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Takumi Kongo and Taisuke Matsubae

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Strategy-proof mechanisms and uniqueness of matching

Takumi Kongo*and Taisuke Matsubae†

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Abstract

We resolve the impossibility introduced by [6] and show that, in marriage markets, any mechanism that is individually rational and respects pairwise unanimity is strategy-proof on a restricted domain of preferences. Respect for pairwise unanimity, defined by [6], is interpreted as the minimally-weakened stability property supported by the farsighted viewpoint. Our restriction on domain of preferences requires each agent to prefer oneself the best or the second best. It is supported by both theoretically and empirically and is related to uniqueness of matching that is individually rational and respects pairwise unanimity.

JEL Classification - C71, C72, C78, D71, D78

Keywords - matching, mechanism, strategy-proofness, uniqueness, respect for pairwise unanimity

1 Introduction

Matching is one of the important functions of markets. Who gets which jobs, which school admits which applicants, who marries whom and etc, these problems are very important for those people who are involved in the problems since the outcomes of the problems have a great effect on their lives and careers. The theory of two-sided matching, introduced by [2], analyzes such matching problems between two types of agents, such as between workers and firms, between applicants and universities and between men and women.

One of the most attractive application of the matching theory is to design mechanisms which determine a matching in any markets. Empirical studies have shown that mechanisms satisfying some properties often succeed whereas ones not satisfying the properties often fail in the real world applications. See [4] for evidence.

In designing mechanisms, the stability and strategic properties have critical importance. A mechanism is stable if no agent and no pair of agent have incentive to deviate from the outcome produced by the mechanism and is strategy-proof if no agent can manipulate the outcomes by misrepresenting one's preference. If some agents deviate from the outcome, or can manipulate the outcome,

*Graduate school of Economics, Waseda University. 1-6-1, Nishi-Waseda, Shinjuku-ku, Tokyo 169-8050, Japan. e-mail: kongo_takumi@toki.waseda.jp

†Graduate school of Economics, Waseda University. 1-6-1, Nishi-Waseda, Shinjuku-ku, Tokyo 169-8050, Japan. e-mail:fwin6544@nifty.com

the other agents may not participate in the market thus the two properties are significant for designing mechanisms.

Unfortunately, [3] showed that, for designing matching mechanisms, these two important properties are incompatible. This is often called "impossibility theorem".[1] Even if we weaken the stability property as efficiency and individual rationality, the impossibility still holds ([1]). In order to solve the impossibility, they restricted the domain of preference profiles and proved that if and (essentially) only if the preference profiles satisfy the top dominance condition, any mechanism that produces a stable matching is strategy-proof on the restricted domain. Another approach is introduced by [6]. They introduced the property called respect for unanimity that is weaker than efficiency and showed that there exists a mechanism that respects unanimity, is individually rational and is strategy-proof on the domain of all possible preference profiles. This seems an positive result, but the mechanism obtained here selects the matching in which all agents remain single in most cases. To improve this point, they replaced respect for unanimity as respect for pairwise unanimity that is a stronger property than respect for unanimity but weaker than efficiency, then again, they obtained an impossibility result, that is, there exists no mechanism that respects pairwise unanimity and is strategy-proof on the domain of all possible preference profiles.

In this paper, we resolve the impossibility introduced by [6] by restricting the domain of preference profiles. In our restriction, we pay attention to how each agent evaluates oneself in one's preferences. Previous researches pay little attention to the point, however, in the marriage market, the point is important for the following two reasons. The first, in some situations, agents may prefer remaining single unless they match their ideal partners. The second is the recent trend of a rise of unmarried rate. According to census figures in Japan, the unmarried rates of men in their early 30s/women in their late 20s in 2000 is triple/more than two times and a half than those of in 1975, respectively. The rates of men and women in the other generations has been rising and, in countries in the EU, the same trend is seen. One of the cause of the rise of the rate is that, for many people, the evaluation of staying single is getting higher and higher.

Reflecting these situations, we restrict agents' preference profiles and obtain the positive result, that is, any mechanism that respects pairwise unanimity and is individually rational is strategy-proof on the restricted domain. In addition, we show that there exist no domain of preference profiles that strictly includes the above mentioned domain and on which any mechanism that respects pairwise unanimity and is individually rational is strategy-proof.

