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Common Grounds as Multiple Information States

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The main goal of this paper is to show how to derive a common ground. In order to do this, first we need to know what a common ground should be like. In traditional dynamic semantics, a common ground is given as a single information state which is shared by the people in the context. In this paper I am going to show that a common ground must include more than one information state, and that this is necessary to deal with conversational implicatures, which require distinction between speakers and audience. In this sense, the new common ground will be the basis for a conversational model of dynamic semantics. The traditional dynamic semantics is not appropriate for conversations, but rather a model for text processing.

1. MOTIVATION FOR MULTIPLE INFORMATION STATES

The possibility operator is taken to be a test of an information state, and thus it does not actually increase information. When a common ground is updated with a statement $[may\,.\phi]$, the statement just tests the common ground only as to whether it allows for the possibility that ϕ. If it passes the test, the common ground remains as it is. If it doesn't, it collapses into the absurd state. Given the semantics of *may* as a test, the following dialogue does not lead to the absurd state.

(1) A: John is ill.
C: ??John may be ill.
(2) A: ??John is ill. And he may be ill.

When John is ill, it is also true that John may be ill. These two examples nevertheless sound odd. The reason is that the second sentences are not informative at all with respect to the common ground after the first sentence is uttered. Since informativeness is checked with respect to the common ground, it does not make any difference who makes the utterance. We can see this condition as the one on the **dynamic** side of information.

There are, however, cases where a speaker satisfies the informativeness condition, but still the dialogue is odd. If we change the order of the two sentences in (2), it still sounds odd. Let's consider the following examples:

(3) A(lice): John may need help.
C(hris): Actually he does. (or 'Actually he doesn't.')
(4) C: ??John may need help. Actually he does.
C: ??John may need help. Actually he doesn't.'

In the first dialogue, A's statement implies that she does not know for sure whether or not John needs help. On the other hand, C knows that he actually needs help (or does not). Therefore, C's statement increases information in the common ground. Compare this with the next two monologues, where the two sentences are uttered by the same speaker. The sequences of the sentences are awkward. In this context, the first sentence implies that the speaker does not know for sure whether John needs help, but he continues with the statement which indicates that the speaker knows for sure that he actually does (or does not). When considering only the dynamic side of information, these sequences of statements should be fine since the second statement increases information. The reason for the awkwardness is that there is conflict between conditions for appropriate uses of expressions. The first statement reveals that the speaker's information state does not tell whether John needs help. This information state does not change while he continues other statements in a sequence. However his second statement tells that the speaker already knows that John does (or does not), which indicates that the speaker violated the maxim of quantity (or the maxim of quality) in uttering the first sentence.

A similar observation can be made in the following dialogues:

(5) A: John may not have stolen the car.
C: Actually he did.

(6) A: ??John may not have stolen the car. Actually he did.

The first dialogue is fine since the two sentences are consistent and the second statement is informative with respect to its immediate common ground. This can be compared with the second example, however. If the second sentence is stated truthfully in an information state, it turns out that the first sentence is false in that information state. A told a lie: A violated the maxim of quality.

This kind of observation is not limited to possibility sentences. Let's look at a conditional sentence.

(7) a. ??If Mary is at home, Susan is out. Actually Susan is at home.
b. ??If Mary is at home, Susan is out. Actually Susan is out.

When the speaker knows that the second sentence of (7a) is true, the first sentence of (7a) is false in that information state. In (7b), if the second sentence is true, the first sentence is too weak for the speaker to satisfy the maxim of quantity. These two examples would be fine if the two sentences were uttered by two different speakers. Finally we can add disjunction to this kind of observation:

(8) ??John is the owner or the manager of the apartment. Actually he is (not) the owner.

This example, where ϕ *or* ψ is followed by (*not*) ϕ, is odd. When the second sentence is true in an information state, uttering the first sentence in that information state is the violation of the maxim of quantity. If the two sentences were uttered by two different speakers, they would be fine.

It depends on a speaker's information state, not on the common ground, whether he makes a statement truthfully or faithfully. Thus the maxims of quality and quality can be seen as conditions on the speaker's information from the **static** aspect of information. This does not, however, mean that such conditions are not imposed in common grounds. In the examples discussed so far, it is clear to both the speaker and the audience that the speaker violates the maxim of quality or quantity. This means that the common ground also tells that the speaker does so. More important, the differences in the naturalness of the dialogues come from whether the two statements are made by one speaker or two different speakers. When one speaker makes a statement, the maxim of quality or quantity imposes conditions only on the speaker's information state, not on the audience's. And the common ground must reflect the same restrictions so that it can tell whether the speaker violates the maxim of quality or quantity. Thus the common ground must keep the speaker's information state and the hearers' information states separate: that is, the common ground must have information states as many as the number of the participants in a conversation. Furthermore, information states in the common ground must be distinguished from the personal information states of the participants in a conversation. They are all the information states of the individuals in the conversation, but the difference is that the former are information states which only include information that is shared by every participant in the conversation while the latter are information states which include all information every participant actually possesses, irrespective of whether it is shared. We will call the actual **personal information state** of an individual the individual's **PIS** and the information state of that individual in the common ground simply his **"information state"**. Only with common grounds with multiple information states can we capture both dynamic and static sides of information in conversations.

