroom behavior: Its short-and long-term consequences for school performance. *Child development*, *64*(3), 801-814.

Alexander, K. L., Entwisle, D. R., Blyth, D. A., & McAdoo, H. P. (1988). Achievement in the first 2 years of school: Patterns and processes. *Monographs of the society for Research in Child Development*, i-157.

Allen, G. (2011). *Early intervention: the next steps, an independent report to Her Majesty's government by Graham Allen MP*. The Stationery Office. http://www.gov.uk/government/publications/early-intervention-the-next-steps--2.

American Psychological Association (APA). (1995). Standards for educational and psychological testing. Washington, DC: Author.

Anders, Y., Rossbach, H. G., Weinert, S., Ebert, S., Kuger, S., Lehrl, S., & von Maurice, J. (2012). Home and preschool learning environments and their relations to the development of early numeracy skills. Early Childhood Research Quarterly, 27(2), 231-244.

Anderson, A. (1997). Families and mathematics: A study of parent-child interactions. *Journal for Research in Mathematics Education*, 484-511.

Anderson, A. G. (1998). Parents as partners: Supporting children's mathematics learning prior to school. *Teaching Children Mathematics*, *4*(6), 331.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. Nature Reviews Neuroscience, 9, 278–291.

Archbold, S., Lutman, M., Gregory, S., O'Neil, C., & Nikolpoulos, T. P. (2002). Parents and their deaf child: Three years after cochlear implantation. Deafness and Education International, 4, 12–40.

Arrindell, W. A., & Van der Ende, J. (1985). An empirical test of the utility of the observations-to-variables ratio in factor and components analysis. Applied Psychological Measurement, 9(2), 165-178.

Ascher, M., & D'Ambrosio, U. (1994). Ethnomathematics: A dialogue. *For the learning of mathematics*, *14*(2), 36-43.

Ashcraft, M. H., & Moore, A. M. (2012). Cognitive processes of numerical estimation in children. *Journal of experimental child psychology*, *111*(2), 246-267.

Asparouhov, T., & Muthén, B. (2014). Auxiliary variables in mixture modeling: Three-step approaches using M plus. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(3), 329-341.

Attout, L., Noël, M. P., & Majerus, S. (2014). The relationship between working memory for serial order and numerical development: A longitudinal study. *Developmental psychology*, *50*(6), 1667.

Aubrey, C., Bottle, G., & Godfrey, R. (2003). Early mathematics in the home and out-of-home contexts. *International Journal of Early Years Education*, *11*(2), 91-103.

Aulet, L. S., & Lourenco, S. F. (2018). The Developing Mental Number Line: Does its Directionality Relate to 5-to 7-Year-Old Children's Mathematical Abilities?. *Frontiers in Psychology*, *9*, 1142.

Aunio, P., & Räsänen, P. (2016). Core numerical skills for learning mathematics in children aged five to eight years–A working model for educators. European Early Childhood Education Research Journal, 24(5), 684-704.

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to Grade 2. *Journal of educational psychology*, *96*(4), 699.

Awang, Z. (2012). *Structural equation modeling using AMOS graphic*. Penerbit Universiti Teknologi MARA.

Ayoola, A. B., Adams, Y. J., Kamp, K. J., Zandee, G. L., Feenstra, C., & Doornbos, M. M. (2017). Promoting the Future of Nursing by Increasing Zest for Research in Undergraduate Nursing Students. *Journal of Professional Nursing*, *33*(2), 126-132.

Baddeley, A. (2003). Working memory: looking back and looking forward. *Nature reviews neuroscience*, *4*(10), 829.

Baillargeon, R., & Carey, S. (2012). Core cognition and beyond: The acquisition of physical and numerical knowledge. *Early childhood development and later outcome*, 33-65.

Baker, T.L. (1994), Doing Social Research (2nd Edn.), New York: McGraw-Hill Inc.

Bakermans-Kranenburg, M. J., van IJzendoorn, M. H., & Bradley, R. H. (2005). Those who have, receive: The Matthew effect in early childhood intervention in the home environment. *Review of Educational Research*, *75*(1), 1-26.

Ball, C. (1994). *Start Right: The Importance of Early Learning*. Lesley James, Royal Society for the Encouragement of Arts, Manufactures and Commerce.

Barchard, K. A., & Russell, J. A. (2006). Bias in consensus scoring, with examples from ability emotional intelligence tests. *Psicothema*, *18*.

Barchard, K. A., & Verenikina, Y. (2013). Improving data accuracy: Selecting the best data checking technique. Computers in Human Behavior, 29(5), 1917-1922.

Barkley, R. A. (1997). Behavioral inhibition, sustained attention, and executive functions: constructing a unifying theory of ADHD. *Psychological bulletin*, *121*(1), 65.

Baroody, A. J., & Ginsburg, H. P. (1986). The relationship between initial meaningful and mechanical knowledge of arithmetic.

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency? *Journal of experimental child psychology*, *117*, 12-28.

Barth, H., Beckmann, L., & Spelke, E. S. (2008). Nonsymbolic, approximate arithmetic in children: Abstract addition prior to instruction. *Developmental psychology*, *44*(5), 1466.

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics*, *92*(3), 331-345.

Batchelor, S., Inglis, M., & Gilmore, C. (2015). Spontaneous focusing on numerosity and the arithmetic advantage. *Learning and Instruction*, *40*, 79-88.

Batchelor, S., Keeble, S., & Gilmore, C. (2015). Magnitude representations and counting skills in preschool children. *Mathematical Thinking and Learning*, *17*(2-3), 116-135.

Bayley, N. (1993). *Bayley scales of infant development: manual*. Psychological Corporation.

Beldhuis, H. (2012). The Proceedings of the 11th European Conference on e-Learning.

Belsky, J., Vandell, D. L. Burchinal, M., Clarke-Stewart, K. A., McCartney, K., Owen, M. T., & the NICHD Early Child Care Research Network. (2007). Are there long-term effects of early child care? Child Development, 78, 681–701.

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of pre-schoolers' counting in middle-income families. *Early Childhood Research Quarterly*, *19*(1), 4-20.

Bentler, P. M. (1990). Comparative fit indexes in structural models. *Psychological bulletin*, *107*(2), 238.

Bergman, L. R., & Magnusson, D. (1997). A person-oriented approach in research on developmental psychopathology. *Development and psychopathology*, *9*(2), 291-319.

Bergman, L. R., Magnusson, D., & El Khouri, B. M. (2003). Studying individual development in an interindividual context: A person-oriented approach. Psychology Press.

Berk, L. E., & Winsler, A. (1995). Scaffolding Children's Learning: Vygotsky and Early Childhood Education. NAEYC Research into Practice Series. Volume 7. National Association for the Education of Young Children, 1509 16th Street, NW, Washington, DC 20036-1426 (NAEYC catalog #146).

Bermejo, V. (1996). Cardinality development and counting. *Developmental Psychology*, *32*(2), 263.

Bertram, T., & Pascal, C. (2002). *Early years education: An international perspective*. London: Qualifications and Curriculum Authority.

Best, J. R., Miller, P. H., & Naglieri, J. A. (2011). Relations between executive function and academic achievement from ages 5 to 17 in a large, representative national sample. *Learning and individual differences*, *21*(4), 327-336.

Bielaczyc, K., Pirolli, P. L., & Brown, A. L. (1995). Training in self-explanation and self-regulation strategies: Investigating the effects of knowledge acquisition activities on problem solving. *Cognition and instruction*, *13*(2), 221-252.

Bisanz, J., Sherman, J. L., Rasmussen, C., Ho, E., & Campbell, J. I. D. (2005). Handbook of mathematical cognition.

Bjorklund, D. F., Hubertz, M. J., & Reubens, A. C. (2004). Young children's arithmetic strategies in social context: How parents contribute to children's strategy development while playing games. *International Journal of Behavioral Development*, *28*(4), 347-357.

Blair, C. (2006). Toward a revised theory of general intelligence: Further examination of fluid cognitive abilities as unique aspects of human cognition. *Behavioral and Brain Sciences*, *29*(2), 145-153.

Blair, C., & Raver, C. C. (2015). School readiness and self-regulation: A developmental psychobiological approach. *Annual review of psychology*, *66*, 711-731.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child development*, *78*(2), 647-663.

Blair, C., Ursache, A., Greenberg, M., & Vernon-Feagans, L. (2015). Multiple aspects of self-regulation uniquely predict mathematics but not letter–word knowledge in the early elementary grades. *Developmental psychology*, *51*(4), 459.

Blevins-Knabe, B. (2016). Early mathematical development: How the home environment matters. In Early Childhood Mathematics Skill Development in the Home Environment (pp. 7-28). Springer International Publishing.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting*, *5*(1), 35-45.

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family Home Care Providers' and Parents' Beliefs and Practices Concerning Mathematics with Young Children. *Early Child Development and Care*, *165*(1), 41-58.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in psychology*, *3*(2), 77-101.

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, *20*(4), 607.

