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New Results in Disturbance Decoupled Fault Reconstruction in Linear Uncertain Systems Using Two Sliding Mode Observers in Cascade

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Abstract: This paper presents a disturbance decoupled fault reconstruction (DDFR) scheme using two sliding mode observers in cascade. Measurable signals from the first observer are found to be the output of a fictitious system that is driven by the fault and disturbances. Then the signals are fed into a second observer which will reconstruct the fault. Sufficient conditions which guarantee DDFR are investigated and presented in terms of the original system matrices, and they are found to be less conservative than if only one single observer is used; therefore DDFR can be achieved for a wider class of systems using two sliding mode observers. A simulation example validates the claims made in this paper.

Keywords: Disturbance decoupling, fault reconstruction, robustness, sliding mode observer.

1. INTRODUCTION

Fault detection and isolation (FDI) is an important area of research activity. A fault is deemed to occur when the system being monitored is subject to an abnormal condition [2]. The fundamental purpose of an FDI scheme is to generate an alarm when a fault occurs (detection) and also to identify the nature and location of the fault (isolation). A special class of problem within the field of FDI is the problem of *fault reconstruction* [4,5,13], which not only detects and isolates, but provides an estimate of the fault so that its shape and magnitude can be better understood and more precise corrective action can be taken. However, most fault reconstruction schemes are designed about a model, which usually does not perfectly represent the system - as certain dynamics are either unknown or do not fit exactly into the framework of the model. These dynamics are usually represented as a class of disturbances within the model [11] and could corrupt the reconstruction; producing a nonzero reconstruction when there are no faults, or worse, mask the effect of a fault. Therefore, schemes need to be designed so that the reconstruction is robust to disturbances.

Edwards *et al.* [4,5] used a sliding mode observer [3] to reconstruct faults, in which there was no explicit consideration of the disturbances or uncertainty. Tan & Edwards [15] built on the work in [4,5] and presented a design algorithm for the observer, using Linear Matrix Inequalities (LMIs) [1], such that the $\mathscr{L}_2$ gain from the disturbances to the fault reconstruction is minimized. Saif & Guan [13] aggregated the faults and disturbances to form a new 'fault' vector and used a linear unknown input observer to reconstruct the new 'fault' vector. Although this successfully decouples the disturbances from the fault reconstruction, it requires very stringent conditions to be fulfilled, and is conservative because the disturbance does not need to be reconstructed, only rejected/decoupled. Edwards & Tan later [6] compared the fault reconstruction performances of [5] and [13], and found that it was not necessary to reconstruct the disturbance in order to generate a disturbance decoupled fault reconstruction (DDFR). A counter example was presented in [6] to demonstrate this, but the conditions for disturbance decoupling were not formally investigated. Ng *et al.* [8,9] built on the work of [6] and analyzed theconditions that guarantee DDFR using the sliding mode observer [3]. It was also found in [8,9] that the sliding mode observer can achieve DDFR with weaker conditions compared with the linear observer.

This paper further builds on the work in [8,9] by using two sliding mode observers in cascade, where measurable signals from the first observer are found to be the output of a fictitious system that is driven by the faults and disturbance, and fed into a second sliding mode observer. The second observer then reconstructs the fault. The conditions that guarantee DDFR are then investigated, and it was found that the conditions are less conservative than those found in [8,9], which meant that the scheme proposed in this paper are applicable to a wider class of systems compared to if only one observer was used [8,9]. In addition, the sufficient conditions arefound to be easily testable in terms of the original system matrices, which means that the user can know

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immediately from the outset whether the scheme proposed in this paper can achieve DDFR or not.

This paper is organized as follows: Section 2 outlines the problem statement, presents previous work as the basis of the work in this paper and states the main result; Section 3 presents the 2-observer DDFR scheme in this paper; Section 4 investigates thesufficient conditions in terms of the original system matrices; Section 5 presents a numerical example to validate the scheme and Section 6 concludes the paper.

2. PRELIMENARIES AND PROBLEM STATEMENT

Consider the following system

$$\dot{x} = Ax + Mf + Q\xi, \tag{1}$$

$$y = Cx, \tag{2}$$

where $x \in \mathbb{R}^n$ are the states, $y \in \mathbb{R}^p$ are the outputs and $f \in \mathbb{R}^q$ are unknown faults. The signals $\xi \in \mathbb{R}^h$ are uncertainties or dynamics that represent the mismatch between the linear model (1)-(2) and the real plant. Without loss of generality assume that $rank(M) = q$, $rank(C) = p$ and $rank(CQ) = k_1 < h.$ Assume also the following

N1. $rank(CM) = rank(M),$

N2. $rank\left(C\begin{bmatrix}M & Q\end{bmatrix}\right) = rank(CM) + rank(CQ).$

The objective is to generate a reconstruction of f that is not affected by ξ.

Proposition 1: If Assumptions N1 - N2 hold, then there exists a change of coordinates $x \mapsto T_1 x$, $\xi \mapsto T_\xi^{-1}\xi$ such that the matrices (A, M, Q, C) have the structure

$$A = \begin{array}{ccl} \overset{n-p}{\longleftrightarrow} & \overset{p}{\longleftrightarrow} & \\ A_1 & A_2 & \updownarrow n-p \\ A_3 & A_4 & \updownarrow p, \end{array} \quad M = \begin{array}{cl} \overset{q}{\longleftrightarrow} & \\ 0 & \updownarrow n-p \\ M_2 & \updownarrow p, \end{array} \tag{3}$$

$$C = \begin{array}{ccl} \overset{n-p}{\longleftrightarrow} & \overset{p}{\longleftrightarrow} & \\ \big[0 & C_2\big] & \updownarrow p, \end{array} \quad Q = \begin{array}{cl} \overset{h}{\longleftrightarrow} & \\ Q_1 & \updownarrow n-p \\ Q_2 & \updownarrow p, \end{array} \tag{4}$$

where M_2 can be further partitioned to be

$$M_2 = \begin{bmatrix} 0 \\ M_o \end{bmatrix} \begin{array}{l} \updownarrow p-q \\ \updownarrow q, \end{array} \quad Q_1 = \begin{bmatrix} 0 & 0 \\ Q_{11} & 0 \end{bmatrix} \begin{array}{l} \updownarrow n-p-h+k_1 \\ \updownarrow h-k_1, \end{array} \tag{5}$$

$$Q_2 = \begin{bmatrix} 0 & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{bmatrix} \begin{array}{l} \updownarrow p-q-k_1 \\ \updownarrow k_1 \\ \updownarrow q, \end{array} \tag{6}$$

where C_2, M_o, Q_{11} and Q_{22} are square and invertible.

Proof: The proof is taken from [8,9] and is available in Appendix A.

In the coordinates of (3)-(4), further partition A to be

$$\left[\begin{array}{c|c} A_1 & \star \\ \hline A_{31} & \star \\ A_{32} & \star \\ A_{33} & \star \end{array}\right] \begin{array}{l} \updownarrow n-p \\ \updownarrow k_1 \\ \updownarrow k_1 \\ \updownarrow q, \end{array} \tag{7}$$

where $\star$ are matrices with p columns that play no role in the following analysis. Then further partition A in (7) as

$$\left[\begin{array}{cc|c} A_{11}^* & A_{12}^* & \star \\ A_{13}^* & A_{14}^* & \star \\ \hline A_{31}^* & A_{32}^* & \star \\ A_{33}^* & A_{34}^* & \star \\ A_{35}^* & A_{36}^* & \star \end{array}\right] \begin{array}{l} \updownarrow n-p-h+k_1 \\ \updownarrow h-k_1 \\ \updownarrow p-q+k_1 \\ \updownarrow k_1 \\ \updownarrow q, \end{array} \tag{8}$$

where A_{11}^*, A_{14}^* are square matrices.

In [8,9], it is possible to generate a reconstruction of f that is independent of ξ (and achieve DDFR) using a sliding mode observer [3] if the following conditions are satisfied

A1. $\left(A, \begin{bmatrix}M & Q\end{bmatrix}, C\right)$ is minimum phase

A2. $rank(A_{32}^*) = rank\begin{bmatrix} A_{12}^* \\ A_{32}^* \\ A_{36}^* \end{bmatrix}.$

2.1. Previous work

A sliding mode observer [3] for the system (1)-(2) of the form

$$\dot{\hat{x}} = A\hat{x} - G_l e_y + G_n \nu, \tag{9}$$

$$\hat{y} = C\hat{x}, \tag{10}$$

where $\hat{x} \in \mathbb{R}^n$ is the estimate of the state x and $e_y = \hat{y} - y$ is the output estimation error. The term ν is a nonlinear discontinuous term defined by

$$\nu = -\rho \frac{e_y}{\| e_y \|} \quad \text{if } e_y \neq 0, \tag{11}$$

where the positive scalar function ρ is an upper bound of f and ξ. The matrices $G_l, G_n \in \mathbb{R}^{n \times p}$ are the observer gains to be designed. In the coordinates of (3) - (14), G_n is assumed to have the structure

$$G_n = \begin{bmatrix} -L \\ I_p \end{bmatrix} (P_o C_2)^{-1}, \; L = \begin{bmatrix} L_1 & 0 \end{bmatrix}, \tag{12}$$

where $P_o \in \mathbb{R}^{p \times p}$ is a symmetric positive definite (s.p.d.) matrix and $L_1 \in \mathbb{R}^{(n-p) \times (p-q)}$.

