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Yasuhiko Nakamura

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# Endogenous Timing in a Mixed Duopoly with Managerial Delegation: A Quadratic Cost Case

Yasuhiko Nakamura*

*Graduate School of Economics, Waseda University, Japan*

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**Abstract**

This paper analyzes the endogenous timing problems in various mixed duopolistic industries with managerial delegation, where the technology of each firm is represented by a quadratic cost function. We show that in price competition the result is unchanged against the cost function type, whereas in quantity competition, we obtain the result that the equilibrium is unique and then the public firm tends to become the follower.

*Keywords*: Mixed duopoly; Managerial delegation; Endogenous timing; Quadratic cost function

*JEL classification*: D43; L13; L32

*Corresponding address: Graduate School of Economics, Waseda University, 1-6-1, Nishi-waseda, Shinjuku-ku, Tokyo 169-8050, Japan; E-mail: `yasu-net.system@asagi.waseda.jp`; *Tel.* (+81) 3-3208-8560.

# 1 Introduction

This paper provides a theoretical analysis on managerial delegation in the oligopoly composed not only of a private firm that maximizes its own profit but also of a public firm that is a welfare maximizer. This type of industry is referred to as *mixed duopoly.* We analyze various competitive environments in the mixed duopoly where the technologies of both public and private firms are represented by quadratic cost functions, and attempt to explain the question of whether the endogenous choice of roles played by managerial firms may actually lead to situations in which these firms move sequentially in the market game.

Today, we observe oligopolistic markets in which private and public firms compete in many industries across many countries, regardless of the worldwide privatization of public enterprises. Research on mixed oligopoly has burgeoned since the pioneering paper by **?**. In the traditional analyses on mixed oligopoly similar to them, the structure of market competition is assumed beforehand, and thus depending on the order of moves based on the assumption, public and private firms set either output or price. However, it is more realistic to assume that firms not only choose the necessary actions to be taken but also decide on the time that these actions need to be taken. In particular, on the case of mixed oligopoly (duopoly), since the sequential or simultaneous order of firms' moves may give rise to significantly different results, it is important to investigate the model in which the time at which output or price should be chosen by each firm is endogenized. **?** first discussed endogenous timing in mixed markets using the observable delay game of **?**, wherein each firm chooses the time to take necessary actions before producing and cannot produce over more than one period. According to **?**, private firms take the lead in an equilibrium for the game with more than two periods; further, in the case of a two-period mixed duopoly, a public firm can be the leader. **?** and **?** extended Pal's seminal work (1998) to consider that the public firm competes with foreign private firms. Although the three papers mentioned above examine quantity competition, **?** analyzed price competition with differentiated goods wherein firms chose to set prices either sequentially or simultaneously. This paper also focuses on the role of the endogenous timing of firms' moves using the observable delay game along the lines of these works.

In the literature on private oligopoly, recent researchers have criticized the view that firms are entities whose sole objectives are to maximize their own profits. **?** and **?**, the pioneering works in this field, explicitly investigated the effects of delegation and distortion of managerial preferences on the competitive performance on firms. They considered profit maximizing firms that hire managers who do not maximize profits. Concretely, they explored two-stage duopoly models where in the first stage, each firm's owner provides a delegation contract for his or her manager, which is a linear combination of profits and sales. On the other hand, in the literature on mixed markets, the analyses on separating the ownership and management within a firm have become increasingly popular in recent years. **?** first modeled the incentive contracts, à la Fershtman and Judd and Sklivas, into a mixed duopolistic industry. **?** solely emphasized the strategic benefits of managerial incentive contracts, whereas Barros considered both the strategic effects and asymmetric information. **?**, as another recent work in the context of managerial delegation contracts modeled after contracts proposed by Fershtman and Judd and Sklivas, analyzed the intersection between the length of incentive contracts and market behavior with a two-period mixed oligopolistic framework. In particular, White considered a two-stage game of mixed oligopoly with a particular order of exogenous moves wherein managerial incentive parameters were chosen before making the choice with regard to quantity. More precisely, he investigated that both owners of firms first choose incentive parameters for their managers simultaneously, and then the managers simultaneously choose their quantities. In this paper, we basically formulate the model based on the one suggested in **?**. The distinction between our model and his model is that the stage in which firms' owners choose the periods of when to set their outputs or prices is added to our model as the pre-play stage in the observable delay game.

As mentioned above, it is only recently that the topic of how managerial firms can improve their own objectives began to be seriously considered in the context of managerial incentive contracts. In particular, for a long time, no attention has been devoted to the question concerning the type of competition that occurs in the equilibrium when endogenous timing choices made by managerial firms are taken into account. In the literature on private oligopoly (duopoly), **?** first addressed the issue of timing in a game between managerial firms. He modeled the situation in which owners provide their managers with incentive contracts that deviate from owners' objectives, *i.e.*, maximizing their own profits, and then the managers choose their outputs or prices in the period that their corresponding owners announce, using the observable delay game. Just like **?**, **?** and **?** examined quantity competition and price competition, respectively, in the context of mixed duopoly. In both the papers, it was assumed that technologies of both public and private firms are represented by constant marginal cost functions.

The purpose of this paper is to examine whether the results derived in Nakamura and Inoue (2007*b*, 2007*c*) are robust when the cost functions of both the firms are quadratic. Moreover, we consider various environments in mixed duopoly. To be more precise, we examine the quantity competition with homogeneous goods under the moderate cost condition and the price and quantity competitions with differentiated goods under the somewhat specific cost condition. **?** obtained the same result as that in **?** in that public and private firms choose quantities sequentially in equilibrium. Thus, their model had the two equilibria in which the public firm became the leader or the follower. In contrast, we show that the case in which the public firm is the follower tends to be a unique equilibrium in the two types of quantity competitions with homogeneous and differentiated goods when the cost functions of both the firms are symmetric and quadratic. Note that in the competition with homogeneous goods, the case in which the public firm takes the lead can also be an equilibrium if both the public firm and private firm are adequately efficient; in other words, parameter $k$ (described later) is sufficiently low. In such a case, we find that there are multiple equilibria, and thus, the result coincides with that in **?**. With respect to price competition, **?** showed that both the public firm and the private firm take the action as late as possible; therefore, in equilibrium, they simultaneously set their prices at the late period. Their result was strikingly different from that in **?** who considered the case in which owners directly manage their respective firms. In the analysis of the case in which technologies of both the firms are represented by symmetric and quadratic cost functions, we obtain the same result as that in **?**. Thus, in the mixed duopoly with managerial delegation wherein both the firms choose their prices, we find that the result of the observable delay game is robust against the change in production technologies, whereas the result of quantity competition is sensitive to such a change.

The remainder of the paper is organized as follows. In Section 2, we formulate the basic setting on the managerial delegation and the observable delay game of the three types of models considered in this paper. In Section 3, we consider the case of quantity competition with homogeneous goods under the weak cost condition. In Section 4, we investigate the two cases of price and quantity competition with differentiated goods. Section 5 concludes with some remarks.

