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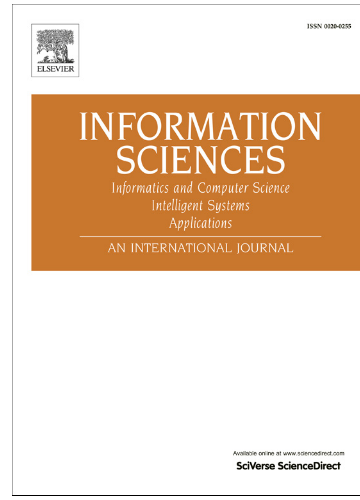

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Enhancing Gait based Person Identification using Joint Sparsity Model and ℓ_1-norm Minimization

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Abstract

We consider the problem of person identification using gait sequences under normal, carrying bag and different clothing conditions as the main concern. It has been demonstrated that Gait Energy Image (GEI) can attain a better gait recognition rate under normal conditions. However, it has been shown that GEI is not robust enough to handle the carrying bags and different clothing conditions. Instead of GEI, there are several appearance based gait features in the available literature to reduce the effect of covariate factors by keeping dynamic parts and removing the static parts of the gait features under the assumption that the carrying bags and different clothing conditions affect mostly the static parts. It is however shown in the literature that the static parts also contain valuable information and removal of certain static parts such as head and thigh certainly decreases the recognition rate.

Our main objective has been to increase the gait recognition rate on different clothing and carrying bag covariate gait sequences. Therefore instead of removing static parts, the Joint Sparsity Model (JSM) is applied to identify the carrying bags and different clothings conditions from GEI features. If a set of GEI feature vectors is submitted to **JSM** model then a common component and an innovations component for each GEI feature are obtained. The innovations component that has unique characteristic to each of features is considered to identify the covariate conditions. The identified covariate conditions are removed from GEI features and a novel gait feature called $\mathbf{GEI_{JSM}}$ is generated. The dimension of $\mathbf{GEI_{JSM}}$ is reduced using Random Projection (RP) approach and ℓ_1-norm minimization technique based sparse representation is used for classification. It is demonstrated that the RP and ℓ_1-norm minimization based sparse representation approach provides statistically significant better results than that of the existing individual identification approaches.

Keywords: Gait Recognition; Individual Identification; covariate factors; CDA; ℓ_1-norm; Joint Sparsity Model (JSM); RP-based dimensional reduction.

1. Introduction

Human identification using gait is a challenging Computer Vision task, and many methods have been proposed in the literature [30, 23, 32]. Gait refers to the walking style of a human. Gaits have unique distinctive features that vary from one person to another. The way an individual normally walks is one of those distinctive features that can be used for person recognition. The term gait recognition is typically used to signify the identification of individuals in image sequences by the way they walk. Gait recognition methods can be mainly classified into three categories: spatiotemporal-based, model-based and appearance-based.

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Spatiotemporal-based methods uncover gait shape variation information in both the spatial and temporal domains, and one such related work includes shape variation-based frieze features [27, 24]. Model-based methods [15, 41] aim to model the body and shape of the person when he/she is walking. Appearance-based methods focus on extracting the static (i.e. head and torso) and/or dynamic (i.e. motion of each arm, hand and leg) information of a walking person from a sequences of binary silhouettes. In appearance-based methods, GEI feature is one of the methods that is used to represent the human motion sequence of a gait cycle as a single gray scale image. The appearance-based gait recognition methods use static features [39], dynamic features [14] or fusion of static and dynamic features [23] for recognition. Various studies have shown that both static and dynamic information of gait features are valuable information for gait recognition. The computational cost of the model-based methods is relatively high compared to the appearance-based methods [43]. Therefore, model-based methods are not considered in this paper. Instead a cost effective appearance-based features are considered for gait recognition.

Even though the appearance-based methods are cost effective, these methods suffer from the problem of covariate factors. Covariate factors can be related either to the subject itself (e.g. carrying bags and different clothing conditions) or to the environment (e.g. different walking surface). Covariate factors are not part of the gait information; therefore, they should be effectively removed from the gait features. Figure 1 shows individuals appearing with normal and covariate factors such as different clothing and carrying bags in the CASIA-B dataset [12].

Sarkar et al. [38] described a baseline algorithm for gait recognition to examine the effects of different covariates: clothing and carrying objects as well as viewpoint, footwear, walking surface and time. But Bouchrika et al. [8] argued that the work of [38] lacked exploratory analysis of the different gait features under covariate data due to the use of an appearance-based approach that includes covariate factors information. Matovski et al. [28] reported that different clothing drastically affects the appearance-based gait recognition performance than other covariate factors such as footwear, speed and viewpoint. The experimental results detailed in [9] also reported that covariate factors such as different clothing and carrying bag conditions highly reduced the gait recognition rate .

Figure 1. Sample from the CASIA-B dataset: First row represents sample of individuals with normal walking sequences and second row represents sample of individuals with covariate factors such as different clothing and carrying bags [12].

To account for the aforementioned issues, a novel robust appearance-based gait feature is required to get a better recognition rate and this can be achieved by removing or reducing most of the covariate conditions from the appearance-based features. The GEI based appearance-based methods operate directly on the gait sequences without assuming any specific model for the walking human and focus on extracting the static (i.e. head and torso) and/or dynamic (i.e. motion of each arm, hand and leg) information of a walking person from the sequence of binary silhouettes. Different appearance-based methods are developed by reducing the effects of covariate factors to get a higher similarity score between the training and testing gait sequences. To reduce the effects of covariate factors, the appearance-based methods remove mainly the static parts under the assumption that covariate factors are mostly attached to the static parts [20], [4], [6], [46], [48], [5]. However, it is well known in the literature that removing certain static parts will decrease the recognition rate [26].

In order to address the above discussed problem, the work presented here considers the problem of covariate factors associated to the subjects itself. By reducing or removing the underlying effects of covariate factors a better gait recognition rate could be achieved in appearance-based methods. The aim of this paper is to construct similar feature for the same individual even if the person appears with different clothing or carrying bags. In the proposed approach instead of removing the static body parts, we propose an appearance-based gait feature extraction method to remove much of the covariate factors information from a given GEI and keep most of the static body parts. The covariate factors such as different clothing or carrying bags information is removed from the given GEI.

Under the effect of covariate factors the static body part in the given GEI will reflect irrelevant information that does not belong to the gait and should be judiciously removed. Our aim is to identify covariate factors of a given GEI against a set of available GEIs and then remove the identified covariate factors from the given GEI. Earlier sparse representation has been used to identify the noise aspect of an image. The idea has been further extended in this paper and covariate factors are considered as noise appearing in the normal gait sequences and are removed from the given GEI. The work presented here deploys $\mathbf{JSM_1}$ [3] to identify the covariate factor information. In the $\mathbf{JSM_1}$ model, each signal consists of a sum of two components: a *common* component that is present in all the signals and an *innovations* component that is unique to each signal. In our case, the *innovations* component that is unique to each GEI is considered as covariate factors. By removing identified covariate factors from the GEI, a novel covariate free gait feature is generated and named as $\mathbf{GEI_{JSM}}$. The work presented here thus aims to identify the body related covariate factors such as different clothing and carrying bag from a given GEI using the JSM approach and our contribution to the knowledge being presented in the paper can therefore be considered as follows:

1. We propose a novel gait feature representation approach for gait recognition using $\mathbf{JSM_1}$. To the best of our knowledge we are the first to propose $\mathbf{JSM_1}$ to remove covariate factors from GEI for gait recognition.
2. It has been demonstrated in the literature that Canonical Discriminant Analysis (CDA) (i.e., multiclass LDA) is the best linear dimensional reduction method for gait based individual identification [4, 6]. Through performance evaluation, we demonstrate that the proposed RP-based dimensional reduction outperforms the CDA approach.

The evaluation of the proposed methodology is performed on the commonly used CASIA-B dataset [12]. As a benchmark for comparison of our proposed approach of using $\mathbf{GEI_{JSM}}$ the new results are compared with the results of other commonly used appearance-based approaches on the CASIA-B dataset. The obtained result shows the superior ability to reduce the effect of covariate factors from the resulting GEI and thus giving a better recognition rate. In addition, to demonstrate the extensibility of the proposed approach in real environments, we evaluate the $\mathbf{GEI_{JSM}}$ on HumanID Gait Challenge dataset available from the University of South Florida (USF). The rest of the paper is organised as follows: the related literature is discussed in Section 2. The proposed methodology and the approach followed are described in Section 3. Section 4 provides details of the evaluation carried out to show the robustness of the proposed methodologies while Section 5 discusses the experimental results obtained. Finally Section 6 gives the conclusion to the work and describes the approaches which will be considered for future work.

2. Related work

The appearance-based gait representation approaches can be categorised into time variant and time invariant approaches. In the case of time variant approach temporal information of the gait is retained in the resulting gait representation image. An example of one such representation is illustrated in [40] proposed Chrono-Gait Image for effectively storing the time variant parameters into the single template image by encoding the temporal information using the multichannel technique. In the case of time invariant approach the temporal information is normalised and the resulting gait feature template is time invariant. An example of one such representation method is GEI and it is one of the most commonly used appearance-based methods and can attain good performance under normal gait sequences [24].

The gait features used for recognition can be broadly classified into two categories, namely static and dynamic features. In a recent research, Wang et al. [42] proved that a promising recognition rate can be achieved using static gait features. On the other hand Cutting et al. [14] argued that dynamic features contribute significantly more in individual recognition than static features. Instead, Lam et al. [23] have preferred to fuse both static and dynamic cues with a belief that fusion would yield the optimal gait recognition rate. However under the effect of covariate factors, such as different clothing and carrying objects, use of GEI features does not attain good recognition rate as they are sensitive to the variation occurring due to different clothing and carrying bag conditions. To alleviate the effect caused by different clothing and carrying bags, Pratheepan et al. [34, 33] proposed a covariate factor removal method under the assumption that the static features can be extracted as clean silhouettes (i.e. without holes or noise). However this assumption may not be applicable in all the real scenarios.