Define the state estimation error as $e := \hat{x} - x.$ Combining (1)-(2) and (9)-(10), results in

$$\dot{e} = (A - G_l C)e + G_n \nu - Mf - Q\xi. \tag{13}$$

Lemma 1 [15]**:** If there exists a value of G_l that satisfies $P(A-G_lC)+(A-G_lC)^T P<0$ where

$$P=\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}>0 \text{ with } P_{12}=\begin{bmatrix} \underset{p-q}{\underset{\leftrightarrow}{P_{121}}} & \underset{q}{\underset{\leftrightarrow}{0}} \end{bmatrix} \updownarrow n-p$$

then if $P_o=P_{22}-P_{12}^T P_{11}^{-1} P_{12}$, and for a large enough choice of ρ, an ideal sliding motion takes place on $\mathscr{S}=\{e : Ce=0\}$ in finite time.

Apply a change of coordinates such that $e_L := col(e_1, e_y)=T_L e$ where

$$T_L := \begin{bmatrix} I_{n-p} & L \\ 0 & C_2 \end{bmatrix}. \tag{14}$$

Then assume a sliding motion has taken place at $\mathscr{S}$, and therefore (13) in the new coordinates can be partitioned to be (see Section 2.2 of [15])

$$\dot{e}_1=(A_1+LA_3)e_1-(Q_1+LQ_2)\xi, \tag{15}$$

$$0=C_2A_3e_1+P_o^{-1}\nu_{eq}-C_2M_2f-C_2Q_2\xi, \tag{16}$$

where ν_{eq} is the equivalent output error injection term required to maintain the sliding motion [5] and can be approximated to any degree of accuracy [5] by replacing ν with

$$\nu=-\rho\frac{e_y}{\|e_y\|+\delta}, \tag{17}$$

where δ is a small positive scalar. Since e_y is measurable, ν_{eq} can be computed online. See [5] for full details.

In [8,9], a fault reconstruction $\hat{f}$ was defined as $\hat{f}:=WC_2^{-1}P_o^{-1}\nu_{eq}$, $W:=\begin{bmatrix} W_1 & 0 & M_o^{-1}\end{bmatrix}$ where $W_1 \in \mathbb{R}^{q\times(p-q-k_1)}$ is design freedom. Define $v:=-e_1$, $e_f:=\hat{f}-f$, pre-multiply (16) with WC_2^{-1} and re-arrange (15)-(16) to obtain the pair of equations

$$\dot{v}=\mathscr{A}v+\mathscr{B}\xi, \tag{18}$$

$$e_f=\mathscr{C}v, \tag{19}$$

where

$$\mathscr{A}:=A_1+LA_3^* \equiv \begin{bmatrix} \mathscr{A}_1 & \mathscr{A}_2 \\ \mathscr{A}_3 & \mathscr{A}_4 \end{bmatrix}, \tag{20}$$

$$\mathscr{B}:=Q_1+LQ_2=\begin{bmatrix} 0 & L_{12}Q_{22} \\ Q_{11} & L_{14}Q_{22} \end{bmatrix} \equiv \begin{bmatrix} 0 & \mathscr{B}_2 \\ \mathscr{B}_3 & \mathscr{B}_4 \end{bmatrix}, \tag{21}$$

$$\mathscr{C}:=\begin{bmatrix} W_1A_{31}^*+M_o^{-1}A_{35}^* & W_1A_{32}^*+M_o^{-1}A_{36}^* \end{bmatrix} \tag{22}$$

$$\equiv\begin{bmatrix} \mathscr{C}_1 & \mathscr{C}_2 \end{bmatrix}, \tag{23}$$

where $\mathscr{A}_1=A_{11}^*+L_{11}A_{31}^*+L_{12}A_{33}^*$, $\mathscr{A}_2=A_{12}^*+L_{11}A_{32}^*+L_{12}A_{34}^*$, $\mathscr{A}_3=A_{13}^*+L_{13}A_{31}^*+L_{14}A_{33}^*$ and $\mathscr{A}_4=A_{14}^*+L_{13}A_{32}^*+L_{14}A_{34}^*$.

If the system (18)-(19) is made zero, then $e_f\equiv 0$ and disturbance decoupling fault reconstruction is achieved. Partition $v:=\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{matrix} \updownarrow n-p-h+k_1 \\ \updownarrow h-k_1 \end{matrix}$. It can be seen that v_2 will always be affected by ξ because $\mathscr{B}_3$ is full rank. However, v_1 can be decoupled from ξ if $\mathscr{B}_2$ and $\mathscr{A}_{12}=0$; this would require $L_{12}=0$ and $A_{12}^*+L_{11}A_{32}^*=0$, which in turn requires $rank(A_{32}^*)=rank\begin{bmatrix} A_{12}^* \\ A_{32}^* \end{bmatrix}$. Then e_f can be decoupled from v_2 (and therefore from ξ) if $\mathscr{C}_2=0$ which requires $W_1A_{32}^*+M_o^{-1}A_{36}^*=0$ which in turn requires $rank(A_{32}^*)=rank\begin{bmatrix} A_{36}^* \\ A_{32}^* \end{bmatrix}$. Combining the rank requirements results in Condition A2. Then Condition A1 guarantees that the remaining degrees of freedom in L can be chosen such that $\mathscr{A}$ is stable.

2.2. Main result

This paper proposes a scheme to achieve DDFR when Condition A2 is not satisfied. The main result of this paper is summarized in the following theorem:

Theorem 1: DDFR can be achieved using a 2-observer structure in Figure 1 if the following conditions are satisfied

B1. $(A,[M \quad Q],C)$ is minimum phase

B2. $rank(X_1)-rank(X_2)=rank(Q)$ where

$$X_1=\begin{bmatrix} AQ & 0 & 0 & Q & 0 & 0 \\ CQ & 0 & 0 & 0 & 0 & 0 \\ CAQ & CQ & 0 & 0 & CM & 0 \\ CA^2Q & CAQ & CQ & 0 & CAM & CM \end{bmatrix},$$

$$X_2=\begin{bmatrix} CQ & 0 & 0 & 0 & 0 \\ CAQ & CQ & 0 & CM & 0 \\ CA^2Q & CAQ & CQ & CAM & CM \end{bmatrix}.$$

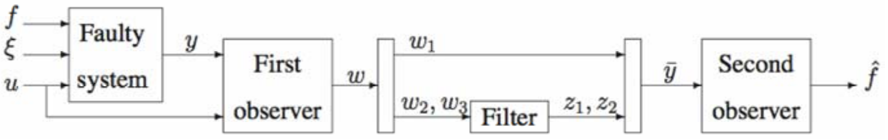

Fig. 1. Schematic diagram of the scheme proposed in this paper.

The remainder of this paper provides a constructive proof of Theorem 1.