## 2 Basic Setting

This paper focuses on the managerial aspects of firms. We consider a well-known principal-agent framework in which each owner hires a manager. Suppose that each firm comprises an owner and a manager. Thus, we consider the situation in which the firms' owners decide to delegate control over his or her own assets to managers. To formalize managerial delegation, we follow **?** and **?**. We assume that the manager of Firm $i$ maximizes the following function $V_i(\Pi_i, q_i)$:

$$V_i(\Pi_i, q_i) = \Pi_i + \theta_i q_i \qquad \theta_i \in \mathbb{R}, \quad i = 0, 1,$$

where parameter $\theta_i$ represents the degree of the relevance of sales. In this delegation regime, the manager of Firm $i$ can maximize his or her payoff by choosing $q_i$ that maximizes $V_i$. This can be supported by the assumption that the payoff to the manager of Firm $i$ is represented as $\lambda_i + \mu_i V_i$ for some real number $\lambda_i$ and some positive number $\mu_i$. This type of delegation scheme functions as a commitment device, since it is common knowledge before the managers compete against each other. Similar to most literature on managerial delegation, we assume that the effect of the managers' payoffs on profits is negligible, since we emphasize the influence of incentive contracts on market outcomes.

We adopt the observable delay game in the context of a mixed duopoly. The observable delay game consists of a a pre-play stage and a subsequent mixed duopolistic game. In the pre-play stage, the firms' owners simultaneously announce whether their managers will choose their own quantities or prices late or early and the managers are committed to the choice before the market competition stages. After both the owners' announcements, each manager sets his or her own quantity or price according to the choice of his or her owner in the pre-play stage, estimating the time when the opponent will set his or her quantity or price.

Formally, the game runs as follows: Note that only the third stage consists of two periods, Period 1 and Period 2. In the first stage, Owner $i$ independently chooses $t_i \in \{1, 2\}$, where $t_i$ indicates the time of the third stage at which his or her strategic variables should be set ($i = 0, 1$). $t_i = 1$ implies that Firm $i$'s manager sets his or her quantity or price early, and $t_i = 2$ implies that he or she sets his or her quantity or price late. In the second stage, after observing the opponent's timing decision, Owners 0 and 1 simultaneously set their respective firm's value of $\theta_i$. In the third stage, the manager of Firm $i$ selects his or her quantity or price at Period $i$ that Owner $i$ chooses in the first stage ($i = 0, 1$). If both the owners decide to choose the same period, then the simultaneous competition in quantity or price follows. Otherwise, there occurs the Stackelberg competition in which the firm having chosen Period 1 becomes the leader. At the end of the game, the market opens and each firm sells its own product. We adopt a subgame perfect Nash equilibrium, and thus, the game is solved backward.

# 3 Competition with Homogeneous Goods

In this section, we consider the competition with homogeneous goods. The duopolists produce perfectly substitutable commodities for which the market demand function is linear: $P = a - Q$ (price as a function of quantity). Let $q_i$ denote the output of Firm $i$ and both firms share identical quadratic cost functions: $C(q_i) = (1/2)\, kq_i$, $i = 0, 1$, and $k > 0$. In the remainder of this paper, we often refer to the public firm as Firm 0 (private firm as Firm 1) and the owner of the public firm as Owner 0 (owner of private firm as Owner 1). The profit function of Firm $i$ is denoted by

$$\Pi_i = (a - q_0 - q_1)q_i - \frac{1}{2}k\,(q_i)^2 \qquad i = 0, 1.$$

Social welfare, denoted by $W$, is measured as the sum of consumer surplus (denoted by $CS$) and producer surplus (denoted by $PS$).

$$W = CS + PS,$$

where $CS = Q^2/2$ and $PS = \Pi_0 + \Pi_1$. Owner 0 (the public firm's owner) is assumed to be a welfare maximizer, while Owner 1 (the private firm's owner) is assumed to maximize his or her own profit.

## 3.1 Fixed Timing Games

We explore the three types of competitions considered in this paper, two Stackelberg (one is the case of the public firm's leader and the other is the case of the private firm's leader) and one Cournot duopolistic case, distinguished by the two owners' choices of timings at which their firms will produce.

To begin with, we consider the Cournot duopoly (denoted by superscript $S$) that occurs when both the owners select the same periods with each other, *i.e.*, the case of $(t_0, t_1) = (1, 1)$ or $(2, 2)$. In this case, each manager independently chooses $q_i$ to maximize $V_i$ $(i = 0, 1)$. We obtain the reaction function of Firm $i$ from the first-order condition for Firm $i$ as follows:

$$q_i = \frac{a - q_j + \theta_0}{2 + k} \qquad i, j = 0, 1 \ ; \ i \neq j. \tag{1}$$

Solving the system of (**??**) for $q_0$ and $q_1$, we obtain the following equilibrium for given $\theta_i$ $(i = 0, 1)$.

$$q_i^S(\theta_0, \theta_1) = \frac{a(1 + k) + \theta_i(2 + k) - \theta_j}{(1 + k)(3 + k)} \qquad i, j = 0, 1 \ ; \ i \neq j.$$

In the second stage, Owner 0 maximizes the reduced welfare function $W\left(q_i^S(\theta_0, \theta_1); i = 0, 1\right) = \widehat{W}^S(\theta_0, \theta_1)$, while the objective of Owner 1 is to maximize $\widehat{\Pi}_1^S(q_i(\theta_0, \theta_1); i = 0, 1) = \Pi_1(\theta_0, \theta_1)$. Owner 0's first-order condition with respect to $\theta_0$ is given as:

$$\frac{\partial \widehat{W}^S(\theta_0, \theta_1)}{\partial \theta_0} = \frac{a(1 + k)^2 - \theta_0\left(1 + 7k + 5k^2 + k^3\right) - \theta_1\left(1 - 2k - k^2\right)}{(3 + 4k + k^2)^2} = 0, \tag{2}$$

while Owner 1's counterpart is given as:

$$\frac{\partial \widehat{\Pi}_1^S(\theta_0, \theta_1)}{\partial \theta_1} = \frac{a(1 + k) - \theta_0 - \theta_1\left(4 + 10k + 6k^2 + k^3\right)}{(3 + 4k + k^2)^2} = 0. \tag{3}$$

Solving (**??**) and (**??**) with respect to $\theta_0$ and $\theta_1$ simultaneously, we obtain the equilibrium incentive parameter of each firm as follows:

$$\theta_0^{S*} = \frac{a(1 + 5k + 4k^2 + k^3)}{1 + 12k + 16k^2 + 7k^3 + k^4}, \quad \theta_1^{S*} = \frac{ak(2 + k)}{1 + 12k + 16k^2 + 7k^3 + k^4},$$

where the asterisk (*) represents the equilibrium outcomes in each subgame. The equilibrium incentive parameters of both Owners 0 and 1 are positive for all $k > 0$, and $\theta_0^{S*} > \theta_1^{S*}$. This is because Owner 0 cares about consumer surplus and thus tends to make his or her firm produce aggressively. In particular, the former property is different from that seen in **?**, who investigated the case in which technologies of both the firms are represented by constant marginal cost functions, and the public firm is less efficient than the private firm.

