A three-state dynamical model for religious affiliation

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Abstract

In the last century the western world has seen a rapid increase in the number of people describing themselves as affiliated with no religious group. We construct a set of models using coupled differential equations in which members of a society can be in one of three groups; religiously committed, religiously affiliated or religiously not affiliated. These models are then used to analyse post World War II census data for Northern Ireland.

Key words: religion; group behaviour; nonlinear models

Introduction

The decline in religious belief, and corresponding rise in religious non-affiliation, in the western world over the last century is well attested. However, even though in decline, religion has shown, as Chaves[1] puts it, a 'stubborn refusal to disappear'. This 'stubborn refusal' has encouraged studies over the last 20 years to investigate a number of aspects of religious belief, by a range of economists, sociologists, mathematicians and physicists. Iannacone[2] and Stark and Iannaccone[3] have modelled religious groups as a religious market analogous to an economic market and argued that this explains why nations with state established denominations which have a monopoly on religion exhibit much lower rates of church attendance than countries with a 'competitive religious market' of multiple denominations. Indeed, Iannaccone[2] suggests that this 'religious market' approach explains the high figures for religious believe in the United States 'where the first amendment's anti-establishment clause has left the religious market virtually unregulated for the past 2 centuries'. Uecker *et al*[4] have analysed the decline of religious belief in American early adults. While decline in religious belief in this group has previously been linked with entry into higher education, with accompanying exposure to alternative worldviews and erosion of the plausibility of religious belief, their data from the National Longitudinal Study of Adolescent Health suggested that there is 'little support' for such an assumption. Rather they suggested that early adults adopting behaviours such as non-marital sexual activity, frequent alcohol consumption, or drug use, may lead to dissociation from religious groups which teach that such behaviour is wrong. McCleary and Barro[5] have sought to find statistical links between specific religious beliefs and the work-ethic of believers, and both Herteliu[6] and Hertiliu and Isaic-Maniu[7] classify a broad range of indicators which are of potential relevance to the modelling of religion. Tilley[8], and Voas and Crockett[9] analyse longitudinal data from the British Household Panel Survey which they conclude shows a major factor in decline in religious belief is its failure to efficiently transfer between generations. Voas and Crockett's results suggest that in Britain institutional religion has 'a half-life of one generation' i.e. the children of the current generation are half as likely to attend church as their parents. Hayward[10,11] has developed a model of how Christian churches grow, particularly in the context of religious revival, which is inspired by the classic mathematical model of the spread of epidemics introduced by Kermack and McKendrick[12]. This more general applicability of epidemiological models is emphasised by the fact that Bettencourt *et al*[13] also successfully used such models to study the spread of a scientific idea - namely Feynman diagrams- in the USA, Japan and USSR in the late 1940s and 1950s. Ausloos and Petroni[14 15]have used the Johnson-Mehl-Avrami-Kolmogorov equation for crystal growth to model the change in size of a number of world religions. Instead of using the numbers of members of a group, Ausloos[1617] has examined the dynamics of a small religious group, the Belgian Antionists, via data regarding their finances and number of temples. Further, Clippe and Ausloos[18] have applied Benford's law of leading digits to the finances of the Belgian Antionists, and Mir[19] has applied Benford's law to the size of seven religious faiths in countries across the world. Vitanov *et al*[20] have used a Lotka-Volterra like model to consider competing ideologies, investigating cases of societies with up to three ideologies.

In contrast to these studies, many of which have investigated specific facets of the dynamics of religious belief, a recent paper by Abrams *et al*[21] has extended earlier work by Abrams and Strogatz[22] on language death by using a simple two-state model for group dynamics to model conversion between those who declare themselves to be religious and those who do not. Labelling these groups X (religiously affiliated) and Y (not religiously affiliated), with the fraction of the total population in each group being *x* and *y* respectively they proposed a model of the form

$$\frac{dx}{dt} = yR_{yx}\left(x,u_x\right) - xR_{xy}\left(y,u_y\right) \tag{1}$$

where $R_{yx}\left(x,u_x\right)$ is the rate per unit time that an individual converts from group Y to group X, and $0 \le u_x \le 1$ is the perceived utility of group X. Abrams *et al* proposed

$$R_{yx}\left(x,u_x\right) = cx^a u_x \tag{2}$$

and further noting that the entire population is divided into the complementary sets of religiously affiliated and not religiously affiliated,

$$x + y = 1 \tag{3}$$

and requiring that the utilities are of the form

$$u_x + u_y = 1 \tag{4}$$

the model was fitted to a range of data sets with the result that a best fit was found for *a*=1. This has the important consequence that (1) reduces to

$$\frac{dx}{dt} = c\left(2u_x - 1\right)x\left(1 - x\right) \tag{5}$$

i.e. – the model becomes one of logistic growth. Although the Abrams *et al* model unifies a significant number of data sets, a restriction of the model is that it divides the social group into only two sub-groups. While a two state system is economic in terms of modelling, it could be argued that a religious group can usefully be divided to distinguish between committed (or core) and non-committed (or peripheral) members. Thus, in this paper we investigate a class of three state models which allow for a greater range of behaviours within a society.

