Optimization of Discrete Power and Resource Block Allocation for Achieving Maximum Energy Efficiency in OFDMA Networks

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***Abstract*—Most of the resource allocation literature on energy-efficient orthogonal frequency division multiple access (OFDMA)-based wireless communication systems assume continuous power allocation/control, while in practice the power levels are discrete (such as in 3GPP LTE). This convenient continuous power assumption has mainly been due to either the limitations of the used optimization tools and/or the high computational complexity involved in addressing the more realistic discrete power allocation/control.**

In this paper, we introduce a new optimization framework to maximize the energy efficiency of the downlink transmission of cellular OFDMA networks subject to power budget and quality-of-service (QoS) constraints, while considering discrete power and resource blocks (RBs) allocations. The proposed framework consists of two parts: 1) we model the predefined discrete power levels and RBs allocations by a single binary variable, and 2) we propose a close-to-optimal semidefinite relaxation (COS) algorithm with Gaussian randomization to efficiently solve this non-convex combinatorial optimization problem with polynomial time complexity. We notice that a small number of power levels suffices to approach the energy efficiency performance of the continuous power allocation. Based on this observation, we propose an iterative suboptimal heuristic (SOH) to further reduce the computational complexity. Simulation results show the effectiveness of the proposed schemes in maximizing the energy efficiency, while considering practical discrete power levels.

***Index Terms*—Energy efficiency, OFDMA, convex optimization, semidefinite relaxation, Gaussian randomization.**

I. Introduction

Cellular communications plays an undeniable role in the daily lives of millions of people worldwide. The demands on data rates are growing exponentially mainly due to smartphones, which are always connected to the cellular network during the day. Due to the hike in energy consumption costs, and ecological, and environmental reasons the increasing demands of data rate cannot be achieved by simply scaling up the transmit power. Instead, this has to be achieved at similar or lower energy consumptions. That said, energy-efficient communications have received a lot of attention from both industry and academia in recent years [1]–[3].

Orthogonal frequency division multiple access (OFDMA) is adopted in many contemporary wireless standards [4] due to multi-user and frequency diversities. In OFDMA, the frequency spectrum is divided into multiple subcarriers where different groups of subcarriers may be allocated for the transmission of different users depending on the varying channel conditions [5]. Frequency diversity is achieved by activating only subcarriers that can support high quality transmission and nulling subcarriers with poor channel conditions. On the other hand, multi-user diversity is achieved with appropriate user mapping, i.e., a subcarrier may not be assigned to a certain user if the channel between this user and the BS on this subcarrier is in deep fade and in this case the subcarrier should be assigned to a different user.

Most of the energy-efficient resource allocation algorithms reported in the literature, for various OFDMA system models, are based on continuous power allocation rather than allocation of discrete power levels [6]–[8]. Additionally, their solution techniques are mainly based on the fractional properties of the energy efficiency maximization problem [9]. In particular, the energy efficiency fractional objective function is transformed into an equivalent weighted sum of rate and power objectives. Then, dual Lagrangian method is applied to achieve the global energy efficiency optimal solution in an iterative manner. However, the complexity of finding the Lagrange multipliers (associated with the continuous power allocation) of the dual Lagrangian method is in general of unknown computation complexity [10].

Discrete power control/allocation simplifies the transmitter design, and also, significantly reduces the signalling overhead among nodes [11]. The authors in [12] considered the discrete power and subcarrier allocations to maximize the transmission rate of an OFDMA system, and solved the binary rate maximization problem using concepts of dynamic programming. In [13], the authors investigated the uplink transmission of contention-based synchronization in OFDMA systems, and formulated a constrained finite non-cooperative game to maximize the energy efficiency, where each mobile station has its own discrete power levels. For single carrier transmission [14], the authors considered predefined discrete power levels at the BS and proposed a reduced complexity algorithm that maximizes the energy efficiency of multi-cell networks. The formulated problem is classified as a fractional discrete optimization problem that is NP-hard to solve. The structure of the fractional problem is investigated and a suboptimal algorithm was proposed to attain an acceptable solution with polynomial time complexity.

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A. Contribution of the Paper

In this paper, we investigate the energy efficiency resource allocation problem of the downlink transmission of OFDMA networks subject to power budget and per user quality-of-service (QoS)[1] constraints, while considering practical design issues, i.e., discrete power levels. Such a constraint adds another dimension to the difficulty of the energy efficiency maximization problem and we are going to efficiently address in this paper. The main contributions of this paper are summarized as follows.

- We introduce a novel optimization framework to efficiently handle such energy efficiency maximization problem with discrete power levels and RBs. This framework consists of the following two parts:
  - We model the discrete power levels and discrete RBs by a single binary variable. We then show that the formulated energy efficiency maximization problem is combinatorial non-convex problem that turns out to be NP-hard to solve.
  - To tackle such a non-convexity, we propose a two stage close-to-optimal semidefinite relaxation (SDR)-based algorithm with Gaussian randomization, named COS, to efficiently solve this NP-hard problem with polynomial time complexity. In the first stage, the SDR generates a positive semidefinite covariance matrix together with an upper bound on the energy efficiency of the downlink transmission. In the second stage, using Gaussian randomization, we exploit the outputs of the first stage to compute good approximate solutions for the non-convex energy efficiency maximization problem with provable approximation accuracy.
- We notice that a small number of the discrete power levels is sufficient to approach the optimal energy efficiency performance of the continuous power allocation in [15]. Based on this observation, we propose a reduced-complexity iterative suboptimal heuristic, named SOH, that adopts a single power level.
- Extensive simulations results are provided to show the effectiveness of the proposed schemes in maximizing the energy efficiency of the downlink transmission. Results reveal that COS achieves the optimal performance of the exhaustive search. Additionally, results show that SOH strikes a balance between complexity and energy efficiency performance.

B. Paper Organization

The rest of this paper is organized as follows. Section II presents the system model. In Section III, the optimization framework for the joint optimization of the RBs and the power allocation is provided. Section IV discusses a two-stage close-to-optimal algorithm to solve the energy efficiency maximization problem. Then, a low-complexity suboptimal heuristic is proposed in Section V. In Section VI, simulation results are provided, and Section VII concludes the paper.

[1]In this paper, the QoS is defined in terms of the minimum data rate that a user requires.

C. Notation

Throughout the paper we use bold-faced upper case letters, e.g., $\boldsymbol{X}$, to denote matrices, bold-faced lower case letters, e.g., $\boldsymbol{x}$, to denote column vectors, light-faced italics letters, e.g., x, to denote scalars, and calligraphic letters, e.g., $\mathcal{X}$, to denote sets. $\boldsymbol{I}$ denotes the identity matrix. The vectors of all-ones and all-zeros are denoted by $\mathbf{1}$ and $\mathbf{0}$, respectively, and for ease of exposition, we drop the subscript indicating the dimension of the all-one and the all-zero vectors and matrices. The trace, the rank, and the column vector consisting of the diagonal elements of matrix $\boldsymbol{X}$ are respectively denoted by $\mathrm{Tr}(\boldsymbol{X})$, $\mathrm{rank}(\boldsymbol{X})$, and $\mathrm{diag}(\boldsymbol{X})$. Lastly, $[\cdot]^{\mathrm{T}}$ denotes the transpose operator, $\mathrm{sgn}(\cdot)$ denotes the element-wise signum function, $\mathcal{N}(\cdot,\cdot)$ denotes Gaussian distribution with a particular mean and variance, and $\mathrm{vec}(\cdot)$ denotes the operator that stacks the columns of a matrix on top of each other.

