# CORNER DETECTION ON HEXAGONAL PIXEL BASED IMAGES

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## ABSTRACT

*Corner detection is used in many computer vision applications that require fast and efficient feature matching. In addition, hexagonal pixel based images have been recently investigated for image capture and processing due to their ability to represent curved structures that are common in real images better than traditional rectangular pixel based images. Therefore, we present an approach to corner detection on hexagonal images and demonstrate that accuracy is comparable to well-known existing corner detectors applied to rectangular pixel based images.*

Keywords: Corner detection; Hexagonal images

## 1. INTRODUCTION

Traditionally, images are captured and displayed using rectangular pixels, and several corner detection algorithms have been developed for such images. Recently there has been an increased interest in using hexagonal pixels for image representation for many reasons, including their ability to better represent curved structures. Additional advantages of the hexagon image structure include both spatial and spectral advantages: equidistance of all pixel neighbours and improved spatial isotropy of spectral response. Pixel spatial equidistance facilitates the implementation of circular symmetric kernels that is associated with an increase in accuracy when detecting edges, both straight and curved [1, 2]. To date, work done using hexagonal images has focussed on the development of hexagonally structured architectures for image representation and addressing [3, 4, 5], and a theoretical framework for signal modelling and transforms [6]; however, much less research has been undertaken on the development and application of image processing techniques for direct use on such image structures. Some standard algorithms have been extended from rectangular to hexagonal arrays in simple cases and recently edge detection operators have been presented in [7, 8] but to date corner detection has not been investigated using hexagonal pixel based images.

Corner detectors can be classified into two main types: heuristic techniques [9] and gradient or curvature based techniques [10, 11, 12, 13]. A number of methods extract edges first and then determine corners as points of maximal curvature or search for points where edge segments intersect [14], leading to ambiguous structure of corner points. The Moravec operator [15] has an anisotropic response as the intensity variation is calculated in only eight principal directions. To overcome this limitation, a function is needed that allows intensity variation to be measured in any direction, and Harris and Stephens [16] expanded the Moravec operator to achieve this. Kitchen and Rosenfeld use a *corner measure*, based on the product of gradient direction change along an edge and local gradient magnitude [10]; Smith and Brady's SUSAN corner detector is based on brightness comparisons over neighbourhoods [17]. Discussions on other corner detectors may be found in [14]. However, none of these corner detectors may be readily applied directly to hexagonal pixel based images.

In this paper we extend the work in [8] to present procedure for the corner detection on hexagonal images. In Section 2, we present an overview of the hexagonal image representation followed by the hexagonal operator design in Section 3. Section 4 provides a brief description of corner detection with Section 5 presenting corner detection results in comparison with traditional rectangular based approaches.

## 2. HEXAGONAL IMAGE REPRESENTATION

An image is typically represented by an array of samples of a continuous function $u(x, y)$ of image intensity on a domain $\Omega$. Fig. 1 represents an image compiled of hexagonal pixels with nodes placed in the centre of each pixel. These nodes are the reference points for finite element computation throughout the domain $\Omega$, where the vertices of each triangular finite element are the pixel centres. The operator design is then based on the use of a triangular mesh, also illustrated in Fig. 1, consisting of equilateral triangular elements that overlay the hexagonal pixel array.

With any node, say node *i*, with co-ordinates $(x_i, y_i)$ we associate a piecewise linear basis function $\phi_i(x, y)$ which has the properties $\phi_i(x_j, y_j) = 1$ if *i=j* and $\phi_i(x_j, y_j) = 0$ if $i \neq j$ and $(x_j, y_j)$ are the co-ordinates of the nodal point *j*. $\phi_i(x, y)$ is thus a "tent-shaped" function with support restricted to a small neighbourhood centred on node $i$ consisting of only those elements that have node $i$ as a vertex. We then may approximately represent the image u over a neighbourhood $\Omega_i^\sigma$ by a function

$$U(x, y) = \sum_{j \in \Omega_i^\sigma} U_j \phi_j(x, y) \qquad (1)$$

in which the parameters $\{U_j\}$ are the sampled image intensity values, giving a piecewise linear representation on the neighbourhood $\Omega_i^\sigma$.