Bronfenbrenner, U. (1976). The Experimental Ecology of Educatim. *Educational researcher*, *5*(9), 5-15.

Bronfenbrenner, U. (1977). Toward an experimental ecology of human development. *American psychologist*, *32*(7), 513.

Bronfenbrenner, U. (1979). *The Ecology of Human Development: Experiments by Nature and Design*. Cambridge, Massachusetts: Harvard University Press. (ISBN 0-674-22457-4)

Bronfenbrenner, U. (1994). Ecological models of development. International encyclopedia of education, 3, 2.

Bronfenbrenner, U. (2005). Making human beings human: Bioecological perspectives on human development. Sage.

Bronfenbrenner, U. (2005). The bioecological theory of human development. In U. Bronfenbrenner (Ed.), Making human beings human: Bioecological perspectives on human development (pp. 3 – 15). Thousand Oaks, CA: Sage. (Original work published in 2001)

Bronfenbrenner, U., & Morris, P. (1998). The ecology of developmental processes. Handbook of child psychology: Volume 1: Theoretical models of human development (pp. 993–1028). Hoboken.

Bronfenbrenner, U., & Morris, P. A. (2006). The bioecological model of human development. *Handbook of child psychology*.

Bruyer, R., & Brysbaert, M. (2011). Combining speed and accuracy in cognitive psychology: is the inverse efficiency score (IES) a better dependent variable than the mean reaction time (RT) and the percentage of errors (PE)?. *Psychologica Belgica*, *51*(1), 5-13.

Buckley, P. B., & Gillman, C. B. (1974). Comparisons of digits and dot patterns. *Journal of experimental psychology*, *103*(6), 1131.

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental neuropsychology*, *33*(3), 205-228.

Burchinal, M., Howes, C., Pianta, R., Bryant, D., Early, D., Clifford, R., & Barbarin, O. (2008). Predicting child outcomes at the end of kindergarten from the quality of pre-kindergarten teacher–child interactions and instruction. *Applied Development Science*, *12*(3), 140-153.

Burgess, S. R., Hecht, S. A., & Lonigan, C. J. (2002). Relations of the home literacy environment (HLE) to the development of reading-related abilities: A one-year longitudinal study. Reading Research Quarterly, 37(4), 408-426.

Butler, J. K. (1991). Toward understanding and measuring conditions of trust: Evolution of a conditions of trust inventory. Journal of management, 17(3), 643-663.

Butterworth, B. (1999). *What counts: How every brain is hardwired for math* (p. pp). The Free Press.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry*, *46*(1), 3-18.

Cahoon, A., Cassidy, T., & Simms, V. (2017). Parents' views and experiences of the informal and formal home numeracy environment. Learning, Culture and Social Interaction, 15, 69-79.

Caldwell, B. M., & Bradley, R. H. (1984). Home observation for measurement of the environment. Little Rock: University of Arkansas at Little Rock.

Cankaya, O., & LeFevre, J. A. (2016). The home numeracy environment: What do cross-cultural comparisons tell us about how to scaffold young children's mathematical skills?. In Early childhood mathematics skill development in the home environment (pp. 87-104). Springer, Cham.

Cargnelutti, E., Tomasetto, C., & Passolunghi, M. C. (2017). The interplay between affective and cognitive factors in shaping early proficiency in mathematics. Trends in Neuroscience and Education, 8, 28-36.

Carle, E. (1974). *The Very Hungry Caterpillar*. London: Puffin

Carlson, S. M. (2005). Developmentally sensitive measures of executive function in preschool children. *Developmental neuropsychology*, *28*(2), 595-616.

Casey, P. H., Bradley, R. H., Nelson, J. Y., & Whaley, S. A. (1988). The clinical assessment of a child's social and physical environment during health visits. *Journal of Developmental and Behavioral Pediatrics*.

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS one*, *7*(4), e33832.

Cattell, R. B. (1978). The scientific use of factor analysis.

Chiswick, B. R., Lee, Y. L., & Miller, P. W. (2003). Patterns of immigrant occupational attainment in a longitudinal survey. *International Migration*, *41*(4), 47-69.

Chong, S. L., & Siegel, L. S. (2008). Stability of computational deficits in math learning disability from second through fifth grades. *Developmental Neuropsychology*, *33*(3), 300-317.

Churchill, G. A. (1979). A paradigm for developing better measures of marketing constructs. Journal of Marketing Research, 16, 64-73.

Claessens, A., & Engel, M. (2013). How Important Is Where You Start? Early Mathematics Knowledge and Later School Success. Teachers College Record, 115 (6), n6.

Clark-Wilson, A., Sutherland, R., & Oldknow, A. (2011). Digital technologies and mathematics education 11-19.

Clark, L. A., & Watson, D. (1995). Constructing validity: Basic issues in objective scale development. Psychological assessment, 7(3), 309.

Clements, D. H. (2004). Major themes and recommendations. Engaging young children in mathematics: Standards for early childhood mathematics education, 7-72.

Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly*, *36*, 79-90.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychosocial Measurement* 20:37–46.

Coles, A., & Sinclair, N. (2018). Re-Thinking ‘Normal’Development in the Early Learning of Number. *Journal of Numerical Cognition*, *4*(1), 136-158.

Collins, L. M., & Lanza, S. T. (2010). Latent class analysis with covariates. *Latent class and latent transition analysis: With applications in the social, behavioral, and health sciences*, 149-177.

Collins, L. M., & Lanza, S. T. (2013). *Latent class and latent transition analysis: With applications in the social, behavioral, and health sciences* (Vol. 718). John Wiley & Sons.

Condry, K. F., & Spelke, E. S. (2008). The development of language and abstract concepts: The case of natural number. *Journal of Experimental Psychology: General*, *137*(1), 22.

Connolly, A. J. (2000). KeyMath – Revised. Richmond Hill, Ontario, Canada: PsyCan.

Cook, D. (1996). Mathematical sense making and role play in the nursery. *Early Child Development and Care*, *121*(1), 55-66.

Cook, J. D., Hepworth, S. J., & Warr, P. B. (1981). The experience of work. San Diego: Academic Press.

Cordes, S., & Gelman, R. (2005). The young numerical mind: when does it count?.

Coroiu, A., Kwakkenbos, L., Moran, C., Thombs, B., Albani, C., Bourkas, S., ... & Körner, A. (2018). Structural validation of the Self-Compassion Scale with a German general population sample. *PloS one*, *13*(2), e0190771.

Cowan, N. (2008). What are the differences between long-term, short-term, and working memory?. *Progress in brain research*, *169*, 323-338.

Cowan, R., Donlan, C., Shepherd, D. L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology*, *103*(4), 786.

Cunningham, J., & Sood, K. (2018). How effective are working memory training interventions at improving maths in schools: a study into the efficacy of working memory training in children aged 9 and 10 in a junior school?. *Education 3-13*, *46*(2), 174-187.

Davidson, M. C., Amso, D., Anderson, L. C., & Diamond, A. (2006). Development of cognitive control and executive functions from 4 to 13 years: Evidence from manipulations of memory, inhibition, and task switching. *Neuropsychologia*, *44*(11), 2037-2078.

De Smedt, B., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: evidence from developmental dyslexia. *Neuropsychologia*, *48*(14), 3973-3981.

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*(2), 48-55.

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of experimental child psychology*, *103*(4), 469-479.

Dearing, E., Casey, B. M., Ganley, C. M., Tillinger, M., Laski, E., & Montecillo, C. (2012). Young girls’ arithmetic and spatial skills: The distal and proximal roles of family socioeconomics and home learning experiences. Early Childhood Research Quarterly, 27(3):458–70.

DeFlorio, L., & Beliakoff, A. (2015). Socioeconomic status and preschoolers’ mathematical knowledge: The contribution of home activities and parent beliefs. Early Education and Development, 26, 319–341.

Dehaene S (1997) The Number Sense. New York: Oxford University Press.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42.

Dehaene, S. (2001). Subtracting Pigeons: Logarithmic or Linear? Psychological Science, 12(3), 244–246.

Dehaene, S. (2007). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. *Sensorimotor foundations of higher cognition*, *22*, 527-574.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical cognition*, *1*(1), 83-120.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive neuropsychology*, *20*(3-6), 487-506.

Dennis, D. J. (2010). The development of boys' aggressive behaviour: a Process-Person-Context-Time model.

Department of Education. (2014). Retrieved from http://www.deni.gov.uk/news/news-de-290414-school-meals-in.htm

Dimitrov, D. M. (2010). Testing for factorial invariance in the context of construct validation. *Measurement and Evaluation in Counseling and Development*, *43*(2), 121-149.

Dowker, A. (2005). Early identification and intervention for students with mathematics difficulties. *Journal of learning disabilities*, *38*(4), 324-332.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. *Developmental science*, *11*(5), 650-654.

Droit-Volet, S., Clément, A., & Fayol, M. (2008). Time, number and length: Similarities and differences in discrimination in adults and children. *Quarterly journal of experimental psychology*, *61*(12), 1827-1846.