II. System Model

We consider a single-cell OFDMA network, in which a base station (BS) is located at the center of the cell. In this network, K uniformly distributed users communicate with the BS over N RBs[2], each of them with a bandwidth of W_0, and L discrete power levels available to the BS. We respectively denote the set of all users by $\mathcal{K} = \{1, \ldots, K\}$, the set of all RBs by $\mathcal{N} = \{1, \ldots, N\}$, and the set of power levels by $\mathcal{P} = \{p^1, \ldots, p^\ell, \ldots, p^L\}$, where $L = |\mathcal{P}|$ is the cardinality of $\mathcal{P}$. Moreover, we denote the channel gain between the BS and the k-th user on the n-th RB as h_k^n, which includes the path loss, shadowing, and small scale fading.

For such a network, we consider a centralized design, in which a central node collects network parameters and decides on the allocation of RBs between the users, as well as the allocation of power levels over the RBs. Due to such a central node, we assume that each RB is exclusively assigned to a user throughout the signalling interval [7], [17]. Hence, the signal-to-noise ratio (SNR) of the received signal by the k-th user on the n-th RB using the ℓ-th power level can be given as

$$\Gamma_k^{n\ell} = \frac{p^\ell |h_k^n|^2}{W_0 N_0}, \tag{1}$$

where N_0 is the power spectral density of the additive white Gaussian noise (AWGN). Hence, the maximum data rate that can be reliably communicated between the BS and the k-th user on the n-th RB using ℓ-th power level is expressed as

$$r_k^{n\ell} = W_0 \log_2\left(1 + \Gamma_k^{n\ell}\right). \tag{2}$$

We note that for a known triplet, (k, n, ℓ), the SNR, $\Gamma_k^{n\ell}$, and the data rate, $r_k^{n\ell}$, can be readily calculated, and they have deterministic values.

III. Energy Efficiency Maximization Problem

In this section, we propose an optimization framework that jointly optimizes the RBs and the discrete power allocations to maximize the energy efficiency of the downlink transmission

[2]In 3GPP LTE networks [16], the BS allocates two-dimensional time-frequency resource units, among the users, i.e., scheduling RBs. An RB has a frequency bandwidth of 180 kHz and a time duration of one slot of 0.5 ms.

of OFDMA systems. We first introduce the RB usage, the power budget and the QoS constrains, and the design objective considered in the system. We then present the energy efficiency maximization problem formulation.

A. System Constraints

1) RB Usage Constraints: We use an indicator variable, $\phi_k^{n\ell}$, and let it represent whether the triplet of (k, n, ℓ), $k \in \mathcal{K}$, $n \in \mathcal{N}$, and $\ell \in \mathcal{P}$, is used for communication or not. If the k-th user is associated with the BS on the n-th RB using the ℓ-th power level, then $\phi_k^{n\ell} = 1$; otherwise, $\phi_k^{n\ell} = 0$. Hence,

$$\phi_k^{n\ell} \in \{0,1\}, \quad \forall k \in \mathcal{K},\ \forall n \in \mathcal{N}, \text{and } \forall \ell \in \mathcal{P}. \tag{3}$$

Using the defined binary variable, the total usage of the n-th RB across the entire network can be shown to be $\sum_{k\in\mathcal{K}} \sum_{\ell\in\mathcal{P}_b} \phi_k^{n\ell}$. To avoid interference, in the system model considered herein, each RB is constrained to be used at most once, and this constraint can be expressed as

$$\sum_{k\in\mathcal{K}} \sum_{\ell\in\mathcal{P}} \phi_k^{n\ell} \leq 1, \quad \forall n \in \mathcal{N}. \tag{4}$$

This constraint implies that at a given time instant, at most one power level can be used on each RB, and each RB can at most be used on one BS-to-user link.

2) Power Allocation Constraint: In a practical system, the total power consumption of the BS cannot exceed a maximum total power budget, $P^{\max}$. The total power consumed by the BS can be expressed as $\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} \phi_k^{n\ell} p^\ell$. Hence, the power budget constraint is expressed as

$$\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} \phi_k^{n\ell} p^\ell \leq P^{\max}. \tag{5}$$

Implicit in (5) and the one in (4) is that at most one non-zero power level is allowed on an RB at a given time instant.

3) QoS Constraints: Utilizing the expression given in (2), the maximum total data rate that the BS can reliably communicate with the k-th user can be expressed as $\sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} r_k^{n\ell} \phi_k^{n\ell}$. To ensure that the QoS requirement of the k-th user is met, the following minimum supported rate constraint should be satisfied:

$$\sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} r_k^{n\ell} \phi_k^{n\ell} \geq r_k^{\min}, \quad \forall k \in \mathcal{K}, \tag{6}$$

where the QoS demanded by k-th user is denoted by $r_k^{\min}$.

B. System Design Objective

The energy efficiency of the network is defined as the ratio of the total data rate and the total consumed power, with unit of bits/Joule. The total BS power consumption is obtained as

$$P_{\mathrm{T}} = P_{\mathrm{C}} + \epsilon \sum\nolimits_{k\in\mathcal{K}} \sum\nolimits_{n\in\mathcal{N}} \sum\nolimits_{\ell\in\mathcal{P}} \phi_k^{n\ell} p^\ell, \tag{7}$$

where P_{C} is the total circuitry power consumption required to deliver the information from the BS to the users, and ϵ is a constant defined by the inverse of the power amplifier efficiency. The total transmission data rate can be written as

$$r_{\mathrm{T}} = \sum\nolimits_{k\in\mathcal{K}} \sum\nolimits_{n\in\mathcal{N}} \sum\nolimits_{\ell\in\mathcal{P}} \phi_k^{n\ell} r_k^{n\ell}. \tag{8}$$

Hence, the energy efficiency metric of the downlink transmission can be expressed as

$$\eta_{\mathrm{EE}} = \frac{r_{\mathrm{T}}}{P_{\mathrm{T}}}. \tag{9}$$

The design objective in this work is to maximize the energy efficiency metric given in (9).