In the available literature a number of novel gait feature representations are proposed to increase the person recognition rate on different clothing and carrying bags covariate gait sequences. Instead of GEI, different gait feature representations such as Gait Entropy Image (GEnI) [4], Enhanced Gait Energy Image (EGEI) [46], Active Energy Image (AEI) [48], Masked Gait Image M_G [6] and Gait Flow Image (GFI) [5] have been shown in the literature to reduce the effect of different clothing and carrying bag covariate factors. Figure 2 illustrates different gait feature representation methods that tend to reduce the effect of different clothing and carrying bags covariate factors evaluated on the CASIA-B dataset.

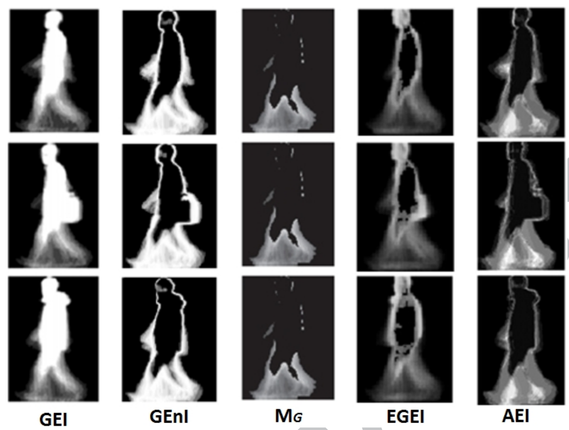

Figure 2. Different gait representation methods for a particular individual from CASIA-B gait dataset. Columns from left to right represent GEI [20], GEnI [4], M_G [6], EGEI [46] and AEI [48] respectively. Rows from top to bottom represent normal, carrying bag and different clothing walking conditions.

In the evaluation of these gait representation methods, the normal gait sequences are considered for training and normal, carrying bag and different clothing covariate gait sequences are considered for testing. All these gait feature representations try to overcome the issues of covariate factors; however, the following important issues are not adequately addressed. These methods produced lower recognition rates for different clothing covariate gait sequences and reasonable recognition rate for carrying bags covariate gait sequences. This makes their overall average recognition rates low. As stated earlier, these methods try to remove the static parts under the assumption that the covariate factors like different clothing and carrying bag are attached to the upper body parts. However, the random removal of the static information such as head and thigh reduces the recognition rate.

In the past, the sparse representation based techniques have been used for classification purposes in gait recognition [37, 45]. In general, the sparse representation based classification methods aim to recover the sparse linear representation of any query sample with respect to a set of reference samples. In this paper, the sparse representation is used for classification as well as feature extraction. We use a specific case of sparse modelling where the signal are represented using the JSM for feature extraction. This concept is used to segregate the normal body of the individual from the covariate factors.

3. Methodology

This Section gives a detailed description of the proposed methodology and the approach followed.

3.1. Our approach

In this paper we propose a methodology for gait recognition under different covariate conditions, in particular carrying bag and different clothing covariates. The key idea is that given a test GEI image with covariate factors and a set of training GEI images belonging to known individuals, JSM is used to identify the unique aspects of the test image against each known individual in the gallery set. The unique aspects (covariate factors) of the given test image are then averaged which specifically identifies the parts of the GEI for the test image that are contributed by the covariate conditions. The identified parts are then removed and a sparse representation-based classification (SRC) model is employed for recognition.

The full details of JSM approach are discussed in [18, 3]. JSM is advanced by Dror Baron et al. [3] who present a new theory for Distributed Compressed Sensing (DCS). Three different JSMs are defined in [3]. $\mathbf{JSM_1}$ models the case when all the signals share a *common* sparse component and at the same time each signal has an *innovation* sparse component that is unique to it. $\mathbf{JSM_2}$ models the case when signals are projected into the same sparse index set of basis functions. The last model is $\mathbf{JSM_3}$ where signals share a *common* component that is not sparse and each signal has its own unique sparse component. To identify covariate conditions in GEI feature the $\mathbf{JSM_1}$ is more appropriate than $\mathbf{JSM_2}$ or $\mathbf{JSM_3}$ as in $\mathbf{JSM_1}$ both the *common* and *innovations* components are sparsely representable in some basis function and it can be solved using ℓ_1-norm minimization. Therefore $\mathbf{JSM_1}$ is considered for our approach.

Recently different versions of $\mathbf{JSM_1}$ are used to overcome certain problems in face recognition. An expression-invariant based face recognition model, **B-JSM**, is developed in [29] using $\mathbf{JSM_1}$. In [19], an improved **B-JSM** model is developed to handle the partial occlusion in face recognition. In our approach, $\mathbf{JSM_1}$ is used to identify covariate factors using innovations component. After the $\mathbf{GEI_{JSM}}$ feature extraction an ℓ_1-norm minimization based sparse representation approach is considered for classification.

The ℓ_1-norm minimization based sparse representation approach to classification is applied in [44] for face recognition. It has been shown that ℓ_1-norm minimization based sparse representation classifier performed better than the Nearest Neighbour (NN) classifier and is comparable to linear SVM classifier in face recognition [44]. Recently ℓ_1-norm minimization based sparse representation classifier is also used in image retrieval [10]. However, the main constraint with ℓ_1-norm minimization based sparse representation is that number of features (N) must be greater than or equal to the dimension of a feature (D).

The calculated $\mathbf{GEI_{JSM}}$ feature is very high dimensional data, i.e. $D > N$. Therefore, a dimensional reduction approach is needed to reduce the dimension of $\mathbf{GEI_{JSM}}$ to satisfy the constraint $D \leq N$. RP has recently appeared as a very good tool for dimensionality reduction. It has been shown that the computation cost of RP is low compared to PCA and LDA. In addition to this, it preserves the structure of the data without introducing significant distortion [7]. Therefore, in this paper, RP is applied on $\mathbf{GEI_{JSM}}$ to reduce its dimension. Following this, ℓ_1-norm minimization based sparse representation classifier is applied for individual identification.

3.2. Gait representation

Given a human walking sequence, a human silhouette is extracted from each frame using the method in [38]. The extracted silhouette is resized to a fixed size of 64×50 pixels. The purpose of resizing is to eliminate the scaling effect. After applying horizontal alignment with respect to its horizontal centroid to each resized silhouette image, the gait cycles are segmented by estimating gait frequency using a maximum entropy estimation technique presented in [38]. Then GEI is computed as:

$$G(a, b) = \frac{1}{L} \sum_{t=1}^{L} I(a, b, t) \tag{1}$$

where L is the number of frames in a complete gait cycle, a and b are the image coordinates, I is the silhouette image and t is the frame number in the gait cycle. Figure 3 shows the samples of extracted silhouettes and the computed GEI image. Next, the $\mathbf{JSM_1}$ approach is used to identify the covariate factors from a GEI.

Figure 3. Sample of extracted silhouettes and computed GEI (fifth image).

3.3. **JSM** *for Decomposing Sparse Common and Innovations Components*

A set of GEI images can be divided into two sets of feature images using $\mathbf{JSM_1}$: one is called *common* component and it is the common feature of all the images, and the other one is called *innovations* component and it is the unique feature of each image [3, 29, 19].

Suppose a GEI image is represented by a D dimensional column vector $\mathbf{x}$, where $D = W \times H$, W and H are the width and height of a GEI respectively. We assume that a group contains M GEI images. So the j-th GEI image of group g can be represented as $\mathbf{x}_j^g$, where $j = 1, 2, ..., M$, and the ensemble of all the group g images can be represented as a column vector $\mathbf{y}_g = [\mathbf{x}_1^g; \mathbf{x}_2^g; ...; \mathbf{x}_M^g]$. According to $\mathbf{JSM_1}$, $\mathbf{x}_j^g$ can be divided into *common* and *innovation* features as:

$$\mathbf{x}_j^g = \mathbf{z}_c^g + \mathbf{z}_j^g \tag{2}$$

where $\mathbf{z}_c^g$ is the common feature of all the images in group g, $\mathbf{z}_j^g$ is the unique feature of the j-th GEI image of group g. If it is possible to generate the system of linear equations then the solution for $\mathbf{z}_c^g$ and $\mathbf{z}_j^g$ can be found. Therefore the both sides of Equation (2) is multiplied by the orthonormal matrix Ψ (e.g., Discrete Cosine Transform (DCT)) and the Equation (2) can be rewritten as:

$$\Psi\mathbf{x}_j^g = \Psi\mathbf{z}_c^g + \Psi\mathbf{z}_j^g \tag{3}$$

Let the projection coefficient replace the right hand side of the Equation (3),

$$\Psi\mathbf{x}_j^g = \theta_c^g + \theta_j^g \tag{4}$$

Both sides of the Equation (4) is multiplied by the transposed matrix Ψ^T as:

$$\begin{aligned} \Psi^T\Psi\mathbf{x}_j^g &= \Psi^T\theta_c^g + \Psi^T\theta_j^g \\ \mathbf{x}_j^g &= \Psi^T\theta_c^g + \Psi^T\theta_j^g \end{aligned} \tag{5}$$

where $\Psi^T\Psi = I$. And all the GEI images of group g can be jointly represented by the matrix [2]:

$$\begin{bmatrix} \mathbf{x}_1^g \\ \mathbf{x}_2^g \\ \vdots \\ \\ \mathbf{x}_M^g \end{bmatrix} = \begin{bmatrix} \Psi^T & \Psi^T & 0 & \cdots & 0 \\ \Psi^T & 0 & \Psi^T & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \\ \Psi^T & 0 & 0 & \cdots & \Psi^T \end{bmatrix} \times \begin{bmatrix} \theta_c^g \\ \theta_1^g \\ \theta_2^g \\ \vdots \\ \\ \theta_M^g \end{bmatrix} \tag{6}$$

We can represent the above system of linear equations as $\mathbf{y}_g = \widetilde{\Psi}\Theta_g$, where $\widetilde{\Psi} = [B, E]$ is formed by concatenating two matrices given by $B = [\Psi^T\Psi^T \cdots \Psi^T]^T$, $E = diag(B)$ and the column vector $\Theta_g = [\theta_c^g;\ \theta_1^g;\ \theta_2^g; \cdots; \theta_M^g]$. We know that $\widetilde{\Psi}$ is the compound matrix corresponding to an over-complete dictionary, i.e. number of rows is less than the number of columns, and hence the system of above linear equations is under-determined and has no unique solution. It has been shown that the sparsest Θ_g can be recovered by solving the following optimization problem [17, 11, 1]:

$$\hat{\Theta}_g = \underset{\Theta_g}{\operatorname{argmin}} \|\Theta_g\|_1 \quad subject\ to \quad \mathbf{y}_g = \widetilde{\Psi}\Theta_g \tag{7}$$

where $\|.\|_1$ denotes the ℓ_1-norm. This problem is often known as Basis Pursuit (BP) and can be solved in polynomial time [13].