A three state model

We consider a society divided into three groups with regard to religious affiliation – the religiously committed, X, the religiously affiliated, or non-committed, Z and the religiously not affiliated, Y.

The division of the religious group into the committed and non-committed corresponds to the observation that although individuals may declare themselves as belonging to a particular religious group, this may not be reflected in active involvement – such as regular attendance at the group's acts of worship.

A general form of this three state model can be given as

$$\begin{aligned}\frac{dx}{dt} &= -x\left(R_{XY}\left(y,\alpha_{XY}\right)+R_{XZ}\left(z,\alpha_{XZ}\right)\right)+yR_{YX}\left(x,\alpha_{YX}\right)+zR_{ZX}\left(x,\alpha_{ZX}\right)\\ \frac{dy}{dt} &= \ xR_{XY}\left(y,\alpha_{XY}\right)-y\left(R_{YX}\left(x,\alpha_{YX}\right)+R_{YZ}\left(z,\alpha_{YZ}\right)\right)+zR_{ZY}\left(y,\alpha_{ZY}\right)\\ \frac{dz}{dt} &= \ xR_{XZ}\left(z,\alpha_{XZ}\right)+yR_{YZ}\left(z,\alpha_{YZ}\right)-z\left(R_{ZX}\left(x,\alpha_{ZX}\right)+R_{ZY}\left(y,\alpha_{ZY}\right)\right)\end{aligned} \qquad (6)$$

where, as before, $R_{IJ}\left(x,\alpha_{IJ}\right)$ is the rate per unit time that members convert from group I to group J, and α_{IJ} is a constant .

Further noting that the three groups X,Y,Z are mutually exclusive and so

$$x+y+z=1 \qquad (7)$$

and considering a class of models where

$$R_{XY}\left(y,\alpha_{XY}\right)=R_{YZ}\left(z,\alpha_{YZ}\right)=0 \qquad (8)$$

i.e. where members of the religiously committed group X, do not move directly to the non-affiliated group Y, and members of the non-affiliated group do not move directly to the religiously non-committed group Z, then (6) becomes

$$\begin{aligned}\frac{dx}{dt} &= -xR_{XZ}\left((1-\mathrm{x}-\mathrm{y}),\alpha_{XZ}\right)+yR_{YX}\left(x,\alpha_{YX}\right)+(1-x-y)R_{ZX}\left(x,\alpha_{ZX}\right)\\ \frac{dy}{dt} &= \ -yR_{YX}\left(x,\alpha_{YX}\right)+(1-x-y)R_{ZY}\left(y,\alpha_{ZY}\right)\end{aligned} \qquad (9)$$

A flow diagram summarising (9) is given in Figure 1. Thus we permit members to move between the religiously committed and religiously non-committed groups, but members moving from the religious group (defined as the union of X and Z) to the religiously unaffiliated group, Y, only do so from the non-committed group, Z. Further if a member of the religiously unaffiliated group moves (or 'converts') to the religious group, s/he will move directly to the committed group, X.

In what follows we consider a range of possible forms of R_{XZ}, R_{YX}, R_{ZX} and R_{ZY}. In order to reduce the number of degrees of freedom for data fitting purposes we restrict ourselves to models which contain only three free parameters. The simplest form of such a model is where the rates are constants:

(i) Model 1

$$R_{XZ} = p, R_{YX} = q, R_{ZX} = q, R_{ZY} = u \tag{10}$$

which has a single fixed point at

$$\left(\frac{q}{p+q}, \frac{pu}{(p+q)(q+u)}, \frac{pq}{(p+q)(q+u)} \right) \tag{11}$$

or,

(ii) Model 2

$$R_{XZ} = p, R_{YX} = q, R_{ZX} = u, R_{ZY} = u \tag{12}$$

which has a single fixed point at

$$\left(\frac{2qu}{p(q+u)+2qu}, \frac{pu}{p(q+u)+2qu}, \frac{pq}{p(q+u)+2qu} \right). \tag{13}$$

The key difference between these two models is that for model 1 members of the uncommitted group *Z* join the committed group at a rate, *q*, equal to that of members of the unaffiliated group, whereas in model 2 members of group *Z* have an equal rate, *u*, of flow into the committed and unaffiliated groups *X* and *Y*.