C. Optimization Problem Formulation

Subsequently, we can cast the joint RB and power allocation problem for energy efficiency maximization in the downlink of an OFDMA network in the following form:

$$\max_{\phi_k^{n\ell}} \quad \eta_{\mathrm{EE}} = \frac{\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} r_k^{n\ell} \phi_k^{n\ell}}{\epsilon \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} p^\ell \phi_k^{n\ell} + P_{\mathrm{C}}}, \tag{10a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}} \sum_{\ell\in\mathcal{P}} \phi_k^{n\ell} \leq 1, \quad \forall n \in \mathcal{N}, \tag{10b}$$

$$\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} \phi_k^{n\ell} p^\ell \leq P^{\max}, \tag{10c}$$

$$\sum_{n\in\mathcal{N}} \sum_{\ell\in\mathcal{P}} r_k^{n\ell} \phi_k^{n\ell} \geq r_k^{\min}, \quad \forall k \in \mathcal{K}, \tag{10d}$$

$$\phi_k^{n\ell} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \text{and } \forall \ell \in \mathcal{P}. \tag{10e}$$

The formulation in (10) is an integer non-linear program. In particular, it is an integer linear fractional program due to the constraint in (10e), in which the optimization variables are restricted to be integer. For solving such a problem optimally, branch-and-bound type algorithms can be used. However, these algorithms have exponential complexity. We note that if the restriction on the variables is removed, then the energy efficiency problem in (10) becomes a linear fractional program and belongs to the class of quasiconvex programs[3] [18]. Hence, it can be solved efficiently using bisection method, in which a sequence of feasibility problems need to be solved.

IV. Close-to-Optimal Resource Allocation: A Semidefinite Relaxation-Based Approach

For solving the non-convex problem in (10), we use the SDR technique with Gaussian randomization. This technique obtains close-to-optimal solutions for the energy efficiency problem in (10) with polynomial-time complexity[4]. A similar approach employed herein was considered in [20] for the user association problem in heterogeneous networks to maximize the number of the accommodated users while to minimize the number of RBs used in the network. The problem considered in [20] was integer linear programming, in which apart from the integer constraints, the objective function and the constraints were

[3]Since all constraints are linear, they comprise a convex set. Moreover, the objective function is quasi-linear in the variable $\phi_k^{n\ell}$ as its superlevel and sublevel sets are convex.

[4]In some particular cases, the SDR technique with Gaussian randomization can have a provable approximation accuracy. Finding the bound (quantifying the gap between the performance of the SDR-based technique and the optimal one) is in general an involved problem and it is out of the scope of the current paper. However, a summary of some of the major approximation accuracy results is given in Tables I and II in [19].

linear. However, in this paper, we consider joint optimization of RB and power allocation to maximize the energy efficiency in a macro-only network. Here, the problem in (10) is integer linear fractional programming, in which the objective function is quasi-convex, the constraints are linear, and the optimization variables are integer.

Before discussing the SDR technique with Gaussian randomization, we first express the energy efficiency, the RB usage, and the power budget, and the QoS constraints in vector form. For this purpose, we introduce a 3-dimensional tensor $\mathbf{\Phi}$ with entries denoted by $\phi_k^{n\ell}$. We express this tensor in the form of a $N \times KL$ block-partitioned matrix, and particularly, it is written as a matrix of $1 \times K$ blocks,

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_1 & \ldots & \mathbf{\Phi}_K \end{bmatrix}, \tag{11}$$

where each blocks of that matrix has $N \times L$ entries, and for $j = 1, \ldots, K$, $\mathbf{\Phi}_j$ is given as

$$\mathbf{\Phi}_j = \begin{bmatrix} \phi_j^{11} & \ldots & \phi_j^{1,\ell} \\ \vdots & \ddots & \vdots \\ \phi_j^{N1} & \ldots & \phi_j^{NL} \end{bmatrix}. \tag{12}$$

We also introduce four additional 3-dimensional tensors: $\boldsymbol{A}_n$, $\boldsymbol{B}_k$, $\boldsymbol{C}$, and $\boldsymbol{D}$. Similar to $\mathbf{\Phi}$, these tensors are expressed in the form of a $N \times KL$ block-partitioned matrix with $1 \times K$ blocks, each with $N \times L$ entries. We define these four tensors as follows:

The tensor $\boldsymbol{A}_n$ can be written as

$$\boldsymbol{A}_n = \begin{bmatrix} \boldsymbol{A}_{n_1} & \ldots & \boldsymbol{A}_{n_K} \end{bmatrix}, \quad n = 1, \ldots, N, \tag{13}$$

where, for all $j = 1, \ldots, K$, $\boldsymbol{A}_{n_j} = \boldsymbol{e}_n \mathbf{1}_L^T$, where $\boldsymbol{e}_n$ is the n-th column of the $N \times N$ identity matrix $\boldsymbol{I}_N$. Likewise, the tensor $\boldsymbol{B}_k$ can be expressed as

$$\boldsymbol{B}_k = \begin{bmatrix} \boldsymbol{B}_{k_1} & \ldots & \boldsymbol{B}_{k_{\hat{k}}} & \ldots & \boldsymbol{B}_{k_K} \end{bmatrix}, \quad k = 1, \ldots, K, \tag{14}$$

where, for $j = 1, \ldots, K$, $\boldsymbol{B}_{k_j} = \mathbf{0}_{N \times L}$ when $j \neq \hat{k}$, and, when $j = \hat{k}$,

$$\boldsymbol{B}_{k_{\hat{k}}} = \begin{bmatrix} r_{\hat{k}}^{11} & \ldots & r_{\hat{k}}^{1\ell} \\ \vdots & \ddots & \vdots \\ r_{\hat{k}}^{N1} & \ldots & r_{\hat{k}}^{NL} \end{bmatrix}. \tag{15}$$

Similar to the tensor $\boldsymbol{B}_k$, the tensor $\boldsymbol{C}$ can be expressed as

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_1 & \ldots & \boldsymbol{C}_K \end{bmatrix}, \tag{16}$$

where, for $j = 1, \ldots, K$, $\boldsymbol{C}_j$ can be given as

$$\boldsymbol{C}_j = \begin{bmatrix} r_j^{11} & \ldots & r_j^{1\ell} \\ \vdots & \ddots & \vdots \\ r_j^{N1} & \ldots & r_j^{NL} \end{bmatrix}. \tag{17}$$

The tensor $\boldsymbol{G}$ can be written as

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_1 & \ldots & \boldsymbol{G}_K \end{bmatrix}, \tag{18}$$

where, for $j = 1, \ldots, K$, $\boldsymbol{G}_j$ is given as

$$\boldsymbol{G}_j = \begin{bmatrix} p^1 & \ldots & p^L \\ \vdots & \ddots & \vdots \\ p^1 & \ldots & p^L \end{bmatrix}. \tag{19}$$

Finally, we make these definitions: $\boldsymbol{\phi} \triangleq \mathrm{vec}(\mathbf{\Phi}^T)$, $\boldsymbol{a}_n \triangleq \mathrm{vec}(\boldsymbol{A}_n^T)$, $\boldsymbol{b}_k \triangleq \mathrm{vec}(\boldsymbol{B}_k^T)$, $\boldsymbol{c} \triangleq \mathrm{vec}(\boldsymbol{C}^T)$, and $\boldsymbol{g} \triangleq \mathrm{vec}(\boldsymbol{G}^T)$.