Using calculated Θ_g, the common ($\mathbf{z}_c^g = \Psi^T\theta_c^g$) and innovation ($\mathbf{z}_j^g = \Psi^T\theta_j^g, j = 1, 2, ..., M$) components are acquired as follows:

$$\begin{bmatrix} \mathbf{z}_c^g \\ \mathbf{z}_1^g \\ \vdots \\ \\ \mathbf{z}_M^g \end{bmatrix} = \begin{bmatrix} \Psi^T & 0 & \cdots & 0 \\ 0 & \Psi^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \\ 0 & 0 & \cdots & \Psi^T \end{bmatrix} \times \begin{bmatrix} \theta_c^g \\ \theta_1^g \\ \vdots \\ \\ \theta_M^g \end{bmatrix} \tag{8}$$

It is shown in [3] that the *innovations* component represents the unique characteristic of a signal of an ensemble of signals. Therefore in this paper, the *innovations* component is considered to identify the covariate factors in GEI images. Thus our novel gait feature, $\mathbf{GEI_{JSM}}$, is extracted following this approach.

3.3.1. Proposed Gait Feature: $\mathbf{GEI_{JSM}}$

Suppose m GEIs from a known class k (i.e. a particular subject) are represented as column vectors $\mathbf{x}_1^k, \mathbf{x}_2^k, .., \mathbf{x}_m^k$ and a test GEI from an unknown class t is represented as a column vector $\mathbf{x}_t$. The ensemble of all these vectors are considered as a group kt and the column vector $\mathbf{y}_{kt}$ is represented as $\mathbf{y}_{kt} = [\mathbf{x}_t; \mathbf{x}_1^k; \mathbf{x}_2^k; ..; \mathbf{x}_m^k]$. We can represent $\mathbf{y}_{kt}$ as

$$\mathbf{y}_{kt} = \widetilde{\Psi}\Theta_{kt} \tag{9}$$

where $\Theta_{kt} = [\theta_c^{kt}; \theta_t^k; \theta_1^{kt}; \theta_2^{kt}; ...; \theta_m^{kt}]$. Then $\mathbf{JSM_1}$ is applied to extract the *common* and *innovations* components, $w_{kt} = [\mathbf{z}_c^{kt}; \mathbf{z}_t^k; \mathbf{z}_1^{kt}; \mathbf{z}_2^{kt}; ..; \mathbf{z}_m^{kt}]$, where $\mathbf{z}_c^{kt}$ is the common component of GEIs from the known class k and a test GEI from an unknown class t. The *innovations* components are represented as $[\mathbf{z}_t^k; \mathbf{z}_1^{kt}; \mathbf{z}_2^{kt}; ..; \mathbf{z}_m^{kt}]$, where $\mathbf{z}_i^{kt}, i = 1...m$, represent *innovations* components of i^{th} GEI of class k against the rest of the GEIs of class k and the test GEI of unknown class t, and $\mathbf{z}_t^k$ represents the *innovations* component of the test GEI of unknown class t against the GEIs of the known class k. The calculated $\mathbf{z}_t^k$ contains the unique characteristic of test GEI against class k and this column vector is reshaped to the original size of GEI and defined as Z_t^k. Figure 4(a) shows four (i.e. $m = 4$) normal GEI images from a known class k.

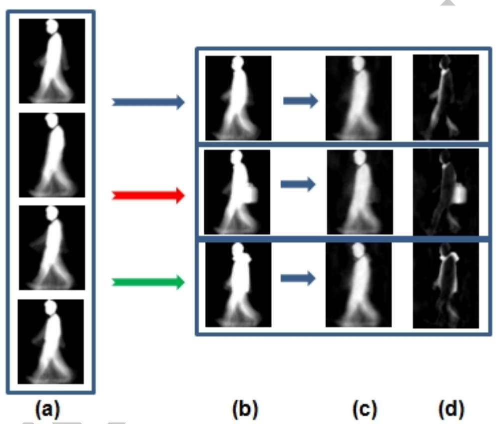

Figure 4. (a) Represents normal GEI images of a known individual (i.e. known class). (b) Represents GEI images with normal, carrying bag and different clothing of an unknown individual. (c)Represents *common* components of the known and the unknown individuals. (d) represents *innovations* components of the unknown individual against the known individual.

The normal, carrying bags and different clothing GEIs from an unknown class t are shown from top to bottom in Figure 4(b). The gait representation methods described in the literature based on CASIA dataset use four gait feature vectors from each class for training Therefore $m = 4$ is selected in our approach to keep the same experimental set-up.

The extracted *common* and *innovations* components using $\mathbf{JSM_1}$ approach are shown in Figure 4(c) and (d) respectively. The top part of Figure 4(d) represents the *innovations* components of normal GEI of unknown class t against known class k. Similarly the middle and bottom figures of Figure 4(d) represent the *innovations* components of carrying bag and different clothing GEIs of unknown class t against the known class k. The *innovation* components shown in Figure 4(d) are the unique characteristic of the unknown class t against the known class k.

The Figure 4(d) represents the *innovations* component of a GEI of an unknown class t against a known class k. Instead of considering only one class k, all the available classes are considered to find the *innovations* component of a GEI of an unknown class t against all known individuals. This gives the unique characteristic of the GEI image of the unknown class against all other known classes. Therefore the GEI image of an unknown class t is considered separately against all known classes, $k = 1, .., K$ where K is total number of classes, and K different *innovations* components are extracted. The Figure 5(a) represent the GEI images of an unknown class t. The sample of *innovations* components of the GEI of an unknown class t against different classes are shown in Figure 5(b)-(f).

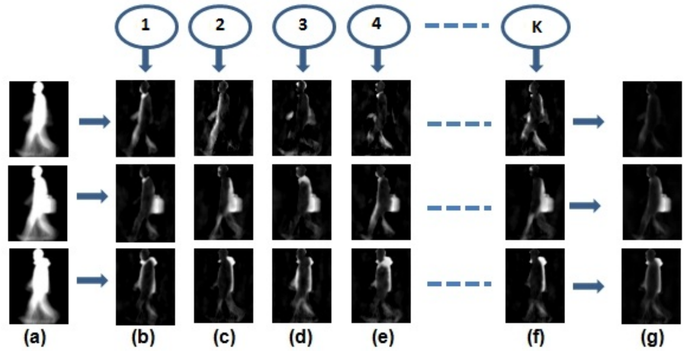

Figure 5. (a) Represents GEI images of an unknown class t. (b)-(f) represent *innovations* components of GEI images of an unknown class t against the different classes $1, 2, ..., K$ respectively. (g) Represents average value of *innovations* components.

Using all the K extracted *innovations* components of the GEI of an unknown class t, the Average Innovation Component (i.e. AIC_t) image is calculated as follows,

$$AIC_t(a,b) = \frac{1}{K}\sum_{k=1}^{K} Z_t^k(a,b) \tag{10}$$

where a and b are the image coordinates, k represents k^{th} class and K represents the total number of classes. Figure 5(g) shows the AIC_t of an unknown normal, carrying bags and different clothing GEI images. It can be seen from Figure 5(g) that AIC_t prominently shows the covariate factors of the given GEI image. It is to be noted that the training images of each gallery person will identify some unique aspect of the test image. After averaging over all gallery people, the persistent bit would naturally correspond to something that is not affected by the difference between different people's gait or the static appearance and therefore represents the corresponding to the covariate conditions. Our idea is to remove the extracted covariate factors from the original GEI image. The new gait feature with less covariate information is obtained using AIC_t and named as $\mathbf{GEI_{JSM}}$:

$$GEI_{JSM}(a,b) = \begin{cases} GEI(a,b) & \text{if } AIC_t < thr, \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where thr is a threshold value and is used to handle the noise in AIC_t. Our new gait feature $\mathbf{GEI_{JSM}}$ is shown in Figure 6(b)

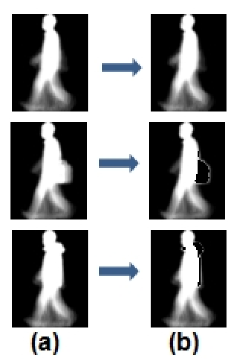

Figure 6. (a) and (b) represent GEI and $\mathbf{GEI_{JSM}}$ respectively.