Duncan, G. J., Brooks-Gunn, J., & Klebanov, P. K. (1994). Economic deprivation and early childhood development. *Child development*, *65*(2), 296-318.

Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., Pagani, L.S., Feinstein, L., Engel, M., Brooks-Gunn, J. & Sexton, H. (2007). School readiness and later achievement. *Developmental psychology*, *43*(6), 1428.

Duncan, J., Emslie, H., Williams, P., Johnson, R., & Freer, C. (1996). Intelligence and the frontal lobe: The organization of goal-directed behavior. *Cognitive psychology*, *30*(3), 257-303.

Dunn, L. M., & Dunn, D. M. (2009). *The British picture vocabulary scale*. GL Assessment Limited.

Dunn, L. M., Dunn, D. M., Styles, B., & Sewell, J. (2009). The British Picture Vocabulary Scale III – 3rd Edition. London: GL Assessment

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number word use. British Journal of Developmental Psychology, 4(3), 269-288.

Early, D. M., Iruka, I. U., Ritchie, S., Barbarin, O. A., Winn, D. M. C., Crawford, G. M., ... & Bryant, D. M. (2010). How do pre-kindergarteners spend their time? Gender, ethnicity, and income as predictors of experiences in pre-kindergarten classrooms. *Early Childhood Research Quarterly*, *25*(2), 177-193.

Eccles, J. S., Arberton, A., Buchanan, C. M., Janis, J., Flanagan, C., Harold, R., ... Reuman, D. (1993). School and family effects on the ontogeny of children's interests, self-perceptions, and activity choices. In J. E. Jacobs, & R. M. Ryan (Eds.), Nebraska Symposium on Motivation, 1992: Developmental perspectives on motivation (pp. 145–208). Lincoln, NE: University of Nebraska Press.

Eisenhardt, K. M., & Graebner, M. E. (2007). Theory building from cases: Opportunities and challenges. Academy of management journal, 50(1), 25-32.

Elliot, C., Smith, P., & McCulloch, K. (1996). *British Ability Scales Second Edition (BAS II)*. Windsor, UK: NFER-Nelson Publishing Company Limited.

Entwisle, D. R., Alexander, K. L., & Olson, L. S. (2005). Urban teenagers: Work and dropout. *Youth & Society*, *37*(1), 3-32.

Ericsson, K. A., & Kintsch, W. (1995). Long-term working memory. *Psychological review*, *102*(2), 211.

Eslinger, P. J. (1996). Conceptualizing, describing, and measuring components of executive function: A summary. In: Lyons GR, Krasnegor NA, editors. Attention, memory and executive function. Baltimore, MD: Brooks, 1996:367–95.

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics*, *5*(1), 1-4.

Evans, M. A., Shaw, D., & Bell, M. (2000). Home literacy activities and their influence on early literacy skills. *Canadian Journal of Experimental Psychology/Revue canadienne de psychologie expérimentale*, *54*(2), 65.

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of experimental child psychology*, *123*, 53-72.

Feigenson, L. (2011). Predicting sights from sounds: 6-month-olds' intermodal numerical abilities. *Journal of experimental child psychology*, *110*(3), 347-361.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in cognitive sciences*, *8*(7), 307-314.

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. Child development perspectives, 7(2), 74-79.

Field, A. (2013). *Discovering statistics using IBM SPSS statistics*. sage.

Field, F. (2010). *The Foundation Years: preventing poor children becoming poor adults, The report of the Independent Review on Poverty and Life Chances*. The Stationery Office. http://www.creativitycultureeducation.org/the-foundation-years-preventing-poor-children-becoming-poor-adults.

Fisher, P. H., Dobbs-Oates, J., Doctoroff, G. L., & Arnold, D. H. (2012). Early math interest and the development of math skills. *Journal of Educational Psychology*, *104*(3), 673.

Fitzpatrick, R. (1991). Surveys of patients satisfaction: I--Important general considerations. *BMJ: British Medical Journal*, *302*(6781), 887.

Floyd, F. J., & Widaman, K. F. (1995). Factor analysis in the development and refinement of clinical assessment instruments. Psychological assessment, 7(3), 286.

Forza, C., & Filippini, R. (1998). TQM impact on quality conformance and customer satisfaction: a causal model. *International journal of production economics*, *55*(1), 1-20.

Fox, N. (2009). Using interviews in a research project. *The NIHR RDS for the East Midlands/Yorkshire & the Humber*.

Frijters, J. C., Barron, R. W., & Brunello, M. (2000). Direct and mediated influences of home literacy and literacy interest on prereaders' oral vocabulary and early written language skill. Journal of Educational psychology, 92(3), 466.

Fryer, L. K. (2017). (Latent) transitions to learning at university: A latent profile transition analysis of first-year Japanese students. *Higher Education*, *73*(3), 519-537.

Fusch, P. I., & Ness, L. R. (2015). Are we there yet? Data saturation in qualitative research. *The Qualitative Report*, *20*(9), 1408.

Fuson, K. C. (1988). Counting and concepts of number.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*(1-2), 43-74.

Galobardes, B., Shaw, M., Lawlor, D. A., Lynch, J. W., & Smith, G. D. (2006). Indicators of socioeconomic position (part 1). *Journal of Epidemiology & Community Health*, *60*(1), 7-12.

Garavan, H., Ross, T. J., & Stein, E. A. (1999). Right hemispheric dominance of inhibitory control: an event-related functional MRI study. *Proceedings of the National Academy of Sciences*, *96*(14), 8301-8306.

Garbarino, J. (1976). The meaning and implications of school success. In *The Educational Forum* (Vol. 40, No. 2, pp. 157-167). Taylor & Francis Group.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of learning disabilities*, *37*(1), 4-15.

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current directions in psychological science*, *22*(1), 23-27.

Geary, D. C. (2018). Growth of symbolic number knowledge accelerates after children understand cardinality. *Cognition*, *177*, 69-78.

Geary, D. C., Berch, D. B., & Koepke, K. M. (2015). Evolutionary origins and early development of number processing. San Diego, CA: Elsevier Academic Press.

Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study. *Journal of educational psychology*, *109*(5), 680.

Geiser, C. (2012). *Data analysis with Mplus.* Guilford press.

Gelman, R., & Gallistel, C. (1978). Young children's understanding of numbers. *Cambridge, MA*.

Gerardi, K., Goette, L., & Meier, S. (2013). Numerical ability predicts mortgage default. *Proceedings of the National Academy of Sciences*, *110*(28), 11267-11271.

Ghiselli, E. E., Campbell, J. P., & Zedeck, S. (1981). Measurement theory for the behavioral sciences. WH Freeman.

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., ... & Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PloS one*, *8*(6), e67374.

Gilmore, C., Göbel, S. M., & Inglis, M. (2018). *An Introduction to Mathematical Cognition*. Routledge.

Göbel, S. M., & Snowling, M. J. (2010). Number-processing skills in adults with dyslexia. *The Quarterly Journal of Experimental Psychology*, *63*(7), 1361-1373.

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children's arithmetic development: It is number knowledge, not the approximate number sense, that counts. *Psychological science*, *25*(3), 789-798.

Grammer, J. K., Coffman, J. L., Ornstein, P. A., & Morrison, F. J. (2013). Change over time: Conducting longitudinal studies of children's cognitive development. *Journal of Cognition and Development*, *14*(4), 515-528.

Gray, S. A., & Reeve, R. A. (2016). Number-specific and general cognitive markers of preschoolers' math ability profiles. *Journal of experimental child psychology*, *147*, 1-21.

Gray, S. A., Rogers, M., Martinussen, R., & Tannock, R. (2015). Longitudinal relations among inattention, working memory, and academic achievement: testing mediation and the moderating role of gender. *PeerJ*, *3*, e939.

Guilford, J. P. (1954). Psychometric methods. New York: McGraw-Hill.

Gunn, B. K., Simmons, D. C., & Kameenui, E. J. (1995). Emergent literacy: Synthesis of the research. National Center to Improve the Tools of Educators, College of Education, University of Oregon.

Guy, J., Rogers, M., & Cornish, K. (2013). Age-related changes in visual and auditory sustained attention in preschool-aged children. *Child Neuropsychology*, *19*(6), 601-614.

Hair Jr, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2010). Multivariate Data Analysis; a global perspective (ed.): Pearson Education Inc. *New Jersey, USA*, *5*.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the" Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, *44*(5), 1457.

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences*, *109*(28), 11116-11120.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665.

Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological science in the public interest*, *8*(1), 1-51.

Hannant, P. (2018). Receptive language is associated with visual perception in typically developing children and sensorimotor skills in autism spectrum conditions. Human movement science, 58, 297-306.

Hannula, M. M. (2005). Spontaneous focusing on numerosity in the development of early mathematical skills.

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. Learning and Instruction, 15(3), 237-256.

Hart, S. A., Ganley, C. M., & Purpura, D. J. (2016). Understanding the Home Math Environment and Its Role in Predicting Parent Report of Children's Math Skills. PloS one, 11(12), e0168227.