Using the defined vectors, the problem in (10) can be expressed as in the following form:

$$\begin{aligned} \max_{\boldsymbol{\phi}} \quad & \frac{\boldsymbol{c}^T \boldsymbol{\phi}}{\epsilon \boldsymbol{g}^T \boldsymbol{\phi} + P_{\mathrm{C}}}, && \text{(20a)} \\ \text{subject to} \quad & \boldsymbol{a}_n^T \boldsymbol{\phi} \leq 1, \quad \forall n \in N, && \text{(20b)} \\ & \boldsymbol{g}^T \boldsymbol{\phi} \leq P^{\max}, && \text{(20c)} \\ & \boldsymbol{b}_k^T \boldsymbol{\phi} \geq r_k^{\min}, \quad \forall k \in K, && \text{(20d)} \\ & \boldsymbol{\phi} \in \{0,1\}^{KNL}. && \text{(20e)} \end{aligned}$$

To express this problem in a form amenable to SDR, we transform the binary optimization variables to antipodal ones. In particular, we introduce the vector $\boldsymbol{\theta} = 2\boldsymbol{\phi} - \mathbf{1}$, which implies that $\boldsymbol{\theta} \in \{-1,1\}^{KNL}$, and

$$\boldsymbol{\phi} = \frac{1}{2}(\boldsymbol{\theta} + \mathbf{1}). \tag{21}$$

Using (21), the formulation in (20) can be rewritten as:

$$\begin{aligned} \max_{\boldsymbol{\theta}} \quad & \frac{\boldsymbol{c}^T (\boldsymbol{\theta} + \mathbf{1})}{\epsilon \boldsymbol{g}^T (\boldsymbol{\theta} + \mathbf{1}) + 2P_{\mathrm{C}}}, && \text{(22a)} \\ \text{subject to} \quad & \frac{1}{2} \boldsymbol{a}_n^T (\boldsymbol{\theta} + \mathbf{1}) \leq 1, \quad \forall n \in N, && \text{(22b)} \\ & \frac{1}{2} \boldsymbol{g}^T (\boldsymbol{\theta} + \mathbf{1}) \leq P^{\max}, && \text{(22c)} \\ & \frac{1}{2} \boldsymbol{b}_k^T (\boldsymbol{\theta} + \mathbf{1}) \geq r_k^{\min}, \quad \forall k \in K, && \text{(22d)} \\ & \boldsymbol{\theta} \in \{-1,1\}^{KNL}. && \text{(22e)} \end{aligned}$$

To use the SDR-based technique, we consider a homogeneous reformulation of the problem in (22), and the optimization variables in (22) are constrained to be in the cone of symmetric positive semidefinite (PSD) matrices [19]. To do so, we define the following vectors in $\mathbb{R}^{KNL+1}$, $\hat{\boldsymbol{c}} \triangleq [\boldsymbol{c}^T \ \boldsymbol{c}^T \mathbf{1}]^T$, $\hat{\boldsymbol{g}} \triangleq [\epsilon \boldsymbol{g}^T \ \epsilon \boldsymbol{g}^T \mathbf{1} + 2P_{\mathrm{C}}]^T$, $\hat{\boldsymbol{a}}_n \triangleq [\boldsymbol{a}_n^T \ \boldsymbol{a}_n^T \mathbf{1}]^T$, $n = 1, \ldots, N$, $\hat{\boldsymbol{b}}_k \triangleq [\boldsymbol{b}_k^T \ \boldsymbol{b}_k^T \mathbf{1}]^T$, $k = 1, \ldots, K$, $\hat{\mathbf{1}} \triangleq [\mathbf{1}^T \ \mathbf{1}^T \mathbf{1}]^T$, $\hat{\boldsymbol{\theta}} \triangleq [\boldsymbol{\theta} \ 1]^T$ and $\hat{\boldsymbol{f}} \triangleq [\mathbf{0}^T \ 1]^T$. We also define the symmetric matrices $\mathbf{\Theta} \in \mathbb{R}^{KNL \times KNL}$ and $\mathbf{\Omega} \in \mathbb{R}^{(KNL+1) \times (KNL+1)}$ to be $\mathbf{\Theta} \triangleq \boldsymbol{\theta}\boldsymbol{\theta}^T$ and $\mathbf{\Omega} = \hat{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}^T$, in particular, $\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Theta} & \boldsymbol{\theta} \\ \boldsymbol{\theta}^T & 1 \end{bmatrix}$. Finally, we define the following $(KNL+1) \times (KNL+1)$ matrices $\boldsymbol{H}_1 \triangleq \hat{\boldsymbol{f}} \hat{\mathbf{1}}^T$, $\boldsymbol{H}_{\hat{\boldsymbol{c}}} \triangleq \hat{\boldsymbol{f}} \hat{\boldsymbol{c}}^T$, $\boldsymbol{H}_{\hat{\boldsymbol{a}}_n} \triangleq \hat{\boldsymbol{f}} \hat{\boldsymbol{a}}_n^T$, and $\boldsymbol{H}_{\hat{\boldsymbol{g}}} \triangleq \hat{\boldsymbol{f}} \hat{\boldsymbol{g}}^T$, $\boldsymbol{H}_{\hat{\boldsymbol{b}}_k} \triangleq \hat{\boldsymbol{f}} \hat{\boldsymbol{b}}_k^T$.

Using the defined matrices, it can be verified that the problem in (10) is equivalent to the following optimization problem:

$$\begin{aligned} \max_{\mathbf{\Omega}} \quad & \frac{\mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{c}}} \mathbf{\Omega})}{\mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{g}}} \mathbf{\Omega})}, && \text{(23a)} \\ \text{subject to} \quad & \frac{1}{2} \mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{a}}_n} \mathbf{\Omega}) \leq 1, \quad \forall n \in N, && \text{(23b)} \\ & \frac{1}{2} \mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{g}}} \mathbf{\Omega}) \leq \epsilon P^{\max} + P_{\mathrm{C}} && \text{(23c)} \\ & \frac{1}{2} \mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{b}}_k} \mathbf{\Omega}) \geq r_k^{\min}, \quad \forall k \in K, && \text{(23d)} \\ & \mathbf{\Omega} \succeq \mathbf{0}, && \text{(23e)} \\ & \mathrm{diag}(\mathbf{\Omega}) = 1, && \text{(23f)} \\ & \mathrm{rank}(\mathbf{\Omega}) = 1. && \text{(23g)} \end{aligned}$$

The formulation in (23) is non-convex due to the rank-1 constraint in (23g). To find a close-to-optimal solution, we consider a relaxed version of (23) by dropping rank-1 constraint. Let $\boldsymbol{z}$, $\boldsymbol{Z}$ and $\boldsymbol{M}$ be the optimization variables of the relaxed problem corresponding to $\boldsymbol{\theta}$, $\boldsymbol{\Theta}$ and $\boldsymbol{\Omega}$ in the original problem in (23), respectively. We then end up with the following formulation

$$\max_{\boldsymbol{M}} \quad \frac{\mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{c}}}\boldsymbol{M})}{\mathrm{Tr}(\boldsymbol{H}_{\hat{\boldsymbol{g}}}\boldsymbol{M})}, \tag{24a}$$
$$\text{subject to} \quad (23\text{b}) - (23\text{f}).$$

Though the formulation in (24) is still non-convex in $\boldsymbol{M}$, it is quasi-convex due to its linear fractional objective structure. It is known that a quasi-convex problem can be optimally solved using the bisection method. Hence, we solve a sequence of convex feasibility problems, and these feasibility problems are in the following form:

$$\text{find} \quad \boldsymbol{M}, \tag{25a}$$
$$\text{subject to} \quad \mathrm{Tr}((\eta_0 \boldsymbol{H}_{\hat{\boldsymbol{g}}} - \boldsymbol{H}_{\hat{\boldsymbol{c}}})\boldsymbol{M}) \le 0, \tag{25b}$$
$$(23\text{b}) - (23\text{f}).$$

For each instance of this problem, the value of η_0 is fixed and represents the energy efficiency in the network. The optimal η_0 must lie in $[0, \eta_{\max}]$, where $\eta^{\max} = \boldsymbol{c}^T \boldsymbol{1}/P_{\mathrm{C}}$.