Moreover, the above mentioned domain is closely related to uniqueness of matching. We consider the relationships between strategy-proofness of mechanisms and uniqueness of matching.

The paper is constructed as follows. Definitions and notations are provided in Section 2. Main results which resolve the impossibility are presented in Section 3. The relationships between the results and uniqueness of matching are considered in Section 4. A comparison between our result and previous results are mentioned in Section 5. Some remarks are given in Section 6.

[1] We sometimes call a various kind of impossibility theorem "a impossibility."

2 Preliminaries

We consider marriage markets. Let M and W are finite and disjoint sets of men and women, respectively. Each man $m \in M$ has a strict, transitive and complete preference P_m over $W \cup \{m\}$ and each woman $w \in W$ has a strict, transitive and complete preference P_w over $M \cup \{w\}$. For each $m \in M$, Let $\mathcal{P}_m$ be a set of all possible preference on $W \cup \{m\}$ and similarly, for each $w \in W$, let $\mathcal{P}_w$ be a set of all possible preferences on $M \cup \{w\}$. A triple (M, W, P) is a marriage market where $P \in \mathcal{P} = \prod_{i \in M \cup W} \mathcal{P}_i$.

A *matching* μ is a mapping from the set $M \cup W$ onto itself satisfying (i) for all $m \in M$ $\mu(m) \in W \cup \{m\}$, (ii) for all $w \in W$, $\mu(w) \in M \cup \{w\}$ and (iii) for all $i \in M \cup W$, $\mu(\mu(i)) = i$. We call $\mu(i)$ the *mate* of i in μ.

Definition 1 (Individual Rationallity; IR)**.** *A matching μ is individually rational (IR) at preference profile P if $\mu(i) P_i i$ or $\mu(i) = i$ for all $i \in M \cup W$.*

Definition 2 ((Pareto) Efficiency; EF)**.** *A matching μ is (Pareto) Efficient (EF) at preference profile P if there exist no matching $\mu' \neq \mu$ such that for all $i \in M \cup W$ with $\mu'(i) \neq \mu(i)$, $\mu'(i) P_i \mu(i)$.*

Definition 3 (Blocking pair)**.** *A blocking pair of μ at preference profile P is a pair $\{m, w\} \in M \times W$ such that $w P_m \mu(m)$ and $m P_w \mu(w)$.*

Definition 4 (Stablity)**.** *A matching μ is stable at preference profile P if it is individually rational and there exist no blocking pair of μ.*

Note that stability implies IR and EF. But the following example illustrates that the converse does not hold.

Example 1. *Let $M = \{m, m'\}$, $W = \{w, w'\}$ and*

$$P = \{\ w P_m w' P_m m,\ w P_{m'} w' P_{m'} m',\ m' P_w m P_w w,\ m P_{w'} m' P_{w'} w'\ \}.$$

A matching

$$\mu = \begin{pmatrix} m & m' \\ w & w' \end{pmatrix}$$

is IR and EF but not stable since a pair $\{m', w\}$ blocks μ.[2]

Let $\mathcal{M}$ be the set of all possible matching on $M \cup W$. A *mechanism* is a procedure to determine a matching for each marriage market, that is, a mechanism on $\mathcal{P}$ is a mapping ϕ from $\mathcal{P}$ to $\mathcal{M}$.

A mechanism is IR/EF/stable if an outcome yielded by the mechanism is IR/EF/stable for each marriage market, respectively.

For each $i \in M \cup W$, let $\mathcal{Q}_i \subseteq \mathcal{P}_i$ be a subset of all possible preferences of i and let $\mathcal{Q} = \prod_{i \in M \cup W} \mathcal{Q}_i$. By definition, $\mathcal{Q} \subseteq \mathcal{P}$.

Definition 5 (Manipulability)**.** *A mechanism ϕ is manipulable on $\mathcal{Q}$ by a agent $i \in M \cup W$ at $P \in \mathcal{Q}$ via $P'_i \in \mathcal{Q}_i$ if $(\phi(M, W, P/P'_i))(i) P_i (\phi(M, W, P))(i)$ where P/P'_i is a preference profile obtained from P by changing i's preference from P_i to P'_i and keeping all the other preferences.*

[2] These representations of matchings is the same as that of in [5]. In μ, a man and a women on the same vertical are matched to each other and an agent with no mate on its vertical remains single.

