

A Very Low Complexity Successive Symbol-by-Symbol Sequence Estimator for Faster-than-Nyquist Signaling

Ebrahim Bedeer, Mohamed Hossam Ahmed, and Halim Yanikomeroglu

Abstract—In this paper, we investigate the sequence estimation problem of binary and quadrature phase shift keying faster-than-Nyquist (FTN) signaling and propose two novel low-complexity sequence estimation techniques based on concepts of successive interference cancellation. To the best of our knowledge, this is the first approach in the literature to detect FTN signaling on a symbol-by-symbol basis. In particular, based on the structure of the self-interference inherited in FTN signaling, we first find the operating region boundary—defined by the root-raised cosine (rRC) pulse shape, its roll-off factor, and the time acceleration parameter of the FTN signaling—where perfect estimation of the transmit data symbols on a symbol-by-symbol basis is guaranteed, assuming noise-free transmission. For noisy transmission, we then propose a novel low-complexity technique that works within the operating region and is capable of estimating the transmit data symbols on a symbol-by-symbol basis. To reduce the error propagation of the proposed successive symbol-by-symbol sequence estimator (SSSSE), we propose a successive symbol-by-symbol with go-back- K sequence estimator (SSSgb K SE) that goes back to re-estimate up to K symbols, and subsequently improves the estimation accuracy of the current data symbol. Simulation results show that the proposed sequence estimation techniques perform well for low intersymbol interference (ISI) scenarios and can significantly increase the data rate and spectral efficiency. Additionally, results reveal that choosing the value of K as low as 2 or 3 data symbols is sufficient to significantly improve the bit-error-rate performance. Results also show that the performance of the proposed SSSgb K SE, with $K = 1$ or 2, surpasses the performance of the lowest complexity equalizers reported in the literature, with reduced computational complexity.

Index Terms—Faster-than-Nyquist (FTN) signaling, intersymbol interference (ISI), Mazo limit, self-interference, sequence estimation, successive interference cancellation

I. INTRODUCTION

There is a need to design better spectral efficient digital communication systems, as data rate requirements are conservatively doubling each year. The term Faster-than-Nyquist (FTN) signaling was coined in late 60s and early 70s [1]–[3] to refer to digital transmission of pulses beyond the Nyquist limit. The pioneering work of J. E. Mazo in 1975 [4] was the first to prove that FTN signaling does not affect the minimum distance of binary sinc pulses when transmitted at rate $\frac{1}{\tau}$, $\tau \in [0.802, 1]$, higher than the Nyquist signaling; this is called the Mazo limit. In other words, Mazo proved that almost 25% more

bits, compared to the Nyquist signaling, can be transmitted in the same bandwidth and at the same signal-to-noise ratio (SNR) without degrading the bit error rate (BER), assuming perfect processing at the receiver.

Despite the doubts raised by Foschini on the benefits of FTN signaling in [5], its potential to improve the transmission rates was revealed in [6], [7]. Although, implementations of FTN signaling in practical communication systems pose several challenges mainly due to the high complexity involved to remove the self-interference. In [8], the binary FTN signaling was viewed as a convolutionally encoded transmission and a Viterbi algorithm (VA) was used for detection. To reduce the complexity of the FTN signaling detection problem in [8], truncated VA [9] and reduced states Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [10] are investigated; yet, the works in [9], [10] are still complex and more effective for strong ISI scenarios. For low ISI scenarios, a frequency domain equalizer (FDE) has been proposed in [11] to detect FTN signaling with reasonable complexity. However, due to the insertion of a guard interval, e.g. cyclic prefix, at the transmitter, the effective transmission rate is reduced depending on the relative length of the cyclic prefix and the transmitted data block. For instance, for a data block transmission of 1000 symbols and a cyclic prefix of 10 symbols, the effective transmission rate is reduced by 1%. The work in [11] was extended in [12] to provide soft decisions about the data symbols using FDE-aided three-stage concatenated turbo decoder. The works in [11], [12] were extended to produce soft-decision of the estimated data symbols while considering the correlated noise samples after the receiver matched filter in [13]. In [14], the authors proposed an iterative block decision feedback frequency domain equalizer in addition to a hybrid automatic repeat request to detect FTN signaling with reduced computational complexity.

Other important aspects of FTN signaling have been recently studied as well. For instance, the authors in [15] studied the tradeoff between increasing the spectral efficiency (SE) of FTN signaling, as a result of the accelerated pulses' transmission in time, and increasing the FTN signaling peak to average power ratio. In [16], the effectiveness of multicarrier FTN signaling is investigated for coexistence scenarios. In particular, it was shown that multicarrier FTN signaling can compensate for the loss in SE due to using guard bands between different systems.

To the best of the authors' knowledge, this is the first approach in the literature to detect FTN signaling on a symbol-by-symbol basis. This is in contrast to all the previous sequence estimation techniques reported in the literature that estimate the transmit data symbols based on the reception of a block of length N [8]–[14]. The main contributions of this paper are summarized as follows:

This work is supported in part by DragonWave Inc., in part by the Mathematics of Information Technology and Complex Systems (MITACS) Canada, and in part by Natural Science and Engineering Research Council of Canada (NSERC) through Discovery program.

E. Bedeer and H. Yanikomeroglu are with the System and Computer Engineering Department, Carleton University, Ottawa, ON K1S 5B6, Canada (e-mails: {e.bedeer, halim.yanikomeroglu}@sce.carleton.ca).

M. H. Ahmed is with the Faculty of Engineering and Applied Science, Memorial University, St. Johns, NL, A1B 3X5, Canada (e-mail: mhahmed@mun.ca).

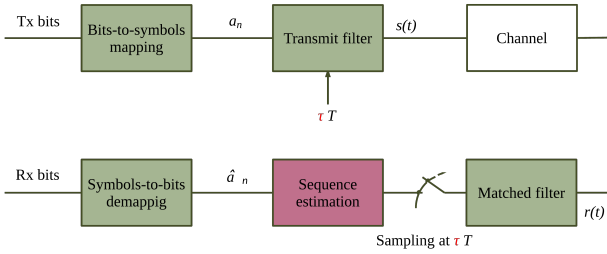


Fig. 1: Block diagram of FTN signaling.