Another piece of evidence for proposing common grounds with multiple information states is the interaction of specific indefinites with the possibility operator. The use of a specific indefinite is characterized as implying that the speaker has some individual in mind. To see the effect of this, let's assume the situation depicted in (9). In this situation the dialogues in (10) and (11) take place:

(9) A(lice) teaches three students, st1, st2, and st3, which C(hris) knows.
A and C both know that st3 has no watch.
While A and C are talking, D(avid) has come to talk to A.

(10) D: I found a watch on the floor in your office.
A: A certain student of mine was there. He might have lost it.

(11) D: I found a watch on the floor in your office.
A: A certain student of mine was there.
C: He might have lost it.

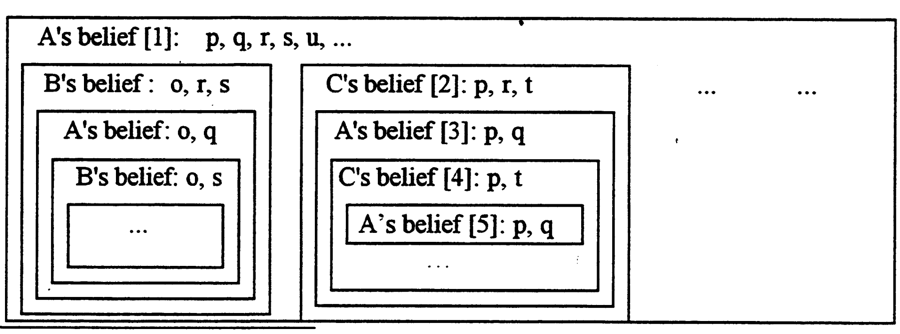
When D has left and A and C resume their talk, we can calculate the common ground between A and C. In dialogue (10), A's last statement excludes the possibility that st3 was in A's office for the following reason. When A uses a specific indefinite, she has some student in mind. This is known to C. C thinks that one of the three students were in her office. When A makes the next statement, C thinks that the student who was in her office was not st3. If it had been st3, A would not make such a statement when she knows that it is impossible that st3 lost the watch. In (11), where C makes the last statement instead of A, he does not know who he is talking about. Thus the possibility that the student d3 was in A's office is not excluded. Considering that the two dialogues are identical except for the speaker who made the last statement, the different results depend only on who utters the last sentence.[1] Note that the possibility operator is a test for an information state.[2] The different results imply that the last possibility statement must be interpreted with respect to different information states, depending on who makes the statement. Therefore the common ground must keep A's and C's information states separate.

2. DERIVATION OF COMMON GROUNDS WITH ONE INFORMATION STATE

A common ground is taken to be a sum of information shared by conversational participants. When you start to talk with others, you need to calculate what information is shared with other participants. In doing this, however, you never ask other participants what is shared. This implies that you calculate the common ground by yourself according to your PIS. In order to derive a common ground, first we need to represent your PIS. For simplicity, we will assume that there are two participants in a conversation, Alice and Chris.

When Alice starts to talk with Chris, she is supposed to make a statement based on her PIS. But first she needs to check whether the statement she is going to make is informative to Chris. For this purpose, she needs to know what information Chris has. Of course it is impossible to know what information he really has. The only thing she can do is to guess whether the information she is going to convey is in his information state as far as she knows. This requires her PIS to include the information she believes/knows he has. Such information lands in her PIS by communications with him in various channels directly or indirectly. By being in the same speech community, they know that they share the same language, the same cultural background, and so on. Previous direct contacts between them also give rise to some information shared by them. Alice also has information she does not share with Chris but with other people. Furthermore, she may have some secrets which are not known to others. Thus her PIS can be assumed to be represented as in (12) below.

(12) A's PIS



[1] Such differences do not arise when A uses a nonspecific indefinite. If A does not have any particular student, the possibility that st3 was in her office is not excluded even when she makes the last statement.

[2] If the possibility operator is such a test as is formulated in (1), the resulting common ground should be the original information state or the absurd state. In this context, this is not the case. We cannot go into this. To see the reason for this result, see Yeom (1997).

In the representation, o, p, q, ..., t, u are propositions which are BELIEVED to be facts. This is why we label each embedded information state "belief". Between knowledge and belief, the following holds:

(13) $K_\alpha \phi = \phi \wedge B_\alpha \phi$

Thus whether or not a belief can be part of knowledge depends on whether it is established as a fact in the actual world. When dealing with information, we do not know what the actual world is like. We only have our beliefs about the world. When A makes a statement, however, she makes an intuitive distinction between belief and knowledge. When she believes that a belief of hers is knowledge, she says it as such. When she is not sure whether it is a fact, she indicates it with expressions like 'I believe', 'I think', or 'It seems to me that'. We are not considering belief statements in this paper, so we will assume that A considers all propositions in her beliefs to be facts.