3. DDFR USING TWO OBSERVERS

For ease of analysis, a coordinate transformation is introduced as follows in the sequel. Define $k_2 := rank\,(A_{32}^*)$ and $p_2 := rank\begin{bmatrix} A_{31}^* & A_{32}^* \end{bmatrix} + q + k_1$ where $p_2 \leq p$. Then let $R_1 \in \mathbb{R}^{(p-q-k_1)\times(p-q-k_1)}$ be an orthogonal matrix such that

$$R_1\begin{bmatrix} A_{31}^* & A_{32}^* \end{bmatrix} = \begin{array}{c} \left[\begin{array}{cc} 0 & 0 \\ A_{31}^o & A_{32}^o \end{array}\right] \\ \begin{array}{cc} \overset{\leftrightarrow}{n-p-h+k_1} & \overset{\leftrightarrow}{h-k_1} \end{array} \end{array} \begin{array}{l} \updownarrow p-p_2 \\ \updownarrow p_2-q-k_1 \\ \end{array} \tag{24}$$

and $R_2 \in \mathbb{R}^{(p_2-q-k_1)\times(p_2-q-k_1)}$ and $R_3 \in \mathbb{R}^{(h-k_1)\times(h-k_1)}$ to be orthogonal matrices such that

$$R_2A_{32}^oR_3 = \begin{array}{c} \left[\begin{array}{cc} 0 & 0 \\ 0 & A_{322} \end{array}\right] \\ \begin{array}{cc} \overset{\leftrightarrow}{h-k_1-k_2} & \overset{\leftrightarrow}{k_2} \end{array} \end{array} \begin{array}{l} \updownarrow p_2-q-k_1-k_2 \\ \updownarrow k_2, \\ \end{array} \tag{25}$$

where A_{322} has full rank, and assume the following general partitions:

$$\begin{aligned} A_{12}^*R_3 &= \begin{bmatrix} A_{121} & A_{122} \\ A_{123} & A_{124} \end{bmatrix} \begin{array}{l} \updownarrow n-p-p_2-h+k_2+q+2k_1 \\ \updownarrow p_2-k_2-q-k_1 \end{array} \\ A_{36}^*R_3 &= \begin{array}{c} \begin{bmatrix} A_{361} & A_{362} \end{bmatrix} \\ \begin{array}{cc} \phantom{A_{361}} & \overset{\leftrightarrow}{k_2} \end{array} \end{array} \begin{array}{l} \updownarrow q. \\ \end{array} \end{aligned} \tag{26}$$

If Condition A2 is satisfied, then A_{121}, A_{123} and/or A_{361} will all be zero. However, in this paper, no such constraint is in place and A_{121}, A_{123} and/or A_{361} are general matrices.

Then let $R_4 \in \mathbb{R}^{(n-p-h+k_1)\times(n-p-h+k_1)}$ be an orthogonal matrix such that

$$R_2A_{31}^oR_4 = \begin{array}{c} \left[\begin{array}{cc} 0 & A_{3112} \\ A_{3121} & A_{3112} \end{array}\right] \\ \begin{array}{cc} \overset{\leftrightarrow}{n-p-p_2-h+k_2+q+2k_1} & \overset{\leftrightarrow}{p_2-q-k_1-k_2} \end{array} \end{array} \begin{array}{l} \updownarrow p_2-q-k_1-k_2 \\ \updownarrow k_2, \\ \end{array} \tag{27}$$

where A_{3112} is full rank. It is straightforward to show that

$$\begin{bmatrix} I_{p-p_2} & 0 \\ 0 & R_2 \end{bmatrix} R_1\begin{bmatrix} A_{31}^* & A_{32}^* \end{bmatrix}\begin{bmatrix} R_4 & 0 \\ 0 & R_3 \end{bmatrix}$$

$$= \begin{array}{c} \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & A_{3112} & 0 & 0 \\ A_{3121} & A_{3122} & 0 & A_{322} \end{array}\right]. \\ \begin{array}{cccc} \phantom{A_{3121}} & \overset{\leftrightarrow}{p_2-q-k_1-k_2} & \overset{\leftrightarrow}{h-k_1-k_2} & \overset{\leftrightarrow}{k_2} \end{array} \end{array}$$

Define a coordinate transformation $Z := Z_1Z_2Z_3$ where

$$Z_3 := \left[\begin{array}{cccc|c} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & 0 & I \end{array}\right] \begin{array}{l} \updownarrow n-p-p_2-h+k_2+q+2k_1 \\ \updownarrow h-k_1-k_2 \\ \updownarrow p_2-k_2-q-k_1 \\ \updownarrow k_2 \\ \updownarrow p \end{array}$$

$$Z_2 := \left[\begin{array}{ccc|c} I_{n-p-p_2-h+k_2+q+2k_1} & 0 & 0 & 0 \\ 0 & I_{p_2-2k_2-q-2k_1+h} & 0 & 0 \\ A_{3121}A_{322}^{-1} & 0 & I_{k_2} & 0 \\ \hline 0 & 0 & 0 & I_p \end{array}\right]$$

$$Z_3 := \left[\begin{array}{cc|cc} R_4^{-1} & 0 & 0 & 0 \\ 0 & R_3^{-1} & 0 & 0 \\ \hline 0 & 0 & \begin{bmatrix} I_{p-p_2} & 0 \\ 0 & R_2 \end{bmatrix} R_1 & 0 \\ 0 & 0 & 0 & I_{q+k_1} \end{array}\right]$$

in order to obtain

$$ZAZ^{-1} = \left[\begin{array}{c|c} \breve{A}_1 & \breve{A}_2 \\ \hline \breve{A}_3 & \breve{A}_4 \end{array}\right] \begin{array}{l} \updownarrow n-p \\ \updownarrow p \end{array} = \left[\begin{array}{c|c} \breve{A}_1 & \star \\ \hline 0 & \star \\ \breve{A}_{31} & \star \\ \breve{A}_{32} & \star \\ \breve{A}_{33} & \star \end{array}\right] \begin{array}{l} \updownarrow n-p \\ \updownarrow p-p_2 \\ \updownarrow p_2-q-k_1 \\ \updownarrow k_1 \\ \updownarrow q \end{array}$$

$$= \left[\begin{array}{cc|cc|c} A_{111} & A_{121} & A_{112} & A_{122} & \star \\ A_{131} & A_{141} & A_{132} & A_{142} & \star \\ A_{113} & A_{123} & A_{114} & A_{124} & \star \\ A_{133} & A_{143} & A_{134} & A_{144} & \star \\ \hline 0 & 0 & 0 & 0 & \star \\ 0 & 0 & A_{3112} & 0 & \star \\ 0 & 0 & A_{3122} & A_{322} & \star \\ A_{331} & A_{341} & A_{332} & A_{342} & \star \\ A_{351} & A_{361} & A_{352} & A_{362} & \star \end{array}\right] \begin{array}{l} \\ \updownarrow h-k_1-k_2 \\ \updownarrow p_2-k_2-q-k_1 \\ \updownarrow k_2 \\ \updownarrow p-p_2 \\ \updownarrow p_2-q-k_1-k_2 \\ \updownarrow k_2 \\ \updownarrow k_1 \\ \updownarrow q, \end{array} \tag{28}$$

where $\star$ are matrices with p columns and play no role in the following analysis. It is clear that $\breve{A}_{31} = [0 \quad \breve{A}_{312}]$ where $\breve{A}_{312} := \begin{bmatrix} A_{3112} & 0 \\ A_{3122} & A_{322} \end{bmatrix}$ is square and invertible. In addition, Q and M are transformed to be

$$ZQ = \begin{bmatrix} \breve{Q}_1 \\ \breve{Q}_2 \end{bmatrix} = \left[\begin{array}{cc} 0 & 0 \\ Q_{111} & 0 \\ 0 & 0 \\ Q_{112} & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{array}\right], \tag{29}$$

$$ZM = \begin{bmatrix} 0 \\ \breve{M}_2 \end{bmatrix} = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ M_o \end{array}\right], \tag{30}$$

$$CZ^{-1} = [0 \quad \breve{C}_2], \tag{31}$$

where

$$\begin{bmatrix} Q_{111} \\ Q_{112} \end{bmatrix} = R_3^{-1} Q_{11}, \tag{32}$$

$$\breve{C}_2 = C_2 \begin{bmatrix} R_1^{-1} \begin{bmatrix} I_{p-p_2} & 0 \\ 0 & R_2^{-1} \end{bmatrix} & 0 \\ 0 & I_{q+k_1} \end{bmatrix}. \tag{33}$$

3.1. The system for the second observer

Implement the first sliding mode observer as described in Section 2.1, except that the matrix L_1 in L from (12) has a different dimension as follows

$$L \in \mathbb{R}^{(n-p)\times p} = [L_1 \quad 0], \; L_1 \in \mathbb{R}^{(n-p)\times(p-q-k_1)}. \tag{34}$$

Choose a matrix L_1 such that $\breve{A}_1 + L_1 \breve{A}_{31}$ is stable. Define $v := -e_1$ and $w := (P_o \breve{C}_2)^{-1} v_{eq}$ and re-arrange (15) - (16) to respectively obtain

$$\dot{v} = (\breve{A}_1 + L\breve{A}_3)v + (\breve{Q}_1 + L\breve{Q}_2)\xi, \tag{35}$$

$$w = \breve{A}_3 v + \breve{Q}_2 \xi + \breve{M}_2 f. \tag{36}$$

From the structures of $\breve{A}_3$, $\breve{Q}_2$ and $\breve{M}_2$ from (28)-(30), it is clear that the top $p - p_2$ components of w. Hence w can be partitioned as follows

$$w = \begin{bmatrix} 0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix} \begin{array}{l} \updownarrow p - p_2 \\ \updownarrow p_2 - k_2 - q \\ \updownarrow k_2 \\ \updownarrow q. \end{array} \tag{37}$$

