Adaptive Rate and Power Transmission for OFDM-based Cognitive Radio Systems

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***Abstract*—This paper studies the joint rate and power allocation problem for OFDM-based cognitive radio systems where secondary users (SUs) can opportunistically access the spectrum of primary users (PUs). We propose a novel algorithm that jointly maximizes the OFDM-based cognitive radio system throughput and minimizes its transmit power, while guaranteeing a target bit error rate per subcarrier and restricting both co-channel interference (CCI) and adjacent channel interference (ACI) to existing primary users. Since estimating the instantaneous channel gains on the links between the SU transmitter and the PUs receivers is impractical, we assume only knowledge of the path loss on these links. Closed-form expressions are derived for the close-to-optimal bit and power distributions. Simulation results are described that illustrate the performance of the proposed scheme and show its closeness to that of an exhaustive search for the discrete optimal allocations. Further, the results quantify the violation ratio of both the CCI and ACI constraints at the PUs receivers due to the partial channel information. The effect of adding a fading margin to reduce the violation ratio is also studied.**

***Index Terms*—Cognitive radio, joint optimization, OFDM systems, rate and power allocation, resource allocation, spectrum sharing.**

I. Introduction

The wireless radio spectrum has become a scarce resource due to the ceaseless demands for spectrum by new applications and services. However, this spectrum scarcity happens while most of the allocated spectrum is under-utilized, as reported by many jurisdictions [1]. This paradox occurs only due to the inefficiency of the traditional static spectrum allocation policies. Cognitive radio (CR) [2] provides a solution to the spectrum utilization inefficiency by allowing unlicensed/secondary users (SUs) to opportunistically access voids in licensed/primary users (PUs) frequency bands/time slots under the condition that no harmful interference occurs to PUs.

Orthogonal frequency division multiplexing (OFDM) is recognized as an attractive modulation technique for CR due to its flexibility, adaptivity in allocating vacant radio resources, and underlying sensing and spectrum shaping capabilities [3]. The performance of the OFDM-based SUs can be ameliorated by dynamically adapting the transmission parameters to the changing quality of the wireless channel and the imposed PUs interference constraints.

To date, most of the research literature has focused on the *single* objective function of maximizing the OFDM SU capacity with constraints on the total transmit power and the interference introduced to adjacent PUs, while less attention was given to guarantee a certain OFDM SU bit error rate (BER) [4]–[9]. In [4], Bansal *et al.* investigated the optimal power allocation problem in CR networks to maximize the SU downlink transmission capacity under a constraint on the instantaneous interference to PUs. The proposed algorithm was complex and several suboptimal algorithms were developed to reduce the computational complexity. Zhang and Leung [5] proposed a low complexity suboptimal algorithm for an OFDM-based CR system in which SUs may access both nonactive and active PU frequency bands, as long as the total co-channel interference (CCI) and adjacent channel interference (ACI) are within acceptable limits. The work in [4]–[6] assumes perfect knowledge of the instantaneous channel gains on the links between the SU transmitter and the PUs receivers, which is a challenging assumption for practical scenarios. On the other hand, the work in [7], [8] assumes only knowledge of the path loss for these links. This partial knowledge of the link causes the proposed algorithms in [7], [8] to violate the CCI and ACI constraints at the PUs receivers when applied in practice.

In this paper, we propose a close-to-optimal algorithm for OFDM-based CR systems that jointly maximizes the OFDM SU throughput and minimizes its transmit power[1], while guaranteeing a SU target BER per subcarrier, total transmit power limit, and an acceptable interference power to adjacent PUs. The problem is formulated as a multi-objective optimization problem by introducing a weighting factor that reflects the importance of the competing OFDM SU throughput and power objectives in accordance with the CR system and application requirements. We adopt the more practical assumption of only knowing the path loss [7], [8] on the links between the SU transmitter and the PUs receivers. Closed-form expressions are derived for the close-to-optimal bit and power distributions. Additionally, we quantify the violation of both the CCI and the ACI constraints that results at the PUs receivers due to the partial link information between the SU transmitter and the PUs receivers. The effect of adding a fading margin to compensate this violation is further studied. Simulation results indicate that the proposed algorithm performance approaches that of the exhaustive search for the optimal discrete allocations, with significantly reduced computational effort.

The remainder of the paper is organized as follows. Section II presents the system model and Section III depicts the proposed joint bit and power loading algorithm. Simulation

[1]In a non-CR environment, jointly maximizing the throughput and minimizing the transmit power show a significant performance improvement, in terms of the achieved throughput and transmit power, compared to other work in the literature that separately maximizes the throughput (while constraining the transmit power) or minimizes the transmit power (while constraining the throughput), respectively [10], [11].

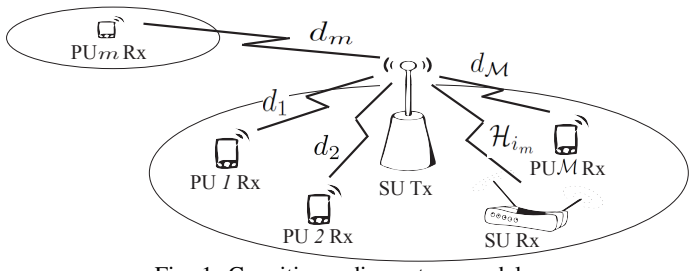

Fig. 1: Cognitive radio system model.

results are presented in Section IV, while conclusions are drawn in Section V.

