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# Mixed Oligopoly with a Vertical Structure of Government: Implications of Strategic Privatization and Other Policies

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## Abstract

This paper incorporates the vertical structure of governments into a mixed oligopoly model by taking into consideration the public firms owned by different levels of governments, namely, a state-owned public firm and a local public firm. We find that only the state-owned public firm should be privatized when the central and local governments independently consider whether to privatize their respective public firms. Furthermore, we shed light on the roles played by the central government apart from that entailed by the ownership and control over state-owned public firms by introducing certain existing policies implemented by the central government into our mixed oligopoly model. In this regard, decentralization, location choice with respect to the state-owned public firm, and subsidization are considered in this paper.

**JEL Classification:** D21, L13, L33

**Keywords:** Mixed Oligopoly, Vertical Structure, Strategic Privatization, Decentralization, Subsidization

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# I Introduction

This paper presents a theoretical analysis of mixed oligopoly with a vertical structure such that a country run by the central government is divided into two local regions run by local governments. This vertical structure creates the possibility of vertical externalities between the different levels of government (e.g., federal, state, and local). Furthermore, the representative model in which vertical externalities are best articulated and explicitly formulated is that of tax competition. This model deals with the situation in which different levels of government impose taxes on the same tax base. Accordingly, the private sector's response to the tax policy decisions of one level affects the tax bases of the other levels. This sort of externality between them does not arise only due to their co-occupation of the same tax base. For example, several state universities compete with national universities over competent students, which implies that one level of the government can indirectly affect another as a result of the market competition between the public firms owned by them and through the privatization of such firms. Thus, we examine the interaction between different levels of government by using a mixed oligopoly model in which public firms compete against private firms.

The modern game theoretical analysis of mixed oligopoly can be traced back to the pioneering work of DeFraja and Delbono (1989).<sup>1</sup> This in addition to many existing works have assumed the theoretical model comprising one public firm and any finite number of private firms in one country or region and have concentrated on intra regional competition. Recently, the number of papers analyzing inter regional competition has increased, for instance, Fjell and Pal (1996), Pal and White (1998) and Fjell and Heywood (2002). These works assume that only private firms export to a country or region containing one public firm and several private firms. However, a public firm in a particular country or region can also compete with and supply to other countries or regions. In the context of international trade, we can cite the example of airline industry. In this industry, national airline enterprises compete against not only the domestic and foreign private enterprises but also the national enterprises of other countries.<sup>2</sup> The competition among state universities and among state hospitals might also exemplify inter regional competition. The existing research mentioned above has failed to capture the existence of such competition between the public firms

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<sup>1</sup>Research on mixed oligopoly models has become increasingly popular in recent years. See for instance, DeFraja and Delbono (1990) and Nett (1993) for general reviews of the models. For recent literature on mixed oligopoly, see Matsumura and Kanda (2005), Lu (2006), Lu and Poddar (2007), and Ogawa and Sanjo (2007).

<sup>2</sup>For further detail on this example, see Dadpay and Heywood (2006).

located in different regions and the fact that public firms also export to other regions.

It is surprising to find that almost all of the formal literature on mixed oligopoly has neither regarded nor analyzed the competition between public firms of different nationalities and the relationships that form between governments as a result of this competition. Nevertheless, there are some exceptions. Bárcena-Ruiz and Garzón (2005) and Dadpay and Heywood (2006) considered the strategic interaction between public firms located in different countries based on an international mixed oligopoly model and examined the effects of the privatization of these public firms on the welfare of these countries. We can reinterpret their model in a manner that allows us to analyze the competition existing between public firms owned by the same level of government in a particular country. Thus, the analytical focus of their models is on the horizontal relationships and possible externalities between states that result from the competition between the public firms owned by them, the federal or central government being notable by its absence. In the mixed oligopoly models employed in earlier literature and the studies mentioned above, the central government makes its appearance, if it does so at all, as an enforcement mechanism or as the enforcer of the relevant industrial policies, such as production subsidies for firms in industries and deregulation to inter regional and intra regional mergers among firms.

The first and most general purpose of this paper is to argue the existence of a vertical relationship between the central and local governments in the context of mixed oligopoly. For this purpose, we explore the mixed oligopoly model in which the central government runs a country in which consists of distinct regions and each region is run by the local government. Specifically, we consider the situation in which one country is divided into two regions, such that a public firm owned by the central government is located in one region and a public firm owned by the local government is located in the other region. Similar to most of the existing works on mixed oligopoly, a public firm owned by the central government, that is, a state-owned public firm is assumed to maximize the total social welfare of the entire country, while a public firm owned by the local government, that is, a local public firm, maximizes only its own region's social welfare.

The second and more particular purpose is to develop some implications of the privatization policies and several policies that are employed in mixed oligopoly with a vertical relationship between different levels of government. In our specific model, similar to Dadpay and Heywood (2006), the central and local governments simultaneously and independently consider whether to privatize their own public firms. Al-

though they revealed that both the governments never have an incentive to privatize their respective public firms in a mixed oligopoly with a horizontal relationship between the levels of government, it is demonstrated in our paper that when the vertical relationship is taken into consideration, one of the governments can have an incentive to privatize its public firm. Furthermore, we argue that an alternative role can be played by the central government apart from that of ownership of the state enterprise. In this regard, we consider the following three policies implemented by the central government: its location choice with respect to a state-owned firm, decentralization, and output subsidization. In addition, we investigate the effects of these policies on the state-owned and local public firms' behaviors and social welfare. With regard to subsidization policies, we examine not only the effects of the subsidies provided by the central government on market outcomes but also that of the devolution of subsidization policies to the local governments.

The remainder of this paper is organized as follows. Section 2 describes the basic model used throughout this paper. Section 3 analyzes the vertical relationship between the central and local governments through competition between public firms owned by them and derives the equilibrium in a mixed oligopoly model. In Section 4, we examine the strategic privatization policies implemented by the governments. Section 5 presents some extensions and Section 6 concludes the paper.

## II Model

We consider a country that is composed of two regions denoted by A and B. These regions are identical in terms of demand and contain the same number of firms. The demand in each country is given by  $P = a - 2Q_i$  ( $i = A, B$ ), where  $P$  is the price and  $Q_i$  is the demand in region  $i$ . We assume that these markets are integrated, which implies that the total demand is given by  $P = a - Q_T$ . The number of firms in each country is  $n + 1$ , and these firms are assumed to possess identical technologies. Let the cost of a firm be  $C(x_{ij}) = f + \frac{1}{2}kx_{ij}^2$ , where  $f$  is a fixed cost,  $k > 0$  is a constant, and  $x_{ij}$  is the output of firm  $j$  in region  $i$  ( $i = A, B; j = 0, 1, \dots, n$ ). For simplicity, we assume that  $f = 0$ , since we do not examine an entry of firms in this paper.

