Gradient Magnitude Based Normalised Convolution

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Abstract

Although image data can often be sparse for a variety of different reasons, standard image processing techniques require the use of complete image data. Therefore sparse image data must undergo reconstruction to yield complete images prior to any subsequent processing. Highly accurate image reconstruction techniques tend to be expensive to implement whilst simpler techniques, such as image interpolation, are usually not adequate to support subsequent reliable image processing. A common approach to image reconstruction is normalised convolution; we present a modified approach to normalised convolution which is based on the sparse image content and we demonstrate that accurate reconstruction is achieved yielding better image processing results than the current standard normalised convolution.

Keywords: Image reconstruction, Normalised convolution.

1 Introduction

Sparsity is a well-documented problem in all branches of science; numerous methods including statistical approaches and multi-resolution analysis do exist in the literature to overcome this problem whether in a time series, a 2-D planar image or an unordered set [Rybicki and Press, 1992, Ford and Etter, 1998]. One of the key points to be addressed when thinking of a solution for data incompleteness is the nature of such incompleteness. As usual, regularity is always easy to deal with and when the image sparsity is regular conventional interpolation techniques can reconstruct the image successfully [Jain, 1989]. However, this is not the case when the data are irregularly distributed. This situation is often encountered in computer vision applications; for example the use of omnidirectional cameras [Yagi and Kawato, 1990, Hong et al., 1992, Yamazawa et al., 1993] where unwarping the omnidirectional images to planar images results in incomplete projections [Scotney et al., 2006]. This sparsity, which should be handled before proceeding with feature detection and managing the process of feature detection on such sparse un-warped images, has attracted considerable research efforts over the last two decades. Normalised convolution is one of the methods used for interpolating irregularly sampled images. It was first introduced in [Knutsson and Westin, 1993] and has shown superiority over conventional grid-based techniques [Foster and Evans, 2008]. It also has illustrated remarkable capabilities in dealing with low subsampling rates [Piroddi and Petrou, 2004].

Feature detection is at the heart of all computer vision applications. There has been an incessant desire to develop reliable and stable feature detectors that could reveal the actual informative content of an image while withstanding possible image distortions and scale variations. In [Kerr et al., 2008], a novel technique was presented for corner detection adopting the finite-element method and working directly on sparse images. Within that framework, an adaptive gradient operator is formulated where an irregular image is represented by an irregular mesh of triangular elements and piece-wise linear basis functions. We have extended the work in [Kerr et al., 2008] by presenting a normalised convolution based approach called Gradient Magnitude-Weighted Normalised Convolution (GMWNC). Using the gradient operator proposed in [Kerr et al., 2008] as a weighting

function, GMWNC improves the reconstruction of sparse images compared with the conventional normalised convolution (NC). For comparative purposes, feature detection evaluation methods have been applied to the reconstructed images using GMWNC as well as standard normalised convolution (NC). While maintaining low computational complexity, the proposed algorithm results in improved feature detection with consistently improved root mean square error (RMSE) than the traditional NC approach.

The paper is organised as follows: in Section 2, an overview on the conventional NC technique is presented. In Section 3, the GMWNC is proposed. Results are shown in Section 4 and Section 5 presents a conclusion and suggested future work.

2   Normalised Convolution

Normalised convolution is an algorithm that operates on irregularly sampled or sparse data sets in order to fill-in the missing information. Accordingly, it has been used extensively to deal with image data incompleteness as a spatial interpolation technique. NC adopts a standard convolution filter, classically a Gaussian filter, in addition to a certainty map. The idea of constructing a certainty map was suggested to differentiate between the locations where we have a zero-valued pixel and the locations where we have missing data; this map is a simple binary filter [Piroddi and Petrou, 2004].

Conventional NC is called normalised averaging and it involves two convolutions and an element-wise division [Knutsson and Westin, 1993]. The first convolution is where the standard filter, namely the applicability filter is convolved with the sampled sparse data. This filter is responsible for the process of diffusing the information from the areas where it exists to the areas where it is missed, according to a certain profile. According to its properties, it defines the vicinity over which it operates. In the case of a Gaussian filter, interpolation takes place within the function support according to a Gaussian profile. Equations (1), (2) and (3) shows the three operations which are carried out in the simplest NC algorithm. The first step is to calculate

$$D(x, y) = f(x, y) \otimes g(x, y) \tag{1}$$

where $f(x, y)$ is the sampled input data and $g(x, y)$ is the applicability filter. The second step is to calculate

$$N(x, y) = c(x, y) \otimes g(x, y) \tag{2}$$

where $c(x, y)$ is the binary valued certainly map.