A basic problem in gait based individual identification is to use labelled training samples from K distinct individual classes to correctly determine the class to which a new test sample belongs. In the training phase for K classes, collect gait feature vectors $\{\mathbf{x}_1^1, ..., \mathbf{x}_j^1, ..., \mathbf{x}_m^1\}, ..., \{\mathbf{x}_1^K, ..., \mathbf{x}_j^K, ..., \mathbf{x}_m^K\}$ and in the testing phase, present a new test gait feature vector x_t, solve for $class(\mathbf{x}_t) \in [1, 2, ..., K]$.

3.4. Classification based on Sparse Representation

NN method is one of the most simple and intuitive methods for gait based individual identification. The Nearest Subspace (NS) method [25, 21] generalizes NN method in the sense that it classifies the test sample based on the best linear representation in terms of all the training samples in each class. The sparse representation-based classification (SRC) method [44] is a further generalization of NS by representing the test sample using the training samples(atoms) adaptively selected from all the candidate training samples (a structured dictionary) from both within and across different classes. SRC based classification showed a higher recognition rate for face recognition than NN and NS methods [44]. Therefore a sparse representation-based classification is applied in our approach.

The sparse representation of Face and Iris recognition is reported in [44, 31]. Following [44, 31], in this section, we briefly describe the sparse representation of gait features. Let us consider that we have K distinct classes and a set of m training D-dimensional $\mathbf{GEI_{JSM}}$ gait feature vectors per class.

Let $\Phi_k = [\mathbf{x}_1^k, ..., \mathbf{x}_j^k, ..., \mathbf{x}_m^k]$ be a $D \times m$ matrix of features from the k^{th} class, where $\mathbf{x}_j^k$ denote the $\mathbf{GEI_{JSM}}$ feature from the j^{th} training image of the k^{th} class. Define a new matrix or dictionary Φ, as the concatenation of training samples from all the classes as:

$$\Phi = [\Phi_1, ..., \Phi_K] \in \mathbb{R}^{D\times(m\times K)} \tag{12}$$

Given sufficient training features vectors of the k^{th} class, $\Phi_k = [\mathbf{x}_1^k, ..., \mathbf{x}_j^k, ..., \mathbf{x}_m^k]$, any test feature vector $\mathbf{x}_t \in \mathbb{R}^D$ from the same class will approximately lie in the linear span of the training features vectors associated with object k:

$$\mathbf{x}_t = \alpha_1^k \mathbf{x}_1^k + ... + \alpha_j^k \mathbf{x}_j^k + ... + \alpha_m^k \mathbf{x}_m^k \tag{13}$$

for some scalars, $\alpha_j^k \in \mathbb{R}$, where $j = 1, 2, ..., m$. If we consider $\alpha_k = [\alpha_1^k, ..., \alpha_m^k] \in \mathbb{R}^m$ then $\mathbf{x}_t$ can be written as $\mathbf{x}_t = \Phi_k \alpha_k$. If entire training feature vectors from all classes are considered then the linear representation of $\mathbf{x}_t$ can be written in terms of all the training feature vectors as:

$$\begin{aligned} \mathbf{x}_t &= [\Phi_1, \Phi_2, ..., \Phi_k, ..., \Phi_K][\alpha_1, \alpha_2, ..., \alpha_k, ..., \alpha_K]^T \\ \mathbf{x}_t &= \Phi\alpha \end{aligned} \tag{14}$$

where $\alpha = [\alpha_1, \alpha_2, ..., \alpha_k, ..., \alpha_K]^T$. Assume $\mathbf{x}_t$ belongs to class k then $\alpha = [0, ..., 0, \alpha_k, 0, ..., 0]^T$ Clearly, solving α would recognise the test feature class. If the system of Equation (14) is underdetermined (i.e. $D < (m \times K)$) then the sparsest α can be recovered by solving the following optimization problem [17, 11, 1, 13]:

$$\hat{\alpha} = \arg\min \|\alpha\|_1 \quad subject\ to \quad \mathbf{x}_t = \Phi\alpha \tag{15}$$

where $\|\alpha\|_1 = |\alpha_1| + |\alpha_2| + ... + |\alpha_K|$. This problem is often known as BP and can be solved in polynomial time [13]. Donoho et al. [17] guaranteed that under certain conditions BP will find the sparsest possible representation.

We perform the classification based on the fact that high values of the coefficients $\hat{\alpha}$ will be associated with the columns of Φ from a single class. We do this by comparing how well the different parts of the estimated coefficients, $\hat{\alpha}$, represent $\mathbf{x}_t$. The minimum of the representation error or the residual error is then used to identify the correct class. The residual error of class k is calculated by keeping the coefficients associated with that class and setting the coefficients not associated with class k to zero. This can be done by introducing a characteristic function, δ_k, that selects the coefficients associated with the k^{th} class as follows:

$$r_k(\mathbf{x}_t) = \|\mathbf{x}_t - \Phi\delta_k(\hat{\alpha})\|_2 \tag{16}$$

where $\delta_k(\hat{\alpha}) = [0, ..., 0, \hat{\alpha}_k, 0, ..., 0]^T$, for $k = 1, ..., K$. The correct class q corresponding to k that gives smallest residual error is calculated as follows:

$$q = \arg \min_{k=1,...,K} r_k(\mathbf{x}_t) \tag{17}$$

where $q \in [1, 2, ..., K]$. We now summarize the sparse recognition algorithm as follows:

Given a matrix of training samples $\Phi \in \mathbb{R}^{D\times(m\times K)}$ for K classes and a test sample $\mathbf{x}_t \in \mathbb{R}^D$:

1. Solve the BP problem of the Equation (15).

2. Compute the residual using the Equation (16).
3. Identify the class label of $\mathbf{x}_t$ using the Equation (17).

Consider the matrix equation $\mathbf{x}_t = \Phi\alpha$, where Φ is a matrix of size $D \times (m \times K)$ and α is the solution of the above system. In our case, feature dimension, D, is greater than the number of training samples, $(m \times K)$, the problem becomes overdetermined and a sparse solution cannot be obtained. Therefore a Random Projection technique is applied to reduce the dimensionality of D to $d << D$, which transforms the system of Equation (14) into an underdetermined system. The RP technique approach is explained as below.

3.4.1. Random Projections

Let Γ be a $d \times D$ random matrix with $d < (m \times K)$. Applying Γ to both sides of the Equation (14) yields:

$$\hat{\mathbf{x}}_t = \Gamma\mathbf{x}_t = \Gamma\Phi\alpha \tag{18}$$

Thus the matrix Φ of training $\mathbf{GEI_{JSM}}$ features is now replaced by the matrix $\Gamma\Phi$ of d dimensional features and $\hat{\mathbf{x}}_t$ can be thought of as a transformed version of $\mathbf{x}_t$. As d is smaller than $(m \times K)$, the system of Equation (18) is underdetermined and a sparse solution can be obtained.

Some examples of random matrices that satisfy the above conditions are listed in [16, 35]. In our approach, the following three matrices are used as as random projections matrices.

- **RM1**: $d \times D$ random matrix Γ whose entries $\Gamma_{i,j}$ are independent realizations of Gaussian random variables $\Gamma_{i,j} \sim N(0, \frac{1}{d})$.
- **RM2**: Multiplication of a $d \times D$ Gaussian random matrix Γ with a deterministic orthogonal $D \times D$ matrix Ω. It has been illustrated in [37] that the DCT matrix is the best sparsifying matrix when compared to the wavelet and Fourier transforms. Therefore, the orthogonal DCT is used as a deterministic orthogonal matrix in our experiments.
- **RM3**: $d \times D$ random matrix Γ whose entries $\Gamma_{i,j}$ are independent random variables such that:

$$\Gamma_{i,j} = \begin{cases} \frac{1}{\sqrt{d}} & \text{with} \quad prob = 0.5 \\ \frac{-1}{\sqrt{d}} & \text{with} \quad prob = 0.5 \end{cases} \tag{19}$$

The next Section describe the evaluation of the proposed methodologies on the standard datasets.

4. Evaluation

For evaluating the merits of the proposed $\mathbf{GEI_{JSM}}$ feature for gait recognition, the proposed methodology is evaluated on the two most standard gait dataset available in the public domain. Firstly, the CASIA dataset developed by The Chinese Academy of Sciences, Institute of Automation (CASIA) [12] to evaluate the proposed algorithms. As our work is based on covariate factors, we used the CASIA Dataset-B covariate dataset for our experiments. The Dataset-B is a large covariate gait database and there are 124 subjects. Secondly, HumanID Gait Challenge dataset available from the University of South Florida is additionally used for demonstrating the extensibility of the proposed $\mathbf{GEI_{JSM}}$ in more realistic environments. For evaluating the proposed RP-based, the second contribution of the paper, we only use the CASIA-B dataset.

4.1. Experimental set-up of CASIA-B dataset

In our experiments, we used fronto parallel view sequences (i.e. perpendicular to the walking direction) with normal, different clothing and carrying bag conditions. For each subject there are ten gait sequences consisting of six normal gait sequences where the subject does not wear a bulky coat or carry a bag (CASIASetA), two carrying bag sequences (CASIASetB) and two wearing coat sequences (CASIASetC). The first four of the six normal gait sequences were used as the gallery set. The probe set included the rest of the normal gait sequences (CASIASetA2), CASIASetB and CASIASetC. As a benchmark comparison, the proposed methodology is compared against the other appearance based gait representation methods: direct Template Matching (TM) method [47], GEI [4], GEnI [4], M_G [6], AEI [48] and GFI [5].

4.2. Results for RP-based reduction methods

First, the robustness analysis of RP-based reduction methods: RM1, RM2 and RM3 approaches are experimented against the CDA. Here, the GEI gait feature representation is considered for the experiment. The normal, carrying bags, different clothing and average (i.e. average rate of normal, carrying bags and different clothing) recognition rates using the random matrices RM1, RM2 and RM3 are shown in Figure 7. Different values of dimension, d, are plotted on the x-axis and the y-axis represents the obtained recognition rate under normal and different covariate conditions. The high average recognition rates are achieved using d values 430, 380 and 430 of RM1, RM2 and RM3 approaches respectively as illustrated in the Figure 7.