Let  $X_i$  denote the total output produced in region  $i$  and  $X_T = X_A + X_B$  denote the total output in an entire

country. Since the market-clearing condition is  $Q_T = X_T$ , the total inverse demand function is rewritten as  $P = a - X_T$ . Thus, the profits of firm  $j$  located in region  $i$  are represented as the function of the outputs of all the firms as follows:

$$\Pi_{ij} = (a - X_A - X_B) x_{ij} - \frac{1}{2} k x_{ij}^2 = \left( a - x_{A0} - \sum_{j=1}^n x_{Aj} - x_{B0} - \sum_{j=1}^n x_{Bj} \right) x_{ij} - \frac{1}{2} k x_{ij}^2. \quad (1)$$

The social welfare in region  $i$ , denoted by  $W_i$ , is measured as the sum of the consumer surplus ( $CS_i$ ) and producer surplus ( $PS_i$ ) as follows:

$$W_i = CS_i + PS_i = \frac{1}{4} X_T^2 + \Pi_{i0} + \sum_{j=1}^n \Pi_{ij}, \quad (2)$$

and the total social welfare in the country,  $W_T$ , is represented as

$$W_T = W_A + W_B = \frac{1}{2} X_T^2 + \sum_{i=A,B} \sum_{j=0}^n \Pi_{ij}. \quad (3)$$

According to the definition of local welfare in equation (2), the state-owned public firm's profit, if any, is included in welfare of the region in which the firm is located. In other words, the central government transfers the pooled profit or the governmental surplus only to the region in which the state-owned firm is located.<sup>3</sup>

### III Vertical mixed oligopoly

As stated in the Introduction, Bárcena-Ruiz and Garzón (2005) and Dadpay and Heywood (2006) present the mixed oligopoly model with two regions wherein one public firm owned by the local government of one region competes with the other public firm owned by that of the other region. Thus, in their model, the same level of government influences market outcomes by holding the right of ownership to the public firms and by making these public firms compete with private firms and maximize the local welfare only. Henceforth, we call this type of mixed oligopoly a *horizontal mixed oligopoly*.

<sup>3</sup>Readers may consider this assumption to be somewhat unnatural and that the pooled profits of the central government should be transferred to both the regions. Even though we consider that the central government transfers a part of the profits to the other region, the results obtained later in the paper are not influenced provided that the size of the transfer to the other region is not large.

Our paper develops another mixed oligopoly model incorporating the competition between public firms. In this model, a state-owned public firm and a local public firm compete against each other. In the tax competition theory, vertical tax competition is defined as the competition wherein the different levels of government (e.g., federal, state, and local) impose their respective taxes on the same tax base. In imitation of the tax competition theory, we term such type of a mixed oligopoly as a *vertical mixed oligopoly* because the public firms owned by the different levels of governments compete over the demand for the same good. As in DeFraja and Delbono (1989), we assume that the central government wants its public firm to maximize the total welfare of the entire country. On the other hand, we also assume that local governments do not take into account the welfare of the other region and, consequently, its public firm maximizes only the local welfare.

Suppose that a state-owned public firm is located in region  $A$  and a local public firm is located in region  $B$ . In other words, the public firm in region  $A$  is owned by the central government and the public firm in region  $B$  is owned by the local government of region  $B$ .<sup>4</sup> We assume that firm 0 in each region is the public firm. For analysis, we first derive all the firms' reaction functions. Private firms maximize their profits and, thus, their first order conditions are given as

$$\frac{\partial \Pi_{ij}}{\partial x_{ij}} = a - X_T - x_{ij} - kx_{ij} = 0, \quad i = A, B, \quad j = 0, 1, \dots, n, \quad (4)$$

while that of a state-owned public firm as a total-welfare-maximizer is

$$\frac{\partial W_T}{\partial x_{A0}} = a - X_T - kx_{A0} = 0. \quad (5)$$

As in the previous mixed oligopoly models in which a country does not consist of several regions, the state-owned firm's marginal cost is equalized with the price. This is because the trade between the two regions is, in fact, an internal trade in terms of total welfare.

The local public firm in region  $B$  maximizes the welfare of the region such that

$$\frac{\partial W_B}{\partial x_{B0}} = \frac{1}{2}X_T + (a - X_T) - X_B - kx_{B0} = (a - X_T) + (Q_B - X_B) - kx_{B0} = 0. \quad (6)$$

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<sup>4</sup>In Section 5, we also consider the case where both the public firms are located in the same region.

The second equality follows from the symmetry of demand in the two regions. This equation states that the local public firm selects its output such that its marginal cost becomes higher (lower) than the price if region  $B$  is an importing (exporting) region. If the local public firm in an exporting region increases its output, the terms of trade for this region worsens, which in turn, reduces the social welfare of the region. Thus, it refrains from expanding its output.

It follows from equations (4), (5), and (6) that

$$X_T^{NN} = \frac{2a[(n+1)k+1]}{k^2+(2n+3)k+2}, \quad Q_i^{NN} = X_i^{NN} = \frac{a[(n+1)k+1]}{k^2+(2n+3)k+2}, \quad x_{ij}^{NN} = \frac{ak}{k^2+(2n+3)k+2},$$

$$x_{i0}^{NN} = \frac{a(k+1)}{k^2+(2n+3)k+2}, \quad i = A, B; \quad j = 1, 2, \dots, n,$$

where  $NN$  denotes that firm 0 in both regions  $A$  and  $B$  are not private but rather public firms. These results demonstrate that the outputs of state-owned and local public firms are the same despite the fact that they have different objective functions. This can be explained as follows. Since both regions contain the same number of private firms, region  $A$  exports to (imports from) region  $B$  when the state-owned firm produces more (less) than the local public firm. Suppose that the state-owned firm decides to produce a large amount of output. Given this fact, based on equation (6), the local public firm also increases its output. If region  $A$  remains as an exporting country even after this adjustment, the difference in output between the two public firms is maintained. This implies that the allocation of production is inefficient and that reducing this production differential enhances total welfare. Thus the state-owned public firm has an incentive to produce a lower amount of output than previously, which results in the equalization of the public firms' outputs.

