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Repayment Frequency and Repayment Performance in Microfinance

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Abstract

This paper explains why most microfinance institutions (MFIs) employ equal installment schedules rather than a single repayment one, which has been a puzzle for economic theorists. Equal installment contracts, however, pose a large financial burden on poor borrowers. The paper therefore investigates a flexible installment contract and establishes how the amount of each installment should be varied according to the interest rate, the probability with which an economic shock arises, and the future discount rate of the borrower, among others. It is shown that under certain conditions both MFIs and borrowers are better off under the proposed flexible installment contract.

JEL classification: O12; G21

Keywords: microfinance; frequent installments; flexible installments; full repayment

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1 Introduction

Microfinance has been regarded as one of the most promising means to alleviate poverty around the world. Following the success of the Grameen bank founded by Nobel Peace Prize laureate Muhammad Yunus, there are now at least 3,589 microfinance institutions (MFIs) serving more than 190 million clients, 128 million of which are poorest (Reed, 2011). This means that microfinance is now affecting the lives of one in some 37 people on earth. The questions naturally arise: What made it possible for MFIs to expand their outreach to the poorest whom traditional commercial banks had been unable to reach? What distinguish MFIs from other commercial banks? Economists have been trying to answer these questions over the last two decades (Armendariz and Morduch, 2005; Banerjee and Duflo, 2010; Karlan and Morduch, 2010).

Among the distinctive features of microfinance, group lending was the first to attract the attention of economic theorists, and has been at the center of theoretical research until recently (Besley and Coate, 1995; Ghatak, 1999; Ghatak and Guinnane, 1999; Stiglitz, 1990; Varian, 1990).[1] They argue that group lending raises repayment rates because, for example, it places each client under repayment pressure from other group members and/or it excludes problematic borrowers by letting borrowers, who have sufficient knowledge about their neighbors' personality and ability, voluntarily select other group members. Group lending, however, is not the only feature that distinguishes microfinance from traditional commercial banks. More recently, economists' attention has been directed to other features, most notably to dynamic aspects of microfinance (Fischer and Ghatak, 2011). They include sequential lending, where a few members of each group receive a loan first and the others can borrow only if those who borrowed successfully repay their loans (Roy Chowdhury, 2005, 2007); progressive lending, where loan size increases as a borrower repays more loans (Egli, 2004); frequent repayment, where borrowers are required to repay in small and frequent installments. The first aim of this paper is to explain why frequent installment schedules are adopted by MFIs, which remain largely unsettled theoretically.

In fact, frequent repayment schedules are quite common among MFIs; most of them require weekly repayments beginning immediately after loan disbursement. Despite its prevalence, frequent repayment has been a theoretical puzzle, because economic theory suggests that it would increase default and delinquency by depriving borrowers of financial liquidity. To our knowledge, there are two papers that attempt to provide a solution to this puzzle. One of them is Jain and Mansuri (2003). They argue that frequent repayment schedules force borrowers to borrow from informal lenders who have superior knowledge about borrowers, thereby enabling MFIs to indirectly take advantage of their knowledge and deal with moral hazard problems.

[1] Theoretical results, which still await empirical scrutiny, tend to emphasize the importance of group lending in alleviating asymmetric information problems. One of few and innovative empirical studies is Gine and Karlan (2010), in which, using randomized trials in the Philippines, they find no change in default between group and individual liability loan contracts.

Another is Fischer and Ghatak (2010). They show that if a borrower is present-biased, frequent repayment reduces the temptation to default by reducing the size of each installment payment, and thus permits larger loans.[2]

This paper makes another attempt to solve the puzzle. It shows that the expected repayment as well as the probability of full repayment under an equal installment contract, which is most often observed in reality, are higher than those under a single repayment contract, even when borrowers are rational and no informal lender exists. There are two reasons behind this result. First, in the case of the equal installment contract, individuals who have borrowed and invested in their projects usually pay earlier installments (before their project returns are realized) out of existing income sources and/or savings. On the other hand, since repayment under the single repayment contract is made at a later time, repayment rates under that contract are more often affected by outcomes of borrowers' projects. By offering the equal installment contract, therefore, MFIs can ensure a certain amount of repayment before the repayment date for the single repayment contract comes, making the effects on repayment of exogenous economic shocks and/or outcomes of borrowers' projects less serious. Second, the equal installment plan reduces the amount of each repayment without much affecting borrowers' incentive to raise project returns. Thus, borrowers are more likely to be able to repay in full after project returns are realized under the equal installment contract than under the single repayment contract.

Equal installment schedules, however, are often too much of a burden for the poor who cannot afford to pay installments before their project returns are realized, which either deprives them of credit access or forces them to borrow from other lenders and thereby pushes them to multiple debts. In fact, to ease financial burden on poor borrowers, the Grameen Bank has introduced a new system called the "Grameen Generalized System (GGS)," in which its staff are allowed to offer a wider variety of repayment schedules as they gather experience and information about borrowers:

> "...size of weekly installments can be varied. A borrower can pay more each week during peak business season, and pay less during lean period. In an extreme case, each installment can be of different size."
> (http://www.grameen-info.org/index.php?option=com_content&task=view&id=30&Itemid=764)

For borrowers who face serious problems such as sickness, economic shocks, or natural disaster, the Grameen Bank also offers a "flexible loan" in which the installment size is reduced so that even those borrowers can afford to pay. However, the design of repayment schedules is left to the discretion of each staff member. More generally, although it has been argued that more flexible installment contracts will be able to improve borrowers' repayment performance

[2]Empirical investigation into this feature has also begun, though still quite rare. For example, Field and Pande (2008) find no significant difference in delinquency and default between a weekly and a monthly repayment schedule. In contrast, Field et al. (2010) find that the introduction of a grace period raises both investment and default.

and welfare, how flexible each installment should be and in what way have not been analyzed in theoretically rigorous terms. The second aim of this paper is to investigate how the amount of each installment should vary under the flexible installment contract, which may depend on factors such as the interest rate, the timing of installments, the probability with which an economic shock or unexpected expenses arise, as well as the borrower's performance under the previous loan contract. It turns out that the proposed flexible installment contract benefits both the borrower and the MFI; the borrower needs to pay a smaller amount of installments than under the equal installment contract until her project return is realized, and the MFI can maintain a higher probability of full repayment than under the single repayment contract and, under certain conditions, the equal installment contract.

The remainder of this paper is organized as follows. Our setup is presented in Section 2. In Section 3, the single repayment contract is analyzed. The equal installment contract is studied in Section 4. In Section 5, we propose and investigate the flexible installment contract. Comparisons are also made with the single-repayment and equal-installment contracts. Section 6 concludes the paper and discusses possible extensions of our basic model.

2 The Model

We consider the situation in which an MFI offers a loan repeatedly to a risk-neutral borrower provided the borrower has repayed all prior loans.[3] The borrower has a source of income, from which she receives a constant income $w(>0)$ at the end of each period.[4] The borrower also has a project requiring an investment of $K(>0)$, returns from which are realized two periods after investment. Thus, if the borrower invests at the beginning of period 1, the project return is realized at the end of period 2. The gross return from the project depends on the total time spent on the project. If the borrower spends a_1 hours in period 1 and a_2 hours in period 2 on the project, the gross return is $\lambda(a_1 + a_2)K$.[5] The function λ satisfies $\lambda(0) = 0$, and is assumed to be strictly increasing ($\lambda' > 0$) and strictly concave ($\lambda'' < 0$) for all $a_1 + a_2 \geq 0$. If the borrower works a_t hours on the project in period t ($t = 1, 2, ...$), her money-equivalent disutility is $\psi(a_t)$. The function ψ is normalized so that $\psi(0) = 0$, and is assumed to be strictly increasing ($\psi' > 0$) and strictly convex ($\psi'' > 0$) for all $a_t \geq 0$. We suppose that the borrower discounts future utilities at a constant rate $\delta(\in (0, 1))$ per period and that she cannot save.[6] The

[3]We make this assumption for analytical simplicity. Tedeschi (2006) shows that in her model, a limited number of punishment periods are enough to discourage strategic default.

[4]As long as the individual can pay earlier installments under equal- and flexible- installment contracts, her income need not necessarily be constant. Again, we make this assumption for simplicity.

[5]If we suppose that the borrower cannot make full repayment when her project fails, an introduction of a probability π with which the project fails does not affect our major results.

[6]By "cannot save", we mean that earnings cannot be set aside for one period or longer. Hence, the fixed income and net project return, which become available at the end of a period, are used for needs that occur when they become available or during the next period before the next earnings are realized. Armendariz and Morduch (2005) and Karlan and Morduch (2010) point out that it is in fact difficult for the poor in developing countries to save because of various saving constraints, including self-control problems and lack of quality savings products,

borrower consumes a numeraire good. In addition, with probability $p(\in (0,1))$, the borrower faces unexpected expenses $D(>0)$ at the end of even periods, which are imposed by, say, house repairs, medical emergencies, or economic shocks.[7] Consumption in period 1 is financed by initial wealth $w_0(>0)$, which is exogenously given. Consumption in period t ($t = 2, 3, ...$) is financed by the fixed income and, if any, the project return net of repayment and unexpected expenses, all of which are realized at the end of the previous period. We assume $w_0 < K$ and $w < K$ so that the borrower can invest in her project only if she can borrow. The functions λ and ψ are time-invariant.