Now let's see what conditions must be satisfied in order for some information to be in the common ground. Suppose that A is going to use a proper name. Its proper use requires that both A and C mutually know who its referent is. In this situation, she does not ask C whether he knows the reference. A's information state must tell her that C knows it. Hence the second condition below is necessary. But this is not sufficient. When C does not know that A knows the reference and is pretty sure that A does not know, he will be surprised at the use of the name. Hence the third condition. Finally, if C believes that A does not know that C knows the reference, it will be surprising to C that A uses the proper name by assuming that C knows the reference. Thus if A is to use a proper name, her information must tell that C knows that A knows that C knows the reference. In addition to the three conditions, A herself should know the reference if she is truthful in using the proper name.[3] These four conditions are summarized as follows:

(14) A) A knows ϕ ($K_A\phi$),
B) A knows that C believes ϕ ($K_AB_C\phi$), and
C) A knows that C believes that A believes ϕ ($K_AB_CB_A\phi$).
D) A knows that C believes that A believes that C believes ϕ ($K_AB_CB_AB_C\phi$).[4]

Based on the conditions in (14), A can derive a common ground from her own PIS. When A talks with C, all information which A shares with individuals other than C is not relevant. We only consider information in [1], [2], [3], and [4]. Notice that these four places in A's PIS correspond to the four conditions in (14). By applying the four conditions to A's PIS, we can derive <u>A's</u> common ground between A and C, which contains all information (A believes) A and C share.

(15) A's common ground between A & C

main context(I): p	
A's belief (II): p, q	C's belief (III): p, t

It is easy to see that p is in the common ground. (K_Ap in [1], K_AB_Cp in [2], $K_AB_CB_Ap$ in [3], and $K_AB_CB_AB_Cp$ in [4]). It is also part of the common ground that A believes q, as the following shows:[5]

(16)
i) $\underline{K_A}B_Aq$	(See [1] and fn. 6)
ii) $\underline{K_AB_C}B_Aq$	(q in [3])
iii) $\underline{K_AB_CB_A}B_Aq$	(q in [3] & 'B_Aq virtually implies B_AB_Aq')
iv) $\underline{K_AB_CB_AB_C}B_Aq$	(q in [5])

[3] This condition is necessary only in order for A to be truthful. If A is telling a lie, or pretending to believe what C said, she herself does not believe it, but it must be in the common ground. So this condition is not necessary for a statement to be in the common ground.

[4] More intuitively plausible conditions are as follows:

(i) $K_A\phi$ & $K_AK_C\phi$ & $K_AK_CK_A\phi$ & $K_AK_CK_AK_C\phi$

These four conditions can be derived from the conditions in (14), but I will not go into this.

[5] We do not consider belief statements, but B_Aq occurs when A's statement that q is not accepted by C. This will be discussed shortly.

Since A knows q, as in [1], she knows that she believes q.[6] [2] shows that A also knows that C believes that $B_A q$, where q occurs in [3]. This is what the second condition says. For the third condition, it should be noted that $B_A q$ virtually implies $B_A B_A q$, as mentioned in fn. 6. (See Hintikka (1962) for the definition of 'virtually imply'.) This must hold even when the belief is embedded in another person's belief context. Therefore (16ii) virtually implies (16iii). The fourth condition is satisfied by $B_A q$ in [4], that is, q in [5]. The discussion so far shows that every proposition in [1], [2], [3], and [4], including someone's belief, is in the common ground. Thus we can see that $B_C t$ is also in the common ground. The proposition u is not in the common ground since that is what only A knows. The proposition r both A and C know, but A knows that C does not know that A knows it. This can happen when A hears from someone else that C knows r, but C has no evidence to assume that A knows r.[7] Therefore it is not part of the common ground. Neither is the proposition s since A knows it, but C does not. Such information as the proposition s can be obtained from direct experience of the world, or from hearing it from someone else.

As the box notation or the use of many K-/B-operators is sometimes clumsy, we introduce another representation format. The two information states in (12) and (15) can be represented respectively as follows :

(17) i) $<[1]\{p,q,r,s\}<[2]\{p,r,t\}<[3]\{p,q\}<[4]\{p,t\}>_C>_A>_C>_A$

ii) $<(I)\{p\}<(II)\{p,q\}>_A, <(III)\{p,t\}>_C>_{A,C}$

(18) $<\{\phi, \psi\}>_A$ is interpreted as $K/B_A\phi \wedge K/B_A\psi$.