A. Gaussian Randomization

In the previous section, the optimal value of η_0 is obtained using bisection search. Let $\boldsymbol{z}^*$, $\boldsymbol{Z}^*$ and $\boldsymbol{M}^*$ be the optimal solution of the convex problem in (25) corresponding the optimal value of η_0. Since the solution of the relaxed problem is not rank-1 in general, we use the Gaussian randomization approach to obtain a close-to-optimal solution for the problem in (23). In the Gaussian randomization approach, the vector $\boldsymbol{z}^*$ generated by solving the relaxed program is considered as the mean of a multivariate Gaussian KNL-dimensional random vector, and $\boldsymbol{Z}^* - \boldsymbol{z}^*\boldsymbol{z}^{*T}$ is considered as the covariance matrix of this random vector. Specifically, a set of J random vector samples is drawn from the Gaussian distribution with mean $\boldsymbol{z}^*$ and covariance $\boldsymbol{Z}^* - \boldsymbol{z}^*\boldsymbol{z}^{*T}$. We denote the set by $\mathcal{R} = \{\boldsymbol{\nu}^j\}_{j=1}^J$, where $\boldsymbol{\nu}^j \sim \mathcal{N}(\boldsymbol{z}^*, \boldsymbol{Z}^* - \boldsymbol{z}^*\boldsymbol{z}^{*T})$, $j = 1, \ldots, J$. Letting $\hat{\boldsymbol{\nu}} = [\boldsymbol{\nu}^T \ 1]^T$, and $\hat{\boldsymbol{z}}^* = [\boldsymbol{z}^{*T} \ 1]^T$, it can be seen that the vectors in $\mathcal{R}$ provide an approximate solution to the following stochastic optimization problem:

$$\max_{\substack{\mathbb{E}\{\hat{\boldsymbol{\nu}}\hat{\boldsymbol{\nu}}^T\} = \boldsymbol{M}^*,\\ \mathbb{E}\{\hat{\boldsymbol{\nu}}\} = \hat{\boldsymbol{z}}^*}} \quad \frac{\mathbb{E}\{\hat{\boldsymbol{\nu}}^T \boldsymbol{H}_{\hat{\boldsymbol{c}}} \hat{\boldsymbol{\nu}}\}}{\mathbb{E}\{\hat{\boldsymbol{\nu}}^T \boldsymbol{H}_{\hat{\boldsymbol{g}}} \hat{\boldsymbol{\nu}}\}}, \tag{26a}$$
$$\text{subject to} \quad \frac{1}{2}\mathbb{E}\{\hat{\boldsymbol{\nu}}^T \boldsymbol{H}_{\hat{\boldsymbol{a}}_n} \hat{\boldsymbol{\nu}}\} \le 1, \quad \forall n \in \mathcal{N}, \tag{26b}$$
$$\frac{1}{2}\mathbb{E}\{\hat{\boldsymbol{\nu}}^T \boldsymbol{H}_{\hat{\boldsymbol{g}}} \hat{\boldsymbol{\nu}}\} \le \epsilon P^{\max} + P_{\mathrm{C}}, \tag{26c}$$
$$\frac{1}{2}\mathbb{E}\{\hat{\boldsymbol{\nu}}^T \boldsymbol{H}_{\hat{\boldsymbol{b}}_k} \hat{\boldsymbol{\nu}}\} \ge r_k^{\min}, \quad \forall k \in \mathcal{K}, \tag{26d}$$
$$\mathbb{E}\{\hat{\boldsymbol{\nu}}_i^2\} = 1, \ i = 1, \ldots, KNL. \tag{26e}$$

It is important to mention that the Schur complement of the matrix $\boldsymbol{M}^*$ is $\boldsymbol{Z}^* - \boldsymbol{z}^*\boldsymbol{z}^{*T}$, and it is PSD.

We use the vectors in $\mathcal{R}$ to obtain candidate binary solutions $\{\tilde{\boldsymbol{\theta}}^j\}_{j=1}^J$ for the problem in (22) by quantizing the entries of each realization of $\{\boldsymbol{\nu}^j\}_{j=1}^J$. Specifically,

$$\tilde{\boldsymbol{\theta}}^j = \mathrm{sgn}(\boldsymbol{\nu}^j), \qquad j = 1, \ldots, J, \tag{27}$$

Using (27) and (21), we obtain candidate binary solutions of (20), $\tilde{\boldsymbol{\phi}}^j$. The candidate that yields the largest objective and satisfies the constraints in (20) is used for allocating the RBs and the discrete power levels, i.e.,

$$\boldsymbol{\phi}^* = \arg\max_{\mathcal{D}_j} \tilde{\boldsymbol{\phi}}^j, \ \mathcal{D}_j \triangleq \{\tilde{\boldsymbol{\phi}}^j : \ \tilde{\boldsymbol{\phi}}^j \text{ satisfying (20b)–(20d)}\}. \tag{28}$$

The proposed close-to-optimal algorithm based on SDR with randomization, viz., COS, is summarized in Algorithm 1.

Algorithm 1: Proposed COS.

Input: P_C, ϵ, $P^{\max}$, $r_k^{\min}$, J, κ, $\eta^{\min} = 0$, and $\eta^{\max} = \boldsymbol{c}^T\boldsymbol{x}/P_C$
Output: $\boldsymbol{\phi}^*$
```
while η^max − η^min ≥ κ do
    η_0 = 0.5 (η^min + η^max)
    Solve (25) to find M.
    if feasible then
        η^min = η^max
        M* = M
    else
        η^max = η^min
Find z*, and Z* from M*.
for j = 1 : J do
    ν^j ~ N(z*, Z* − z* z*^T)
    θ̃^j = sgn(ν^j)
    φ̃^j = 0.5(θ̃^j + 1)
    if (20b) – (20d) satisfied then
        Record φ̃^j.
φ* = arg max_{D_j} φ̃^j
```

B. Computational Complexity Analysis

The joint design problem in (20) can be optimally solved using exhaustive search with complexity $\mathcal{O}(2^{KNL})$, which is computationally prohibitive. In contrast with exhaustive search, the two-stage algorithm proposed herein, COS, has a polynomial-time complexity, and hence, it is suitable for solving large-scale RB and power allocation problems. More specifically, the complexity of solving the relaxation of (25), a PSD-constrained convex problem, is $\mathcal{O}\left((KNL)^{3.5}\right)$[5].