At the beginning of period 1, the MFI offers a two-period loan contract. The contract is either single-repayment, (K, R), or frequent-installment $\{K, (T, d), B, R\}$, where K is the size of the loan as well as the borrower's investment, $R(>1)$ a per-period gross interest rate, $T(\in [0, 2))$ the time between loan disbursement and the first installment, d the amount of the first installment, and B the amount of the final installment which is to be payed at the end of the second period.[8] Under the single repayment contract, the borrower is required to repay R^2K at the end of the second period. As for frequent installment contracts, we consider two types of two-installment contracts; an equal installment contract and a flexible installment contract. In the equal installment contract, the borrower repays the same amount at the end of the first and the second period. In the flexible installment contract, the second installment payment is required at the end of the second period, while the timing of the first installment (T) can be set freely. The amounts of the two installments can also be varied flexibly. In order to compare the effects of different repayment schedules on the borrower's behavior and repayment performance, the values of K and R are set to be identical in all types of contracts. As a first step toward a better understanding of flexible installment contracts, we consider the case in which the MFI has sufficient knowledge necessary to devise flexible installment plans. In particular, we suppose that, possibly through repeated transactions with the same borrower, the MFI can correctly infer from the project return how long the borrower has worked on the project. Alternatively, the MFI could gather such information from the borrower's neighbors or other group members, if any. Likewise, the MFI is assumed to know the borrower's earnings, the future discount rate, and the probability and amount of unexpected expenses. The welfare of the borrower falls (at least weakly) if the MFI imposes a minimum time input on her. If the resulting welfare decrease is large, the individual would rather not borrow from the MFI, regardless of potential benefits for the both parties that would have arisen had the MFI not imposed such restrictions. Thus, in this paper, we suppose that the MFI does not impose direct restrictions on time input and let the borrower work as much as she wants. If the borrower has fully repaid the loan, the MFI offers the next loan contract.

Given a loan contract offered by the MFI, the borrower decides whether or not to take it

among others.

[7]Unexpected expenses at the other timings do not affect the borrower's repayment performance, though they reduce consumption in the corresponding period.

[8]As we will see later (Eq. (17)), T, d and B satisfy a certain condition given R and K.

and invest in her project at the beginning of period 1. Since the MFI is supposed to know the borrower's earnings, the borrower repays fully whenever possible. Otherwise, she repays all her disposable earnings if any. When the borrower could fully repay her first loan, she decides whether or not to take a new loan from the MFI. Otherwise, contracts will be terminated permanently and zero consumption results in the next period. In the flexible installment contract, the borrower is assumed to know how the MFI will change the amounts of installments under the next contract.

Let C_t and U_t respectively denote the borrower's consumption and utility in period $t(=1, 2, 3, ...)$. Since the borrower is risk neutral, let $U_t = C_t - \psi(a_t)$. The borrower maximizes her total expected discounted utility (TEDU). Thus, her maximization problem is given by

$$max \;\; E\left(\sum_{t=1}^{\infty} \delta^{t-1} U_t\right).$$

In the benchmark case of no borrowing, the individual cannot invest in her project by assumption and thus her consumption in even periods is w. In period 1, the borrower consumes the initial wealth, w_0. In odd periods except period 1, the borrower's consumption, which is equal to her utility in this case, is w if unexpected expenses do not arise, which occurs with probability $1-p$, $\max\{0, w-D\}$ if unexpected expenses arise, which occurs with probability p. In our analysis, we restrict ourselves to unexpected expenses that cannot be covered by the fixed income: $w < D$.[9]

Let $\bar{v}$ denote the TEDU from period 2 evaluated in period 2. If unexpected expenses do not arise at the end of period 2, the individual can consume w in periods 2 and 3, and the TEDU from period 4 evaluated in period 4 is $\bar{v}$. On the other hand, if unexpected expenses arise at the end of period 2, the individual consumes w in period 2 and nothing in period 3, and, again, her TEDU from period 4 (evaluated in period 4) is $\bar{v}$. By discounting future utilities by δ per period, $\bar{v}$ is expressed as:

$$\bar{v} = (1-p)(w + \delta w + \delta^2 \bar{v}) + p(w + 0 + \delta^2 \bar{v}).$$

By solving for $\bar{v}$, we have

$$\bar{v} = \frac{[1 + \delta(1-p)]w}{1-\delta^2}.$$

In period 1, the borrower consumes the initial wealth w_0. Therefore, the TEDU from period 1 in the benchmark case of no borrowing is given by

$$\begin{aligned}\bar{V}(w_0) &= w_0 + \delta\bar{v} \\ &= w_0 + \frac{\delta[1 + \delta(1-p)]w}{1-\delta^2}.\end{aligned} \tag{1}$$

[9]This could be interpreted that if unexpected expenses of such extent arise, the borrower may rely on her relatives or friends to meet emergency needs but her consumption falls to (nearly) zero.

From the above expression, it can be seen that $\bar{V}$ is increasing in initial wealth and its marginal increase is one: $\bar{V}'(w_0) = 1(> 0)$. $\bar{V}(w_0)$ given above is also expressed as

$$\bar{V}(w_0) = w_0 + \delta w + \delta^2[(1-p)\bar{V}(w) + p\bar{V}(0)]. \tag{2}$$

3 The Single Repayment Contract

Let us first analyze the single repayment contract, (K, R). If the borrower agrees to the contract, she receives and invests K at the beginning of period 1, decides how long to work on the project in periods 1 and 2, and makes full, partial or no repayment at the end of period 2, depending on her disposable earnings at that time. The order of decisions and events is summarized in Figure 1.

Suppose that the borrower devotes a_1 hours to the project in period 1. Then she consumes the initial wealth, w_0, and incurs disutility from working for a_1 hours, $\psi(a_1)$. Thus, the borrower's utility in period 1 is $U_1 = w_0 - \psi(a_1)$. Likewise, if the borrower spends a_2 hours on the project in period 2, her utility in that period is her income net of the disutility of working on the project: $U_2 = w - \psi(a_2)$. The borrower's consumption in period 3, which is equal to the initial wealth at the beginning of period 3, is the sum of the fixed income and the project return net of repayment and, if any, unexpected expenses, provided that the borrower could make full repayment at the end of period 2. Otherwise, consumption in period 3 is zero. Formally, with probability $1 - p$, consumption in period 3 is

$$max\{0, C_3^H(a_1, a_2)\},$$

where

$$C_3^H(a_1, a_2)(= w_2^H(a_1, a_2)) = w + \lambda(a_1 + a_2)K - R^2K, \tag{3}$$

while with probability p

$$max\{0, C_3^L(a_1, a_2)\},$$

where

$$C_3^L(a_1, a_2)(= w_2^L(a_1, a_2)) = w + \lambda(a_1 + a_2)K - R^2K - D. \tag{4}$$

w_2^L and w_2^H denote the initial wealth at the beginning of period 3 with and without unexpected expenses, respectively.

As we will see in more detail below, how much the borrower works on her project depends on how likely it is that she will be able to fully repay her loan; as it becomes more likely that the borrower will be able to repay in full, she works longer (in total). Suppose that the borrower works for a_1^i hours in period 1 and a_2^i hours in period 2 if she expects that she will be able to repay fully only when unexpected expenses do not arise. Similarly, suppose that the borrower works for a_1^{ii} hours in period 1 and a_2^{ii} hours in period 2 if she expects that she will always be able to repay successfully regardless of unexpected expenses. Likewise, let a_1^{iii} (respectively, a_2^{iii}) denote the time input in period 1 (respectively, period 2) when the borrower expects that she will never be able to repay in full whether or not unexpected expenses arise. Let us restrict ourselves to situations wherein the borrower has rational expectations in the sense that her decisions on time input based on her expectations concerning successful repayment give rise to the expected outcomes. For example, if the borrower chooses the set of time inputs (a_1^{ii}, a_2^{ii}) expecting that she will always be able to repay her loan successfully, she can actually repay her loan fully regardless of unexpected expenses.

Throughout our analysis, we focus on cases wherein the following conditions are met:

Assumption 1: The borrower is (ex ante) better off borrowing from the MFI if she can fully repay her loan at least when unexpected expenses do not arise.

Assumption 2: The borrower's utility rises the more likely that she can successfully repay her loan.

In such cases, the borrower chooses to work longer if there exists another rational expectation that is associated with a higher probability of full repayment. Hence, the borrower chooses the set of time inputs (a_1^{i}, a_2^{i}) if she expects that she will be able to repay successfully only when unexpected expenses do not arise, which is correct if she chooses these levels of time input, *and* if she cannot make full repayment when unexpected expenses arise even if she works longer expecting that she will be able to repay in full regardless of unexpected expenses. Analogously, the borrower chooses the set of time inputs (a_1^{iii}, a_2^{iii}) if the following conditions are met: (a) she forms the self-fulfilling expectation that she will never be able to repay in full, (b) she cannot repay fully even if unexpected expenses do not arise when she chooses (a_1^{i}, a_2^{i}) expecting otherwise, and (c) she cannot repay successfully either with or without unexpected expenses when she chooses (a_1^{ii}, a_2^{ii}) expecting otherwise.

