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Bargaining over Managerial Contracts in Delegation Games: The Generalized Oligopolistic Case

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Abstract

This paper analyzes a differentiated-products oligopoly model with two stages in which profit-maximizing owner-shareholders first negotiate compensation with their managers. In the second stage, the managers compete either in quantities or in prices. We show that in the case of sales delegation, market competition increases with an increase in the bargaining power of managers, irrespective of each firm's strategic variables, the degree of product differentiation, and the number of existing firms in the industry. Thus, we find that so long as the technologies of all the firms are represented by constant marginal cost functions and each firm simultaneously chooses its own strategic variable, managers' bargaining power is positively associated with social welfare.

Keywords: Oligopoly with differentiated goods; Managerial delegation; Bargaining

JEL classification: D43; L13; L22

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1 Introduction

This paper presents a case of theoretical bargaining over managerial delegation contracts between owner-shareholders and managers in a differentiated-products market. Recently, codes of corporate governance have prescribed clauses requiring the disclosure of managerial compensation for the protection of shareholders' interests. The protection of the interests of owners is needed in order to restrict the opportunistic behavior of managers, such that their power in the bargaining process is not misused to boost their own remuneration without considering the interests of the shareholders. Although the bargaining power of the managers influences market outcomes, such as the profit of each firm, consumer surplus, and social welfare, as seen in Witteloostuijn et al. (2007) and Nakamura (2007), the literature on the managerial delegation theory has not sufficiently considered the effects on market outcomes through the bargaining process between an owner and a manager in individual firms. This paper aims to fill this gap by contributing to the literature on this subject.

In two well-known articles, Vickers (1985) and Fershtman and Judd (1987) showed that when managers in oligopolistic markets are provided with managerial contracts based on both the firms' profit and sales, owners strategically manipulate incentive parameters toward sales maximization. More precisely, they consider the separation of owners as principals and managers as agents, and analyze a two-stage game, where, the first stage, the owner provides a delegation contract based on the profit and sales to manager, and in the second stage, quantity competition is observed and each manager chooses the level of his or her output. This idea has been developed in several directions after the pioneering works by Vickers, Fershtman, Judd, and Sklivas. Salas Fumas (1992) and Miller and Pazgal (2002) expanded the style of the compensation scheme to that based on a combination of own profit and the total profits of competing firms. In short, they considered the situation in which each manager explicitly takes into account the performance of other firms and found that managers are concerned with the relative performance of their own firms. Subsequently, Jansen et al. (2007) introduced the compensation scheme related to own profit and the market share of the firm's output.

On the other hand, in the literature on managerial delegation, the *Nash bargaining theory* has been applied primarily in the context of wage negotiation ever since the seminal work of Fershtman (1985). Szymanski (1994) and Bughin (1995) extended the scope of this theory to managerial negotiation with labor unions. In his paper, Szymanski showed that owners want managers to act as profit maximizers when cost control through the bargaining is relatively im-

portant. Bughin (1995) found that a wage-maximizing union captures a portion of the rents that accrues from managerial delegation and agrees with the owner's policy to pursue goals apart from profit maximization. In accordance with, Witteloostuijn et al. (2007), in their path-breaking work, considered the effect of the disclosure of managerial compensation on a homogeneous goods market outcomes. For this purpose, they investigated the model that included bargaining between owners and managers in a duopolistic market and showed that market competition is promoted in the case of a sales delegation à la Vickers, Fershtman, Judd, and Sklivas, while it deteriorates in the case of a relative profit, in a Salas Fumas, Miller and Pazgal fashion. Recently, Nakamura (2007) considered the cases of duopolistic competition with regard to both quantity and price in a differentiated-products market and extended the model of Witteloostuijn et al. (2007).

In the present paper, we work under the framework established by Witteloostuijn et al. (2007) and extended by Nakamura (2007). The purpose of the paper is to examine whether or not Nakamura's result (2007) obtained for the duopolistic case will change in accordance with the generalization, with respect to the number of the firms. The literature on the theoretical analysis of the managerial delegation – which focused on the separation between an owner and a manager in a firm, such as in Miller and Pazgal (2002) – has extensively considered not only the duopolistic case but also the oligopolistic case more than two firms. We consider the model in which each firm simultaneously chooses its own strategic variable from both the prospects of both quantity and price competitions. Similar to Witteloostuijn et al. (2007) and Nakamura (2007), in this paper, we find that as the relative bargaining power of managers increases, the equilibrium profit of each firm decreases and the social welfare increases. Thus, we can state that extensive use of the sales delegation generally promotes market competition, irrespective of the firms' strategic variables.