The second convolution can be thought of as the certainties associated with the interpolations that took place in the first convolution [Foster and Evans, 2008]. In order to normalise the first convolution, the reconstructed image $\hat{f}$ is determined by

$$\hat{f} = \frac{D(x, y)}{N(x, y)} \tag{3}$$

By using the available information and a map for data certainty, we are able to generate interpolated pixel values in the locations where none were originally present. The NC technique is superior to many conventional in-filling techniques such as the bi-linear and bi-cubic interpolation [Foster and Evans, 2008].

3   Gradient Magnitude-Weighted Normalised Convolution

Gradient Magnitude-Weighted Normalised Convolution GMWNC is a NC-based technique that enhances the performance of the conventional NC for reconstructing sparse images. The approach in based on that presented in [Kerr et al., 2008] and an overview is presented in the following subsections.

3.1   Overview of Gradient Operator Design

As in [Kerr et al., 2008], we consider sparse image to be represented by a spatially irregular sample of values of a continuous function $u(x, y)$ of image intensity on a domain Ω. The operator design procedure is then based

on the use of a mesh generated using Delaunay triangulation. With each node i in the mesh is associated a piecewise linear basis function $\phi_i(x,y)$ which has the properties $\phi_i(x_j, y_j) = 1$ if $i = j$ and $\phi_i(x_j, y_j) = 0$ if $i \neq j$ where (x_j, y_j) are the co-ordinates of the nodal point j in the mesh. We then approximately represent the image function u by a function $U(x,y) = \sum_{j=1}^{N} U_j \phi_j(x,y)$ in which the parameters $\{U_1, \ldots, U_N\}$ are mapped from the sparse image intensity values. The approximate image function representation is therefore piecewise linear on each triangle and has value U_j at node j.

The gradient operator design in [Kerr et al., 2008] is based on weak forms of operators in the finite element method [Becker et al., 1981, Scotney and Coleman, 2007]. In order to be directly applicable to sparse image data, the operator design needs to explicitly embrace the concept of operator size and shape, thus the design procedure explicitly embodies a size parameter σ that is determined by the local point density. Therefore, we use sets of Gaussian test functions $\psi_i^\sigma(x,y)$, $i = 1, \ldots, N$, when defining the derivative based operators; for first order operators respectively, this provides the functionals

$$E_i^\sigma(U) = \int_{\Omega_i^\sigma} \underline{b}_i \cdot \underline{\nabla} U \psi_i^\sigma d\Omega_i \qquad (4)$$

where $\underline{b}_i$ is the basis function and U is the image. Each Gaussian function $\psi_i^\sigma(x,y)$ is restricted to have support over a neighbourhood Ω_i^σ, centred on node i, consisting of those triangular elements that have node i as a vertex and therefore that the integral in the definition of the functional E_i^σ need be computed by integration over only the neighbourhood Ω_i^σ rather than the entire image domain Ω. This process enables us to compute the gradient magnitude at each point within the sparse image for subsequent use in the reconstruction process.

3.2 Weighted Normalised Convolution

The algorithm commences by calculating the gradient magnitude responses across the sparse image using the gradient operators presented. Referring to equations (1) and (2) in Section 2, the gradient magnitude is equivalent to the applicability filter denoted here as the weighting function, and the presence of a gradient magnitude value implies a value of 1 in the certainty map in equation (2). Depending on whether the current location is non-zero-valued or not, $D(x,y)$ and $N(x,y)$ will be calculated and weighted by the gradient magnitude response. Equations (5) and (6) demonstrate the steps of carrying out the GMWNC algorithm. First, $D(x,y)$ is calculated as follows

$$D(x,y) = f(x,y) \otimes [g(x,y) * (\nu - GM(x,y))] \qquad (5)$$

where ν is the maximum gradient magnitude response and $GM(x,y)$ is the gradient magnitude at the point (x,y). Similar to NC, $N(x,y)$ is identical to $D(x,y)$ with the sampled input data is replaced by the certainty filter $c(x,y)$.

$$N(x,y) = c(x,y) \otimes [g(x,y) * (\nu - GM(x,y))]. \qquad (6)$$

This ensures that when the gradient magnitude is high (i.e. a potential feature point) the smoothing is reduced and when the gradient magnitude is low (i.e. a background point) the smoothing is increased, thus retaining the key images features during image reconstruction whilst removing noise. The reconstructed image is then calculated in the same way as indicated by equation (3).