Using equilibrium output above, we obtain equilibrium profits and welfare.

$$\Pi_{ij}^{NN} = \frac{a^2 k^2 (k+2)}{2[k^2+(2n+3)k+2]^2}, \quad \Pi_{i0}^{NN} = \frac{a^2 k (k+1)^2}{2[k^2+(2n+3)k+2]^2},$$

$$W_i^{NN} = \frac{a^2 [(n+1)k^3 + 2(n^2+3n+2)k^2 + (4n+5)k+2]}{2[k^2+(2n+3)k+2]^2}, \quad W_T^{NN} = 2W_i^{NN}, \quad i = A, B; \quad j = 1, 2, \dots, n.$$

Similar to the results obtained for a typical mixed oligopoly model with increasing marginal costs, the public firms earn higher profits than do the private firms due to their excess production. Note that provided that the local public firm is privatized, our model is the same as that in DeFraja and Delbono (1989) in



which one public firm competes against the private firms in a particular market or region. Based on this fact, we compare our equilibrium profits and welfare to those in DeFraja and Delbono (1989). We denote  $NP$  as the case where only the local public firm is privatized, that is, DeFraja and Delbono's case. All the equilibrium outcomes for this case  $NP$  are presented in the Appendix.

$$\begin{aligned}\Pi_{ij}^{NN} - \Pi_{ij}^{NP} &= -\frac{a^2 k^2 (k+2) [2k^2 + 2(2n+3)k + 3]}{2 [k^2 + (2n+3)k + 1]^2 [k^2 + (2n+3)k + 2]^2} < 0, \\ \Pi_{A0}^{NN} - \Pi_{A0}^{NP} &= -\frac{a^2 k (k+1)^2 [2k^2 + 2(2n+3)k + 3]}{2 [k^2 + (2n+3)k + 1]^2 [k^2 + (2n+3)k + 2]^2} < 0, \\ W_T^{NN} - W_T^{NP} &= \frac{a^2 k [k^4 + 5k^3 - (4n^2 + 2 - 9)k^2 + 7k + 2]}{2 [k^2 + (2n+3)k + 1]^2 [k^2 + (2n+3)k + 2]^2}.\end{aligned}$$

Private and state-owned firms' profits are lower in the vertical mixed oligopoly model than in the typical mixed oligopoly model employed in Defraja and Delbono (1989). Excess production by the two public firms decreases the price; consequently, all the firms in the vertical oligopoly earn lower profits. Although the above equations indicate that profits are always lower in the vertical mixed oligopoly, it is not certain whether the equilibrium welfare is also lower than in the typical mixed oligopoly. When parameter  $k$  is sufficiently low or the number of private firms  $n$  is sufficiently large, welfare in the vertical mixed oligopoly model is lower than in DeFraja and Delbono (1989). A smaller  $k$  implies that all the firms including the public firms are more efficient, and thus, public firms expand their production. This increase the discrepancy between the outputs of the private and public firms. Furthermore, this large discrepancy results in inefficient production allocation, which in turn, results in the deterioration of welfare. Similarly, when the number of private firms is large, the effect of inefficient production allocation on welfare becomes large.

#### IV Privatization of public firms

In the previous section, we analyzed the vertical mixed oligopoly model and compared it with the typical mixed oligopoly model employed by DeFraja and Delbono (1989). We now examine the effect of the privatization of state-owned and/or local public firms. In our model, the decision-makers in this regard

are the central government and the local government of region  $B$ . They independently determine whether to privatize their public firms in order to maximize their respective objectives, that is, total welfare and the local welfare of region  $B$ . Note that one government is strategically related to the other government. For example, suppose that the central government privatizes the state-owned public firm. The change in the objective function of the firm affects market outcomes and, as a result, not only total welfare but also the local welfare of region  $B$  are changed. Thus, one government's decision to privatize its public firm influences the payoff of the other government.

To describe this situation, we formulate the following two-stage game. In the first stage, both the central and local governments simultaneously determine whether to privatize their public firms. We name their actions "privatization" and "non-privatization," denoted by  $P$  and  $N$ , respectively. We then define the strategies of the central and local governments as  $s_A \in \{P, N\}$  and  $s_B \in \{P, N\}$ , respectively. In the second stage, all the firms simultaneously select their outputs. In accordance with the choices of the two governments, the following four subgames in the second stage can be considered:  $PP$ , in which both the central and local governments privatize their public firms;  $PN$ , in which only the central government privatizes its public firm;  $NP$ , in which only the local government of region  $B$  privatizes its public firm;  $NN$ , in which neither the government privatizes its public firm. We solve for the subgame perfect Nash equilibrium (SPNE) of this game by backward induction.

Let us solve the equilibrium of the second stage. Since we have already obtained the equilibrium outcomes of the subgame  $NN$  in the previous section, we only derive the outcomes of the three remaining subgames. The equilibrium outcomes of the subgames  $PP$ ,  $PN$ , and  $NP$  follow from the first order conditions (4), (4) and (6), and (4) and (5), respectively. These outcomes are presented in the Appendix.

Before proceeding to the analysis of the first stage, we compare outputs among the four subgames. The results of this comparison are summarized in the following lemma.

**Lemma 1.**

- (a)  $X_T^{NN} > X_T^{NP} > X_T^{PN} > X_T^{PP}$ ,
- (b)  $X_A^{NP} > X_A^{NN} > X_A^{PP} > X_A^{PN}$ ,  $X_B^{PN} > X_B^{NN} > X_B^{PP} > X_B^{NP}$ ,
- (c)  $x_{A0}^{NP} > x_{A0}^{NN} > x_{A0}^{PP} > x_{A0}^{PN}$ ,  $\min\{x_{B0}^{NN}, x_{B0}^{PN}\} > x_{B0}^{PP} > x_{B0}^{NP}$ ,  $x_{B0}^{PN} \geq x_{B0}^{NN} \iff n \leq \frac{k+1}{2}$ ,
- (d)  $x_{ij}^{PP} > x_{ij}^{PN} > x_{ij}^{NP} > x_{ij}^{NN}$ ,  $i = A, B$ ;  $j = 1, 2, \dots, n$ .

The intuition behind this lemma is as follows. With respect to (a), it is clear that  $X_T^{NP} > X_T^{PP}$ ,  $X_T^{PN} > X_T^{PP}$ , and  $X_T^{NN} > X_T^{PP}$  hold due to the fact that aggressive behaviors by public firms increase the total output. Although both the public firms have an incentive to produce more than the private firms, in equilibrium, the incentive of the local public firm is not stronger than that of the state-owned public firm. Suppose that there is one public firm in the market. If this firm is the local public one, then region  $B$  exports to region  $A$  and, thus, output expansion by this firm worsens the terms of trade for region  $B$ . As a consequence of its fear that local welfare will deteriorate due to this terms of trade effect, the local public firm restrains its output expansion, that is,  $X_T^{NP} > X_T^{PN}$ . Furthermore, the reverse is true with regard to the outputs of the private firms. It follows from this fact that each private firm's quasi reaction function is negatively related to  $X_T$ , as seen in equation (4).

