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Public debt and economic growth: what do neoclassical growth models teach us?

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ABSTRACT

This paper aims to quantify the crowding-out effect of public debt and the related loss in long-run output in neoclassical growth models. To accomplish this task, we incorporate the government sector into the Ramsey–Cass–Koopmans (RCK) model, the Blanchard model and the Solow model, which differ only in their assumptions concerning the consumption behaviour of households. We also introduce a general framework that is capable of gauging the burden of public debt in a neoclassical world in the case of any type of consumption behaviour. Our results are threefold. First, contrary to the RCK model, public debt reduces long-run output in the Blanchard model and the Solow model, although to a different extent: the crowding-out effect is marginal in the former, whereas it can be very large in the latter. Second, the burden of public debt is country-specific depending crucially on the saving rate and the population growth rate. Finally, in developed countries the upper limit of the output loss related to public debt is moderate at best even if distortionary taxes are taken into account.

KEYWORDS

Public debt; neoclassical growth models; crowding-out effect; Blanchard model; Solow model

JEL CLASSIFICATION

E13; H31; H63; O40



I. Introduction

This paper investigates the burden of public debt in neoclassical growth models. Our goal is to quantify the crowding out of physical capital by public debt and the related loss in long-run output under different assumptions regarding the consumption behaviour of households. The relevance of the issue can be traced back to the historical indebtedness of developed countries in the wake of the latest global financial crisis (Figure 1). There is much concern that the high public debt-to-GDP ratios, which are not expected to decrease significantly in the foreseeable future, will have adverse effects on growth prospects (Reinhart and Rogoff 2013). Moreover, in the recent past, a common argument against fiscal easing in the core countries in the frame of the euro area crisis management was the possible negative effect of public debt on economic growth.

There are many channels through which public debt might affect economic output either positively or negatively. The most frequently cited negative

effect is the crowding out of private investments (Elmendorf and Mankiw 1999). A further adverse effect is macroeconomic vulnerability. Two major positive effects of public debt are the Keynesian effect and the hysteresis effect, which refer to the ability of expansionary fiscal policy to mitigate both the actual rate and the natural rate of unemployment during recessions (DeLong and Summers 2012). To sum up, the main message of economic theory is that the debt–growth nexus is country- and time-specific, being conditional on several factors, such as the business cycle and institutional quality (e.g. Krugman 2012; Reinhart, Rogoff, and Savastano 2003). This conditionality of the debt–growth nexus is also confirmed by the latest empirical results (e.g. Eberhardt and Presbitero 2015; Égert 2015).

Given the importance of the issue, it is urgent that economic theory improves our understanding of the complex relationship between public debt and economic growth. Our paper contributes to this mission by thoroughly investigating the magnitude of the crowding-out effect of public debt under different consumption behaviours. The

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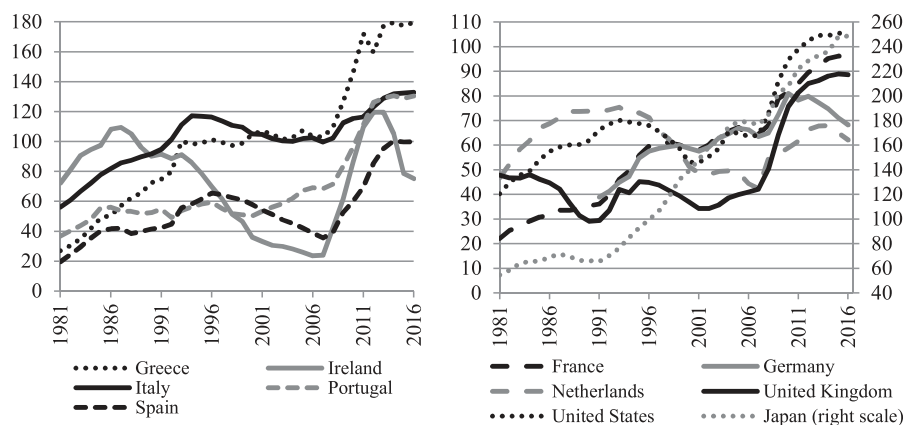


Figure 1. The ratio of public debt to the GDP in some developed countries (%).