4 Performance Evaluation

Improving the reconstruction of features is the main goal of the proposed technique. Therefore for evaluation purposes we apply a feature detector and use the well-known Figure of Merit (FoM) technique [Abdou and Pratt, 1979]. Taking into account the fairness of evaluation, FoM has been calculated over a range of signal to noise ratios, typically 100, 50, 20, 10, 5 and 1. In addition, the assessment was made using different percentages of sparsity. FoM was calculated using synthetic images for diagonal, vertical and horizontal edges. However, the results obtained for 75% of the image data were similar for both the proposed GMWNC and NC

techniques and are therefore not shown here; the significant improvements are found when the percentage data is reduced as low as 25%.

Figure 1 and Figure 2 present graphs for the Figure of Merit versus Signal to noise ratio using only 25% of the original image data for three edge types (diagonal, horizontal and vertical) using 3×3 and 7×7 filters respectively. In all cases we can see that the proposed GMWNC technique outperforms the traditional NC approach with respect to subsequent image processing, i.e., edge detection.

Edge image	Method	RMSE	
		25% data	75% data
Diagonal	NC	25.845	26.6573
	GMWNC	26.5877	26.0140
Horizontal	NC	28.0025	28.3764
	GMWNC	28.2964	28.1106
Vertical	NC	25.896	26.3009
	GMWNC	27.124	26.1031

Table 1: RMSE values for different sparsity ratios and edge orientations

In addition, Table 1 presents root mean squared errors (RMSE) for both approaches, comparing the reconstructed images with the original images using 25% and 75% data. The RMSE values demonstrate that both approaches have similar accuracy with respect to reconstruction; however the FoM results in Figure 1 and Figure 2 illustrate that our proposed approach yields better edge detection and hence feature reconstruction.

5   Conclusion

Techniques that are based on image reconstruction without prior image knowledge do not generally provide reliable mechanisms for accurate feature extraction. The most accurate reconstruction technique currently available that is also computationally efficient is the Normalised Convolution approach. In [Scotney et al., 2006], a design procedure was presented for edge detection operators for direct use on sparse image data. Here we extend this existing approach by applying it to sparse image data to derive knowledge of the image content that can be subsequently used to enhance the image reconstruction process. We have demonstrated the success of this approach by presenting root mean squared errors that are similar to those obtained using the standard Normalised Convolution method but also by demonstrating via the Figure of Merit technique that the images reconstructed using the proposed GMWNC approach yield better results with respect to edge detection than images reconstructed using standard NC even when using as little as 25% of the original image data. Having obtained accurate results for image reconstruction the focus of our further work will be the use of real omnidirectional images for robot localisation.