Next, we proceed to the exposition of (b) and (c). We only elucidate the relationship between  $x_{B0}^{NN}$  and  $x_{B0}^{PN}$  because the other relationships in (b) and (c) can be directly explained by the aggressive behaviors of public firms and the negative slopes of the reaction functions. Suppose that  $n$  is sufficiently large. In the case of  $NN$ , the state-owned firm produces less, since substituting the outputs of more efficient private firms for its own output is likely to generate the higher total welfare in spite of a decrease in price through excess production by the state-owned firm, as demonstrated by DeFraja and Delbono (1989). As a result, the local public firm also decreases its output to maintain the equivalence between the outputs of the two public firms in  $NN$ . On the other hand, in the case of  $PN$ , not only substituting the output of the local public firm but also avoiding deterioration in the terms of trade increases the local welfare of region  $B$ . Thus, the output of the local public firm is less in  $PN$  than in  $NN$ .

We now investigate the first stage in which both the central and local governments determine whether to privatize their respective public firms. The local government always selects non-privatization of its public

firm, as indicated in the following equations:

$$W_B^{PN} - W_B^{PP} = \frac{a^2 [2k^3 + (4n + 15)k^2 + 4(3n + 8)k + 4n^2 + 22n + 31]}{(k + 2n + 5)^2 [2k^2 + (4n + 11)k + 2n + 7]^2} > 0,$$

$$W_B^{NN} - W_B^{NP} = \frac{a^2 k [2k^4 + (4n + 15)k^3 + 2(5n + 16)k^2 + (8n + 27)k + 6]}{4 [k^2 + (2n + 5)k + 1]^2 [k^2 + (2n + 5)k + 2]^2} > 0,$$

which implies that non-privatization is a dominant strategy for the local government in region  $B$ . On the other hand, depending on  $k$  and  $n$ , the strategy of the central government is determined as follows:

$$W_T^{PP} - W_T^{NP} = -\frac{a^2 [k^3 + 3k^2 - (4n^2 + 6n - 1)k + 1]}{2(k + 2n + 3) [k^2 + (2n + 3)k + 1]^2},$$

$$W_T^{PN} - W_T^{NN} = \frac{a^2 [4k(2k^2 + 4k + 1)n^2 + 4k(2k^2 + 5k + 3)n - 2k^5 - 12k^4 - 27k^3 - 28k^2 - 13k - 2]}{[k^2 + (2n + 3)k + 2]^2 [2k^2 + (4n + 7)k + 2n + 5]^2}.$$

These equations demonstrate that the reaction of the central government is complicated. Nevertheless, an examination of the sign of  $W_T^{PN} - W_T^{NN}$  is sufficient for deriving the SPNE, since the local government always selects non-privatization, that is,  $s_B = N$ . In the second equation, the answer to whether the privatization of state-owned public firm worsens the total welfare depends on the sign of the numerator when the local public firm is not privatized. Based on this, we obtain the following:

$$W_T^{PN} \gtrless W_T^{NN} \iff n \gtrless \tilde{n} := \frac{(k + 1) [-2k^2 - 3k + \sqrt{2k(k + 1)(2k^4 + 10k^3 + 18k^2 + 12k + 1)}]}{2k(2k^2 + 4k + 1)} (> 1).$$

The above indicates that if the integrated market in our model is competitive and the local government of region  $B$  decides not to privatize its public firm, the privatization of a state-owned public firm decreases the total welfare. This can be explained as follows. DeFraja and Delbono (1989) demonstrate that when the number of private firms is large an improvement in cost efficiency resulting from an adjustment to production allocation through privatization outweighs a decrease in price due to excess production by the public firm when the number of private firms is large. This can even be applied to our model. According to Lemma 1, a larger  $n$  means a smaller  $x_{B0}^{PN}$ . As the number of private firms becomes larger, the total welfare obtained in the equilibrium outcome of  $PN$  (that is,  $W_T^{PN}$ ) approaches closer to that obtained in DeFraja

and Delbono's model (that is,  $W_T^{NP}$ ). Thus, from the results in the previous section, we find that  $W_T^{PN}$  is larger than  $W_T^{NN}$  if  $n$  is large.

All the results above yield the SPNE, which are summarized in Proposition 1.

**Proposition 1.** *In equilibrium, the privatization policies of both the central and local governments  $(s_A, s_B)$  are given as follows: (i)  $(s_A, s_B) = (P, N)$  if  $n > \tilde{n}$ , (ii)  $(s_A, s_B) = (P, N), (N, N)$  if  $n = \tilde{n}$ , and (iii)  $(s_A, s_B) = (N, N)$  if  $n < \tilde{n}$ .*

Two remarks on Proposition 1 are made here. First, Dadpay and Heywood (2005) reveal that neither of the local governments has an incentive to privatize its local public firm. However, if one of the public firms is owned by the central government, then the state-owned public firm should be privatized. Bárcena-Ruiz and Garzón (2005) explain the existence of an equilibrium in which one public firm is privatized by introducing cost inefficiencies of the public firms. Proposition 1 provides an explanation for this by taking the different levels of governments into consideration. Second, in the real world, local governments are likely to follow the policies set by the central government. Once this fact is taken into account, it might be natural to develop a model where the central government is the first mover and the local government is the second mover in the decisions regarding the privatization of public firms. Nevertheless, the equilibrium outcomes of this new game are the same as those of Proposition 1, since a strategy of non-privatization for the local public firm, that is,  $s_B = N$ , is the dominant strategy for the local government of region  $B$ .

## V Some extensions and policy issues

In this section, we present some extensions and policy issues. As stated in the Introduction, we shed light on an alternative role that can be played by the central government apart from that entailed by the ownership and control over state-owned public firms. The central government as the higher level of government can influence local governments by implementing certain policies or ensuring that they comply with the relevant laws and rules. We consider the implications of these policies in the mixed oligopoly model. For this purpose, we consider three policies that are implemented by the central government. First, we examine the effect of transferring the right of ownership to a state-owned firm from the central government to the local government. Second, we consider the policy by which the central government decides which region to locate its public firm in. Finally, we analyze the effectiveness of output subsidies in our vertical mixed

oligopoly model.