Setting the rates to be constant means that the likelihood of moving to another group is not influenced by the group's size. This can be interpreted as meaning that movement to a particular group is not influenced by the group's popularity and/or that groups mix socially or spread information in a way which insures that a group and its beliefs are widely known. An obvious generalisation of this is to assume that where groups are widely separated in terms of their beliefs they are less likely to interact socially and hence the chance of social interaction between such widely-separated groups will be dependent on the size of the group moved to, with the simplest way to model this being to include a linear factor. Thus, taking model 1 and assuming a wide social-separation between the unaffiliated group, Y, and committed group, X we write,

(iii) Model 3

$$R_{XZ} = p, R_{YX} = qx, R_{ZX} = q, R_{ZY} = u \tag{14}$$

which has fixed points

$$\begin{aligned} &(0,1,0) && \text{if } pu > q(q+u) \\ &\left(\frac{q(q+u)-pu}{q(p+q+u)}, \frac{pu}{q(q+u)}, \frac{p\left(q(q+u)-pu\right)}{q(q+u)(p+q+u)}\right) && \text{if } pu \le q(q+u). \end{aligned} \tag{15}$$

(iv) Model 4

If we further assume a wide social-separation between the unaffiliated group, Y, and committed group, X *and* between the unaffiliated group Y and the uncommitted group Z we obtain

$$R_{XZ} = p, R_{YX} = qx, R_{ZX} = q, R_{ZY} = uy \tag{16}$$

which has fixed points

$$\begin{aligned} &\left(0,1,0\right) && \text{if } pu > q(q+u) \\ &\left(\frac{q}{p+q}, 0, \frac{p}{p+q}\right) && \text{if } pu < q^2 \\ &\left(\frac{q\left(q+u\right)-pu}{q\left(q+u\right)}, \frac{pu-q^2}{qu}, \frac{q\left(q+u\right)-pu}{u\left(q+u\right)}\right) && \text{if } q^2 \le pu \le q(q+u). \end{aligned} \tag{17}$$

We note that both models 3 and 4 support a fixed point where the entire population is religiously unaffiliated, and in both cases this is given by the same condition, namely $pu > q(q+u)$.

Models 3 and 4 can be thought of as mathematical generalisations of Hayward's 1999 model[11]. Hayward divides his population into enthusiasts, susceptibles and post-enthusiasts – which within the current models may be thought to correspond to the groups X, Y and Z respectively. However, in Hayward's model post-enthusiasts do not return either to the enthusiastic or susceptible state, and as such it is of the form

$$R_{XZ} = p, R_{YX} = qx, R_{ZX} = 0, R_{ZY} = 0\,. \tag{18}$$

(v) Model 5

Next we assume that all groups have a wide social separation and we modify model 2 so that

$$R_{XZ} = p(1-x-y), R_{YX} = qx, R_{ZX} = ux, R_{ZY} = uy \tag{19}$$

which, assuming $p,q,u > 0$ has a fixed point at

$$(1,0,0) \text{ if } \quad p \le u \tag{20}$$

and a centre at

$$\left(\frac{u}{p+q},\frac{p-u}{p+q},\frac{q}{p+q}\right) \text{ if } p>u\,. \tag{21}$$

(vi) Model 6

Finally, we modify model 1 so that all groups are assumed to have wide social separation, giving,

$$R_{XZ} = p(1-x-y), R_{YX} = qx, R_{ZX} = qx, R_{ZY} = uy \tag{22}$$

which, assuming $p,q,u>0$ has a fixed point at

$$(1,0,0) \text{ if } p \le q \tag{23}$$

and a centre at

$$\left(\frac{u}{p+u},\frac{p-q}{p+u},\frac{q}{p+u}\right) \text{ if } p>q\,. \tag{24}$$

Models 5 and 6 possess the notable properties of admitting periodic solutions, and a fixed point where the entire population becomes religiously committed. In this latter case the stability of the point is determined by the ratios of the constants of proportionality , p/u and p/q respectively, of movement between the committed and uncommitted groups. While it is doubtful that such a society, with *all* three groups being socially separated, is realistic, we include these models here as they are natural extensions of models 1 to 4.

The models 1 to 6 above may be broadly considered to represent an increasing social separation and corresponding reduction in group mixing/ information spread between the groups. Models 1 and 2 represent scenarios where all three groups are well integrated, models 3 and 4 where there is increased social separation between the religiously unaffiliated and the other two groups, and models 4 and 6 modelling cases where all three groups are socially separated.