Define $\breve{Q}_{22} := \begin{bmatrix} 0_{k_2 \times (h-k_1)} & Q_{22} \end{bmatrix}$ and substituting for $\breve{A}_3$, $\breve{Q}_2$ and $\breve{M}_2$ from (28)-(30) into (36), then w_1 - w_3 in (37) can be expanded to be

$$w_1 = \breve{A}_{31} v, \tag{38}$$

$$w_2 = \breve{A}_{32} v + \breve{Q}_{22} \xi, \tag{39}$$

$$w_3 = \breve{A}_{33} v + M_o f. \tag{40}$$

Then define z$_1$, z$_2$ to be filtered versions of w_2, w_3 representing

$$\dot{z}_1 = -\alpha_1 z_1 + \alpha_1 w_2, \dot{z}_2 = -\alpha_2 z_2 + \alpha_2 w_3, \tag{41}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}_+$. Substituting from (39)-(40) into (41) to get the following analytical expressions for z_1 and z_2:

$$z_1 = -\alpha_1 z_1 + \alpha_1 \breve{A}_{32} v + \alpha_1 \breve{Q}_{22} \xi, \tag{42}$$

$$z_2 = -\alpha_2 z_2 + \alpha_2 \breve{A}_{33} v + \alpha_2 M_o f. \tag{43}$$

Then equations (35), (38), (42) and (43) can be combined to form another system of order $n_2 := n - p + q + k_1$ with a *measurable* output of dimension p_2

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{Q}\xi + \bar{M}f, \tag{44}$$

$$\bar{y} = \bar{C}\bar{x}, \tag{45}$$

where $\bar{x} := \begin{bmatrix} v \\ z_1 \\ z_2 \end{bmatrix}$, $\bar{y} := \begin{bmatrix} w_1 \\ z_1 \\ z_2 \end{bmatrix}$ and

$$\bar{A} = \begin{bmatrix} \breve{A}_1 + L\breve{A}_3 & 0 & 0 \\ \alpha_1 \breve{A}_{32} & -\alpha_1 I_{k_1} & 0 \\ \alpha_2 \breve{A}_{33} & 0 & -\alpha_2 I_q \end{bmatrix}, \; \bar{Q} = \begin{bmatrix} \breve{Q}_1 \\ \breve{Q}_{22} \\ 0 \end{bmatrix}, \tag{46}$$

$$\bar{M} = \begin{bmatrix} 0 \\ 0 \\ M_o \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} \breve{A}_1 & 0 & 0 \\ 0 & I_{k_1} & 0 \\ 0 & 0 & I_q \end{bmatrix}. \tag{47}$$

Remark 1: Note that the system (44) - (45) is not a physical system; rather it is a 'fictitious' system that treats the faults ξ and disturbances f as its unknown inputs. The key point is that it possesses a measurable 'output' which is $\bar{y}$; hence an observer can be constructed for (44)-(45) to estimate f. This approach of estimating faults using a measurable output of a fictitious system is not new and has been used in [10,14].

Remark 2: The purpose of the filtering in (42)-(43) is to achieve the structure in (44) where the fault and disturbance vectors have been forced to be in the 'state equation' (44), which is the framework where the fault reconstruction technique can be applied to. This technique has been widely used in the published literature, for example [15,7]. If the filters have not been used and w_1, w_2 have been used directly as the output $\bar{y}$, then there will be faults and disturbances in the 'output equation' which is not the structure where the fault reconstruction technique can be applied to.

Further expanding $\bar{A}$, $\bar{M}$, $\bar{C}$, $\bar{Q}$ using (28)-(31) and (34) to get

$$\bar{A} = \left[\begin{array}{cc|c} A_{111} & A_{121} & \star \\ A_{131} & A_{141} & \star \\ \hline A_{113} & A_{123} & \star \\ A_{133} & A_{143} & \star \\ A_{331} & A_{341} & \star \\ A_{351} & A_{361} & \star \end{array}\right] \begin{array}{l} \updownarrow n - p - p_2 - h + k_2 + q + 2k_1 \\ \updownarrow h - k_1 - k_2 \\ \updownarrow p_2 - q - k_1 - k_2 \\ \updownarrow k_2 \\ \updownarrow k_1 \\ \updownarrow q, \end{array} \tag{48}$$

$$\bar{Q} = \begin{bmatrix} 0 & 0 \\ Q_{111} & 0 \\ \hline 0 & 0 \\ Q_{112} & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} 0 \\ \hline 0 \\ M_o \end{bmatrix}, \tag{49}$$

$$\bar{C} = \left[\begin{array}{cc|cccc} 0 & 0 & A_{3112} & 0 & 0 & 0 \\ 0 & 0 & A_{3122} & A_{322} & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{array}\right] \begin{array}{l} \updownarrow p_2 - q - k_1 - k_2 \\ \updownarrow k_2 \\ \updownarrow k_1 \\ \updownarrow q. \end{array} \tag{50}$$

Note that Q_{111}, Q_{112} will form a square and invertible matrix. Therefore there exists a matrix $\bar{T}_\xi \in \mathbb{R}^{(h-k_1)\times(h-k_1)}$ such that

$$\begin{bmatrix} Q_{111} \\ Q_{112} \end{bmatrix} \bar{T}_\xi^{-1} = \begin{bmatrix} \bar{Q}_{11} & 0 \\ 0 & Q_x \end{bmatrix} \begin{array}{l} \updownarrow h - k_1 - k_2 \\ \updownarrow k_2, \end{array}$$

where $\bar{Q}_{11}$, Q_x are square and invertible. By performing the a transformation on $\xi \mapsto \begin{bmatrix} \bar{T}_\xi & 0 \\ 0 & I_{k_1} \end{bmatrix} \xi$ results in

$$\bar{Q} \mapsto \bar{Q} \begin{bmatrix} \bar{T}_\xi & 0 \\ 0 & I_{k_1} \end{bmatrix}^{-1} = \begin{bmatrix} \bar{Q}_1 \\ \bar{Q}_2 \end{bmatrix} \tag{51}$$

$$= \begin{bmatrix} 0 & 0 \\ \bar{Q}_{11} & 0 \\ \hline 0 & 0 \\ 0 & \bar{Q}_{22} \\ 0 & 0 \end{bmatrix} \begin{array}{l} \updownarrow n_2 - p_2 - h + k_1 + k_2 \\ \updownarrow h - k_1 - k_2 \\ \updownarrow p_2 - k_1 - k_2 - q \\ \updownarrow k_1 + k_2 \\ \updownarrow q, \end{array} \tag{52}$$

$$\bar{Q}_{22} := \begin{bmatrix} Q_x & 0 \\ 0 & Q_{22} \end{bmatrix}. \tag{53}$$

The matrices $\bar{A}$, $\bar{C}$, $\bar{M}$ remain unaltered by the transformation, but can also be re-expressed so that they are partitioned conformably with $\bar{Q}$ in (53), as follows

$$\bar{A} = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix} = \left[\begin{array}{cc|c} A_{111} & A_{121} & \star \\ A_{131} & A_{141} & \star \\ \hline A_{113} & A_{123} & \star \\ \bar{A}_{33} & \bar{A}_{34} & \star \\ A_{351} & A_{361} & \star \end{array}\right] \begin{array}{l} \updownarrow n_2 - p_2 - h + k_1 + k_2 \\ \updownarrow h - k_1 - k_2 \\ \updownarrow p_2 - k_1 - k_2 - q \\ \updownarrow k_1 + k_2 \\ \updownarrow q, \end{array} \tag{54}$$

$$\bar{M} = \begin{bmatrix} 0 \\ \bar{M}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \hline 0 \\ \bar{M}_o \end{bmatrix} \begin{array}{l} \updownarrow n_2 - p_2 \\ \updownarrow p_2 - q, \\ \updownarrow q \end{array} \quad \bar{C} = \begin{bmatrix} 0 & \breve{A}_{312} \end{bmatrix}, \tag{55}$$

where $\star$ are matrices of p_2 columns that do not play any role in the following analysis.

Notice that the structure of $\bar{A}$, $\bar{M}$, $\bar{C}$, $\bar{Q}$ in (53)-(55) is identical to the structure of A, M, C, Q in (3)-(6). Therefore, using the results of [8,9], it is possible to achieve DDFR if the following conditions are satisfied

C1. $(\bar{A}, [\bar{M} \quad \bar{Q}], \bar{C})$ is minimum phase

C2. $rank(A_{123}) = rank \begin{bmatrix} A_{121} \\ A_{123} \\ A_{361} \end{bmatrix}$

C3. The first observer has a stable sliding motion.

