

Puzzles about Conjunction and Disjunction
 in Certain Intensional Contexts

Makoto Kanazawa
 Stanford University

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The fairly standard account of the connectives *and* and *or* treats them as standing for the boolean meet and join operations, respectively, in the appropriate denotation spaces (Keenan and Faltz 1978, 1985; Gazdar 1980; Partee and Rooth 1983). According to this view, the set of possible denotations of each conjoinable category is equipped with the structure of a boolean algebra, and a phrase of the form [*A and B*], where *A* and *B* are expressions of a common conjoinable category, denotes the boolean meet of the denotations of *A* and *B*, and similarly, [*A or B*] denotes the boolean join of the denotations of *A* and *B*. Although it is clear that this approach cannot cover such uses of *and* as in group-denoting conjoined NPs (there are no corresponding uses of *or*), it seems to have been viewed as successful and unproblematic for other cases.

Now this approach makes a certain interesting prediction. Take a context $U \underline{\quad} V$, where the underlined position is to be occupied by a phrase of some fixed conjoinable category (VP, NP, PP, etc.), such that if *A* is an expression of that category, $U A V$ makes up a declarative sentence. Suppose that this context allows *elimination of conjunction*, that is, an inference of the following form is valid:

$$(1) \quad \frac{U [A \text{ and } B] V}{U A V}$$

Given the above treatment of *and* and *or* and assuming compositional semantics, it follows that the same context supports another principle, *introduction of disjunction*:

$$(2) \quad \frac{U A V}{U [A \text{ or } B] V}$$

The reason is this. An instance of the schema (1) is

$$(3) \quad \frac{U [[A \text{ or } B] \text{ and } A] V}{U [A \text{ or } B] V}$$

But the denotations of $[[A \text{ or } B] \text{ and } A]$ and *A* must coincide, since $(x \vee y) \wedge x = x$ is one of the laws of boolean algebra (indeed of any lattice).¹ Since, by the compositionality assumption, the semantic value of $U X V$ is functionally determined by the semantic value of *X*, the semantic value of $U [[A \text{ or } B] \text{ and } A]$

V must also coincide with that of $U \wedge V$. Then (3) becomes the same principle as (2).

This is a necessary consequence of the boolean account of the two connectives (plus compositionality). The boolean account predicts that there is no context of the form $U _ V$ such that (1) is a valid form of inference and (2) is not.²

Now it turns out that there are a few constructions that seem to be counterexamples to this prediction. Consider the following inferences:

- (4) a. John must write a paper and take an exam

 John must write a paper
- b. John must write a paper

 John must write a paper or take an exam
- (5) a. John may take an apple and take a pear

 John may take an apple
- b. John may take an apple

 John may take an apple or take a pear

In both cases, our intuition is that (a) is a perfectly valid inference. Indeed, we are inclined to assert with confidence that the following are valid forms of inference:

- (4) a'. C must A and B

 C must A
- (5) a'. C may A and B

 C may A

where C is a proper name and A and B are any verb phrases of the appropriate sort. On the other hand, we strongly feel that there is something wrong with the (b) inferences. This intuition can be reinforced by considering the following:

- (4) c. John must write a paper or take an exam

 John may take an exam
- (5) c. John may take an apple or take a pear

 John may take a pear

These inferences are felt intuitively to be valid. However, combining these with the (b) inferences above, we get

- (4) d. John must write a paper

 John may take an exam
- (5) d. John may take an apple

 John may take a pear

which are utterly absurd. Thus, we cannot accept the (b) inferences as valid.³

We have seen that the contexts *John must*___ and *John may*___ seem to constitute counterexamples to a certain prediction of the boolean account of the connectives *and* and *or*, namely, that validity of elimination of conjunction implies validity of introduction of disjunction. These contexts have been traditional puzzles in deontic logic. The problem associated with *must* is called the Ross paradox and that associated with *may* is called the Paradox of the Free Choice Permission (Ross 1941; von Wright 1968).⁴

Insofar as these puzzles are genuine, they show an inadequacy of the standard semantics of *and* and *or*. We will, beginning next section, look at some proposed solutions to these deontic paradoxes, but, before doing so, a couple of remarks about these contexts involving *must* and *may* are in order.

First, it should be immediately noted that sentences of the form *C must/may [A and/or B]* can have multiple readings. Since each of *must/may* and *and/or* has its own scope, there are (at least) two scope possibilities for such sentences: one possibility has *must/may* scoping over *and/or*, and the other has *and/or* scoping over *must/may*. In the case of *or*, it seems obvious that these different scopings give rise to different truth conditions. In the following, *or* takes wide scope with respect to *must/may*:

- (6) John must write a paper or take an exam, but I don't know which.
(7) John may take an apple or take a pear, but I don't know which.

The first part of these sentences seem to be equivalent to the following, respectively:

- (8) Either John must write a paper or he must take an exam.
(9) Either John may take an apple or he may take a pear.

