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COMBINING THE LEARNING CURVE CONCEPT
WITH ECONOMIC LOT SIZING

Abstract

Simple concepts familiar to most operations management students are frequently not integrated as a result of the complexity generated by their combination. This expository note demonstrates a method for combining the economic lot size concept with the learning curve and using a simple computer algorithm for solution purposes. It avoids the traditional trade-off of reality and accuracy for expediency.

Introduction

Traditionally model-builders have faced a serious no-win situation in their attempts to model reality. Reality involves so many variables that it is impossible to consider them all without reaching an intolerable level of complexity. To avoid making a model unworkably large, basic assumptions are usually made to simplify the working equations. The resulting dilemma is that the model-builder may have an extremely difficult model that is more realistic, and hence more accurate, or a simplified model that is less realistic.

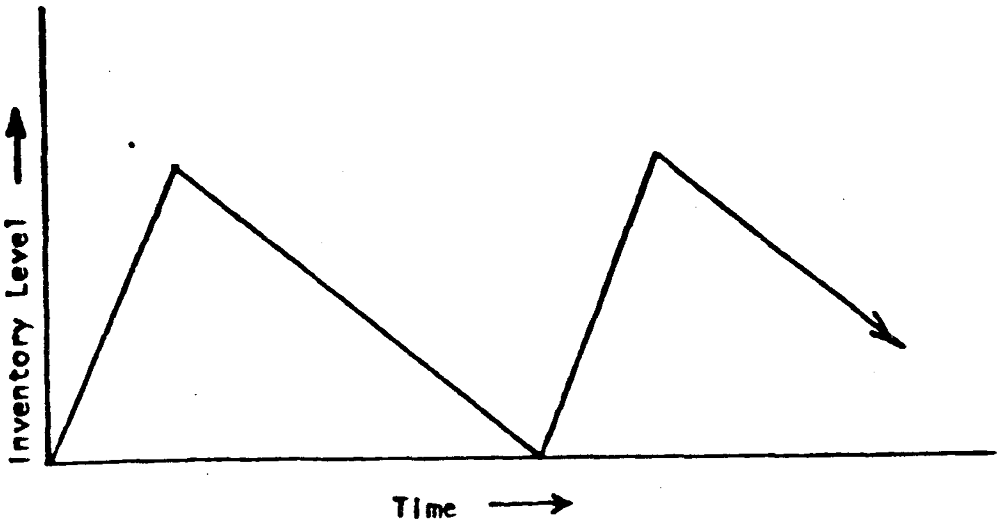
Computer programming offers an appealing answer to this dilemma by providing the ability to solve the more complicated, realistic models with relative ease. This is illustrated by relaxing the constant production rate assumption in the economic lot size (ELS) model and solving the resulting model using a BASIC program.

The ELS Model

One of the simplest approaches to the ELS problem is a model suggested by Buffa [1]. A graphical illustration of this model appears in Figure 1.

FIGURE 1

The ELS Model



The assumptions of the model include: (a) a constant rate of production, (b) a constant rate of sales or usage, and (c) simultaneous production and usage.

In order to construct the basic mathematical model, let:

D = Annual demand (in units)
U = Usage (or sales) per day
P = Production per day
H = Holding cost as a percent of unit cost
S = Setup cost per production run
C = Cost per unit
TC = Total cost per year
TMC = Total manufacturing cost per year
THC = Total holding cost per year
TSC = Total setup cost per year
X = Optimal number of units per production run

Now:

$$TC = TSC + THC + TMC$$

Substituting as appropriate:

$$TC = DS/X + XHC/2(1-U/P) + DC \quad (1)$$

In order to find the number of units per production run that will minimize total costs we set the first derivative with respect to X equal to zero, as:

$$TC' = -DS/X^2 + HC/2(1-U/P) = 0 \quad (2)$$

Solving for X yields the well known ELS formula:

$$X = [2DS/HC(1-U/P)]^{1/2} \quad (3)$$

Checking the second derivative:

$$TC'' = 2DS/X^3 \quad (4)$$

TC" will be positive for positive values of D, S and X, thus verifying X in equation 3 is a minimum.