- We exploit the structure of the self-interference inherited in binary and quadrature phase shift keying (BPSK and QPSK) FTN signaling to find the operating region boundary where perfect data symbols estimation on a symbol-by-symbol basis is guaranteed, assuming noise-free transmission. The operating region boundary is defined by the root-raised cosine (rRC) pulse shape, its roll-off factor β , and the time acceleration parameter τ of the FTN signaling.
- For noisy transmission, we propose a successive symbol-by-symbol sequence estimator (SSSSE) that works within the operating region and is capable of estimating the transmit data symbols in a low-complexity manner.
- We additionally propose a successive symbol-by-symbol with go-back- K sequence estimator (SSSgb K SE) to reduce the error propagation effect of the proposed SSSSE. The proposed SSSgb K SE can go back to re-estimate up to K data symbols (based on the knowledge of the current data symbol and its previous $K-1$ data symbols), and subsequently improves the estimation accuracy of the current data symbol (based on the re-estimation of the previous K data symbols).
- Simulation results show the effectiveness of the proposed SSSSE and SSSgb K SE techniques for low ISI scenarios to considerably increase the data rate and SE. Additionally, results reveal that for low ISI scenarios choosing the value of K as low as 2 or 3 data symbols is sufficient to significantly improve the BER performance. Results also show that the proposed SSSgb K SE, with $K = 1$ or 2, outperforms the lowest complexity equalization techniques reported in the open literature, with reduced computational complexity.

The remainder of this paper is organized as follows. Section II presents the system model of the FTN signaling. The proposed SSSSE is discussed in Section III, while the proposed SSSgb K SE is introduced in Section IV. Section V provides the performance results of our proposed sequence estimation techniques, and finally the paper is concluded in Section VI

II. FTN SIGNALING SYSTEM MODEL

Fig. 1 shows a block diagram of a communication system employing FTN signaling. Data bits to be transmitted are gray mapped¹ to data symbols through the bits-to-symbols mapping

¹It is worthy to mention that there may exist other bits-to-symbol mapping schemes that result in better performance of the FTN signaling; however, investigating such schemes are out of the scope of this paper.

block. Data symbols are transmitted, through the rRC transmit filter block, faster than Nyquist signaling, i.e., every τT , where $0 < \tau \leq 1$ is the time packing/acceleration parameter and T is the symbol duration. A possible receiver structure is shown in Fig. 1, where the received signal is passed through a filter matched to the rRC transmit filter followed by a sampler. Since the transmission rate of the transmit pulses carrying the data symbols intentionally violate the Nyquist criterion, ISI occurs between the received samples. Accordingly, sequence estimation techniques are needed to remove the ISI and to estimate the transmitted data symbols. The estimated data symbols are finally gray demapped to the estimated received bits.

The transmitted signal $s(t)$ of the FTN signaling shown in Fig. 1 can be written in the form

$$s(t) = \sqrt{E_s} \sum_{n=1}^N a_n p(t - n\tau T), \quad 0 < \tau \leq 1, \quad (1)$$

where N is the total number of transmit data symbols, a_n , $n = 1, \dots, N$, is the independent and identically distributed data symbols, E_s is the data symbol energy, $p(t)$ is a unit-energy pulse, i.e., $\int_{-\infty}^{\infty} |p(t)|^2 dt = 1$, and $1/(\tau T)$ is the signaling rate. The received FTN signal in case of additive white Gaussian noise (AWGN) channel is written as

$$y(t) = s(t) + n(t), \quad (2)$$

where $n(t)$ is a zero mean complex valued Gaussian random variable with variance σ^2 . A possible receiver architecture for FTN signaling is to use a filter matched to $p(t)$; thus the received signal after the matched filter can be written as

$$y(t) = \sqrt{E_s} \sum_{n=1}^N a_n g(t - n\tau T) + w(t), \quad (3)$$

where $g(t) = \int p(x)p(x-t)dx$ and $w(t) = \int n(x)p(x-t)dx$. Assuming perfect timing synchronization between the transmitter and the receiver, the received FTN signal $y(t)$ is sampled every τT and the k th received sample can be expressed as

$$\begin{aligned} y_k &= y(k\tau T) \\ &= \sqrt{E_s} \sum_{n=1}^N a_n g(k\tau T - n\tau T) + w(k\tau T) \\ &= \underbrace{\sqrt{E_s} a_k g(0)}_{\text{desired symbol}} + \underbrace{\sqrt{E_s} \sum_{n=1, n \neq k}^N a_n g((k-n)\tau T)}_{\text{ISI from adjacent symbols}} + w(k\tau T). \end{aligned} \quad (4)$$

The optimal detector of the FTN signaling in (4) in terms of minimizing the block-error-rate is the maximum likelihood sequence estimation; however, its NP-hard computational complexity is prohibitive for practical implementations. In the following, we discuss and propose very low complexity symbol-by-symbol sequence estimators for BPSK and QPSK FTN signaling.

III. SUCCESSIVE SYMBOL-BY-SYMBOL SEQUENCE ESTIMATION (SSSSE)

As discussed earlier, all the previous FTN signaling sequence estimation techniques reported in the literature estimate

$$\mathbf{y} = \mathbf{G} \mathbf{a}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_k \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} G_{1,1} & G_{1,2} & G_{1,3} & \dots & G_{1,L} & 0 & 0 & 0 & 0 \\ G_{1,2} & G_{1,1} & G_{1,2} & G_{1,3} & \dots & G_{1,L} & 0 & 0 & 0 \\ G_{1,3} & G_{1,2} & G_{1,1} & G_{1,2} & G_{1,3} & \dots & G_{1,L} & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \dots & \dots & G_{1,3} & G_{1,2} & G_{1,1} & G_{1,2} & G_{1,3} & \dots & \dots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & G_{1,L} & \dots & G_{1,3} & G_{1,2} & G_{1,1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_k \\ \vdots \\ a_N \end{bmatrix}, \quad (5)$$

the transmit data symbols based on the reception of a block of length N [8]–[14]. In this section, we propose a novel SSSSE that estimates the transmit data symbols on a symbol-by-symbol basis.