The next examples are different:

- (10) John must write a paper or take an exam, but the choice is up to him.
(11) John may take an apple or take a pear; he can take whichever he likes.

It is clear that *John must write a paper or take an exam* has different truth conditions in (6) and (10). In fact, *John must write a paper or take an exam*, read in the way of (6), seems to be almost inconsistent with (10), although (10) itself is of course perfectly consistent. Similarly, as for (7) and (11), *John may not take a pear*

seems to be consistent with *John may take an apple or take a pear* if the latter is read in the way of (7), but not if it is read in the way of (11). It must be made clear that the reading we are interested in is not the wide scope reading found in (6) and (7), but rather the reading found in (10) and (11). Indeed, (4b) and (5b) are perfectly valid if it is interpreted in such a way that *or* takes wide scope. In the case of *and*, the difference in truth conditions between different scope possibilities in the context of *C must/may*— is harder to discern,⁵ but nonetheless it must be borne in mind that multiple scope possibilities exist and that we are only interested in the reading where *and* takes narrow scope with respect to *must* and *may*, when we consider inferences of the form (4a') and (5a').

Secondly, it should be emphasized that the two 'paradoxes' are related, involving the dual notions of obligation and permission, but they are not exactly parallel, despite the fact that some people have viewed them as one and the same phenomenon (cf. section 4). In the case of *may*, what appears to be narrow scope disjunction seems to be equivalent to wide scope conjunction:

$$(12) \quad C \text{ may } A \text{ and } B \iff C \text{ may } A \text{ and } C \text{ may } B,$$

where *C* is a proper name, but this does not hold if *may* is replaced by *must*. (Note the asymmetry between (4c) and (5c).) In fact, in the case of *may* (but not in the case of *must*), the same kind of problem arises in the 'reverse' direction as well. If it is true that inference of the form (13) is valid, then it must be the case that inference of the form (14) is also valid, under the boolean account:

$$(13) \quad \frac{C \text{ may } A \text{ or } B}{C \text{ may } A}$$

$$(14) \quad \frac{C \text{ may } A}{C \text{ may } A \text{ and } B}$$

Now we turn to some of the solutions to these deontic paradoxes proposed so far in the literature. These solutions are all in a sense inadequate in that they consider only a few contexts other than those involving deontic notions like *must* and *may*. We will see below (sections 4 and 5) that the problem is more general and arises in a variety of contexts.

1.

There have been attempts to explain away the paradoxes in terms of Grice's theory of conversational implicature (e.g. Hare 1967). Roughly, the explanation goes as follows.⁶ *Must* and *may* do allow introduction of disjunction. The (b) inferences in (4) and (5) are really valid, despite our first reactions to them. (4c) and (5c) are not genuine entailments, but rather instances of Gricean conversational implicature. Indeed, the implicature does not always arise:

- (15) John must buy an apple or buy a pear, but he must choose the cheaper one.
- (16) John may take an apple or take a pear, but he may not take the more expensive one.

We cannot conclude from (16) that John may take a pear, since it might be more expensive than an apple. The reading of *John may take an apple or take a pear* in (16) is not the wide scope reading, since (16) says that John is allowed to take whichever turns out to be the cheaper of an apple and a pear. A similar thing can be said about (15). If this shows that the implications in (4c) and (5c) can be defeasible and therefore are just conversational implicatures, (4b) and (5b) need not lead to such absurdities as (4d) and (5d), and hence are innocuous. (But see footnote 3.)

A problem with this account is that the conversational principle alone cannot explain why the implications in (4c) and (5c) are more strongly felt than in other cases where similar conversational implicatures arguably arise. In fact, the *may* of possibility also seems to create a similar implicature:

- (17) John may be working in his office or playing tennis outside
-
- John may be playing tennis outside

However, the following inference, unlike (5b), does not have a paradoxical flavor and is felt to be valid:

- (18) John may be working in his office
-
- John may be working in his office or playing tennis outside

The conversational principle cannot account for the difference in this respect between the *may* of permission and the *may* of possibility (Hintikka 1979, see section 3 below).

Another problem with the conversational account is pointed out by Kamp (1978). The strong semantic force of *you may take an apple or take a pear* (it is stronger than *you may take an apple* or *you may take a pear* in the sense that it implies them) does not disappear when it gets embedded in a larger sentence:

- (19) Usually you may only take an apple. So if you may take an apple or take a pear, you should bloody well be pleased. (Kamp 1978, p. 279.)