Applying the models to Northern Irish church and census data

Data on religious belief in Northern Ireland is available via the national census[23] and gives detailed information on the religious affiliations of those within the province. A graphical representation of this data for those claiming to religious affiliation from 1861-2011 is given in Figure 2. Attempting to model the census data from 1861-2011 using models 1 to 6 would implicitly include the (strong) assumption that the parameters *p*, *q* and *u* remain constant over the 150 year time period. However, one interpretation of the census data for Northern Ireland is that a change occurred post World War II – at which point religious non-affiliation, which before 1951 had been at most 0.2% of the population, began to rise dramatically. This would imply that it would be appropriate to apply the models to only the post war data. We note that the 1981 census return gave an anomalously high value for religious non-affiliation. This results from the 1981 census taking place during a time of particular unrest within Northern Ireland which gave rise to a 'protest' census return by some members of the community[24]. Hence the 1981 data point is excluded from the analysis in this paper.

The census provides data for the non-affiliated group Y in the class of models described above, but it does not give any information of commitment levels for religious believers.

To generate estimates of religious commitment we use data from the Presbyterian Church in Ireland (PCI), published in its annual reports[25] on the number of communicant members of the church (i.e. the number of people who have formally joined the denomination), and the number of such communicant members who have attended a communion service (a significant religious ceremony within the church which occurs typically between once a year, and once a month depending on the individual congregation) on at least one occasion during the year. The Presbyterian Church of Ireland is the second largest religious denomination in Northern Ireland, accounting for 23% of those who identified as being members of a religious group in the 2011 census. Further, the religious population of Northern Ireland is overwhelmingly Christian , with 99% of those who identified as being members of a religious group in the 2011 census declaring themselves in terms which the census defined to be Christian, and 93% of the Christian group declaring themselves as part of one of the four main denominations of Roman Catholic, Presbyterian, Church of Ireland or Methodist. Thus we assume that the PCI data can be used as a reasonable surrogate for the entire religious population for the province and hence can be used to estimate the size of the committed group across all religious groups in the province. Table 1 presents estimates for the sizes of the committed group, X, using the measures noted above over the period 1951-2011.

As a comparison, we note that a recent survey of church attendance in the UK[26] states that 30% of adults in NI attended church 'once a week or more' in 2006, a figure which is comparable with a linearly interpolated value of f_1=0.279 from Table 1.

Results

We solve the coupled equations for each of the models 1-6 above numerically, using the 4th order Runge-Kutta method, and perform a brute force least absolute difference fit to the census data and the ratios f_1 and f_2 over the parameter space unit cube $0 \leq p, q, u \leq 1$ with a parameter increment of $\Delta = 5 \times 10^{-3}$. We take the unit of time to be 10 years. Sample fits are presented in Figures 3 and 4, and results are summarised in Tables 2 and 3. If we focus on the predicted size of the unaffiliated groups at the next census point of 2021, and in 2041it is clear that the models may be placed into two groups. Models 1-3 predict that by 2021 the non-affiliated group in Northern Ireland will be between 19.0% and 19.6% of the population and models 4-6 that it will be between 20.6% and 21.8% of the population. This divergence increases when we examine the predictions for 2041 with models 1-3 predicting that the non-affiliated group in Northern Ireland will be between 24.0% and 24.9% of the population and models 4-6 that it will be between 29.8% and 33.8% of the population. For comparison we note that that the Abram's *et al* model predicts non-affiliated group sizes of 20.6% and 29.4% respectively for 2021 and 2041. We note that the key model difference between 1-3 and 4-6 is that the latter assume social separation between more of the groups. Further we note that while the use of f_1 or f_2 to represent the fraction of the population which is religiously committed does change the optimal parameter values *p,q,u* for each model, it does not significantly change the predictions for the size of the religiously unaffiliated group at the time points considered. In 9 of the 12 of scenarios given in Tables 2 and 3 the parameter q = 0, and in the other 3 cases it takes on the value of q = 0.005 – i.e. the minimum non-zero value allowable on the search grid. In all 6 models q is associated with movement from group Y to group X (models 2 and 5), or from Y or Z to group X (models 1,3,4,6) – i.e. q measures flow to the religiously committed group. Thus, the overwhelming thrust of the models is that people are leaving religious belief, and not re-joining. The mathematical effect of q=0 in models 5 and 6 should be noted. The fixed points of these models given in (20), (21) and (23) , (24) only hold if q>0. However, analysing the stability of both models in the cases where q=0 shows that the best fit trajectories approach the fixed line $x+y=1$.

Further note that in the cases of models 1-3 where the rates of flow $R_{XZ} = p$ and $R_{ZY} = u$, $p>u$ in all cases, suggesting that the dominant change is people leaving the religiously committed group X to move to the religiously affiliated group Z.