The number of iterations required for the convergence of bisection method is $\log(\eta_{\max}/\kappa)$, where $\kappa > 0$, is the solution accuracy of the bisection method. Lastly, for the Gaussian

[5]We note that this technique relies on constructing PSD matrices with a number of entries that scales with the square of the number of variables. The number of variables in a network with K users, N RBs, and L power levels is KNL. Hence, as the network size enlarges, simulation of the network becomes more challenging. Unless special-purpose computers are used, it can be time-consuming

randomization, the complexity of generating and evaluating the objective function corresponding to the J random samples is $\mathcal{O}\left((KNL)^2 J\right)$. Hence, the overall complexity of COS is $\mathcal{O}\left((KNL)^{3.5}\log(\eta_{\max}/\kappa) + (KNL)^2 J\right)$.

V. Suboptimal Resource Allocation: A Heuristic-Based Approach

In the previous section, we described the proposed COS that provides a lower computation complexity compared to exhaustive search. However, to further reduce the computational complexity, we propose an iterative suboptimal heuristic SOH that offers a trade-off between the computational complexity and performance.

To reduce the computational complexity involved in solving the energy efficiency maximization problem, SOH performs RBs and discrete power allocations, separately. For discrete power allocation, SOH considers a uniform single power level that is obtained with an offline search. The rationale behind this consideration is that as the transmit power of the BS increases power allocation across RBs tends to become uniform. It is intuitive that when the BS has a low transmit power, power allocation has a pivotal role and increasing the number of power levels affect the performance significantly, whereas it becomes less critical when the BS has a high transmit power. In [21] and [22], it is shown that uniform power allocation is sufficient to approach the maximum transmission rate and maximum energy efficiency. Based on our simulation results in Section VI, we verify that a single power level can provide a desired level of energy efficiency. For instance, in Section VI, we provide an example for obtaining a tuned power level that can be then used for allocating the RBs among the users instead of large sets of power levels. Using this observation, when power allocation is assumed to be fixed, the overall complexity of COS given in IV-B can be reduced to $\mathcal{O}\left((KN)^{3.5}\log(\eta_{\max}/\kappa) + (KN)^2 J\right)$. However, with the aim of further reducing computational complexity, we use a heuristic approach in SOH to determine the RB allocation between the users with QoS requirements. We first assign each RB to the user with highest SNR on that RB until meeting the QoS requirements of all users, as long as the BS power budget is not violated. Afterwards, the remaining RBs are allocated among the users in a greedy manner to maximize the energy efficiency. In other words, the RB having the highest energy efficiency is assigned to corresponding user to maximize energy efficiency.

The proposed suboptimal heuristic, SOH, is summarized in Algorithm 2.

Algorithm 2: Proposed SOH.

Input: P_C, ϵ, p^1, $P^{\max}$, $\Gamma_k^{n\ell}$, $r_k^{\min}$, $\bar{\mathcal{K}} = \mathcal{K}$, and $\bar{\mathcal{N}} = \mathcal{N}$
Output: ϕ^*
1 **for** $n = 1 : |\bar{\mathcal{N}}|$ **do**
2   **if** $P^{\max} \geq p^1$ and $|\bar{\mathcal{K}}| \geq 0$ **then**
3     $k^* = \arg\max_{k\in\bar{\mathcal{K}}} \Gamma_k^{n\ell}$
4     $k^* \leftarrow n$
5     $\phi_{k^*}^{n1} = 1$
6     $P^{\max} = P^{\max} - p^1$
7     $\bar{\mathcal{N}} = \bar{\mathcal{N}} \backslash_n$
8   **if** $r_{k^*}^{n1} \geq r_{k^*}^{\min}$ **then**
9       $\bar{\mathcal{K}} = \bar{\mathcal{K}} \backslash_{k^*}$
10 **for** $n = 1 : |\bar{\mathcal{N}}|$ **do**
11   **if** $P^{\max} \geq p^1$ **then**
12   **for** $k = 1 : |\bar{\mathcal{K}}|$ **do**
13       $k \leftarrow n$
14       $\phi_k^{n1} = 1$
15   Calculate $\eta_{\mathrm{EE}}^{(k\leftarrow n)}$ using (10a).
16     $k^* = \arg\max_{k\in\bar{\mathcal{K}}} \eta_{\mathrm{EE}}^{(k\leftarrow n)}$
17     $k^* \leftarrow n$
18     $\phi_{k^*}^{n1} = 1$
19     $P^{\max} = P^{\max} - p^1$

A. Computational Complexity Analysis

The computational complexity of SOH can be analyzed as follows:

- Step 1 requires a complexity of $\mathcal{O}(N)$ to evaluate the **for** loop.
- Step 2 requires a complexity of $\mathcal{O}(K)$ to calculate the cardinality of the set of the unsatisfied users with QoS requirements, $\bar{\mathcal{K}}$.
- Step 3 requires a complexity of $\mathcal{O}(K)$ to find the user with the highest SNR on the respective RB.
- Step 7 requires a complexity of $\mathcal{O}(N)$ to remove the used RB from the set of the unused RBs, $\bar{\mathcal{N}}$.
- Step 9 requires a complexity of $\mathcal{O}(K)$ to remove the satisfied user with QoS requirement from the set of the unsatisfied users with QoS requirement.
- Step 10 requires a complexity of $\mathcal{O}(N-K)$ to evaluate the **for** loop for the remaining $(N-K)$ RBs at most.
- Step 12 requires a complexity of $\mathcal{O}(K)$ to evaluate the **for** loop.
- Step 16 requires a complexity of $\mathcal{O}(K)$ to find the user with the highest energy-efficiency on the respective RB.

Finally, Steps 1 to 9 require a complexity of $\mathcal{O}(KN(2K+N))$, and Steps 10 to 19 require a complexity of $\mathcal{O}(2(N-K)K)$. Hence, the worst case computational complexity of SOH is calculated as $\mathcal{O}(KN(2K+N)+2(N-K)K)$. As it can be seen that SOH has less complexity than COS.

VI. Simulation Results and Discussions

In this section, we will present the simulation results to verify the proposed algorithms, and provide discussions regarding the obtained results.

A. Simulation Models and Parameters

We consider a single-cell network, in which the BS is located in the center of the cell, and the location of users are randomly generated and uniformly distributed over 500×500 m^2 square. For brevity, we assume the QoS requirements of all users are equal. We use the 3GPP propagation model [16].

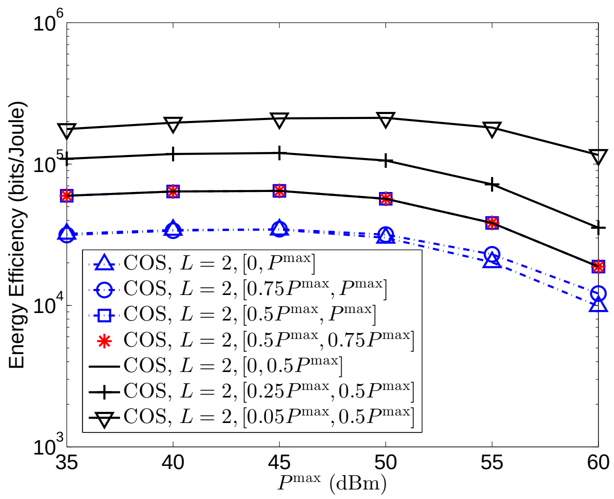


Fig. 1. Performance of COS with different set of two power levels.