Heckman, J. J. (2006). Skill formation and the economics of investing in disadvantaged children. *Science*, *312*(5782), 1900-1902.

Heckman, J. J., & LaFontaine, P. A. (2008). The declining American high school graduation rate: Evidence, sources, and consequences. *NBER Reporter: Research Summary*, *1*, 1-4.

Hickendorff, M., Edelsbrunner, P. A., McMullen, J., Schneider, M., & Trezise, K. (2017). Informative tools for characterizing individual differences in learning: Latent class, latent profile, and latent transition analysis. *Learning and Individual Differences*.

High, P. C. (2008). School readiness. *Pediatrics*, *121*(4), e1008-e1015.

Hill, R. (1998). What sample size is "enough" in internet survey research? Interpersonal Computing and Technology: An Electronic Journal for the 21st Century, 6(3-4).

Hinkin, T. R. (1998). A brief tutorial on the development of measures for use in survey questionnaires. *Organizational research methods*, *1*(1), 104-121.

Hinkin, T. R., & Schriesheim, C. A. (1989). Development and application of new scales to measure the French and Raven (1959) bases of social power. *Journal of applied psychology*, *74*(4), 561.

Hipp, J. R., & Bauer, D. J. (2006). Local solutions in the estimation of growth mixture models. *Psychological methods*, *11*(1), 36.

Hobbs, G., & Vignoles, A. (2007). Is free school meal status a valid proxy for socio-economic status (in schools research)?. Centre for the Economics of Education, London School of Economics and Political Science.

Hoff, E. (2003). The specificity of environmental influence: Socioeconomic status affects early vocabulary development via maternal speech. *Child development*, *74*(5), 1368-1378.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of experimental child psychology*, *103*(1), 17-29.

Holmes, W., & Dowker, A. (2013). Catch up numeracy: a targeted intervention for children who are low-attaining in mathematics. *Research in mathematics education*, *15*(3), 249-265.

Howard-Jones, P., Ioannou, K., Bailey, R., Prior, J., Yau, S. H., & Jay, T. (2018). Applying the science of learning in the classroom. *Teacher journal*, *2*, 6.

Howe, N., Adrien, E., Della Porta, S., Peccia, S., Recchia, H., Osana, H. P., & Ross, H. (2015). 'Infinity Means it Goes on Forever': Siblings' Informal Teaching of Mathematics. *Infant and Child Development*.

Howe, N., Ross, H. S., & Recchia, H. (2011). Sibling relations in early and middle childhood. *The Wiley-Blackwell Handbook of Childhood Social Development, Second Edition*, 356-372.

Howse, R. B., Lange, G., Farran, D. C., & Boyles, C. D. (2003). Motivation and self-regulation as predictors of achievement in economically disadvantaged young children. *The Journal of Experimental Education*, *71*(2), 151-174.

Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). Improving mathematics at work: The need for techno-mathematical literacies. Routledge.

Hruschka, D. J., Schwartz, D., John, D. C. S., Picone-Decaro, E., Jenkins, R. A., & Carey, J. W. (2004). Reliability in coding open-ended data: Lessons learned from HIV behavioral research. *Field Methods*, *16*(3), 307-331.

Hu, L. T., & Bentler, P. M. (1999). Cut-off criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural equation modeling: a multidisciplinary journal*, *6*(1), 1-55.

Hughes, C., Ensor, R., Wilson, A., & Graham, A. (2009). Tracking executive function across the transition to school: A latent variable approach. *Developmental Neuropsychology*, *35*(1), 20-36.

Hughes, C., White, N., Foley, S., & Devine, R. T. (2018). Family support and gains in school readiness: A longitudinal study. *British Journal of Educational Psychology*, *88*(2), 284-299.

Huizinga, M., Dolan, C. V., & van der Molen, M. W. (2006). Age-related change in executive function: Developmental trends and a latent variable analysis. *Neuropsychologia*, *44*(11), 2017-2036.

Huntley-Fenner, G., & Cannon, E. (2000). Preschoolers' magnitude comparisons are mediated by a preverbal analog mechanism. *Psychological Science*, *11*(2), 147-152.

Huntsinger, C. S., Jose, P. E., & Luo, Z. (2016). Parental facilitation of early mathematics and reading skills and knowledge through encouragement of home-based activities. Early Childhood Research Quarterly, 37, 1-15.

Huntsinger, C. S., Jose, P. E., Larson, S. L., Balsink Krieg, D., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. Journal of Educational Psychology, 92(4), 745–760.

Huntsinger, C. S., Jose, P. E., Liaw, F., & Ching, W. (1997). Cultural differences in early mathematics learning: A comparison of Euro-American, Chinese-American, and Taiwan-Chinese families. International Journal of Behavioral Development, 21(2), 371–388.

Hutcheson, G. D., & Sofroniou, N. (1999). The multivariate social scientist: Introductory statistics using generalized linear models. Sage.

Hutchison, J. & Phillips, D. (2018, March, 27). Executive Functions: Supporting Foundational Skills for Early Math Learning. Retrieved from https://dreme.stanford.edu/news/executive-functions-supporting-foundational-skills-early-math-learning?

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: But only in children. *Psychonomic bulletin & review*, *18*(6), 1222-1229.

Irwin, L. G., Siddiqi, A., & Hertzman, G. (2007). *Early child development: A powerful equalizer*. Vancouver, BC: Human Early Learning Partnership (HELP).

Isaac, S., & Michael, W. B. (1995). Handbook in research and evaluation. San Diego, CA: Educational and Industrial Testing Services.

Isman, E., Ekéus, C., & Berggren, V. (2013) Perceptions and experiences of female genital mutilation after immigration to Sweden: An explorative study. Sexual & Reproductive Healthcare, 4, 93–98.

Isman, E., Mahmoud Warsame, A., Johansson, A., Fried, S., & Berggren, V. (2013) Midwives' experiences in providing care and counselling to women with female genital mutilation (FGM) related problems. *Obstetrics and gynecology international*, *2013*.

Izard, V., Dehaene-Lambertz, G., & Dehaene, S. (2008). Distinct cerebral pathways for object identity and number in human infants. *PLoS biology*, *6*(2), e11.

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences*, *106*(25), 10382-10385.

Jodl, K. M., Michael, A., Malanchuk, O., Eccles, J. S., & Sameroff, A. (2001). Parents' roles in shaping early adolescents' occupational aspirations. *Child development*, *72*(4), 1247-1266.

Jordan, J. A., Mulhern, G., & Wylie, J. (2009). Individual differences in trajectories of arithmetical development in typically achieving 5-to 7-year-olds. *Journal of Experimental Child Psychology*, *103*(4), 455-468.

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Disabilities Research and Practice, 22,* 36–46.

Jordan, N. C., Kaplan, D., Nabors Oláh, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child development*, *77*(1), 153-175.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental psychology*, *45*(3), 850.

José-Luis, Á. G. (2015). *OECD Reviews of Vocational Education and Training A Skills beyond School Review of Egypt*. OECD Publishing.

Kam, C., Morin, A. J., Meyer, J. P., & Topolnytsky, L. (2016). Are commitment profiles stable and predictable? A latent transition analysis. *Journal of Management*, *42*(6), 1462-1490.

Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Goebel, S., Grabner, R., ... & Rubinsten, O. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in psychology*, *4*, 516.

Kaufmann, L., Vogel, S. E., Starke, M., Kremser, C., & Schocke, M. (2009). Numerical and non-numerical ordinality processing in children with and without developmental dyscalculia: Evidence from fMRI. *Cognitive Development*, *24*(4), 486-494.

Kenny, D. A. (2015). Measuring model fit. 2012. Accessed 11 March 2018 http://davidakenny.net/cm/fit.htm

Kenny, D. A., & McCoach, D. B. (2003). Effect of the number of variables on measures of fit in structural equation modeling. *Structural equation modeling*, *10*(3), 333-351.

Kerlinger, F. (1986). Foundations of behavioral research (3rd ed.). New York: Holt, Rinehart & Winston.

Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students’ discourses on infinity. *International Journal of Educational Research*, *51*, 86-108.

Kipnis, D., Schmidt, S. M., & Wilkinson, I. (1980). Intraorganizational influence tactics: Explorations in getting one's way. Journal of applied psychology, 65(4), 440.

Kirby, J. R., & Hogan, B. (2008). Family literacy environment and early literacy development. Exceptionality Education International, 18(3), 112-130.

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly*, *27*(3), 471-477.

Kluczniok, K., Lehrl, S., Kuger, S., & Rossbach, H. G. (2013). Quality of the home learning environment during preschool age–Domains and contextual conditions. *European Early Childhood Education Research Journal*, *21*(3), 420-438.

Knops, A., Nuerk, H. C., & Goebel, S. M. (2017). Domain-General Factors Influencing Numerical and Arithmetic Processing. *Journal of Numerical Cognition*, 112-132.