Source: AMECO Database

Notes: General government consolidated gross debt

framework of the analysis is provided by three basic neoclassical growth models: the Ramsey–Cass–Koopmans (henceforth RCK) model (Ramsey 1928; Cass 1965; Koopmans 1965), the Blanchard model (Blanchard 1985) and the Solow model (Solow 1956). We start with the RCK model and assume that households pursue dynamic optimization and are connected by altruistic intergenerational links. The RCK model and its implications concerning the effect of public debt are well understood in the literature, so we discuss them only briefly to provide a theoretical baseline for the subsequent analyses. After the RCK model, we drop the assumption of intergenerational links in the Blanchard model and also, later, the assumption of dynamic optimization in the Solow model. In each case, we consider a closed economy, assume exogenous technological change, and focus on the long run.

Of course, we are not the first to investigate the crowding-out effect of public debt in neoclassical growth models. In his seminal paper, Diamond (1965) deals with the effect of public debt in a life-cycle OLG model. Based on the Diamond model, Auerbach and Kotlikoff (1987) reveal significant crowding out of private investments by public debt. Inspired by the results of life-cycle models, Barro (1974) demonstrates that if intergenerational links prevail, government bonds do not represent net wealth for the households and therefore Ricardian equivalence holds. Blanchard (1985) constructs an OLG model that neglects the

life-cycle aspect of life in a continuous time setting in order to provide a tractable framework for analysing the effect of public debt on long-run output when the time horizon of households is finite. Weil (1989) and Buiter (1988) reveal that the failure of Ricardian equivalence in the Blanchard model is caused by the disconnectedness of new and old dynasties and not by the finite time horizon of households. Ball and Mankiw (1995) introduce the parable of the debt fairy, a back-of-the-envelope-type calculation of the burden of public debt in the Solow model.

The main contribution of the paper to the *debt-growth debate* is to provide an overview on the magnitude of the crowding-out effect as a function of households' behaviour in a neoclassical growth framework. Although our knowledge about consumption and saving has improved a lot, there is still considerable confusion about the extent to which intergenerational links and dynamic optimization might characterize the behaviour of households (Romer 2012). Thus, to provide an approximate range on the possible burden of public debt is of first-order importance from the point of view of economic policy. Our results show that in a neoclassical world the long-run output loss related to public debt can vary on a large scale. Beyond this major policy conclusion, the paper also provides three additional contributions to the theory. First, it is often argued that the burden of public debt through distortionary taxation can be considerable (e.g. Mankiw 2000). We prove that in fact this is not true, at least

in the RCK model. Second, we present a new formula for the crowding-out effect in the Blanchard model according to which it is straightforward to perform the calculation. Third, we introduce a general framework that is capable of gauging the burden of public debt in a neoclassical world in the case of any type of households' consumption behaviour.

The remainder of the paper is organized as follows. Section 2 discusses briefly the role of public debt in the RCK model. Section 3 considers the crowding-out effect and the resulting output loss in the Blanchard model, while section 4 discusses these issues in the Solow model. Section 5 provides the general framework for examining the burden of public debt in neoclassical growth models. Section 6 concludes the paper.

II. Government debt in the Ramsey–Cass–Koopmans model

This section investigates the impact of government debt on steady-state output in the RCK model (Ramsey 1928; Cass 1965; Koopmans 1965). We consider a closed economy that consists of three sectors: households, firms, and government.

The representative household supplies labour inelastically and decides only on consumption. As intergenerational links are operative, households maximize utility on the infinite time horizon:

$$\max_{c(t)} U = \int_0^{\infty} u[c(t)] e^{nt} e^{-\rho t} dt, \quad (1)$$

where ρ is the subjective discount rate, $c = C/L$ is the per capita consumption, C is the aggregate consumption, $L(t) = e^{nt}$ is the population growing at rate n and $\rho > n$. The initial value of the population is normalized to one. For simplicity, we assume the logarithmic utility function: $u(c) = \ln(c)$.