Then a secondary sliding mode observer [3] can be implemented on the system (44)-(45) similar to what was done in Section 2.1. Let $\bar{L} = \begin{bmatrix} \bar{L}_{11} & \bar{L}_{12} & 0 \\ \bar{L}_{13} & \bar{L}_{14} & 0 \end{bmatrix}$ be such that $\bar{A}_1 + \bar{L}\bar{A}_3$ is stable (where $\bar{L} \in \mathbb{R}^{(n_2-p_2)\times p_2}$, $\bar{L}_{13} \in \mathbb{R}^{(h-k_1-k_2)\times(p_2-k_1-k_2-q)}$, $\bar{L}_{14} \in \mathbb{R}^{(h-k_1-k_2)\times(k_1+k_2)}$) and $\bar{\nu}_{eq}$ be the equivalent output error injection required to maintain sliding motion for the second observer. Then let there be a pair $\bar{P}_o \in \mathbb{R}^{p_2 \times p_2}$, $\bar{G}_l \in \mathbb{R}^{n_2 \times p_2}$ such that the condition in Lemma 1 is satisfied. Then define the fault reconstruction signal $\hat{f} := \bar{W}\bar{A}_{312}^{-1}\bar{P}_o^{-1}$ where $\bar{W} := [\bar{W}_1 \quad 0 \quad M_o^{-1}]$ and do the necessary re-arrangements as in Section 2.1; it results in the fault reconstruction error (from the second observer) being excited through a state-space system with the triple $(\bar{\mathscr{A}}, \bar{\mathscr{B}}, \bar{\mathscr{C}})$ (in the same way as (18) - (23)), where

$$\bar{\mathscr{A}} := \bar{A}_1 + \bar{L}\bar{A}_3 \equiv \begin{bmatrix} \bar{\mathscr{A}}_1 & \bar{\mathscr{A}}_2 \\ \bar{\mathscr{A}}_3 & \bar{\mathscr{A}}_4 \end{bmatrix}, \tag{56}$$

$$\bar{\mathscr{B}} := \bar{Q}_1 + \bar{L}\bar{Q}_2 = \begin{bmatrix} 0 & \bar{L}_{12}\bar{Q}_{22} \\ \bar{Q}_{11} & \bar{L}_{14}\bar{Q}_{22} \end{bmatrix} \equiv \begin{bmatrix} 0 & \bar{\mathscr{B}}_2 \\ \bar{\mathscr{B}}_3 & \bar{\mathscr{B}}_4 \end{bmatrix}, \tag{57}$$

$$\bar{\mathscr{C}} \equiv \begin{bmatrix} \bar{\mathscr{C}}_1 & \bar{\mathscr{C}}_2 \end{bmatrix}, \tag{58}$$

where $\bar{\mathscr{A}}_1 = A_{111} + \bar{L}_{11}A_{113} + \bar{L}_{12}\bar{A}_{33}$, $\bar{\mathscr{A}}_2 = A_{121} + \bar{L}_{11}A_{123}^{*} + \bar{L}_{12}\bar{A}_{34}$, $\bar{\mathscr{A}}_3 = A_{131} + \bar{L}_{13}A_{113} + \bar{L}_{14}\bar{A}_{33}$, $\bar{\mathscr{A}}_4 = A_{141} + \bar{L}_{13}A_{123} + \bar{L}_{14}\bar{A}_{34}$, $\bar{\mathscr{C}}_1 = \bar{W}_1A_{113} + M_o^{-1}A_{351}$ and $\bar{\mathscr{C}}_2 = \bar{W}_1A_{123} + M_o^{-1}A_{361}$.

If Conditions C1-C3 are satisfied, then $\bar{W}_1$, $\bar{L}_{11}$, $\bar{L}_{12}$, $\bar{L}_{13}$, $\bar{L}_{14}$ can be chosen such that the system $(\bar{\mathscr{A}}, \bar{\mathscr{B}}, \bar{\mathscr{C}})$ will be made zero (see [8,9]).

4. EXISTENCE CONDITIONS IN TERMS OF ORIGINAL SYSTEM MATRICES

This section seeks to recast Conditions C1-C3 in terms of the original system matrices so that it is easy for the user to immediately determine from the outset whether or not it is possible to achieve DDFR using the scheme in this paper.

4.1. Condition C1

Proposition 2: The systems $(\bar{A}, [\bar{M} \quad \bar{Q}], \bar{C})$ and $(A, [M \quad Q], C)$ have the same invariant zeros.

Proof: The Rosenbrock system matrix [12] of (A, [M Q],C) is as follows

$$U_1(s):=\begin{bmatrix} sI-A & -M & -Q \\ C & 0 & 0 \end{bmatrix}$$

and the invariant zeros of a system are the values of s that make its Rosenbrock matrix lose rank [12]. Substituting for A, M, Q, C from (28)-(31) results in

$$U_1(s)=\left[\begin{array}{c|c} U_{1a} & U_{1b} \\ \hline U_{1c} & U_{1d} \end{array}\right],$$

where

$$U_{1a}=\begin{bmatrix} sI-A_{111} & -A_{121} & -A_{112} & -A_{122} & \star \\ -A_{131} & sI-A_{141} & -A_{132} & -A_{142} & \star \\ -A_{113} & -A_{123} & sI-A_{114} & -A_{124} & \star \\ -A_{133} & -A_{143} & -A_{134} & sI-A_{144} & \star \\ 0 & 0 & 0 & 0 & \star \\ 0 & 0 & -A_{3112} & 0 & \star \\ 0 & 0 & -A_{3122} & -A_{322} & \star \\ -A_{331} & -A_{341} & -A_{332} & -A_{342} & \star \\ -A_{351} & -A_{361} & -A_{352} & -A_{362} & \star \end{bmatrix},$$

$$U_{1b}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & -Q_{111} & 0 \\ 0 & 0 & 0 \\ 0 & -Q_{112} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -Q_{22} \\ -M_o & 0 & 0 \end{bmatrix},$$

$$U_{1c}=[0 \quad 0 \quad 0 \quad 0 \quad \breve{C}_2], \quad U_{1d}=[0 \quad 0 \quad 0].$$

Since Q_{22}, M_o, $\breve{C}_2$, $\begin{bmatrix} Q_{111} \\ Q_{112} \end{bmatrix}$ and $\begin{bmatrix} A_{3112} & 0 \\ A_{3122} & A_{322} \end{bmatrix}$ are square and invertible, then $U_1(s)$ loses rank if and only if $U_{12}(s)$ loses rank, where

$$U_{12}(s):=\begin{bmatrix} sI-A_{111} & -A_{121} \\ -A_{113} & -A_{123} \end{bmatrix}.$$

Using (51)-(55), the Rosenbrock matrix of ($\bar{A}$,[$\bar{M}$ $\bar{Q}$],$\bar{C}$) is as follows

$$U_1(s):=$$
$$\left[\begin{array}{ccc|ccc} sI-A_{111} & -A_{121} & \star & 0 & 0 & 0 \\ -A_{131} & sI-A_{141} & \star & 0 & -\bar{Q}_{11} & 0 \\ -A_{113} & -A_{123} & \star & 0 & 0 & 0 \\ -\bar{A}_{33} & -\bar{A}_{34} & \star & 0 & 0 & -\bar{Q}_{22} \\ -A_{351} & -A_{361} & \star & -M_o & 0 & 0 \\ \hline 0 & 0 & \breve{A}_{322} & 0 & 0 & 0 \end{array}\right].$$

Since $\bar{Q}_{11}$, $\bar{Q}_{22}$, M_o and $\breve{A}_{322}$ are square and invertible, then $U_2(s)$ loses rank if and only if $U_{22}(s)$ loses rank, where

$$U_{22}(s):=\begin{bmatrix} sI-A_{111} & -A_{121} \\ -A_{113} & -A_{123} \end{bmatrix}.$$

Therefore, it is straightforward to show that $U_{21}(s) = U_{22}(s)$ and the proof is complete.

Therefore, C1 can be recasted in terms of the original system matrices as (A,[M Q],C) being minimum phase, which is identical to Condition B1.