Definition 6 (Strategy-proofness; SP)**.** *A mechanism is strategy-proof (SP) on $\mathcal{Q}$ if it is not manipulable at any profile $P \in \mathcal{Q}$ by any agent via any preference.*

In other words, a mechanism is SP if it is a dominant strategy for each agent to announce its true preference.[3]

For designing mechanisms in which agents voluntarily participate, the above properties are of quite importance. If mechanisms are not IR, agents do not participate in the mechanisms. If mechanisms are not EF nor stable, agents deviate from the outcome produced by the mechanisms. If mechanisms are not SP, agents who desire to obtain better outcomes have to consider preferences of the others, but in general, they know nothing about the others' preferences. The following is, however, impossibility of designing such "ideal" mechanisms.

Remark 1 ([3])**.** *There exist no mechanism that is stable and SP on $\mathcal{P}$.*

[1] showed that there exists a possibility of designing a stable and strategy-proof mechanism by restricting the domain of preference profiles. Without restricting the domain of preference profiles, [6] studied the possibility of strategy-proof mechanisms. They relaxed efficiency to the following weaker property.

Definition 7 (Respect for pairwise unanimity; RPU ([6]))**.** *A matching μ respects pairwise unanimity (RPU) if $m \in M$ prefers $w \in W$ the best and w prefers m the best then $\mu(m) = w$ and if $i \in M \cup W$ prefers oneself the best then $\mu(i) = i$.*

Note that EF implies RPU. But, the following example illustrates that the converse does not hold.

Example 2. *Let $M = \{m, m'\}$, $W = \{w, w'\}$ and*

$$P = \{\ wP_m mP_m w',\ wP_{m'} w'P_{m'} m',\ mP_w wP_w m',\ mP_{w'} m'P_{w'} w'\ \}.$$

A matching

$$\mu = \begin{pmatrix} m & m' & - \\ w & - & w' \end{pmatrix}$$

is RPU but not EF since, in a matching

$$\mu' = \begin{pmatrix} m & m' \\ w & w' \end{pmatrix},$$

$\mu'(m') \neq \mu(m)$ and $\mu'(m') P_{m'} \mu(m)$.

To give an interpretation of RPU, let us consider the following example.

Example 3. *Let $M = \{m, m'\}$, $W = \{w, w'\}$ and*

$$P = \{\ w'P_m wP_m m,\ w'P_{m'} wP_{m'} m',\ m'P_w mP_w w,\ m'P_{w'} mP_{w'} w'\ \}.$$

[3] A dominant strategy for an agent is one's preference that gives the agent better outcomes than the outcomes obtained by any other preference for any possible preference profile of the others'.

Let

$$\mu = \begin{pmatrix} m & m' & - \\ w & - & w' \end{pmatrix}.$$

There exist three blocking pairs of μ: $\{m, w'\}, \{m', w\}$ and $\{m', w'\}$. When we consider the stability property, any of the three blocking pairs have the possibility of blocking and deviate from matching μ. However, as we mention the below, the first two blocking pairs actually does not block the matching μ, if agents are not myopic but farsighted, that is, agents concern with not the immediate state obtained from their deviation but the eventual consequence obtained from their immediate deviation.

A deviation of $\{m, w'\}$ from μ induces the matching

$$\mu' = \begin{pmatrix} m & m' & - \\ w' & - & w \end{pmatrix}.$$

But, a matching μ' is blocked by a pair $\{m', w'\}$ hence it becomes

$$\mu'' = \begin{pmatrix} m & m' & - \\ - & w' & w \end{pmatrix}.$$

A matching μ'' is also blocked by a pair $\{m, w\}$ and becomes

$$\mu''' = \begin{pmatrix} m & m' \\ w & w' \end{pmatrix}.$$

There are no blocking pair of μ''' thus, a deviation of $\{m, w'\}$ from μ induces further deviations and the matching obtained in the eventual consequence is the same for m. For the blocking pair $\{m', w\}$, the similar argument holds and the matching obtained in the eventual consequence is the same for w. On the other hand, a deviation of $\{m', w'\}$ from μ induces μ''' directly and this deviation makes both m' and w' strictly better off.

Generally, a deviation of a blocking pair who prefer each other the best makes both of them strictly better off in any eventual consequence, however, a deviation of the other blocking pairs may eventually the same for some of them. Therefore, RPU is the minimally-weakened stability property for a matching.