The key enabler behind the proposed SSSSE is an observation about an operation region of BPSK and QPSK FTN signaling, where perfect estimation of data symbols on a symbol-by-symbol basis is guaranteed for noise-free transmission. The boundary of this operation region is defined by the rRC pulse shape, its roll-off factor β , and the time acceleration parameter τ . To explain the basic idea of the observation that lead to the proposed SSSSE, let us rewrite (4) in a vector form for noise-free transmission as where \mathbf{G} is the ISI matrix, $G_{n,n'} = g((n - n')\tau T)$ represents the ISI between data symbols n and n' , and $L - 1$ is the length of the one-sided ISI. As can be seen in (5), each received sample value, i.e., y_k , is affected by ISI in the form of an accumulated weighted sum of up to $2L - 2$ adjacent data symbols. This ISI depends on the rRC pulse shape, its roll-off factor β , and the time acceleration parameter τ of the FTN signaling. Following (5), the k th received sample is expressed as

$$y_k = \underbrace{G_{1,L} a_{k-L+1} + \dots + G_{1,2} a_{k-1}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1} a_k}_{\text{Current symbol to be estimated}} + \underbrace{G_{1,2} a_{k+1} + \dots + G_{1,L} a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}}. \quad (6)$$

Hence, to detect the k th transmit symbol a_k from the k th received sample y_k , we need to remove the interference from the previously detected $L - 1$ data symbols $a_{k-1}, \dots, a_{k-L+1}$ and from the upcoming $L - 1$ (and yet undetected) data symbols $a_{k+1}, \dots, a_{k+L-1}$. That said, the main challenge is how to handle the interference from the upcoming $L - 1$ data symbols, while still estimating the current data symbol a_k based on a symbol-by-symbol basis. In the following, we explain how to handle the interference from the upcoming $L - 1$ data symbols for the case of BPSK and QPSK FTN signaling. The same idea can be extended to higher constellations as well.

A. Binary Phase Shift Keying FTN Signaling

For BPSK FTN signaling, the perfect reconstruction condition is outlined in **Lemma 1** below.

Lemma 1. *Perfect estimation condition for BPSK FTN signaling for noise-free transmission.*

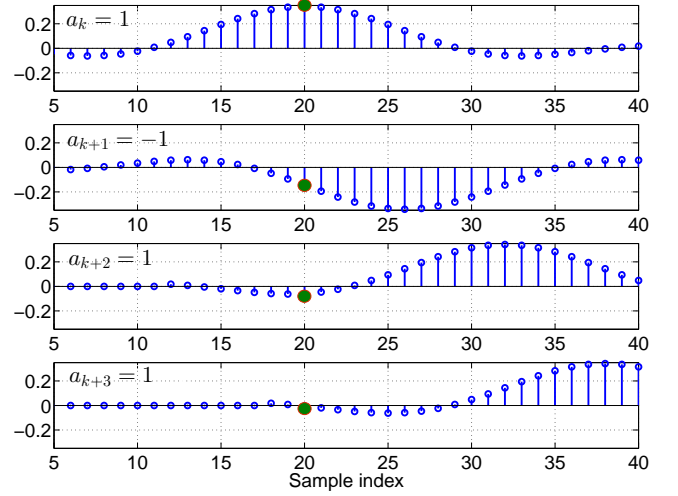


Fig. 2: Explanation of the basic idea of the proposed SSSSE.

Regardless the value of the current data symbol a_k , the upcoming $L - 1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$, and the value of L , the following inequality holds for a certain range of τ and β :

$$|G_{1,1} a_k| > |G_{1,2} a_{k+1} + \dots + G_{1,L} a_{k+L-1}|. \quad (7)$$

Proof: the range of τ and β that satisfies the perfect estimation condition in (7) can be found by a simple numerical search on the upcoming $L - 1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$ that will result in the worst ISI contribution to the current data symbol a_k as follows. The worst ISI scenario for the estimation of a_k occurs when the upcoming $L - 1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$ are chosen such that $G_{1,2} a_{k+1}, \dots, G_{1,L} a_{k+L-1}$ has an opposite sign to $G_{1,1} a_k$. In other words, based on the signs of $G_{1,2}, \dots, G_{1,L}$, the data symbols $a_{k+1}, \dots, a_{k+L-1}$ are chosen such that the ISI to the k th received sample is maximized. This can be illustrated with the help of Fig. 2, where the k th transmit data symbol a_k is affected by the interference from the upcoming $L - 1 = 3$ transmit data symbols. Without loss of generality, we assume that the k transmit data symbol a_k in Fig. 2 (a) is $+1$. Given that the signs of $G_{1,2}, G_{1,3}$, and $G_{1,4}$ are positive, negative, and negative respectively, then the worst interference affecting a_k will occur when $a_{k+1} = -1$, $a_{k+2} = 1$, and $a_{k+3} = 1$ such that $G_{1,2} a_k$, $G_{1,3} a_{k+1}$, and $G_{1,4} a_{k+2}$ are all negative values

(i.e., opposite to a_k), and hence, the interference to the k th data symbol, i.e., $G_{1,2} a_k + G_{1,3} a_{k+1} + G_{1,4} a_{k+2}$, is maximized. On the other hand, if the k th transmit data symbol a_k is -1 , then we choose $a_{k+1} = 1$, $a_{k+2} = -1$, and $a_{k+3} = -1$ such that $G_{1,2} a_k$, $G_{1,3} a_{k+1}$, and $G_{1,4} a_{k+2}$ have all positive values (i.e., opposite to a_k), and hence, the interference to the k th data symbol, i.e., $G_{1,2} a_k + G_{1,3} a_{k+1} + G_{1,4} a_{k+2}$, is maximized. ■

B. Quadrature Phase Shift Keying FTN Signaling

Similar to the discussion of the BPSK FTN signaling, the perfect estimation condition for QPSK FTN signaling is outlined in **Lemma 2** below.