When a common ground is derived from A's own PIS in talking with C, we will call it **A's common ground** between A and C. Their correspondence relations are summarized as follows:

(19)

A's PIS	A's common ground (A&C)
The information in [1],[2],[3],[4]	The information in (I)
The information only in [1], [3]	The information in (II)
The information only in [2], [4]	The information in (III)

From this relation, we can characterize the information in the common ground. The information in (I) is the information shared by the participants of the conversation. When A's or C's statement is accepted by the other participant, the information it conveys goes to (I). All information in (I) is also in (II) and (III) since each participant accepted the information in (I) as facts. Recall that if $K_\alpha\phi$, then (ϕ and) $B_\alpha\phi$. In addition to this, (II) contains some information that is conveyed by A's statement, but which is rejected by C. Similarly (III) includes what is asserted by C, but which is rejected by A.

So far we have discussed how to derive A's common ground. At the beginning of the conversation, C also derives his own common ground. If we suppose that C's PIS is like (20), his common ground will be the same as A's.

(20) $<[1]\{p,r,t,v, \ldots\}<[2]\{p,q\}<[3]\{p,t\}<[4]p,q\}>_A>_C>_A>_C$

In some cases, it is possible that C's common ground is actually different from A's. This happens when one of them or both made mistakes in the process of accumulating information or in the derivation of common grounds. We will just assume that they will derive the same common ground.

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[6] This can be proved as follows:

(i) (a) $K_A q \Rightarrow B_A q$, where '$\Rightarrow$' stands for 'virtually imply'.

(b) $B_A q \Rightarrow B_A B_A q$ (See Hintikka (1962: 109-110))

(c) $\therefore B_A q \wedge B_A B_A q \Rightarrow K_A B_A q$

[7] This kind of information could be useful in interpreting some statement, but it is excluded from the common ground. For example, suppose that there are twins, Mary and Jane. A knows that they are twins. A asks them to behave consistently in front of B in order to deceive B. So B believes that he knows Mary, and actually regards both of the twins as Mary. In this situation, if B makes a statement about something which A did not expect Mary to do, A has to consider the possibility that "Mary" may refer to Jane. In this situation, the common ground derived from A's information state and that derived from B's are still the same. More interestingly, A can calculate what B's common ground is like, too.

3. CHANGES OF PIS'S AND COMMON GROUNDS

Now let us turn to the effect of communication. Suppose that A makes a statement. Then the proposition must be in [1] of A's PIS, but not in [2], [3], or [4], like *s*, because according to A's PIS, it must be informative for C, and thus to her common ground. When C comes across the statement, he assigns one of the two operators, the K(now)-operator or the B(elief)-operator, to it. When he assigns the K-operator, all he does is not to argue against the statement. A considers it as C's agreeing to it. In this case, C interprets the statement as $K_A s$, which is again paraphrased as $[s \wedge B_A s]$. To A, it is interpreted as $B_C[s \wedge B_A s]$, where the first and the second *s* go to [2] and [3] respectively in A's PIS. Since $B_C[s \wedge B_A s]$ and this is conveyed by A's overt statement, it also holds that $B_C B_A B_C[s \wedge B_A s]$, where the first and the second *s* go to [4] and [5] respectively. This process can be repeated infinitely. Then A's PIS changes as follows:

(21) i') <[1]{p,q,r,s,u, ...}<[2]{p,r,t,s}<[3]{p,q,s}<[4]{p,t,s,...}$>_C>_A>_C>_A$

Therefore the proposition s goes to (I), and thus to (II) and (III), in A's common ground.

When C does not agree to the statement, he must express it overtly. In this case, updating is suspended until it is decided whether it is considered to be a fact. If C does not agree with A and A does not cancel her statement, it is understood as C's assigning the B-operator to it. It is interpreted by A as $B_C B_A s$, $B_C B_A B_C B_A s$, ..., and so on. Then the proposition *s* goes to all A's embedded beliefs, but not to C's beliefs, in A's PIS. A's PIS changes like the following:

(22) i") <[1]{p,q,r,s}<[2]{p,r,t}<[3]{p,q,s}<[4]{p,t}$>_C>_A>_C>_A$

Therefore the proposition *s* goes only to A's belief (II) in A's common ground. A similar change occurs in C's PIS, too. The only difference is that the proposition *s* is expected not to be in C's PIS.

The use of the B-/K-operators seems natural. When C assigns the K-operator to A's statement *s*, he regards it as a fact, and he knows that A knows that *s*. This is just like the PIS in which he can make a statement that A *knows* that *s*. When C assigns the B-operator to *s*, he gets into a PIS in which he himself does not believe that *s*, but in which he knows that A believes that *s*. This is just like the PIS in which he would say that A *believes* that *s*. Even in this case, what A believes must be in the common ground since both A and C know it. The discussion so far also shows that changes in common grounds are reflections of changes in PISs.

4. COMMON GROUNDS WITH MULTIPLE INFORMATION STATES

We have seen that a common ground is derived from a PIS. This must be true of other forms of common grounds. At the outset, we showed that in order to account for phenomena related to some semantic operators, we need a common ground with multiple information states. If such a common ground is derived from a PIS, it is implies that when a participant in a conversation comes across a statement, he changes his PIS and thus the common ground he calculates at the same time. Information to be added to the common ground will change all information states in the common ground unless it is known to all the participants in the conversation that the statement or part of its meaning affects only one participant's information state. In deriving such a common ground, we can use the same conditions as we have used to derive a common ground with a single information state again.