The remainder of this paper is organized as follows. In Section 2, we formulate the basic setting for the delegation game of the two types of models considered in this paper. In Section 3, we consider the case of quantity competition. Section 4 explores the case of price competition, and Section 5 concludes the paper. All proofs of the Propositions are provided in the Appendix.

2 Model

There is an oligopoly in which n firms produce differentiated goods. Let q_i and p_i be the output and price of the i th firm's good. On the demand side of the market, the representative

consumer's utility is denoted as

$$U(\mathbf{q}) = a \left(\sum_{i=1}^n q_i \right) - \frac{1}{2} \left[\sum_{i=1}^n (q_i)^2 + 2b \sum_{i \neq j} q_i q_j \right] + q, \quad b \in (0, 1),$$

where $\mathbf{q} = (q_1, q_2, \dots, q_n; q)$ and b represents the degree of product differentiation.¹ Note that q denotes a numeraire good. The utility function generates the system of linear demand functions:

$$q_i = \frac{1}{(1-b)[1+b(n-1)]} \left\{ (1-b)a - [1+b(n-2)]p_i + b \sum_{j=1, j \neq i}^n p_j \right\}, \quad b \in (0, 1),$$

$$i = 1, 2, \dots, n;$$

and then, the inverse demand functions can be inverted to obtain

$$p_i = a - q_i - b \sum_{j=1, j \neq i}^n q_j, \quad b \in (0, 1), \quad i = 1, 2, \dots, n.$$

We assume that the technologies of all the firms are represented by an identical constant marginal cost, $c (< a)$. The profits of firms i are given by

$$\begin{aligned} \Pi_i &= \left(a - q_i - b \sum_{j=1, j \neq i}^n q_j - c \right) q_i, \\ &= \frac{1}{(1-b)[1+b(n-1)]} \left\{ (1-b)a - [1+b(n-2)]p_i + b \sum_{j=1, j \neq i}^n p_j \right\} (p_i - c), \\ & \quad i, j = 1, 2, \dots, n; \quad i \neq j. \end{aligned}$$

Social welfare, denoted by W , is measured as the sum of consumer surplus (CS) and producer surplus (PS):

$$W = CS + PS,$$

where $PS = \sum_{i=1}^n \Pi_i$, and consumer surplus is given by

$$CS = \frac{1}{2} \left[\sum_{i=1}^n (q_i)^2 + 2b \sum_{i \neq j} q_i q_j \right].$$

An owner's objective is to maximize his or her firm's profit. To this end, the owner of each firm specifies a compensation scheme for the manager that pays him or her the following combination of the firm's profit and its sales:

$$U_i = \Pi_i + \theta_i q_i, \quad \theta_i \in \mathbb{R}, \quad i = 1, 2, \dots, n,$$

¹We assume that $b < 1$ to ensure that the function $U(\mathbf{q})$ is strictly concave; however, the results in quantity competition hold even though each of the two firms produces a single homogeneous good, that is, $b = 1$.

where parameter θ_i measures the relevance of sales. The incentive scheme corresponding to each owner-manager pair becomes known to the rival pair before any production decision is taken. The manager of firm i , given his or her incentive scheme, chooses q_i or p_i that maximizes U_i .

We propose the following two-stage delegation game. In the first stage, the owner and manager in each firm bargain over the incentive parameter θ_i ($i = 1, 2, \dots, n$); subsequently, in the second stage, each of the managers simultaneously decide their outputs or prices according to the type of competition, knowing the level of their own incentive parameters.

The outcomes of owner-manager bargaining over the incentive parameter θ_i are modeled in terms of the generalized two-person Nash bargaining solution which, as shown by Binmore et al. (1986), can be interpreted as the outcomes of a noncooperative (offer/counter-offer) bargaining process in the style proposed by Rubenstein (1982) ($i = 1, 2, \dots, n$). The negotiated settlement of the owner-manager bargaining in firm i coincides with the following asymmetric Nash bargaining solution in terms of each incentive parameter θ_i ($i = 1, 2, \dots, n$):

$$B_i = U_i^\beta \cdot \Pi_i^{1-\beta} \quad i = 1, 2, \dots, n,$$

where $\beta \in [0, 1)$ denotes a measure for the relative bargaining power of the manager. In this paper, we consider the situation in which all managers have identical bargaining powers, β . Moreover, the disagreement point of both the owner and manager is zero; if the bargaining process breaks down, the managers are unable to find new jobs, and the owners cannot run their firms since they do not have enough time or managerial skills.