This discourse only makes sense when *you may take an apple or take a pear* in the second sentence is read in the strong sense which implies *you may take a pear*. In fact, if it is really the case that *you may take an apple* entails *you may take an apple or take a pear*, as the conversational implicature account holds, and if we assume that indicative conditionals like the one in (19) support the principle of strengthening of the antecedent, then the second sentence of (19) should entail *if you may take an apple, you should bloody well be pleased*, which contradicts

what (19) presupposes. The conversational account is unable to explain why we understand (19) in the way we do. Since *you may take an apple or take a pear* is not asserted in (19), and hence does not inform the addressee of his or her actual permissible actions, an account which makes use of the information it conveys cannot be straightforwardly applied to a case like this. A similar example can be constructed for *must* as well:

(20) If John must write a paper or take an exam, he will probably choose to take an exam.

(20) makes sense only if *John must write a paper or take an exam* is meant to imply *John may take an exam*.⁷

Overall, Hintikka (1979) seems to be right in claiming that the conversational implicature may in fact contribute to the paradoxes, but is certainly 'not the whole story' (p. 336).

2.

Hilpinen (1982), building on Kamp (1973), emphasizes the ambiguity of 'disjunctive permission sentences' like *John may take an apple or take a pear*.⁸ Besides the wide scope reading, disjunctive permission sentences can have two readings.⁹ On one reading, it is equivalent to *John may take an apple and John may take a pear*, and is stronger than *John may take an apple* (free choice permission). On the other reading, it is weaker than *John may take an apple* and indeed implied by it (weak permission). The free choice permission reading is illustrated by (11), and the weak permission reading is the one found in (16). He represents the two readings by the following formulae:

- (21) a. $may(F(\text{John take an apple}) \vee F(\text{John take a pear}))$
 b. $may F(\text{John take an apple} \vee \text{John take a pear})$

(a) is the free choice permission reading and (b) is the weak permission reading. *F* is what Kamp (1973) called the 'focus operator'. Its function is "to 'move' the sub-formula which stands within its scope into focus and thereby subject it (as a whole) to the particular semantic-pragmatic operation which is associated with the mode, or pragmatic function, of the speech act in question" (Kamp 1973, pp. 69–70). Hilpinen gives semantics to his formal language in such a way that *may FA* implies *may F(A ∨ B)*, but not *may(FA ∨ FB)*.

Hilpinen also emphasizes the similarity between permission sentences and subjunctive counterfactual conditionals. *C may A or B* usually seems to imply *C may A*, but in spite of what this fact would imply under the standard treatment of *and* and *or* (this is the 'reverse direction' mentioned at the end of section 0), *C may A* does not imply *C may A and B*. (Let *C* be a proper name.) Parallel to this, although *if A or B, then C* seems to imply *if A, then C*, you can't generally infer from *if A, then C* to *if A and B, then C*, in the case of counterfactual conditionals (failure of the principle of strengthening of the antecedent).

(22) If this match were struck, it would light.

does not imply

(23) If this match were struck and no oxygen were present, it would light.

(an example from Hilpinen 1982). Also, just as in some cases *C may A or B* fails to imply *C may A*, there are cases in which you cannot infer from *if A or B, then C* to *if A, then C*. Compare:

(24) If Spain had fought in World War II on the Axis side or on the Allied side, she would have fought on the Allied side. (Hilpinen 1982.)

Hilpinen argues that both disjunctive permission sentences and counterfactuals with disjunctive antecedents are ambiguous, and the scope of the focus operator is responsible for this ambiguity. In the case of counterfactuals, the focus operator has a natural language counterpart, namely *if*. Compare (24) with:

(25) If Spain had fought in World War II on the Axis side or if Spain had fought on the Allied side, she would have fought on the Axis side. (Hilpinen 1982.)

One could hold that (24) is true, but (25) is definitely false.

Now Hilpinen can avoid the paradox in the reverse direction in the following way. Although *John may A or B*, on one reading, entails *John may A*, this does not lead to such an absurdity as *John may A* entailing *John may A and B*, because the relevant reading of a sentence of the first form is represented as *may(F(John A) \vee F(John B))* and, in particular, *John may [[A and B] or A]*, under this reading, becomes *may(F(John A \wedge John B) \vee F(John A))*, not *may F((John A \wedge John B) \vee John may A)*, and hence cannot be equivalent to *John may A*, the only representation of which is *may F(John A)*. The case of conditionals is completely parallel. He does not discuss the Paradox of Free Choice Permission in our original direction, but it can be avoided in a similar way.

Although his treatment of counterfactuals with disjunctive antecedents looks quite successful and he offers a plausible explanation to the fact that counterfactuals with conjunctive antecedents are unambiguous unlike counterfactuals with disjunctive antecedents, his definition of permission in terms of counterfactual conditionals:

(26) $may A \equiv_{def} if A, then I^{10}$

(I stands for 'immunity to punishment') looks problematic. The logics of permission sentences and of counterfactuals are not quite parallel. For one thing, permission sentences allow 'elimination of conjunction', but counterfactuals in general do not allow elimination of conjunction in the antecedent. This fact is totally neglected in his paper. In order to reduce the logic of permission sentences to the logic of counterfactuals by the above definition, it must be shown that the particular form *If A, then I* of conditionals generally allow elimination of con-

junction in the antecedent. If I can be possibly false, I can find no simple way to guarantee this.¹¹ If the logic of permission sentences cannot be reduced to the logic of counterfactuals, then the fact that sentences of the form C *may* A and B is unambiguous (in the sense that there is only one truth condition associated with it) despite the fact that it can be rendered either as *may* $F(CA \wedge CB)$ or as *may* $(F(CA) \wedge F(CB))$, needs an independent explanation.¹²

Also, it is not at all clear how Hilpinen's account can be extended to cover the case of *must*. Certainly, much more needs to be said to maintain his account as a solution to all the paradoxes we have looked at.