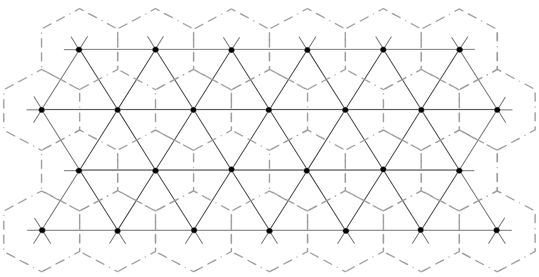


Figure 1. Hexagonal array of pixels and overlying triangular mesh

## 3. HEXAGONAL OPERATOR DESIGN

We formulate operators that correspond to weak forms of operators in the finite element method. Operators used for smoothing may be based simply on a weak form of the image function. In this case it is assumed that the image function $u \equiv u(x, y)$ belongs to the Hilbert space $H^0(\Omega)$; that is, the integral $\int u^2 d\Omega$ over $\Omega$ is finite.

Edge detection and enhancement operators are often based on first or second derivative approximations, for which it is necessary that the image function $u \equiv u(x, y)$ belongs to the Hilbert space $H^1(\Omega)$. We are currently concerned only with first order derivative operators and therefore to obtain a weak form of the first directional derivative $\partial u / \partial b \equiv \underline{b} \cdot \underline{\nabla} u$ the derivative term is multiplied by a test function $v \in H^1$, and the result is integrated on the image domain $\Omega$ to give

$$E(u) = \int_\Omega \underline{b} \cdot \underline{\nabla} u v d\Omega \qquad (2)$$

where $\underline{b} = (\cos\theta, \sin\theta)$ is the unit direction vector. This enables us to design our hexagonal operator using either a Cartesian coordinate system or the three axes of symmetry of the hexagon. Our current operator design uses the Cartesian coordinate system as the three axes of symmetry introduces redundancy.

In the finite element method a finite-dimensional subspace $S^h \subset H^1$ is used for function approximation; in our design procedure $S^h$ is defined by the virtual finite element mesh that overlays the hexagonal pixel structure as illustrated in Figure 1. Our general design procedure incorporates a finite-dimensional test space $T_\sigma^h \subset H^1$ that explicitly embodies a scale parameter σ, enabling the operators to be readily scaled. The test space $T_\sigma^h$ comprises a set of Gaussian basis functions $\psi_i^\sigma(x, y)$, $i$=1,…, $N$ of the form

$$\psi_i^\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right)} \qquad (3)$$

Each test function $\psi_i^\sigma(x, y)$ is restricted to have support over the neighbourhood $\Omega_i^\sigma$, centred on node $i$. In general the size of $\Omega_i^\sigma$ may be explicitly related to the scale parameter σ. The sets of test functions $\psi_i^\sigma(x, y)$, $i$=1, …, $N$, are then used in the weak forms of the first derivative in (2). In particular we note that the integrals need to be computed only over the neighbourhood $\Omega_i^\sigma$, rather than the entire image domain Ω, since $\psi_i^\sigma$ has support restricted to $\Omega_i^\sigma$. Hence the approximate image representation over $\Omega_i^\sigma$ may be used, providing the functional

$$E_i^\sigma(U) = \int_{\Omega_i^\sigma} \underline{b}_i \cdot \underline{\nabla} U \psi_i^\sigma d\Omega_i \qquad (4)$$

Using this design procedure in Section 3, we can develop operators of any size and in this paper we use the 7-point, 19-point and 37-point hexagonal operators which are equivalent to the $3 \times 3$, $5 \times 5$ and $7 \times 7$ standard rectangular operators. The hexagonal operators are therefore denoted as $H_3$, $H_5$ and $H_7$ throughout the reminder of the paper.

