Optimality and robustness of a biophysical decision-making model under norepinephrine modulation: Supplementary Materials

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1 The spiking neuron model

Here we describe the architecture and specify parameters of the computational model developed in [S1] and used in the present paper.

1.1 Integrate and fire dynamics

A leaky integrate-and-fire neuronal model is used to simulate the pyramidal cells and interneurons. Each model neuron has a resting membrane potential of $V_L = -70$ mV, a firing threshold of $V_{th} = -50$ mV, and a reset potential of $V_{reset} = -55$ mV. The membrane leak conductances for pyramidal cells and interneurons are $g_L = 25$ nS and 20 nS respectively, and the membrane time constants for pyramidal cells and interneurons are $\tau_m = 20$ ms and 10 ms, respectively. The corresponding membrane capacitances are given by $C_m = g_L/\tau_m$.

Below the firing threshold of each cell, the dynamical equation for its cellular membrane potential V_m is:

$$C_m \frac{dV_m(t)}{dt} = -g_L(V_m(t) - V_L) - I_{syn}(t),$$

where I_{syn} is the total synaptic input current, as described below. When threshold is reached ($V_m(t) = V_{th}$), the cell fires an action potential that is approximated by a Dirac delta function. V_m is then immediately reset to V_{reset} and clamped for an absolute refractory period of 2 ms for pyramidal cells and 1 ms for interneurons.

1.2 Synapses

The synaptic current I_{syn} entering each cell consists of both external (ext) inputs and recurrent (rec) synaptic connections. Recurrent inputs include both excitatory and inhibitory currents. Excitatory inputs are made up of AMPA and NMDA

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receptor-mediated (glutamatergic) synapses, while inhibitory inputs are $GABA_A$ receptor-mediated. In all, the input currents are as follows:

$$
\begin{aligned}
I_{syn}(t) &= I_{ext,AMPA}(t) + I_{rec,AMPA}(t) + I_{rec,NMDA}(t) + I_{rec,GABA}(t), \\
I_{ext,AMPA}(t) &= g_{ext,AMPA}(V(t) - V_E)S_{j,ext,AMPA}(t), \\
I_{rec,AMPA}(t) &= g_{rec,AMPA}(V(t) - V_E)\sum_{j=1}^{N_E} w_j S_{j,rec,AMPA}(t), \\
I_{rec,NMDA}(t) &= \frac{g_{rec,NMDA}(V(t) - V_E)}{(1 + [Mg^{2+}]exp(-0.062V(t))/3.57)}\sum_{j=1}^{N_E} w_j S_{j,NMDA}(t), \\
I_{rec,GABA}(t) &= g_{GABA}(V(t) - V_I)\sum_{j=1}^{N_I} S_{j,GABA}(t).
\end{aligned}
$$

Here, with appropriate subscripts, g denotes the peak synaptic conductance, S the synaptic gating variable (fraction of open channels), $V_E = 0$ the reversal potential of excitatory connectivity, and $V_I = -70$ mV the reversal potential for inhibitory synapses. The sum over j is a sum over presynaptic neurons. The voltage-dependent NMDA current is controlled by extracellular magnesium concentration $[Mg^{2+}]$, which is set at 1 mM. The peak conductances for excitatory synapses to pyramidal cells, in units of μS, are $g_{rec,AMPA} = 0.0005$, $g_{ext,AMPA} = 0.0021$, $g_{NMDA} = 0.000165$, and they are $g_{rec,AMPA} = 0.00004$, $g_{ext,AMPA} = 0.00162$, $g_{NMDA} = 0.00013$ to the interneurons. Peak conductances g_{GABA} for inhibitory synapses to pyramidal cells and interneurons are $0.0013\,\mu S$ and $0.001\,\mu S$ respectively. All peak conductance values are comparable to experimental measurements.

A dimensionless potentiation factor w is included to represent the structure of excitatory synapses. Those within the two selective populations are chosen to be relatively stronger ($w = w_+ = 1.7 > 1$) than excitatory synapses to cells outside the selective populations or across populations. To ensure that all excitatory neurons maintain the same spontaneous mean firing rate, compensation from unpotentiated synapses is required such that $w = w_- = 1 - f(w_+ - 1)/(1 - f) < 1$ for the synapses between two different selective populations, and for synapses between the nonselective population to selective ones [S2]. For all other connections, $w = 1$.