A mechanism is RPU if an outcome yielded by the mechanism is RPU for each marriage market. As mentioned before, RPU is the minimally-weakened stability property on mechanisms. The following shows that, even if we consider the minimally-weakened stability property, SP and such stability are incompatible in marriage markets.

Remark 2 ([6])**.** *There exists no mechanism that is RPU and SP on $\mathcal{P}$.*

3 Strategy-proof mechanisms

In order to resolve the impossibility introduced by [6], we restrict the domain of preference profiles. It is very important since RPU is the minimally-weakened stability property in marriage market as we mentioned in the previous section.

In our restriction of domain of preference, we pay attention to how each agent evaluates oneself in one's preference. Previous researches pay little attention to the point, however, in marriage market, the point is important for the following two reasons. The first, in some situations, agents may prefer remaining single unless they match their ideal partners. The second is the recent trend of a rise of unmarried rate. According to census figures in Japan, the unmarried rates of men in their early 30s/women in their late 20s in 2000 is triple/more than two times and a half than those of in 1975, respectively. The rates of men and women in the other generations have been rising and the same trend is also observed in countries in the EU. One of the cause of the rise of the rates is that, for many people, the evaluation of staying single is getting higher and higher. Reflecting these situations, we restrict agents' preferences as follows.

For each $i \in M \cup W$, let $\bar{\mathcal{P}}_i \subsetneq \mathcal{P}_i$ be a set of preferences which satisfy "i prefers oneself the best or the second best in i's preference." Let $\bar{\mathcal{P}} = \prod_{i \in M \cup W} \bar{\mathcal{P}}_i$. In this preference profiles, all agents evaluate themselves high rank.

The above restriction of domain of preference profiles induces the following positive result of designing mechanisms in marriage markets.

Theorem 1. *Any mechanism that is IR and RPU is SP on $\bar{\mathcal{P}}$.*

Proof. For any $i \in M \cup W$, if i prefers oneself the best, then by RPU, i matches to oneself when i states i's true preference. This is the best outcome for i, thus i is not manipulable.

If i prefers oneself the second best, then i prefers a member of the opposite sex the best in i's preference. Let j be the member. If i states i's true preference, by RPU, i matches to j if j states that i is the best of j's preference. If j does not state that i is the best of j's preference, by IR, i matches to j or remain single. However, by the restriction on preferences, in this case, j must state that j prefers oneself to i and by IR, j is not matched to i. Consequently, i remains single in this case.

If i represents other preferences with keeping j is the best of i's preference, we can show that the outcomes of the mechanisms are the same as the case of stating true preference in the same manner.

If i changes the other member, say $k \neq j$ of opposite sex as the best of i's preference, by RPU, i matches to k if k states that i is the best of k's preference and, in the same argument of the above, remains single for any other cases. In the cases that j states that i is the best or k states that i is the best, this misrepresentation makes i's outcome worse off. For other cases, the outcome for i is remaining single and this is the same as the case of stating true preference. Thus, this misrepresentation is weakly worse off from stating stating true preference.

If i changes oneself as the best of i's preference, by RPU, i remains single for any others' preferences profile. In the case that j states i is the best, this misrepresentation makes i's outcome worse off. In other cases, the outcome for i is remaining single and it is the same as the case of stating true preference. Thus, this misrepresentation is weakly worse off from stating the true preference.

Therefore, any $i \in M \cup W$ is not manipulable the mechanisms on $\bar{\mathcal{P}}$. □

Note that, the above theorem mentions not a *specific* but *any* mechanism that is IR and RPU. In this sense, the above theorem is stronger than so-called a possibility theorem. Examples of the mechanisms which are IR and RPU are the deferred acceptance mechanism introduced by [2] and the mutually best mechanism mentioned in the proof of Theorem 2 below.

One may consider that $\bar{\mathcal{P}}$ is too restrictive. The following theorem, however, shows that $\bar{\mathcal{P}}$ is the maximal set on which mechanisms that are IR and RPU is SP.

Theorem 2. *There exist no set of preference profile $\hat{\mathcal{P}}$ which strictly includes $\bar{\mathcal{P}}$ and on which any mechanism that is IR and RPU is SP.*

Proof. First, we define the mechanism used in the following proof.