3 Quantity Competition

In this section, we consider the case of quantity competition. Nakamura (2007) considered the case of duopolistic differentiated-products and showed the robustness of the results of the case involving homogeneous goods in Witteloostuijn et al. (2007). According to Nakamura (2007), there exists a unique subgame perfect equilibrium and, as the incentive parameter β increases, the equilibrium profit of each firm decreases, while the equilibrium consumer surplus and social welfare increases. We confirm the robustness of these results against the generalized number of firms.

First, we analyze the second stage of the game. In this stage, each manager maximizes his or her objective function U_i . Taking into account the problem's symmetry, $\theta_2 = \theta_3 = \dots = \theta_n = \theta$, the first-order condition of the manager of Firm 1 and those of the other firms' managers are

obtained as follows:

$$\begin{cases} \frac{\partial U_1}{\partial \theta_1} = 0 \Leftrightarrow 2q_1 + bQ_{-1} = a - c + \theta_1, \\ \frac{\partial U_i}{\partial \theta_i} = 0 \Leftrightarrow 2q_i + bQ_{-i} = a - c + \theta, \quad i = 2, 3, \dots, n, \end{cases}$$

where $Q_{-i} = \sum_{j \neq i} q_j$. By adding the above conditions for $i \geq 2$, we obtain the following system for q_1 , Q_{-1} , θ_1 and θ :

$$\begin{cases} 2q_1 + bQ_{-1} = a - c + \theta_1, \\ b(n-1)q_1 + [2 + b(n-2)]Q_{-1} = (n-1)(a - c + \theta). \end{cases}$$

Given the values of θ_1 and θ , we obtain the following results from this system.

$$q_1 = \frac{a(2-b) - c(2-b) + b\theta(1-n) + \theta_1(2-2b+bn)}{(2-b)[2+b(n-1)]},$$

$$Q_{-1} = \frac{(n-1)[a(2-b) - c(2-b) + 2\theta - b\theta_1]}{(2-b)[2+b(n-1)]}.$$

Moreover, taking into consideration the first-order condition of the manager of Firm 1, the objective function of the owner of Firm 1 and the profit of Firm 1 are obtained as follows:

$$U_1 = (q_1)^2 \text{ and } \Pi_1 = (q_1)^2 - \theta_1 q_1, \text{ respectively.}$$

Thus, we obtain

$$\frac{\partial U_1}{\partial \theta_1} = \frac{2[2+b(n-2)]}{(2-b)[2+b(n-1)]} q_1 > 0,$$

$$\frac{\partial^2 \Pi_1}{\partial \theta_1^2} = -\frac{8+8b(n-2)+2b^2(n^2-6n+6)-2b^3(n-2)(n-1)}{(2-b)^2[2+b(n-1)]^2} < 0,$$

$$\forall b \in (0, 1], \quad \forall n \in [2, \infty).$$

From the above comparative statics of the respective objective function of an owner and a manager, we find that U_1 is strictly increasing with respect to θ_1 , and Π_1 is a strictly concave function of θ_1 . Since the interest of an owner is different from that of a manager, it is necessary to account for such bargaining between two.

We now move to the analysis of the first stage. From the above analysis of the second stage, the Nash product in first stage is rewritten as follows:

$$B_1 = U_1^\beta \cdot \Pi_1^{1-\beta} = (q_1)^{2\beta} \cdot [(q_1)^2 - \theta_1 q_1]^{1-\beta}.$$

Thus, the first-order condition of the bargaining over Firm 1's incentive parameter is

$$\frac{\partial B_1}{\partial \theta_1} = 0 \Leftrightarrow [2q_1 - \theta_1(1+\beta)] \frac{\partial q_1}{\partial \theta_1} - (1-\beta)q_1 = 0.$$

Utilizing the fact that $\partial q_1/\partial \theta_1 = [2 + b(n - 2)] / (2 - b)[2 + b(n - 1)]$ and the symmetry of the firms, $\theta_1 = \theta$, we obtain the following equilibrium incentive parameter of each firm:

$$\theta^* = \theta_i^* = \frac{4\beta + 2b\beta(n - 2) + b^2(n - 1)(1 - \beta)}{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]} (a - c), \quad i = 1, 2, \dots, n.$$