## Decentralization

We have analyzed the privatization of public firms and investigated the effect of this privatization on welfare. Privatization is a significant policy issue worldwide. However, we should not ignore another equally important issue regarding governance and administration, namely, decentralization. Presently, decentralization is highly popular all around the world, and it has been at the center stage of policy experiments for the last two decades in a large number of developing and transition economies. Such a decentralization policy is often suggested as a means of reducing the role of the state in general, since it dilutes central authority and furthers intergovernmental competition. Thus, decentralization is widely believed to promise a wide range of benefits. In this subsection, we investigate whether or not decentralization is effective, and consequently, whether or not it improves the total welfare of the entire country.

For this purpose, we summarize the various meanings of *decentralization* as the transfer of the right of ownership to a state-owned firm from the central government to the local government. We use our mixed oligopoly model to examine whether this transfer is desirable in terms of total welfare. In order to maximize total welfare, the central government decides whether or not it should transfer its right of ownership to the state-owned firm to the local government of region  $A$  and then commits to its decision. If it transfers this right, horizontal mixed oligopoly prevails, that is, two different local public firms and private firms compete in our integrated market. If otherwise, the vertical mixed oligopoly of the previous section prevails.

To conduct a comparison between vertical and horizontal mixed oligopoly, we derive the equilibrium outcomes in the horizontal mixed oligopoly model. In this case, each local public firm selects its output  $x_{i0}$  to maximize the local welfare of the corresponding region according to equation (2). On the other hand, the other private firms maximize their profits only. Thus, we obtain the following equilibrium outcomes:

$$X_T^H = \frac{2a[(n+1)k+1]}{k^2+(2n+3)k+2}, \quad Q_i^H = X_i^H = \frac{a[(n+1)k+1]}{k^2+(2n+3)k+2}, \quad x_{ij}^H = \frac{ak}{k^2+(2n+3)k+2},$$

$$x_{i0}^H = \frac{a(k+1)}{k^2+(2n+3)k+2}, \quad i = A, B; \quad j = 1, 2, \dots, n,$$

where  $H$  denotes horizontal mixed oligopoly. From these outcomes, we can observe that allocation in the

horizontal mixed oligopoly is the same as that in the vertical mixed oligopoly. This implies that the central government cannot increase total welfare by transferring the ownership of the state-owned firm to the local government. This is summarized in Proposition 2.

**Proposition 2.** *Decentralization as the transfer of a state-owned firm to the local government does not improve total welfare.*

Proposition 2 implies that decentralization is not desirable in terms of total welfare when the decentralization incurs some cost. In addition, when the central government can decide whether it should privatize its public firm or transfer the right of ownership to the local government, privatization is the better policy provided the market is competitive.

### **Location choice regarding a state-owned firm**

Next, we consider the issue of location choice with regard to a state-owned public firm by the central government. If the central government decides that its state-owned public firm should be located in region  $A$ , the situation under the setting of  $NN$  in Section 3 occurs. Thus, the market consists of one state-owned public firm and  $n$  private firms located in region  $A$  and one local public firm and  $n$  private firms located in region  $B$ . The equilibrium market outcomes in this case have been already described in Section 3. On the other hand, if the central government decides that its state-owned public firm should be located in region  $B$ , there exist  $n$  private firms in region  $A$ , while one state-owned public firm, one local public firm and  $n$  private firms are located in region  $B$ . Thus, the two regions are asymmetric in terms of the number of existing firms. For tractability of the analysis, we assume the following condition.

**Assumption 1.**  *$k$  is sufficiently large, that is,  $k > \frac{1}{2}$ .*

This assumption ensures that the local public firm is active in the case where the state-owned public firm is located in region  $B$ . Under the above assumption, we obtain the following equilibrium market

outcomes:

$$\begin{aligned}
x_{B0}^L &= \frac{a(2k^2 + k - 1)}{k[2k^2 + (4n + 7)k + (2n + 5)]}, & x_{ij}^L &= \frac{a(2k + 1)}{2k^2 + (4n + 7)k + 2n + 5}, \\
x_{Bn+1}^L &= \frac{a(2k^2 + 3k + 1)}{k[2k^2 + (4n + 7)k + (2n + 5)]}, & X_A^L &= \frac{an(2k + 1)}{2k^2 + (4n + 7)k + 2n + 5}, \\
X_B^L &= \frac{a[2(n + 2)k + n + 4]}{2k^2 + (4n + 7)k + 2n + 5}, & X_T^L &= \frac{2a[2(n + 1)k + n + 2]}{2k^2 + (4n + 7)k + 2n + 5}, \\
W_T^L &= \frac{a^2[4(n + 1)k^4 + 4(2n^2 + 7n + 5)k^3 + (8n^2 + 33n + 27)k^2 + 2(n^2 + 5n + 5)k - 1]}{k[2k^2 + (4n + 7)k + 2n + 5]^2}, \\
i &= A, B; \quad j = 1, 2, \dots, n,
\end{aligned} \tag{7}$$

where the superscript  $L$  denotes the equilibrium market outcomes in the case where the state-owned public firm is located in region  $B$ , and  $x_{Bn+1}^L$  denotes the equilibrium output of the state-owned public firm located in region  $B$ . We compare the two public firms' outputs in the following two cases: when the state-owned firm is located in region  $A$  and when it is located in region  $B$ . This is useful toward comparing the total welfare in the two cases.

$$\begin{aligned}
x_{B0}^{NN} - x_{B0}^L &= \frac{2a(k + 1)[k^2 + 2(n + 1)k + 1]}{k[k^2 + (2n + 3)k + 2][2k^2 + (4n + 7)k + 2n + 5]} > 0, \\
x_{A0}^{NN} - x_{Bn+1}^L &= -\frac{2a(k + 1)^2}{k[k^2 + (2n + 3)k + 2][2k^2 + (4n + 7)k + 2n + 5]} < 0.
\end{aligned}$$

When the state-owned firm is relocated from region  $A$  to  $B$ , the output of the local public firm decreases, whereas that of the state-owned firm increases. Consequently, region  $B$ , with two public firms, becomes an exporting region. Excess production by the state-owned firm in region  $B$  leads to deterioration in the terms of trade of the region. Subsequently, the local public firm decreases its output to offset this negative effect on the local welfare of region  $B$ .

Comparing the equilibrium total social welfare in the case where the state-owned public firm is located



in region  $A$  with that in the case where it is located in region  $B$ , we obtain the following result:

$$\begin{aligned}
W_T^L - W_T^{NN} &= -\frac{2a^2 \left[ k^5 + 6k^4 - (4n^2 - 14)k^3 + 4(n+4)k^2 + (4n+9)k + 2 \right]}{k \left[ k^2 + (2n+3)k + 2 \right]^2 \left[ 2k^2 + (4n+7)k + 2n+5 \right]^2} > 0, \\
\iff n > n^L &:= \frac{(k+1) \left( \sqrt{k^4 + 4k^3 + 5k^2 + 2k + 1} + 1 \right)}{2k^2}. \tag{8}
\end{aligned}$$

Note that  $n^L > 1$  when  $k > 0$ . We obtain the following proposition directly from this result.