Turning to the long term (fixed point) behaviour of the models we note that models 1-4 all predict the long term growth of religious non-affiliation, and a corresponding decline in the committed group X, with all models predicting the ultimate extinction of religious belief. The best fit parameter values for models 5 and 6 give rise to different behaviour. In the case of model 5 for the f_2 data, the fixed point is in fact the centre given by (21), with an estimate of the period of the orbit being approximately 26,000 years, whereas in the other cases the fixed line $x+y=1$ forms the attractor, leading to long term co-existence of the committed religious group X and the religiously unaffiliated group Y. However, such long term extrapolation of the models has no predictive merit. Tables 2 and 3 also give census points by which the models predict the religiously committed group X has fallen below 10% of the population and the religiously unaffiliated group Y has risen beyond 50% of the population. There is good consistency across f_2 and f_3 ,with in both cases models 1-3 suggesting Y will exceed 50% by the second half of the next century and models 4-6 suggesting that this will occur by the second half of this century.

In terms of overall choice between the models the final column of both Table 2 and 3 show that the average absolute difference between the solution curves and data is consistently less for models 1-3, suggesting that they may be better representing the group behaviours. Assuming this in what follows, and focussing on models 1-3 we make the following observations: First, the form of these models (with constant conversion rates in models q and 2 and only one group-size dependant rate in model 3) suggest that the information about the beliefs of the various groups are well known and/or social interaction between groups is considerable. This of course, in not surprising in a society as small (a population of 1.8 million at the 2011 census) and geographically compact as Northern Ireland. Secondly, if current trends continue then the religiously non-affiliated group in Northern Ireland will continue to grow monotonically. Finally, in only one of the six scenarios (model 3 with data set f_1 is the parameter q>0, and even in that case it is 0.005 (i.e. the smallest non-zero value allowable on the search lattice). This suggests that the social movement to the religiously unaffiliated position is effectively one way – with none of this group being converted or re-converted to the religiously committed group. In ecclesiastical terms this could be interpreted as implying that evangelistic strategies currently in place by Northern Irish churches are ineffective. In particular, the fixed points of the models, given by (11), (13) and (15) mean that if $q>0$, for models 1 and 2 or if $q>0.041$ (for the f_1 data set) or $q>0.045$ (for the f_2 data set) for model 3, then there would be long term co-existence of religious and non-religious groups.

Conclusions

The class of models considered here have the advantage of representing a wider range of religious views that a binary divide of religious/not religious. That such a wider range exists is justified by, for example, the data contained in Table 1 which indicates that only a subset of people who return in the census as being part of a particular religious denomination are in fact members of that denomination. In the context of the data set considered, the three state models considered here agree with the Abram's *et al* two-state model in predicting the continued growth of religious non-affiliation. Our models suggest that if current societal changes continue then the religiously non-affiliated group in Northern Ireland will continue to grow, reaching between 19.0 to 22.1% of the population by the next census point of 2021 and to between 24.0 and 33.8% by 2041. They also suggest that conversion rates from a non-religiously affiliated group to a religious group are negligible.

In future work we intend to investigate further the Presbyterian Church data to see if it reflects the Voas and Crockett[7] suggestion of a generational 'half-life' noted in the introduction, and to incorporate this into a more general class of model.

Extension of the work investigated here to classes of models with four free parameters would also be an interesting direction for future work. However, to do so robustly would require more data to be available. In this regard if more denominations followed the recent practise of the Church of England by producing statistics on joining, leaving, and death rates within the church this could enable more detailed modelling to be completed.

Finally, the success of the three state model used in this paper suggests that this class of model may form the basis of a useful strategy for investigating other aspects of social diffusion which have hitherto only been investigated using two state models.

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year	f_1	f_2
1951	0.387	0.262
1961	0.415	0.300
1971	0.382	0.267
1981	0.384	0.270
1991	0.345	0.234
2001	0.314	0.202
2011	0.244	0.148

Table 1 Fraction of Northern Irish population who are deemed to be religiously committed (group X). These fractions have been estimated by (a) comparing the total number of people describing themselves as Presbyterian in the NI census with the total number of communicant members of the Presbyterian Church as recorded in church reports (resulting in f_1) and (b) comparing the total number of people describing themselves as Presbyterian in the NI census with the total number of communicant members of the Presbyterian Church who attended a communion service at least once during the year as recorded in church reports (resulting in f_2). In both cases data was normalised to account for the fact that communicant membership excludes children.