Lemma 2. *Perfect estimation condition for QPSK FTN signaling for noise-free transmission.*

Regardless the value of the current data symbol a_k , the upcoming $L-1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$, and the value of L , the following inequality holds for a certain range of τ and β :

$$|G_{1,1} \Re\{a_k\}| > |G_{1,2} \Re\{a_{k+1}\} + \dots + G_{1,L} \Re\{a_{k+L-1}\}|, \quad (8)$$

$$|G_{1,1} \Im\{a_k\}| > |G_{1,2} \Im\{a_{k+1}\} + \dots + G_{1,L} \Im\{a_{k+L-1}\}|, \quad (9)$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary parts of a complex number.

Proof: **Lemma 2** can be proved similar to **Lemma 1**; hence, the proof is omitted to avoid unnecessary repetitions. ■

C. Finding the Operation Region Boundary

To find the range of β and τ such that the perfect estimation conditions in **Lemma 1** and **Lemma 2** hold, and hence, perfect estimation of data symbols on symbol-by-symbol basis is guaranteed for noise-free transmission, we perform the following offline search. For BPSK FTN signaling and for a certain value of β and τ and the ISI samples between adjacent symbols, i.e. $G_{1,1}, \dots, G_{1,L}$, we select the upcoming $L-1$ symbols $a_{k+1}, \dots, a_{k+L-1}$ according to the signs of $G_{1,2}, \dots, G_{1,L}$, respectively. For instance, for the case of $a_k = +1$, the upcoming $L-1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$ are selected to be of opposite sign to $G_{1,2}, \dots, G_{1,L}$, respectively. On the other hand, for the case of $a_k = -1$, the upcoming $L-1$ data symbols $a_{k+1}, \dots, a_{k+L-1}$ are selected to be of the same sign to $G_{1,2}, \dots, G_{1,L}$, respectively. We note that $G_{1,2} a_{k+1} + \dots + G_{1,L} a_{k+L-1}$ represents the worst ISI that can affect the k th transmit data symbol a_k . Then, we gradually decrease the value of τ until **Lemma 1** and **Lemma 2** are violated. We follow similar approach for the case of QPSK FTN signaling to find the value of τ . This is formally expressed as follows:

Algorithm 1: Finding the Operation Region Boundary

- 1) **Input:** rRC pulse shape and its roll-off factor β .
- 2) Initialize the value of $\tau = 1$.
- 3) Calculate the values of $G_{1,1}, \dots, G_{1,L}$.
- 4) Select $a_{k+1}, \dots, a_{k+L-1}$ to have the same signs as $G_{1,1}, \dots, G_{1,L}$, respectively.
- 5) Decrease the value of τ until the perfect estimation conditions in **Lemma 1** and **Lemma 2** are violated.

TABLE I: Operating region boundary and the SE bound.

β	τ	SE bound (bits/sec/Hz)	SE bound (bits/sec/Hz)
		BPSK FTN	QPSK FTN
0	0.68	1.47	2.94
0.1	0.63	1.44	2.89
0.2	0.59	1.41	2.82
0.3	0.49	1.57	3.14
0.4	0.47	1.52	3.03
0.5	0.45	1.48	2.96
0.6	0.43	1.45	2.90
0.7	0.41	1.43	2.87
0.8	0.39	1.42	2.85
0.9	0.37	1.42	2.85
1	0.35	1.43	2.86

6) Output: Final value of τ .

Following *Algorithm 1*, the operation region and the SE bound, where perfect data symbols estimation on a symbol-by-symbol basis is guaranteed for noise-free transmission, are summarized in Table I. It is worthy to emphasize that the operating region is found for the noise-free transmission scenario. It is expected that the proposed schemes working in a noisy transmission will give unsatisfactory performance if the operating parameters τ and β are selected to be at the edge of the operating region. This is as the noise can easily violate the perfect reconstruction conditions and move the proposed schemes operation outside the operating region. As expected, the operation region boundaries match for both BPSK and QPSK FTN signaling. For the reader's convenience, the operation region and the SE bound of QPSK FTN signaling is plotted in Fig. 3.

Under these operating conditions, the k th data symbol a_k can be estimated on a symbol-by-symbol basis as follows. First, the contribution from the previous $L-1$ data symbols are subtracted from the k th received sample y_k . Then, the k data symbol a_k is estimated in the presence of the ISI from the upcoming $L-1$ data symbols and thermal noise as

$$\hat{a}_k = \text{quantize} \{y_k - (G_{1,L} \hat{a}_{k-L+1} + \dots + G_{1,2} \hat{a}_{k-1})\}, \quad (10)$$

where $\text{quantize}\{x\}$ rounds x to the nearest BPSK/QPSK symbol².

D. Proposed SSSSE and Complexity Analysis

The proposed SSSSE can formally be expressed as follows: *Algorithm 2:* Proposed SSSSE

- 1) **Input:** rRC pulses shape, its roll-off factor β , and the time acceleration parameter τ .
- 2) **If** β and τ belong to the operation region shown in Table I **then**
- 3) \hat{a}_k is given as in (10)
- 4) **End**

When compared to Nyquist signaling, the proposed SSSSE requires to subtract the effect of the ISI of the previous $L-$

²It is worthy to mention that the proposed schemes provide only hard decisions about the data symbols. One possible way to provide soft decisions about the data symbols is to approximate the ISI (from previous and upcoming symbols) as a zero-mean Gaussian process with a given variance according to the ISI term in (4). Then assume that the received samples are affected by zero-mean Gaussian process with variance equals to the sum of noise and ISI variances.

$$\hat{a}_k = \text{quantize} \left\{ y_k - \underbrace{\left(G_{1,L} \hat{a}_{k-L+1} + \dots + \underbrace{G_{1,K+1} \hat{a}_{k-K} + \dots + G_{1,2} \hat{a}_{k-1}}_{\text{ISI from previous } K \text{ data symbols with improved estimation accuracy}} \right)}_{\text{ISI from the previous } L-1 \text{ data symbols}} \right\}. \quad (13)$$

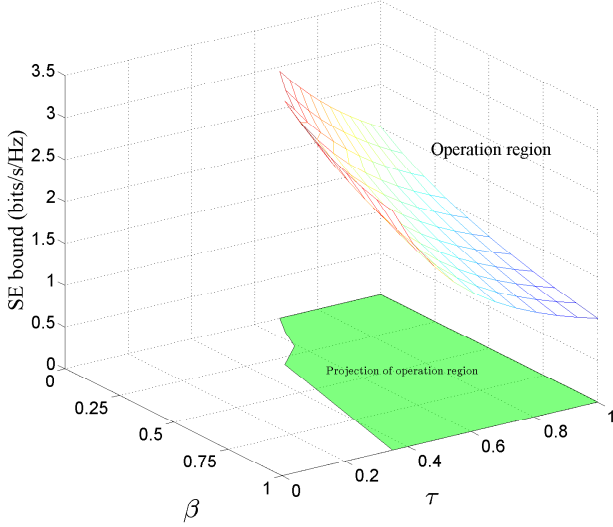


Fig. 3: Operation region and SE bound of **Lemma 2**, where perfect data symbols estimation on a symbol-by-symbol basis is guaranteed for noise-free transmission.