## 4. CORNER DETECTION

The hexagonal $x$- and $y$- directional derivative operators can now be applied for the purpose of corner detection using hexagonal pixel based images. As in [16], for each gradient operator, $X$ and $Y$, applied to image $I$, we calculate the following:

$$I_X = X \otimes I \qquad (5)$$

$$I_Y = Y \otimes I \qquad (6)$$

$$S_{I_X I_Y} = I_X I_Y \otimes \psi^{\sigma_s} \qquad (7)$$

$$S_{(I_X)^2} = (I_X)^2 \otimes \psi^{\sigma_s} \qquad (8)$$

$$S_{(I_Y)^2} = (I_Y)^2 \otimes \psi^{\sigma_s} \qquad (9)$$

In this formulation, $\psi^{\sigma_s}$ represents a post-smoothing Gaussian kernel that is considered to be a noise-suppressant in [16], but which Rockett [18] points out is fundamental to the operation of the detector in that it isotropically modifies the frequency spectra. The corner strength response is then calculated as in [16], using the cornerness measure

$$C = \left(S_{I_X^2} S_{I_Y^2} - \left(S_{I_X I_Y}\right)^2\right) - k\left(S_{I_X^2} + S_{I_Y^2}\right)^2 \qquad (10)$$

We choose the parameter $k = 0.04$, to be consistent with the analysis presented in [18].

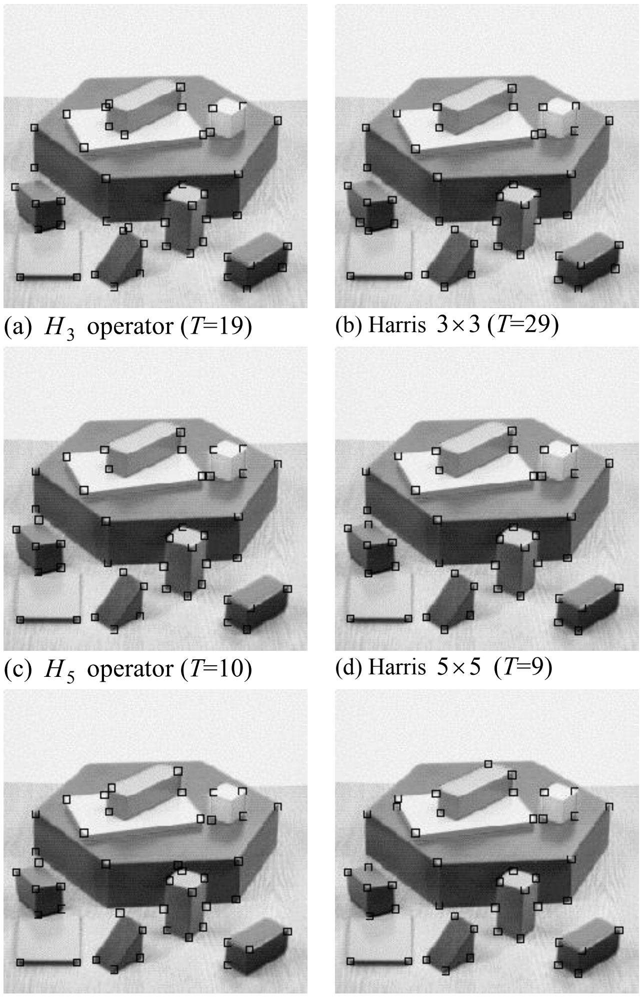
## 5. RESULTS

Currently, there are no hexagonal camera sensors commercially available and therefore hexagonal pixel-based images are created via resampling. The resampling technique that we use is the 56 sub-pixel approach in [19] which is based on the technique of Wuthrich [20]. In order to create a sub-pixel effect to enable the sub-pixel clustering, each pixel of the original rectangular pixel based image is represented by a $7 \times 7$ pixel block of equal intensity in the new image. This creates a resized image of the same resolution as the original image with the ability to display each pixel as a group of $n \times n$ sub pixels and limits the loss of image resolution. Another motivation for image resizing is to enable the display of sub pixels, which is not otherwise possible. With this structure now in place, a cluster of 56 sub pixels in the new image, closely representing the shape of a hexagon, can be created that represents a single hexagonal pixel in the resized image. Using this approach to create hexagonal pixel based images, we can compare the proposed technique with the use of the well known Harris algorithm applied to rectangular pixel based images.