Let's derive a common ground with A's and C's information states in it. From the conditions in (14), if ϕ is in the standard common ground, $[\phi \wedge B_C\phi]$ is in the common ground. This can be shown as follows:

(23) a. $K_A\phi \wedge K_A B_C\phi = K_A[\phi \wedge B_C\phi]$ (14A & 14B)
b. $K_A B_C[\phi \wedge B_C\phi]$ (14B & $K_A B_C\phi = K_A B_C B_C\phi$)
c. $K_A B_C B_A[\phi \wedge B_C\phi]$ (14B & 14D)
d. $K_A B_C B_A B_C[\phi \wedge B_C\phi]$ (14D & $K_A B_C B_A B_C\phi = K_A B_C B_A B_C B_C\phi$)

We have already seen that *p*, $B_A q$, and $B_C t$ satisfy the four conditions. According to this result, we can derive the common ground in A's information state as follows:

(24) A's common ground: $<\{p,B_Aq, B_Ct\}<\{p,B_Aq, B_Ct\}>_C>_A$

Likewise, C will derive his common ground in his own PIS.

(25) C's common ground: $<\{p,B_Aq,B_Ct\}<\{p, B_Aq,B_Ct\}>_A>_C$

A's and C's common grounds show different embedding structures, but this can be modified as in (26), and thus A's and C's common grounds become those in (27).[8]

(26) $< \phi , < \phi >_C>_A = K_A[\phi \wedge B_C\phi] = K_A[\phi \wedge B_C\phi] = K_A\phi \wedge K_AB_C\phi$
$\Rightarrow K_AB_A\phi \wedge K_AB_C\phi = K_A[B_A\phi \wedge B_C\phi] = << \phi >_A, < \phi >_C>_A$ [9]

(27) A's new common ground: $<<\{p,B_Aq, B_Ct\}>_A,< \{p,B_Aq, B_Ct\}>_C>_A$
C's new common ground: $<<\{p,B_Aq, B_Ct\}>_A,< \{p,B_Aq, B_Ct\}>_C>_C$

The two common grounds in (27) are actually private hypothetical common grounds in the sense that they are formed based on A's and C's PISs. This is more realistic since this is what really happens in a conversation. However, we can suppose that there is an expert in the interpretation of statements exchanged between A and C. Before the conversation starts, he adjusts the common ground between A and C, and interprets each sentence uttered by A and C, as an expert. He can be supposed to keep the public record of the common ground. In an ideal situation, in which A and C have the same competence as the expert, A's and C's hypothetical private common ground will be the same as the common ground kept by the expert. Under the assumption of this ideal situation, we can regard the common ground as publicly declared expertise of A's and C's, and represent it as follows:

(28) $<<\{p,B_Aq, B_Ct\}>_A,< \{p,B_Aq, B_Ct\}>_C>_{A,C}$

The separation of the two information states in the common ground opens the possibility to change. the structure of A's information state without affecting C's information state, or vice versa, provided that the asymmetric information is known to every participant in the conversation. When a speaker makes a statement *may p*, it is known to every one in the conversation that the speaker does not know for sure that *p*.

We have said that changes in common grounds are reflections of changes in PISs. This does not mean that common grounds are not necessary. In order to capture anaphora and the use of pronouns, we introduce **discourse referents** in common grounds. This does not seem to be the case with PISs. Suppose that A and C remember a man they saw walking backwards yesterday. A cannot utter sentence (29) out of the blue, intending to refer to the man with the pronoun *he*. Pronouns are contrasted with definite descriptions in this context. A can utter sentence (30).

(29) He was hit by a car.
(30) The man we saw walking backwards was hit by a car.

Discourse referents belong to discourse information. This is kept only during the conversation. We may assume that we keep something like discourse referents (or **pegs**, as Landman (1986) suggests) in our PISs, but it must be noted that they are not accessible for pronouns in the immediate conversational context.[10]

5. NEW INFORMATION THEORY

So far we have derived a common ground with multiple information states. In dynamic semantics, the meaning of a sentence is taken to be the common ground change potential: a partial function from the set of common grounds to the set of common grounds. In a new theory, this has to be modified since the type of common

[8]We have been using numbers for the convenience of referring to the information states. It is used only when we need to refer to some embedded information states.
[9]K_Ap virtually implies K_AK_Ap. See Hintikka (1962: 104-105) for details.
[10] This makes some implication on presupposition theories. In the satisfaction theory of presupposition projection (Heim 1983), if a presupposition is entailed by the immediate context, it is not projected. The discussion implies that even in that case, the discourse referent must be newly introduced in the context. In anaphora analysis of presuppositions (van der Sandt 1992), the antecedent of the presupposition cannot be found in terms of discourse referents, but the conditions on the discourse referent can be matched with the information in the common ground.

grounds varies with the number of the participants in a conversation. In order to maintain a consistent interpretation function $||\cdot||$, this should be defined with respect to one information state. Instead, the update of a common ground must be defined with a separate function which is different from the interpretation function. So far an information state has been represented as a set of propositions. In order to capture the dynamic nature of information, we will assume that an information state *s* is represented as a set of possible worlds in which all the propositions are true. A common ground is updated with a sentence as follows:

(31) $S = \{s | s \subseteq W\}$, where S is the set of all possible information states *s*'s, and W is the set of all possible worlds, i.e., w's.