1 data symbols, as can be seen in (10). This translates to additional $L-2$ additions and $L-1$ multiplications operations, when compared to the Nyquist signaling detection.

IV. SUCCESSIVE SYMBOL-BY-SYMBOL WITH GO-BACK- K SEQUENCE ESTIMATION (SSSGB K SE)

The proposed SSSSE suffers from the error propagation effect, i.e., an incorrectly estimated data symbol will affect the estimation accuracy of all the upcoming data symbols, with the strongest effect being on the adjacent data symbols. To address this problem, in this section we introduce the SSSgb K SE to reduce the error propagation effect of the proposed SSSSE, and hence, to improve its estimation accuracy.

The basic idea of the proposed SSSgb K SE can be explained as follows. Upon the estimation of the current data symbol a_k , one can go back and improve the estimation accuracy of the previous K data symbols a_{k-1}, \dots, a_{k-K} based on the knowledge of the current data symbol a_k . Subsequently, we can re-estimate the current k th data symbol based on the improved estimation of the previous K data symbols a_{k-1}, \dots, a_{k-K} . To explain the proposed SSSgb K SE in more

details, let us rewrite the received k th sample value y_k as

$$y_k = \underbrace{G_{1,L} a_{k-L+1} + \dots + G_{1,K+1} a_{k-K} + \dots + G_{1,2} a_{k-1}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1} a_k}_{\text{Current symbol to be re-estimated}} + \underbrace{G_{1,2} a_{k+1} + \dots + G_{1,L} a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}}. \quad (11)$$

Hence, the improved re-estimation of the $(k-K)$ th data symbol can be written as

$$\hat{a}_{k-K} = \text{quantize} \left\{ y_{k-K} - \underbrace{\left(G_{1,L} \hat{a}_{k-K-L+1} + \dots + G_{1,2} \hat{a}_{k-K-1} \right)}_{\text{ISI from previous } L-1 \text{ data symbols of the } (k-K) \text{th data symbol}} - \underbrace{\left(G_{1,2} \hat{a}_{k-K+1} + \dots + G_{1,K+1} \hat{a}_k \right)}_{\text{ISI from upcoming } K \text{ data symbols of the } (k-K) \text{th data symbol}} \right\}. \quad (12)$$

Similarly, the $k-1, k-2, \dots, k-K+1$ data symbols are re-estimated to improve their estimation accuracy. Accordingly, the k th data symbol can be re-estimated as

A. Proposed SSSgb K SE and Complexity Analysis

The proposed SSSgb K SE is formally expressed as follows:
Algorithm 3: Proposed SSSgb K SE

- 1) **Input:** rRC pulses shape, its roll-off factor β , the time acceleration parameter τ , and K .
- 2) **If** β and τ belong to the operation region shown in Table I **then**
- 3) \hat{a}_k is given as in (13)
- 4) **End**

As discussed earlier, the key idea of the proposed SSSgb K SE is to re-estimate up to K previous data symbols to improve the estimation accuracy of the current k th data symbol. The computational complexity of the proposed SSSgb K SE can be analyzed as follows:

- To re-estimate the $(k-1)$ th data symbol, we need $L-2$ additions and $L-1$ multiplications operations to remove the ISI from the previous $L-1$ data symbols; this is similar to the complexity of the proposed SSSSE. Additionally, 1 multiplication operation is required to subtract the ISI from the upcoming k th data symbol.
- To re-estimate the $(k-2)$ th data symbol, we need $L-2$ additions and $L-1$ multiplications operations to remove the ISI from the previous $L-1$ data symbols in addition to 1 addition and 2 multiplication operations to subtract the ISI from the upcoming $(k-1)$ th and k th data symbols.

TABLE II: Computational complexity of the SSSSE and SSSgbKSE algorithms.

Algorithm	addition operations No.	multiplication operations No.
SSSSE	$L - 2$	$L - 1$
SSSgbKSE	$K(L - 2) + \frac{K(K-1)}{2}$	$K(L - 1) + \frac{K(K+1)}{2}$

- The complexity of re-estimating up to previous K data symbol can be done in the same manner. For instance, to re-estimate the $(k - K)$ th data symbol we need $L - 2$ additions and $L - 1$ multiplications operations to remove the ISI from the previous $L - 1$ data symbols in addition to $K - 1$ additions and K multiplications operations.

Hence, the required number of iterations for the proposed SSSgbKSE can be summarized as $K(L-2)+1+2+\dots+(K-1)$ additions and $K(L-1)+1+2+\dots+K$ multiplications operations. The computational complexities of the proposed SSSSE and SSSgbKSE algorithms are summarized in Table II.

The works in [11], [13] require a complexity of $\mathcal{O}(N)$ to calculate the MMSE coefficients of the FDE. This is in addition to a complexity of $\mathcal{O}(N \log(N))$ to perform the FFT and iFFT. Hence, the complexity of the FDEs in [11], [13] equals $\mathcal{O}(N) + \mathcal{O}(N \log(N)) = \mathcal{O}(N \log(N))$, i.e., the complexity is dominated by the FFT and iFFT operations. Such complexity is much higher than its counterparts of the proposed SSSSE and SSSgbKSE algorithms.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed SSSSE and SSSgbKSE in estimating transmit data symbols of FTN signaling. We employ a rRC filter with roll-off factors $\beta = 0.3$ and 0.5 (unless otherwise mentioned), and we consider the data symbols to be drawn from the constellation of QPSK. We consider a transmission of data block of length $N = 1000$ data symbols and a cyclic prefix of length $\nu = 10$ symbols when simulating the works in [11], [13]. Hence, there is a loss of 1% of the SE of the works in [11], [13] and it is considered in our simulations to have a fair comparison with the proposed SSSSE and SSSgbKSE schemes. The SE is calculated as $\frac{\log_2 M}{(1+\beta)\tau} \frac{N-\nu}{N}$, where M is the constellation size.