For comparison, the visual responses from each of the two corner detectors are illustrated in Figure 2 and Figure 3; the block image contains 64 ground truth corners and the bricks image contains 8; in each case the threshold value $T$ is selected as the visual best. Summaries of the corner points detected by each method as illustrated in Figure 2 and Figure 3 are provided in Table 1 and Table 2 respectively and illustrates that the hexagonal operator detects a similar number of true corners to the Harris corner detection methods.

| Corner Detector | # True detected corners | # False detected corners |
|---|---|---|
| $H_3$ operator | 45 | 2 |
| $H_5$ operator | 45 | 0 |
| $H_7$ operator | 44 | 1 |
| Harris $3 \times 3$ | 44 | 1 |
| Harris $5 \times 5$ | 44 | 0 |
| Harris $7 \times 7$ | 44 | 1 |

Table 1: Corner point detection rates for blocks image



(a) $H_3$ operator ($T$=19) (b) Harris $3 \times 3$ ($T$=29)

(c) $H_5$ operator ($T$=10) (d) Harris $5 \times 5$ ($T$=9)

(e) $H_7$ operator ($T$=11) (f) Harris $7 \times 7$ ($T$=8)

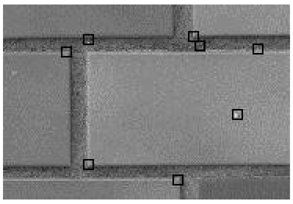
Figure 2. Illustration of detected corners for various techniques using the blocks image

| Corner Detector | # True detected corners | # False detected corners |
|---|---|---|
| $H_3$ operator | 4 | 4 |
| $H_5$ operator | 7 | 1 |
| $H_7$ operator | 7 | 2 |
| Harris $3 \times 3$ | 5 | 3 |
| Harris $5 \times 5$ | 7 | 3 |
| Harris $7 \times 7$ | 7 | 1 |

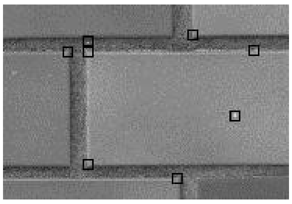
Table 2: Corner point detection rates for bricks image

## 6. SUMMARY

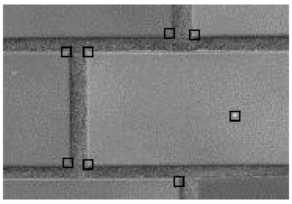
Detection of corners and general points of interest plays an important role in computer vision, particularly with respect to real-time vision.



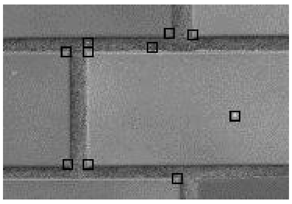
(a) $H_3$ operator ($T$=340)



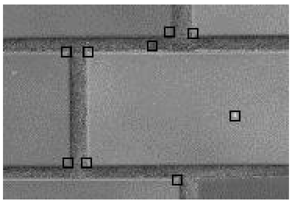
(b) Harris $3 \times 3$ ($T$=133)



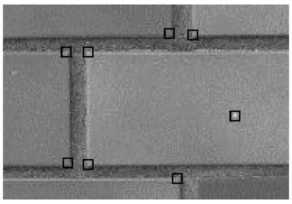
(c) $H_5$ operator ($T$=324)



(d) Harris $5 \times 5$ ($T$=198)



(e) $H_7$ operator ($T$=104)



(f) Harris $7 \times 7$ ($T$=238)

Figure 3. Illustration of detected corners for various techniques using the bricks image

We have presented an approach to corner detection on hexagonal pixel based images and demonstrated that it is comparable with the existing commonly used Harris corner detector applied to rectangular pixel based images. As well as both the spatial and spectral advantages of using hexagonal pixel based digital images, there is a significant computational gain. Hexagonal pixel-based images contain 13% less pixels than a standard rectangular pixel-based image and in addition the hexagonal operators designed on a Cartesian axis contain less operator values than the corresponding square operators, thus generating an overall significant reduction in computation. For example, for a given $256 \times 256$ image, removing boundary pixels, 63504 pixels will be processed. Using a $5 \times 5$ operator there will be $63504 \times 25$ multiplications totalling 1587600. If the same image is re-sampled on to a hexagonal based image there will be 55566 pixels processed by an equivalent hexagonal gradient operator containing only 19 values. Therefore there will be only 1055754 multiplications; this is only 66.5% of the computation required for corner detection as an equivalent traditional square pixel-based image. Similar computational gain it achieved in the post-smoothing stage using an hexagonal Gaussian filter..

## 7. REFERENCES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