In sum, there are the following three possible cases:

Case (i): The borrower can make full repayment only if unexpected expenses do not arise if she so expects. In addition, she cannot repay fully when unexpected expenses arise even if she works longer expecting that she will always be able to repay in full. $\Longleftrightarrow$ $w_2^L(a_1^i, a_2^i) < 0 \leq w_2^H(a_1^i, a_2^i)$ and $w_2^L(a_1^{ii}, a_2^{ii}) < 0$

Case (ii): The borrower can always make full repayment regardless of unexpected expenses if she so expects. $\Longleftrightarrow w_2^L(a_1^{ii}, a_2^{ii}) \geq 0$

Case (iii): The borrower can never repay in full with or without unexpected expenses if she so expects. In addition, no other rational expectations exist, i.e., either she can never make full repayment even if she works longer expecting that she will be able to repay fully if unexpected expenses do not occur, or there are cases in which she cannot repay in full even if she works even longer expecting that she will always be able to repay successfully. $\Longleftrightarrow w_2^H(a_1^{iii}, a_2^{iii}) < 0$, $w_2^H(a_1^{i}, a_2^{i}) < 0$, and $w_2^L(a_1^{ii}, a_2^{ii}) < 0$ or $w_2^H(a_1^{ii}, a_2^{ii}) < 0$, where we consider strictly positive a_1^j and a_2^j ($j = i, ii, iii$).[10] In case (iii), the condition $w_2^H(a_1^{iii}, a_2^{iii}) < 0$ is redundant because $w_2^H(a_1^{iii}, a_2^{iii}) < w_2^H(a_1^i, a_2^i)$ holds. Also, the condition $w_2^H(a_1^{ii}, a_2^{ii}) < 0$ is not needed, for the condition $w_2^L(a_1^{ii}, a_2^{ii}) < 0$ alone prevents case (ii) from occurring.

First, in case (iii), the borrower can never repay in full and thus her consumption in period 3 is zero for certain. Since she loses borrowing opportunities from period 3, the TEDU from period 3 evaluated in period 3 is the TEDU in the benchmark case of no borrowing with zero initial wealth, $\bar{V}(0)$. The utilities in the first two periods are the income net of disutility from working. Hence, the TEDU in this case, $V^{iii}(w_0)$, is of the form

$$V^{iii}(w_0) = (w_0 - \psi(a_1^{iii})) + \delta(w - \psi(a_2^{iii})) + \delta^2 \bar{V}(0).$$

In the no-borrowing case, the present value of the first-period (respectively, the second-period) utility is w_0 (respectively, δw), which is greater than the first (respectively, the second) term in the right-hand side of the above expression. In addition, the initial wealth at the beginning of period 3 in the no-borrowing case is w with probability $1 - p$, 0 with probability p (see Eq. (2)), while that in case (iii) is zero for certain. Since $\bar{V}$ is increasing in initial wealth, we can conclude that $V^{iii}(w_0)$ given above is less than the utility without borrowing opportunities. Therefore, unless the borrower can make full repayment at least when unexpected expenses do not arise, she will not borrow from the MFI in the first place. In that sense, adverse selection is partially alleviated. The same applies to the equal-installment and flexible-installment contracts.

Second, in case (i), the borrower can make full repayment only if unexpected expenses do not arise. Thus, with probability $1 - p$, the borrower can successfully repay her loan at the end of period 2, in which case the initial wealth at the beginning of period 3 is $w_2^H(a_1, a_2)$ if she has worked for a_1 and a_2 hours in the first two periods. At the beginning of period 3, the borrower is placed in the same situation as she was at the beginning of period 1. The only difference is initial wealth. In contrast, if unexpected expenses arise, the borrower cannot make full repayment at the end of period 2, ending up with the no-borrowing situation with zero initial wealth at the beginning of period 3. In the first and the second period, the borrower consumes the initial wealth and the fixed income, respectively, and incurs disutility from working. Therefore, the value function in case (i) with initial wealth w_0, $V^i(w_0)$, satisfies:

$$V^i(w_0) = max\ \ (w_0 - \psi(a_1)) + \delta(w - \psi(a_2)) + \delta^2[(1-p)V^i(w_2^H(a_1, a_2)) + p\bar{V}(0)]. \quad (5)$$

Before solving for V^i, let us turn to case (ii), wherein the borrower can always make full repayment. In this case, the borrower's initial wealth at the beginning of period 3 is $w_2^L(a_1, a_2)$ with probability p, $w_2^H(a_1, a_2)$ with probability $1 - p$, which are both (at least weakly) positive.

[10] Conditions for interior optimums will be provided later (see footnotes 12, 15, and 16).

In either event, the borrower can receive the next loan at the beginning of period 3. The value function in case (ii) with initial wealth w_0, $V^{ii}(w_0)$, satisfies:

$$V^{ii}(w_0) = max\ (w_0 - \psi(a_1)) + \delta(w - \psi(a_2)) + \delta^2[(1-p)V^{ii}(w_2^H(a_1, a_2)) + pV^{ii}(w_2^L(a_1, a_2))]. \tag{6}$$

It is clear from (5) and (6) that both V^i and V^{ii} are increasing in initial wealth, and their marginal increase is unity. Put differently, changes in the initial wealth do not affect the borrower's subsequent decisions. Since w_0, w, δ, p, K, R, and D are all given, the Bellman equations (5) and (6) respectively reduce to the following maximization problems:

$$max_{a_1,a_2}\ \ -\psi(a_1) - \delta\psi(a_2) + \delta^2(1-p)\lambda(a_1 + a_2)K \tag{7}$$

$$max_{a_1,a_2}\ \ -\psi(a_1) - \delta\psi(a_2) + \delta^2\lambda(a_1 + a_2)K.$$

Since changes in the initial wealth at the beginning of any odd period do not affect the borrower's subsequent decisions, as long as loan contracts continue, the solutions to the above maximization problem, a_1^j and a_2^j ($j = i, ii$), are optimal in any odd and even periods, respectively.

In case (i), the borrower solves (7) by backward induction. First, given the time input in period 1, a_1, she maximizes the sum of the second and third terms. From the first-order condition, the optimal a_2 given a_1, $a_2^i(a_1)$, satisfies:[11]

$$\psi'(a_2^i(a_1)) = \delta(1-p)\lambda'(a_1 + a_2^i(a_1))K, \tag{8}$$

assuming an interior solution (see Figure 2).[12] This condition states that, given a_1, the optimal time spent on the project in period 2 equalizes the marginal increase in disutility in period 2 to the marginal increase in period-3 consumption (through an increase in the project return) discounted by the discount rate and for uncertainty.[13] As a_1 increases, the marginal increase in the project return with an increase in a_2 falls, with the marginal increase in disutility with a rise in a_2 being unchanged. It follows that $a_2^i(a_1)$ is decreasing in a_1: $a_2^{i\prime}(a_1) < 0$. In addition, differentiating (8) with respect to a_1 yields

$$a_2^{i\prime}(a_1) = -\frac{\delta(1-p)\lambda''(a_1 + a_2^i(a_1))K}{\delta(1-p)\lambda''(a_1 + a_2^i(a_1))K - \psi''(a_2^i(a_1))},$$

which is greater than -1. Thus, total time spent on the project increases as the borrower devotes more time in period 1:

$$\frac{d}{da_1}(a_1 + a_2^i(a_1)) = 1 + a_2^{i\prime}(a_1) > 0. \tag{9}$$

[11] The second-order condition is satisfied.

[12] Precisely, we assume $\psi'(0) < \delta(1-p)\lambda'(a_1)K$.

[13] It should be remembered that in case (i), all the earnings in period 2 are taken away if unexpected expenses arise.

Given $a_2^i(a_1)$, the borrower solves (7) with respect to a_1:

$$max_{a_1} \quad -\psi(a_1) - \delta\psi(a_2^i(a_1)) + \delta^2(1-p)\lambda(a_1 + a_2^i(a_1))K.$$

The first-order condition combined with (8) gives:[14]

$$\begin{aligned}\psi'(a_1) &= \delta^2(1-p)\lambda'(a_1 + a_2^i(a_1))K \\ &= \delta\psi'(a_2^i(a_1)),\end{aligned} \tag{10}$$

assuming again an interior solution.[15] The above condition ensures that the marginal increase in disutility in period 1 with an increase in the time input in that period is just compensated by the increase in period-3 consumption discounted by the discount rate and for uncertainty, taking into account the effect on the period-2 time input of changes in the period-1 time input. Conditions (8) and (10) characterize the optimal time inputs in periods 1 and 2 in case (i), a_1^i and $a_2^i \equiv a_2^i(a_1^i)$.