The households' flow budget constraint is

$$\dot{a}(t) = (1 - \tau_W)w(t) + (1 - \tau_A)r(t)a(t) - na(t) - c(t), \quad (2)$$

where a and w are the per capita assets and wage, respectively, r is the interest rate, and τ_W and τ_A are the tax rates levied on wage income and capital income. The dot above the variables denotes derivation with respect to time. Because the economy is closed, the total assets ($A = aL$) equal the sum of physical capital (K) and government debt (B): $A = K + B$.

The intensive form of the $Y = F(K, EL)$ neoclassical production function is $\hat{y} = f(\hat{k})$, where $\hat{y} = Y/(EL)$, $\hat{k} = K/(EL)$, Y is the output, and $E = e^{gt}$ is the level of technology growing at a constant exogenous rate of g .¹ Firms are supposed to operate in competitive markets; thus, production factors are rewarded by their marginal products:

$$r + \delta = \partial f(\hat{k})/\partial \hat{k} = f'(\hat{k}) \text{ and } \hat{w} = f(\hat{k}) - \hat{k}f'(\hat{k}) \quad (3)$$

where δ is the depreciation rate of physical capital and \hat{w} is the wage per effective labour.²

The government collects revenues by imposing taxes on labour and capital income, while its outlays consist of government expenditures and interest payments on debt. For simplicity, the consumption tax is neglected. Thus, the government obeys the following flow budget constraint:

$$\begin{aligned} \dot{B}(t) &= r(t)B(t) + G(t) - \tau_W W(t) \\ &\quad - \tau_A r(t)(K(t) + B(t)) \\ &= r(t)B(t) - \Gamma_{RCK}(t) \end{aligned} \quad (4)$$

where G is government expenditures, W is aggregate wages, and $\Gamma_{RCK} = \tau_W W + \tau_A rA - G$ is the primary balance of the budget.

For simplicity, government expenditures are assumed to affect neither the utility of households nor the production of firms. Government expenditures as a share of GDP ($\phi = G(t)/Y(t)$) are considered to be constant, therefore tax rates are also constant for any given debt-to-GDP ratio. In other words, tax rates are set to run the necessary primary budget surplus in order to achieve the target of the government for the long-run debt-to-GDP ratio. Of course, the latter implies that larger government debt results

¹Releasing the exogeneity of technological change may alter the results on the burden of public debt to some extent since capital intensity (K/Y) positively covaries with the technological level (Klenow and Rodríguez-Clare 1997). This implies that higher public debt might lead to lower per capita income not just through the crowding out of physical capital but through the lower level of technology as well. Therefore, endogenous technological change provides an additional channel through which the burden of public debt can manifest itself. However, the investigation of this topic is far beyond the scope of the paper and is the subject of future research.

²In what follows, we neglect the t time index when no confusion emerges.

in higher tax rates. This increase of tax rates raises the well-known issue of distortionary taxation related to public debt.

The crowding out of capital brought about by distortionary taxes must be treated separately from the classical crowding-out effect of public debt. Although the rising tax rates crowd out capital indirectly, the mechanism is completely different from the classical crowding-out effect, which is related to the consumption–saving behaviour of households and the wealth effect of government bonds. Consequently, we do not deal with distortionary taxes triggered by public debt in the main text of the paper. Nevertheless, an appendix is devoted to the issue, especially as to some authors ‘...substantial steady-state crowding out can occur simply because of distortionary taxation’ (Mankiw 2000, 123). In the appendix, we demonstrate that in fact this is not the case.

The steady state

The representative household maximizes utility (equation.1) subject to its budget constraint (equation.2). The optimal time path of consumption is

$$\dot{\hat{c}}/\hat{c} = (1 - \tau_A)r - \rho - g, \quad (5)$$

where $\hat{c} = C/(EL)$. The dynamics of physical capital (per effective) labour can be derived according to the households’ budget constraint, taking into account that $\dot{a} = \dot{k} + \dot{b}$, where $b = B/L$, and using equations (3) and (4):

$$\dot{\hat{k}} = (1 - \phi)f(\hat{k}) - \hat{c} - (n + g + \delta)\hat{k} \quad (6)$$