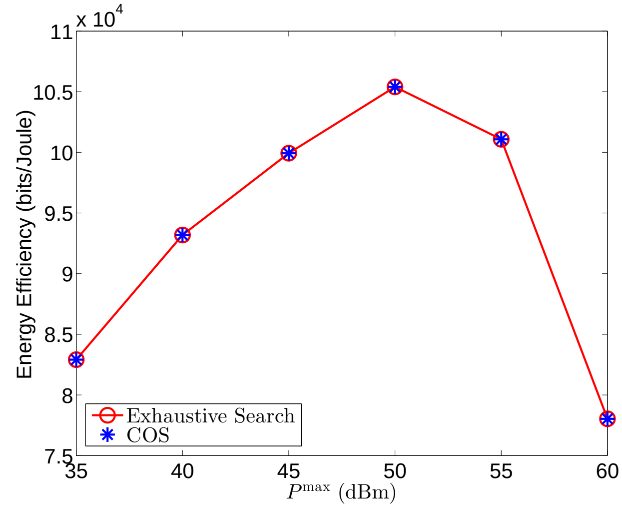


Fig. 2. Performance of COS in comparison with exhaustive search.

According to [16], the path loss is assumed to be given by $PL(d) = 128.1 + 37.6 \log_{10}(d)$ for the link between the BS and the users, where d is the distance in kilometers. The shadowing component is assumed to have a log-normal distribution with a standard deviation of $\sigma_s = 8$ dB. The bandwidth of each RB is assumed to be 180 kHz, and the noise power spectral density is assumed to be -174 dBm/Hz. In addition, the efficiency of the power amplifiers for the BS is assumed to be 38% [23]. Furthermore, the set of power levels used by the BS is assumed to consist of L equally spaced points in the interval $[P^{\text{lower}},\ P^{\text{upper}}]$, where P^{lower} and P^{upper} are the lower and upper bounds of that set.

We consider Monte Carlo simulations to evaluate the network performance. The reported simulation results are averaged over 100 independent channel realizations. For each channel realization, the solution to the problem of energy efficiency maximization in Section IV is obtained using the CVX [24] with the SDPT3 solver, and the number of Gaussian samples used in the randomization stage is set to be $J = 10^4$.

B. Lower and Upper Limits for the Set of Power Levels

In Fig. 1, we consider a scenario, where the minimum rate requirement for all users is zero, the BS static power consumption is $P_C = 50$ dBm, the number of users is set to be $K = 4$, the number of RBs is set to be $N = 8$, and the number of power levels is $L = 2$.

Fig. 1 investigates the performance of COS with the given different sets of the two power levels. From this figure, it can be seen that the choice of the set of the power levels heavily influences the network performance, and a well-chosen set of power levels can enhance the energy efficiency of the system in a positive way. For instance, when $P^{\max} = 45$ dBm, using the set of $\mathcal{P} = \{0.05P^{\max},\ 0.5P^{\max}\}$ rather than the set of $\mathcal{P} = \{0,\ P^{\max}\}$ can improve the energy efficiency of the system from 3.436×10^4 bits/Joule to 2.104×10^5 bits/Joule. Another important observation is that the sets of $\mathcal{P} = \{0,\ 0.5P^{\max}\}$, $\mathcal{P} = \{0.5P^{\max},\ P^{\max}\}$, and $\mathcal{P} = \{0.75P^{\max},\ P^{\max}\}$ show very close performances, since they tend to employ $0.5P^{\max}$ for transmission. Moreover, the best performance for all values of the transmit power can be achieved using the set of $\mathcal{P} = \{0.05P^{\max},\ 0.5P^{\max}\}$. Based on these observations, for the following simulations, unless otherwise stated, the lower and upper bounds for the set of power levels is assumed to be $P^{\text{lower}} = 0.05P^{\max}$, and $P^{\text{upper}} = 0.5P^{\max}$, respectively.

C. Performance Comparison With Optimal Solution

In Fig. 2, we consider a relatively small scenario, where the minimum rate requirement for all users is zero, the BS static power consumption is $P_C = 50$ dBm, the number of users is $K = 3$, the number of RBs is $N = 4$, and the number of power levels is $L = 2$, viz., $\mathcal{P} = \{0.05P^{\max},\ 0.5P^{\max}\}$.

Fig. 2 compares the performance of COS with the optimal energy efficiency solution obtained by exhaustive search in order to validate COS. It is worth to mention that the baseline exhaustive search method looks at every possible triplets of (k, n, ℓ), $k \in \mathcal{K}$, $n \in \mathcal{N}$, and $\ell \in \mathcal{P}$, in order to find which one yields the maximum energy efficiency. As it can be seen from Fig. 2, the algorithm proposed herein attains the optimal solutions for the entire range of $P^{\max}$.

D. Effect of Number of Power Levels

In Fig. 3, we consider a scenario with the different number of power levels. In this scenario, the minimum rate requirement for all users is zero, the BS static power consumption is $P_C = 50$ dBm, the number of users is set to be $K = 4$ and the number of RBs is set to be $N = 8$.

Fig. 3 illustrates the effect of the number of discrete power levels on the energy efficiency. Here, we evaluate the performance of COS through numerical comparisons with the energy efficiency maximization scheme proposed in [15] that considers a joint design of the RBs and the continuous power allocation. From this figure, it can be observed that as the number of power levels available at the BS increases, the performance of COS is enhanced, and the gap between the discrete power allocation and the continuous one becomes

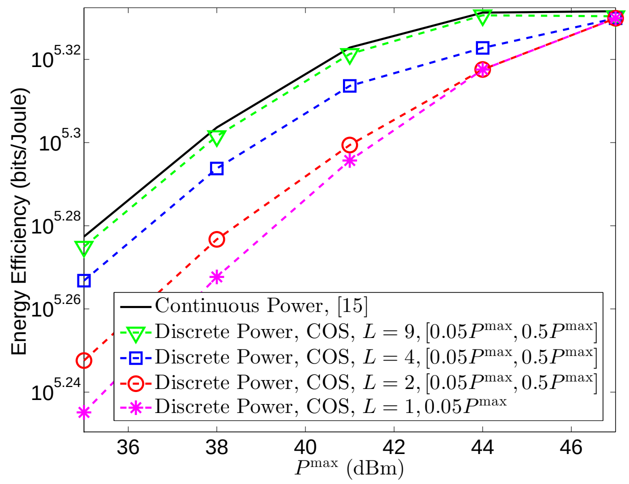


Fig. 3. Effect of the number of power levels on the energy efficiency.