Mutually Best Mechanism

Step 1. Agent who prefers a member of opposite sex the best proposes to the member.

Step 2. Any pair of man and woman who propose with each other get married. Any other man and woman remain single.

It is straightforward that the above mechanism is RPU and IR.

By the fact that $\hat{\mathcal{P}}$ strictly includes $\bar{\mathcal{P}}$, there exist $i \in M \cup W$ whose preference P_i is not in $\bar{\mathcal{P}}_i$, that is, there exists an agent i who prefers at least two members of opposite sex j, k to himself. Without loss of generality, let jP_ik.

In the mutually best mechanism, if i states that i's true preference, i matches to j if j states that i is j's best and remains single otherwise. But, in the case that j states that i is not j's best and k states that i is k's best, i matches to k if i misrepresents i's preference as "k is i's best." Consequently, the mutually best mechanism is manipulable by i at a preference profile in $\hat{\mathcal{P}}$. □

Together with Theorem 1, we prove the necessary and sufficient conditions of the domain of preference profiles on which any mechanism that is IR and RPU is SP.

As we mentioned before, census figures in Japan shows that the number of people who prefer oneself has increased in recent years. Thus, our restriction of preference profiles are justified in both theoretically and empirically.

4 Uniqueness of matching

The positive results obtained in the previous section are closely related to uniqueness of matching. In the domain of preference profiles $\bar{\mathcal{P}}$ that we mentioned in the previous section, the matching that is IR and RPU is unique and, as shown in the end of this section, it implies that the domain of preference profiles on which mechanisms that is IR and RPU is SP is sufficient condition of the domain in which matching that is IR and RPU is unique.

To prove the result, we need the following lemma.

Lemma 1. *In a marriage market (M, W, P) with $P \in \bar{\mathcal{P}}$, any pair of man and woman who match with each other in a matching that is IR at P prefer each other the best.*

Proof. Let μ be a matching that is IR. For any $i \in M \cup W$ with $\mu(i) = j$ and $j \neq i$, IR implies that $\mu(i)P_ii$ and $\mu(j)P_jj$. By the fact that $P \in \bar{\mathcal{P}}$, i prefers oneself to any other $k \neq j, i$ and j prefers oneself to any other $k \neq i, j$ and consequently i and j prefer each other the best. □

By Lemma 1, the following holds.

Theorem 3. *In any marriage market (M, W, P) with $P \in \bar{\mathcal{P}}$, a matching μ that is IR and RPU at P is unique.*

Proof. In a marriage market (M, W, P) with $P \in \bar{\mathcal{P}}$, suppose that there exist two matchings μ, μ' $(\mu \neq \mu')$ each of which is IR and RPU at P. Then, there exists an agent $i \in M \cup W$ such that $\mu(i) \neq \mu'(i)$. Without loss of generality, let $\mu(i) P_i \mu'(i)$.

If $\mu'(i) \neq i$, by Lemma 1 i prefers $\mu'(i)$ the best. This contradicts the fact that there exists another member $\mu(i)$ such that $\mu(i) P_i \mu'(i)$.

If $\mu'(i) = i$, (i) i prefers oneself the best or (ii) there exist a member k in the opposite sex whom i prefers the best does not prefer i the best. In the case of (i), $\mu(i) P_i \mu'(i) = i$ is a contradiction. In the case of (ii), by the supposition $\mu(i) \neq \mu'(i)$, $\mu(i) \neq i$. By Lemma 1, i and $\mu(i)$ prefer each other the best at P. This contradicts that μ' is RPU. □

Unlike Theorem 2, the following example shows that there exists a preference profiles $P \notin \bar{\mathcal{P}}$ on which a matching that is IR and RPU at P is unique.

Example 4. *Let $M = \{m, m'\}$, $W = \{w, w'\}$ and*

$$P = \{\ w P_m w' P_m m,\ w' P_{m'} m' P_{m'} w,\ m P_w w P_w m',\ m' P_{w'} w' P_{w'} m\ \}.$$

In this example, $P_m \notin \bar{\mathcal{P}}_m$ but,

$$\mu = \begin{pmatrix} m & m' \\ w & w' \end{pmatrix}$$

is a unique matching that is IR and RPU at P.

By Theorems 1,2 and 3, we obtain the following.