3.

Hintikka (1979) offers an ingenious solution to the paradoxes within the framework of his game-theoretical semantics, which seems to me to be more satisfactory than other solutions proposed so far. According to Hintikka, the peculiar nature of sentences like:

(27) John must write a paper or take an exam

and

(28) John may take an apple or take a pear

results from 'the tendency to confuse moves in a semantical game with the choices of real-life persons'. Let us sketch his solution.

In game-theoretical semantics, the semantic property of a sentence is determined by reference to the two-person game which is associated with the sentence. The game is played between players called Myself and Nature according to certain prescribed rules. The game is considered as my attempt to verify the given sentence against all the counterexamples which Nature might produce. The game starts with the given sentence and the actual world and continues until it reaches an atomic sentence which contains only proper names and simple verbs. This atomic sentence is either true or false in the world under consideration. If the former, I win, and if the latter, Nature wins. The sentence is true if I have a winning strategy that leads me to verify the sentence no matter how Nature makes his moves, false otherwise. Let us look at how the game proceeds for the paradoxical sentences which are the focus of this paper. The relevant rules are the following:

(G.phrasal or) When the game has reached a sentence of the form

$$X - Y_1 \text{ or } Y_2 - Z$$

where Y_1 and Y_2 are phrases of a coccjoinable category, then I can choose Y_i ($i = 1$ or 2) and the game is continued with respect to

$$X - Y_i - Z$$

and the same world.

(G.may) When the game has reached a world W and a sentence of the form

a may X,

where 'a' is a proper name, I can choose a world *W'* which is a deontic *a*-alternative to *W*. The game is continued with respect to *W'* and *X'*, where *X'* is the result of changing the main infinitive in *X* to third person singular and prefixing 'a' to it.

(G.must) Similar to above, except that the choice is made by Nature instead of Myself.

Here is how the game-theoretical semantics of (27) works out. Starting with the actual world *W*, Nature will choose a deontic alternative *W'* to *W* by (G.must), and the game is continued with respect to *W'* and the sentence

(29) John writes a paper or takes an exam.

Now according to (G.phrasal or) I can choose from

(30) John writes a paper

and

(31) John takes an exam

in each such *W'* so that a true sentence be produced. But 'everybody keenly feels it to be wrong for me to pretend to make a choice on behalf of another agent, in this case John, of what he is to do. Hence John's real-life choice is naturally transmuted into a choice by him in my semantical games.' (Hintikka 1979, p. 334.) The consequence of this is that since the choice between (30) and (31) is John's, not mine, John must be permitted to do either. This is what radically changes the semantics of (27).

In the case of (28), the semantical games associated with it are just like those associated with (27), except that the choice of the deontic alternative is made by Myself instead of Nature. But after the choice of the deontic alternative, the same confusion between John's choice and mine arises. This gives (28) the force of

(32) John may write a paper and he may take an exam.

This solution has several implications. Among other things, it predicts that the same peculiarity should arise when the sentence involves an existential quantifier in place of disjunction, since the rule associated with existentials also involves a choice to be made by Myself in much the same way as the rule of disjunction. This prediction seems to be borne out:

- (33) a. John must read *War and Peace*.
b. John must read some book (or other).

It seems strange to infer from (33a) to (33b), just like the case of disjunction (Hintikka 1979, p. 335).

Another prediction of this solution is that the paradox manifests itself only when a 'disjunction between options of an agent' is present. Other kinds of disjunction should not bring about any paradox. According to Hintikka, there is nothing paradoxical with the following inferences; they are perfectly valid (p. 339).¹³

- (34) John may mail the letter.
Therefore: John or Bill may mail the letter.
- (35) This letter may be mailed by John.
Therefore: This letter may be mailed by John or Bill.

Even when different options of an agent are being discussed, the paradox does not arise if 'we are focusing not on his decisions but impersonally on the different options open to him.' (36) is felt to imply (37) only if the *may* is taken to be the *may* of possibility (Hintikka 1979, pp. 337–338):

- (36) John may come on Tuesday.
- (37) John may come on Tuesday or Wednesday.

Although Hintikka does not show how to get the 'weak' sense of sentences like (27) and (28), it's possible to get the wide scope disjunction reading. If (G.phrasal or) applies first to (28), it yields either

- (38) John may take an apple
- or
- (39) John may take a pear.