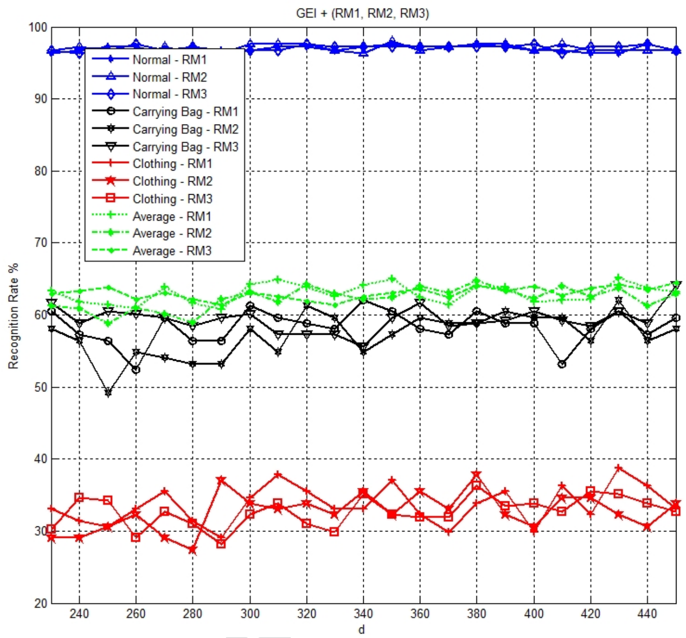

Figure 7. Represents gait recognition rate of different d values for normal, carrying bag and different clothing GEI features using RM1, RM2 and RM3 random projection matrices.

Table 1 shows the performance of the CDA and RP-based reduction methods using GEI feature on CASIA-B (covariate) dataset. It can be seen from Table 1 that in the case of normal (CASIASetA) gait sequences our proposed approaches [GEI+RM1], [GEI+RM2] and [GEI+RM3] are comparable to the [GEI+CDA] approach.

Table 1. Performance of the CDA and RP-based reduction methods on CASIA-B (covariate) dataset using GEI.

Dataset	GEI + CDA [4]	GEI + RM1 [Proposed]	GEI + RM2 [Proposed]	GEI + RM3 [Proposed]
CasiaSetA2	**99.4**%	96.4%	97.6%	96.8%
CasiaSetB	60.2%	60.5%	58.9%	**64.1**%
CasiaSetC	30.0%	**38.7**%	37.9%	32.7%
Average	63.2%	**65.6**%	65.1%	64.5%

At the same time, in the case of different clothing (CASIASetC) gait sequence, our approaches outperform the [GEI+CDA] and [GEI+RM1] showed highest recognition rate of 38.7%. In the case of carrying bag sequences,[GEI+RM3] showed highest recognition rate of 64.1%. The average recognition results 65.6%, 65.1% and 64.5% show that our RM1, RM2 and RM3 approaches produced better recognition results than that from the CDA approach. This shows that our RM1, RM2 and RM3 approaches are better than CDA approach in handling carrying bag and different clothing gait sequences.

4.3. *Results with* **GEI$_{\mathbf{JSM}}$**

Next, our novel gait feature **GEI$_{\mathbf{JSM}}$** using RP-based approaches: RM1, RM2 and RM3 and CDA approaches are considered for gait recognition. In our approach the random matrices RM1, RM2 and RM3 are applied to reduce the feature vector dimension from D to d, where $d < (m \times K)$. An experiment is conducted to analyse the influence of the value of d on recognition rate. Based on the experimental set-up, the number of training samples, $m \times K$ equals to 496 (i.e. 4×124). Therefore value of d should be less than 496. The values from 230 to 450 are considered to find a suitable value of d for higher recognition rate. In our experiments, d values of 360, 370 and 330 provide highest average recognition rates for approaches RM1, RM2 and RM3 respectively.

4.3.1. *Results with* **GEI$_{\mathbf{JSM}}$** *using* **RM1**

We use our proposed **GEI$_{\mathbf{JSM}}$** feature with RM1 random matrix for gait recognition. The normal, carrying bags, different clothing and average recognition rates using the random matrix RM1 is illustrated in Figure 8.

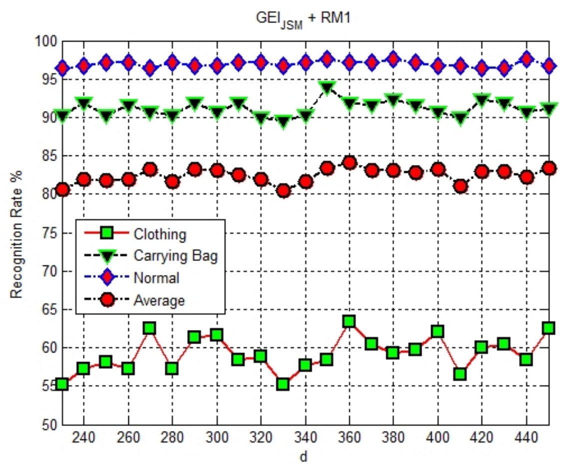

Figure 8. represents gait recognition rate of different d values for the normal, carrying bag and different clothing **GEI$_{\mathbf{JSM}}$** features using RM1 random projection matrix.

Table 2 shows the obtained performance accuracy using the **GEI$_{\mathbf{JSM}}$** and RM1 approach applied to the CASIA-B (covariate) dataset. It can be seen from Table 2 that in the case of carrying bags (CASIASetB) and different clothing (CASIASetC) gait sequences our **GEI$_{\mathbf{JSM}}$** using RM1 outperforms the rest of the approaches. It shows a better recognition rate of 97.2% for normal gait sequences and it is comparable to other methods in the Table 2. The average recognition result of 84.1% shows that our **GEI$_{\mathbf{JSM}}$** using RM1 based gait recognition produced better recognition result than any other method shown in the Table 2.

Table 2. Performance on the CASIA-B (covariate) dataset using [**GEI$_{\mathbf{JSM}}$** + RM1] approach.

Dataset	GEI+TM [47]	GEI+CDA [4]	GEnI+CDA [4]	AEI+LDA [48]	M_G+CDA [6]	GFI+CDA [5]	**GEI$_{\mathbf{JSM}}$**+RM1 [proposed]
CasiaSetA2	97.6%	99.4%	98.3%	88.7%	**100**%	97.5%	97.2%
CasiaSetB	52.0%	60.2%	80.1%	75.0%	78.3%	83.6%	**91.9**%
CasiaSetC	32.7%	30.0%	33.5%	57.3%	44.0%	48.8%	**63.3**%
Average	60.8%	63.2%	70.6%	73.7%	74.1%	76.6%	**84.1**%

4.3.2. *Results with* **GEI$_{\mathbf{JSM}}$** *using* **RM2**

Next, we use our proposed **GEI$_{\mathbf{JSM}}$** feature with RM2 random matrix for gait recognition. The normal, carrying bags, different clothing and average recognition rates with the **GEI$_{\mathbf{JSM}}$** using the random matrix RM2 is illustrated in Figure 9.

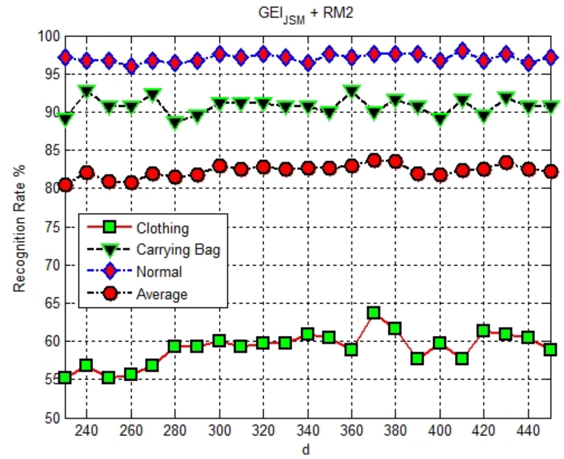

Figure 9. Represents gait recognition rate of different d values for normal, carrying bag and different clothing $\mathbf{GEI_{JSM}}$ features using RM2 random projection matrix.

Table 3 shows the obtained performance accuracy using the $\mathbf{GEI_{JSM}}$ and RM2 approach applied to the CASIA-B (covariate) dataset. It can be seen from Table 3 that in the case of carrying bags (CASIASetB) and different clothing (CASIASetC) gait sequences our $\mathbf{GEI_{JSM}}$ using RM2 outperforms the rest of the approaches. It shows a very high recognition rate, 97.6% for normal gait sequences and it is comparable to other methods in the Table 3. The average recognition result of 83.7% shows that our $\mathbf{GEI_{JSM}}$ using RM2 based gait recognition produced better recognition results than any other method shown in Table 3.

Table 3. Performance on CASIA-B (covariate) dataset using [$\mathbf{GEI_{JSM}}$ + RM2] approach.

Dataset	GEI+TM [47]	GEI+CDA [4]	GEnI+CDA [4]	AEI+LDA [48]	M_G+CDA [6]	GFI+CDA [5]	$\mathbf{GEI_{JSM}}$+RM2 [proposed]
CasiaSetA2	97.6%	99.4%	98.3%	88.7%	**100**%	97.5%	97.6%
CasiaSetB	52.0%	60.2%	80.1%	75.0%	78.3%	83.6%	**89.9**%
CasiaSetC	32.7%	30.0%	33.5%	57.3%	44.0%	48.8%	**63.7**%
Average	60.8%	63.2%	70.6%	73.7%	74.1%	76.6%	**83.7**%

4.3.3. *Results with* $\mathbf{GEI_{JSM}}$ *using* RM3

Next, we use our proposed $\mathbf{GEI_{JSM}}$ feature with RM3 random matrix for gait recognition. The normal, carrying bags, different clothing and average recognition rates using the random matrix RM3 is illustrated in Figure 10.