The equilibrium outputs, prices, profits, consumer surplus, and social welfare are as follows:

$$q_0^{S*} = \frac{a\left(1 + 6k + 5k^2 + k^3\right)}{1 + 12k + 16k^2 + 7k^3 + k^4}, \quad q_1^{S*} = \frac{ak(2 + k)^2}{1 + 12k + 16k^2 + 7k^3 + k^4},$$

$$p^{S*} = \frac{ak\left(2 + 7k + 5k^2 + k^3\right)}{1 + 12k + 16k^2 + 7k^3 + k^4}, \quad \Pi_0^{S*} = \frac{a^2k\left(3 + 26k + 68k^2 + 74k^3 + 39k^4 + 10k^5 + k^6\right)}{2\left(1 + 12k + 16k^2 + 7k^3 + k^4\right)^2},$$

$$\Pi_1^{S*} = \frac{a^2 k^2 (2+k)^3 \left(2+4k+k^2\right)}{2\left(1+12k+16k^2+7k^3+k^4\right)^2}, \quad CS^{S*} = \frac{a^2 \left(1+10k+9k^2+2k^3\right)^2}{2\left(1+12k+16k^2+7k^3+k^4\right)^2},$$
$$W^{S*} = \frac{a^2 \left(1+23k+160k^2+308k^3+263k^4+113k^5+24k^6+2k^7\right)}{2\left(1+12k+16k^2+7k^3+k^4\right)^2}.$$

Next, we consider the subgame in which the manager of Firm $i$ takes the lead ($i = 0, 1$). In the market stage, by symmetry, we can confine ourselves to the case in which the manager of Firm $i$ takes the lead, while Firm $j$'s manager follows, *i.e.*, $(t_i, t_j) = (1, 2)$, $(i, j = 0, 1 \; ; \; i \neq j)$. Since the manager of Firm $i$ takes Firm $j$'s reaction function $q_j(q_i)$ in (**??**) into account, the objective function for him or her to maximize is simplified as follows:

$$\widehat{V}_i(q_i) = V_i(q_i, q_j(q_i)) \qquad i, j = 0, 1 \; ; \; i \neq j. \tag{4}$$

Thus, the leader's first-order condition is given by:

$$\frac{d\widehat{V}_i(q_i)}{dq_i} = \frac{a(1+k) - q_i\left(2+4k+k^2\right) + \theta_i(2+k) - \theta_j}{2+k} = 0,$$

yielding

$$q_i^L(\theta_i, \theta_j) = \frac{a(1+k) + \theta_i(2+k) - \theta_j}{2+4k+k^2} \qquad i, j = 0, 1 \; ; \; i \neq j, \tag{5}$$

where superscript $L$ denotes the leader of the firm's manager in outputs. Substituting (**??**) into (**??**), we obtain the follower's output:

$$q_j^F(\theta_i, \theta_j) = \frac{a\left(1+3k+k^2\right) - \theta_i(2+k) + \theta_j\left(3+4k+k^2\right)}{4+10k+6k^2+k^3} \qquad i, j = 0, 1 \; ; \; i \neq j,$$

where superscript $F$ denotes the follower.

Since the objective functions of Owner 0 and 1 are different, the stage in which Owner $i$ independently chooses his or her incentive parameter $\theta_i$ is asymmetric, dependent on the order of moves of firms' managers ($i = 0, 1$). First, we consider each owner's decision of his or her incentive parameter in the subgame where Firm 0's manager becomes the leader in the market stage. Owner 0 chooses $\theta_0$ to maximize $W\left(q_0^L(\theta_0, \theta_1), q_1^F(\theta_0, \theta_1)\right) = \widehat{W}^L(\theta_0, \theta_1)$, while Owner 1 chooses $\theta_1$ to maximize $\Pi_1\left(q_0^L(\theta_0, \theta_1), q_1^F(\theta_0, \theta_1)\right) = \widehat{\Pi}_1^F(\theta_0, \theta_1)$, independent of each other. Each of their first-order conditions is represented as follows:

$$\frac{\partial \widehat{W}^L(\theta_0, \theta_1)}{\partial \theta_0} = \frac{a\left(1+2k+3k^2+k^3\right) - \theta_0\left(2+15k+17k^2+7k^3+k^4\right) - \theta_1\left(1-3k-4k^2-k^3\right)}{(2+k)\left(2+4k+k^2\right)^2}$$
$$= 0, \tag{6}$$

$$\frac{\partial \widehat{\Pi}_1^F(\theta_0, \theta_1)}{\partial \theta_1} = \frac{a\left(1+3k+k^2\right) - \theta_0(2+k) - \theta_1\left(3+16k+20k^2+8k^3+k^4\right)}{(2+k)\left(2+4k+k^2\right)^2} = 0. \tag{7}$$

Solving (**??**) and (**??**) with respect to $\theta_0$ and $\theta_1$ simultaneously, we obtain the equilibrium incentive parameters as follows:

$$\theta_0^{L*} = \frac{a\left(1+9k+18k^2+17k^3+7k^4+k^5\right)}{2+37k+96k^2+97k^3+47k^4+11k^5+k^6}, \quad \theta_1^{F*} = \frac{ak(2+k)^2}{1+18k+39k^2+29k^3+9k^4+k^5}.$$

We find that the incentive parameters of both the owners are positive and $\theta_0^{L*} > \theta_1^{F*}$, analogous to the simultaneous-move case.

The respective values in the equilibrium are as follows:

$$
\begin{aligned}
q_0^{L*} &= \frac{a\left(1+10k+15k^2+7k^3+k^4\right)}{1+18k+39k^2+29k^3+9k^4+k^5}, \quad q_1^{F*} = \frac{ak\,(2+k)\left(3+4k+k^2\right)}{1+18k+39k^2+29k^3+9k^4+k^5}, \\
p^{L*} &= \frac{ak\,(2+k)\left(1+6k+5k^2+k^3\right)}{1+18k+39k^2+29k^3+9k^4+k^5}, \\
\Pi_0^{L*} &= \frac{a^2k\left(3+46k+222k^2+438k^3+441k^4+250k^5+81k^6+14k^7+k^8\right)}{2\left(1+18k+39k^2+29k^3+9k^4+k^5\right)^2}, \\
\Pi_1^{F*} &= \frac{a^2k^2\,(2+k)^3\left(3+16k+20k^2+8k^3+k^4\right)}{2\left(1+18k+39k^2+29k^3+9k^4+k^5\right)^2}, \\
CS^{L*} &= \frac{a^2\left(1+16k+26k^2+13k^3+2k^4\right)^2}{2\left(1+18k+39k^2+29k^3+9k^4+k^5\right)^2}, \\
W^{L*} &= \frac{a^2\left(1+35k+378k^2+1244k^3+1904k^4+1584k^5+763k^6+213k^7+32k^8+2k^9\right)}{2\left(1+18k+39k^2+29k^3+9k^4+k^5\right)^2}.
\end{aligned}
$$

Note that in the remainder of the paper, the equilibrium price, consumer surplus, and social welfare are denoted by the superscript based on the move of the manager of Firm 0.