Since in this case the choice between (38) and (39) is not between different options open to John, but rather between different permissions that might have been given to him, the confusion between my choice and John's arguably does not arise.¹⁴

4.

Hintikka's solution seems to be more satisfactory than others in that it explains why the puzzling behavior of *and* and *or* arises in the case of the *may* of permission, but not in the case of the *may* of possibility, and it has a clear prediction as to the range of contexts in which similar puzzles arise: validity of elimination of conjunction fails to imply validity of introduction of disjunction just in case what the sentence expresses is somehow related to choices of real-life individuals, and choices made by the players of the semantical game associated with the sentence can get confused with such choices of real-life individuals.

However, his solution is not without problems. Firstly, although this is not necessarily an inadequacy, it should be pointed out that his solution amounts to denial of both the boolean semantics of *or* and compositionality. (G.phrasal or), which corresponds to the usual boolean interpretation, does not apply in the usual way just in case there has been a previous application of (G.must) or

(G.may) (or game rules associated with other similar expressions). This means that the meaning of *write a paper or take an exam or take an apple or take a pear* cannot be given independently of the context in which it is embedded. Even if the failure of compositionality is independently motivated in game-theoretical semantics, it is not clear whether this particular problem necessitates denial of both the boolean semantics of *or* and compositionality. In other words, his solution might be unnecessarily radical. If compositionality is given up, there must be a way to avoid the paradoxes without giving up boolean semantics, and if boolean semantics is to be denied, there must be a compositional semantics that avoids the paradoxes.

Secondly, it is not obvious that the problem associated with *may* and that associated with *must* are one and the same. As for *may*, the strong 'free choice permission' reading has an affinity with the phenomenon that *or* behaves as wide scope conjunction in some constructions:¹⁵

- (40) John is taller than Bill or Mary.
- (41) Either an apple or a pear satisfies John.
- (42) The doctor makes house calls in either village A or village B.

These are contexts that naturally allow *any*:

- (43) John is taller than any other student.
- (44) Any fruit satisfies John.
- (45) The doctor makes house calls in any village within 5 miles.

The *may* of permission also allows *any* in its complement:

- (46) John may take any fruit.
- (47) John may come any day next week.

There is some reason to believe *any* is basically an existential quantifier, rather than a universal quantifier:

- (48) I wonder if anyone has come.

(48) means "neither 'I wonder if everyone has come' nor 'It is true of everyone that I wonder if he has come', but rather 'I wonder if someone has come'." (Kamp 1973, p. 69.)

Parallelism between (40), (41), (42), (28), and (37) (the last two repeated below), on the one hand, and (43), (44), (45), (46), and (47), on the other, is obvious.¹⁶

- (28) John may take an apple or take a pear.
- (37) John may come on Tuesday or Wednesday.

Suppose that these contexts can have an effect of somehow 'converting' existential quantification into wide scope universal quantification. (*Any* is particularly liable to undergo this process, while *some* cannot.) Now given the parallelism between existential quantification and disjunction, and between universal quantification and conjunction (existential quantification is a generalized disjunction, and universal quantification is a generalized conjunction), it is naturally expected that these contexts can also convert disjunction into wide scope conjunction.

Must does not create a context of this kind:

(49) John must read any paper in this list.

sounds odd. Compare:

(50) John may skip any paper in this list.

Also, when *must* is replaced by *may* in inference like (33), the problem seems to disappear:

(51) John may read *War and Peace*.

Therefore: John may read some book (or other).

These considerations suggest the following conclusion: The Paradox of Free Choice Permission can be viewed as a general phenomenon that disjunction and existential quantification behave like wide scope conjunction and universal quantification, respectively, in certain contexts in which *any* naturally occur.¹⁷ Since *some*, although an existential quantifier, cannot become a wide scope universal quantifier even in such contexts, the *may-some* combination does not give rise to the paradox. The Ross paradox is quite distinct from this general phenomenon, since sentences containing *must* and *or* do not have wide scope conjunction readings, and the same paradox arises with sentences containing *some*. Therefore, the Ross paradox and the Paradox of Free Choice Permission are not one and the same paradox. Hintikka is mistaken in trying to give a single solution to both.

5.

Thus, I assume that the Paradox of Free Choice Permission can be solved by applying to disjunction whatever mechanism is needed to account for the wide scope universal quantificational force of *any*, and that it has nothing to do with the particular concept of permission. Intensionality is not essential to this, either, as can be seen from (40), although it will probably be a contributing factor in many cases.¹⁸ What exactly is responsible for the behavior of *or* and *any* as wide scope conjunction and universal quantifier, respectively, is an interesting question itself, but cannot be answered here.¹⁹ From now on, let us put aside the Paradox of Free Choice Permission, and concentrate on the Ross paradox and its kin.