Similarly, the optimal time inputs in case (ii), a_1^{ii} and $a_2^{ii} \equiv a_2^{ii}(a_1^{ii})$, are determined by:[16]

$$\psi'(a_2^{ii}(a_1)) = \delta\lambda'(a_1 + a_2^{ii}(a_1))K \tag{11}$$

and

$$\begin{aligned}\psi'(a_1) &= \delta^2\lambda'(a_1 + a_2^{ii}(a_1))K \\ &= \delta\psi'(a_2^{ii}(a_1)).\end{aligned} \tag{12}$$

By differentiating (11) with respect to a_1, we have

$$-1 < a_2^{ii\prime}(a_1) = -\frac{\delta\lambda''(a_1 + a_2^{ii}(a_1))K}{\delta\lambda''(a_1 + a_2^{ii}(a_1))K - \psi''(a_2^{ii}(a_1))} < 0.$$

Therefore, as in case (i), total time spent on the project increases as a_1 rises:

$$\frac{d}{da_1}(a_1 + a_2^{ii}(a_1)) = 1 + a_2^{ii\prime}(a_1) > 0. \tag{13}$$

Several findings can be obtained here. First, by comparing (8) and (11), the optimal a_2 given a_1 is greater in case (ii) than in case (i):

$$a_2^i(a_1) < a_2^{ii}(a_1). \tag{14}$$

[14] The second-order condition is satisfied.

[15] The required assumption is: $\psi'(0) < \delta^2(1-p)\lambda'(a_2^i(0))K = \delta\psi'(a_2^i(0))$.

[16] The second-order conditions for optimal a_2 (given a_1) and a_1 are both satisfied. Conditions for the interior solutions are $\psi'(0) < \delta\lambda'(a_1)K$ and $\psi'(0) < \delta^2\lambda'(a_2^{ii}(0))K = \delta\psi'(a_2^{ii}(0))$.

This is because an increase in a_2 raises consumption in period 3 (through an increase in the project return) regardless of unexpected expenses in case (ii), while that is the case only when unexpected expenses do not occur in case (i). On the other hand, an increase in a_2 raises disutility in period 2 for certain in either case. Therefore, given the same a_1, the borrower spends more time on the project in period 2 in case (ii) than in case (i). Second, from (10) and (12), the borrower works longer on the project in period 2 than in period 1 in either case:

$$a_1^i < a_2^i(a_1^i) \quad \text{and} \quad a_1^{ii} < a_2^{ii}(a_1^{ii}).$$

The above results hold because an increase in period-3 consumption (due to a rise in the project return) is discounted less heavily in period 2 than in period 1. Third, comparing (10) and (12) reveals that the time input in period 1 is higher in case (ii) than in case (i):

$$a_1^i < a_1^{ii}. \tag{15}$$

The reason for this is similar to that for (14); an increase in a_1 raises the borrower's consumption in period 3 for certain in case (ii), only when unexpected expenses do not arise in case (i). Fourth, from (9), (14) and (15), the borrower works on the project longer in case (ii) than in case (i):

$$a_1^i + a_2^i(a_1^i) < a_1^{ii} + a_2^i(a_1^{ii}) < a_1^{ii} + a_2^{ii}(a_1^{ii}). \tag{16}$$

4 The Equal Installment Contract

In reality, most MFIs require equal installment repayment. In this section, we consider an equal installment plan as a special case of frequent repayment schedules. As stated in Section 2, two-installment contracts can be expressed as $\{K, (T, d), B, R\}$, where $T(< 2)$ is the time between loan disbursement and the first installment payment, $d(> 0)$ the amount of the first installment, and $B(> 0)$ the amount of the second and final installment, which is to be payed two periods after loan disbursement. In order to compare frequent- and single- repayment schedules, the loan size (K) and the interest rate (R) are kept identical to those in the case of single repayment. T, d, and B satisfy the following equation(s):

$$R^{2-T}d + B = R^2K \quad \Leftrightarrow \quad B = R^2K - R^{2-T}d. \tag{17}$$

From the above expression(s), it is obvious that the amount of repayment at the end of the second period, B, is smaller than that under the single repayment contract, R^2K, which is a common feature of frequent repayment schedules.

In most cases, borrowers must pay in equal installments on a regular basis, which corresponds to $T = 1$ and $d^e = B^e = R^2K/(1 + R)$ in our setup. The borrower should manage to pay the first installment from her disposable income at the end of the first period. We suppose that the individual can borrow from the MFI only when she can pay the first installment. Variables associated with the equal installment contract will be denoted using ˜(tilde).

The utility in period 1 when the borrower works for a_1 hours is $\tilde{U}_1 = w_0 - \psi(a_1)$, as in the case of single repayment. In contrast, since the borrower should now pay the first installment ($d^e (= B^e)$) at the end of period 1, the utility in period 2 becomes $\tilde{U}_2 = w - B^e - \psi(a_2)$. If the borrower can fully pay the final installment at the end of period 2, her consumption in period 3 is the sum of the fixed income and the project return, both of which become available at the end of period 2, net of the final installment payment and, if any, unexpected expenses. Otherwise, her consumption in period 3 is zero. More formally, period-3 consumption is

$$max\{0, \tilde{C}_3^H(a_1, a_2)\}$$

with probability $1 - p$,

$$max\{0, \tilde{C}_3^L(a_1, a_2)\}$$

with probability p, where

$$\tilde{C}_3^H(a_1, a_2)(= \tilde{w}_2^H(a_1, a_2)) = w + \lambda(a_1 + a_2)K - B^e$$

and

$$\tilde{C}_3^L(a_1, a_2)(= \tilde{w}_2^L(a_1, a_2)) = w + \lambda(a_1 + a_2)K - B^e - D.$$

$\tilde{w}_2^L$ and $\tilde{w}_2^H$ respectively denote the initial wealth at the beginning of period 3 with and without unexpected expenses. Since $B^e < R^2 K$ holds, $\tilde{w}_2^H(a_1, a_2)$ and $\tilde{w}_2^L(a_1, a_2)$ are greater than the corresponding functions in the case of single repayment, (3) and (4), respectively:

$$w_2^H(a_1, a_2) < \tilde{w}_2^H(a_1, a_2) \text{ and } w_2^L(a_1, a_2) < \tilde{w}_2^L(a_1, a_2). \tag{18}$$

This means that if the borrower works the same number of hours, she is more likely to be able to repay successfully under the equal installment contract than under the single repayment contract.

As in the previous section, the following three cases could occur:

Case (i): The borrower can make full repayment only if unexpected expenses do not arise if she so expects. In addition, she cannot repay fully when unexpected expenses arise even if she works longer expecting otherwise. $\Longleftrightarrow \tilde{w}_2^L(a_1^i, a_2^i) < 0 \leq \tilde{w}_2^H(a_1^i, a_2^i)$ and $\tilde{w}_2^L(a_1^{ii}, a_2^{ii}) < 0$

Case (ii): The borrower can always make full repayment if she so expects. $\Longleftrightarrow \tilde{w}_2^L(a_1^{ii}, a_2^{ii}) \geq 0$

Case (iii): The borrower cannot repay in full regardless of unexpected expenses if she so expects. In addition, no other rational expectations exist. $\Longleftrightarrow \tilde{w}_2^H(a_1^i, a_2^i) < 0$ and

$$\tilde{w}_2^L(a_1^{ii}, a_2^{ii}) < 0,$$

where a_1^j and a_2^j ($j = i, ii, iii$) are the optimal time inputs in periods 1 and 2 in each case, respectively. In case (iii), the borrower devotes some time to her project in periods 1 and 2 but can never repay in full, resulting in the no-borrowing situation with no initial wealth at the beginning of period 3 for certain. Hence, the borrower would rather choose not to borrow from the MFI in the first place. Since the timing and the amount of installment payments are fixed, the borrower's decisions on time input are unchanged from those under the single repayment contract. In the other two cases, therefore, the borrower chooses exactly the same levels of time input for the first two periods as those in the case of single repayment.

By the end of period 2, the borrower has already paid the first installment. In addition, since the borrower chooses the same levels of a_1 and a_2 under the equal-installment and single-repayment contracts, her total earnings available for repayment at the end of period 2 are also the same. This means that the amount of total repayment is always higher in the case of equal installments than in the case of single repayment. Moreover, since the amount of the required repayment at the end of period 2 is smaller in the former case, if the borrower can fully repay her loan under the single repayment contract, so can she under the equal installment contract, but not vice versa. Therefore, we have the following proposition:

Proposition 1. *Suppose that if the equal installment contract is offered, the borrower can pay the first installment from her existing income source: $B^e \leq w$. Then the expected repayment as well as the probability of full repayment under the equal installment contract are higher than those under the single repayment contract.*

The above proposition indicates that there exist cases in which (a) the borrower can fully repay her loan only when unexpected expenses do not occur under the single repayment contract, while she can always do so under the equal installment contract, and (b) the borrower cannot repay in full regardless of unexpected expenses with single repayment, while she can repay successfully at least when unexpected expenses do not arise with equal installments. The following corollary gives the precise conditions under which (a) and (b) occur:

Corollary 1. **(a)** *Suppose that*

$$R^2 K \leq w + \lambda(a_1^i + a_2^i)K$$

and

$$B^e + D \leq w + \lambda(a_1^{ii} + a_2^{ii})K < R^2 K + D$$

hold. Then the borrower can repay in full only when unexpected expenses do not arise under the single repayment contract, while she can always do so under the equal installment contract.