Second, we consider each owner's decision of his or her incentive parameter in the subgame where Firm 1 is the leader. Owner 0 chooses $\theta_0$ to maximize $W\left(q_0^F(\theta_0,\theta_1), q_1^L(\theta_0,\theta_1)\right) = \widehat{W}^F(\theta_0,\theta_1)$, while Owner 1 chooses $\theta_1$ to maximize $\Pi_1\left(q_0^F(\theta_0,\theta_1), q_1^L(\theta_0,\theta_1)\right) = \widehat{\Pi}_1^L(\theta_0,\theta_1)$, independent of each other. Each of their first-order conditions is represented as follows:

$$
\begin{aligned}
\frac{\partial \widehat{W}^F(\theta_0,\theta_1)}{\partial \theta_0} &= \frac{a\left(1+8k+12k^2+6k^3+k^4\right)-\theta_0\left(1+19k+39k^2+29k^3+9k^4+k^5\right)-\theta_1\left(2-5k-11k^2-6k^3-k^4\right)}{\left(4+10k+6k^2+k^3\right)^2} \\
&= 0, \qquad (8)
\end{aligned}
$$

$$
\frac{\partial \widehat{\Pi}^L(\theta_0,\theta_1)}{\partial \theta_1} = -\frac{\theta_1\,(2+k)}{2+4k+k^2} = 0. \qquad (9)
$$

From (**??**), we directly obtain $\theta_1^F = 0$ as the equilibrium incentive parameter of Firm 1. On the other hand, substituting $\theta_1^F = 0$ into (**??**), we obtain the equilibrium incentive parameters as follows:

$$
\theta_0^{F*} = \frac{a\left(1+7k+5k^2+k^3\right)}{1+18k+21k^2+8k^3+k^4}, \qquad \theta_1^{L*} = 0.
$$

We find that Owner 1 sets $\theta_1^{L*} = 0$ independent of $\theta_0$. Since Owner 1, a profit maximizer is the leader in the market stage, he or she cannot increase the profit of Firm 1 by adjusting his or her incentive parameter. On the other hand, the equilibrium incentive parameter of Owner 0 is positive, similar to the above two cases.

$$
\begin{aligned}
q_0^{F*} &= \frac{a\left(1+9k+6k^2+k^3\right)}{1+18k+21k^2+8k^3+k^4}, \quad q_1^{L*} = \frac{ak\left(6+5k+k^2\right)}{1+18k+21k^2+8k^3+k^4}, \\
p^{F*} &= \frac{ak\left(3+10k+6k^2++k^3\right)}{1+18k+21k^2+8k^3+k^4}, \quad \Pi_0^{F*} = \frac{a^2k\left(5+56k+135k^2+126k^3+56k^4+12k^5+k^6\right)}{2\left(1+18k+21k^2+8k^3+k^4\right)^2},
\end{aligned}
$$

$$\Pi_1^{F*} = \frac{a^2k^2(3+k)^2\left(4+10k+6k^2+k^3\right)}{2\left(1+18k+21k^2+8k^3+k^4\right)^2}, \quad CS^{F*} = \frac{a^2\left(1+15k+11k^2+2k^3\right)^2}{2\left(1+18k+21k^2+8k^3+k^4\right)^2},$$
$$W^{F*} = \frac{a^2\left(1+17k+12k^2+2k^3\right)}{2\left(1+18k+21k^2+8k^3+k^4\right)}.$$

## 3.2 Equilibrium in the Observable Delay Game

In this subsection, we intend to investigate the equilibria in our observable delay game in which the move of each firm's manager is endogenized. For this purpose, we compare the equilibrium market outcomes among the three subgames.

**Lemma 1.** *The equilibrium values of three subgames are ranked as follows:*

*(i)* $\theta_0^{S*} > \theta_0^{F*} > \theta_0^{L*}$ *and* $\theta_1^{F*} > \theta_1^{S*} > \theta_1^{L*}$.

*(ii)* $q_0^{S*} > q_0^{L*} > q_0^{F*}$ *and* $q_1^{F*} > q_1^{L*} > q_1^{S*}$.

*(iii)* $p^{F*} > p^{S*} > p^{L*}$.

*(iv)* $\Pi_0^{F*} > \Pi_0^{S*} > \Pi_0^{L*}$.

*(v)* $\Pi_1^{L*} > \Pi_1^{F*} \geq \Pi_1^{S*}$, $\forall k \in (0,\ 0.324929]$ *and* $\Pi_1^{L*} > \Pi_1^{S*} > \Pi_1^{F*}$, $\forall k \in (0.324929,\ \infty)$.

*(vi)* $CS^{L*} > CS^{S*} > CS^{F*}$ *and* $W^{L*} > W^{F*} > W^{S*}$.

If the owners directly manage their respective firms (in other words, $V_0$ and $V_1$ are replaced by $W$ and $\Pi_1$, respectively), the payoff ranking of the three subgames is given as follows:

$$\text{Owner 0:} \quad \widetilde{W}^{F*} > \widetilde{W}^{L*} > \widetilde{W}^{S*},$$
$$\text{Owner 1:} \quad \begin{cases} \widetilde{\Pi}_1^{L*} \geq \widetilde{\Pi}_1^{F*} > \widetilde{\Pi}_1^{S*} & \text{if} \quad k \in (0, 0.0921936], \\ \widetilde{\Pi}_1^{F*} > \widetilde{\Pi}_1^{L*} > \widetilde{\Pi}_1^{S*} & \text{if} \quad k \in (0.0921936, \infty), \end{cases}$$

where the tilde ($\tilde{}$) represents the case in which there is no delegation and the firms are managed by their owners. The ranking of Owner 0 is different from that in the introduction of the delegation. In the no-delegation case, Owner 0 whose objective is to maximize social welfare, wishes to become the follower in order to increase the opponent's output and profit by reducing his or her own firm's output slightly. However, in the delegation case, Owner 0 desires to take the lead since he or she cannot directly control his or her firm's output. On the other hand, the ranking of Owner 1 also changes in the sufficiently wide interval of parameter $k$ since the increase of Firm 1's output reduces its profit because of the convexity of the cost function.

Taking into account Lemma 1, we obtain the following proposition.

**Proposition 1.** *In the three-stage game with managerial delegation, the subgame perfect Nash equilibria are classified into two cases, dependent on the value of parameter $k$,* *(i)* $(t_0^*, t_1^*) = (1,2)$ *and* $(2,1)$ *if* $0 < k \leq 0.324929$,

*(ii)* $(t_0^*, t_1^*) = (2,1)$ *if* $k > 0.324929$.