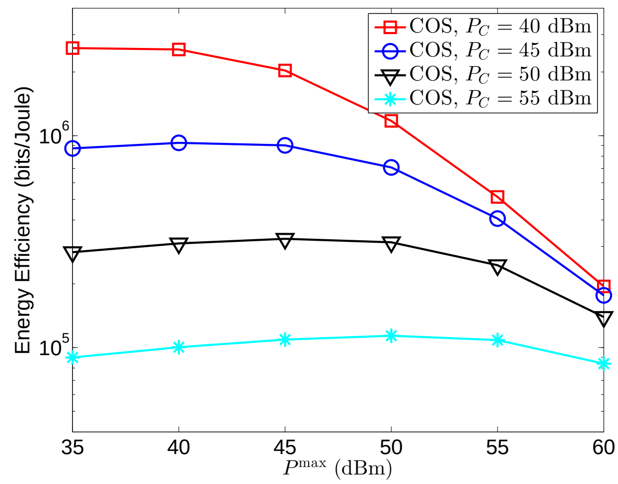


Fig. 4. Effect of circuitry power consumption on the energy efficiency.

smaller. It is worth to mention that an increase in the number of power levels helps to improve the performance, especially when the number of power levels is low. For instance, when the number of power level is increased from two to four, the gain in the energy efficiency is 4% at $P^{\max} = 38$ dBm, whereas when the number of power level is increased from four to eight, the gain in the energy efficiency is only 1.8% at $P^{\max} = 38$ dBm. Hence, in the subsequent figure, we consider four number of power levels at the BS. Another interesting observation is that the algorithm with a carefully chosen single power level reveals a quite good performance even at low $P^{\max}$ values, and its performance improves as $P^{\max}$ increases. Using this observation, we propose SOH that has not only a good performance, but also, a lower complexity. We will show the performance of SOH in comparison to COS below.

E. Effect of Circuitry Power Consumption

In Fig. 4, we consider a scenario, where the minimum rate requirement for all users is zero, the number of users is $K = 8$, the number of RBs is $N = 12$, and the number of power levels is $L = 4$.

Fig. 4 investigates the impact of the BS static power consumption on the energy efficiency. From this figure it can be seen that the energy efficiency decreases with the increase of the BS circuitry power consumption, because transmitting data requires more total power. For instance, when $P^{\max} = 50$ dBm, increasing the circuitry power consumption from 40 dBm to 55 dBm decreases the energy efficiency from 1.175×10^6 bits/Joule to 1.137×10^5 bits/Joule for COS. Another important observation is that as the static power consumption increases, the network performance becomes less sensitive to a change in the transmit power, especially at the medium and high transmit power values.

F. Performance Comparison of Proposed Algorithms

In Fig. 5, we consider a scenario, where the number of users is $K = 4$, the number of RBs is $N = 8$, and the number of power levels is $L = 4$.

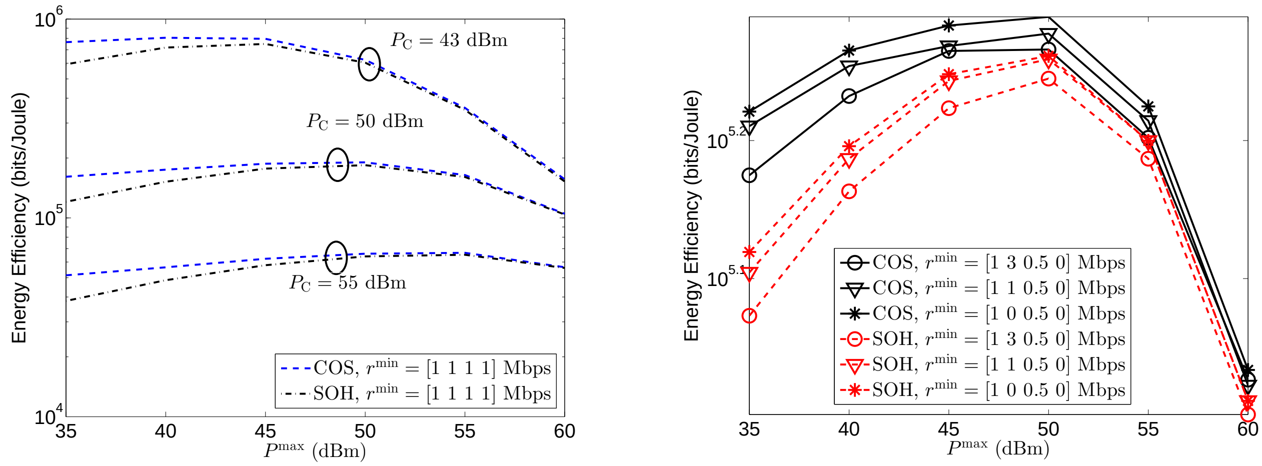
Fig. 5 investigates the effect of increasing the transmit power on the performance of the proposed algorithms, viz., COS and SOH. Particularly, in Fig. 5(a), we study the effect of increasing the transmit power at different circuitry power consumptions without the effect being obscured by potentially different minimum rate requirements per users, while in Fig. 5(b), we study the effect of increasing the transmit power at different minimum rate requirements per users without the effect being obscured by potentially different circuitry power consumptions.

In Fig. 5(a), the energy efficiency is plotted as a function of the available power budget at the BS for different static power consumption values. In this figure, we assume that the minimum rate requirement is the same for all users, i.e., $r_k^{\min} = 1$ Mbps, $\forall k \in \mathcal{K}$. One can see that for high power budget values both COS and SOH provide almost identical results, for all static power consumption values. On the other hand, for low power budget values COS outperforms SOH. For instance, when $P^{\max} = 45$ dBm and $P_C = 50$ dBm, the gain achieved using COS over SOH is only 6%. Such trend suggests that the uniform power allocation assumed by SOH is a good approximation, especially for high BS transmit power.

In Fig. 5(b), the energy efficiency is plotted as a function of the available power budget at $P_{\mathrm{C}} = 50$ dBm and for different minimum rate requirements per users. As it can be seen, the performance of the SOH scheme is close to that of the COS scheme for high power budget (although users have different minimum rate requirements), while for low power budget the energy efficiency of the COS scheme is higher than its counterpart of the SOH scheme. Fig. 5(b) with Fig. 5(a) suggest that the uniform power allocation assumption is directly related to the BS power budget. Hence, regardless of both the circuitry power consumption and the minimum rate requirements. the gap between COS and SOH becomes smaller as the BS transmit power increases.

VII. Conclusion

This paper presented a new optimization framework that efficiently handle practical design issues, i.e., discrete power



(a) Comparison of the proposed algorithms, considering different circuitry power consumption values.

(b) Comparison of the proposed algorithms, considering different minimum rate requirements for the users.

Fig. 5. Performance comparison of COS and SOH.

levels, in resource allocation problems in OFDMA networks. The proposed framework maximized the energy efficiency of the downlink transmission of cellular OFDMA networks subject to RB usage, power budget and per-user QoS constraints. In particular, we modelled both the discrete power levels and discrete RBs by a single binary variable. Then, we used SDR technique with Gaussian randomization to efficiently solve the combinatorial non-convex problem, with polynomial time complexity. We noticed from the solution of the close-to-optimal SDR-based (COS) algorithm that a small number of discrete power levels is sufficient to approach the maximum energy efficiency performance of the continuous power allocation solution. Based on this observation, we proposed a low-complexity iterative suboptimal heuristic (SOH) algorithm that relies on a single power level. Simulation results showed that the energy efficiency of the COS algorithm approaches that of the exhaustive search. Additionally, the simulation results revealed that the SOH strikes a balance between the performance and complexity.

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