4.2. Condition C2

Proposition 3: Define $A_o := A-KC$, where $K := K_1C_2^{-1}$ with K_1 being the last p columns of A (therefore A_o is identical to A except that the last p columns of A_o are all zero). Then it can be shown that

$$rank(X_1)=rank(X_{10}), \quad rank(X_2)=rank(X_{20}), \tag{59}$$

where

$$X_{10}:=\begin{bmatrix} AQ & 0 & 0 & Q & 0 & 0 \\ CQ & 0 & 0 & 0 & 0 & 0 \\ CA_oQ & CQ & 0 & 0 & CM & 0 \\ CA_o^2Q & CA_oQ & CQ & 0 & CA_oM & CM \end{bmatrix},$$

$$X_{20}:=\begin{bmatrix} CQ & 0 & 0 & 0 & 0 \\ CA_oQ & CQ & 0 & CM & 0 \\ CA_o^2Q & CA_oQ & CQ & CA_oM & CM \end{bmatrix}.$$

Proof: Define the following square and invertible matrices

$$T_{10}:=\begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_p & 0 & 0 \\ 0 & -CK & I_p & 0 \\ 0 & -CAK & -CK & I_p \end{bmatrix},$$

$$T_{20}:=\begin{bmatrix} I_p & 0 & 0 \\ -CK & I_p & 0 \\ -CAK & -CK & I_p \end{bmatrix}.$$

It is straightforward to show that $X_1 = T_{10}X_{10}$ and $X_2 = T_{20}X_{20}$. Since T_{10} and T_{20} are both square and invertible, then $rank(X_1)=rank(X_{10})$, $rank(X_2)= rank(X_{20})$

Proposition 4: It can be shown that

$$rank(X_1)-rank(X_2)=rank\begin{bmatrix} A_{121} \\ A_{123} \\ A_{361} \end{bmatrix} - rank(A_{123})+rank(Q). \tag{60}$$

Proof: Using (28)-(33), the following can be established:

$$CM = \breve{C}_2 \begin{bmatrix} 0 \\ M_o \end{bmatrix} \begin{matrix} \updownarrow p-q \\ \updownarrow q, \end{matrix}$$

$$CQ = \breve{C}_2 \begin{bmatrix} 0 & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{bmatrix} \begin{matrix} \updownarrow p-k_1-q \\ \updownarrow k_1 \\ \updownarrow q, \end{matrix} \tag{61}$$

$$CA_oQ = \breve{C}_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{322} & 0 \\ A_{341} & A_{342} & 0 \\ A_{361} & A_{362} & 0 \end{bmatrix} X_3, \tag{62}$$

where $X_3 = \begin{bmatrix} R_3^{-1}Q_{11} & 0 \\ 0 & I_{k_1} \end{bmatrix}$.

$$CA_o^2Q = \breve{C}_2 \begin{bmatrix} I_{p-p_2} & 0 & 0 \\ 0 & \breve{A}_{312} & 0 \\ 0 & 0 & I_{q+k_1} \end{bmatrix} \left[\begin{array}{cc|c} 0 & 0 & 0 \\ \hline A_{123} & A_{124} & 0 \\ A_{143} & A_{144} & 0 \\ \hline \star & \star & 0 \end{array}\right] X_3, \tag{63}$$

where $\star$ are matrices with $q+k_1$ rows that do not play any role in the following analysis.

Then, define

$$C_{10} := diag\{I_n, \breve{C}_2, \breve{C}_2, \breve{R}\},$$
$$Q_{10} := diag\{R_3^{-1}Q_{11}, I_{k_1}, R_3^{-1}Q_{11}, I_{k_1}, I_{2h+2q}\},$$

where $\breve{R} := \breve{C}_2\ diag\{I_{p-p_2}, \breve{A}_{312}, I_{q+k_1}\}$. It can be shown that X_{10} from Proposition 3 can be expanded to be

$$C_{10} \left[\begin{array}{c|c|c} \mathscr{X}_a & \mathscr{X}_b & 0 \\ \hline \mathscr{X}_c & 0 & \mathscr{X}_d \end{array}\right] Q_{10},$$

where

$$\mathscr{X}_a = \left[\begin{array}{ccc|ccc|cc} A_{121} & A_{122} & \star & 0 & 0 & 0 & 0 & 0 \\ A_{141} & A_{142} & \star & 0 & 0 & 0 & 0 & 0 \\ A_{123} & A_{124} & \star & 0 & 0 & 0 & 0 & 0 \\ A_{143} & A_{144} & \star & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \star & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{322} & \star & 0 & 0 & 0 & 0 & 0 \\ A_{341} & A_{342} & \star & 0 & 0 & 0 & 0 & 0 \\ A_{361} & A_{362} & \star & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$\mathscr{X}_b = \begin{bmatrix} 0 & 0 \\ Q_{111} & 0 \\ 0 & 0 \\ Q_{112} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & Q_{22} \\ 0 & 0 \end{bmatrix},$$

$$\mathscr{X}_c = \left[\begin{array}{ccc|ccc|cc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{322} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{341} & A_{342} & 0 & 0 & 0 & Q_{22} & 0 & 0 \\ A_{361} & A_{362} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{123} & A_{124} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{143} & A_{144} & 0 & 0 & A_{322} & 0 & 0 & 0 \\ \star & \star & 0 & A_{341} & A_{342} & 0 & 0 & Q_{22} \\ \star & \star & 0 & A_{361} & A_{362} & 0 & 0 & 0 \end{array}\right],$$

$$\mathscr{X}_d = \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ M_o & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & M_o \end{array}\right].$$

Since C_{10}, Q_{10} are square and invertible, and recalling that $rank(Q_{22}) = k_1$, $rank(M_o) = q$, $rank(A_{322}) = k_2$ and $rank\begin{bmatrix} Q_{111} \\ Q_{112} \end{bmatrix} = h - k_1$, then it can be shown that

$$\begin{aligned} rank(X_{10}) &= 2q + 3k_1 + 2k_2 + h + rank\begin{bmatrix} A_{121} \\ A_{123} \\ A_{361} \\ A_{123} \end{bmatrix} \\ &= 2q + 3k_1 + 2k_2 + h + rank\begin{bmatrix} A_{121} \\ A_{123} \\ A_{361} \end{bmatrix}. \end{aligned} \tag{64}$$

Then, define

$$C_{20} := diag\{\breve{C}_2, \breve{C}_2, \breve{R}\},$$
$$Q_{20} := diag\{R_3^{-1}Q_{11}, I_{k_1}, R_3^{-1}Q_{11}, I_{k_1}, I_{h+2q}\}.$$

It can be shown that X_{20} from Proposition 224 can be expanded to be

$$C_{20}\begin{bmatrix} \mathscr{X}_c & \mathscr{X}_d \end{bmatrix} Q_{20}.$$

Since C_{20}, Q_{20} are square and invertible, then it follows that

$$rank(X_{20}) = 2q + 3k_1 + 2k_2 + rank(A_{123}). \quad (65)$$

Then from (64) and (65), using the result of Proposition 3, and recalling that $rank(Q) = h,$ the proof is complete.

Hence, from (60), $rank\begin{bmatrix} A_{121} \\ A_{123} \\ A_{361} \end{bmatrix} = rank(A_{123}) \Leftrightarrow rank$ $(X_1) - rank(X_2) = rank(Q)$ which is identical to Condition B2.

4.3. Condition C3

Proposition 5: A stable sliding motion exists for the first observer if $\left(A, \begin{bmatrix} M & Q \end{bmatrix}, C\right)$ is minimum phase.

Proof: From the structure of L in (34), it is clear that $(\breve{A}_1 + L\breve{A}_3)$ is stable if and only if the pair $\left(\breve{A}_1, \breve{A}_{31}\right)$ is detectable.

From the Popov-Hautus-Rosenbrock (PHR) rank test [12], the unobservable modes of $\left(\breve{A}_1, \breve{A}_{31}\right)$ are the values of s that make the following matrix pencil lose rank

$$U_3(s) = \begin{bmatrix} sI - \breve{A}_1 \\ \breve{A}_{31} \end{bmatrix}.$$

Substituting for the pair $\left(\breve{A}_1, \breve{A}_{31}\right)$ from (28) results in

$$U_3(s) = \left[\begin{array}{cccc} sI - A_{111} & -A_{121} & -A_{112} & -A_{122} \\ -A_{131} & sI - A_{141} & -A_{132} & -A_{142} \\ -A_{113} & -A_{123} & sI - A_{114} & -A_{124} \\ -A_{133} & -A_{143} & -A_{134} & sI - A_{144} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & -A_{3112} & 0 \\ 0 & 0 & -A_{3122} & -A_{322} \end{array}\right].$$

It is then clear that $U_3(s)$ loses rank if and only if $U_{32}(s)$ loses rank, where

$$U_{32}(s) = \begin{bmatrix} sI - A_{111} & -A_{121} \\ -A_{131} & sI - A_{141} \\ -A_{113} & -A_{123} \\ -A_{133} & -A_{143} \end{bmatrix}.$$

It is easy to see that if $U_{12}(s)$ is full rank, then $U_{32}(s)$ is also full rank. If $\left(\breve{A}_1, \breve{A}_{31}\right)$ is minimum phase, then $U_{12}(s)$ is full rank (and $U_{32}(s)$ will also be full rank) when $s \in \mathbb{C}_+$ which implies that any values of s that make $U_{32}(s)$ lose rank will be stable, resulting in the detectability of $\left(\breve{A}_1, \breve{A}_{31}\right)$, and as a consequence a stable sliding motion existing for the first observer.