Throughout this paper we use bold-faced lower case letters for vectors, e.g., $\mathbf{x}$, and light-faced letters for scalar quantities, e.g., x. $[.]^T$ denotes the transpose operation, ∇ represents the gradient operator, and $\mathbb{E}[.]$ is the statistical expectation operator. $[x, y]^-$ represents $\min(x, y)$ and $\bar{\bar{\mathbb{X}}}$ is the cardinality of the set $\mathbb{X}$.

II. System Model

The available spectrum is assumed to be divided into $\mathcal{M}$ subchannels that are licensed to $\mathcal{M}$ PUs. A subchannel m, of bandwidth B_m, has N_m subcarriers and i_m denotes subcarrier i in the subchannel m, $i_m = 1, ..., N_m$. A PU does not occupy its licensed spectrum all the time and/or at all its coverage locations; hence, an SU may access such voids as long as no harmful interference occurs to adjacent PUs due to ACI, or to other PUs operating in the same frequency band at distant locations due to CCI.

A typical CR system is shown in Fig. 1. An SU first obtains the surrounding PUs' information[2], such as the PUs' positions and spectral band occupations. Then, it makes a decision on the possible transmission subchannels. We consider Fig. 1 where the SU has all the required information of the existing $\mathcal{M}$ PUs, and it decides to use the vacant PU m subchannel, $m \in \{1, ..., \mathcal{M}\}$.

While it is possible to estimate the instantaneous channel gains between the SU transmitter and receiver pairs, it is more challenging or even impossible to estimate the instantaneous channel gains from the SU transmitter to the PUs receivers without the PU cooperation. That being said, we assume perfect channel state information (CSI) between the SU transmitter and receiver pairs, while only the path loss is assumed to be known between the SU transmitter and PUs receivers. Estimating the path loss is practically possible especially in wireless applications with stationary nodes, where the path loss exponent and the node locations can be estimated with high accuracy [12].

In the following, we model both types of interference from the SU to the PUs (CCI and ACI). The interference, $\mathcal{J}_{i_m}$, from all the PUs to subcarrier i_m of the SU is considered as in [4], [7]–[9], [13], which depends on the SU receiver windowing function and power spectral density (PSD) of the PUs. $\mathcal{J}_{i_m}$ is not presented here due to space limitations.

[2]This is done by visiting a database administrated by a government or third party, or by optionally sensing the PUs' radio frequency [7].

A. Interference from the SU to the PUs

1) Co-channel interference (CCI): When the SU uses the m subchannel, the total transmit power on this subchannel $\mathcal{P}_{T,m}$ should be less than a certain threshold $\mathcal{P}_{th,m}$ at the location of the distant PU m receiver. To further reflect the SU transmitter's power amplifier limitations or/and to satisfy regulatory maximum power limits, the total SU transmit power should be limited to a certain threshold $\mathcal{P}_{th}$. Hence, the condition on the total transmit power is formulated as

$$\mathcal{P}_{T,m} = \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)} 10^{0.1\,\text{FM}}\, \mathcal{P}_{th,m}\right]^-, \quad (1)$$

where $L(d_m)$ is the log-distance path loss in dB at distance d_m [14] from the SU and FM is the fading margin in dB.

2) Adjacent channel interfernce (ACI): The ACI interference introduced to the PUs is caused by the sidelobe leakage of the SU subcarriers. Hence, this amount of interference depends on the power allocated to each SU subcarrier and the spectral distance between the SU subcarriers and the PUs. The total ACI from subcarrier i_m of the SU to PU ℓ receiver can be formulated as follows [4], [7]–[9], [13]

$$\mathcal{I}_{i_m \to \ell} = \mathcal{P}_{i_m} T_{s,m} 10^{-0.1L(d_\ell)} 10^{-0.1\,\text{FM}} \int_{f_{i_m,\ell} - \frac{B_\ell}{2}}^{f_{i_m,\ell} + \frac{B_\ell}{2}} \text{sinc}^2(T_{s,m} f) df, \quad (2)$$

where $T_{s,m}$ is the duration of the OFDM symbol of the SU, d_ℓ is the distance from the SU to the PU ℓ receiver, $f_{i_m,\ell}$ is the spectral distance between the SU subcarrier i_m and the PU ℓ frequency band, B_ℓ is the bandwidth of the PU ℓ, and $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. Consequently, the total ACI from the SU to the PU ℓ receiver should be kept below a certain threshold $\mathcal{P}_{\text{ACI},\ell}$ as follow

$$\sum_{i_m=1}^{N_m} \mathcal{I}_{i_m \to \ell} = \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \mathcal{P}_{\text{ACI},\ell}. \quad (3)$$

where $\varpi_{i_m}^{(\ell)} = T_{s,m} 10^{-0.1L(d_\ell)} 10^{-0.1\,\text{FM}} \int_{f_{i_m,\ell} - \frac{B_\ell}{2}}^{f_{i_m,\ell} + \frac{B_\ell}{2}} \text{sinc}^2(T_{s,m} f) df$.