Table 4 shows the obtained performance accuracy using the $\mathbf{GEI_{JSM}}$ and RM3 approach applied to the CASIA-B (covariate) dataset. It can be seen from Table 4 that in the case of carrying bags (CASIASetB) and different clothing (CASIASetC) gait sequences our $\mathbf{GEI_{JSM}}$ using RM3 outperforms the rest of the approaches.

Table 4. Performance on the CASIA-B (covariate) dataset using [$\mathbf{GEI_{JSM}}$ + RM3] approach.

Dataset	GEI+TM [47]	GEI+CDA [4]	GEnI+CDA [4]	AEI+LDA [48]	M_G+CDA [6]	GFI+CDA [5]	$\mathbf{GEI_{JSM}}$+RM3 [proposed]
CasiaSetA2	97.6%	99.4%	98.3%	88.7%	**100**%	97.5%	97.2%
CasiaSetB	52.0%	60.2%	80.1%	75.0%	78.3%	83.6%	**92.7**%
CasiaSetC	32.7%	30.0%	33.5%	57.3%	44.0%	48.8%	**60.9**%
Average	60.8%	63.2%	70.6%	73.7%	74.1%	76.6%	**83.6**%

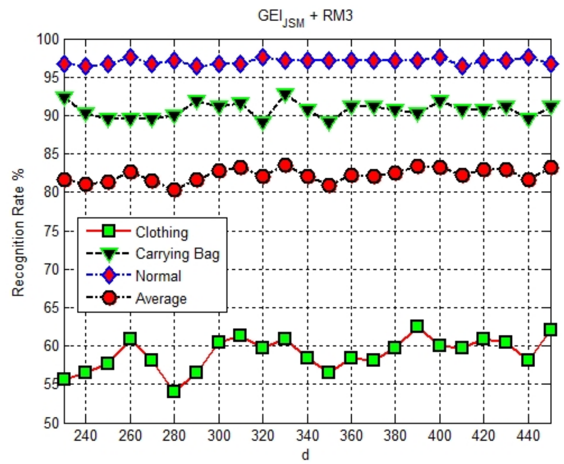

Figure 10. Represents gait recognition rate of different d values for normal, carrying bag and different clothing $\mathbf{GEI_{JSM}}$ features using the RM3 random projection matrix.

It shows a better recognition rate, 97.2% for normal gait sequences and it is comparable to other methods in the Table 4. The average recognition result of 83.6% shows that our $\mathbf{GEI_{JSM}}$ using RM3 based gait recognition produced better recognition results than any other method shown in the Table 4.

4.3.4. *Results with* $\mathbf{GEI_{JSM}}$ *using* **CDA**

Next, the robustness of our proposed $\mathbf{GEI_{JSM}}$ feature is experimented. To test the robustness of the $\mathbf{GEI_{JSM}}$ feature against the features showed in the existing literature, the similar dimensional reduction and classification approaches proposed in the other methods are used. Therefore, the CDA is chosen for dimensional reduction and 1-NN is selected as classifier for the recognition process. Table 5 shows the obtained performance accuracy using the $\mathbf{GEI_{JSM}}$ and CDA approach applied to the CASIA-B (covariate) dataset.

Table 5. Performance on the CASIA-B (covariate) dataset using [$\mathbf{GEI_{JSM}}$ + CDA] approach.

Dataset	GEI+TM [47]	GEI+CDA [4]	GEnI+CDA [4]	AEI+LDA [48]	M_G+CDA [6]	GFI+CDA [5]	$\mathbf{GEI_{JSM}}$+CDA [proposed]
CasiaSetA2	97.6%	99.4%	98.3%	88.7%	**100**%	97.5%	99.2%
CasiaSetB	52.0%	60.2%	80.1%	75.0%	78.3%	83.6%	**88.7**%
CasiaSetC	32.7%	30.0%	33.5%	**57.3**%	44.0%	48.8%	49.2%
Average	60.8%	63.2%	70.6%	73.7%	74.1%	76.6%	**79.0**%

It can be seen from the Table 5 that in the case of carrying bags gait sequences (CASIASetB) our $\mathbf{GEI_{JSM}}$ method outperforms the rest of the approaches. It shows a better recognition rate of 88.7%. At the same time, in the case of different clothing gait sequences (CASIASetC), the $\mathbf{GEI_{JSM}}$ is comparable to others. The average recognition rate 79.0% shows that our $\mathbf{GEI_{JSM}}$ method using CDA based gait recognition produced better recognition results than any of the methods shown in Table 5.

4.3.5. *Statistical significance of the proposed methods*

A two-way ANOVA test [36] is applied to find the statistically significant evidence of a difference between our proposed methods $\mathbf{GEI_{JSM}}$+RM1, $\mathbf{GEI_{JSM}}$+RM2, $\mathbf{GEI_{JSM}}$+RM3 and $\mathbf{GEI_{JSM}}$+CDA. Based on our experimental set-up, each testing set (i.e. normal, carrying bags and different clothing) contains two gait sequences. To find significant evidence of a difference between our proposed methods, we considered six test cases as $\mathbf{GEI_{JSM}}$+RM1 & $\mathbf{GEI_{JSM}}$+RM2, $\mathbf{GEI_{JSM}}$+RM1 & $\mathbf{GEI_{JSM}}$+RM3, $\mathbf{GEI_{JSM}}$+RM1 & $\mathbf{GEI_{JSM}}$+CDA, $\mathbf{GEI_{JSM}}$+RM2 & $\mathbf{GEI_{JSM}}$+RM3, $\mathbf{GEI_{JSM}}$+RM2 & $\mathbf{GEI_{JSM}}$+CDA and $\mathbf{GEI_{JSM}}$+RM3 & $\mathbf{GEI_{JSM}}$+CDA.

Table 6 summarises the results obtained using our proposed methods $\mathbf{GEI_{JSM}}$+RM1, $\mathbf{GEI_{JSM}}$+RM2, $\mathbf{GEI_{JSM}}$+RM3 and $\mathbf{GEI_{JSM}}$+CDA. The recognition rate values from the Table 6 are considered to find significant evidence of a difference between methods $\mathbf{GEI_{JSM}}$+RM1, $\mathbf{GEI_{JSM}}$+RM2, $\mathbf{GEI_{JSM}}$+RM3 and $\mathbf{GEI_{JSM}}$+CDA. The methods are represented by columns in the Table 6.

Table 6. Recognition rates using our proposed methods.

Dataset	$\mathbf{GEI_{JSM}}$+ RM1	$\mathbf{GEI_{JSM}}$+RM2	$\mathbf{GEI_{JSM}}$+RM3	$\mathbf{GEI_{JSM}}$+CDA
CasiaSetA2	98.4%	98.4%	98.4%	99.2%
	96.0%	96.8%	96.0%	99.2%
CasiaSetB	91.1%	87.9%	90.3%	87.9%
	92.7%	91.9%	95.2%	89.5%
CasiaSetC	66.9%	64.5%	58.9%	49.2%
	59.7%	62.9%	62.9%	49.2%

First the significant evidence of a difference between methods $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+RM2 is tested. An ANOVA table is generated using the values of columns $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+RM2 from the Table 6 as shown by the Table 7. Here SS, df, MS, F and p columns represent Sum of Squares, degrees of freedom, Mean Squares and test statistics and p value respectively. The p value corresponding to the source "Columns" (cf. Table 7) equals to 0.799 and which is greater than 0.05. Therefore we can conclude that there is no significant evidence of a difference between $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+RM2. We also ran other five tests and found that corresponding p values as 0.7734, 0.0083, 0.9346, 0.0012 and 0.0075, see Tables 8-12.

Table 7. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+RM2.

Source	SS	df	MS	F	p
Columns	0.48	1	0.48	0.07	0.799
Rows	2589.63	2	1294.81	191.16	0
Interaction	3.84	2	1.92	0.28	0.7627
Error	40.64	6	6.77		
Total	2634.59	11			

Table 8. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+RM3.

Source	SS	df	MS	F	p
Columns	0.8	1	0.8	0.09	0.7734
Rows	2892.44	2	1446.22	163.83	0
Interaction	5.68	2	2.84	0.32	0.7366
Error	52.96	6	8.83		
Total	2951.88	11			

Based on the p values 0.799, 0.7734 and 0.9346, we can conclude that there is no significant evidence of a difference between methods $\mathbf{GEI_{JSM}}$+RM1, $\mathbf{GEI_{JSM}}$+RM2 and $\mathbf{GEI_{JSM}}$+RM3. At the same time the p values 0.0083, 0.0012 and 0.0075 which are less than 0.05 illustrates that methods $\mathbf{GEI_{JSM}}$+RM1, $\mathbf{GEI_{JSM}}$+RM2 and $\mathbf{GEI_{JSM}}$+RM3 are significantly different from $\mathbf{GEI_{JSM}}$+CDA. Table 13 summarises the percentage accuracy achieved using our proposed methodology and the percentage accuracy obtained in the existing literature tested on the CASIA-B (covariate) dataset.

4.3.6. Additional results with $\mathbf{GEI_{JSM}}$

In order to further test the robustness of the proposed a feature $\mathbf{GEI_{JSM}}$, we carried out further evaluation by mixing the training dataset with each subjects covariate conditions also included in the training phase. Instead of

Table 9. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM1 and $\mathbf{GEI_{JSM}}$+CDA.

Source	SS	df	MS	F	p
Columns	78.03	1	78.03	14.93	0.0083
Rows	3975.49	2	1987.74	380.31	0
Interaction	135.02	2	67.51	12.92	0.0067
Error	31.36	6	5.23		
Total	4219.9	11			

Table 10. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM2 and $\mathbf{GEI_{JSM}}$+RM3.