By Lemma 1, Owner 0 wishes to set his or her firm's output in the period that is different from that when Owner 1 does. On the other hand, the strategy of Owner 1 as the optimal reaction against Owner 0's strategy changes dependent on the value of $k$. Thus, in our model, the subgame perfect Nash equilibria also depend on $k$. When the value of $k$ is sufficiently low, $k \in (0, 0.324929]$, there are two equilibria, $(t_0^*, t_1^*) = (2,1)$ and $(1,2)$. Otherwise, $(t_0^*, t_1^*) = (2,1)$ is the unique equilibrium. In the very wide area of $k$, we obtain the result that the equilibrium is unique. In particular, the latter result is strikingly different from that in obtained **?** who analyzed the case in which the technologies of both the firms are represented by the constant marginal cost functions and the public firm is less efficient than the private firm.

# 4 Product Differeniation

The next structure we consider is differentiated products mixed duopoly with linear demand and quadratic cost. The basic structure of the model follows Dixit (1979) and Singh and Vives (1984). On the demand side of the market, the representative consumer's utility is a quadratic function of two differentiated goods, $q_0$ and $q_1$, and a linear function of a numeraire good, $q$.

$$U = a(q_0 + q_1) - \frac{1}{2}\left[(q_0)^2 + 2bq_0q_1 + (q_1)^2\right] + q, \qquad b \in (0,1).$$

where the parameter $b$ measures the degree of product differentiation. The utility function implies the system of linear demand functions:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \qquad i,j = 0,1\ ;\ i \neq j.$$

Under the condition of $b < 1$, these direct demand functions can be inverted to obtain

$$p_i = a - q_i - bq_j, \qquad i,j = 0,1\ ;\ i \neq j.$$

We assume that the technologies of both the firms are specified by the quadratic functions with respect to their own outputs, *i.e.*, $(q_i)^2$ $(i = 0,1)$. Note that in the context of mixed duopoly with both of homogeneous and differentiated goods, this type of cost function is used in **?** and **?**. The profit function of Firm $i$ is given by:

$$\begin{aligned}
\Pi_i &= (a - q_i - bq_j)\,q_i - (q_i)^2, \\
&= \frac{\left[a(-1+b) + p_i - bp_j\right]\left[a(1-b) - \left(2-b^2\right)p_i + bp_j\right]}{\left(1-b^2\right)^2}, \qquad i,j = 0,1\ ;\ i \neq j.
\end{aligned}$$

Social welfare is defined by the sum of consumer surplus (denoted by $CS$) and producer surplus (denoted by $PS$). The producer surplus is represented by $PS = \Pi_0 + \Pi_1$, while the consumer surplus is denoted as:

$$\begin{aligned} CS &= \frac{1}{2}\left[ (q_0)^2 + 2bq_0q_1 + (q_1)^2 \right], \\ &= \frac{2a^2(1-b) + (p_0)^2 - 2bp_0p_1 + (p_1)^2 - 2a(1-b)(p_0+p_1)}{2(1-b^2)}. \end{aligned}$$

Thus, Social welfare $W$ is indicated by

$$W = CS + PS.$$

## 4.1 Price Competition

In this subsection, we consider a model of differentiated products mixed duopoly wherein the firms compete in terms of prices. To begin with, we consider the simultaneous-move game of firms' managers. Given the incentive parameters of both the firms, from the maximization problem of each firm's manager, we obtain the reaction function of Firm $i$ as follows:

$$p_i = \frac{a\left(3 - 3b - b^2 + b^3\right) + bp_j\left(3 - b^2\right) - \theta_i\left(1 - b^2\right)}{2\left(2 - b^2\right)}, \qquad i, j = 0, 1\ ;\ i \neq j, \tag{10}$$

yielding

$$p_i^S = \frac{a\left(12 - 3b - 7b^2 + b^3 + b^4\right) - 2\theta_i\left(2 - b^2\right) - b\theta_j\left(3 - b^2\right)}{16 - 9b^2 + b^4}, \qquad i, j = 0, 1\ ;\ i \neq j.$$

In the second stage, Owner 1 maximizes $W\left(p_i\left(\theta_0, \theta_1\right); i = 0, 1\right) = \widehat{W}^S\left(\theta_0, \theta_1\right)$, while the objective of Owner 1 is to maximize $\Pi\left(p_i\left(\theta_0, \theta_1\right); i = 0, 1\right) = \widehat{\Pi}\left(\theta_0, \theta_1\right)$. Solving these two problems simultaneously, we obtain the equilibrium incentive parameters of both the firms as follows:

$$\begin{aligned} \theta_0^S &= \frac{a\left(64 - 32b - 96b^2 + 36b^3 + 39b^4 - 11b^5 - 5b^6 + b^7\right)}{192 - 128b^2 + 25b^4 - b^6}, \\ \theta_1^S &= -\frac{ab^2\left(36 - 12b - 21b^2 + 7b^3 + 3b^4 - b^5\right)}{192 - 128b^2 + 25b^4 - b^6}. \end{aligned}$$

The equilibrium incentive parameter of Owner 1 is negative for any $b \in (0, 1)$, whereas that of Owner 0 is positive in the wide area of $b$ ($0 < b < 0.91131$). Owner 1 aims to maximize his or her own profit, and thus, he or she attempts to raise his or her price by reducing the incentive parameter. On the other hand, Owner 0 takes consumer surplus into consideration, since his or her objective is to maximize social welfare. Accordingly, Owner 0 reduces his or her price by increasing the incentive parameter. However, when the value of $b$ is very high (in other words, each of the two goods produced by Firms 0 and 1 is nearly homogeneous), Owner 0 attempts to maintain a certain level of his or her price by decreasing the incentive parameter, since intense price competition results in excessive decrease in producer surplus.

In this case, the respective values in the equilibrium are as follows:

$$q_0^{S*} = \frac{a\left(64-20b-36b^2+10b^3+5b^4-b^5\right)}{192-128b^2+25b^4-b^6}, \quad q_1^{S*} = \frac{a\left(3-b\right)\left(4-b^2\right)^2}{192-128b^2+25b^4-b^6},$$
$$p_0^{S*} = \frac{2a\left(64-14b-38b^2+7b^3+6b^4-b^5\right)}{192-128b^2+25b^4-b^6}, \quad p_1^{S} = \frac{4a\left(36-12b-21b^2+7b^3+3b^4-b^5\right)}{192-128b^2+25b^4-b^6},$$
$$\Pi_0^{S*} = \frac{a^2\left(4096-1792b-4704b^2+1984b^3+2084b^4-852b^5-384b^6+166b^7+21b^8-12b^9+b^{10}\right)}{\left(192-128b^2+25b^4-b^6\right)^2},$$
$$\Pi_1^{S} = \frac{a^2\left(3-b\right)^2\left(4-b^2\right)^3\left(8-3b^2\right)}{\left(192-128b^2+25b^4-b^6\right)^2},$$
$$CS^{S*} = \frac{a^2\left(3200+1024b-5112b^2-816b^3+3120b^4+164b^5-914b^6+22b^7+129b^8-11b^9-7b^{10}+b^{11}\right)}{\left(192-128b^2+25b^4-b^6\right)^2},$$
$$W^{S*} = \frac{a^2\left(11904-3840b-14488b^2+4624b^3+6752b^4-2128b^5-1454b^6+452b^7+133b^8-41b^9-3b^{10}+b^{11}\right)}{\left(192-128b^2+25b^4-b^6\right)^2}.$$