(32) $<s_1, ..., s_n> + \phi = <s_1||\phi||, ..., s_n||\phi||>$

The common ground is defined as a tuple of information states, i.e., $<s_1, ..., s_n>$. The operator '+' is defined for updating a common ground with an arbitrary number of information states and a sentence ϕ in such a way that each of the information state is updated with the meaning of the sentence $||\phi||$. We have already seen that if a proposition is in the standard common ground, it is in all the information states in the common ground with multiple information states. Therefore if a statement is made, it must update all the information states at the same time. This is shown in (32).

Now we need to define interpretation rules for propositional logic.

(33) a. $s||\phi|| = \{w \in s | \phi \text{ is true in } w\}$
b. $s||\text{not } \phi|| = s - s||\phi||$
c. $s||\phi \text{ and } \psi|| = (s||\phi||)||\psi||$ (This is simply written as $s||\phi|| \, ||\psi||$.)
d. $s||\text{may } \phi|| = s \quad \text{if } s||\phi|| \neq \varnothing$
$= \varnothing \quad \text{if } s||\phi|| = \varnothing$
e. $s||\text{if } \phi, \text{ then } \psi|| = s||\text{not } (\phi \text{ and not } \psi)||$
f. $s||\phi \text{ or } \psi|| = s||\phi|| \cup s||\psi||$[11]

(33a) says that an information state is updated with the meaning of a sentence in such a way that only the possible worlds in which the sentence is true remain in the information state. In this sense this semantics is eliminative. The negation of a sentence *not* ϕ updates an information state into a set of possible worlds which we get by eliminating all possible worlds in which the sentence ϕ is true. This is what (33b) means. If two sentences are conjoined, the information state is updated with the first conjunct and then with the second. Rule (33c) does this. As for the interpretation of a possibility sentence, we already characterized it as a test of an information state. If a sentence is true in at least one possible world in an information state, the information state remains as it is. Otherwise, it becomes the absurd state. The rule for conditionals (33e) just follows the logical equivalence. The interpretation of disjunction is clear.

6. EXPLANATION OF THE DATA

Once the two persons' information states are separated in the common ground, each information state can be updated in a different way considering conversational implicatures. When we do this, we can treat each information state in the common ground as if it were a PIS: In other words, A's information state in the common ground can be regarded as a "common ground version" of her own PIS, admitting that her actual PIS contains more information than her information state in the common ground. Thus if her statement imposes a condition on her own PIS and it is known to every one in the conversation, the same condition can be imposed on her information state in the common ground.

Let's consider the examples again.

(34) A: John may need help.
C: Actually he does.

[11] Logically, *ϕ or ψ* is equivalent to *not(not ϕ and not ψ)*. This may cause some problems in dealing with presuppositions. I just want to avoid such problems and make the rule as simple as possible.

Dynamically, the dialogue in (34) is perfect in the sense that the information state does not become the absurd state. If an information state allows the possibility that John needs help, there is at least one possible world in which the proposition that John needs help is true. This also opens the possibility that there are other possible worlds in which the proposition is false. In this context, C's statement reduces the set of possible worlds in the information state into the set of possible worlds in which John needs help. This contrasts with the following example, in which the two sentences are uttered by the same speaker.

(35) ??John may need help. Actually he does.

This is related to the conditions on the information state in which he can make the utterance truthfully and faithfully.

According to Grice's (1975) maxim of quality, A is supposed to say only what she believes is true. The maxim of quantity says that A is supposed to make the strongest possible statement which she can defend. These two can be formulated in terms of the relation 'support' as follows:

(36) ϕ is **supported** by an information state s iff $s||\phi|| = s$.[12]

(37) ϕ **entails** ψ iff for every information state s in S, if ϕ is supported in s, then ψ is supported in s

(38) A speaker *sp* follows the **maxim of quality** in making a statement ϕ in *sp*'s PIS
iff ϕ is supported by *sp*' PIS.

(39) A speaker *sp* follows the **maxim of quantity** in making a statement ϕ in *sp*'s PIS
iff there is no statement ψ such that ψ entails ϕ and is supported by *sp*'s PIS.

If a statement is supported by an information state, it must already be true in every possible world in it: the statement does not reduce the information state. Entailment must be defined irrespective of any particular information state, and thus it is defined with respect to the set of all possible information states S. The maxims of quality and quantity are defined with respect to speakers' PISs, but as we mentioned above, the same conditions can be imposed on the speaker's information state in the common ground.