The dynamics of the system are determined by equations (4), (5) and (6). In steady state, \hat{c} , \hat{k} and $\hat{b} = B/(EL)$ are constant, which yield the determining equations of the long-run equilibrium:

$$\hat{c}^* = 0 \rightarrow (1 - \tau_A)(f'(\hat{k}^*) - \delta) = \rho + g, \quad (7)$$

$$\hat{k}^* = 0 \rightarrow \hat{c}^* = (1 - \phi)f(\hat{k}^*) - (n + g + \delta)\hat{k}^*, \quad (8)$$

$$\hat{b}^* = 0 \rightarrow \frac{\dot{\hat{b}}^*}{\hat{b}^*} = \mu^* = \frac{\Gamma_{RCK}^*/Y^*}{r^* - g - n} = \frac{d}{g_Y^*}, \quad (9)$$

where the asterisk refers to the steady state, μ is the debt-to-GDP ratio, d is the budget deficit over the GDP, and $g_Y^* = g + n$ is the steady-state growth rate of output.³

The intertemporal budget constraint of households and the government

Based on the solution of the flow budget constraint (equation.2.) and the transversality condition of the dynamic optimization, the intertemporal budget constraint of the representative household is

$$\begin{aligned} & \int_0^\infty c(t)e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt \\ & = a(0) \\ & \quad + \int_0^\infty (1 - \tau_W)w(t)e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt \\ & = a(0) + \bar{W} - \bar{T}_W, \end{aligned} \quad (10)$$

where $\bar{r}(t) = \int_0^t r(s)ds/t$, \bar{W} is the present value of the wage income and \bar{T}_W is the present value of the taxes levied on the wages. Regarding the fact that the value of any asset equals the present value of its future net incomes, the initial stock of assets is

$$\begin{aligned} a(0) & = k(0) + b(0) \\ & = \int_0^\infty (R_K(t) + R_B(t) - T_K(t) - T_B(t))e^{-(1-\tau_A)\bar{r}(t)t} dt \\ & = (\bar{R}_K - \bar{T}_K) + (\bar{R}_B - \bar{T}_B), \end{aligned} \quad (11)$$

where R_i is the income earned on asset i ($i = K, B$), T_i is the tax imposed on R_i , and \bar{R}_i and \bar{T}_i are the present values of the respective future asset incomes and taxes. Substituting equation (11) into equation (10), the intertemporal budget constraint of the representative household becomes

$$\begin{aligned} & \int_0^\infty c(t)e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt \\ & = k(0) + b(0) + \bar{W} - \bar{T}_W \\ & = (\bar{W} + \bar{R}_K + \bar{R}_B) - (\bar{T}_W + \bar{T}_K + \bar{T}_B) \end{aligned} \quad (12)$$

According to equation (12), the present value of future consumption equals the present value of all types of future income net of taxes.

To derive the intertemporal budget constraint of the government (IBCG), one has to first solve

³Strictly speaking, d stands for the operational (inflation-adjusted) rather than the total budget deficit.

the flow budget constraint (equation.4), then take the limit of the solution and use the no-Ponzi-game (NPG) condition⁴:

$$\begin{aligned} B(0) &= \int_0^{\infty} e^{-(1-\tau_A)\bar{r}(t)t} [\tau_W W(t) \\ &\quad + \tau_A r(t)K(t) - G(t)] dt \\ &= \bar{T}_W + \bar{T}_K - \bar{G}, \end{aligned} \quad (13)$$

where \bar{G} is the present value of future government spending. Equation (13) claims that the initial public debt must equal the present value of future primary budget surpluses exclusive of the tax revenues from interest payment on government bonds ($\tau_A rB$).⁵ As the initial value of population is normalized to one, the equation also determines the initial per capita debt ($b(0) = B(0)$).

The effect of public debt on steady-state output

To find out how public debt affects steady-state output, we must combine the government's intertemporal budget constraint with that of the representative household. To do so, substitute equation (13) for $b(0)$ in equation (12):

$$\int_0^{\infty} c(t) e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt = \bar{W} + \bar{R}_K - \bar{G} \quad (14)$$

Equation (14) conveys the key results of the RCK model with regard to the impact of public debt on steady-state output. First, the present value of future incomes stemming from government bonds disappears from the intertemporal budget constraint; that is, government bonds do not represent net wealth for households (Barro 1974). Second, government purchases enter in place of taxes.