III. Proposed Algorithm

A. Optimization Problem Formulation

We propose a low complexity close-to-optimal algorithm that jointly maximizes the OFDM SU throughput and minimizes its transmit power, while satisfying a target BER per subcarrier[3] and guaranteeing certain levels of CCI/total transmit power and ACI to adjacent PUs. The optimization problem is formulated as

$$\underset{\mathcal{P}_{i_m}}{\text{Minimize}}\ \mathcal{P}_{T,m} = \sum_{i_m=1}^{N} \mathcal{P}_{i_m} \ \text{ and } \ \underset{b_{i_m}}{\text{Maximize}}\ b_{T,m} = \sum_{i_m=1}^{N} b_{i_m},$$

subject to

[3]The constraint on the BER per subcarrier is a suitable formulation that results in similar BER characteristics compared to an average BER constraint, especially at high signal-to-noise ratios (SNR) [15]. Further, it enables obtaining closed-form expressions for the optimal bit and power solutions.

$$\text{BER}_{i_m} \leq \text{BER}_{th,i_m}, \qquad i_m = 1, ..., N_m, \tag{4a}$$

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \left[\mathcal{P}_{th}, 10^{0.1L(d_m)} 10^{0.1\,\text{FM}}\, \mathcal{P}_{th,m}\right]^{-}, \qquad i_m = 1, ..., N_m, \tag{4b}$$

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \mathcal{P}_{\text{ACI},\ell}, i_m = 1, ..., N_m, \ell = 1, ..., \mathcal{M}, \tag{4c}$$

where $b_{T,m}$ and b_{i_m} are the throughput, and number of bits per subcarrier i_m, respectively, and BER_{i_m} and BER_{th,i_m} are the BER per subcarrier i_m and the threshold value of the BER per subcarrier i_m, i_m = 1, ..., N_m, respectively. An approximate expression for the BER per subcarrier i_m in the case of M-ary QAM [16], while taking the interference from the PUs into account, is given by

$$\text{BER}_{i_m} \approx 0.2 \exp\left(-1.6 \frac{\mathcal{P}_{i_m}}{(2^{b_{i_m}} - 1)} \frac{|\mathcal{H}_{i_m}|^2}{(\sigma_n^2 + \mathcal{J}_{i_m})}\right), \tag{5}$$

where $\mathcal{H}_{i_m}$ is the channel gain of subcarrier i_m between the SU transmitter and receiver pair and σ_n^2 is the variance of the additive white Gaussian noise (AWGN).

The multi-objective optimization problem can be rewritten as a linear combination of the multiple objective functions as follows

$$\underset{\mathcal{P}_{i_m}, b_{i_m}}{\text{Minimize}} \quad \mathcal{F}(\mathbf{p}_m, \mathbf{b}_m) = \alpha \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - (1-\alpha) \sum_{i_m=1}^{N_m} b_{i_m}, \quad \text{subject to} \quad g_\varrho(\mathbf{p}_m, \mathbf{b}_m) \leq 0, \tag{6}$$

where α ($0 < \alpha < 1$) is a constant whose value indicates the relative importance of one objective function relative to the other, i.e., minimum power versus maximum throughput, $\varrho = 1, ..., N_m + 2$ is the number of constraints, $\mathbf{p}_m = [\mathcal{P}_{1_m}, ..., \mathcal{P}_{N_m}]^T$ and $\mathbf{b}_m = [b_{1_m}, ..., b_{N_m}]^T$ are the N_m-dimensional power and bit distribution vectors, respectively, and

$$g_\varrho(\mathbf{p}_m, \mathbf{b}_m) = \begin{cases} 0.2 \sum_{i_m=1}^{N_m} b_{i_m} \exp\left(\frac{-1.6\,\mathcal{C}_{i_m}\mathcal{P}_{i_m}}{2^{b_{i_m}}-1}\right) - \text{BER}_{th,i_m} \leq 0, & \varrho = i_m = 1, ..., N_m, \\ \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)} 10^{0.1\,\text{FM}}\, \mathcal{P}_{th,m}\right]^{-} \leq 0, & \varrho = N_m + 1, \\ \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \mathcal{P}_{\text{ACI},\ell} \leq 0, & \varrho = N_m + 2, \end{cases} \tag{7}$$

where $\mathcal{C}_{i_m} = \frac{|\mathcal{H}_{i_m}|^2}{\sigma_n^2 + \mathcal{J}_{i_m}}$ is the channel-to-noise-plus-interference ratio for subcarrier i_m.

B. Optimization Problem Analysis and Solution

The optimization problem in (6) can be solved by the method of Lagrange multipliers. Accordingly, the inequality constraints are transformed to equality constraints by adding non-negative slack variables, $\mathcal{Y}_\varrho^2$, $\varrho = 1, ..., N_m + 2$ [17]. Hence, the constraints are given as

$$\mathcal{G}_\varrho(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}) = g_\varrho(\mathbf{p}_m, \mathbf{b}_m) + \mathcal{Y}_\varrho^2 = 0, \tag{8}$$

where $\mathbf{y} = [\mathcal{Y}_1^2, ..., \mathcal{Y}_{N_m+2}^2]^T$ is the vector of slack variables, and the Lagrangian function $\mathcal{L}$ is expressed as