Source	SS	df	MS	F	p
Columns	0.04	1	0.04	0.01	0.9346
Rows	2815.16	2	1407.58	252.52	0
Interaction	16.08	2	8.04	1.44	0.3079
Error	33.45	6	5.57		
Total	2864.72	11			

Table 11. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM2 and $\mathbf{GEI_{JSM}}$+CDA.

Source	SS	df	MS	F	p
Columns	66.27	1	66.27	33.58	0.0012
Rows	3895.65	2	1947.82	987.07	0
Interaction	147.98	2	73.99	37.49	0.0004
Error	11.84	6	1.97		
Total	4121.74	11			

Table 12. ANOVA Table for $\mathbf{GEI_{JSM}}$+RM3 and $\mathbf{GEI_{JSM}}$+CDA.

Source	SS	df	MS	F	p
Columns	63.02	1	63.021	15.65	0.0075
Rows	4254.01	2	2127.003	528.12	0
Interaction	94.27	2	47.136	11.7	0.0085
Error	24.16	6	4.027		
Total	4435.46	11			

Table 13. Performance comparison of our proposed gait feature representations and other reported methods tested on the CASIA-B (covariate) dataset.

Methods	Recognition Rate
Template Matching [47]	60.8%
GEI [4]	63.2%
GEnI [4]	70.6%
AEI [48]	73.7%
M_G [6]	74.1%
GFI [5]	76.6%
Proposed : [$\mathbf{GEI_{JSM}}$ + CDA]	**79.0**%
Proposed : [$\mathbf{GEI_{JSM}}$ + RM3]	**83.6**%
Proposed : [$\mathbf{GEI_{JSM}}$ + RM2]	**83.7**%
Proposed : [$\mathbf{GEI_{JSM}}$ + RM1]	**84.1**%

considering four normal GEI images from the known class k, a set of four GEIs are considered as a combination of normal, carrying bag and different clothing GEI images to test the robustness of $\mathbf{JSM_1}$ approach in discriminating covariate conditions from the given GEI images.

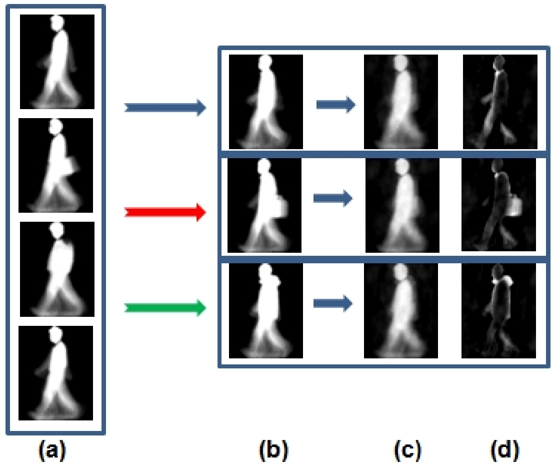

Figure 11. (a) Represents GEI images with normal, carrying bag and different clothing of a known individual (i.e. known class). (b) represents GEI images with normal, carrying bag and different clothing of an unknown individual. (c) represents *common* components of the known and the unknown individual. (d) represents *innovations* components of the unknown individual against the known individual.

Figure 11(a) shows a known class k of two normal (i.e. top and bottom), one carrying bags (i.e. second top) and one different clothing (i.e. third top) GEI images. The normal, carrying bag and different clothing GEIs of the unknown class t are shown from top to bottom in Figure 11(b). The extracted *common* and *innovations* components are shown in Figures 11(c) and (d) respectively. The Figure 11(d) represent the *innovations* component of a GEI of an unknown class t against a known class k.

Figure 11 illustrates the robustness of the proposed $\mathbf{GEI_{JSM}}$ in distinguishing the covariate conditions from the subject available training dataset, where the combination of normal, carrying bags or clothing condition GEIs are available for training. Here the known class k is considered with the four GEIs of two normal, one carrying bag and one different clothing GEIs. Then JSM_1 approach is applied to acquire *common* and *innovations* components. The proposed $\mathbf{GEI_{JSM}}$ is still robust and is able to differentiate the covariate conditions from the body parts. This illustrates that the proposed approach can work in uncontrolled environments, where the subjects may or may not appear with covariate factors and then also we can train the system under uncooperative settings.

It can be seen from the Figures 4(d) and 11(d) that the extracted *innovations* component of the GEI image of an unknown class t are almost similar even if the known class k contains normal GEI images or combination of normal, carrying bag and different clothing GEI images. An experiment is conducted based on the same set-up proposed in [6], where the gallery set include a mixture of normal, carrying bag and wearing coat sequences. This gives us a challenging condition for gait recognition with uncooperative subjects.

In the conducted experiment, the gallery set is selected by using the first one third of the sequences from CASIASetC, the second one third from CASIASetB and the last one third from CASIASetA. The probe sets consist of the rest of the dataset and are referred to as CASIASetA3, CASIASetB2 and CASIASetC2. In this scenario, now both the gallery and probe GEIs contain the covariate factors, therefore, we need to remove the covariate factors and obtain the final $\mathbf{GEI_{JSM}}$ for all the given GEIs. We have tested the CASIA-B dataset under the uncontrolled set-up using the $\mathbf{GEI_{JSM}}$ with CDA and 1-NN classifier. Table 14 shows the obtained performance accuracy using the $\mathbf{GEI_{JSM}}$ and CDA approach applied to the CASIA-B (covariate) dataset under uncontrolled set-up along with comparison to the existing approach described in [6]. The results shown in Table 14 indicate that our proposed method outperformed the approach described in [6].

Table 14. Performance on the CASIA-B (covariate) dataset in uncooperative set-up using [$\mathbf{GEI_{JSM}}$ + CDA] approach.

	M_G+ACDA [6]	$\mathbf{GEI_{JSM}}$+CDA [proposed]
CasiaSetA3	69.1%	78.7%
CasiaSetB2	55.6%	65.4%
CasiaSetC2	34.7%	40.1%

4.4. *Evaluation on USF HumanID Gait Dataset*

For obtaining more comprehensive set of results and to demonstrate the extensibility of our proposed $\mathbf{GEI_{JSM}}$ feature, a more realistic gait benchmark dataset: HumanID Gait dataset is used. The $\mathbf{GEI_{JSM}}$ feature is used for segregating the covariate factor from the body part and then used for recognition. The USF HumanID dataset (Version 2.1) consists of 122 subjects walking in an elliptical path captured under outdoor conditions. This is one of the most challenging dataset consisting of a range of covariate conditions including carrying briefcase, surface, shoe, view and time. For benchmarking purposes 12 experiments A-L have been designed to test the performance of state of the art algorithms. 1870 sequences of the 122 subjects are divided into 1 gallery set for training and 12 probes labelled from **A** to **L** for testing. Dividing rule is based on five covariates: surface [C/G], camera position [L/R], shoe [A/B], carrying condition [NB/BF] and recording time [M/N]. The gallery for all of the experiments is (G, A, R, NB M/N). The Probe set is shown in the Table 15.

Table 15. The Probe set of HumanID USF dataset (Version 2.1).

Exp.	Probe	no. of People	Difference
A	(G,A,L,NB,M/N)	122	View
B	(G,B,R,NB,M/N)	54	Shoe
C	(G,B,L,NB,M/N)	54	Shoe, View
D	(C,A,R,NB,M/N)	121	Surface
E	(C,B,R,NB,M/N)	60	Surface, Shoe
F	(C,A,L,NB,M/N)	121	Surface, View
G	(C,B,L,NB,M/N)	60	Surface, Shoe, View
H	(G,A,R,BF,M/N)	120	Briefcase
I	(G,B,R,BF,M/N)	60	Shoe, Briefcase
J	(G,A,L,BF,M/N)	120	View, Briefcase
K	(G,A/B,R,NB,N)	33	Time, Shoe, Clothing
L	(C,A/B,R,NB,N)	33	Surface, Time, Shoe, Clothing

We have tested the USF HumanID dataset (Version 2.1) using the $\mathbf{GEI_{JSM}}$ with CDA and 1-NN classifier. The dataset provides the silhouettes extracted from the video sequences, which were used to compute $\mathbf{GEI_{JSM}}$. The HumanID gait dataset is captured in an elliptical view. Therefore in the resulting videos at the different instances the captured view of the walking person will be different. In order to calculate the final GEI in these kinds of scenarios, we consider number of GEIs from all the calculated gait cycles in a given video.

Following this, a final GEI is calculated by averaging all the GEIs obtained from the gait cycles in the given video. Figures 12 (a)-(e) illustrates the GEI image of an individual at different instances in the elliptical view walking sequence and Figure 12 (f) represents the final GEI. The final GEI obtained from each video is used to extract our proposed $\mathbf{GEI_{JSM}}$ feature.

4.4.1. $\mathbf{GEI_{JSM}}$ *extraction on USF HumanID gait dataset*

Following, the approach described in the Section 3.3, the JSM is used for extracting the *common* component and the *innovations* component using the final GEIs. $\mathbf{GEI_{JSM}}$ feature is generated by applying an appropriate threshold value in Equation (11). Figure 13 (a) represent the final GEI images of an unknown class t. The sample of *innovations* components of the GEI of an unknown class t against different classes are shown in Figure 13 (b)-(f).

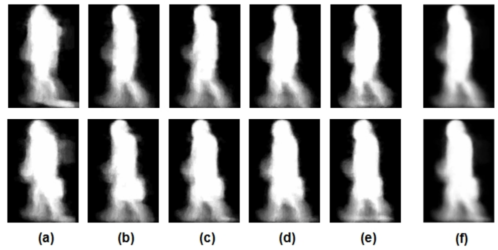

Figure 12. (a)-(e) represent GEI images of an individual with id '03532' from the gallery sequence (top row) and from the experiment 'H' sequence. (f) Represents the final GEI image of (a) to (e).