If taxes were lump-sum, the preceding two points would imply that only the magnitude of government purchases, but not their financing method, affects the consumption of households and thereby the steady-state output. Consequently, the Ricardian equivalence holds: public debt is neutral to long-run output. On the other hand, if taxes are proportional, as in our case, the above considerations continue to hold

almost completely. As equation (14) shows, government bonds still do not represent net wealth for households. However, the financing of government expenditures is no longer neutral. If government expenditures are financed temporarily by budget deficit, tax rate will be higher after the debt-to-GDP ratio stabilizes again at its new long-run value. Higher tax rate on capital income results in higher interest rate and thus in lower steady-state capital and income (equation.7). Nonetheless, in the appendix, we demonstrate that the output loss triggered by distortionary taxes stemming from government debt amounts to only a few percentage points.

III. Public debt in the Blanchard model

The results of the RCK model depend crucially on the assumed behaviour of households. However, intergenerational links and dynamic optimization can both be challenged by the empirics. As regards the intergenerational links, '...many people leave no bequest and, therefore, are not economically linked to future generations' (Mankiw 1995, 279). Moreover, even if bequests are present, they are unintended in many cases; that is, altruism does not play a role in determining them (Bernheim 1987). Thus, in this section, we drop the assumption of intergenerational links and investigate the effect of public debt on long-run output in the OLG model of Blanchard (1985).

The structure of the economy is the same as in the RCK model. The only difference is the lack of intergenerational links. In OLG models, individuals do not care about the utility of their descendants and make consumption decisions solely with respect to their own life cycle. This means that individuals optimize their consumption on finite time horizons. We only introduce the main structure of the model.⁶

Let p be the probability of death per unit of time, which is independent of age. Then, the probability of a person born at time j being alive at time $t > j$ is $e^{-p(t-j)}$. The expected lifetime is $1/p$.

⁴The NPG condition is $\lim_{t \rightarrow \infty} B(t) e^{-(1-\tau_A)\bar{r}(t)t} = 0$.

⁵The IBCG is fulfilled if the debt-to-GDP ratio is stable at an arbitrary level (Greiner 2011). Concerning the debt-trajectories of some highly indebted developed countries it is obvious that they are struggling to comply with their IBCG. In spite of this, we do not deal with the important issue of debt-sustainability. Instead, we follow the common practise of the debt-growth literature by supposing that the government is always able to run the necessary budget surplus to stabilize the debt-to-GDP ratio at the desired level.

⁶The full model is provided upon request. See, also Acemoglu (2009, ch.9.8).

The population grows at a constant n rate, so $L = e^{nt}$. Given this and the p death rate, the size of the cohort born at time t is $(n + p)e^{nt}$.

Regarding the capital market, savings are held in the form of life insurance that pays an annuity (z) over the riskless interest rate (r). If the individual dies, his or her assets will be left to the life insurance company. The expected profit of a life insurance company at time t with respect to an individual born at time j and with assets $a(j, t)$ is $\pi(a(j, t)) = p \cdot a(j, t) - z \cdot a(j, t)$. Thus the zero-profit requirement of competitive markets ensures that the annuity must equal the probability of death: $z = p$. This implies that the rate of return on households' assets is $(r + p)$.

The individual (household) maximizes the expected utility of lifetime consumption as follows:

$$\max_{c(j, \nu)} E[U(t)] = \int_t^\infty \ln[c(j, \nu)] e^{-(\rho+p)(\nu-t)} d\nu, \quad (15)$$

where $E[.]$ refers to the expected value, and $e^{-p(\nu-t)}$ is the probability of being alive at time ν , provided that the person was alive at time t .