model	*p*	*q*	*u*	*x(2021)*	*y(2021)*	*x(2041)*	*y(2041)*	x_∞	y_∞	*x(T)<0.1*	*y(T)>0.5*	*Average difference*
1	0.070	0.000	0.050	0.270	0.196	0.234	0.249	0.000	1.000	2171	2161	0.0144
2	0.140	0.000	0.050	0.286	0.195	0.261	0.246	0.000	1.000	>2251	2171	0.0152
3	0.075	0.005	0.050	0.276	0.195	0.243	0.246	0.000	1.000	2201	2161	0.0142
4	0.075	0.005	0.410	0.276	0.214	0.243	0.316	0.000	1.000	2171	2081	0.0192
5	0.515	0.000	0.400	0.278	0.211	0.249	0.309	0.188	0.812	-	2071	0.0191
6	0.120	0.000	0.405	0.276	0.213	0.246	0.314	0.186	0.814	-	2081	0.0189

Table 2 Results of best fit analysis to NI census data at religious commitment fractions f_1 for models 1-6. For each model the predicted size of the committed and unaffiliated groups, *x(t)* and *y(t)* are given for the years *t*=2021 and *t* =2041. The values x_∞ and y_∞ give the corresponding fixed points for the model. The columns *x(T)<0.1* and *y(T)>0.5* give the census decade T for which the statement is first true. For model 2 the condition *x(T)<0.1* had not been met by T=2251. The last column gives the average absolute difference between the data points and the solution curves, and as such is a measure of comparative goodness of fit.

model	*p*	*q*	*u*	*x(2021)*	*y(2021)*	*x(2041)*	*y(2041)*	x_∞	y_∞	*x(T)<0.1*	*y(T)>0.5*	*Average difference*
1	0.090	0.000	0.050	0.170	0.191	0.142	0.241	0.000	1.000	2081	2171	0.0144
2	0.200	0.000	0.040	0.180	0.190	0.161	0.240	0.000	1.000	2171	2181	0.0159
3	0.095	0.000	0.040	0.167	0.191	0.138	0.241	0.000	1.000	2081	2161	0.0144
4	0.090	0.000	0.355	0.170	0.218	0.142	0.338	0.000	1.000	2081	2071	0.0191
5	0.490	0.005	0.350	0.169	0.217	0.144	0.324	-	-	>2251	2071	0.0189
6	0.140	0.000	0.310	0.169	0.206	0.143	0.298	0.091	0.909	2111	2071	0.0188

Table 3 Results of best fit analysis to NI census data at religious commitment fractions f_2 for models 1-6. For each model the predicted size of the committed and unaffiliated groups, *x(t)* and *y(t)* are given for the years *t*=2021 and *t* =2041. The values x_∞ and y_∞ give the corresponding fixed points for the model. In the case of model 5 there is no fixed point for the given paraments (*p*,*q*,*u*), rather there is a centre. The columns *x(T)<0.1* and *y(T)>0.5* give the census decade T for which the statement is first true. For model 5 the condition *x(T)<0.1* had not been met by T=2251. The last column gives the average absolute difference between the data points and the solution curves, and as such is a measure of comparative goodness of fit.