(b) *Suppose that*

$$B^e \leq w + \lambda(a_1^i + a_2^i)K < R^2 K$$

and

$$w + \lambda(a_1^{ii} + a_2^{ii})K < R^2K + D$$

hold. Then the borrower cannot repay in full whether or not unexpected expenses occur if the single repayment contract is offered, while she can fully repay her loan at least when unexpected expenses do not occur under the equal installment contract. If, in addition,

$$B^e + D \leq w + \lambda(a_1^{ii} + a_2^{ii})K$$

holds, the borrower can always repay her loan successfully under the equal installment contract.

Proof. The borrower can fully repay her loan only when unexpected expenses do not arise under the single repayment contract if she falls into case (i) of the corresponding contract, in which case $w_2^L(a_1^i, a_2^i) < 0 \leq w_2^H(a_1^i, a_2^i)$ and $w_2^L(a_1^{ii}, a_2^{ii}) < 0$ hold. The borrower can always repay fully under the equal installment contract if she is in case (ii) of that contract, in which case $\tilde{w}_2^L(a_1^{ii}, a_2^{ii}) \geq 0$ holds. By rearranging these inequalities, we have the conditions stated in (a). The conditions in part (b) can be obtained in an analogous way. □

The above results indicate possible reasons why MFIs could lend to relatively poor borrowers without requiring collateral. It should be remembered, however, that under the equal installment contract, the borrower must be able to pay the first installment out of her existing income source, which may be an impossible requirement for the very poor. Hence, it is worth noting that borrowers whose fixed incomes fall below the amount of the first installment are left out of consideration when analyzing the equal installment contract.

5 The Flexible Installment Contract

Let us next investigate a more flexible installment plan in which the MFI varies the amounts of two installments more freely. Precisely, we suppose that the MFI offers a two-installment contract, $\{K, (T, d), B, R\}$, where the amount of the first installment, d, is not necessarily the same as that of the final repayment, B. We allow the MFI to set the value of d based on how long the borrower worked under the previous loan contract. Since d and B satisfy Eq. (17), as the amount of the first installment increases, that of the final installment decreases, and vice versa. In what follows, we mainly consider the case in which the first installment payment is required during the first period, i.e., $0 \leq T < 1$. The other case $1 \leq T < 2$ can be analyzed in a similar way, and so will be briefly referred to after the main case has been studied. The final installment is due at the end of the second period. The timing of decisions and events is summarized in Figure 3. Values associated with the flexible installment contract are denoted using a “hat”.

In period 1 (before the fixed income earned in that period becomes available), the borrower is required to pay the first installment, d_0, which is exogenously given. Since she must do so using her initial wealth, w_0, her consumption in period 1 is given by $\hat{C}_1 = w_0 - d_0$. Consumption in period 2 is financed by the fixed income that the borrower earned in period 1, as is the case under the single repayment contract: $\hat{C}_2 = w$. The initial wealth at the beginning of period 3, $\hat{w}_2$, is the sum of the period-2 income and the project return net of the final installment payment and, if any, unexpected expenses, provided the net sum is weakly positive. Otherwise, the borrower cannot repay successfully and thus her wealth at the beginning of period 3 is zero. We suppose that the amount of the first installment under the subsequent contract is set to be sufficiently small so that the borrower can pay it whenever she has successfully repaid the current loan ($d \leq \hat{w}_2$). It follows that consumption in period 3 is the initial wealth at the beginning of period 3 subtracted by the first installment payment if the initial wealth is positive, zero otherwise. To be more precise, period-3 consumption is

$$\hat{C}_3^H(a_1, a_2) = max\{0, \hat{w}_2^H(a_1, a_2) - d(a_1 + a_2)\}$$

with probability $1 - p$,

$$\hat{C}_3^L(a_1, a_2) = max\{0, \hat{w}_2^L(a_1, a_2) - d(a_1 + a_2)\}$$

with probability p, where

$$\hat{w}_2^H(a_1, a_2) = w + \lambda(a_1 + a_2)K - B_0$$

and

$$\hat{w}_2^L(a_1, a_2) = w + \lambda(a_1 + a_2)K - B_0 - D, \tag{19}$$

with $B_0 = R^2K - R^{2-T}d_0$. From $B_0 < R^2K$, $\hat{w}_2^H$ and $\hat{w}_2^L$ given above are greater than the corresponding functions in the single repayment contract, (3) and (4), respectively:

$$w_2^H(a_1, a_2) < \hat{w}_2^H(a_1, a_2) \text{ and } w_2^L(a_1, a_2) < \hat{w}_2^L(a_1, a_2).$$

Thus, given the same (a_1, a_2), it is more likely that the borrower can make full repayment under the flexible installment contract than under the single repayment contract.

As in the previous sections, there are three possible cases:

Case (i): The borrower can make full repayment only if unexpected expenses do not arise if she so expects. In addition, she cannot repay in full when unexpected expenses arise even if she works longer expecting otherwise. $\iff \hat{w}_2^L(\hat{a}_1^i, \hat{a}_2^i) < 0 \leq \hat{w}_2^H(\hat{a}_1^i, \hat{a}_2^i)$ and $\hat{w}_2^L(\hat{a}_1^{ii}, \hat{a}_2^{ii}) < 0$

Case (ii): The borrower can always make full repayment if she so expects. $\iff \hat{w}_2^L(\hat{a}_1^{ii}, \hat{a}_2^{ii}) \geq$

0

Case (iii): The borrower cannot repay in full regardless of unexpected expenses if she so expects. In addition, no other rational expectations exist. $\Longleftrightarrow \hat{w}_2^H(\hat{a}_1^i, \hat{a}_2^i) < 0$ and $\hat{w}_2^L(\hat{a}_1^{ii}, \hat{a}_2^{ii}) < 0$,

where $\hat{a}_1^j$ and $\hat{a}_2^j$ $(j = i, ii, iii)$ are the optimal time inputs in periods 1 and 2 in each case, respectively.[17] As is the case under the single-repayment and equal-installment contracts, the borrower will not borrow in case (iii), for the TEDU in this case is

$$\hat{V}^{iii}(w_0) = (w_0 - d_0 - \psi(\hat{a}_1^{iii})) + \delta(w - \psi(\hat{a}_2^{iii})) + \delta^2 \bar{V}(0),$$

which is less than that in the benchmark case with no borrowing (see Eq. (2)). Hence, we consider only cases (i) and (ii) below.

Suppose that the borrower could successfully repay her first loan. Then at the beginning of period 3, the amounts of the two installments under the new contract, which depend on the total time input under the first contract, are already given for her. Hence, the borrower's decisions on time input from period 3 on are not affected by the time inputs in periods 1 and 2. On the other hand, at the beginning of period 1, the borrower should decide how long to work in the first two periods by taking into account not only disutility from working and the project return but also the effect on the amounts of two installments under the next contract. This is because the amount of the first installment under the next contract, which is due during period 3, affects her period-3 consumption, and the amount of the final installment under the next contract, which is due at the end of period 4, affects her period-5 consumption (provided the borrower can fully repay the next loan). Therefore, by choosing the levels of time input in periods 1 and 2, the borrower maximizes the sum of the expected discounted utilities in the first two periods, the expected discounted consumption in periods 3 and 4, and the expected discounted gain in period 5 from a reduction in the final installment payment under the next contract, which is due to the first installment payment of the next contract. Thus, the borrower's utility maximization problem in case (i) is given by

$$max_{a_1,a_2} \ -\psi(a_1) - \delta\psi(a_2) + \delta^2(1-p)(\lambda(a_1+a_2)K - d(a_1+a_2)) + \delta^4(1-p)^2 R^{2-T} d(a_1+a_2), \tag{20}$$

while that in case (ii) is

$$max_{a_1,a_2} \ -\psi(a_1) - \delta\psi(a_2) + \delta^2[\lambda(a_1+a_2)K - d(a_1+a_2)] + \delta^4 R^{2-T} d(a_1+a_2),$$

in which the constants are removed.

[17] For simplicity, we suppose that the borrower can make full repayment if and only if she can do so for the exogenously given B_0.