Fig. 4 depicts the BER of QPSK FTN signaling as a function of $\frac{E_b}{N_0}$ for the proposed SSSSE, SSSgbKSE for $K = 1, 2, 3$, and the FDEs in [11], [13] for $\beta = 0.3$ and a SE of 1.71 bits/sec/Hz. This means that the value of τ used for our proposed SSSSE and SSSgbKSE is 0.9 and its value for the works in [11], [13] is set to 0.891. As can be seen in Fig. 4, increasing the value of K improves the BER performance of the proposed SSSgbKSE as expected. Additionally, going back up to $K = 3$ data symbols approaches the optimal performance of the Nyquist ISI-free transmission for the given β and SE values. Fig. 4 reveals that the proposed SSSgbKSE can achieve $\frac{1.71-1.54}{1.54} = 11\%$ increase in the transmission rate without increasing the BER, the bandwidth, or the symbol energy, when compared to the Nyquist signaling (i.e., no ISI case). Additionally, Fig. 4 shows the the proposed SSSgbKSE with $K = 1$ and $K = 2$ outperforms the works in [11] and [13], respectively.

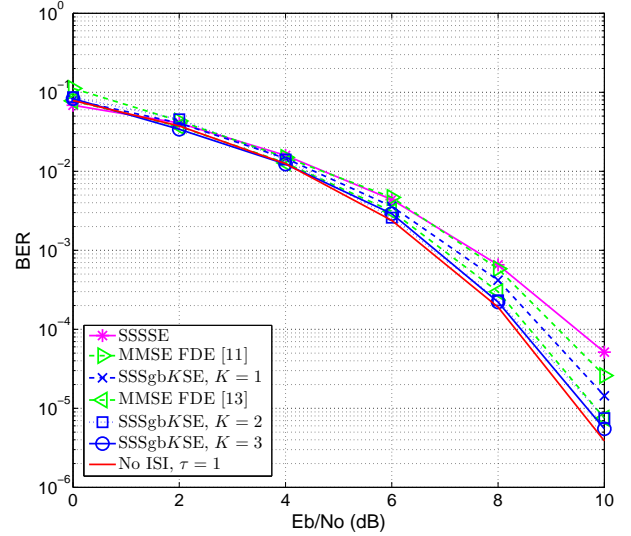


Fig. 4: BER performance of QPSK FTN sequence estimation as a function of $\frac{E_b}{N_0}$ using the proposed SSSSE, proposed SSSgbKSE, and FDEs in [11], [13] at $\beta = 0.3$ and SE of 1.71 bits/sec/Hz.

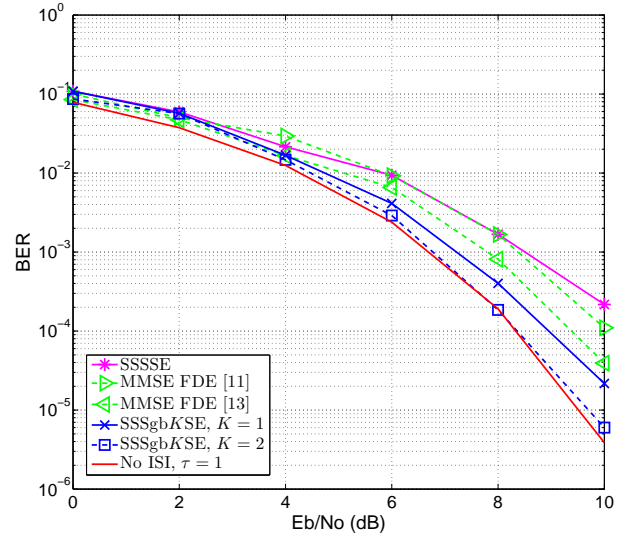


Fig. 5: BER performance of QPSK FTN sequence estimation as a function of $\frac{E_b}{N_0}$ using the proposed SSSSE, proposed SSSgbKSE, and FDEs in [11], [13] at $\beta = 0.5$ and spectral efficiency bound of 1.67 bits/sec/Hz.

Fig. 5 plots the BER of QPSK FTN as a function of $\frac{E_b}{N_0}$ for the proposed SSSSE, SSSgbKSE for $K = 1, 2$, and SDSE for $\beta = 0.5$ and a SE of 1.67 bits/sec/Hz. This means that the value of τ used for our proposed SSSSE and SSSgbKSE is 0.8 and its value for the works in [11], [13] is set to 0.792. Similar to the previous discussion on Fig. 4, going back for $K = 2$ data symbols at $\beta = 0.5$ and a SE of 1.67 bits/sec/Hz is enough to approach the performance of the Nyquist ISI-free transmission. One can infer from Fig. 5 that the proposed SSSgbKSE can achieve 25% increase in the transmission rate without deteriorating the BER or

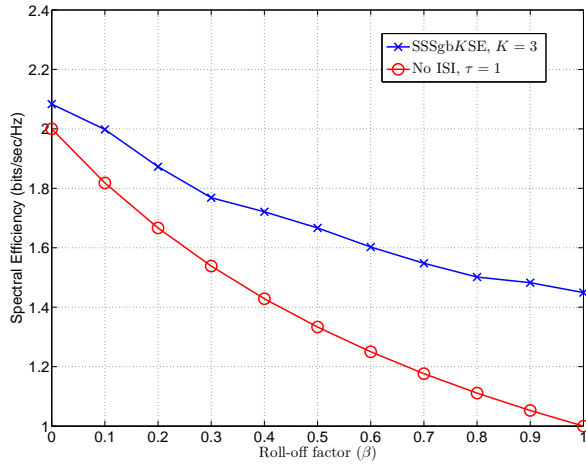


Fig. 6: Spectral efficiency of QPSK Nyquist and FTN signaling as a function of β using the proposed SSSgbKSE at BER = 10^{-4} .

increasing the bandwidth or the SNR, when compared to the Nyquist signaling. Additionally, the performance of the proposed SSSgbKSE with $K = 2$ surpasses the performance of the works in [11], [13].