The households' budget constraint is similar to equation (2) with two exceptions. First, it does not contain the growth rate of population because of the absence of intergenerational links. Second, the returns on assets are the sum of the interest rate and the annuity. Thus, we have

$$\begin{aligned} \dot{a}(j, \nu) &= (1 - \tau_A)(r(\nu) + p)a(j, \nu) \\ &+ (1 - \tau_W)w(\nu) - c(j, \nu) \end{aligned} \quad (16)$$

Based on equations (15) and (16), the optimal path of an individual's consumption is

$$\begin{aligned} \frac{\dot{c}(j, \nu)}{c(j, \nu)} &= (1 - \tau_A)(r(\nu) + p) - (\rho + p) \\ &= r_{ne}(\nu) - (\rho + p), \end{aligned} \quad (17)$$

where $r_{ne}(\nu) = (1 - \tau_A)(r(\nu) + p)$ is the net effective interest rate at time ν , which adjusts both for the tax rate and the risk premium resulting from death. Solving equation (16) and taking into account the transversality condition, we obtain the intertemporal budget constraint of households:

$$\begin{aligned} \int_t^\infty c(j, \nu) e^{-\bar{r}_{ne}(\nu)(\nu-t)} d\nu \\ &= a(j, t) \\ &+ \int_t^\infty (1 - \tau_W)w(\nu) e^{-\bar{r}_{ne}(\nu)(\nu-t)} d\nu, \end{aligned} \quad (18)$$

where $\bar{r}_{ne}(\nu) = \int_t^\nu r_{ne}(s) ds / (\nu - t)$. Solving equation (17) and substituting for $c(j, \nu)$ in equation (18), we arrive at the present consumption of the individual born at time j :

$$c(j, t) = (\rho + p)(a(j, t) + (1 - \tau_W)\bar{w}(t)), \quad (19)$$

where $\bar{w}(t)$ is the present value of the individual's future wage incomes.

Aggregating equation (19) over the population and differentiating it with respect to time yields the dynamics of aggregate consumption (per effective labour):

$$\frac{\dot{\hat{c}}}{\hat{c}} = r_{ne} - (g + \rho + p) - (\rho + p)(p + n) \frac{\hat{a}}{\hat{c}} \quad (20)$$

In steady state, $\dot{\hat{c}} = 0$; thus, equation (20) provides the following relationship between steady-state consumption and assets (\hat{c}^*, \hat{a}^*) :

$$\begin{aligned} \hat{c}^* &= \frac{x\hat{a}^*}{r_{ne}^* - (g + \rho + p)} \\ &= \frac{x\hat{a}^*}{(1 - \tau_A)(f'(\hat{k}^*) - \delta + p) - (\rho + p) - g}, \end{aligned} \quad (21)$$

where $x = (\rho + p)(p + n)$ and $\hat{a}^* = \hat{k}^* + \hat{b}^*$. The equation of motion for public debt is as follows:

$$\begin{aligned} \dot{B}(t) &= r(t)B(t) + G(t) - \tau_W W(t) \\ &- \tau_A(r(t) + p)(K(t) + B(t)) \\ &= r(t)B(t) - \Gamma_B(t), \end{aligned} \quad (22)$$

where $\Gamma_B = \tau_W W + \tau_A(r + p)(K + B) - G$. The only difference between equations (4) and (22) is the additional tax revenue term ($\tau_A p A$) in equation (22), thanks to the higher rate of return on assets in the Blanchard model. Following the same steps as in section 2, the intertemporal budget constraint of the government can be expressed as follows:⁷

$$B(t) = \int_t^\infty e^{-(\bar{r}_{ne}(\nu)-p)\nu} [\tau_W W(\nu) + \tau_A(r(\nu) + p)K(\nu) - G(\nu)] d\nu$$

Note, that the discount rate in the intertemporal budget constraint is lower by the p mortality rate in

⁷Now, the NPG condition is $\lim_{t \rightarrow \infty} B(t) e^{-(\bar{r}_{ne}(t)-p)t} = 0$.

the case of the government compared to the case of households. The underlying reason is that the time horizons of the two actors differ from each other: contrary to the households, the time horizon of the government continues to be infinite.