A question we should ask ourselves is: Are there any other contexts where *and* and *or* behave puzzlingly just as in the context created by *must*? In fact, there are many such:

- (52) a. Pierre wants to learn English and study computer science

Pierre wants to learn English
- b. Pierre wants to learn English

Pierre wants to learn English or study computer science
- c. Pierre wants to learn English or study computer science

Pierre has a desire which will be satisfied
if he studies computer science
- (53) a. Pierre is trying to learn English and Spanish

Pierre is trying to learn English
- b. Pierre is trying to learn English

Pierre is trying to learn English or Spanish
- c. Pierre is trying to learn English or Spanish

Pierre is engaging in an attempt which will succeed
if he learns English
- (54) a. John needs to come on Tuesday and Wednesday

John needs to come on Tuesday
- b. John needs to come on Tuesday

John needs to come on Tuesday or Wednesday
- c. John needs to come on Tuesday or Wednesday

If John can't come on Tuesday, he needs to come on Wednesday
- (55) a. John wants a model plane and a video game

John wants a model plane
- b. John wants a model plane

John wants a model plane or a video game
- c. John wants a model plane or a video game

If John is given a video game, he will be happy

- (56) a. John needs a computer and a printer

 John needs a computer
- b. John needs a computer

 John needs a computer or a word processor
- c. John needs a computer or a word processor

 If John cannot afford to get a computer,
 he needs to get a word processor
- (57) a. John is looking for a unicorn and a centaurs

 John is looking for a unicorn
- b. John is looking for a unicorn

 John is looking for a unicorn or a centaurs
- c. John is looking for a unicorn or a centaurs

 John is engaging in a search which will be fulfilled
 if he finds a centaurs
- (58) a. I hope John will catch a unicorn and a centaurs

 I hope John will catch a unicorn
- b. I hope John will catch a unicorn

 I hope John will catch a unicorn or a centaurs
- c. I hope John will catch a unicorn or a centaurs

 I will be happy if John will catch a centaurs

In all cases above, (a) is a valid form of inference, (b) is an inference which is hard to accept, and (c) is an intuitively valid inference which leads to an absurdity in conjunction with (b). Perhaps there might be a difference in degree of paradoxicality among these examples.

These are all intensional contexts created by verbs which (with the exception of *need*) Asher (1987) called 'buletics' or 'conditionally dependent indefinites'. Not all kinds of intensional contexts are paradoxical (e.g. the *may* of possibility). The question is: What exactly is the common feature of these contexts that is responsible for the paradox? It is obvious that not all of these constructions involve an agent's 'decision' to choose between different options given to him. For example, when John is looking for a unicorn or a centaurs, whether to find a

unicorn or a centaurs is not something that he can choose. This reveals another inadequacy of Hintikka's solution.

It does seem, however, that these contexts have some common semantic feature. It seems that the notion of fulfillment or satisfaction is associated with all these contexts. And this fulfillment or satisfaction can be expressed by conditional sentences. For example, in the case of (52), if Pierre wants to learn English or study computer science, his desire will be satisfied if he can manage to learn English or study computer science. In the case of (57), if John is looking for a unicorn or a centaurs, his search will be fulfilled if he finds a unicorn or a centaurs.²⁰ Unfortunately, I have no good answer yet to the question why this semantic feature causes the paradox, and so must leave the question here with these remarks.

6.

I would like to close this paper with some speculation on possible ways to avoid the Ross paradox. (Let us call the puzzles in (4), (52)–(58) generically 'the Ross Paradox'.) The paradox depends on the following assumptions (suppose that the context is $U \text{---} V$):

A1. Elimination of conjunction ((1)) is valid:

$$\frac{U [A \text{ and } B] V}{U A V}$$

A2. Introduction of disjunction ((2)) is not in general valid:

$$\frac{U A V}{U [A \text{ or } B] V}$$

where the conclusion is read in the 'strong' sense.

A3. $U A V$ and $U [[A \text{ or } B] \text{ and } A] V$ are truth-conditionally equivalent.

A1–A3 are sufficient to yield the paradox. A1 and A2 are data based on our intuition. A3 is a consequence of the standard boolean account and compositionality. To avoid the paradox, one must deny at least one of them.

The conversational implicature account was essentially an attempt to deny A2. We have seen that it faces a considerable difficulty.

Is it possible to deny A1? In fact, Asher (1987) claims that conditionally dependent verbs do not in general allow elimination of conjunction. The following is his example:

(59) Fred wants a car and a garage for it. So Fred wants a garage for a car.

'Fred might very well not want a garage unless he gets a car, so the ... inference in [(59)] doesn't seem valid...' (p. 171). Actually, this is not an instance of elimination of conjunction, because the first sentence is different from *Fred wants a car and*

a garage for a car. Let us slightly modify (59) to make it a genuine instance of elimination of conjunction:²¹

(60) Fred wants a car and a garage. So Fred wants a garage.