$$\begin{aligned} \mathcal{L}(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}, \boldsymbol{\lambda}) &= \mathcal{F}(\mathbf{p}_m, \mathbf{b}_m) + \sum_{\varrho=1}^{N_m+2} \lambda_\varrho \mathcal{G}_\varrho(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}), \\ &= \alpha \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - (1-\alpha) \sum_{i_m=1}^{N_m} b_{i_m} \\ &+ \sum_{i_m=1}^{N_m} \lambda_{i_m} \left[0.2\exp\left(\frac{-1.6\mathcal{C}_{i_m}\mathcal{P}_{i_m}}{2^{b_{i_m}}-1}\right) - \text{BER}_{th,i_m} + \mathcal{Y}_{i_m}^2\right] \\ &+ \lambda_{N_m+1}\left[\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)} 10^{0.1\,\text{FM}}\, \mathcal{P}_{th,m}\right]^{-} + \mathcal{Y}_{N_m+1}^2\right] \\ &+ \lambda_{N_m+2}\left[\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \mathcal{P}_{\text{ACI},\ell} + \mathcal{Y}_{N_m+2}^2\right], \end{aligned} \tag{9}$$

where $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_{N_m+2}]^T$ is the vector of Lagrange multipliers associated with the $N_m + 2$ constraints in (7). A stationary point is found when $\nabla\mathcal{L}(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}, \boldsymbol{\lambda}) = 0$, which yields

$$\frac{\partial \mathcal{L}}{\partial \mathcal{P}_{i_m}} = \alpha - \lambda_{i_m} \frac{(0.2)(1.6)\,\mathcal{C}_{i_m}}{2^{b_{i_m}} - 1} \exp\left(\frac{-1.6\,\mathcal{C}_{i_m}\mathcal{P}_{i_m}}{2^{b_{i_m}} - 1}\right) + \lambda_{N_m+1} + \varpi_{i_m}^{(\ell)} \lambda_{N_m+2} = 0, \tag{10a}$$

$$\frac{\partial \mathcal{L}}{\partial b_{i_m}} = -(1-\alpha) + \lambda_{i_m} \frac{(0.2)(1.6)(\ln(2))\,\mathcal{C}_{i_m}\mathcal{P}_{i_m} 2^{b_{i_m}}}{(2^{b_{i_m}} - 1)^2} \times \exp\left(\frac{-1.6\,\mathcal{C}_{i_m}\mathcal{P}_{i_m}}{2^{b_{i_m}} - 1}\right) = 0, \tag{10b}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{i_m}} = 0.2 \exp\left(\frac{-1.6\,\mathcal{C}_{i_m}\mathcal{P}_{i_m}}{2^{b_{i_m}} - 1}\right) - \text{BER}_{th,i_m} + \mathcal{Y}_{i_m}^2 = 0, \tag{10c}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{N_m+1}} = \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)} 10^{0.1\,\text{FM}}\, \mathcal{P}_{th,m}\right]^{-} + \mathcal{Y}_{N_m+1}^2 = 0, \tag{10d}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{N_m+2}} = \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \mathcal{P}_{\text{ACI},\ell} + \mathcal{Y}_{N_m+2}^2 = 0, \tag{10e}$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{i,m}} = 2\lambda_{i_m} \mathcal{Y}_{i_m} = 0, \tag{10f}$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{N_m+1}} = 2\lambda_{N_m+1}\, \mathcal{Y}_{N_m+1} = 0, \tag{10g}$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{N_m+2}} = 2\lambda_{N_m+2}\, \mathcal{Y}_{N_m+2} = 0. \tag{10h}$$

It can be seen that (10) represents $4N_m + 4$ equations in the $4N_m + 4$ unknown components of the vectors $\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}$, and $\boldsymbol{\lambda}$. By solving (10), one obtains the solution $\mathbf{p}_m^*, \mathbf{b}_m^*$. Equation (10f) implies that either $\lambda_{i_m} = 0$ or $\mathcal{Y}_{i_m} = 0$, (10g) implies that either $\lambda_{N_m+1} = 0$ or $\mathcal{Y}_{N_m+1} = 0$, and (10h) implies that either $\lambda_{N_m+2} = 0$ or $\mathcal{Y}_{N_m+2} = 0$. Hence, eight possible cases exist and we are going to investigate each case independently.

— *Cases 1, 2, 3 , and 4*: Setting $\lambda_{i_m} = 0$ in (10) and $\lambda_{N_m+1} = 0$ (case 1)/$\mathcal{Y}_{N_m+1} = 0$ (case 2), or $\lambda_{N_m+2} = 0$ (case 3)/$\mathcal{Y}_{N_m+2} = 0$ (case 4) result in an underdetermined system of $N_m + 4$ equations in $3N_m + 2$ unknowns, and, hence, no unique solution can be reached.

— *Case 5*: Setting $\mathcal{Y}_{i_m} = \lambda_{N_m+1} = \lambda_{N_m+2} = 0$ (i.e., inactive CCI/total transmit power and inactive ACI constraints),

we can relate $\mathcal{P}_{i_m}$ and b_{i_m} from (10a) and (10b) as follows

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\alpha \ln(2)}(1-2^{-b_{i_m}}), \tag{11}$$

with $\mathcal{P}_{i_m} \geq 0$ if and only if $b_{i_m} \geq 0$. By substituting (11) into (10c), one obtains the solution

$$b^*_{i_m} = \frac{1}{\log(2)} \log\left[-\frac{1-\alpha}{\alpha \ln(2)} \frac{1.6\,\mathcal{C}_{i_m}}{\ln(5\,\text{BER}_{th,i_m})}\right]. \tag{12}$$