**Proposition 3.** *Under Assumption 1, with respect to the location of the state-owned public firm, the central government strictly prefers region  $B$  if  $n > n^L$  and region  $A$ , if  $n^L > n \geq 1$ . If  $n = n^L$ , for the central government, the choice between regions  $A$  and  $B$  as the preferred location for its state-owned public firm becomes insignificant.*

Proposition 3 states that the state-owned firm should compete with the local public firm in the same region when both the regions are fairly competitive. The intuition behind this proposition is as follows. All the relevant outputs — namely,  $x_{B0}^{NN}$ ,  $x_{B0}^L$ ,  $x_{A0}^{NN}$  and  $x_{Bn+1}^L$  — of the two public firms decrease with  $n$ . In addition,  $x_{B0}^L$  is the smallest of them and, thus, the closest to the outputs of the private firms. Provided that  $n$  is large, the local public firm whose output is closer to that of each private firm yields a total welfare that is closer to the welfare obtained in the case  $NP$ . Recall that  $W_T^{NP} > W_T^{NN}$  when  $n$  is large. Thus, Proposition 3 holds.

## Subsidization

Several mixed oligopoly models have stressed the importance of implementing subsidization policies in mixed markets. Using the single region model of DeFraja and Delbono (1989), White (1996) reveals that providing uniform production subsidies to all firms can enhance the social welfare of a region or country and that the government can achieve the first-best allocation by employing such policies. Moreover, he demonstrates that the government can attain the first-best allocation in the case of a private oligopoly by providing all the private firms with the same level of subsidies as that provided in the case of a mixed oligopoly. This implies that the privatization of a public firm is fruitless in the sense that it does not improve welfare to the same extent that the provision of optimal subsidies improves welfare. Thus, if alternative industry policies are available, then the government should not utilize the privatization policy.

In this subsection, we investigate whether governments should comply with this suggestion in the case of a vertical mixed oligopoly. For this investigation, we should recall that privatization and subsidization policies are implemented by two types of decision-makers — the central government and the local governments. Thus, we explore the following two cases: (a) only the central government provides subsidies and (b) the governments of both regions provide subsidies. In both the cases, we analyze the incentives for the central government and the local government of region  $B$  to privatize their own public firms.

For the subsequent analysis, we present the first-best allocation in terms of total welfare, which is given as follows:

$$x_i^* = \frac{a}{k + 2n + 2}, \quad X_i^* = \frac{a(n + 1)}{k + 2n + 2}, \quad \Pi_i^* = \frac{a^2(k + 2)}{2(k + 2n + 2)}, \quad W_i^* = \frac{a^2(n + 1)}{2(k + 2n + 2)}, \quad i = A, B.$$

Note that based on White's discussion, this allocation is attainable by subsidy level  $s^* = a/(k + 2n + 2)$  in the cases where the local public firm is privatized, that is,  $NP$  and  $PP$ .

We now start with case (a). In this case, it is sufficient to focus on  $PN$  and  $NN$ . Here, we should note how the central government finances subsidies. For exposition, we assume that the costs required for subsidizing the firms in one region are financed by the lump-sum taxes collected from the residents of the region. Under this setting, firms' profits are rewritten as

$$\Pi_{ij} = P(X_T) - C(x_{ij}) + sx_{ij} = \left( a - x_{A0} - \sum_{j=1}^n x_{Aj} - x_{B0} - \sum_{j=1}^n x_{Bj} \right) x_{ij} - \frac{1}{2}kx_{ij}^2 + sx_{ij},$$

while the total welfare and local welfare of the two regions remain (3) and (2), respectively. Thus, we obtain the following results.

**Lemma 2.** *When the central government provides an output subsidy, the optimal subsidy level is identical to  $s^*$  in all four cases, namely,  $PP$ ,  $PN$ ,  $NP$ , and  $NN$ . Furthermore, all the allocations in these cases are identical to the first-best allocation.*

This lemma demonstrates that under the optimal subsidies provided by the central government, the local government as well as the central government should not privatize their public firms if the privatization incurs some costs. We elucidate the results of Lemma 2 by explaining that subsidy level  $s^*$  yields the first-best allocation in  $NN$ , since the other cases can be explained in the same manner. Suppose that the

central government sets the subsidy level to  $s = -P'(X_T)x_{ij}$  ( $i = A, B, j = 1, 2, \dots, n$ ). Under this subsidy level, the first order conditions of all the private firms and the state-owned public firm are given as

$$P(X_T) - C'(x_{ij}) = 0,$$

which indicates that their outputs are equivalent in equilibrium. Hence, the trade pattern relies on the output of the local public firm. Region  $B$  is an exporting (importing) region when this output is larger (smaller) than the output of another firm. However, the situation wherein the region is either an exporting or importing region is not consistent with the first order condition of the local public firm given by equation (6). Thus, the condition is satisfied when the trade does not occur, which leads to marginal cost pricing by the local public firm.

Next, we examine case (b). In this case, the local governments in both the regions independently select their subsidy level. It is evident that the subsidy level chosen by one government is the same as that chosen by the other in  $PP$  and  $NN$  on the basis of the symmetry of demand and the number of private firms. Surprisingly, we can find that the subsidies of the local governments are the same even in  $PN$  and  $NP$ . Moreover, all the subsidy levels in all the cases are the same as  $s^*$ .