Figure 1

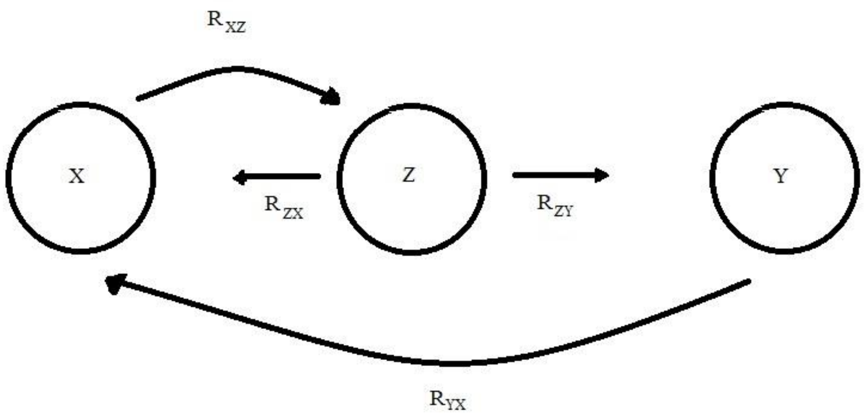
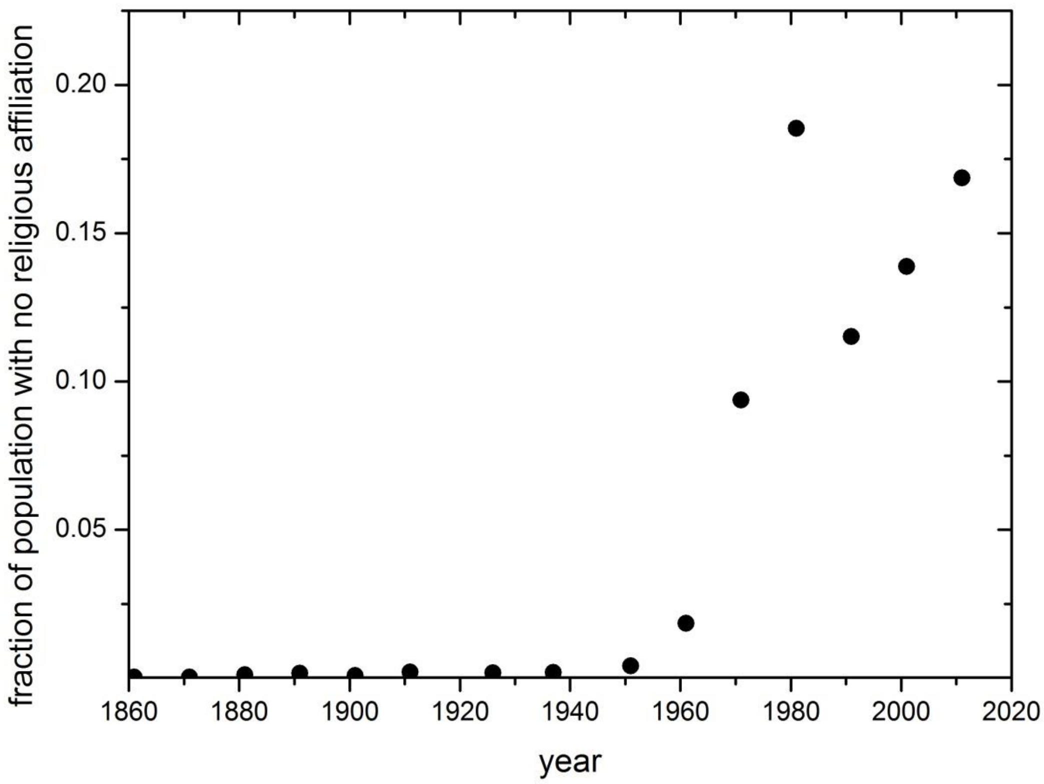
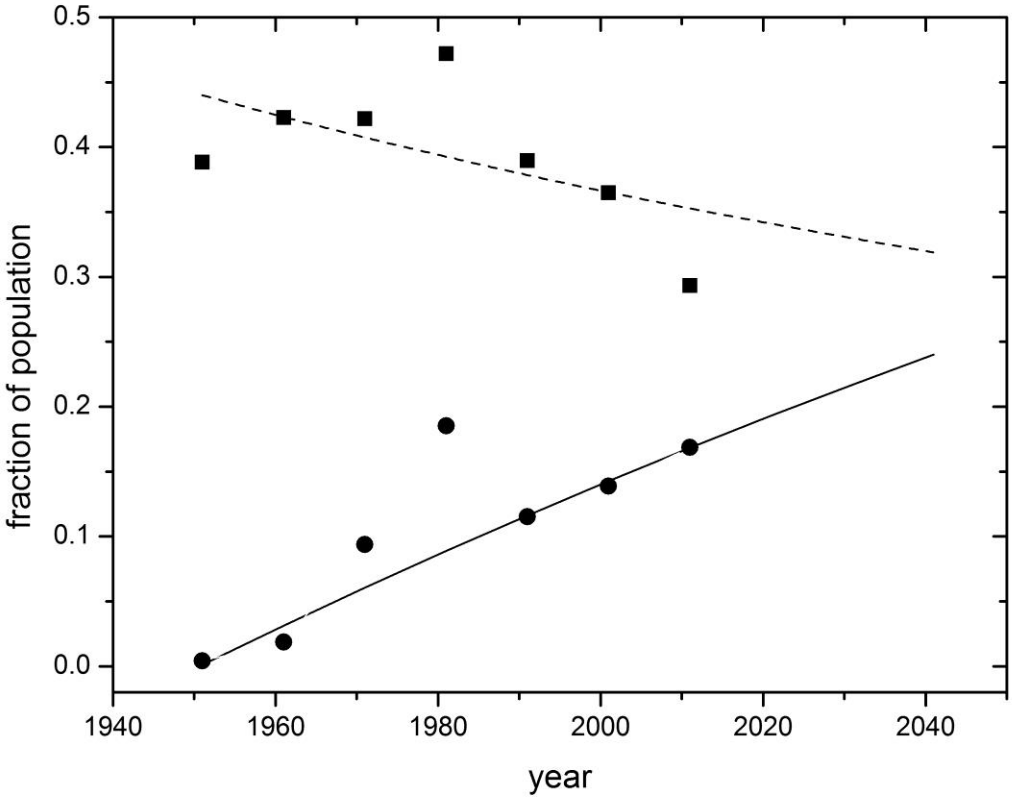