The same remark should apply to (60). It seems possible to say 'Fred wants a car and a garage, but he does not want a garage unless he gets a car'. But does this really show that the inference in (60) is invalid? Consider:

(61) Fred wants a garage, but he doesn't want one unless he gets a car.

(62) Fred doesn't want a garage, but he wants a car and a garage.

(62) sounds to me like a downright contradiction. On the other hand, (61) seems to me to be something that we could say. If so, the mere fact that you can say 'Fred wants a car and a garage, but he doesn't want a garage unless he gets a car' does not show that *Fred wants a car and a garage* fails to entail *Fred wants a garage*.²² Certainly, it is not an easy task to produce counterexamples to elimination of conjunction for all kinds of contexts that apparently give rise to the Ross paradox.²³

It should be noted that the derivation of the paradox does not in fact depend on the fact that elimination of conjunction hold in full generality. What is relevant for the derivation of the paradox is the following special case of elimination of conjunction (=3):

A1'. The following special form of elimination of conjunction is valid:

$$\frac{U \text{ [[} A \text{ or } B \text{] and } A \text{] } V}{U \text{ [} A \text{ or } B \text{] } V}$$

where the conclusion is read in the 'strong' sense.

For example, to derive (57b), it is only required that the following instance of elimination of conjunction is valid:

(63) John is looking for a unicorn or a centaurs and a unicorn

John is looking for a unicorn or a centaurs

Here, the conclusion *John is looking for a unicorn or a centaurs* must be understood in the strong sense that finding a unicorn and finding a centaurs would both satisfy John. We may have no robust intuition about a sentence like the assumption in (63), but it seems that this particular instance, at least, is not immediately objectionable.

The remaining assumption is A3. A3 is a consequence of the standard treatment, but what about our intuition about its validity? Take, for example, *John is looking for a unicorn or a centaurs and a unicorn*. Is it synonymous with *John is looking for a unicorn*? It seems not. If John is looking for a unicorn or a centaurs and a unicorn, John must find either two unicorns or a unicorn and a centaurs

in order to achieve his goal. Then *John is looking for a unicorn or a centaurs and a unicorn* cannot mean the same as *John is looking for a unicorn*. But this is probably because of the indefiniteness of the second occurrence of *a unicorn*. The Ross paradox does not depend on the presence of an indefinite. Take, for a second example, *Pierre wants to learn English or study computer science and learn English*. Unfortunately, it seems that we have no clear intuition about a case like this.

There are various possible ways to avoid A3. One could deny the sameness of meaning between $[[A \text{ or } B] \text{ and } A]$ and A by denying the boolean semantics of *and* and *or*,²⁴ or by invoking something like a structured meaning as their 'hyperintension'. Or one could argue that the surface syntax of $U [[A \text{ or } B] \text{ and } A] V$ is concealing its real 'logical form', by, for example, positing some 'invisible' operator like Kamp-Hilpinen's focus operator. Or one could deny the principle of compositionality and say that although the meanings of $[[A \text{ or } B] \text{ and } A]$ and A are one and the same, the difference in their internal structure causes $U [[A \text{ or } B] \text{ and } A] V$ and $U A V$ to have different meanings (which is equivalent to invoking structured meanings).

Much more work would be needed to diagnose the problem correctly and propose a reasonable treatment, but I must stop here with this incomplete discussion.²⁵

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NOTES

¹ Thus, the present argument would go through even if the logic of *and* and *or* were weaker than classical logic, like intuitionistic logic.

² Similarly, validity of (2) implies validity of (1) under the boolean account. In this sense, (1) and (2) are equivalent.

³ Some people might find reason to object to (4c) and/or (5c). It seems to me, however, that our intuition about the invalidity of (4b) and (5b) do not necessarily depend on acceptance of the validity of (4c) and (5c).

⁴ But the way the puzzles are formulated here is not necessarily the same as in the earlier literature in deontic logic. For example, we are not treating *must* and *may* as sentential operators.

⁵ One could perhaps argue that while a person who *may A and B* (where *and* takes narrow scope) are entitled to do both of *A* and *B*, a person who *may A and may B* might possibly be disallowed to do both, but the intuition is not very clear. I can think of no good reason not to regard *C must A and B* and *C must A and C must B* as always truth-conditionally equivalent. But see section 5 below.

⁶ The following account does not follow Hare (1967) or any others.

⁷ But this might not necessarily constitute a knockdown argument for the conversational account. Consider:

- (i) If John may be working in his office or playing tennis outside, you should check the tennis court first.

This makes sense only if *John may be working in his office or playing tennis outside* implies *John may be playing tennis outside*. Thus, the same kind of phenomenon also occurs in cases which the conversational account is supposed to be able to handle. Then, the notion of conversational implicature should perhaps be adapted to account for cases like this where the sentence in question does not lose its usual implicature when it is embedded in a larger sentence and is not itself asserted.