The equilibrium outcomes are obtained as follows:

$$\begin{aligned} q_i^* &= \frac{[2 - b(2 - n)](1 + \beta)}{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]} (a - c), \\ p_i^* &= \frac{a[2 - b(2 - n) + b^2(1 - n)](1 - \beta) + c[2 - b(4 - 3n) + b^2(2 - 3n + n^2)](1 + \beta)}{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]}, \\ \Pi_i^* &= \frac{[2 + b(n - 2)][2 + b(n - 2) - b^2(n - 1)](1 - \beta)(1 + \beta)}{\{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]\}^2} (a - c)^2, \\ &\quad i = 1, 2, \dots, n, \\ CS^* &= \frac{[2 - b(2 - n)]^2 n(1 + n)(1 + \beta)^2}{2\{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]\}^2} (a - c)^2, \\ W^* &= \frac{[2 + b(n - 2)]n(1 + \beta)\{6 - 2\beta - b[8 - n(5 + \beta)] + b^2(n - 1)[n(1 + \beta) - 4]\}}{2\{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]\}^2} (a - c)^2. \end{aligned}$$

The partial derivative of Π_i^* with respect to β is strictly negative, whereas those of the consumer surplus and social welfare are strictly positive ($i = 1, 2, \dots, n$). Thus, the equilibrium profit of each firm decreases, while the consumer surplus and social welfare increase with an increase in the relative bargaining power of managers. The preceding result is summarized in Proposition 1.

Proposition 1. *If each owner in a symmetric Cournot oligopoly hires a manager who receives a payoff through the managerial incentive contract based on profits and sales, and if the incentive parameter on sales is the outcome of a bargaining process, then the corresponding delegation game will have a unique subgame perfect Nash equilibrium that depends on the bargaining power β of the managers. Moreover, if the bargaining power of the managers increases, the equilibrium profit of each firm will decrease, while consumer surplus and social welfare will increase.*

Proof. See the Appendix. □

With respect to Proposition 1, Witteloostuijn et al. (2007) and Nakamura (2007) obtained similar results. The intuition of this proposition is given as follows: Each firm's equilibrium output increases with an increase in the relative bargaining power of managers, β ; This is because a high value of β leads to more aggressive production by each firm. Thus, the equilibrium consumer surplus increases. Obviously, this fact has a positive impact on the social welfare. On

the other hand, there is a decrease in the equilibrium profit of each firm, i.e., each owner's objective function, because an increase in the value of β implies the manager's bargaining power declines. So long as all firms' technologies are represented by constant marginal cost functions, the former positive effect strongly influences social welfare compared to the latter negative effect, irrespective of the degree of product differentiation and the number of the existing firms in the industry.

4 Price Competition

In this section, we consider the case of price competition. Nakamura (2007) extended the duopolistic quantity competition in which both the firms simultaneously produce a homogeneous good (Witteloostuijn et al. 2007) to the simultaneous-move game of the duopolistic price competition in which each firm produces a differentiated good.

We first analyze the second stage of the game. Exploiting the symmetry of the firms, $\theta_2 = \theta_3 = \dots = \theta_n = \theta$, the first-order condition of each manager of the firm is denoted as

$$\begin{cases} \frac{a(1-b)-p_1[1+b(n-2)]+bP_{-1}-(p_1-c+\theta_1)[1+b(n-2)]}{(1-b)[1+b(n-1)]} = 0, \\ \frac{a(1-b)-p_i[1+b(n-2)]+bP_{-i}-(p_i-c+\theta_i)[1+b(n-2)]}{(1-b)[1+b(n-1)]} = 0 \quad i = 2, 3, \dots, n. \end{cases}$$

By adding the equations for $i \geq 2$, we obtain the following system for p_1 , P_{-1} , θ_1 and θ :

$$\begin{cases} \frac{a(1-b)-p_1[1+b(n-2)]+bP_{-1}-(p_1-c+\theta_1)[1+b(n-2)]}{(1-b)[1+b(n-1)]} = 0, \\ \frac{a(1-b)(n-1)-bp_1(n-1)-P_{-1}[2+b(n-2)]+(n-1)[1+b(n-2)](c-\theta)}{(1-b)[1+b(n-1)]} = 0. \end{cases}$$