According to equation (22), in steady state, the debt-to-GDP ratio is

$$\mu^* = \frac{\Gamma_B^*/Y^*}{r^* - g - n} \tag{23}$$

The dynamics of the system is determined by equations (6),(20) and (22).⁸ The steady state is described by equations (8), (21) and (23).

Substituting equation (8) for \hat{c}^* and $\mu^*f(\hat{k}^*)$ for \hat{b}^* in equation (21), we arrive at the following alternative expression for the long-run debt-to-GDP ratio after some manipulation:

$$\mu^* = \left[1 - \phi - (n + \delta + g) \frac{\hat{k}^*}{f(\hat{k}^*)} \right] \frac{r_{ne}^* - (g + \rho + p)}{x} - \frac{\hat{k}^*}{f(\hat{k}^*)} \tag{24}$$

The advantage of equation (24) over equation (23) is that in the former case the steady-state debt-to-GDP ratio is expressed solely as a function of the steady-state physical capital per effective labour, which is of first-order importance from our point of view. Namely, the crucial step with regard to the derivation of the crowding-out effect is the differentiation of the debt-to-GDP ratio (equation.24) with respect to the physical capital per effective labour:

$$\frac{\partial \mu^*}{\partial \hat{k}^*} = \left[-(n + \delta + g) \frac{f(\hat{k}^*) - \hat{k}^* f'(\hat{k}^*)}{f^2(\hat{k}^*)} \right] \frac{r_{ne}^* - (g + \rho + p)}{x} + \left[1 - \phi - (n + \delta + g) \frac{\hat{k}^*}{f(\hat{k}^*)} \right] \frac{(1 - \tau_A) f''(\hat{k}^*)}{x} - \frac{f(\hat{k}^*) - \hat{k}^* f'(\hat{k}^*)}{f^2(\hat{k}^*)} < 0. \tag{25}$$

Because the sign of the derivative in equation (25) is negative, an increase in the debt-to-GDP ratio leads to lower steady-state capital and vice versa.⁹ Thus, public debt crowds out physical capital and reduces long-run output in the Blanchard model

as opposed to the RCK model. This crucial result was already documented by Blanchard (1985).

Equation (25) proves not only the existence of crowding out but is also appropriate to determine its magnitude. The calculation requires the specification of the production function. Therefore, in what follows, we assume a Cobb–Douglas production function: $\hat{y} = \hat{k}^\alpha$, where $0 < \alpha < 1$. With this end in view, multiplying equation (25) by \hat{k}^* and taking into account that $f(\hat{k}) - \hat{k}f'(\hat{k}) = \hat{w}$, and $f''(\hat{k}) = -\alpha(1 - \alpha) \frac{\hat{y}}{\hat{k}^2}$, we arrive at the basic underlying expression of our calculation exercise:

$$\begin{aligned} \frac{\partial \mu^*}{\partial \hat{k}^*} \hat{k}^* &= -(n + \delta + g) \frac{\hat{w}^* \hat{k}^* \hat{a}^*}{\hat{y}^* \hat{y}^* \hat{c}^*} \\ &\quad - \frac{(1 - \tau_A) \alpha (1 - \alpha) \hat{c}^* \hat{y}^*}{x \hat{y}^* \hat{k}^*} - \frac{\hat{w}^* \hat{k}^*}{\hat{y}^* \hat{y}^*} \\ &= \frac{1}{\Phi} \end{aligned} \tag{26}$$

Equation (26) shows the percentage point change in the debt-to-GDP ratio due to a one per cent change in the capital per effective labour. This is the inverse of the crowding-out effect of public debt (Φ). Considering the latter and the fact that $\Delta \hat{y}/\hat{y} = \alpha \Delta \hat{k}/\hat{k}$ in the case of a Cobb–Douglas production function, we can calculate the burden of public debt as

$$\frac{\partial \hat{y}^*/\hat{y}^*}{\partial \mu^*} = \alpha \frac{\partial \hat{k}^*/\hat{k}^*}{\partial \mu^*} = \alpha \cdot \Phi \tag{27}$$

Blanchard and Fischer (1989) were the first to derive a closed-form solution to the crowding-out effect in the Blanchard model. However, they worked with lump-sum taxes and determined the $\partial K/\partial B$ marginal effect instead of the percentage effect of the debt-to-GDP ratio on long-run output. Consequently, their formula is inappropriate to perform easy and direct calculations on the burden of public debt.