Fig. 6 plots the SE of QPSK Nyquist (i.e., no ISI and $\tau = 1$) and FTN signaling as a function of the roll-off factor β at the same SNR and BER = 10^{-4} . In order to have a fair comparison, the value of τ of the FTN signaling is selected to be the smallest value such that the proposed SSSgbKSE with $K = 3$ achieves the same BER = 10^{-4} of Nyquist signaling at the same SNR. As can be seen, the SE of FTN signaling is higher than its counterpart of Nyquist signaling for all values of β . For instance, at $\beta = 0$ and 0.3 the proposed SSSgbKSE improves the SE by 4% and 20.55%, respectively, for the same BER and SNR values, when compared to Nyquist signaling. One can also infer from Fig. 6 that the FTN signaling exploits the excess bandwidth of the rRC pulse as the gain in SE of the proposed SSSgbKSE increases for increasing the value of β . Additionally, results revealed that the proposed SSSgbKSE can achieve SE higher than the maximum SE of Nyquist signaling (2 bit/s/Hz achieved at $\beta = 0$) for the range of $\beta \in [0, 0.1]$.

VI. CONCLUSION

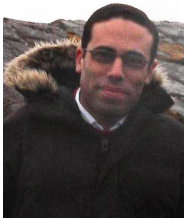
FTN signaling is a promising non-orthogonal transmission technique capable of significantly increasing the spectral efficiency, when compared to the conventional Nyquist signaling. This paper presents the first attempt in the literature to detect FTN signaling on a symbol-by-symbol basis, i.e., with very low computational complexity. We proposed two novel sequence estimation techniques, namely, SSSSE and SSSgbKSE, to estimate the transmit data symbols of BPSK and QPSK FTN signaling. In particular, based on concepts of successive interference cancellation we found the operating region boundary (defined by the rRC pulse shape, its roll-off factor, and the time acceleration parameter), where the proposed SSSSE and SSSgbKSE can perfectly estimate the transmit data symbols for noise-free transmission.

For noisy transmission, the proposed SSSSE estimates the data symbols on a symbol-by-symbol basis, with a significant reduction in the computational complexity when compared to all the sequence estimation techniques from the literature. To overcome the error propagation effect in the SSSSE, the proposed SSSgbKSE can go-back to re-estimate up to K data symbols, based on the knowledge of the current data symbol, and accordingly improves the estimation accuracy of the current data symbol based on the re-estimation of the previous K data symbols. The proposed schemes are of low complexity. More specifically, the proposed SSSSE requires additional $L - 2$ additions and $L - 1$ multiplications operations when compared to Nyquist signaling; while the proposed SSSgbKSE requires additional $K(L - 2) + \frac{K(K-1)}{2}$ additions and $K(L - 1) + \frac{K(K+1)}{2}$ multiplications operations. Simulation results showed that the proposed techniques are suitable for low ISI scenarios and can achieve up to 11.11% and 25% increase in the transmission rate at $\beta = 0.3$ and 0.5, respectively, and up to 4% and 20.55% improvement in the spectral efficiency at $\beta = 0$ and 0.3, respectively, when compared to Nyquist signaling, for the same SNR and BER. We showed that for low ISI scenarios choosing $K = 2$ or 3 data symbols is sufficient to improve the BER performance. Additionally, results revealed that the proposed SSSgbKSE can achieve spectral efficiency higher than the maximum spectral efficiency of QPSK Nyquist signaling (2 bit/s/Hz achieved at $\beta = 0$) for the range of $\beta \in [0, 0.1]$.

REFERENCES

- [1] B. Saltzberg, "Intersymbol interference error bounds with application to ideal bandlimited signaling," *IEEE Trans. Inf. Theory*, vol. 14, no. 4, pp. 563–568, Jul. 1968.
- [2] R. Lucky, "Decision feedback and faster-than-Nyquist transmission," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Jun. 1970, pp. 15–19.
- [3] J. Salz, "Optimum mean-square decision feedback equalization," *Bell Syst. Tech. J.*, vol. 52, no. 8, pp. 1341–1373, Oct. 1973.
- [4] J. Mazo, "Faster-than-Nyquist signaling," *Bell Syst. Tech. J.*, vol. 54, no. 8, pp. 1451–1462, Oct. 1975.
- [5] G. J. Foschini, "Contrasting performance of faster binary signaling with QAM," *Bell Syst. Tech. J.*, vol. 63, no. 8, pp. 1419–1445, Oct. 1984.
- [6] F. Rusek and J. B. Anderson, "On information rates for faster than Nyquist signaling," in *Proc. IEEE Global Communication Conference (GLOBECOM)*, Dec. 2006, pp. 1–5.
- [7] —, "Constrained capacities for faster-than-Nyquist signaling," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 764–775, Feb. 2009.
- [8] A. D. Liveris and C. N. Georghiades, "Exploiting faster-than-Nyquist signaling," *IEEE Trans. Commun.*, vol. 51, no. 9, pp. 1502–1511, Sep. 2003.
- [9] A. Prlja, J. B. Anderson, and F. Rusek, "Receivers for faster-than-Nyquist signaling with and without turbo equalization," in *Proc. IEEE International Symposium on Information Theory*, Jul. 2008, pp. 464–468.
- [10] J. B. Anderson, A. Prlja, and F. Rusek, "New reduced state space BCJR algorithms for the ISI channel," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Jun. 2009, pp. 889–893.
- [11] S. Sugiura, "Frequency-domain equalization of faster-than-Nyquist signaling," *IEEE Wireless Commun. Lett.*, vol. 2, no. 5, pp. 555–558, Oct. 2013.
- [12] S. Sugiura and L. Hanzo, "Frequency-domain-equalization-aided iterative detection of faster-than-Nyquist signaling," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 2122–2128, May 2015.
- [13] T. Ishihara and S. Sugiura, "Frequency-domain equalization aided iterative detection of faster-than-Nyquist signaling with noise whitening," in *Proc. IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–6.