Given the values of θ_1 and θ , we obtain the following result from the above system:

$$p_1 = \frac{a(1-b)[2+b(2n-3)] + [1+b(n-2)]\{c[2+b(2n-3)] - 2\theta_1 - b[\theta_1(n-2) + \theta(n-1)]\}}{4 + 6b(n-2) + b^2(2n^2 - 9n + 9)},$$

$$P_{-1} = \frac{(n-1)(a(1-b)[2+b(2n-3)] + [1+b(n-2)]\{c[2+b(2n-3)] - 2\theta_1 - b[\theta_1(n-2) + \theta(n-1)]\})}{4 + 6b(n-2) + b^2(2n^2 - 9n + 9)}.$$

Furthermore, taking into account the first-order condition of each manager, we obtain the following result:

$$U_1 = \frac{\{a(1-b) - p_1[1+b(n-2)] + bP_{-1}\}^2}{(1-b)[1+b(n-1)][1+b(n-2)]},$$

$$\Pi_1 = \frac{\{a(1-b)-p_1[1+b(n-2)]+bP_{-1}\}\{a(1-b)-p_1[1+b(n-2)]+bP_{-1}-\theta_1[1+b(n-2)]\}}{(1-b)[1+b(n-2)][1+b(n-1)]}.$$

Thus, we get

$$\frac{\partial U_1}{\partial \theta_1} = \frac{2[2 + 3b(n-2) + b^2(n^2 - 5n + 5)]}{4 + 6b(n-2) + b^2(2n^2 - 9n + 9)} q_1 > 0,$$

$$\frac{\partial^2 \Pi_1}{\partial \theta_1^2} = -\frac{2[1+b(n-2)]^2[4+8b(n-2)+b^2(5n^2-22n+22)+b^3(n^3-7n^2+15n-10)]}{(1-b)[1+b(n-1)][4+6b(n-2)+b^2(2n^2-9n+9)]} < 0,$$

$$\forall b \in (0, 1), \quad \forall n \in [2, \infty).$$

In short, we find that U_1 is strictly increasing with respect to θ_1 , and Π_1 is a concave function of θ_1 . The interest of the owner of Firm 1 is different from that of the manager of Firm 1. Therefore, the bargaining between them over the incentive parameter stands to reason.

Next, we consider the second stage of the game. From the above analysis of the first stage, the bargaining problem in Firm 1 is reduced as follows:

$$\begin{aligned} B_1 &= U_1^\beta \cdot \Pi_1^{1-\beta} \\ &= \left(\frac{\{a(1-b) - p_1[1+b(n-2)] + bP_{-1}\}^2}{(1-b)[1+b(n-2)][1+b(n-1)]} \right)^\beta \\ &\quad \times \left(\frac{\{a(1-b) - p_1[1+b(n-2)] + bP_{-1}\} \{a(1-b) - p_1[1+b(n-2)] + bP_{-1} - \theta_1[1+b(n-2)]\}}{(1-b)[1+b(n-2)][1+b(n-1)]} \right)^{1-\beta}. \end{aligned}$$

Taking into account the problem's symmetry, $\theta_1 = \theta$ again, the first-order condition of the bargaining in Firm 1 is obtained as follows:

$$\begin{aligned} \frac{\partial B_1}{\partial \theta_1} &= \theta [1+b(n-2)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\} \\ &\quad - (1-b)(a-c) \{4\beta+6b\beta(n-2) + b^2[2n^2\beta-n(1+9\beta)+1+9\beta]\} = 0, \end{aligned}$$

yielding

$$\begin{aligned} \theta^* &= \theta_i^* = \frac{(1-b) \{4\beta+6b(-2+n)\beta + b^2[1+9\beta+2n^2\beta-n(1+9\beta)]\} (a-c)}{[1+b(n-2)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\}}, \\ &i = 1, 2, \dots, n. \end{aligned}$$