The dynamics of the synaptic gating variables are described by:

$$
\begin{aligned}
\frac{dS_{j,AMPA}(t)}{dt} &= -\frac{S_{j,AMPA}(t)}{\tau_{AMPA}} + \sum_k \delta(t - t_j^k), \\
\frac{dS_{j,GABA}(t)}{dt} &= -\frac{S_{j,GABA}(t)}{\tau_{GABA}} + \sum_k \delta(t - t_j^k), \\
\frac{dS_{j,NMDA}(t)}{dt} &= -\frac{S_{j,NMDA}(t)}{\tau_{NMDA,decay}} + \alpha x_j(t)(1 - S_{j,NMDA}(t)), \\
\frac{dx_j(t)}{dt} &= -\frac{x_j(t)}{\tau_{NMDA,rise}} + \sum_k \delta(t - t_j^k),
\end{aligned}
$$

where the summation of delta functions denotes the sum of presynaptic spikes. The time constants are $\tau_{AMPA} = 2$ ms, $\tau_{NMDA,decay} = 100$ ms, $\tau_{NMDA,rise} = 2$ ms, $\tau_{GABA} = 5$ ms, and $\alpha = 0.5$ ms^{-1}. The rise time for AMPA and GABA (< 1 ms) are assumed to be instantaneous.

Spikes from cells external to the local network, with a combined presynaptic mean firing rate $f_{ext} = 2.4$ kHz, and from upstream neurons (e.g. visual motion stimulus via MT/V5 cells), are assumed to go through AMPA receptors. Following [S1], we approximate the spike times t_j^k by drawing independent samples from Gaussian distributions with mean and variance

$$\begin{aligned}
\mu_{S,ext,AMPA} &= 0.001\, f_{ext}\, g_{ext,AMPA}\, \tau_{AMPA}, \\
\sigma_{S,ext,AMPA} &= \sqrt{0.5\, g_{ext,AMPA}\, \mu_{S,ext,AMPA}},
\end{aligned}$$

which are derived from the asymptotic values of the conductances. This approximation is adequate for high spike rates, but we are currently developing a method that applies over a broader range of firing rates. Finally, the external conductances $S_{j,ext,AMPA}(t)$ are calculated as

$$dS_{j,ext,AMPA}(t) = dt \frac{-S_{j,ext,AMPA} + \mu_{S,ext,AMPA}}{\tau_{AMPA}} + \sigma_{S,ext,AMPA}\sqrt{\frac{2dt}{\tau_{AMPA}}}\mathcal{N}(0,1),$$

where $\mathcal{N}(0,1)$ is a standard normal distribution. This is the source of variability and noise in the simulation.

1.3 Simulations

The firing rate of each neural population is computed by averaging the total number spikes over an exponentially decaying, sliding time window with a 20 ms decay time constant. This computation is repeated every 2 ms. The Euler-Maruyama method with fixed step size of 0.05 ms was used for numerical integration of all the dynamical equations. For each selected parameter set, the neural and behavioral data predicted by the model were averaged over 500 trials. Simulations were run on a Linux workstation.

# 2 Linear approximation of NE concentration in terms of tonic LC activity

Microdialysis measurements of NE concentrations in cortical cells resulting from a range of LC discharge rates [S3] allow us to quantify this relationship. Supplementary Fig. 1 shows that a linear fit is acceptable, as noted in the main text. However, as also noted there, data that would allow us to describe the relationship between NE levels and synaptic conductances are lacking. In the absence of such data we assume a linear relationship in that case also.

References

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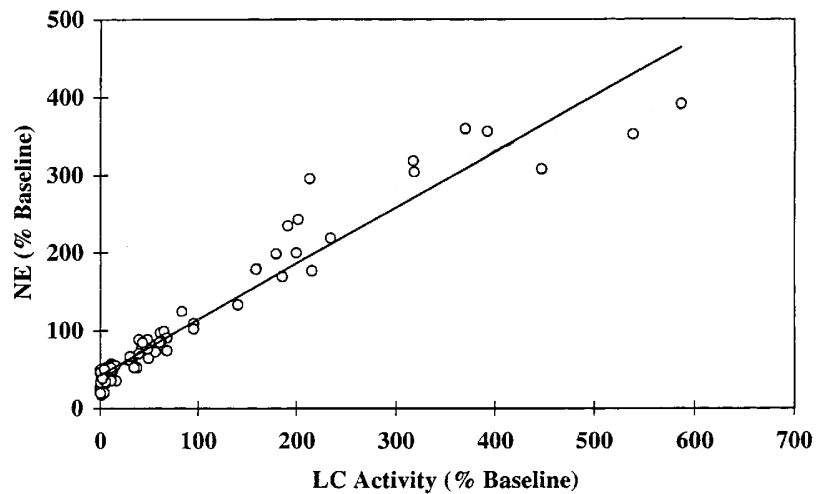
Supplementary Figure 1: Linear fit to experimental data relating NE concentration to tonic LC neuronal activity. Data adapted from [*S*3, Fig. 8].