**Lemma 3.** *When the local governments employ output subsidies, the optimal subsidy levels are identical to  $s^*$  in all four cases, namely,  $PP$ ,  $PN$ ,  $NP$ , and  $NN$ . Furthermore, all the allocations in these cases are identical to the first-best allocation.*

We explain the intuition behind this lemma. Similar to the exposition of Lemma 2, we consider only the case of  $NN$ . Let  $\widehat{W}_i(s_A, s_B)$  denote the equilibrium welfare of region  $i$  in  $NN$ . Since each local government sets a level of subsidy such that  $\widehat{W}_i$  is maximized, the following equation system is derived.

$$\frac{\partial \widehat{W}_k}{\partial s_k} = \sum_{i=A,B} \sum_{j=0}^n \frac{\partial W_k}{\partial x_{ij}} \cdot \frac{\partial x_{ij}^{NN}}{\partial s_k} = 0, \quad k = A, B.$$

Using the first order conditions of all the firms, we obtain

$$\frac{\partial \widehat{W}_A}{\partial s_A} = P'(X_T)(X_A - Q_A) \cdot \frac{\partial X_T^{NN}}{\partial s_A} - \sum_{j=1}^n (s_A + P'(X_T)x_{Aj}) \cdot \frac{\partial x_{Aj}^{NN}}{\partial s_A} = 0, \quad (9)$$

$$\frac{\partial \widehat{W}_B}{\partial s_B} = P'(X_T)(X_B - Q_B) \cdot \frac{\partial X_T^{NN}}{\partial s_B} - \sum_{j=1}^n (s_B + P'(X_T)x_{Bj}) \cdot \frac{\partial x_{Bj}^{NN}}{\partial s_B} + (P(X_T) - C'(x_{B0}^{NN})) \cdot \frac{\partial x_{B0}^{NN}}{\partial s_B} = 0. \quad (10)$$

Suppose that the local government of region  $i$  sets  $s_i = -P'(X_T)x_{ij}$ , which in turn, causes all the private firms in both the regions to equalize their marginal costs with the price. Consequently, no trade between the two regions follows from (9), and the local public firm also uses the marginal-cost pricing in accordance with equation (10). Thus, this equation system in (9) and (10) is consistent with the first-best allocation  $(x_{A0}^*, \dots, x_{An}^*, x_{B0}^*, \dots, x_{Bn}^*)$  and  $s_A = s_B = s^* = -P'(X_T^*)x_{ij}^*$ . In sum, we can establish the following proposition.

**Proposition 4.** *In the first stage, regardless of whether (i) the central government sets the output subsidy or (ii) both the local governments simultaneously set the output subsidy in each region, the equilibrium market outcomes are identical, and the first-best allocation is achieved.*

This result is an extension of White (1996) in two directions. First, we verify whether White's results hold when a type of public firm that is different from that used in earlier mixed oligopoly models is taken into consideration. The local public firm in our model has an objective function that is distinct from those of the private and state-owned public firms. However, this heterogeneity of objectives does not undermine White's results.<sup>5</sup> Second, subsidization policies are available for the different levels of governments. Proposition 4 implies that decentralization of subsidization by the central government does not increase total welfare and local welfare. Although it is an important suggestion, we should not emphasize it too much. When the governments subsidize firms, some transaction costs might be generated to some extent and firms may make false statements on their technologies and outputs. The amount of such costs and the accuracy of the information provided by firms might depend on the accessibility of the subsidizing governments to the firms in each region. If the lower levels of government subsidize the firms, these transaction

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<sup>5</sup>There are several other works on the extensions of White (1996) with regard to objective functions of firms. For example, Tomaru (2006) focuses on a public firm that maximizes not rather the welfare but weighted average of welfare and profits. Further, Kato and Tomaru (2007) analyze the case where private firms have the objective functions that include other factor apart from profits.

costs might be relatively low and the lack of accurate information — which contributes to the failure of the central government’s subsidization policies — might be less acute. In this sense, decentralization could be desirable.

## **VI Concluding Remarks**

In this paper, we model the competition that exists between public firms owned by different levels of government, namely, that between a state-owned public firm and a local public firm. Dadpay and Heywood (2007) and Bárcena-Ruiz and Garzón (2005) have developed a model in which local public firms owned by the respective local governments of the two regions where they are located compete against each other, and examine the effects of the privatization of these firms on each region’s local welfare and the total welfare. They demonstrate that both the local governments should not preferably privatize their public firms when all the public and private firms have the same increasing marginal cost. On the other hand, this paper finds that the decision depends on the ownership of the public firms. Provided that one of the public firms is owned by the central government, this firm should be privatized, though the other local public firm is never privatized.

The effects of several policies other than privatization are also examined in our paper. First, we treat decentralization as the transfer of the right of ownership to a state-owned public firm from the central government to the local government. Surprisingly, this transfer policy yields the same allocation as the one that is yielded when the state-owned public firm competes against the local public firm. In this sense, the policy is shown to be fruitless. Second, we consider the location choice with respect to the state-owned public firm in terms of total welfare. When the market is highly competitive, the state-owned public firm should be established in the region where the local public firm is located, since this would achieve cost efficiency due to the adjustment of an improved production allocation and increase total welfare. Third, we investigate the effects of the subsidization policies of the central government and the local governments and find that the first-best allocation can be attained irrespective of which government subsidizes and whether both the state-owned and local public firm are privatized.

Finally, we should mention the limitations of our paper and its possible extensions. Our model assumes that both the regions contain the same number of private firms. It also assumes that the profits of the state-owned public firm are completely transferred to the region in which the firm is located. Nevertheless, we

believe that the results obtained from our model include instructive implications in that it incorporates elements that stem from ownership by different levels of government and sheds light on some issues that cannot be handled in more conventional models. However, this paper was not addressed several important issues. One such issue is that we should investigate some sequential move structures in output competition. In reality, public firms tend to enjoy leadership in the markets. Thus, we might need to consider some additional cases where, for example, (a) only the state-owned public firm leads the other firms, (b) only the local public firm leads the other firms, and (c) both the public firms lead the private firms. Another issue is that foreign firms should also be taken into consideration. When foreign firms enter our vertical mixed oligopoly, the local public firm's actions are not affected, however, the state-owned public firm's action might be affected to a certain extent. It might be of interest to examine how this entry by foreign firms influences the equilibrium outcomes and implications of privatization. Thus, these remain as questions for future research.

## Appendix

### The equilibrium outcomes in subgame $PP$

$$X_T^{PP} = \frac{2a(n+1)}{k+2n+3}, \quad Q_A^{PP} = Q_B^{PP} = X_A^{PP} = X_B^{PP} = \frac{2a(n+2)}{k+2n+3}, \quad \Pi_{ij}^{PP} = \frac{a^2(k+2)}{2(k+2n+3)^2},$$

$$W_A^{PP} = W_B^{PP} = \frac{a^2(n+1)(k+2n+4)}{2(k+2n+3)^2}, \quad W_T^{PP} = \frac{a^2(n+1)(k+2n+4)}{(k+2n+3)^2}.$$