We obtain the equilibrium outcomes as follows:

$$\begin{aligned} p_i^* &= \frac{a(1-b)[2+b(n-2)](1-\beta) + c[2+3b(n-2)+b^2(n^2-5n+5)](1-\beta)}{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]}, \\ q_i^* &= \frac{[2+3b(n-2)+b^2(n^2-5n+5)](1+\beta)(a-c)}{[1+b(n-1)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\}}, \\ \Pi_i^* &= \frac{(1-b)[2+b(n-2)][2+3b(n-2)+b^2(n^2-5n+5)](1-\beta)(1+\beta)(a-c)^2}{[1+b(n-1)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\}^2}, \\ &i = 1, 2, \dots, n, \\ CS^* &= \frac{n[2+3b(n-2)+b^2(n^2-5n+5)](1+\beta)^2(a-c)^2}{2[1+b(n-1)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\}^2}, \\ W^* &= \frac{n[2+3b(n-2)+b^2(n^2-5n+5)](1+\beta) \{6-2\beta+b[n(5+\beta)-2(7-\beta)] + b^2[n^2(1+\beta)-n(7+3\beta)+9+\beta]\}}{2[1+b(n-1)] \{4+2b[n(2+\beta)-5-\beta] + b^2[n^2(1+\beta)-2n(3+2\beta)+7+3\beta]\}^2} (a-c)^2. \end{aligned}$$

Similar to the quantity competition, the partial derivative of Π_i^* with respect to β is strictly negative, whereas those of the consumer surplus and social welfare are strictly positive ($i = 1, 2, \dots, n$). Thus, the equilibrium profit of each firm decreases, while the consumer surplus and social welfare increase with an increase in the relative bargaining power of managers. The preceding result is summarized in Proposition 2.

Proposition 2. *If each owner in a symmetric Bertrand oligopoly hires a manager who receives a payoff through the managerial delegation contract based on profits and sales, and if the incentive parameter on sales is the outcome of a bargaining process, then the corresponding delegation game has a unique subgame perfect Nash equilibrium, which depends on the bargaining power β of the managers. Moreover, if the bargaining power of the managers increases, the equilibrium profit of each firm decreases, while consumer surplus and social welfare increases.*

Proof. See the Appendix. □

Compared to Proposition 1, Proposition 2 shows that the property of the sales delegation - that the increase of the bargaining power of managers implies the promotion of market competition - is robust against a change in the competition structure and the number of firms.

5 Conclusion

This paper has examined the bargaining between owners and managers over managerial incentive contracts for the case of sales delegation in two types of differentiated-products competitions where the generalized numbered symmetric firms exist. We investigate whether the results of Nakamura (2007), who considered the case of the duopolistic competition are in accordance with those of this study. With respect to the number of firms, we showed that in both quantity and price competitions for sales delegation under the general condition, the equilibrium profit of each firm decreases while the consumer surplus and social welfare increases with an increase in the relative bargaining power of managers, β . Our findings strengthened the results obtained in Nakamura (2007) and Witteloostuijn et al. (2007), which the bargaining power of managers is positively associated with the equilibrium consumer surplus and social welfare, while it is negatively associated with the profit of each firm.

Analogous to many existing literatures in this field, this present study assumed that the technologies of all the firms are represented by constant marginal cost functions. We realize that this assumption is very restrictive. It is necessary to explore the case of increasing marginal

cost and confirm whether or not the results in this paper are robust against such an extension. Moreover, we should focus on the other compensation schemes, for example, the cases of the relative profit delegation and market share delegation. These are left for future research.

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Appendix

Proof of Proposition 1

We first show that Π^* is strictly decreasing in the relative bargaining power of the manager, β , irrespective of the values of $b \in (0, 1]$ and $\beta \in [0, 1)$. The partial derivative of Π^* with respect to β is given as follows:

$$\frac{\partial \Pi^*}{\partial \beta} = \frac{-2[2+b(n-2)][2+b(n-2)-b^2(n-1)]\{4\beta+2b[n-1+\beta(2n-3)]+b^2(n-1)[n-1+\beta(n-3)]\}}{\{4+2b[n(2+\beta)-3-\beta]+b^2(n-1)[n-3+\beta(n-1)]\}^3} (a-c)^2.$$

Taking $n \geq 2$ into consideration, we obtain the following result:

$$\begin{aligned} & 4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)] \\ & > 4 + 2b(1 + \beta) - b^2(1 - \beta) > 0. \quad \forall b \in (0, 1], \quad \forall \beta \in [0, 1). \end{aligned}$$

Thus, we find that the denominator is strictly positive, Moreover, we obtain

$$\begin{aligned} & 2 + b(n - 2) \geq 2 > 0, \quad 2 + b(n - 2) - b^2(n - 1) \geq 2 - b^2 > 0, \\ & 4\beta + 2b[n - 1 + \beta(2n - 3)] + b^2(n - 1)[n - 1 + \beta(n - 3)] \geq 2b(1 + \beta) + b^2(1 - \beta) > 0, \\ & \forall b \in (0, 1], \quad \forall \beta \in [0, 1). \end{aligned}$$

Thus, we find that the numerator is strictly negative. Therefore, we recognize that $\partial \Pi^* / \partial \beta < 0$ for all $b \in (0, 1]$ and $\beta \in [0, 1)$.