Next, we consider the subgame in which the manager of Firm $i$ takes the lead $(i=0,1)$. The manager of Firm $i$ chooses the value of $p_i$ that maximizes $\widehat{V}_i\left(p_i\right) = V_i\left(p_i, p_j\left(p_i\right)\right)$, taking into account (**??**) $(i,j=0,1;\ i \neq j)$. Solving this problem, we obtain:

$$p_i^L = \frac{2a\left(12-3b-7b^2+b^3+b^4\right)-\theta_i\left(8-6b^2+b^4\right)-2b\theta_j\left(3-b^2\right)}{32-20b^2+3b^4}, \qquad i,j=0,1\ ;\ i \neq j.$$

Furthermore, we obtain the equilibrium follower's price as follows:

$$p_j^F = \frac{a\left(48-12b-31b^2+7b^3+5b^4-b^5\right)-b\theta_i\left(12+7b^2-b^4\right)-\theta_j\left(16-9b^2+b^4\right)}{64-40b^2+6b^4},$$
$$j=0,1;\ j \neq i.$$

Since the objectives of Owners 0 and 1 are different (in other words, the objective of Owner 0 is to maximize social welfare, while that of Owner 1 is to maximize his or her own profit), the two subgames in which the manager of Firm $i$ takes the lead are asymmetric $(i=0,1)$. Thus, in the remainder of this section, we must consider the two subgames as different ones. First, we consider each owner's determination of his or her incentive parameter in the subgame where the manager of Firm 0 takes the lead. Each owner $i$ chooses the value of $\theta_i$, taking both of the firms' prices in the market stage into account $(i=0,1)$. We obtain the equilibrium incentive parameters of both the owners as follows:

$$\theta_0^{L*} = \frac{a\left(1024-512b-1088b^2+496b^3+380b^4-153b^5-47b^6+15b^7+b^8\right)}{3072-3008b^2+1076b^4-165b^6+9b^8},$$
$$\theta_1^{F*} = -\frac{4ab^2\left(36-12b-21b^2+7b^3+3b^4-b^5\right)}{768-560b^2+129b^4-9b^6}.$$

The equilibrium incentive parameter of Owner 1 is negative, while that of Owner 0 is positive for any $b \in (0,1)$. The behavior of Owner 1 is the same as that obtained in the simultaneous-move case. On the other hand, since the manager of Firm 0 takes the lead in this case, Owner 0 always keeps the incentive parameter positive in order to make his or her own firm's price low and to increase consumer surplus, regardless of what the value of $b$ is.

In this case, we obtain the equilibrium market outcomes as follows:

$$q_0^{L*} = \frac{a\left(256 - 80b - 160b^2 + 45b^3 + 28b^4 - 6b^5 - b^6\right)}{768 - 560b^2 + 129b^4 - 9b^6}, \quad q_1^{F*} = \frac{a\,(3-b)\left(4-b^2\right)\left(16-5b^2\right)}{768 - 560b^2 + 129b^4 - 9b^6},$$

$$p_0^{L*} = \frac{a\left(512 - 112b - 336b^2 + 63b^3 + 65b^4 - 9b^5 - 3b^6\right)}{768 - 560b^2 + 129b^4 - 9b^6},$$

$$p_1^{F*} = \frac{a\left(4-b^2\right)\left(144 - 48b - 57b^2 + 19b^3 + 3b^4 - b^5\right)}{768 - 560b^2 + 129b^4 - 9b^6},$$

$$\Pi_0^{L*} = \frac{a^2\left(65536-28672b-83456b^2+35328b^3+41920b^4-16960b^5-10374b^6+3897b^7+1289b^8-414b^9-75b^{10}+15b^{11}+2b^{12}\right)}{\left(768-560b^2+129b^4-9b^6\right)^2},$$

$$\Pi_1^{F*} = \frac{a^2\left(12 - 4b - 3b^2 + b^3\right)^2\left(512 - 384b^2 + 86b^4 - 5b^6\right)}{\left(768 - 560b^2 + 129b^4 - 9b^6\right)^2},$$

$$CS^{L*} = \frac{a^2\left(102400+32768b-176384b^2-30208b^3+119024b^4+7904b^5-39879b^6+336b^7+6819b^8-402b^9-527b^{10}+42b^{11}+11b^{12}\right)}{2\left(768-560b^2+129b^4-9b^6\right)^2},$$

$$W^{L*} = \frac{a^2\left(380928-122880b-511232b^2+163328b^3+271664b^4-85536b^5-71443b^6+21954b^7+9361b^8-2742b^9-515b^{10}+132b^{11}+5b^{12}\right)}{2\left(768-560b^2+129b^4-9b^6\right)^2}.$$

Second, we consider the subgame in which the manager of Firm 1 becomes the leader. Similar to the case in which the manager of Firm 0 is the leader, we obtain the equilibrium incentive parameters of both the owners as follows:

$$\theta_0^{F*} = \frac{a\left(256 - 128b - 400b^2 + 164b^3 + 173b^4 - 61b^5 - 23b^6 + 7b^7\right)}{768 - 560b^2 + 123b^4 - 7b^6}, \quad \theta_1^{L*} = 0.$$

By analogy to the competition with homogeneous goods, when the manager of Firm 1 becomes the leader in the ensuing market stage, Owner 1 decides that it is optimal not to allow any further price increase. With regard to Owner 0, similar to the simultaneous-move case, his or her equilibrium incentive parameter is positive in the wide interval of $b$ $(0 < b < 0.925761)$.