Consequently, from (11) one gets

$$\mathcal{P}^*_{i_m} = \frac{1-\alpha}{\alpha \ln(2)} + \frac{\ln(5\,\text{BER}_{th,i_m})}{1.6\,\mathcal{C}_{i_m}}. \tag{13}$$

Since (5) is only valid for M-ary QAM, b_{i_m} should be greater than 2. From (12), to have $b_{i_m} \geq 2$, the channel-to-noise ratio per subcarrier, $\mathcal{C}_{i_m}$, must satisfy the condition

$$\mathcal{C}_{i_m} \geq \mathcal{C}_{th,i_m} = -\frac{4}{1.6}\frac{\alpha \ln(2)}{1-\alpha} \ln(5\text{BER}_{th,i_m}), i_m = 1, ..., N_m. \tag{14}$$

— *Case 6*: Setting $\mathcal{Y}_{i_m} = \mathcal{Y}_{N_m+1} = \lambda_{N_m+2} = 0$ (i.e., active CCI/total transmit power and inactive ACI constraints), similar to *case 5*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1})}(1-2^{-b_{i_m}}), \tag{15}$$

$$b^*_{i_m} = \frac{1}{\log(2)} \log\left[-\frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1})} \frac{1.6\,\mathcal{C}_{i_m}}{\ln(5\,\text{BER}_{th,i_m})}\right]. \tag{16}$$

$$\mathcal{P}^*_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1})} + \frac{\ln(5\,\text{BER}_{th,i_m})}{1.6\,\mathcal{C}_{i_m}}, \tag{17}$$

where λ_{N_m+1} is calculated to satisfy the active CCI/total transmit power constraint in (10d). Hence, the value of λ_{N_m+1} is found to be

$$\lambda_{N_m+1} = \frac{\bar{\bar{\mathbb{N}}}^a_m \frac{1-\alpha}{\ln 2}}{\left[\mathcal{P}_{th},\, 10^{0.1L(d_m)}10^{0.1\,\text{FM}}\,\mathcal{P}_{th,m}\right]^- - \sum_{i_m \in \mathbb{N}^a_m} \frac{\ln(5\,\text{BER}_{th,i_m})}{1.6\,\mathcal{C}_{i_m}}} - \alpha, \tag{18}$$

where $\bar{\bar{\mathbb{N}}}^a_m$ is the cardinality of the set of active subcarriers $\mathbb{N}^a_m$.

— *Case 7*: Setting $\mathcal{Y}_{i_m} = \lambda_{N_m+1} = \mathcal{Y}_{N_m+2} = 0$ (i.e., inactive CCI/total transmit power and active ACI constraints), similar to *cases 5* and *6*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})}(1-2^{-b_{i_m}}), \tag{19}$$

$$b^*_{i_m} = \frac{1}{\log(2)} \log\left[-\frac{1-\alpha}{\ln(2)(\alpha+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})} \frac{1.6\mathcal{C}_{i_m}}{\ln(5\text{BER}_{th,i_m})}\right]. \tag{20}$$

$$\mathcal{P}^*_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})} + \frac{\ln(5\,\text{BER}_{th,i_m})}{1.6\,\mathcal{C}_{i_m}}. \tag{21}$$

where λ_{N_m+2} is calculated numerically using the Newton's method [18] to satisfy the active ACI constraint in (10e).

— *Case 8*: Setting $\mathcal{Y}_{i_m} = \mathcal{Y}_{N_m+1} = \mathcal{Y}_{N_m+2} = 0$ (i.e., active CCI/total transmit power and active ACI constraints), similar to the previous *cases*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1}+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})}(1-2^{-b_{i_m}}), \tag{22}$$

$$b^*_{i_m} = \frac{1}{\log(2)} \log\left[-\frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1}+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})} \frac{1.6\mathcal{C}_{i_m}}{\ln(5\text{BER}_{th,i_m})}\right], \tag{23}$$

$$\mathcal{P}^*_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha+\lambda_{N_m+1}+\varpi^{(\ell)}_{i_m}\lambda_{N_m+2})} + \frac{\ln(5\,\text{BER}_{th,i_m})}{1.6\,\mathcal{C}_{i_m}}, \tag{24}$$

where λ_{N_m+1} and λ_{N_m+2} are calculated numerically using the Newton's method to satisfy the active CCI/total transmit power and ACI constraints in (10d) and (10e), respectively.

The obtained solution $(\mathbf{p}^*_m, \mathbf{b}^*_m)$ represents a minimum of $\mathcal{F}(\mathbf{p}_m, \mathbf{b}_m)$ as the Karush-Kuhn-Tucker (KKT) conditions [17] are satisfied[4]; the proof is omitted due to space limitations.

C. Proposed Joint Bit and Power Loading Algorithm

The proposed algorithm can be formally stated as follows.