Therefore, the results in this section show that Conditions C1-C3 (which guarantee DDFR using the 2-observer structure in this paper) are guaranteed by Conditions B1 - B2. Hence Theorem 1 is proven.

Remark 3: Notice that Condition B2 is less restrictive than Condition A2 because Condition A2 implies that $A_{121} = 0,$ $A_{123} = 0,$ $A_{361} = 0$ whereas Condition B2 implies that $rank(A_{123}) = rank[A_{121}^T \ A_{123}^T \ A_{361}^T]^T$ which is obviously a weaker condition. Recall that for the work that uses only one observer [8,9], DDFR can be guaranteed if A2 is satisfied. Therefore, the 2-observer algorithm in this paper is able to achieve DDFR for a wider class of systems compared to using only one observer as in [8,9].

Remark 4: The ability of the second observer to achieve DDFR does not depend on the design of the first observer, namely G_l and L_1. This is seen from the fact that the conditions B1 and B2 are in terms of the original system matrices. Therefore, it is possible for the designer to know from the outset whether DDFR can be achieved using the 2-observer method in this paper, without having to a-priori design the first observer.

5. SIMULATION EXAMPLE

The method described in this paper will be demonstrated using a simulation example. Consider a 3rd order general nonlinear system described as:

$$\frac{d^3}{dt^3}\theta + a_1\frac{d^2}{dt^2}\theta + a_2\frac{d}{dt}\theta + a_3\theta + \xi = u, \quad (66)$$

where θ is the position and u is the measurable control input. For simplicity, let $u \equiv 0.$ Without loss of generality, the term ξ will encapsulate any disturbances or nonlinearities present in the system. For example, for a nonlinear uncertain system represented by $\frac{d^3}{dt^3}\theta + a_1\frac{d^2}{dt^2}\theta + a_2\frac{d}{dt}\theta + a_3\theta + \sin(\theta) + \theta^2 + d = u$ where d is an external disturbance, then $\xi = \sin(\theta) + \theta^2 + d.$

Let $a_1 = 2,$ $a_2 = 3,$ $a_3 = 4.$ Assume that $\theta, \dot{\theta}, \ddot{\theta}$ are measurable. However, let the sensors of $\dot{\theta}, \ddot{\theta}$ be assumed to be faulty. Hence the sensor equations can be written as

$$y_1 = \ddot{\theta} + f_{heta}, y_2 = \dot{\theta} + f_{\dot{\theta}}, y_3 = \theta.$$

Filter the signals $y_1,$ y_2 to respectively generate $y_{f,1},$ $y_{f,2}$ as follows:

$$\dot{y}_{f,1} = -y_{f,1} + y_1 = -y_{f,1} + \ddot{\theta} + f_{\ddot{\theta}}, \quad (67)$$

$$\dot{y}_{f,2} = -y_{f,2} + y_2 = -y_{f,2} + \dot{\theta} + f_{\dot{\theta}}. \quad (68)$$

Combine (67), (68) and (69) to obtain the following state-space system in the framework of (1)-(2) where

$$\breve{A}=\left[\begin{array}{cc|ccc}0 & 1 & 0 & 0 & 0\\ -a_2 & -a_1 & -a_3 & 0 & 0\\ \hline 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & 0\\ 1 & 0 & 0 & 0 & -1\end{array}\right],\quad \breve{M}=\left[\begin{array}{cc}0 & 0\\ 0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{array}\right],\qquad (69)$$

$$\breve{Q}=\left[\begin{array}{c}0\\ 1\\ \hline 0\\ 0\\ 0\end{array}\right],\quad \breve{C}=\left[\begin{array}{cc|ccc}0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{array}\right].\qquad (70)$$

Notice that (69)-(70) is already in structure of (3)-(6). From the parameters given above, it can be established that $n=5,\ p=3,\ q=2,\ h=1,\ CQ=0\Rightarrow k_1=0.$ Comparing with (7), it is clear that $A_{32}^*=0,\ A_{36}^*=[1\quad 0]^T$ hence Condition A2 is not satisfied and it is not possible to guarantee DDFR using one observer as described in Section 2.1. Besides, by further analyzing the work in [9,8], it is found that when $A_{32}^*=0,\ A_{36}^*\neq 0\infty$ it is impossible to achieve DDFR using one observer. However, Conditions B1 and B2 are satisfied, hence it is possible to achieve DDFR using the 2-observer method in this paper. The following choice of coordinate transform Z

$$Z=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & I_3\end{bmatrix}$$

will cause A to have the structure in (28) with $A_{322}=$ ϕ (empty matrix) and $A_{3112}=1,$ which is full rank.

5.1. Design of observer 1

It is desired that $\lambda(\breve{A}_1+L\breve{A}_3)=-2,-3,$ hence the appropriate choice of L_1 is $L_1=\begin{bmatrix}3\\ -3\end{bmatrix}.$ The observer gains G_l, P_o are designed using the method in [3] where $\lambda(\breve{A}_1+L\breve{A}_3)$ are a subset of $\lambda(A-G_lC).$ From [3], by choosing the remaining eigenvalues of $A-G_lC$ to be $-3,-4,-5,$ the gain G_l can be obtained, and a suitable choice of P_o was found to be $P_o=I_3.$ From the values of P_o and L obtained, the gain G_n can be determined from (12). The following are the calculated values of the gains (that will guarantee sliding motion of the first observer):

$$G_l=\begin{bmatrix}-16 & 0 & 0\\ 6 & 0 & 0\\ 6 & 0 & 0\\ -3 & 3 & 0\\ 3 & 0 & 4\end{bmatrix},\quad G_n=\begin{bmatrix}-3 & 0 & 0\\ 3 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

5.2. Design of observer 2

Choosing $\alpha_1=\alpha_2=1$ results in the following matrices

$$\bar{A}=\left[\begin{array}{c|ccc}-2 & 0 & 0 & 0\\ \hline 1 & -3 & 0 & 0\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{array}\right],\quad \bar{M}=\left[\begin{array}{cc}0 & 0\\ \hline 0 & 0\\ 1 & 0\\ 0 & 1\end{array}\right],\qquad (71)$$

$$\bar{C}=\left[\begin{array}{c|ccc}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{array}\right],\quad \bar{Q}=\left[\begin{array}{c}0\\ \hline 1\\ 0\\ 0\end{array}\right].\qquad (72)$$

Comparing $\bar{A},\ \bar{M},\ \bar{C},\ \bar{Q}$ with (53)-(55), it can be seen that $\bar{Q}_{11}=1,\ \bar{Q}_{22}=\phi$ empty matrix, $\bar{C}_2=I_3$ and $\bar{M}_o=I_2,$ hence resulting in A_{123}=1 which has full column rank. As a consequence, Condition C2 is satisfied, which verifies the earlier fact that 2 observers are sufficient to achieve DDFR.

In designing the second observer, $\bar{L}_1=\bar{L}=[-1\quad 0\ 0]$ is chosen so that the reduced order sliding motion for the second observer has a pole at -3. (See [8] on how the matrix $\bar{L}_1$ is designed to achieve DDFR for the second system (71)-(72)). The gains $\bar{G}_l$ and $\bar{P}_o$ are designed using the algorithm in [3]; the remaining eigenvalues of $\bar{A}-\bar{G}_l\bar{C}$ are specified to be $-4,-5,-6,$ and $\bar{P}_o=I_3$ is an appropriate choice. The following gains are the resulting appropriate gains to guarantee a sliding motion:

$$\bar{G}_n=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad \bar{G}_l=\begin{bmatrix}2 & 0 & 0\\ 2 & 0 & 0\\ 1 & 4 & 0\\ 1 & 0 & 5\end{bmatrix}.$$

Then choosing $\bar{W}_1=\begin{bmatrix}-1 & 0\end{bmatrix}^T$ to get $\bar{W}=\begin{bmatrix}-1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ results in DDFR being achieved.

5.3. Simulation results

Faults were injected into the faulty sensors, together with the disturbance ξ. The left subfigure of Figs. 2 and 3 show the faults, and Fig. 4 shows the disturbance ξ. The middle subfigure of Figs. 2 and 3 shows the reconstructions of the fault which are visually identical to the fault despite the presence of the disturbance ξ, confirming the achievement of DDFR. The right subfigures of Figs. 2 and 3 shows the fault reconstruction error $\hat{f}-f,$ though non-zero[1], is very small.

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[1] This is due to the sigmoidal approximation to obtain v_{eq} in (17) which will result in a small phase lag between the fault and the fault reconstruction. The bigger the value of δ in (17), the bigger will the phase lag be.