Krajewski, K., & Ennemoser, M. (2009). The impact of domain-specific precursors and visuospatial skills in kindergarten on mathematical school achievement in Grade 9. In biennial meeting of the Society for Research in Child Development Denver, CO.

Krajewski, K., & Schneider, W. (2009a). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*(6), 513-526.

Krajewski, K., & Schneider, W. (2009b). Exploring the impact of phonological awareness, visual–spatial working memory, and preschool quantity–number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of experimental child psychology*, *103*(4), 516-531.

Kritzer, K. L. (2011). Math in Riley's World. *Teaching Children's Mathematics*, *18*(1), 38-45.

Krosnick, J. A. (1999). Maximizing questionnaire quality. Measures of political attitudes, 2, 37-58.

Krosnick, J. A., Thomas, R., & Shaeffer, E. (2003). How Does Ranking Rate?: A Comparison of Ranking and Rating Tasks. In Conference Papers–American Association for Public Opinion Research.

Kuh, D., & Shlomo, Y. B. (Eds.). (2004). *A life course approach to chronic disease epidemiology* (No. 2). Oxford University Press.

Kumar, K., & Beyerlein, M. (1991). Construction and validation of an instrument for measuring ingratiatory behaviors in organizational settings. *Journal of applied psychology*, *76*(5), 619.

Lanza, S. T., & Cooper, B. R. (2016). Latent class analysis for developmental research. *Child Development Perspectives*, *10*(1), 59-64.

Lauer, J. E., & Lourenco, S. F. (2016). Spatial processing in infancy predicts both spatial and mathematical aptitude in childhood. *Psychological science*, *27*(10), 1291-1298.

Lazarsfeld, P., & Henry, N. (1968). Latent Structure Analysis Houghton Mifflin. *New York*.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*(2), 395-438.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive psychology*, *52*(2), 130-169.

Lee, D., Romine, C., Wolfe, M. & Wong, S. (2002, March). Everything school psychologists wanted to know about executive function but were afraid to ask. Paper presented at annual meeting of the National Association of School Psychologists, Chicago, IL.

LeFevre, J. A., Clarke, T., & Stringer, A. P. (2002). Influences of language and parental involvement on the development of counting skills: Comparisons of French-and English-speaking Canadian children. *Early Child Development and Care*, *172*(3), 283-300.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010a). Pathways to mathematics: Longitudinal predictors of performance. *Child development*, *81*(6), 1753-1767.

LeFevre, J. A., Polyzoi, E., Skwarchuk, S. L., Fast, L., & Sowinski, C. (2010b). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children?. *International Journal of Early Years Education*, *18*(1), 55-70.

LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children’s math performance in the early school years. *Canadian Journal of Behavioural Science/Revue canadienne des sciences du comportement*, *41*(2), 55.

Lehto, J. E., Juujärvi, P., Kooistra, L., & Pulkkinen, L. (2003). Dimensions of executive functioning: Evidence from children. *British Journal of Developmental Psychology*, *21*(1), 59-80.

Leibovich, T., & Ansari, D. (2016). The symbol-grounding problem in numerical cognition: A review of theory, evidence, and outstanding questions. *Canadian Journal of Experimental Psychology/Revue canadienne de psychologie expérimentale*, *70*(1), 12.

Levine, S. C., Ratliff, K. R., Huttenlocher, J., & Cannon, J. (2012). Early puzzle play: A predictor of preschoolers' spatial transformation skill. Developmental Psychology, 48, 530–542.

Li, C. H. (2016). Confirmatory factor analysis with ordinal data: Comparing robust maximum likelihood and diagonally weighted least squares. *Behavior Research Methods*, *48*(3), 936-949.

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental science*, *14*(6), 1292-1300.

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability?. *Learning and individual differences*, *25*, 126-133.

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta psychologica*, *141*(3), 373-379.

Liebeck, P. 1990. How children learn mathematics: A guide for parents and teachers. 2nd Edition. New York, USA: Penguin

Lindblom-Ylänne, S., Parpala, A., & Postareff, L. (2015). Methodological challenges in measuring change in students' learning processes. *World education research*, 160-183.

Linehan, M. M., Comtois, K. A., Brown, M. Z., Heard, H. L., & Wagner, A. (2006). Suicide Attempt Self-Injury Interview (SASII): development, reliability, and validity of a scale to assess suicide attempts and intentional self-injury. Psychological assessment, 18(3), 303.

Lo, Y., Mendell, N. R., & Rubin, D. B. (2001). Testing the number of components in a normal mixture. *Biometrika*, *88*(3), 767-778.

Long, J. S. (1983). *Confirmatory factor analysis: A preface to LISREL* (Vol. 33). Sage Publications.

Lonigan, C. J., Burgess, S. R., & Anthony, J. L. (2000). Development of emergent literacy and early reading skills in preschool children: evidence from a latent-variable longitudinal study. *Developmental psychology*, *36*(5), 596.

Lourenco, S. F., Bonny, J. W., Fernandez, E. P., & Rao, S. (2012). Nonsymbolic number and cumulative area representations contribute shared and unique variance to symbolic math competence. *Proceedings of the National Academy of Sciences*, *109*(46), 18737-18742.

Lugo-Gil, J., & Tamis-LeMonda, C. S. (2008). Family resources and parenting quality: Links to children's cognitive development across the first 3 years. *Child development*, *79*(4), 1065-1085.

Lukie, I. K., Skwarchuk, S. L., LeFevre, J. A., & Sowinski, C. (2014). The role of child interests and collaborative parent–child interactions in fostering numeracy and literacy development in Canadian homes. *Early Childhood Education Journal*, *42*(4), 251-259.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, *121*(2), 256-261.

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental science*, *17*(5), 714-726.

MacCallum, R. C., Widaman, K. F., Zhang, S., & Hong S. (1999). Sample size in factor analysis. Psychological Methods, 4, 84-99.

MacCann, C., Roberts, R. D., Matthews, G., & Zeidner, M. (2004). Consensus scoring and empirical option weighting of performance-based emotional intelligence (EI) tests. *Personality and Individual Differences*, *36*(3), 645-662.

MacKenzie, S. B., Podsakoff, P. M., & Fetter, R. (1991). Organizational citizenship behavior and objective productivity as determinants of managerial evaluations of salespersons' performance. *Organizational behavior and human decision processes*, *50*(1), 123-150.

Magnusson, D., & Cairns, R. B. (1996). Developmental science: Toward a unified framework. In: Cairns, RB.; Elder, G.; Costello, E., editors. Developmental science. New York, NY: Cambridge University Press, p. 7-30

Magnusson, D., & Stattin, H. (2006). The person in context: A holistic-interactionistic approach. *Handbook of child psychology*.

Mahone, E. M., Pillion, J. P., & Hiemenz, J. R. (2001). Initial development of an auditory continuous performance test for preschoolers. *Journal of attention disorders*, *5*(2), 93-106.

Maio, G. R., Roese, N. J., Seligman, C., & Katz, A. (1996). Rankings, ratings, and the measurement of values: Evidence for the superior validity of ratings. Basic and Applied Social Psychology, 18(2), 171-181.

Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. Early Childhood Research Quarterly, 28, 692–703.

Marsh, H. W., Hau, K. T., & Wen, Z. (2004). In search of golden rules: Comment on hypothesis-testing approaches to setting cutoff values for fit indexes and dangers in overgeneralizing

Marsh, H. W., Lüdtke, O., Trautwein, U., & Morin, A. J. (2009). Classical latent profile analysis of academic self-concept dimensions: Synergy of person-and variable-centered approaches to theoretical models of self-concept. *Structural Equation Modeling*, *16*(2), 191-225.

Mayo, A., & Siraj, I. (2015). Parenting practices and children's academic success in low-SES families. *Oxford Review of Education*, *41*(1), 47-63.

McClelland, M. M., Cameron, C. E., Wanless, S. B., & Murray, A. (2007). Executive function, behavioral self-regulation, and social-emotional competence. *Contemporary perspectives on social learning in early childhood education*, *1*, 113-137.

McClelland, M. M., Morrison, F. J., & Holmes, D. L. (2000). Children at risk for early academic problems: The role of learning-related social skills. *Early childhood research quarterly*, *15*(3), 307-329.

McCormack, T., Simms, V., McGourty, J., & Beckers, T. (2013). Blocking in children's causal learning depends on working memory and reasoning abilities. *Journal of experimental child psychology*, *115*(3), 562-569.

McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science*, *15*(11), 776-781.

McLachlan, G., & Peel, D. (2000). Multivariate normal mixtures. *Finite Mixture Models*, 81-116.

Melchers, P., & Preuss, U. (2003). Kaufman Assessment Battery for Children (KABC). German Version (6th ed.). Göttingen, Germany: Hogrefe.

Melhuish, E. C., Phan, M. B., Sylva, K., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2008). Effects of the home learning environment and preschool center experience upon literacy and numeracy development in early primary school. *Journal of Social Issues*, *64*(1), 95-114.