Proposed Algorithm

INPUT The AWGN variance (σ^2_n), channel gain per subcarrier i_m ($\mathcal{H}_{i_m}$), target BER per subcarrier i_m (BER_{th,i_m}), initial weighting parameter α, $\mathcal{P}_{th}$, $\mathcal{P}_{th,m}$, $\mathcal{P}_{\text{ACI},\ell}$, and PUs information.
for i_m = 1, ..., N_m **do**
  if $\mathcal{C}_{i_m} \geq \mathcal{C}_{th,i_m} = -\frac{4}{1.6}\frac{\alpha\ln(2)}{1-\alpha}\ln(5\,\text{BER}_{th,i_m})$ **then**
    - $b^*_{i_m}$ and $\mathcal{P}^*_{i_m}$ are given by (12) and (13), respectively.
  else
    - Null the corresponding subcarrier i_m.
  end if
end for
if $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m} \geq \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)}10^{0.1\,\text{FM}}\ \mathcal{P}_{th,m}\right]^-$ and $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m}\varpi^{(\ell)}_{i_m} \leq \mathcal{P}_{\text{ACI},\ell}$ **then**
  - $b^*_{i_m}$ and $\mathcal{P}^*_{i_m}$ are given by (16) and (17), respectively.
  - λ_{N_m+1} is given by (18) and $\lambda_{N_m+2} = 0$.
else if $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m} \leq \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)}10^{0.1\,\text{FM}}\ \mathcal{P}_{th,m}\right]^-$ and $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m}\varpi^{(\ell)}_{i_m} \geq \mathcal{P}_{\text{ACI},\ell}$ **then**
  - $b^*_{i_m}$ and $\mathcal{P}^*_{i_m}$ are given by (20) and (21), respectively.
  - $\lambda_{N_m+1} = 0$ and λ_{N_m+2} is calculated to satisfy $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m}\varpi^{(\ell)}_{i_m} = \mathcal{P}_{\text{ACI},\ell}$
else if $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m} \geq \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)}10^{0.1\,\text{FM}}\ \mathcal{P}_{th,m}\right]^-$ and $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m}\varpi^{(\ell)}_{i_m} \geq \mathcal{P}_{\text{ACI},\ell}$
  - $b^*_{i_m}$ and $\mathcal{P}^*_{i_m}$ are given by (23) and (24), respectively.
  - λ_{N_m+1} and λ_{N_m+2} are calculated to satisfy $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m} = \left[\mathcal{P}_{th},\ 10^{0.1L(d_m)}10^{0.1\,\text{FM}}\ \mathcal{P}_{th,m}\right]^-$ and $\sum_{i_m=1}^{N_m}\mathcal{P}_{i_m}\varpi^{(\ell)}_{i_m} = \mathcal{P}_{\text{ACI},\ell}$, respectively.
end if
- $b^*_{i_m,final} \leftarrow$ Round $b^*_{i_m}$ to the nearest integer.
- $\mathcal{P}^*_{i_m,final} \leftarrow$ Recalculate $\mathcal{P}^*_{i_m}$ according to (5).
- If the conditions on the CCI/total transmit power and the ACI are violated due to rounding, decrement the number of bits on the subcarrier that has the largest $\Delta\mathcal{P}_{i_m}(b_{i_m}) = \mathcal{P}_{i_m}(b_{i_m}) - \mathcal{P}_{i_m}(b_{i_m}-1)$ until satisfied.
OUTPUT $b^*_{i_m,final}$ and $\mathcal{P}^*_{i_m,final}$, i_m = 1, ..., N_m.

IV. Numerical Results

In this section, we present illustrative numerical results for the proposed allocation algorithm. Without loss of generality,

[4]Since the optimization problem in (6) is not convex, the obtained solution is not guaranteed to be a global optimum. In the next section, we compare the local optimum results to the global optimum results obtained through an exhaustive search to 1) characterize the gap to the global optimum solution and 2) characterize the gap to the equivalent discrete optimization problem (i.e., with integer constraints on b_{i_m}).

we assume that the OFDM SU coexists with one adjacent PU and one co-channel PU, and both SU and PUs have an equal bandwidth of 1.25 MHz. The OFDM SU transmission parameters are as follows: number of subcarriers $N_m = 128$, symbol duration $T_{s,m} = 102.4$ μsec, and subcarrier spacing $\Delta f_m = 9.7656$ kHz. The propagation log-distance path loss parameters are as follows: exponent $\beta = 4$, wavelength $\lambda = \frac{3\times10^8}{900\times10^6} = 0.33$ meters, distance to PU ℓ $d_\ell = 1$ km, distance to PU m $d_m = 5$ km, and reference distance $d_0 = 0.5$ km. Unless otherwise mentioned, the fading margin is set to FM $= 0$ dB. $\mathcal{P}_{th} = 0.1$ mW and $\mathcal{P}_{th,m} = 10^{-8}\mu$W; hence, $\left[\mathcal{P}_{th}, 10^{0.1L(d_m)}10^{0.1\,\mathrm{FM}}\mathcal{P}_{th,m}\right]^- = \left[0.1\,\mathrm{mW}, 35.53\,\mathrm{mW}\right]^- = 0.1\,\mathrm{mW} = \mathcal{P}_{th}$. The BER constraint per subcarrier, $\mathrm{BER}_{th,i}$, is assumed to be the same for all subcarriers and set to 10^{-4}. A Rayleigh fading environment with average channel power gain $\mathbb{E}\{|\mathcal{H}_{i_m}|^2\}$ equal to 1 is considered. Representative results are presented in this section and were obtained by repeating Monte Carlo trials for 10^4 channel realizations. The PU signal is assumed to be an elliptically filtered white noise process [4], [7]–[9], [13] of variance $10^{-3}\mu$W. Due to space limitations, a certain set of system parameters is chosen to investigate the performance of the proposed algorithm.