Let us consider case (i) first. As in the the case of single repayment, the borrower maximizes the last three terms in the maximand given a_1, and then solves the whole maximization problem with respect to a_1. The first-order condition for the optimal a_2 given a_1, $\hat{a}_2^i(a_1)$, is

$$\psi'(\hat{a}_2^i(a_1)) = \delta(1-p)\lambda'(a_1 + \hat{a}_2^i(a_1))K + \delta(1-p)[\delta^2(1-p)R^{2-T} - 1]d'(a_1 + \hat{a}_2^i(a_1)), \quad (21)$$

assuming an interior solution.[18] The second-order condition is satisfied if

$$[\delta^2(1-p)R^{2-T} - 1]d'' < 0. \quad (22)$$

Differentiating (21) with respect to a_1 gives

$$\frac{d\hat{a}_2^i(a_1)}{da_1} = -\frac{\delta(1-p)\lambda''(a_1 + \hat{a}_2^i(a_1))K + \delta(1-p)[\delta^2(1-p)R^{2-T} - 1]d''(a_1 + \hat{a}_2^i(a_1))}{\delta(1-p)\lambda''(a_1 + \hat{a}_2^i(a_1))K + \delta(1-p)[\delta^2(1-p)R^{2-T} - 1]d''(a_1 + \hat{a}_2^i(a_1)) - \psi''(\hat{a}_2^i(a_1))},$$

which is negative but greater than -1 if (22) holds. Then the total time spent on the project increases as a_1 increases:

$$\frac{d}{da_1}[a_1 + \hat{a}_2^i(a_1)] = 1 + \hat{a}_2^{i'}(a_1) > 0.$$

Given $\hat{a}_2^i(a_1)$, the borrower solves the maximization problem (20) with respect to a_1. From the first-order condition and (21), the optimal a_1 in case (i), $\hat{a}_1^i$, is determined by:[19]

$$\begin{aligned}\psi'(a_1) &= \delta^2(1-p)\lambda'(a_1 + \hat{a}_2^i(a_1))K + \delta^2(1-p)[\delta^2(1-p)R^{2-T} - 1]d'(a_1 + \hat{a}_2^i(a_1)) \\ &= \delta\psi'(\hat{a}_2^i(a_1)). \quad (23)\end{aligned}$$

The optimum values of a_1 and a_2 in case (i), $\hat{a}_1^i$ and $\hat{a}_2^i \equiv \hat{a}_2^i(\hat{a}_1^i)$, are determined by (21) and (23). Let us suppose that the MFI sets the timing and the amount of the first installment such that

$$[\delta^2(1-p)R^{2-T} - 1]d' > 0. \quad (24)$$

The above inequality, the meaning of which will be explained later, plays a critical role in motivating the borrower to work longer.

Likewise in case (ii), the optimal values, $\hat{a}_1^{ii}$ and $\hat{a}_2^{ii} \equiv \hat{a}_2^{ii}(\hat{a}_1^{ii})$, are characterized by:[20]

$$\psi'(\hat{a}_2^{ii}(a_1)) = \delta\lambda'(a_1 + \hat{a}_2^{ii}(a_1))K + \delta(\delta^2 R^{2-T} - 1)d'(a_1 + \hat{a}_2^{ii}(a_1)) \quad (25)$$

[18] The required condition is: $\psi'(0) < \delta(1-p)\lambda'(a_1)K + \delta(1-p)[\delta^2(1-p)R^{2-T} - 1]d'(a_1)$.

[19] The second-order condition is satisfied if (22) holds. The condition for an interior optimum is: $\psi'(0) < \delta^2(1-p)\lambda'(\hat{a}_2^i(0))K + \delta^2(1-p)[\delta^2(1-p)R^{2-T} - 1]d'(\hat{a}_2^i(0)) = \delta\psi'(\hat{a}_2^i(0))$.

[20] The condition for an interior optimum is $\psi'(0) < \delta\lambda'(a_1)K + \delta(\delta^2R^{2-T} - 1)d'(a_1)$ for a_2^{ii}, $\psi'(0) < \delta^2\lambda'(\hat{a}_2^{ii}(0))K + \delta^2(\delta^2R^{2-T} - 1)d'(\hat{a}_2^{ii}(0)) = \delta\psi'(\hat{a}_2^{ii}(0))$ for a_1^{ii}.

and

$$\psi'(a_1) = \delta^2\lambda'(a_1 + \hat{a}_2^{ii}(a_1))K + \delta^2(\delta^2 R^{2-T} - 1)d'(a_1 + \hat{a}_2^{ii}(a_1))$$
$$= \delta\psi'(\hat{a}_2^{ii}(a_1)). \tag{26}$$

As in case (i), we suppose that the MFI changes the amount of the first installment so that the following inequalities hold:

$$(\delta^2 R^{2-T} - 1)d'' < 0 \tag{27}$$

$$(\delta^2 R^{2-T} - 1)d' > 0. \tag{28}$$

Differentiating (25) with respect to a_1 yields

$$-1 < \frac{d\hat{a}_2^{ii}(a_1)}{da_1} = -\frac{\delta\lambda''(a_1 + \hat{a}_2^{ii}(a_1))K + \delta(\delta^2 R^{2-T} - 1)d''(a_1 + \hat{a}_2^{ii}(a_1))}{\delta\lambda''(a_1 + \hat{a}_2^{ii}(a_1))K + \delta(\delta^2 R^{2-T} - 1)d''(a_1 + \hat{a}_2^{ii}(a_1)) - \psi''(\hat{a}_2^{ii}(a_1))} < 0.$$

Thus, again, total time input increases as period-1 time input increases:

$$\frac{d}{da_1}[a_1 + \hat{a}_2^{ii}(a_1)] = 1 + \hat{a}_2^{ii'}(a_1) > 0.$$

Let us briefly mention the case wherein the first installment payment is required during the second period of each contract, $1 \leq T < 2$. The only difference from the case of $0 \leq T < 1$ is that consumption in the second period, rather than that in the first period, decreases by the amount of the first installment. In this case, the function d should satisfy

$$[\delta(1-p)R^{2-T} - 1]d'' < 0$$

and

$$[\delta(1-p)R^{2-T} - 1]d' > 0$$

in case (i) instead of (22) and (24), respectively,

$$(\delta R^{2-T} - 1)d'' < 0$$

and

$$(\delta R^{2-T} - 1)d' > 0$$

in case (ii) instead of (27) and (28), respectively. The sings of d' and d'' may or may not change as the MFI changes the timing of the first installment payment from the first period to the second one. If, for example, the borrower discounts the future so heavily that $\delta^2(1-p)R^{2-T} < 1 < \delta(1-p)R^{2-T'}$ hold with $0 \leq T < 1$ and $1 \leq T' < 2$ in case (i), the sign of d' changes from negative to positive as the MFI changes the timing of the first installment payment from the first to the second period; i.e., the MFI should decrease (respectively, increase) the amount of the first installment as the borrower works longer if it requires the first installment payment during the first (respectively, the second) period. Cases for other values of δ, R, T, and p can be analyzed analogously.

We are now ready to compare the flexible installment contract with the other two types of contracts. To begin with, let us compare the flexible installment contract with the single repayment contract. First, from (8) and (21), the optimal level of a_2 given a_1 in case (i) of the flexible installment contract, $\hat{a}_2^i(a_1)$, is greater than that in case (i) of the single repayment contract, $a_2^i(a_1)$:

$$a_2^i(a_1) < \hat{a}_2^i(a_1). \tag{29}$$

In both types of contracts, as the borrower works longer in period 2, disutility in that period as well as the expected consumption in period 3 rise. In addition, in the flexible installment contract, a marginal increase in period-2 time input changes the amount of the first installment under the next contract by d', which in turn changes the amount of the second installment under the next contract. A unit increase in repayment during period 3 reduces consumption in that period by one but raises period-5 consumption by R^{2-T} through the reduction in the amount of repayment at the end of period 4 (see Eq. (17)), provided the borrower can repay the second loan fully. In case (i), the borrower can repay successfully with probability $1-p$. Hence, the expected discounted sum of the borrower's gains from repaying one more unit of money during period 3 evaluated in period 3 is $\delta^2(1-p)R^{2-T}-1$. If this is positive, the borrower is better off paying more during period 3 as the first installment, and vice versa. The condition (24) thus ensures that the borrower is better off working longer under the current contract. These effects, which are absent in the single repayment contract, motivate the borrower to work longer under the flexible installment contract. By comparing (11) and (25), we can see that given the same period-1 time input, the borrower works longer in period 2 under the flexible installment contract than under the single repayment contract in case (ii) too:

$$a_2^{ii}(a_1) < \hat{a}_2^{ii}(a_1). \tag{30}$$

Second, since (29) holds, comparing (10) and (23) shows that the borrower spends more time on her project in period 1 in case (i) of the flexible installment contract than in case (i) of the single repayment contract:

$$a_1^i < \hat{a}_1^i.$$

From (9), (29) and the above inequality, the borrower devotes more time to her project under the flexible installment contract when she falls into case (i) under both contracts:

$$a_1^i + a_2^i(a_1^i) < \hat{a}_1^i + a_2^i(\hat{a}_1^i) < \hat{a}_1^i + \hat{a}_2^i(\hat{a}_1^i). \tag{31}$$

Likewise in case (ii), (12) and (26) together with (30) show that the borrower works longer on her project in period 1 under the flexible installment contract:

$$a_1^{ii} < \hat{a}_1^{ii}.$$

From (13), (30) and the above inequality, the borrower again devotes more time to her project under the flexible installment contract than under the single repayment contract in case (ii):

$$a_1^{ii} + a_2^{ii}(a_1^{ii}) < \hat{a}_1^{ii} + a_2^{ii}(\hat{a}_1^{ii}) < \hat{a}_1^{ii} + \hat{a}_2^{ii}(\hat{a}_1^{ii}). \tag{32}$$

In short, in the flexible installment contract, if the timing and the amount of the first installment are set so that (22) and (24) hold in case (i), and (27) and (28) hold in case (ii), the borrower's expected discounted consumption increases as she spends a longer time on her project not only through the increase in the project return under the current loan contract but also through the change in the amounts of installment payments under the next contract and the consequent rearrangement of future consumption in the borrower's favor. In contrast, the consumption-rearrangement effect is absent in the single repayment contract. Therefore, the borrower works longer on her project with flexible installments than with a single repayment.