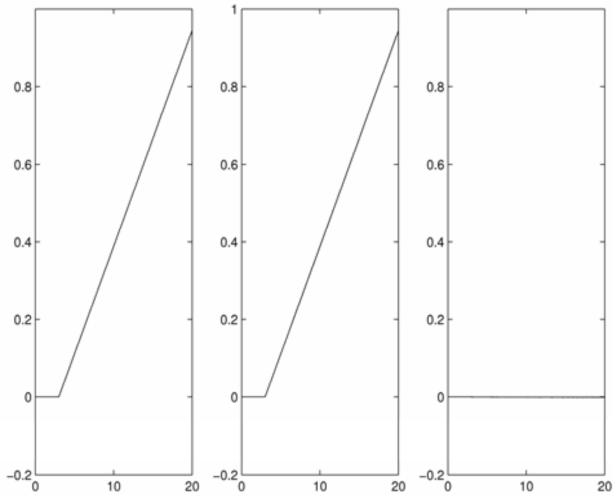

Fig. 2. The left subfigure is the fault in sensor 1, the middle subfigure is its reconstruction, and the right subfigure is the fault reconstruction error.

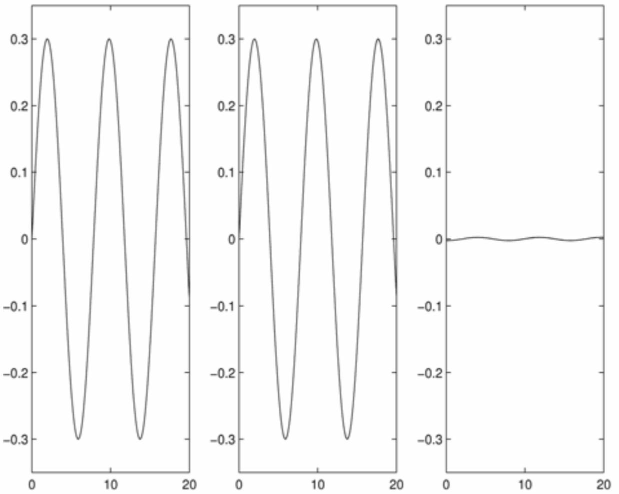

Fig. 3. The left subfigure is the fault in sensor 2, the middle subfigure is its reconstruction, and the right subfigure is the fault reconstruction error.

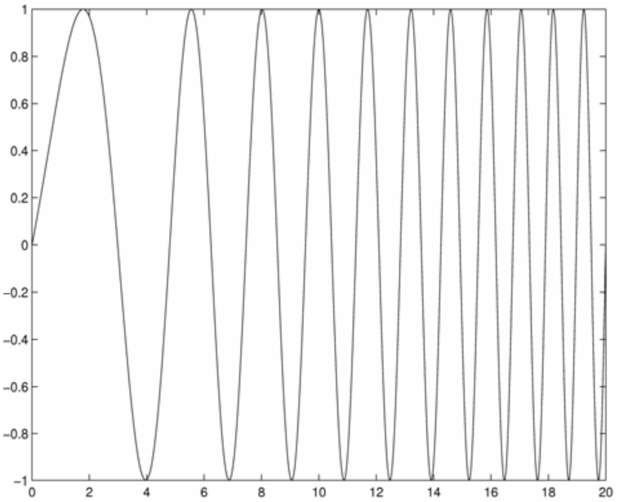

Fig. 4. The disturbance ξ.

6. CONCLUSION

This paper has presented new results in DDFR using two sliding mode observers in cascade. Measurable signals from the first observer are treated as the output for a second observer which will reconstruct the fault. It was found that by using two observers, DDFR could be achieved for a wider class of systems compared when just only one observer being used. This paper also investigated the conditions that guarantee the success of the scheme, which are found to be easily testable in terms of the original system matrices. This is very useful because the user can know from the outset whether the scheme in this paper is applicable to a particular system or not. A simulation example validates the claims made in this paper. The usage of a higher and general number of observers for further enhanced DDFR is under the authors' investigation.

APPENDIX A

A.1. Proof of Proposition 1

From [3], since N1 holds, then there exists a change of coordinates such that (M, C, Q) can be written as

$$\tilde{M} = \begin{bmatrix} 0 \\ \tilde{M}_2 \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} 0 & T \end{bmatrix} \tilde{Q} = \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} \begin{matrix} \updownarrow n-p \\ \updownarrow p, \end{matrix} \ \tilde{M}_2 = \begin{bmatrix} 0 \\ M_o \end{bmatrix}, \tag{A.1}$$

where T is orthogonal and M_o is square and invertible.

Let $\tilde{Q}_2 = \begin{bmatrix} \tilde{Q}_{21} \\ \tilde{Q}_{22} \end{bmatrix} \begin{matrix} \updownarrow p-q \\ \updownarrow q \end{matrix}$. Since $rank(\tilde{C}\tilde{Q}) = k_1$, then $rank(\tilde{Q}_2) = k_1$ as T is orthogonal. From the structure of $\tilde{C}$ in (A.1), it results in

$$\tilde{C}\begin{bmatrix} \tilde{M} & \tilde{Q} \end{bmatrix} = T \begin{bmatrix} 0 & \tilde{Q}_{21} \\ M_o & \tilde{Q}_{22} \end{bmatrix}. \tag{A.2}$$

Assumption N2 then results in $rank(\tilde{Q}_{21}) = k_1$ and hence

$$rank(\tilde{Q}_{21}) = rank \begin{bmatrix} \tilde{Q}_{21} \\ \tilde{Q}_{22} \end{bmatrix}. \tag{A.3}$$

Therefore, there exists a matrix $\tilde{Q}_{21}^{\dagger}$ such that $\tilde{Q}_{21}^{\dagger} Q_{21} = R^T \begin{bmatrix} 0 & 0 \\ 0 & I_{k_1} \end{bmatrix} R$ where R is an orthogonal matrix. It then follows that

$$-\tilde{Q}_{22}\tilde{Q}_{21}^{\dagger}\tilde{Q}_{21} + \tilde{Q}_{22} = 0. \tag{A.4}$$

Hence, applying the following change of coordinates $x \mapsto T_{pre}x$ where

$$T_{pre} := \left[\begin{array}{c|cc} I & 0 & 0 \\ \hline 0 & I & 0 \\ 0 & -\tilde{Q}_{22}\tilde{Q}_{21}^{\dagger} & I \end{array} \right] \begin{matrix} \updownarrow n-p \\ \updownarrow p-q \\ \updownarrow q \end{matrix} \tag{A.5}$$

and the structures in (3)-(4) are achieved.

Since $rank(\tilde{Q}_{21}) = k_1$, there exists orthogonal matrices $N_1 \in \mathbb{R}^{(n-p)\times(n-p)}$, $N_2 \in \mathbb{R}^{(p-q)\times(p-q)}$ such that

$$\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_{21} \end{bmatrix} T_\xi^{-1} = \left[\begin{array}{cc} 0 & Q_{12} \\ Q_{11} & Q_{13} \\ \hline 0 & 0 \\ 0 & Q_{22} \end{array}\right] \begin{array}{l} \updownarrow n-p-h+k_1 \\ \updownarrow h-k_1 \\ \updownarrow p-k_1-q \\ \updownarrow k_1, \end{array}$$

where Q_{11}, Q_{22} are square and invertible.

Then define

$$T_1 = T_{1a} T_{1b} T_{1c},$$

where

$$T_{1a} = \left[\begin{array}{cc|ccc} I_{n-p-h+k_1} & 0 & 0 & -Q_{12}Q_{22}^{-1} & 0 \\ 0 & I_{h-k_1} & 0 & -Q_{13}Q_{22}^{-1} & 0 \\ \hline 0 & 0 & I_{p-k_1-q} & 0 & 0 \\ 0 & 0 & 0 & I_{k_1} & 0 \\ 0 & 0 & 0 & 0 & I_q \end{array}\right],$$

$$T_{1b} = \left[\begin{array}{c|cc} N_1 & 0 & 0 \\ \hline 0 & N_2 & 0 \\ 0 & 0 & I_q \end{array}\right],$$

$$T_{1c} = \left[\begin{array}{c|cc} I_{n-p} & 0 & 0 \\ \hline 0 & I_{p-q} & 0 \\ 0 & -\tilde{Q}_{22}\tilde{Q}_{21}^{\dagger} & 0 \end{array}\right],$$

and performing the transformations $x \mapsto T_1 x$, $\xi \mapsto T_\xi \xi$ results in $Q \mapsto T_1 Q T_\xi^{-1}$, $M \mapsto T_1 M$, $C \mapsto C T_1^{-1}$ and the structures in (3)-(6) are achieved.

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