From the definitions given above, we can give correctness conditions for the operators we are interested in in this paper. First, consider the conditions for the correct use of the operator *may*. The conditions for the correct use of the operator is as follows:

(40) *may* ϕ is correct in the speaker's information state s iff (i) $s||may\ \phi|| = s$ and (ii) $s||\phi|| \neq s$.

In the correctness condition, (i) comes from the maxim of quality and (ii) from the maxim of quantity. The first condition can be simplified as $s||\phi|| \neq \varnothing$ thanks to the interpretation rule of the operator *may*. The second condition holds since ϕ entails *may* ϕ. This can be further simplified as follows:

(41) *may* ϕ is correct in the speaker's information state s iff $\varnothing \subset s||\phi|| \subset s$.

The two conditions in (40) are represented as one condition here, but note that they come from different maxims.

Let's go back to the example above. We have two participants, A and C, in the conversation, and the common ground is represented as $<s_A, s_C>$. When A makes the statement *may p*, where p is 'John needs help', s_A must satisfy the following condition:

(42) $\varnothing \subset s_A||p|| \subset s_A$

After the statement *may p*, if C makes the statement p, the maxim of quality requires the condition (43).

(43) $s_C||p|| = s_C$

In this case, there is no conflict between the two conditions. The condition in (42) is imposed in A's information state in the common ground while the condition in (43) is in C's. This is contrasted with the case where A makes the statement p. The maxim of quality requires the following condition:

[12] Veltman (1985) and Landman (1986) use 'be true' instead of 'be supported'. Groenendijk, Stokhof, and Veltman (1994) use 'support' instead of 'be true'. The reason is that truth is a relation between languages and worlds, but the relation in question is between languages and information states of the worlds.

(44) $s_A||p|| = s_A$

This condition is not compatible with the condition from the maxim of quantity in (42). In other words, the speaker has no way to satisfy the maxim of quantity in the first sentence and that of quantity in the second sentence. If the second sentence in (35) were *not p* and uttered by C, the condition from the maxim of quality would be $s_C||$ *not p* $|| = s_C$: that is, $s_C||p|| = \varnothing$. This would not cause any incompatibility with the condition in (42) since they would be imposed in different information states. On the other hand, if it were uttered by A, the new condition would be $s_A||$*not p*$|| = s_A$: that is, $s_A||p|| = \varnothing$. This conflicts with the condition from the maxim of quality in (42): that is, incompatibility arises between the conditions from the maxim of quality.

A similar (in)compatibility between correctness conditions is relevant'to the following examples, too.

(45) A: John may not have stolen the car.
C: Actually he did.

(46) A: John may not have stolen the car. Actually he did.

As for the statement *may*[*not p*], where *p* is 'John stole the car', the following correctness condition is given:

(47) $\varnothing \subset s_A||$*not p*$|| \subset s_A$: i.e., $\varnothing \subset s_A||p|| \subset s_A$

When the second sentence is uttered by C, as in (45), the condition from the maxim of quality is imposed in C's information state, and thus no conflict occurs. On the other hand, when it is also uttered by A, the condition follows that $s_A||p|| = s_A$, which is incompatible with (47).

Let's look at examples of conditional sentences in which the two sentences are uttered by two different speakers:

(48) A: If Mary is at home, Susan is out. (*If* ϕ, ψ)
C: Actually Susan is at home. (*not* ψ)

First, we will show that the dialogue does not lead the common ground to the absurd state. According to Gazdar (1979), if a conditional sentence *if* ϕ, *then* ψ is to be used correctly, ϕ and ψ are not either supported or forbidden by the speaker's information state.[13] Also the conditional itself must be supported by the same information state. From these, we can derive the following correctness conditions for the first sentence:

(49) the maxim of quality: $s_A||$*if* ϕ, *then* $\psi|| = s_A$: i.e., $s_A||\phi|| = s_A||\phi|| \, ||\psi||$.

(50) the maxim of quantity: $s_A||\phi|| \neq \varnothing$, $s_A||\phi|| \neq s_A$, $s_A||\psi|| \neq \varnothing$, and $s_A||\psi|| \neq s$:
i.e. $\varnothing \subset s_A||\phi||$, $s_A||\psi|| \subset s_A$

In order for s_A to satisfy the conditions in (50), s_A must have four non-empty subsets of the possible worlds with respect to the truth and falsity of ϕ and ψ. Then before the sentence is uttered, A's information state in; the common ground s_A will have been a bigger set of possible worlds. This implies that the common ground as a whole has a larger set of possible worlds than the s_A which is constrained by the conditions in (50). Therefore the common ground will also have four non-empty subsets of possible worlds with respect to the truth and falsity of ϕ and ψ. This is represented as a table as follows:

(51) Four subgroups of possible worlds in the common ground

	W1	W2	W3	W4
ϕ	0	1	0	1
ψ	0	0	1	1

0: false; 1: true

When A's utterance is made in this context, the common ground is updated with it, which results in W1 $\cup$ W3 $\cup$ W4. This is also required in order for A to satisfy the condition from the maxim of quality (= 49). In this context, C's utterance (= *not* ψ) does not change the common ground to the absurd state, but to W1 $\cup$ W3.