The equilibrium market outcomes are obtained as follows:

$$q_0^{F*} = \frac{a\left(256 - 80b - 160b^2 + 48b^3 + 25b^4 - 7b^5\right)}{768 - 560b^2 + 123b^4 - 7b^6}, \quad q_1^{L*} = \frac{a\left(192 - 64b - 108b^2 + 36b^3 + 15b^4 - 5b^5\right)}{768 - 560b^2 + 123b^4 - 7b^6},$$

$$p_0^{F*} = \frac{2a\left(256 - 56b - 168b^2 + 30b^3 + 31b^4 - 4b^5 - b^6\right)}{768 - 560b^2 + 123b^4 - 7b^6},$$

$$p_1^{L*} = \frac{4a\left(144 - 48b - 93b^2 + 31b^3 + 15b^4 - 5b^5\right)}{768 - 560b^2 + 123b^4 - 7b^6},$$

$$\Pi_0^{F*} = \frac{a^2\left(65536-28672b-83456b^2+34560b^3+41536b^4-16176b^5-9952b^6+3628b^7+1113b^8-380b^9-43b^{10}+14b^{11}\right)}{\left(768-560b^2+123b^4-7b^6\right)^2},$$

$$\Pi_1^{L*} = \frac{a^2\left(48 - 16b - 15b^2 + 5b^3\right)^2\left(32 - 20b^2 + 3b^4\right)^2}{\left(768 - 560b^2 + 123b^4 - 7b^6\right)^2},$$

$$CS^{F*} = \frac{a^2\left(51200+16384b-88192b^2-14336b^3+59080b^4+2688b^5-19308b^6+876b^7+3085b^8-367b^9-193b^{10}+35b^{11}\right)}{\left(768-560b^2+123b^4-7b^6\right)^2},$$

$$W^{F*} = \frac{a^2\left(248 - 80b - 152b^2 + 48b^3 + 23b^4 - 7b^5\right)}{768 - 560b^2 + 123b^4 - 7b^6}.$$

By comparing the equilibrium market outcomes in the above three subgames, the following results are obtained.

**Lemma 2.** *The equilibrium values of the three subgames are ranked as follows:*

(i) $\theta_0^{L*} > \theta_0^{F*} > \theta_0^{S*}$ *and* $\theta_1^{L*} > \theta_1^{S*} > \theta_1^{F*}$.

(ii) $q_0^{F*} > q_0^{L*} > q_0^{S*}$ *and* $q_1^{S*} > q_1^{L*} > q_1^{F*}$.

(iii) $p_0^{L*} > p_0^{S*} > p_0^{F*}$ *and* $p_1^{F*} > p_1^{S*} > p_1^{L*}$.

(iv) $\Pi_0^{L*} > \Pi_0^{S*} > \Pi_0^{F*}$ *and* $\Pi_1^{F*} > \Pi_1^{S*} > \Pi_1^{L*}$.

(v) $CS^{F*} > CS^{S*} > CS^{L*}$ *and* $W^{F*} > W^{S*} > W^{L*}$.

The ranking of the equilibrium incentive parameters and market outcomes are the same as those in Nakamura and Inoue (2007) who considered the case in which technologies of both the firms are represented by constant marginal cost functions. Thus, in the context of the observable delay game of a price-setting mixed duopoly with managerial delegation, we find that the results are robust against the cost conditions of both the firms. The rank orders of the equilibrium profits of both the firms are identical to those of the corresponding prices, whereas in consumer surplus and social welfare, the rankings are inverted.

**Proposition 2.** *In the unique subgame perfect Nash equilibrium,* $(t_0^*, t_1^*) = (2, 2)$.

By Lemma 2, Owners 0 and 1 want to set their firms' prices after his or her opponent does. Thus, we obtain the result that it is a dominant strategy for each firm to set price at $t = 2$. Owner 0 prefers the case in which lower prices are observed since he or she wants to increase consumer surplus, while Owner 1 wishes to realize higher prices. Therefore, the pair of each owner's strategy, $(t_0, t_1) = (2, 2)$, is the dominant strategy for each owner, resulting in the pair of equilibrium strategy in this model.

## 4.2 Quantity Competition

In this subsection, we consider differentiated products quantity competition with linear demand and quadratic cost. Analogous to the price competition, the following analysis proceeds. In the subgame in which each of the managers of the firms simultaneously chooses his or her output, the equilibrium incentive parameters are as follows:

$$\theta_0^{S*} = \frac{a\left(8 - 4b + b^2\right)}{24 - b^2}, \quad \theta_1^{S*} = \frac{4ab^2\left(3 - b\right)}{192 - 32b^2 + b^4}.$$

We find that $\theta_i^{S*} > 0$ $(i = 0, 1)$, and $\theta_0^{S*} > \theta_1^{S*}$ for any $b \in (0, 1)$. As is mentioned above, the latter result shows that Owner 0 wishes to expand output more aggressively than Owner 1 because of the difference of their objectives.

The equilibrium market outcomes are as follows:

$$q_0^{S*} = \frac{a\left(64 - 20b - 4b^2 + b^3\right)}{192 - 32b^2 + b^4}, \quad q_1^{S*} = \frac{16a\left(3 - b\right)}{192 - 32b^2 + b^4},$$

$$p_0^{S*} = \frac{a\left(128 - 28b - 12b^2 - b^3 + b^4\right)}{192 - 32b^2 + b^4}, \quad p_1^{S*} = \frac{4a\left(36 - 12b - 3b^2 + b^3\right)}{192 - 32b^2 + b^4},$$

$$\Pi_0^{S*} = \frac{a^2\left(4096 - 1792b - 608b^2 + 128b^3 + 128b^4 - 20b^5 - 6b^6 + b^7\right)}{\left(192 - 32b^2 + b^4\right)^2},$$

$$\Pi_1^{S*} = \frac{64a^2\left(3 - b\right)^2}{\left(24 - b^2\right)^2\left(8 - b^2\right)}, \quad CS^{S*} = \frac{a^2\left(6400 + 2048b - 3824b^2 + 544b^3 + 200b^4 - 40b^5 + b^6\right)}{2\left(192 - 32b^2 + b^4\right)^2},$$

$$W^{S*} = \frac{a^2\left(23808 - 7680b - 5168b^2 + 1568b^3 + 328b^4 - 80b^5 - 11b^6 + 2b^7\right)}{2\left(192 - 32b^2 + b^4\right)^2}.$$

Second, we consider the subgame in which the manager of Firm 0 becomes the leader. In this case, the equilibrium incentive parameters are as follows:

$$\theta_0^{L*} = \frac{a\left(1024 - 512b - 320b^2 + 176b^3 + 12b^4 - 9b^5\right)}{4\left(768 - 176b^2 + 9b^4\right)}, \qquad \theta_1^{F*} = \frac{16ab^2\left(3 - b\right)}{768 - 176b^2 + 9b^4}. \tag{11}$$

Similar to the simultaneous-move case, we find that $\theta_0^{L*}, \theta_1^{F*} > 0$, and $\theta_0^{L*} > \theta_1^{F*}$ for any $b \in (0, 1)$.