- [14] R. Dinis, B. Cunha, F. Ganhao, L. Bernardo, R. Oliveira, and P. Pinto, "A Hybrid ARQ Scheme for Faster than Nyquist Signaling with Iterative Frequency-Domain Detection," in *Proc. IEEE Vehicular Technology Conference (VTC Spring)*, 2015, pp. 1–5.
- [15] J.-A. Lucciardi, N. Thomas, M.-L. Boucheret, C. Poulliat, and G. Mesnager, "Trade-off between spectral efficiency increase and PAPR reduction when using FTN signaling: Impact of non linearities," in *Proc. IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–7.
- [16] C. Le, M. Fuhrwerk, M. Schellmann, and J. Peissig, "Faster than nyquist—an enabler for achieving maximum spectral efficiency in co-existence scenarios?" in *Proc. European Signal Processing Conference (EUSIPCO)*, Aug. 2015, pp. 2142–2146.



Ebrahim Bedeer (S'10, M'14) received the B.Sc. (Hons.) and M.Sc. degrees from Tanta University, Tanta, Egypt and the Ph.D. degree from Memorial University, St. Johns, NL, Canada, all in Electrical Engineering. He was a postdoctoral fellow at University of British Columbia (UBC), BC, Canada. Currently, he is a postdoctoral fellow at Carleton University, Ottawa, ON, Canada. His general research interests are in the area of wireless communications and signal processing, with a current focus of resource management in wireless networks and

applications of optimization in signal processing.

Dr. Bedeer is an Editor of the IEEE COMMUNICATIONS LETTERS. He has served on the Technical Program Committees of numerous major international communication conferences, such as IEEE GLOBECOM, IEEE ICC, IEEE VTC, and CROWNCOM. Dr. Bedeer receives numerous awards including the Exemplary Reviewer of IEEE COMMUNICATIONS LETTERS and IEEE WIRELESS COMMUNICATIONS LETTERS.



Mohamed H. Ahmed obtained his Ph.D. degree in Electrical Engineering in 2001 from Carleton University, Ottawa, where he worked from 2001 to 2003 as a senior research associate. In 2003, he joined the Faculty of Engineering and Applied Science, Memorial University where he works currently as a Full Professor. Dr. Ahmed published more than 135 papers in international journals and conferences. He serves as an Editor for *IEEE Communication Surveys and Tutorials* and as an Associate Editor for *Wiley International Journal of Communication*

Systems and *Wiley Communication and Mobile Computing (WCMC)*. He served as a Guest Editor of a special issue on Fairness of Radio Resource Allocation, *EURASIP JWCN* in 2009, and as a Guest Editor of a special issue on *Radio Resource Management in Wireless Internet*, *Wiley Wireless and Mobile Computing Journal*, 2003. Dr. Ahmed is a *Senior Member* of the IEEE. He served as a cochair of the Signal Processing Track in ISSPIT'14 and served as a cochair of the Transmission Technologies Track in *VTC'10-Fall*, and the multimedia and signal processing symposium in *CCECE'09*. Dr. Ahmed won the Ontario Graduate Scholarship for Science and Technology in 1997, the Ontario Graduate Scholarship in 1998, 1999, and 2000, and the Communication and Information Technology Ontario (CITO) graduate award in 2000. His research interests include radio resource management in wireless networks, multi-hop relaying, cooperative communication, vehicular ad-hoc networks, cognitive radio networks, and wireless sensor networks. Dr. Ahmed's research is sponsored by NSERC, CFI, QNRF, Bell/Aliant and other governmental and industrial agencies. Dr. Ahmed is a registered Professional Engineer (P.Eng.) in the province of Newfoundland, Canada.



Halim Yanikomeroglu was born in Giresun, Turkey, in 1968. He received the B.Sc. degree in electrical and electronics engineering from the Middle East Technical University, Ankara, Turkey, in 1990, and the M.A.Sc. degree in electrical engineering (now ECE) and the Ph.D. degree in electrical and computer engineering from the University of Toronto, Canada, in 1992 and 1998, respectively.

During 1993-1994, he was with the R&D Group of Marconi Kominikasyon A.S., Ankara, Turkey. Since 1998 he has been with the Department of Systems and Computer Engineering at Carleton University, Ottawa, Canada, where he is now a Full Professor. His research interests cover many aspects of wireless technologies with a special emphasis on cellular networks. In recent years, his research has been funded by Huawei, Telus, Allen Vanguard, Blackberry, Samsung, Communications Research Centre of Canada (CRC), and DragonWave. This collaborative research resulted in about 25 patents.

Dr. Yanikomeroglu is a Fellow of the IEEE. He is a Distinguished Lecturer for the IEEE Communications Society and a Distinguished Speaker for the IEEE Vehicular Technology Society in 5G wireless technologies. He has been involved in the organization of the IEEE Wireless Communications and Networking Conference (WCNC) from its inception in 1998 in various capacities including serving as a Steering Committee member, Executive Committee member and the Technical Program Chair or Co-Chair of WCNC 2004 (Atlanta), WCNC 2008 (Las Vegas), and WCNC 2014 (Istanbul). He was the General Co-Chair of the IEEE 72nd Vehicular Technology Conference (VTC2010-Fall) held in Ottawa and is currently serving as the General Co-Chair of the IEEE 86th Vehicular Technology Conference (VTC2017-Fall) to be held in Toronto. He has served in the editorial boards of the IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, and IEEE Communications Surveys & Tutorials. He was the Chair of the IEEE's Technical Committee on Personal Communications (now called Wireless Technical Committee).

Dr. Yanikomeroglu is a recipient of the IEEE Ottawa Section Outstanding Educator Award in 2014, Carleton University Faculty Graduate Mentoring Award in 2010, the Carleton University Graduate Students Association Excellence Award in Graduate Teaching in 2010, and the Carleton University Research Achievement Award in 2009. Dr. Yanikomeroglu spent the 2011-2012 academic year at TOBB University of Economics and Technology, Ankara, Turkey, as a Visiting Professor. He is a registered Professional Engineer in the province of Ontario, Canada.