Figure 2



fraction of population with no religious affiliation
0.20
0.15
0.10
0.05
1860
1880
1900
1920
1940
1960
1980
2000
2020
year

Figure 3



0.5
0.4
0.3
0.2
0.1
0.0
fraction of population
1940
1960
1980
2000
2020
2040
year

Figure 4

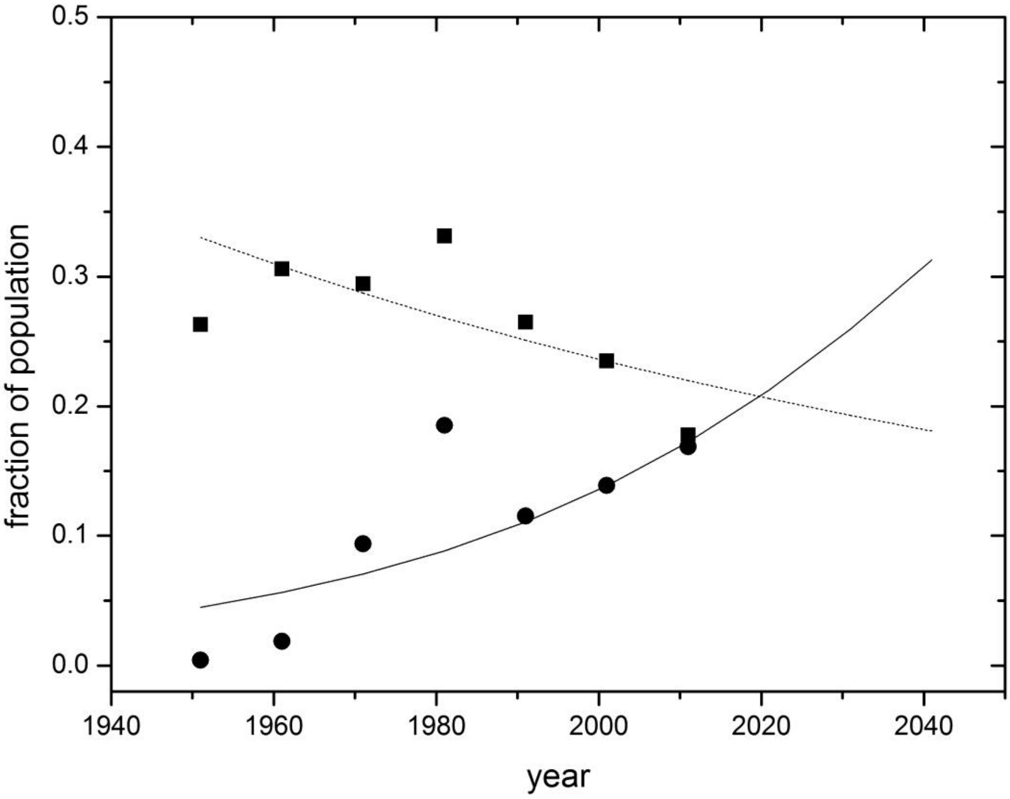

0.5
0.4
0.3
0.2
0.1
0.0
fraction of population
1940
1960
1980
2000
2020
2040
year

Figure Captions

Figure 1. Flow diagram for three state model of change in religious belief. Group X are religiously committed, group Z, religious, but uncommitted, and group Y religiously unaffiliated. We assume that members of the religious group (X,Z) only leave the group via group Z, and that if members of the religiously unaffiliated, Y, 'convert' to the religious group, they do so to the committed group X. Models 1 to 6 described in the paper consider various simple linear or constant forms of the rates of flow between the groups.

Figure 2. Results from NI and Irish Census returns 1861-2011 giving fraction of Northern Irish population declaring no religious affiliation.

Figure 3. Results of best fit analysis for model 3 to NI census data for religious nonaffiliation (circles) and religious commitment fraction f_1 (squares). Solid line non-affiliated, $y(t)$. Dashed line, religiously committed $x(t)$. Best fit parameters p=0.075, q=0.005, u= 0.050.

Figure 4. Results of best fit analysis for model 4 to NI census data for religious nonaffiliation (circles) and religious commitment fraction f_2 (squares). Solid line non-affiliated, $y(t)$. Dashed line, religiously committed $x(t)$. Best fit parameters p=0.090, q=0.000, u= 0.355.

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