Learning

The basic ELS model carries the underlying assumption of a constant production rate which ignores the learning phenomenon. In reality the assumption of a constant production rate is rarely, if ever, satisfied. It is both logically and intuitively reasonable that as a worker repeats a certain task, he becomes more proficient in the performance of that task. Over a period of time, the worker's production rate increases.

According to Hein [2] the learning curve takes on the form of an exponential curve, shown below modified for our use:

Let:

$\bar{L}$ = Average labor input time per unit as a percent of the labor input time to produce the first unit

T = Variable component of the time to produce the first unit

X = Number of units in the lot

b = A constant factor representing the rate of learning

a = Fixed component of the time to produce any unit

Now:

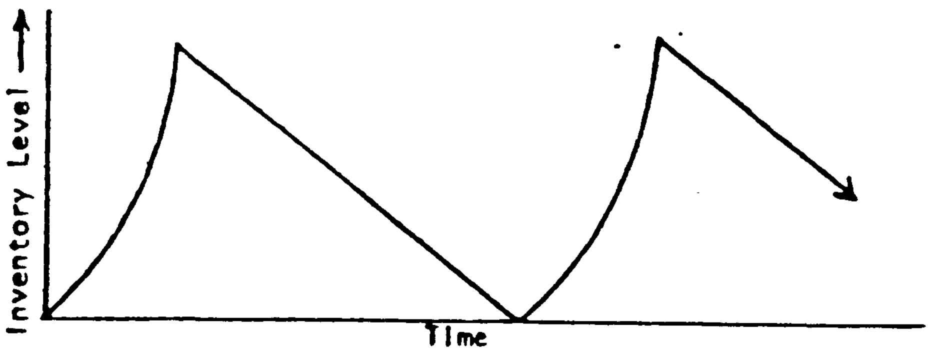
$$\bar{L} = a + TX^{-b}$$

Abernathy and Baloff [3] point out that the values of b found in practice have been in the range of $0 \leq b \leq 1$.

Combining the Concepts

The assumption of a constant production rate in the ELS model is relaxed by including the learning curve concept. A graphical illustration of the resulting model appears in Figure 2.

FIGURE 2
The ELS Model with Learning



This graph infers that the worker returns to his original production rate after each production run. Since the work entails psychomotorability skills, there may be some "dislearning" but probably not a great deal. The model in this paper does not deal with the "dislearning" phenomena, but it is acknowledged as an important concept to be reckoned with.

Remember:

P = Production per day without learning

$\bar{L}$ = Average labor input time per unit as a percent of the labor input time to produce the first unit

Now let:

L = Labor input time per unit without learning

N = Number of labor input units available for the production period

P_L = Production per day under learning

So:

$P = N/L$ and $P_L = N/\bar{L}$

The ratio of the production rate without learning to the production rate with learning is:

$$\frac{P}{P_L} = \frac{N/L}{N/\bar{L}} = \frac{\bar{L}}{L} \qquad \text{or} \qquad P_L = P(L/\bar{L})$$

If:

$\bar{I}$ = Average inventory without learning

and $\bar{I}_L$ = Average inventory under learning

Then:

$$\bar{I} = X/2(1-U/P)$$

and $\bar{I}_L = X/2\ [1-U/(P(L/\bar{L}))]$ or $X/2(1-U\bar{L}/PL)$

So:

$$\bar{I}_L = X/2\ [1-U(a+TX^{-b})/PL] \qquad (5)$$

The cost per unit is also affected by the changes in production rate.