Corollary 1. *Any marriage market (M, W, P) with $P \in \hat{\mathcal{P}}$ has a unique matching that is IR and RPU at P if any mechanism that is IR and RPU is SP on $\hat{\mathcal{P}}$.*

5 Discussion

Table 1: Related Papers

papers \ properties	IR	RPU	Stability	SP	Preferences
Roth(1982)	○	○	○	×	any
Alcalde & Barberà(1994)	○	○	○	○	restricted
Takagi & Serizawa(2006)	○	○	×	×	any
Our paper	○	○	×	○	restricted

Table 1 summarizes our result and previous researches. Since the result provided by [1] also satisfies IR, RPU and SP on a domain of preference profiles, it is worth mentioning the relationships between the domain considered in [1] and ours.

Definition 8 (Top dominance condition ([1])). *For each $i \in M \cup W$, a set of preferences $\tilde{\mathcal{P}}_i \subseteq \mathcal{P}_i$ satisfies top dominance condition if for any $P_i, P'_i \in \tilde{\mathcal{P}}_i$ and any $x, y \in M \cup W$ such that (i) $xP_i i$ or $x = i$, (ii) $yP'_i i$ or $y = i$ and (iii) $xP_i y$ and $yP'_i x$, then there exists no $z \in M \cup W$ such that $zP_i x$ and $zP'_i y$.*

[1] showed that there exist mechanisms which are stable and SP on the domain of preferences profiles in which all agents in one side satisfy the top dominance condition.

The following example illustrates that our domain of preference profiles implies the domain of preference profiles mentioned in [1], but the converse does not hold.

Example 5. *Let $M = \{m, m'\}$ and $W = \{w, w'\}$. A set of m's preferences*

$$\bar{\mathcal{P}}_m = \{\ wP_m mP_m w',\ w'P'_m mP'_m w,\ mP''_m wP''_m w',\ mP'''_m w'P'''_m w\ \}$$

satisfies both our restriction and the top dominance condition. While a set of m's preferences

$$\tilde{\mathcal{P}}_m = \{\ wP^*_m w'P^*_m m,\ w'P^{**}_m wP^{**}_m m,\ mP''_m wP''_m w'\ \}$$

satisfies the top dominance condition but does not satisfy our restriction.

Even if our domain of preference profiles implies the domain of preference profiles mentioned in [1], Theorem 2 implies that there exist no mechanism that is IR, RPU and SP on the domain of preferences profiles mentioned in [1]. This result is derived from the fact that we consider just a weaker condition than stability.

6 Concluding remarks

In this paper, we resolve the impossibility introduced by [6] by restricting agents preferences in marriage markets. Our restriction is justified by both theoretically and empirically and also is related to uniqueness of matching.

Our results can be extended to college admission markets introduced by [2]. In college admission markets, a set of agents (students or workers) in one side are matched to each agent (college or firm) in the other side.

Let S, C be a set of students and colleges, respectively, and let q_c be a quota of each college $c \in C$. Similar as marriage market, each agent in one side has strict, transitive and complete preferences over the other set and oneself. The following is an example of college admission markets.

Example 6. *Let $S = \{s, s', s''\}$, $C = \{c, c'\}$, $q_c = 1$, $q_{c'} = 2$ and*

$$P = \{\ cP_s sP_s c',\ c'P_{s'} s'P_{s'} c,\ c'P_{s''} s''P_{s''} c,\ sP_c cP_c s'P_c s'',\ s'P_{c'} s''P_{c'} cP_{c'} s\ \}.$$

Since marriage markets are considered as college admission markets with the quota of each college is equal to 1, our restriction of the domain of preference profiles is extended to the following way: "Each student prefers oneself the best or the second best" and "each college c with its quota q_c prefers itself at least q_c+1-th best at its preferences." Note that the preference profiles in Example 6 satisfies this condition and the similar results provided in this paper hold, that is, a matching

$$\mu = \begin{pmatrix} s & s's'' \\ c & c' \end{pmatrix}$$

is a unique matching that is IR and RPU and any mechanism that is IR and RPU is SP on the above mentioned domain of preference profiles.[4]

There are many other variations of definition of college admission markets: the preferences of colleges is defined directly on the collection of sets of students that include students less than its quota and satisfy some properties such as responsiveness or substitutability. Generalizations of our results to the various college admission markets is our future research.

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[4]For details of generalizations of each properties to college admission markets, see Section 3 of [6].