Next, we show that W^* is strictly increasing in the relative bargaining power of the manager, β , irrespective of the values of $b \in$ and $\beta \in [0, 1)$. The partial derivative of W^* with respect to β is denoted as

$$\frac{\partial W^*}{\partial \beta} = \frac{2n(1 - \beta)[2 + b(n - 2) - b^2(n - 1)]^2}{\{4 + 2b[n(2 + \beta) - 3 - \beta] + b^2(n - 1)[n - 3 + \beta(n - 1)]\}^3} (a - c)^2.$$

Since the denominator is as same as that of Π^* , we find that it is strictly positive. On the other hand, the numerator is obviously strictly positive. Thus, we find that $\partial W^* / \partial \beta > 0$ for all $b \in (0, 1]$ and $\beta \in [0, 1)$. \square

Proof of Proposition 2

First, we show that Π^* is strictly decreasing in the relative bargaining power of the manager, β , irrespective of the values $b \in (0, 1)$ and $\beta \in [0, 1)$. The partial derivative of Π^* with respect to β is given by

$$\frac{\partial \Pi^*}{\partial \beta} = \frac{\left(\begin{array}{c} -2(1-b)[2+b(n-2)][2+3b(n-2)+b^2(n^2-5n+5)] \\ \times \{b(2n-2)+b^2(n-1)(n-3)+\beta[4+b(4n-10)+b^2(n^2-6n+7)]\} \end{array} \right)}{[1+b(n-1)]\{4+b(4n-10)+b^2(7-6n+n^2)+\beta[b(2n-2)+b^2(n^2-4n+3)]\}^3} (a-c)^2.$$

With respect to the denominator, we obtain the following result:

$$\begin{aligned} & 4 + b(4n - 10) + b^2(7 - 6n + n^2) + \beta[b(2n - 2) + b^2(n^2 - 4n + 3)] \\ & \geq 4 - 2b - b^2 + b\beta(2 - b) > 0, \quad \forall b \in (0, 1), \quad \forall \beta \in [0, 1). \end{aligned}$$

Considering that $n \geq 2$, we find that the denominator is strictly positive. On the other hand, Considering the following results:

$$\begin{aligned} & 2 + b(n - 2) \geq 2 > 0, \\ & 2 + 3b(n - 2) + b^2(n^2 - 5n + 5) \geq 2 - b^2 > 0, \\ & b(2n - 2) + b^2(n - 1)(n - 3) + \beta[4 + b(4n - 10) + b^2(n^2 - 6n + 7)] \\ & \geq 2b - b^2 + \beta(4 - 2b - b^2) > 0. \quad \forall b \in (0, 1), \quad \forall \beta \in [0, 1). \end{aligned}$$

Thus, we find that $\partial \Pi^*/\partial \beta$ is strictly negative for all the values of $b \in (0, 1)$ and $\beta \in [0, 1)$.

Second, we show that W^* is strictly increasing in the relative bargaining power of the manager, β , irrespective of the values $b \in (0, 1)$ and $\beta \in [0, 1)$. The partial derivative of W^* with respect to β is as follows:

$$\frac{\partial W^*}{\partial \beta} = \frac{2n(1-\beta)(1-b)^2[2+b(n-2)]^2[2+3b(n-2)+b^2(n^2-5n+5)]}{[1+b(n-1)]\{4+b(4n-10)+b^2(7-6n+n^2)+\beta[b(2n-2)+b^2(n^2-4n+3)]\}^3} (a-c)^2.$$

Analogous to the case of quantity competition, the denominator is the same as that of Π^* . Thus, we find that it is strictly positive. Moreover, we obtain the following results:

$$2 + 3b(n - 2) + b^2(n^2 - 5n + 5) \geq 2 - b^2 > 0, \quad \forall b \in (0, 1).$$

Thus, we find that the numerator is strictly positive. Therefore, we recognize that $\partial W^*/\partial \beta > 0$ for all $b \in (0, 1)$ and $\beta \in [0, 1)$, similar to the case in the quantity competition. \square