In this case, taking the values of $\theta_0^{L*}$ and $\theta_1^{F*}$ into account, we obtain the equilibrium values in the market as follows:

$$q_0^{L*} = \frac{a\left(256 - 80b - 32b^2 + 9b^3\right)}{768 - 176b^2 + 9b^4}, \quad q_1^{F*} = \frac{4a\left(3 - b\right)\left(16 - b^2\right)}{768 - 176b^2 + 9b^4},$$

$$p_0^{L*} = \frac{a\left(512 - 112b - 80b^2 + 3b^3 + 5b^4\right)}{768 - 176b^2 + 9b^4},$$

$$p_1^{F*} = \frac{4a\left(144 - 48b - 21b^2 + 7b^3\right)}{768 - 176b^2 + 9b^4},$$

$$\Pi_0^{L*} = \frac{a^2\left(65536 - 28672b - 17920b^2 + 5632b^3 + 3008b^4 - 640b^5 - 214b^6 + 45b^7\right)}{\left(768 - 176b^2 + 9b^4\right)^2},$$

$$\Pi_1^{F*} = \frac{32a^2\left(3 - b\right)^2\left(256 - 64b^2 + 3b^4\right)}{\left(768 - 176b^2 + 9b^4\right)^2},$$

$$CS^{L*} = \frac{a^2\left(4 - b\right)^2\left(6400 + 5248b - 2400b^2 - 1240b^3 + 201b^4 + 72b^5\right)}{2\left(768 - 176b^2 + 9b^4\right)^2},$$

$$W^{L*} = \frac{a^2\left(380928 - 122880b - 130304b^2 + 40448b^3 + 14384b^4 - 4128b^5 - 611b^6 + 162b^7\right)}{2\left(768 - 176b^2 + 9b^4\right)^2}.$$

Finally, we consider the subgame in which the manager of Firm 1 is the leader. In this case, we obtain the incentive parameters of the both the firms as follows:

$$\theta_0^{F*} = \frac{a\left(256 - 128b - 16b^2 + 20b^3 - 3b^4\right)}{768 - 176b^2 + 11b^4}, \quad \theta_1^{L*} = 0.$$

By analogy to the above two settings (the quantity competition with homogeneous goods and price competition with differentiated goods) that the manager of Firm 1 takes the lead, we recognize that $\theta_1^{L*} = 0$. Moreover, we recognize that $\theta_0^{F*} > 0$.

The equilibrium values in the market outcomes are obtained as follows:

$$q_0^{F*} = \frac{a\left(256 - 80b - 32b^2 + 8b^3 + b^4\right)}{768 - 176b^2 + 11b^4}, \quad q_1^{L*} = \frac{4a\left(48 - 16b - 3b^2 + b^3\right)}{768 - 176b^2 + 11b^4},$$

$$p_0^{F*} = \frac{2a\left(256 - 56b - 40b^2 + 2b^3 + 3b^4\right)}{768 - 176b^2 + 11b^4}, \quad p_1^{L*} = \frac{a\left(576 - 192b - 84b^2 + 28b^3 + 3b^4 - b^5\right)}{768 - 176b^2 + 11b^4},$$

$$\Pi_0^{F*} = \frac{a^2\left(65536 - 28672b - 17920b^2 + 5888b^3 + 3136b^4 - 688b^5 - 240b^6 + 36b^7 + 5b^8\right)}{\left(768 - 176b^2 + 11b^4\right)^2},$$

$$\Pi_1^{L*} = \frac{4a^2\left(8 - b^2\right)\left(48 - 16b - 3b^2 + b^3\right)^2}{\left(768 - 176b^2 + 11b^4\right)^2},$$

$$CS^{F*} = \frac{a^2\left(102400 + 32768b - 73984b^2 + 4096b^3 + 11024b^4 - 1280b^5 - 560b^6 + 56b^7 + 9b^8\right)}{2\left(768 - 176b^2 + 11b^4\right)^2},$$

$$W^{F*} = \frac{a^2\left(496 - 160b - 56b^2 + 16b^3 + b^4\right)}{1536 - 352b^2 + 22b^4}.$$

We obtain the following ranking orders based on the equilibrium values of several variables in the three subgames.

**Lemma 3.** *The equilibrium values of the three subgames are ranked as follows:*

*(i)* $\theta_0^{S*} > \theta_0^{F*} > \theta_0^{L*}$ *and* $\theta_1^{F*} > \theta_1^{S*} > \theta_1^{L*}$.

*(ii)* $q_0^{S*} > q_0^{L*} > q_0^{F*}$ *and* $q_1^{F*} > q_1^{L*} > q_1^{S*}$.

*(iii)* $p_0^{F*} > p_0^{S*} > p_0^{L*}$ *and* $p_1^{L*} > p_1^{S*} > p_1^{F*}$.

*(iv)* $\Pi_0^{F*} > \Pi_0^{S*} > \Pi_0^{L*}$ *and* $\Pi_1^{L*} > \Pi_1^{S*} > \Pi_1^{F*}$.

*(v)* $CS^{L*} > CS^{S*} > CS^{F*}$ *and* $W^{L*} > W^{F*} > W^{S*}$.

When the cost functions of both of the firms are quadratic in the quantity-setting mixed duopoly with differentiated goods, the rankings of all equilibrium market outcomes are the same as those in the competition with homogeneous goods when $k > 0.324929$. Thus, the intuition of Lemma 3 is almost unchanged with that of Lemma 1 in the case of $k > 0.324929$.

**Proposition 3.** *In the unique subgame perfect Nash equilibrium,* $(t_0^*, t_1^*) = (2, 1)$.

Since, given the strategy of Owner 0, the optimal reaction of Owner 1 is clear, there exists a unique equilibrium in this case, different from the one that exists in the competition with homogeneous goods. Note that the dominant strategy of Owner 1 is $t_1 = 1$, *i.e.*, to move as soon an possible.

# 5 Conclusion

This paper examined a model in which a public firm and a private firm set their own outputs or prices sequentially or simultaneously in the various environments, focusing on the managerial delegation of the firms. In particular, we consider the case in which technologies of both the firms are represented by the quadratic cost functions, whereas Nakamura and Inoue (2007*b*, 2007*c*) assumed that the cost functions of both the firms are linear. In the case of the quantity competition with homogeneous goods, we showed that in equilibrium, the public firm tends to become the follower under the moderate condition. Moreover, in the quantity competition with differentiated goods, we found that in equilibrium the public firm is certainly the follower under the somewhat strong cost condition. These two results are in contrast to that in **?**. Furthermore, in the price competition with differentiated goods, we showed that both the firms choose their prices as late as possible under the same cost condition as in the quantity competition. This result coincides with that in **?**. Thus, in the analysis on the mixed duopoly with managerial delegation, using the observable delay game in **?**, we can say that in price competition, the equilibrium structure of competition is robust against changes in production technologies of both the firms, while in quantity competition, the equilibrium competition structure is changed due to forms of the technologies of them.

Finally, we must accept the fact that our model is restrictive. We considered the situation in which the production technologies of both the public firm and private firm are symmetric, *i.e.*, they are represented by identical quadratic cost functions. Whether all results we derived in this paper survive under the asymmetric situation with regard to the production efficiencies of both the firms needs to be explored. Moreover, our analysis is limited to duopoly and the case in which there exist only two periods that firms' owners can select based on their decisions of when the necessary action should be taken. The preceding researches such as Pal (1998) and Lu (2006) explored the endogenous timing in mixed oligopoly where firms select the timing from $n \geq 2$ periods. These issues will dealt with in future research.

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