Melhuish, E., Quinn, L., Sylva, K., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2013). Preschool affects longer term literacy and numeracy: results from a general population longitudinal study in Northern Ireland. *School Effectiveness and School Improvement*, *24*(2), 234-250.

Mercy, J. A, & Steelman, L. C. (1982). Familial influence on the intellectual attainment of children. American Sociological Review, 47, 532–42.

Methe, S. A., Hintze, J. M., & Floyd, R. G. (2008). Validation and decision accuracy of early numeracy skill indicators. *School Psychology Review*, *37*(3), 359.

Miles, T. R., Haslum, M. N., & Wheeler, T. J. (2001). The mathematical abilities of dyslexic 10-year-olds. *Annals of Dyslexia*, *51*(1), 299-321.

Miller, E. K., & Cohen, J. D. (2001). An integrative theory of prefrontal cortex function. *Annual review of neuroscience*, *24*(1), 167-202.

Missall, K., Hojnoski, R. L., Caskie, G. I., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development*, *26*(3), 356-376.

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1997). Numerical abstraction in infants: Another look. *Developmental psychology*, *33*(3), 423.

Miyake, A., & Shah, P. (Eds.). (1999). *Models of working memory: Mechanisms of active maintenance and executive control*. Cambridge University Press.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex “frontal lobe” tasks: A latent variable analysis. *Cognitive psychology*, *41*(1), 49-100.

Moll, K., Snowling, M. J., Göbel, S. M., & Hulme, C. (2015). Early language and executive skills predict variations in number and arithmetic skills in children at family-risk of dyslexia and typically developing controls. *Learning and instruction*, *38*, 53-62.

Morgan, P. L., Farkas, G., & Wu, Q. (2009). Five-year growth trajectories of kindergarten children with learning difficulties in mathematics. *Journal of Learning Disabilities*, *42*(4), 306-321.

Morin, A. J., Maiano, C., Nagengast, B., Marsh, H. W., Morizot, J., & Janosz, M. (2011). General growth mixture analysis of adolescents' developmental trajectories of anxiety: The impact of untested invariance assumptions on substantive interpretations. *Structural Equation Modeling: A Multidisciplinary Journal*, *18*(4), 613-648.

Morrison, F. J., & Ornstein, P. A. (1996). Cognitive development. In: Cairns, RB.; Elder, Gr; Costello, E., editors. Developmental science. New York, NY: Cambridge University Press, p. 121-134.

Muldoon, K., Towse, J., Simms, V., Perra, O., & Menzies, V. (2013). A longitudinal analysis of estimation, counting skills, and mathematical ability across the first school year. *Developmental psychology*, *49*(2), 250.

Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of experimental child psychology*, *103*(4), 490-502.

Munn, P. (1994). The early development of literacy and numeracy skills. *European Early Childhood Education Research Journal*, *2*(1), 5-18.

Murray, J. (2013). The factors that influence mathematics achievement at the Berbice campus. *International journal of business and social science*, *4*(10).

Mussolin, C., Nys, J., Leybaert, J., & Content, A. (2012). Relationships between approximate number system acuity and early symbolic number abilities. *Trends in Neuroscience and Education*, *1*(1), 21-31.

Muthén, B. (2003). Statistical and substantive checking in growth mixture modeling: comment on Bauer and Curran (2003).

Muthén, B. O. (2001). Latent variable mixture modeling. In *New developments and techniques in structural equation modeling* (pp. 21-54). Psychology Press.

Muthen, L. K., & Muthen, B. O. (1998). Mplus [computer software]. *Los Angeles, CA: Muthén & Muthén*.

Muthén, L. K., & Muthen, B. O. (2017). Mplus. Los Angeles: Muthen & Muthen.

Nanu, C. E., McMullen, J., Munck, P., Hannula-Sormunen, M. M., & Pipari Study Group. (2018). Spontaneous focusing on numerosity in preschool as a predictor of mathematical skills and knowledge in the fifth grade. *Journal of experimental child psychology*, *169*, 42-58.

National Early Literacy Panel. (2008). *Report of the National Early Literacy Panel: Developing early literacy*. Washington, DC: U. S. Department of Health and Human Services.

National Mathematics Advisory Panel. (2008). *Foundations for success: The Final report of the National Mathematics Advisory Panel*. Washington, DC: U. S. Department of Education.

National Research Council. (2009). *Learning science in informal environments: People, places, and pursuits*. National Academies Press.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement?. *Early childhood research quarterly*, *36*, 550-560.

Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, *29*(3), 327-345.

Niklas, F., & Schneider, W. (2017). Intervention in the home literacy environment and kindergarten children's vocabulary and phonological awareness. First Language.

Niklas, F., Cohrssen, C., & Tayler, C. (2016). Improving preschoolers' numerical abilities by enhancing the home numeracy environment. *Early Education and Development, 27*(3), 372-383.

Norris, E. (2012). Solving the maths problem: International perspectives on mathematics education. *London: Royal Society for the Encouragement of Arts. Retrieved May*, *15*, 2012.

Northern Ireland Audit Office (2013). Improving literacy and numeracy achievement in schools. Accessed 02 March 2016. https://www.niauditoffice.gov.uk/a-to-z.htm/improving_literacy_and_numeracy_achievement_in_schools_2

Noss, R. (1997). *New cultures, new numeracies*. London: Institute of Education, University of London.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Wiley-Blackwell.

Nunes, T., Pretzlik, U., & Ilicak, S. (2005). Validation of a parent outcome questionnaire from pediatric cochlear implantation. Journal of Deaf Studies and Deaf Education, 10(4), 330-356.

Nylund-Gibson, K., Grimm, R., Quirk, M., & Furlong, M. (2014). A latent transition mixture model using the three-step specification. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(3), 439-454.

Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. *Structural equation modeling*, *14*(4), 535-569.

O'Donnell, S. (2004). *International Review of Curriculum and Assessment Frameworks: Comparative Tables and Factual Summaries 2004*. INCA.

Oberauer, K. (2005). Binding and inhibition in working memory: individual and age differences in short-term recognition. *Journal of experimental psychology: General*, *134*(3), 368.

OECD (2013), OECD Skills Outlook 2013: First Results from the Survey of Adult Skills, OECD Publishing. *http://dx.doi.org/10.1787/9789264204256-en*

OECD (2016), Survey of Adult Skills (PIAAC) (Database 2012, 2015), www.oecd.org/site/piaac/publicdataandanalysis.htm.

OfCom, U. K. (2013). Children and Parents: Media Use and Attitudes Report. (Report).

OfCom, U. K. (2016). Children and Parents: Media Use and Attitudes Report. (Report).

Ofsted. (2014). “Are You Ready? Good Practice in School Readiness.” Accessed 10 June 2016. http://www.ofsted.gov.uk/resources/140074.

Olson, S. L., Sameroff, A. J., Kerr, D. C., Lopez, N. L., & Wellman, H. M. (2005). Developmental foundations of externalizing problems in young children: The role of effortful control. *Development and psychopathology*, *17*(1), 25-45.

Osei-Boadi, B. (2016). The Role of Working Memory Capacity in Math Performance.

Pan, Y., Gauvain, M., Liu, Z., & Cheng, L. (2006). American and Chinese parental involvement in young children's mathematics learning. Cognitive Development, 21(1), 17-35.

Passolunghi, M. C., Vercelloni, B., & Schadee, H. (2007). The precursors of mathematics learning: Working memory, phonological ability and numerical competence. *Cognitive Development*, *22*(2), 165-184.

Paulsen, D. J., Woldorff, M. G., & Brannon, E. M. (2010). Individual differences in nonverbal number discrimination correlate with event-related potentials and measures of probabilistic reasoning. *Neuropsychologia*, *48*(13), 3687-3695.

Piazza, M. (2011). Neurocognitive start-up tools for symbolic number representations. In *Space, Time and Number in the Brain* (pp. 267-285).

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., ... & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33-41.

Pigott, T. D., & Israel, M. S. (2005). Head Start children's transition to kindergarten: Evidence from the Early Childhood Longitudinal Study. *Journal of Early Childhood Research*, *3*(1), 77-104.

Pomerantz, E. M., Moorman, E. A., & Litwack, S. D. (2007). The how, whom, and why of parents' involvement in children's academic lives: More is not always better. Review of educational research, 77(3), 373-410.

Posner, M. I., & Rothbart, M. K. (2000). Developing mechanisms of self-regulation. *Development and psychopathology*, *12*(3), 427-441.

Price, G. R., & Wilkey, E. D. (2017). Cognitive mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing and their relation to math. *Cognitive Development*, *44*, 139-149.

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). Nonsymbolic numerical magnitude comparison: Reliability and validity of different task variants and outcome measures, and their relationship to arithmetic achievement in adults. *Acta psychologica*, *140*(1), 50-57.

Price, J., & Mueller, W. (1986). Handbook of Organizational Measurement, Marshfield: Pitman. Organization Studies, 9(1), 134-135.