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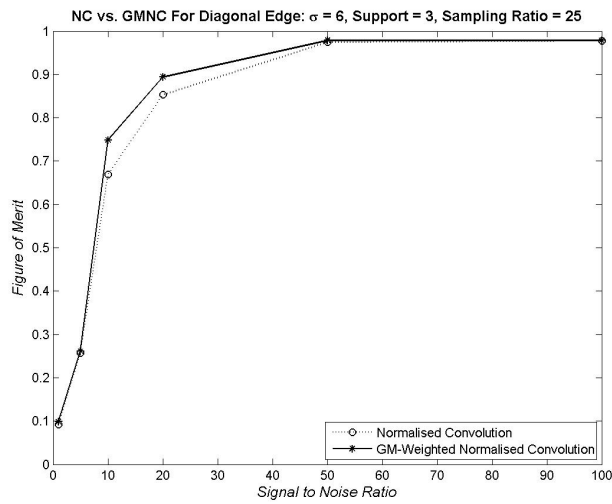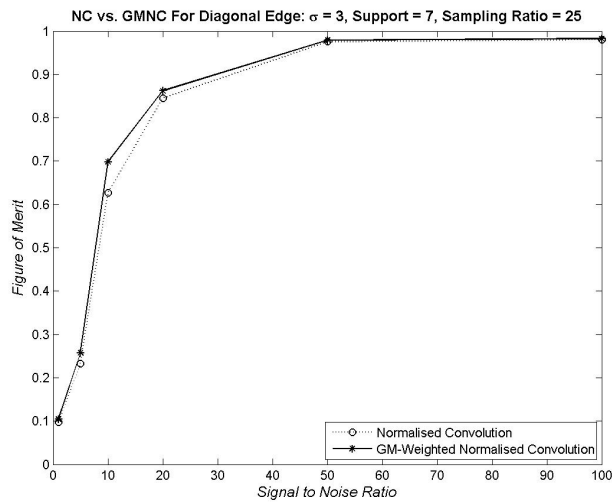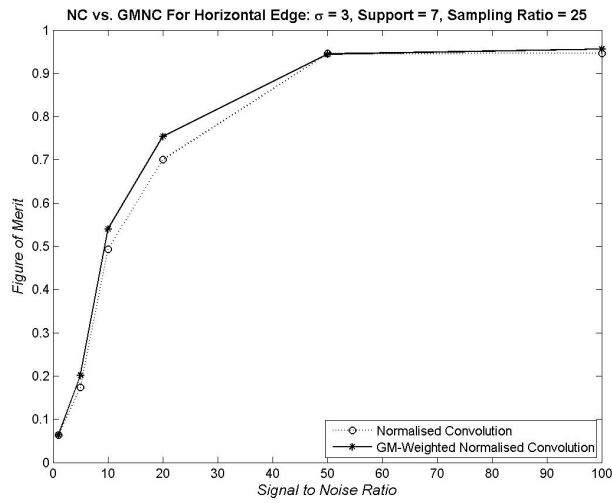
(a) Diagonal Edge
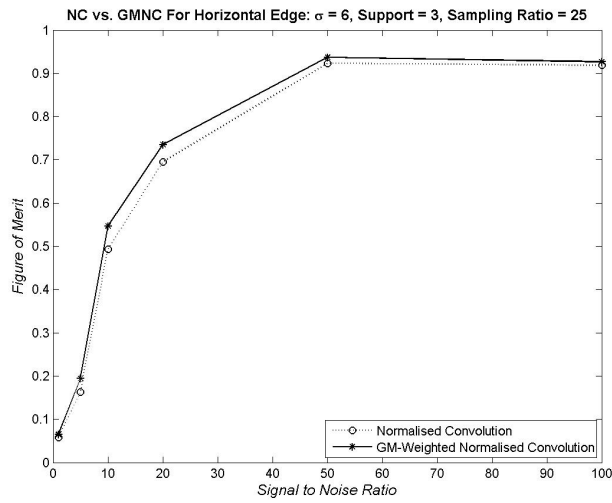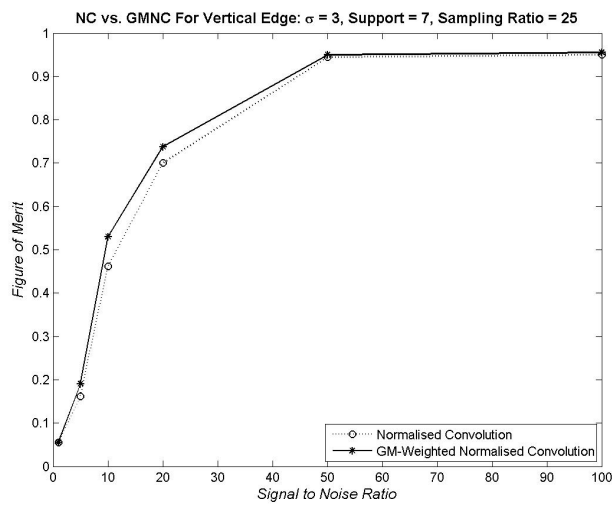

(b) Horizontal Edge
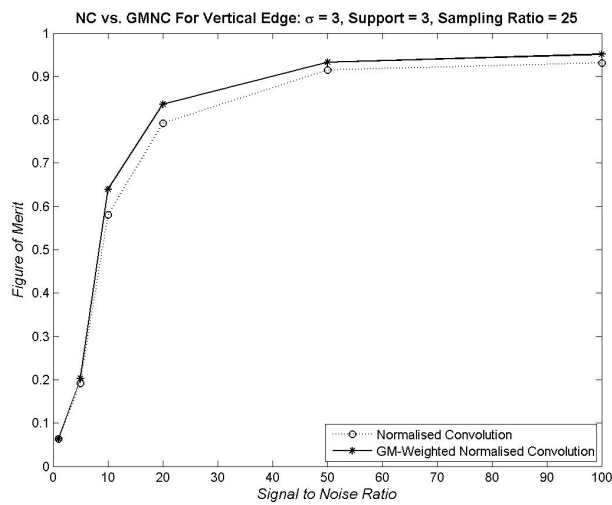
(c) Vertical Edge

Figure 1: FoM Vs SNR using images with 25% data and a 3 × 3 filter

(a) Diagonal Edge

(b) Horizontal Edge

(c) Vertical Edge

Figure 2: FoM Vs SNR using images with 25% data and a 7 × 7 filter