Fig. 2 shows the average throughput and average transmit power as a function of the weighting factor α at $\sigma_n^2 = 10^{-3}$ μW, for different values of $\mathcal{P}_{th}$ and $\mathcal{P}_{\mathrm{ACI}}$. For $\mathcal{P}_{th} = \infty$ (inactive CCI/total transmit power constraint) and $\mathcal{P}_{\mathrm{ACI}} = \infty$ (inactive ACI constraint), one can notice that an increase of the weighting factor α yields a decrease of both the average throughput and average transmit power. This can be explained as follows. By increasing α, more weight is given to the transmit power minimization (the minimum transmit power is further reduced), whereas less weight is given to the throughput maximization (the maximum throughput is reduced), according to the problem formulation. Similarly for $\mathcal{P}_{th} = \infty$ (inactive CCI/total transmit power constraint) and $\mathcal{P}_{\mathrm{ACI}} = 10^{-8}\mu$W (active ACI constraint), the average throughput and transmit power decrease as α increases. On the other hand, for $\mathcal{P}_{th} = 0.1$ mW (active CCI/total transmit power constraint) and $\mathcal{P}_{\mathrm{ACI}} = \infty$ (inactive ACI constraint), the same average throughput and power are obtained if the total transmit power is less than $\mathcal{P}_{th}$, while the average throughput and power saturate if the total transmit power exceeds $\mathcal{P}_{th}$.

In Fig. 3, the average throughput and average transmit power are plotted as a function of the power threshold $\mathcal{P}_{th}$, at $\mathcal{P}_{\mathrm{ACI}} = \infty$ (inactive ACI constraint), α = 0.5, and $\sigma_n^2 = 10^{-3}$ μW. It can be noticed that the average throughput increases as $\mathcal{P}_{th}$ increases, and saturates for higher values of $\mathcal{P}_{th}$; moreover, the average transmit power increases linearly with $\mathcal{P}_{th}$, while it saturates for higher values of $\mathcal{P}_{th}$. This can be explained, as for lower values of $\mathcal{P}_{th}$, the total transmit power is restricted by this threshold value, while increasing this threshold value results in a corresponding increase in both the average throughput and total transmit power. For higher values of $\mathcal{P}_{th}$, the CCI/total transmit power constraint is inactive. In this case, the proposed algorithm essentially minimizes the transmit power by keeping it constant; consequently, the average throughput remains constant.

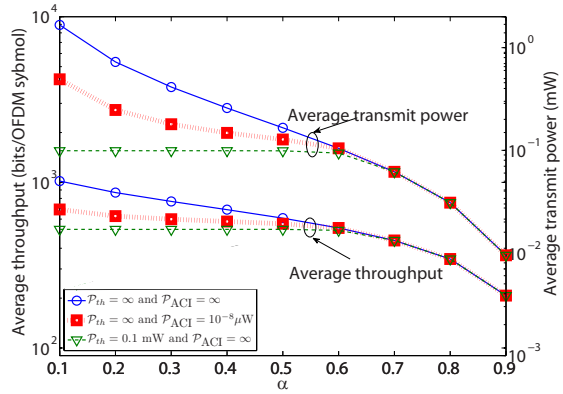

Fig. 2: Effect of the weighting factor α on the OFDM SU performance for different values of $\mathcal{P}_{th}$ and $\mathcal{P}_{\mathrm{ACI}}$ at $\sigma_n^2 = 10^{-3}$ μW.

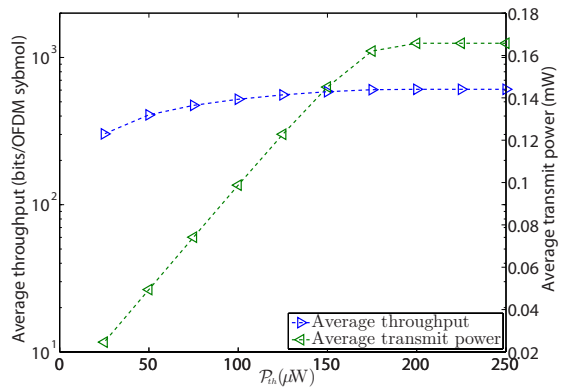

Fig. 3: Effect of $\mathcal{P}_{th}$ on the OFDM SU performance at $\mathcal{P}_{\mathrm{ACI}} = \infty$, α = 0.5, and $\sigma_n^2 = 10^{-3}$ μW.

Table I shows different values of $\mathcal{P}_{\mathrm{ACI}}$ and $\mathcal{P}_{th,m}$ and the corresponding violation ratio of the ACI and CCI, respectively, at the PUs receivers at FM $= 0$ dB (no fading margin is considered). The violation happens when applying the proposed algorithm which is based on the knowledge of the path loss in a real scenario, where the SU transmit signal is additionally affected by fading. As can be seen, for higher values of $\mathcal{P}_{\mathrm{ACI}}$ (inactive ACI constraint), no violation of the ACI was observed at the PU receiver. For lower values of $\mathcal{P}_{\mathrm{ACI}}$, the percentage of trials for which the ACI constraint was violated at the PU receiver was found to vary between 7.87% and 15.15%. Similarly, violation ratios of 24.38% and 27.30% are noticed for small values of $\mathcal{P}_{th,m}$. Since it is practically challenging to estimate the channel between the SU transmitter and the PUs receivers, and the ACI and CCI constraints may be violated in practice when only knowledge of the path loss is available, adding a fading margin becomes crucial to protect the PUs receivers.