Rabbitt, P. (2004). Introduction: Methodologies and models in the study of executive function. In *Methodology of frontal and executive function* (pp. 9-45). Routledge.

Rabbitt, P. Introduction: Methodologies and models in the study of executive function. In: Rabbit, P., editor. Methodology of frontal and executive function. Hove, East Sussex, UK: Psychology Press Ltd; 1997. p. 1-38.

Raghubar, K. P., & Barnes, M. A. (2017). Early numeracy skills in preschool-aged children: a review of neurocognitive findings and implications for assessment and intervention. *The Clinical Neuropsychologist*, *31*(2), 329-351.

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. Cognitive Development, 35, 15-33.

Raver, C. C., Smith-Donald, R., Hayes, T., & Jones, S. M. (2005). Self-regulation across differing risk and sociocultural contexts: Preliminary findings from the Chicago School Readiness Project. In *biennial meeting of the Society for Research in Child Development, Atlanta, GA*.

Rittle-Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of educational psychology*, *93*(2), 346.

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2018). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*.

Roßbach, H. G. (2005). Effekte qualitativ guter Betreuung, Bildung und Erziehung im frühen Kindesalter auf Kinder und ihre Familien.

Rose, D., & Pevalin, D. (2010). Volume 3 The National Statistics Socio-economic Classification:(rebased on the SOC2010) User manual (pp. 1–70).

Rubin, K. H., Burgess, K. B., Dwyer, K. M., & Hastings, P. D. (2003). Predicting preschoolers' externalizing behaviors from toddler temperament, conflict, and maternal negativity. *Developmental psychology*, *39*(1), 164.

Rubinsten, O., & Sury, D. (2011). Processing ordinality and quantity: the case of developmental dyscalculia. *PLoS One*, *6*(9), e24079.

Ryoo, J. H., Wang, C., Swearer, S. M., Hull, M., & Shi, D. (2018). Longitudinal model building using latent transition analysis: an example using school bullying data. *Frontiers in psychology*, *9*.

Sadler, P. M., & Tai, R. H. (2007). The two high-school pillars supporting college science. Science, 317, 457–458.

Sammons, P., Elliot, K., Sylva, K., Melhuish, E. C., Siraj-Blatchford, I., & Taggart, B. (2004). The impact of pre-school on young children’s cognitive attainments at entry to reception. British Educational Research Journal, 30, 691–712.

Sánchez-Pérez, N., Castillo, A., López-López, J. A., Pina, V., Puga, J. L., Campoy, G., ... & Fuentes, L. J. (2018). Computer-Based Training in Math and Working Memory Improves Cognitive Skills and Academic Achievement in Primary School Children: Behavioral Results. *Frontiers in psychology*, *8*, 2327.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662-674.

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of experimental child psychology*, *103*(3), 325-337.

Sarnecka, B. W., & Wright, C. E. (2013). The idea of an exact number: Children's understanding of cardinality and equinumerosity. *Cognitive Science*, *37*(8), 1493-1506.

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology*, *30*(2), 344-357.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement?. *Journal of experimental child psychology*, *114*(3), 418-431.

Saxe, G. B., Guberman, S. R., Gearhart, M., Gelman, R., Massey, C. M., & Rogoff, B. (1987). Social processes in early number development. Monographs of the society for research in child development, i-162.

Scarborough, H. S., & Dobrich, W. (1994). On the efficacy of reading to preschoolers. *Developmental review*, *14*(3), 245-302.

Schmitt, N. W., & Klimoski, R. J. (1991). Research methods in human resources management. Cincinnati, OH: South-Western.

Schmitt, S. A., Geldhof, G. J., Purpura, D. J., Duncan, R., & McClelland, M. M. (2017). Examining the relations between executive function, math, and literacy during the transition to kindergarten: A multi-analytic approach. *Journal of Educational Psychology*, *109*(8), 1120.

Schmitt, S. A., Pratt, M. E., & McClelland, M. M. (2014). Examining the validity of behavioral self-regulation tools in predicting preschoolers' academic achievement. *Early Education and Development*, *25*(5), 641-660.

Schneider, M., & Hardy, I. (2013). Profiles of inconsistent knowledge in children’s pathways of conceptual change. *Developmental Psychology*, *49*(9), 1639.

Schoenfeldt, L. F. (1984). Psychometric properties of organizational research instruments. In T. S. Bateman & G. R. Ferris (Eds.), Method & analysis in organizational research (pp. 68-80). Reston, VA: Reston.

Schwartz, G. (1978). Annals of statistics. *Estimating the Dimension of a Model*, *6*, 461-464.

Segers, E., Kleemans, T., & Verhoeven, L. (2015). Role of Parent Literacy and Numeracy Expectations and Activities in Predicting Early Numeracy Skills. *Mathematical Thinking and Learning*, *17*(2-3), 219-236.

Seltzer, M. H., Frank, K. A., & Bryk, A. S. (1994). The metric matters: The sensitivity of conclusions about growth in student achievement to choice of metric. *Educational evaluation and policy analysis*, *16*(1), 41-49.

Sénéchal, M., & LeFevre, J. A. (2002). Parental involvement in the development of children’s reading skill: A five-year longitudinal study. *Child development*, *73*(2), 445-460.

Sénéchal, M., LeFevre, J. A., Hudson, E., & Lawson, E. P. (1996). Knowledge of storybooks as a predictor of young children's vocabulary. *Journal of Educational Psychology*, *88*(3), 520.

Sénéchal, M., LeFevre, J. A., Thomas, E. M., & Daley, K. E. (1998). Differential effects of home literacy experiences on the development of oral and written language. Reading research quarterly, 33(1), 96-116.

Sénéchal, M., Pagan, S., Lever, R., & Ouellette, G. P. (2008). Relations among the frequency of shared reading and 4-year-old children's vocabulary, morphological and syntax comprehension, and narrative skills. Early Education and Development, 19, 27–44.

Sergeant, J. A., Geurts, H., & Oosterlaan, J. (2002). How specific is a deficit of executive functioning for attention-deficit/hyperactivity disorder?. *Behavioural brain research*, *130*(1-2), 3-28.

Shallice, T., & Burgess, P. (1993). Supervisory control of action and thought selection.

Sharp, C. (2002, November). School starting age: European policy and recent research. In *LGA Seminar'When Should Our Children Start School*.

Sheldon, S., & Epstein, J. (2005). Involvement counts: Family and community partnerships and mathematics achievement. *Journal of Educational Research, 98*(4), 196–207.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child development*, *75*(2), 428-444.

Simmons, F. R., & Singleton, C. (2006). The mental and written arithmetic abilities of adults with dyslexia. *Dyslexia*, *12*(2), 96-114.

Singh, K. (2007). Quantitative social research methods. Sage.

Sirin, S. R. (2005). Socioeconomic status and academic achievement: A meta-analytic review of research. *Review of educational research*, *75*(3), 417-453.

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? Early Childhood Education Journal, 37, 189–197.

Skwarchuk, S. L., & LeFevre, J. A. (2015). The role of the home environment in children's early numeracy development: A Canadian perspective. In Mathematics and Transition to School (pp. 103-117). Springer Singapore.

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of experimental child psychology*, *121*, 63-84.

Slaughter, V., Kamppi, D., & Paynter, J. (2006). Toddler subtraction with large sets: Further evidence for an analog-magnitude representation of number. *Developmental Science*, *9*(1), 33-39.

Song, M. J., & Ginsburg, H. P. (1987). The development of informal and formal mathematical thinking in Korean and US children. *Child Development*, 1286-1296.

St. Clair-Thompson, H. L., & Gathercole, S. E. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory. *Quarterly journal of experimental psychology*, *59*(4), 745-759.

Starkey, P., & Klein, A. (2000). Fostering parental support for children's mathematical development: An intervention with Head Start families. Early Education & Development, 11(5), 659–680.

Starkey, P., & Klein, A. (2008). Sociocultural influences in young children's mathematical knowl- edge. In O. Saracho & B. Spodak (Eds.), Contemporary perspectives on mathematics in early childhood education (pp. 253–276). Charlotte, NC: Information Ag

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99-120.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*(2), 97-127.

Steiger, J. H. (1990). Structural model evaluation and modification: An interval estimation approach. *Multivariate behavioral research*, *25*(2), 173-180.

Stone, E. F. (1978). *Research methods in organizational behavior*. Goodyear Publishing Company.

Sylva, K., Melhuish, E., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2004). The effective provision of pre-school education (eppe) project: Findings from pre-school to end of key stage 1. Department for Edu- cation and Skills.

Sylva, K., Melhuish, E., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2008). Final report from the primary phase: Pre-school, school and family influences on children's development during Key Stage 2 (7-11).

Tickell, C. (2011). *The early years: foundations for life, health and learning*, report of the independent review on the early years foundation stage, Department for Education. http://www.gov.uk/government/publications/the-early-years-foundations-for-life-health-and-learning-an-independent-report-on-the-early-years-foundation-stage-to-her-majestys-government.