⁸ Hilpinen only discusses permission sentences, not sentences containing *must*.

⁹ Hilpinen does not mention the wide scope reading. I simply assume that his 'weak permission' reading is distinct from the wide scope reading.

¹⁰ The free choice permission reading is defined as $(If A) \vee (If B), then I$.

¹¹ For example, if we take a necessarily true proposition as *A* and a necessarily false proposition as *B*, and suppose that *I* is false in a given world, then it should be the case that *If A and B, then I* is true and *If A, then I* is false. Hilpinen suggests that the conditional in the definition (19) might not be an 'ordinary conditional', but a special deontic conditional. However, as long as it is given a kind of Lewis type semantics, it is not clear how it could possibly guarantee the possibility of allowing elimination of conjunction in the antecedent.

¹² But see footnote 5. Consider the following dialogue due to Stanley Peters:

- A. May I take this \$100,000?
B. No.
A. May I take this \$100,000 and deposit it in the bank?
B. Yes.

Does this show that permission sentences can sometimes fail to allow elimination of conjunction? Many things can be said about this example. On hearing B's second reply, A can then say:

- A. So I may take this \$100,000.
B. You're right.

Does B contradict himself when he makes his third reply? Or, his third reply is actually consistent with his first reply because the context of A's final utterance is different from that of A's first question and thus B denies and affirms two different propositions? If the latter view is right, then isn't the context of A's second question also different from that of his first question? Can this example show that *A may B and C* and *A may not B* can be consistent in some context, despite the fact that it seems to be always contradictory to say *A may B and C, but A may not B*?

¹³ This assertion might be questionable. If *may* is replaced by *must*, it becomes obvious that the inference does not go through (on one reading):

John must mail the letter.

Therefore: (Either) John or Bill must mail the letter.

This is a problem for Hintikka's account, in addition to those discussed in the next section.

¹⁴ But Kamp (1978) notes that 'You may take an apple or you may take a pear' *can* be read with the same strong sense as (28), although it sounds less natural read in this way.

¹⁵ These constructions should be distinguished from those cases where it is merely the case that narrow scope disjunction is equivalent to wide scope disjunction:

- (i) If I find an apple or a pear, I will take it.

¹⁶ This kind of parallelism is noted by Kamp (1973): “[In] most statements containing ‘or’ which have natural counterparts with ‘any’, the ‘or’ can be regarded as having the force of ‘and’.” (p. 69.)

¹⁷ It seems that this phenomenon is either lacking or far less general in Japanese. The literal, word-to-word translation of (40) does not mean what the English sentence means; it only means ‘John is taller than Bill or John is taller than Mary’. Correspondingly, the translation of (28) into Japanese does not seem to have the strong free choice permission reading, at least not as readily as in English.

¹⁸ The *can* of ‘ability’ is another intensional context that gives rise to wide scope conjunction reading of *or*:

(i) I can drop you at the next corner or drive you to the bus stop. (Kamp 1978, p. 281.)

¹⁹ Some kind of ‘focus’ might well be relevant.

²⁰ Asher (1987) claims that *buletics* have the logic of *would be happy if*, and *need* has the logic of *would be happy only if*. His characterization does not seem to me to be quite accurate.

²¹ Actually, what Asher denies is the inference he calls ‘simplification’, of which elimination of conjunction is a proper subcase, so his example is not itself inadequate for his purposes.

²² Asher also claims that verbs like *hope* and *want* generally support *elimination of disjunction*:

(i) Alexis hopes that she will have chicken or fish for dinner. (With either she will be equally happy.) So she hopes that she will have chicken for dinner. (Asher 1987, p. 172.)

²³ Kamp (1973) considers a similar example with *ought*. (I assume that *ought* shows an inference pattern similar to *must*.) He says that $O(p \wedge q) \rightarrow Op$ is not valid, “if we read *O* as ‘it ought to be the case that’, and understand ‘ought’ as ‘ought, no matter what’. For it could be the case that independently of whatever else may happen the realization of both *p* and *q* would bring about a better state of affairs than would ensue if the conjunction were not realized, while the realization of *p* in the absense of *q* would mean disaster. Perhaps the man standing under the first floor window of the burnng house ought, no matter what, to tell the frightened girl behind it to jump, and catch her in his arms when she does. But if he cannot catch her, or doesn’t want to risk his bones, he certainly ought *not* to tell her to jump.” (pp. 59–60.)

²⁴ We could perhaps try to generalize the semantics of group-denoting conjoined NPs to all conjoinable categories. For example, under this approach, conjoined sentences would stand for ‘plural’ propositions or groups of propositions, and conjoined VPs would stand for groups of properties.

²⁵ One thing I have left out in this paper is examining inferences like the following, which should follow from the validity of elimination of conjunction under the standard account with a reasonable assumption about the semantics of a certain class of adverbs:

(i) John must speak slowly
 —————
 John must speak

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