Lemma 1. *Suppose that the borrower can fully repay her loan either (i) only when unexpected expenses do not arise, or (ii) whether or not unexpected expenses arise. Suppose also that, under the flexible installment contract, the MFI sets the timing and the amount of the first installment under the next contract so that (22) and (24) hold in case (i), (27) and (28) hold in case (ii). Then in either case, the borrower devotes more time to her project under the flexible installment contract than under the single repayment contract.*

From (4), (19) and (32), if the borrower can fully repay her loan regardless of unexpected expenses under the single repayment contract, so can she under the flexible installment contract. On the other hand, if the borrower cannot repay in full with unexpected expenses under the single repayment contract, there are two possible cases; she still cannot make full repayment under the flexible installment contract in bad times, or she can always repay in full under the flexible installment contract regardless of unexpected expenses. Clearly, in the latter case, the amount of repayment increases under the flexible installment contract. Also, in the former case, from (4), (19) and (31) together with the fact that the borrower pays the first installment in the case of flexible installments, the amount of repayment increases under the flexible installment contract. Therefore, we have the following proposition:

Proposition 2. *Suppose that the assumptions stated in Lemma 1 hold. Then both the probability of full repayment and the expected repayment are higher under the flexible installment contract than under the single repayment contract.*

In particular, the following corollary gives precise conditions under which the borrower's repayment performance is strictly better under the flexible installment contract:

Corollary 2. **(a)** *Suppose that*

$$R^2K \le w + \lambda(a_1^i + a_2^i)K$$

$$w + \lambda(a_1^{ii} + a_2^{ii})K < R^2K + D$$

and

$$B_0 + D \le w + \lambda(\hat{a}_1^{ii} + \hat{a}_2^{ii})K$$

hold. Then the borrower can repay fully only when unexpected expenses do not arise under the single repayment contract, while she can always do so under the flexible installment contract.

(b) *Suppose that*

$$w + \lambda(a_1^i + a_2^i)K < R^2K$$

$$w + \lambda(a_1^{ii} + a_2^{ii})K < R^2K + D$$

and

$$B_0 \le w + \lambda(\hat{a}_1^i + \hat{a}_2^i)K$$

hold. Then the borrower cannot repay in full regardless of unexpected expenses (and thus remains in the no-borrowing situation) if the single repayment contract is offered, while she can fully repay her loan at least when unexpected expenses do not arise (and thus will borrow) under the flexible installment contract. If, in addition,

$$B_0 + D \le w + \lambda(\hat{a}_1^{ii} + \hat{a}_2^{ii})K$$

holds, the borrower can always repay her loan successfully under the flexible installment contract.

Let us next compare the flexible installment contract with the equal installment contract. Under the latter type of contract, the amount of final installment (B^e) might be smaller than that under the flexible installment contract (B_0), which makes it easier for the borrower to pay. On the other hand, the project return is higher under the flexible installment contract than under the equal installment contract. Thus, in general, it is ambiguous whether or not the probability of full repayment is higher under the flexible installment contract. If, however, the borrower's productivity is sufficiently high, harder work under the flexible installment contract improves the borrower's repayment performance. In such cases, under our assumption that the borrower is better off the more likely that she will have credit access in the future, her welfare is higher under the flexible installment contract:

Proposition 3. **(a)** *Suppose that*

$$B^e \le w + \lambda(a_1^i + a_2^i)K$$

$$w + \lambda(a_1^{ii} + a_2^{ii})K < B^e + D$$

and

$$B_0 + D \leq w + \lambda(\hat{a}_1^{ii} + \hat{a}_2^{ii})K$$

hold. Then the borrower can repay in full only when unexpected expenses do not arise under the equal installment contract, while she can always do so under the flexible installment contract. Therefore, the flexible installment contract is better for both the MFI and the borrower.

(b) *Suppose that*

$$w + \lambda(a_1^i + a_2^i)K < B^e$$

$$w + \lambda(a_1^{ii} + a_2^{ii})K < B^e + D$$

and

$$B_0 \leq w + \lambda(\hat{a}_1^i + \hat{a}_2^i)K$$

hold. Then the borrower cannot repay in full regardless of unexpected expenses if the equal installment contract is offered, while she can fully repay her loan at least when unexpected expenses do not occur under the flexible installment contract. If, in addition,

$$B_0 + D \leq w + \lambda(\hat{a}_1^{ii} + \hat{a}_2^{ii})K$$

holds, the borrower can always fully repay her loan under the flexible installment contract, in which case flexible installments are better for both the MFI and the borrower.

So far, we have compared the flexible-installment and the equal-installment contract by assuming that the borrower can pay the first installment of the latter contract out of her income. Thus, poor borrowers who cannot afford to pay it were left out of consideration. If we focus on those borrowers, as long as borrowing opportunities improve borrower welfare, the flexible installment contract benefits the borrower under looser conditions:

Proposition 4. *Suppose that either* (a) $B_0 + D \leq w + \lambda(\hat{a}_1^{ii} + \hat{a}_2^{ii})K$*, or* (b) $w + \lambda(a_1^{ii} + a_2^{ii})K < B^0 + D$ *and* $B_0 \leq w + \lambda(\hat{a}_1^i + \hat{a}_2^i)K$*, are satisfied. Suppose also that the borrower's income falls below the amount of the first installment in the equal installment contract:* $w < B^e$*. Then, the borrower's (ex ante) welfare is higher when the flexible installment contract is offered than when the equal installment contract is offered.*

The conditions on the function d, namely, (22) and (24) for case (i), and (27) and (28) for case (ii), are worth emphasizing. These conditions show that how the amount of the first installment should be changed depends on the future discount rate of the borrower, δ, as well as the interest rate, R. Let us consider case (i) for example. Suppose that the borrower should decide how long to work under the first contract, i.e., during periods 1 and 2. Suppose also that, given the probability of unexpected expenses (p), the borrower does not discount the future much and/or the interest rate is sufficiently high so that $\delta^2(1-p)R^{2-T} > 1$ holds. Then, as we have already seen, an additional unit of repayment during period 3 under the next contract reduces the amount of the final repayment at the end of period 4 by R^{2-T}, which in turn raises period-5 consumption by that amount with probability $1 - p$. Since the expected discounted value of the consumption increase evaluated in period 3 is $\delta^2(1 - p)R^{2-T}$, which is greater than one, the more the borrower pays as the first installment during period 3, the more she gains. Therefore, if the borrower does not discount future much and/or the interest rate is high, the MFI can induce the borrower to work harder by raising the amount of the first installment under the next contract as the borrower works longer under the current contract. In contrast, if the borrower discounts future utilities heavily enough and/or the interest rate is low enough for $\delta^2(1 - p)R^{2-T} < 1$ to hold, the MFI should decrease the amount of the first installment under the next contract as the borrower works longer under the current contract. In this way, the MFI can raise the borrower's period-3 consumption. The amount of the final installment under the next contract (which is due at the end of period 4) increases according to Eq. (17) and, as a result, period-5 consumption falls. Still, as a heavy future discounter, the borrower is induced to work longer during periods 1 and 2 because period-3 consumption is much more important than period-5 consumption.

The timing of the first installment, T, is also important. The sooner the borrower pays the first installment during the first period, the smaller the amount of the final repayment (see, again, Eq. (17)). Suppose that the borrower should decide how long to work during periods 1 and 2 and that T is small enough for $\delta^2(1 - p)R^{2-T} > 1$ to hold. Following the same argument as above, an increase in the amount of repayment during period 3 (under the next contract) reduces period-3 consumption but raises period-5 consumption. Since the latter effect dominates the former, the expected discounted utility from consumption increases. Thus, the borrower works longer under the current contract if she knows that the amount of the first installment under the next contract will increase by doing so. The opposite result holds if T is so large that $\delta^2(1 - p)R^{2-T} < 1$ holds.

Analogously, the probability of unexpected expenses also matters in case (i). If it is highly likely that the borrower will face unexpected expenses at the end of even periods, the probability that the borrower will gain in period 5 due to a decrease in the amount of the final repayment under the next contract (at the end of period 4) is quite low, because that happens only when unexpected expenses do not arise at the end of period 4. In such cases, $\delta^2(1 - p)R^{2-T} < 1$ is more likely to hold. Then the value of a unit of consumption during period 3 is higher than the expected discounted value of the increase in period-5 consumption due to a unit increase of repayment during period 3. Therefore, the MFI should decrease the amount of the first installment under the next contract as the borrower devotes more time to her project under the current contract. We obtain the opposite result if p is small.