Tolchinsky, L. (2003). *The cradle of culture and what children know about writing and numbers before being*. Psychology Press.

Toll, S. W., Van Viersen, S., Kroesbergen, E. H., & Van Luit, J. E. (2015). The development of (non-) symbolic comparison skills throughout kindergarten and their relations with basic mathematical skills. *Learning and Individual Differences*, *38*, 10-17.

Totsika, V., & Sylva, K. (2004). The home observation for measurement of the environment revisited. Child and Adolescent Mental Health, 9(1), 25-35.

Townsend, J. T., & Ashby, F. G. (1978). Methods of modeling capacity in simple processing systems. *Cognitive theory*, *3*, 200-239.

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes*. CUP Archive.

Tucker, L. R., & Lewis, C. (1973). A reliability coefficient for maximum likelihood factor analysis. *Psychometrika*, *38*(1), 1-10.

Tudge, J. R., Mokrova, I., Hatfield, B. E., & Karnik, R. B. (2009). Uses and misuses of Bronfenbrenner's bioecological theory of human development. *Journal of Family Theory & Review*, *1*(4), 198-210.

Tudge, J.R. (1990) Vygotsky, the zone of proximal development, and peer collaboration: Implications for classroom practice. In: Mol LC (eds) *Vygotsky and education: Instructional implications and applications of sociohistorical psychology.* Cambridge, UK: Cambridge University Press, pp.155-172.

Uebersax, J. (2000). *A Brief Study of Local Maximum Solutions in Latent Class Analysis*. Available online at: http://www.john-uebersax.com/stat/local.htm

van der Sluis, S., de Jong, P. F. & van der Leij,A. (2007). Executive functioning in children, and its relations with reasoning, reading, and arithmetic. *Intelligence*. 35(5), 427-49.

Vandermaas-Peeler, M., & Pittard, C. (2014). Influences of social context on parent guidance and low-income preschoolers' independent and guided math performance. Early Child Development and Care, 184(4), 500-521.

Vandermaas-Peeler, M., Boomgarden, E., Finn, L., & Pittard, C. (2012). Parental support of numeracy during a cooking activity with four-year-olds. International Journal of Early Years Education, 20(1), 78-93.

Vandermaas-Peeler, M., Ferretti, L., & Loving, S. (2012). Playing The Ladybug Game: parent guidance of young children's numeracy activities. *Early Child Development and Care*, *182*(10), 1289-1307.

Vandermaas-Peeler, M., Nelson, J., & Bumpass, C. (2007). ‘Quarters are what you put into the bubble gum machine’. Numeracy interactions during parent–child play. Early Childhood Research and Practice 9(1).

Vandermaas-Peeler, M., Nelson, J., Bumpass, C., & Sassine, B. (2009). Numeracy-related exchanges in joint storybook reading and play. International Journal of Early Years Education, 17(1), 67-84.

Vandierendonck, A. (2017). A comparison of methods to combine speed and accuracy measures of performance: A rejoinder on the binning procedure. *Behavior research methods*, *49*(2), 653-673.

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. (2017). *Links between spatial and mathematical skills across the preschool years*. Wiley.

Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: A neural model. *Journal of Cognitive Neuroscience*, *16*(9), 1493-1504.

Vermunt, J. K. (2010). Latent class modeling with covariates: Two improved three-step approaches. *Political analysis*, *18*(4), 450-469.

Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses. *Econometrica: Journal of the Econometric Society*, 307-333.

Vygotsky, L. (1978a). Interaction between learning and development. Readings on the development of children, 23(3), 34-41.

Vygotsky, L. S. (1978b). Mind in society: The development of higher mental process.

Wachs, M. (1989). When planners lie with numbers. American Planning Association. Journal of the American Planning Association, 55(4), 476.

Wagner, J. B., & Johnson, S. C. (2011). An association between understanding cardinality and analog magnitude representations in preschoolers. *Cognition*, *119*(1), 10-22.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. Journal of Educational Psychology, 101, 817–835.

Walton, P. D., & French, D. P. (2016). What do people think about running barefoot/with minimalist footwear? A thematic analysis. *British journal of health psychology*.

Wang, J., & Wang, X. (2012). Structural equation modeling: Applications using Mplus. John Wiley & Sons.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, *43*(7), 352-360.

Wechsler, D., & Kort, W. (2005). *WISC-IIInl: Wechsler intelligence scale for children*. Harcourt Assessment.

Weinert, S. 2006. "Sprachentwicklung." In Entwicklungspsychologie [Developmental Psychology], edited by W. Schneider and B. Sodian. Enzyklopaedie der Psychologie 2, 609–719. Goettingen: Hogrefe.

West, A., & Varlaam, A. (1990). Does it matter when children start school?. *Educational Research*, *32*(3), 210-217.

White, K. R. (1982). The relation between socioeconomic status and academic achievement. Psychological bulletin, 91(3), 461.

Wiebe, S. A., Espy, K. A., & Charak, D. (2008). Using confirmatory factor analysis to understand executive control in preschool children: I. Latent structure. *Developmental psychology*, *44*(2), 575.

Wiggins, R. D., Netuveli, G., Hyde, M., Higgs, P., & Blane, D. (2008). The evaluation of a self-enumerated scale of quality of life (CASP-19) in the context of research on ageing: A combination of exploratory and confirmatory approaches. *Social Indicators Research*, *89*(1), 61-77.

Williams, J. (2003). *The Skills for Life survey: A national needs and impact survey of literacy, numeracy and ICT skills* (No. 490). The Stationery Office.

Williamson, J. B., Karp, D. A., Dalphin, J. R., & Gray, P. S. (1982). *The research craft: An introduction to social research methods*. Little, Brown.

Wilson, A., Hewitt, G., Matthews, R., Richards, S. H., & Shepperd, S. (2006). Development and testing of a questionnaire to measure patient satisfaction with intermediate care. *BMJ Quality & Safety*, *15*(5), 314-319.

Windisch, H. C. (2016). How to motivate adults with low literacy and numeracy skills to engage and persist in learning: A literature review of policy interventions. *International Review of Education*, *62*(3), 279-297.

Wolf, M. H., Putnam, S. M., James, S. A., & Stiles, W. B. (1978). The medical interview satisfaction scale: development of a scale to measure patient perceptions of physician behavior. Journal of behavioral medicine, 1(4), 391-401.

Wolfgang, C. H., Stannard, L. L., & Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. Journal of Research in Childhood Education, 15(2), 173–180.

Wolniak, G. C. (2016). Examining STEM bachelor's degree completion for students with differing propensities at college entry. Journal of College Student Retention: Research, Theory & Practice, 18, 287–309.

Woodcock, R. W., & Johnson, M. B. (1989). Woodcock-Johnson Psycho-Educational Battery-Revised. Allen, TX: DLM Teaching Resources.

Woodhead, M. (1989). ‘School starts at five... or four years old?‘The rationale for changing admission policies in England and Wales. *Journal of Education policy*, *4*(1), 1-21.

Woolfson, L. M., Geddes, R., McNicol, S., Booth, J. N., & Frank, J. (2013). A cross-sectional pilot study of the Scottish early development instrument: a tool for addressing inequality. *BMC public health*, *13*(1), 1187.

World Bank. (2012). World Development Report 2013: Jobs. World Bank, Washington, DC, doi: http://dx.doi.org/10.1596/978-0-8213-9575-2

World Bank. (2015). The World Bank Annual Report. http://www.worldbank.org/en/about/annual-report-2015

Wright, R. J., Martland, J., & Stafford, A. K. (2006). Early numeracy: Assessment for teaching and intervention. Sage.

Wurpts, I. C., & Geiser, C. (2014). Is adding more indicators to a latent class analysis beneficial or detrimental? Results of a Monte-Carlo study. Frontiers in Psychology, 5, 920.

Wynn, K. (1990). Children's understanding of counting. Cognition, 36(2), 155-193.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, *358*(6389), 749.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*(1), B1-B11.

Yen, C. J., Konold, T. R., & McDermott, P. A. (2004). Does learning behavior augment cognitive ability as an indicator of academic achievement?. *Journal of School Psychology*, *42*(2), 157-169.

Young-Loveridge, J. M. (1989). The relationship between children's home experiences and their mathematical skills on entry to school. Early Child Development and Care, 43(1), 43-59.

Zelazo, P. D., & Müller, U. (2002). Executive function in typical and atypical development. *Blackwell handbook of childhood cognitive development*, 445-469.

Zelazo, P. D., & Müller, U. (2002). Executive function in typical and atypical development. In: Goswami U, editor. The Wiley-Blackwell handbook of childhood cognitive development. 2n ed. Oxford, England: Wiley-Blackwell; 2011. pp. 574–603.

Zill, N., & West, J. (2001). Entering Kindergarten: A Portrait of American Children When They Begin School. Findings from the Condition of Education, 2000.

Zimmerman, B. J. (2002). Achieving academic excellence: A self-regulatory perspective. *The pursuit of excellence through education*, 85-110.