Fig. 4 depicts the average throughput and average transmit power as a function of the ACI threshold $\mathcal{P}_{\mathrm{ACI}}$ for different values of FM, at $\mathcal{P}_{th} = \infty$, α = 0.5 and $\sigma_n^2 = 10^{-3}$ μW. As can be seen, both the average throughput and average transmit

TABLE I: ACI AND CCI VIOLATION RATIO AT THE PUS RECEIVERS DUE TO PARTIAL CHANNEL KNOWLEDGE, FOR DIFFERENT VALUES OF $\mathcal{P}_{\mathrm{ACI}}$ AND $\mathcal{P}_{th,m}$.

$\mathcal{P}_{\mathrm{ACI}}(\mu\mathrm{W})$	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
Violation ratio (%)	14.37	15.15	7.87	13.91	15.07	0.00

$\mathcal{P}_{th,m}(\mu\mathrm{W})$	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
Violation ratio (%)	24.38	27.30	0.41	0.00	0.00	0.00

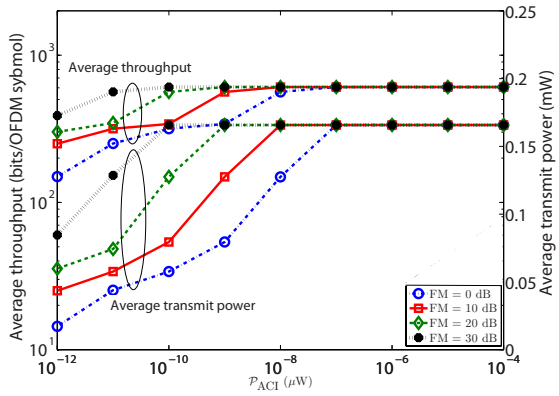

Fig. 4: Effect of $\mathcal{P}_{\text{ACI}}$ on the OFDM SU performance at $\mathcal{P}_{th} = \infty$, α = 0.5, and $\sigma_n^2 = 10^{-3}$ μW.

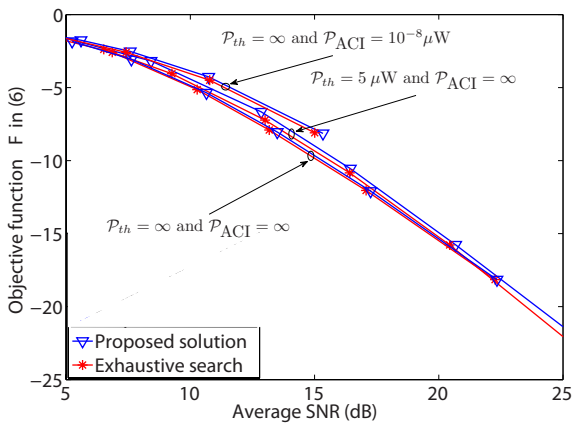

Fig. 5: Objective function for the proposed algorithm and the exhaustive search when N = 8 and $\alpha = 0.5$.

power increase as $\mathcal{P}_{\text{ACI}}$ increases, and saturates for higher values of $\mathcal{P}_{\text{ACI}}$. This can be explained, as for lower values of $\mathcal{P}_{\text{ACI}}$ the ACI constraint is active and, hence, affects the total transmit power. Increasing $\mathcal{P}_{\text{ACI}}$ results in a corresponding increase in both the average throughput and total transmit power. For higher values of $\mathcal{P}_{\text{ACI}}$ (the ACI constraint is inactive), the achieved throughput and transmit power saturate, and, hence, there is no violation of the ACI at the PU receiver as discussed earlier. Increasing the fading margin FM, results in an expected increase in the average throughput and transmit power. Further, it reduces the violation ratio at the PU receiver. This can be explained with the aid of Fig. 4 as follows. For a given $\mathcal{P}_{\text{ACI}}$ the average throughput and transmit power saturate (and hence the ACI constraint is not violated at the PU receivers) at higher values of FM when compared to lower values of FM.

Fig. 5 compares the objective function achieved with the proposed algorithm and an exhaustive search that finds the discredited global optimal allocation for the problem in (6) for different values of $\mathcal{P}_{th}$ and $\mathcal{P}_{\text{ACI}}$. Results are presented for a small number of subcarriers N = 8, such that the exhaustive search is feasible. As one can notice, the proposed algorithm approaches the optimal results of the exhaustive search with significantly reduced computational effort as observed from simulations.

V. Conclusions

In this paper, we proposed a joint bit and power loading algorithm that maximizes the OFDM SU throughput and minimizes its transmit power while guaranteeing a target BER and certain limits on the CCI/total transmit power and ACI interferences to existing PUs. We assume only knowledge of the path loss on the links between the SU transmitter and the PUs receivers, as estimating the instantaneous channel gains on these links are practically challenging. Accordingly, due to this partial channel knowledge, the ACI and CCI constraints may be violated in practice at the PUs receivers. Hence, adding a fading margin is crucial to protect the PUs receivers. Simulation results show that the proposed algorithm approaches that of an exhaustive search for the discrete optimal allocation.

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