Proposition 5. *If* (1) *the borrower does not discount the future much, and/or* (2) *the interest rate is sufficiently high, and/or* (3) *the first installment is required soon after loan disbursement, and/or* (4) *unexpected expenses occur with low probability in case (i) (wherein the borrower* *can fully repay her loan only when unexpected expenses do not arise), then the MFI should raise the amount of the first installment under the next contract as the borrower works longer on her project under the current loan contract, and vice versa.*

One of the major differences between the equal-installment and the flexible-installment contract is that the amount of the first installment could be too much of a burden for the very poor in the case of equal installment, whereas that can be flexibly chosen so that every borrower who could successfully repay the previous loan can pay it in the case of flexible installment. Indeed, Proposition 4 indicates that even if the borrower cannot pay a large amount for the first installment out of her fixed income, she may still be able to repay her loan at the end of the loan period out of her project return. At the same time, the MFI incurs risk if it waits for the whole repayment until the end of the loan period, which is the case in the single repayment contract. The proposed flexible installment contract ensures the MFI higher probability of full repayment than single repayment and, under certain conditions, equal installment, without depriving the very poor of loan access.

6 Discussion and Conclusion

Most MFIs adopt equal installment schedules. Our analysis shows that the expected repayment as well as the probability of full repayment are higher under the equal installment contract than under the single repayment contract. This indicates that the equal installment contract enables MFIs to lend to the poor who are unable to provide collateral. However, equal installment schedules are oftentimes too much of a burden for the very poor, because earlier installments should be paid out of existing income sources or savings, if any. We thus propose a more flexible installment contract. Under this contract, the amount of earlier installments could be sufficiently small so that every borrower who could successfully repay the previous loan can pay them, which enables MFIs to expand their outreach further. In addition, the probability of full repayment under the flexible installment contract is higher than that under the single repayment contract and, under certain conditions, the equal instalment contract. This is because in a dynamic situation wherein the MFI offers loans repeatedly to a borrower who has a long-run project, the flexible installment plan gives the MFI a new device to induce the borrower to make higher effort to raise her project returns. Hence, the borrower has higher disposable earnings at the time of the final repayment and, therefore, she is more likely to successfully repay her loans even when unexpected expenses arise. In the case of two installments, we also derive the way in which the amount of the first installment should vary in the flexible installment contract, which depends on factors such as the interest rate, the probability with which unexpected expenses arise, and the future discount rate of the borrower. To be more precise, if the borrower does not discount the future heavily, and/or the interest rate is high, and/or unexpected expenses arise with low probability, then the MFI can induce the borrower to work longer on her project if it *raises* the amount of the first installment payment under the next contract as the borrower works longer under the current contract, and vice versa. The timing of the first installment also matters. As the time between the loan disbursement and the first installment payment becomes longer by, say, an introduction of a grace period, it becomes more likely that the MFI should decrease the amount of the first installment under the next contract as the borrower puts more effort into her project under the current contract.

Our analysis merely serves as the first step toward the pursuit of more flexible and more desirable loan contracts. In order to provide a benchmark, we assume that the MFI has sufficient knowledge to devise the flexible installment contract. The next step should be to relax this assumption and analyze cases in which the MFI lacks part of the information about the borrower such as the amount of earnings or unexpected expenses. Second, it should be of interest to incorporate savings into the setup by considering a risk averse borrower. Third, more sophisticated installment plans may improve the probability of full repayment and borrower welfare further. For example, how do the borrower's decisions change if the MFI requires n installment payments, d_k $(k = 1, ..., n)$, at T_k $(0 < T_1 < ... < T_n < 2)$, respectively, allowing each d_k to depend on the information available on the previous contract(s) in different ways? Fourth, the number of MFIs and/or borrowers could be increased. In fact, competition between MFIs is found in many microfinance markets (Cull et al., 2009), which may affect borrowers' repayment behavior (Vogelgesang, 2003). We view these enrichments of our model as worthwhile avenues for future research.

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Figure 1: Single Repayment

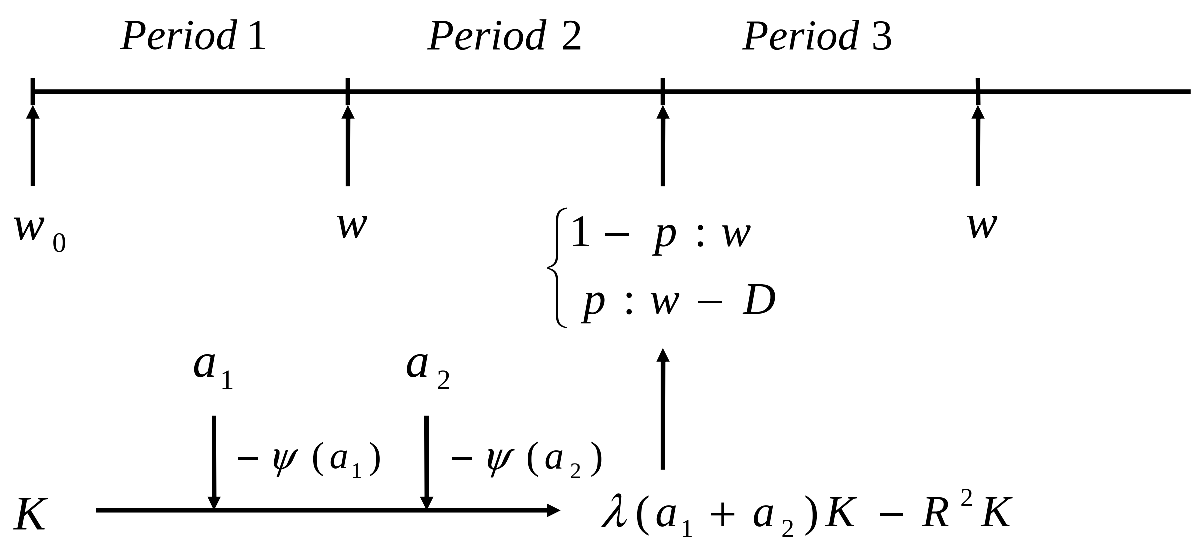

Figure 2: Determination of Optimal a_2 Given a_1

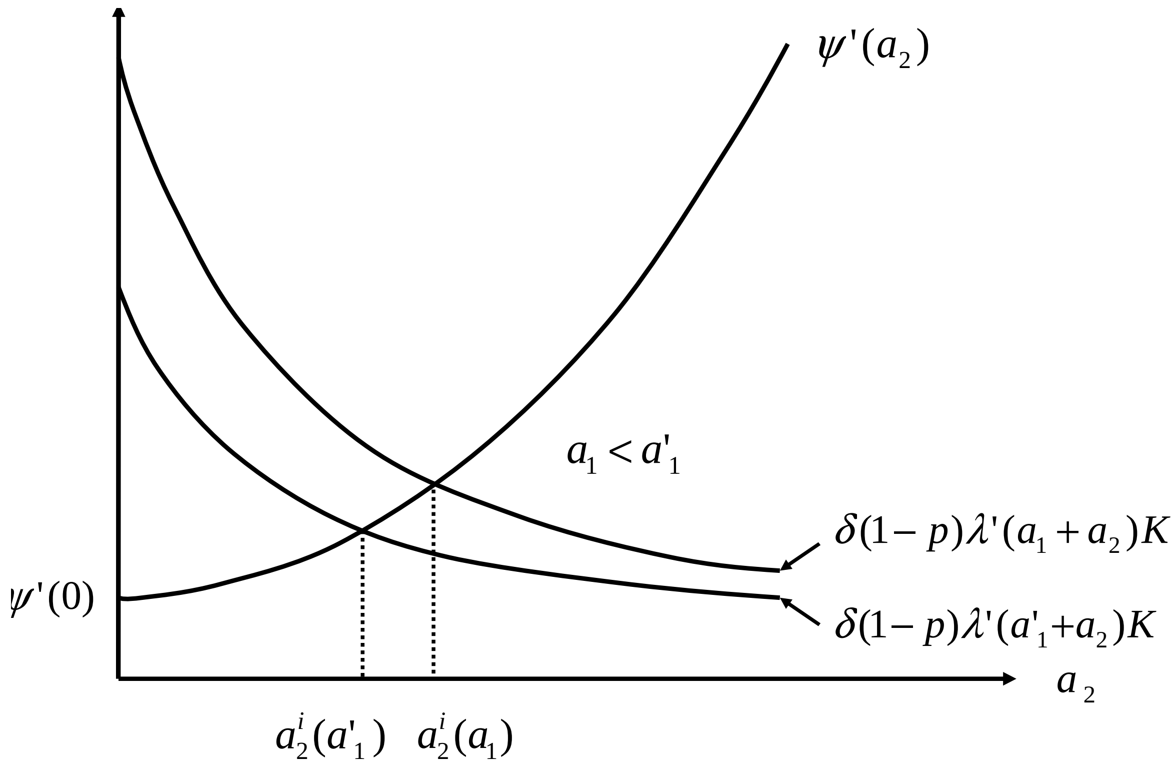

Figure 3: Frequent Repayment