### The equilibrium outcomes in subgame $PN$

$$\begin{aligned}
X_T^{PN} &= \frac{2a[2(n+1)k+n+3]}{2k^2+(4n+7)k+2n+5}, & X_A^{PN} &= \frac{a(2k+1)(n+1)}{2k^2+(4n+7)k+2n+5}, & X_B^{PN} &= \frac{a[2k(n+1)+n+3]}{2k^2+(4n+7)k+2n+5}, \\
x_{ij}^{PN} &= \frac{a(2k+1)}{2k^2+(4n+7)k+2n+5}, & x_{B0}^{PN} &= \frac{a(2k+3)}{2k^2+(4n+7)k+2n+5}, & \Pi_{ij}^{PN} &= \frac{a^2(k+2)(2k+1)^2}{2[2k^2+(4n+7)k+2n+5]^2}, \\
\Pi_{B0}^{PN} &= \frac{a^2(4k^3+12k^2+13k+6)}{2[2k^2+(4n+7)k+2n+5]^2}, \\
W_A^{PN} &= \frac{a^2[4(n+1)k^3+4(2n^2+7n+5)k^2+(8n^2+33n+25)k+2(n^2+5n+5)]}{2[2k^2+(4n+7)k+2n+5]^2}, \\
W_B^{PN} &= \frac{a^2[4(n+1)k^3+4(2n^2+7n+5)k^2+(8n^2+33n+29)k+2(n^2+5n+7)]}{2[2k^2+(4n+7)k+2n+5]^2}, \\
W_T^{PN} &= \frac{a^2[(4(n+1)k^3+4(2n^2+7n+5)k^2+(8n^2+33n+27)k+2(n^2+5n+6))]}{[2k^2+(4n+7)k+2n+5]^2}.
\end{aligned}$$

### The equilibrium outcomes in subgame $NP$

$$\begin{aligned}
X_T^{NP} &= \frac{a[2(n+1)k+1]}{k^2+(2n+3)k+1}, & X_A^{NP} &= \frac{a[(n+1)k+1]}{k^2+(2n+3)k+1}, & X_B^{NP} &= \frac{ak(n+2)}{k^2+(2n+3)k+1}, \\
x_{ij}^{NP} &= \frac{ak}{k^2+(2n+3)k+1}, & x_{A0}^{NP} &= \frac{a(k+1)}{k^2+(2n+3)k+1}, & \Pi_{ij}^{NP} &= \frac{a^2k^2(k+2)}{2[k^2+(2n+3)k+1]^2}, \\
\Pi_{A0}^{NP} &= \frac{a^2k(k+1)^2}{2[k^2+(2n+3)k+1]^2}, \\
W_A^{NP} &= \frac{a^2[2(n+1)k^3+4(n^2+3n+2)k^2+2(2n+3)k+1]}{4[k^2+(2n+3)k+1]^2}, \\
W_B^{NP} &= \frac{a^2[2(n+1)k^3+4(n^2+3n+2)k^2+4(n+1)k+1]}{4[k^2+(2n+3)k+1]^2}, \\
W_T^{NP} &= \frac{a^2[2(n+1)k^3+4(n^2+3n+2)k^2+(4n+5)k+1]}{2[k^2+(2n+3)k+1]^2}.
\end{aligned}$$

### Proof of Lemma 1

For the proof of (a),

$$\begin{aligned}
X_T^{NP} - X_T^{PN} &= \frac{a(k+1)}{[k^2+(2n+3)k+1][2k^2+(4n+7)k+2n+5]} > 0, \\
X_T^{NP} - X_T^{NN} &= -\frac{ak(k+1)}{[k^2+(2n+3)k+1][k^2+(2n+3)k+2]} < 0, \\
X_T^{PN} - X_T^{PP} &= \frac{2a(k+1)}{(k+2n+3)[2k^2+(4n+7)k+2n+5]} > 0.
\end{aligned}$$

Thus, (a) holds. For (b),

$$\begin{aligned} X_A^{PP} - X_A^{PN} &= \frac{2a(n+1)}{(k+2n+3)[2k^2+(4n+7)k+2n+5]} > 0, \\ X_A^{NN} - X_A^{NP} &= -\frac{a[(n+1)k+1]}{[k^2+(2n+3)k+1][k^2+(2n+3)k+2]} < 0, \\ X_A^{NN} - X_A^{PP} &= \frac{a(k+1)}{(k+2n+3)[k^2+(2n+3)k+2]} > 0. \end{aligned}$$

Further,

$$\begin{aligned} X_B^{PP} - X_B^{NP} &= \frac{a(n+1)}{(k+2n+3)[k^2+(2n+3)k+1]} > 0, \\ X_B^{NN} - X_B^{PN} &= -\frac{a[(2n+1)k+1]}{[k^2+(2n+3)k+2][2k^2+(4n+7)k+2n+5]} < 0, \\ X_B^{PP} - X_B^{NN} &= -\frac{a(k+1)}{(k+2n+3)[k^2+(2n+3)k+2]} < 0. \end{aligned}$$

These demonstrate that (b) also holds. Finally, (c) and (d) follow from

$$\begin{aligned} x_{A0}^{PP} - x_{A0}^{PN} &= \frac{2a}{(k+2n+3)[2k^2+(4n+7)k+2n+5]} > 0, \\ x_{A0}^{NP} - x_{A0}^{NN} &= \frac{a(k+1)}{[k^2+(2n+3)k+1][k^2+(2n+3)k+2]} > 0, \\ x_{A0}^{NN} - x_{A0}^{PP} &= \frac{a(k+2n+3)}{(k+2n+3)[k^2+(2n+3)k+2]} > 0, \\ x_{B0}^{NP} - x_{B0}^{PP} &= -\frac{a}{(k+2n+3)[k^2+(2n+3)k+1]} < 0, \\ x_{B0}^{PP} - x_{B0}^{PN} &= -\frac{2a[k+2(n+2)]}{(k+2n+3)[2k^2+(4n+7)k+2n+5]} < 0, \\ x_{B0}^{NN} - x_{B0}^{PN} &= -\frac{a[k-2n+1]}{[k^2+(2n+3)k+2][2k^2+(4n+7)k+2n+5]}, \\ x_{B0}^{NN} - x_{B0}^{PP} &= \frac{a(k+2n+1)}{(k+2n+3)[k^2+(2n+3)k+2]} > 0. \end{aligned}$$



Lastly,

$$\begin{aligned}x_{ij}^{NP} - x_{ij}^{PN} &= -\frac{a}{[k^2 + (2n + 5)k + 1][2k^2 + (4n + 11)k + 2n + 7]} < 0, \\x_{ij}^{PP} - x_{ij}^{PN} &= \frac{2a}{(k + 2n + 5)[2k^2 + (4n + 11)k + 2n + 7]} > 0, \\x_{ij}^{NN} - x_{ij}^{NP} &= -\frac{ak}{[k^2 + (2n + 5)k + 1][k^2 + (2n + 5)k + 2]} < 0.\end{aligned}$$

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