[13] The relation 'forbid' can be defined as follows:
(i) An information state s forbids ϕ iff $s||\phi|| = \varnothing$.

Even if the second sentence were ψ, it would result in W4. Thus there is no inconsistency in the dynamic side of information.

Next, we need to show that correctness conditions do not conflict when the two sentences are uttered by different speakers. For the first sentence, we got the conditions in (50). When the second sentence is uttered by C, the condition from the maxim of quality, which is $s_C||$ *not* $\psi|| = s_C$, does not conflict with any conditions in (50). This can be contrasted with the example below.

(52) A: ??If Mary is at home, Susan is out. Actually Susan is at home.

If the second sentence is also uttered by A, the condition follows that $s_A||$*not* $\psi|| = s_A$: that is, $s_A||\psi|| = \varnothing$. This condition is incompatible with the condition $s_A||\phi|| \neq \varnothing$, which comes from the maxim of quantity for the first sentence. A similar incompatibility is observed in the following example:

(53) A: ??If Mary is at home, Susan is out. Actually Susan is out.

The condition from the maxim of quality would be $s_A||\psi|| = s_A$. This again is incompatible with the condition $s_A||\psi|| \neq s_A$ in (50). Thus A cannot satisfy both the maxim of quantity for the first sentence and the maxim of quality for the second sentence.

Finally let's consider the example of disjunction:

(54) ??John is the owner or the manager of the apartment. Actually he is (not) the owner.

Groenendijk and Stokhof (1975) list the following correctness conditions for disjunction:

(55) For ϕ *or* ψ to be correct for s_A,
a. the maxim of quantity: $s_A||\phi|| \neq \varnothing$, $s_A||\phi|| \neq s_A$, $s_A||\psi|| \neq \varnothing$, and $s_A||\psi|| \neq s$:
i.e. $\varnothing \subset s_A||\phi||, s_A||\psi|| \subset s_A$
b. exhaustiveness: $s_A||\phi|| \cup s_A||\psi|| = s_A$
c. exclusiveness: $s_A||\phi|| \cap s_A||\psi|| = \varnothing$

If A knows that ϕ is true or that ψ is true, she is expected to say it instead of ϕ *or* ψ. This gives rise to the conditions from the maxim of quantity. Exhaustiveness corresponds to the maxim of quality: ϕ *or* ψ must be true in all possible worlds in s_A. Exclusiveness is a characteristic of natural language disjunction. This is illustrated in the following:

(56) ??John is a bachelor or a man.

This sentence is awkward since the two options are not exclusive. If a person is a bachelor, he is a man at the same time. If he is a man, he may be a bachelor. However, this does not seem to constitute an absolute condition. We do not need to be concerned with this condition since it is not relevant to the discussion below. The correctness conditions from the maxim of quantity is the same as those for conditionals. Thus it is easy to see from the discussion of the conditional examples above that if the second sentence is also uttered by A, a conflict arises between correctness conditions. We will not go into this again. If the two sentences are uttered by different persons, say, A and C respectively, there is no conflict between correctness conditions.

The use of specific indefinites is another good illustration of a common ground with multiple information states. A specific indefinite is generally characterized as indicating that the speaker has some individual in mind. Thus the use of a specific indefinite results in an asymmetric information states between the participants in a conversation. In the common ground with two information states, only the speaker's information state within the common ground contains this information. If it affects updating in any information state, it will be only the speaker's, and it will interact with his own utterances. When someone else makes an utterance, there is no interaction between the information of having an individual in mind and the utterance. As the illustration in (9-11) shows, the asymmetric information states result in different results as to whether st3 was possibly in Alice's office. Unfortunately, we cannot discuss the semantics of having someone in mind since it is not the main topic and needs much discussion. For the semantics of specificity, see Yeom (1997).

7. CONCLUSION

In a conversation, there are two main factors that restrict the conversation. One is informativeness. An utterance must be informative to the current common ground, or, more accurately, to the audience. This restricts the conversation as a dynamic aspect of information. The other is truthfulness or faithfulness to the speaker's own information state. This restricts the conversation as a static aspect of information. In order to capture these two sides of information, we need multiple information states in a common ground. Before closing the discussion, we need to mention one thing about the restrictions. It is well-known that correctness conditions do not constitute absolute conditions. Sometimes speakers violate the maxims of quality and quantity on purpose. For these cases the correctness conditions must not have absolute effects on (the information states in) the common ground. One solution for these cases would be default semantics, which does not eliminate possible worlds in which some condition is not satisfied. Instead they are ranked lower in relative ordering between possible worlds. In normal situations, the possible worlds in which the condition is satisfied and which are ranked relatively higher are taken to be the current information state. When the condition is not compatible with the context, the possible worlds which were ranked lower turn into the possible worlds which constitute the current information state.

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