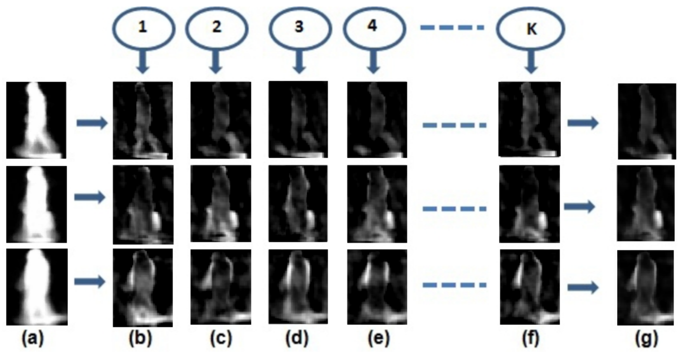

Figure 13. (a) Represents the final GEI images of an unknown class t. (b)-(f) Represent *innovations* components of the final GEI images of an unknown class t against the different classes $1, 2, ..., K$ respectively. (g) Represents average value of *innovations* components.

For each of the experiments A to L, a calculated average GEI image is considered with individual gallery sequences as separate groups and the set of *innovations* components are extracted. The Average Innovation Component image is calculated from the generated *innovations* components. Finally the $\mathbf{GEI_{JSM}}$ is calculated from the generated *innovations* components by applying an appropriate threshold value.

4.4.2. *Results using* $\mathbf{GEI_{JSM}}$ *on HumanID gait dataset*

Following, the extraction of the proposed $\mathbf{GEI_{JSM}}$, the evaluation of the $\mathbf{GEI_{JSM}}$ feature is considered for gait recognition. To test the robustness of the $\mathbf{GEI_{JSM}}$ feature against the features described in the existing literature, the similar dimensional reduction and classification approaches proposed in the other methods are used. Therefore, the CDA is chosen for dimensional reduction and 1-NN is selected as the classifier for the recognition process. Table 16 shows the performance of the $\mathbf{GEI_{JSM}}$ feature using the CDA dimension reduction and the 1-NN classification on the HumanID gait dataset along with the percentage accuracy obtained with other gait representation methods describe in the literature. It can be seen from the Table 16 that the proposed $\mathbf{GEI_{JSM}}$ feature outperformed the other methods.

In [45], Gabor-PDF feature is proposed for gait recognition on the HumanID gait dataset. Table 17 shows the obtained results for the Gabor-PDF and compared with our proposed $\mathbf{GEI_{JSM}}$ feature. It is to be noted that both the methods used NN as a classifier. From the Table 17, it observed that in the experiments J, K and L that involved briefcase and clothing covariate factors in the probe dataset, our proposed $\mathbf{GEI_{JSM}}$ feature outperformed the approach described in [45]. However, on an average the approach described in [45] has a higher recognition rate in comparison to our proposed $\mathbf{GEI_{JSM}}$ feature on the HumanID gait dataset. We believe that this is a particular artifact of the dataset, where other covariate factors may be causing the performance degradation. Nevertheless, the claim made in the work presented in the paper relates to the covariate factors associated to the upper body part. Therefore, in general we would expect that the proposed $\mathbf{GEI_{JSM}}$ feature would be able to robustly segregate the normal body of an individual from the body related covariate factors. However in the case of covariate factors that are externally related may require additional parameters to be considered for obtaining higher gait recognition rate. The problem of externally associated covariate factors will be addressed in the future work.

Table 16. Performance on the USF HumanID dataset (Version 2.1) using [$\mathbf{GEI_{JSM}}$ + CDA] approach.

Probe Set	Baseline [38]	MSCT+SST [23]	GEI [20]	HMM [22]	GEnI [4]	$\mathbf{GEI_{JSM}}$ [proposed]
A	73%	80%	89%	89%	89%	90%
B	78%	89%	87%	88%	89%	88%
C	48%	72%	78%	68%	80%	83%
D	32%	14%	36%	35%	30%	48%
E	22%	10%	38%	28%	38%	44%
F	17%	10%	20%	15%	20%	34%
G	17%	13%	28%	21%	22%	28%
H	61%	49%	62%	85%	82%	88%
I	57%	43%	59%	80%	63%	78%
J	36%	30%	59%	58%	66%	72%
K	3%	39%	3%	17%	6%	19%
L	3%	9%	6%	15%	9%	16%
Average	40.9%	38.3%	50.1%	53.5%	53.5%	57.3%

Table 17. Comparison of our proposed $\mathbf{GEI_{JSM}}$ feature and Gabor-PDF on the HumanID gait dataset.

Probe Set	Gabor-PDF [45]	$\mathbf{GEI_{JSM}}$ [proposed]
A	90%	90%
B	91%	88%
C	85%	83%
D	53%	48%
E	52%	44%
F	32%	34%
G	28%	28%
H	92%	88%
I	86%	78%
J	64%	72%
K	12%	19%
L	15%	16%
Average	62.99%	57.3%

5. Discussion

The advantage of using our proposed methodologies is demonstrated on two different datasets available in the public domain. Initially, we evaluated our proposed RP-based reduction methods: RM1, RM2 and RM3 and compared it with the commonly used CDA approach for the gait based individual identification. The obtained results demonstrated that the proposed RP-based dimensional reduction outperforms the CDA approach as detailed in the Section 4.2.

Next, we evaluated our proposed gait feature: $\mathbf{GEI_{JSM}}$ in recognition along with the RP-based reduction methods. From the Figures 8, 9, 10 in the Section 4.3, it can be observed that the average recognition rate deviates between 80% and 85% (approximately). However, none of the methods in the literature showed the average recognition rate more than 76.6%. The recognition rate results from the Tables 2, 3 and 4 shows the robustness of our proposed novel $\mathbf{GEI_{JSM}}$ feature using the RM1, RM2 and RM3 approaches. The highest average recognition rate is achieved using $\mathbf{GEI_{JSM}}$+RM1 approach under the normal conditions. At the same time, the highest recognition rate for the carrying bags and different clothing gait sequences are achieved using approaches $\mathbf{GEI_{JSM}}$+RM3 and $\mathbf{GEI_{JSM}}$+RM2 respectively. The robustness of the $\mathbf{GEI_{JSM}}$ is further demonstrated using the CDA approach. This is done to show how $\mathbf{GEI_{JSM}}$ outperforms the other existing gait representation methods tested with the CDA in the literature. The recognition rate of $\mathbf{GEI_{JSM}}$ using the CDA is higher than any of the other methods in Table 5. This shows that our $\mathbf{GEI_{JSM}}$ gait feature is more robust and better at handling different clothing and carrying bags covariate factors than any of the other methods in the existing literature.

The results detailed in the Section 4.3.6 illustrated the extensibility of our proposed approach of using $\mathbf{GEI_{JSM}}$ feature in uncontrolled environment and the system can still be trained with the combination of normal, carrying bag or different clothing condition GEIs. An experiment is conducted based on the uncooperative set-up of the CASIA-B dataset and a better recognition result is obtained.

The proposed $\mathbf{GEI_{JSM}}$ feature method is tested on HumanID gait dataset where the sample dataset was collected in the real environment. The results obtained using the $\mathbf{GEI_{JSM}}$ feature for gait recognition outperformed other methods compared on this dataset. The main objective of the proposed the proposed $\mathbf{GEI_{JSM}}$ feature is to remove the different clothing and carrying bags covariate factors from the given GEI images. However in the HumanID gait dataset, there is a huge influence of shadow on the concrete surface. The proposed $\mathbf{GEI_{JSM}}$ feature approach successfully identified and removed the shadow area from the final GEI images in the HumanID gait dataset. This makes our $\mathbf{GEI_{JSM}}$ feature more robust in handling covariate factors and increasing the recognition rate.

6. Conclusion and future work

In this paper, a novel gait recognition algorithm has been proposed for individual identification based on JSM and ℓ_1-norm minimization approach. In this work, our main objective has been to increase the gait recognition rate on different clothing and carrying bag covariate gait sequences. We have proposed a novel robust gait feature representation (i.e. $\mathbf{GEI_{JSM}}$) using the JSM_1 model. If a set of GEI feature vectors are submitted to JSM_1 model then a *common* component for all GEI features and *innovations* components for each GEI features is obtained. These *innovations* components are considered as covariate conditions and removed from the given GEI feature resulting in $\mathbf{GEI_{JSM}}$. Even though the proposed methodology is built on existing ideas but has a novelty. That is, the training images of each gallery person will identify some unique aspect of the test image; after averaging over all gallery people, the persistent bit would naturally correspond to something that is not affected by the difference between different people's gait and static appearance, therefore corresponding to the covariate conditions.

It has been demonstrated that covariate factors could be separated efficiently using JSM approach. By using appropriate experiments it has been shown that JSM based Gait Energy Image: $\mathbf{GEI_{JSM}}$ handled the covariate factors efficiently. Also a ℓ_1-norm minimization approach based sparse approximation technique works well for gait recognition through random projections and outperforms all the existing approaches.

Our proposed methods are thus based on a sparse approximation technique. The solution of sparse approximation is calculated under the concept of BP approach. The BP approach gives better results with noise free features. There are approaches such as Basis Pursuit Denoising that can be applied to handle features that includes noise. We would like to further investigate a sparse approximation solution using approaches that could better handle noisy features to get higher recognition rate on covariate gait sequences as part of our future work. The work presented here is the next step in our efforts to build a more realistic gait recognition system. The work described considered one GEI image for a given video sequence for sparse based recognition. There can be a further variation to the approach where instead of considering a single GEI image for a given video sequence, we can consider multiple GEIs for Group Sparse Representation based gait recognition as described in [45]. This we would like to consider as a part of future work.

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