Remember:

C = Cost per unit

Now let:

C_{UL} = Cost per unit of labor input

C_{VAR} = Variable costs

C_F = Fixed costs

Without learning:

$$C = C_{VAR} + C_F = C_{UL}L + C_F$$

Under learning:

$$C = C_{VAR} + C_F = C_{UL}\bar{L} + C_F = C_{UL}(a+TX^{-b}) + C_F$$

Recalling the basic model:

$$TC = TSC + THC + TMC$$

For our new model including the learning phenomenon:

$$TC = DS/X + HX/2[C_{UL}(a+TX^{-b})+C_F][1-U(a+TX^{-b})/PL] + D[C_{UL}(a+TX^{-b})+C_F]$$

Rewriting this equation:

$$TC = DSX^{-1} + HC_{UL}TX^{1-b}/2 + aHC_{UL}X/2 + HC_{F}X/2 - HC_{UL}UT^{2}X^{1-2b}/2PL$$

$$- aHC_{UL}UTX^{1-b}/2PL - aHUTX^{1-b}/2PL - a^{2}HC_{UL}UX/2PL - HC_{F}UTX^{1-b}/2PL$$

$$- aHC_{F}UX/2PL + DC_{UL}TX^{-b} + aDC_{UL} + DC_{F} \quad (6)$$

Taking the first derivative with respect to X:

$$TC' = -DSX^{-2} + (1-b)HC_{UL}TX^{-b}/2 + aHC_{UL}/2 + HC_{F}/2 - (1-2b)HC_{UL}UT^{2}X^{-2b}/2PL$$

$$- (a-ab)HC_{UL}UTX^{-b}/2PL - (a-ab)HUTX^{-b}/2PL - a^{2}HC_{UL}U/2PL$$

$$- (1-b)HC_{F}UTX^{-b}/2PL - aHC_{F}U/2PL - bDC_{UL}TX^{-1-b} \quad (7)$$

The second derivative with respect to X is:

$$TC'' = 2DSX^{-3} + (b^{2}-b)HC_{UL}TX^{-1-b}/2 + (2b-4b^{2})HC_{UL}UT^{2}X^{-1-2b}/2PL$$

$$+ (ab-ab^{2})HC_{UL}UTX^{-1-b}/2PL + (ab-ab^{2})HUTX^{-1-b}/2PL$$

$$+ (b-b^{2})HC_{F}UTX^{-1-b}/2PL + (b+b^{2})DC_{UL}TX^{-2-b} \quad (8)$$

A quick comparison of equations 1 through 4 from the model without learning and equations 6 through 8 from the model under learning reveals a greatly increased level of complexity for the new model. In fact equation 7, the derivative of TC with respect to X, can no longer be solved explicitly for X when set equal to zero. It is easy to see why the constant production rate assumption in the basic ELS model is so popular.

The Solution

A simple computer program written in VAX 2.2 BASIC was used to solve for X when TC' = 0. A recursive technique was employed; starting with some initial X and allowing an incremental value I to be added or subtracted from X in successive steps to drive the value of TC' to zero. If TC' changes sign or the new value of TC generated is greater than the old value of TC, the incremental step I is reduced to one tenth of its original value and the program continues. In this way TC' is driven to zero while simultaneously reducing the value of TC, thus insuring a local minimum has been found. This is checked by the sign of the second derivative.

Conclusion

It should be noted that when b = 0 the effects of the learning curve on the complex model are negated, and the larger the value of b the more the learning phenomenon will change the optimal solution. Where learning is significant, as in our sample run with b = .4, the cost savings of incorporating it into the ELS model can also be significant. For the values in our sample run:

Without Learning	Under Learning
X = 2001 units	X = 1559 units
TC = $409,994	TC = $401,658

By relaxing the constant production rate assumption of the ELS model for this case a savings of $8336 or 2% was realized.

Computer programming opens a new frontier to the model-builder. Now it is no longer necessary to trade off reality and accuracy for expedience. Where real world variables are significant and can be measured or readily approximated, it may be to the model-builder's advantage to relax the assumptions of the model and incorporate these variables.

References

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