In what follows, we calculate the long-run output loss of public debt based on real data according to equations (26) and (27). The sample covers the period of the ‘great moderation’ (2000–2007) and incorporates the 167 countries of the Penn World Table 8.0.¹⁰

⁸The equation of motion for \hat{k} is the same as in the RCK model.

⁹Remember that $f(\hat{k}^*) - \hat{k}^* f'(\hat{k}^*) = \hat{w}^* > 0$, $\frac{r_{ne}^* - (g + \rho + p)}{x} = \frac{\hat{c}^*}{\hat{y}^*} > 0$, $f''(\hat{k}^*) < 0$ and $(1 - \phi) - \frac{(n + \delta + g)\hat{k}^*}{f(\hat{k}^*)} = \frac{\hat{c}^*}{\hat{y}^*} > 0$.

¹⁰We neglect the years of the global crisis because the crowding-out effect cannot prevail in a balance sheet recession (Krugman 2012). Nevertheless, the results are robust to the sample period.

To obtain robust results, the calculations are performed for six country groups and for six countries as well. The individual countries under consideration are the USA, the United Kingdom (UK), Japan (JPN), Germany (GER), Italy (ITA) and France (FRA). The six groups of countries are the OECD countries, the Eurozone (EURO), the low-income countries (LIC), the lower-middle-income countries (LMIC), the upper-middle-income countries (UMIC) and finally the high-income countries (HIC). In the last four cases, the 167 countries are grouped according to the income classification of the World Bank. During the calculations, we use the median values of the individual variables in the investigated period.

The inputs and the results of the calculations are presented in Table 1. According to our results, neither the magnitude of the crowding out of physical capital nor the resulting loss in the long-run output are significant. The calculations show that a one percentage point change in the debt-to-GDP ratio reduces the steady-state per capita output only by 0.008–0.032 per cent throughout the sample.

The conclusions of the Blanchard model in relation to the crowding-out effect of public debt are reasonable. The presence of the crowding out of physical capital is due to the lack of intergenerational links and the finite horizon of households. Namely, the finite horizon assumption implies that a deficit-financed temporary tax cut increases not just the actual income but to some extent the present value of lifetime income as well. Moreover, because of the absence of intergenerational links, the future tax burden of the present deficit financing will fall to some extent on new households (generations) from which current households (generations) feel themselves disconnected.¹¹ The negligible magnitude of the crowding-out effect can also be explained. Although individuals maximize their utility during their lifetimes, the time horizon is very long; a minimum of 30–40 years even if we correct for inactive years. This means that although government bonds are net wealth for households, as equation (21) suggests, at best they affect the yearly permanent income only marginally.

To test the robustness of our results, we perform sensitivity analyses with respect to the capital tax rate, the depreciation rate, the debt-to-GDP ratio and the expected lifetime. The robustness check with respect to the latter is inspired by the key idea that, ‘If we think of $1/p$ as the horizon index, we can choose it anywhere between zero and infinity and study the effects of the horizon of agents on the behaviour of the economy’ (Blanchard 1985, 224). The underlying country group of the sensitivity analyses is the Eurozone. The results are presented in Figure 2. These results show that the crowding-out effect is robust to the debt-to-GDP ratio and the tax rate, whereas it is moderately sensitive to the depreciation rate and the horizon index in the Blanchard model. However, in the latter cases, the magnitude of the crowding-out effect continues to remain negligible throughout the entire interval of the depreciation rate and throughout the reasonable interval (i.e. $1/p > 10-15$) of the horizon index.

To illustrate the magnitude of the crowding-out effect in the Blanchard model, we calculate the burden of public debt at a 90 per cent debt-to-GDP ratio. Because the crowding-out effect is not sensitive to B/Y , the total impact on output can be calculated by multiplying the marginal burden of debt ($\alpha\Phi$) by the debt-to-GDP ratio: $\Delta\hat{y}^*/\hat{y}^* = 0.9 \cdot (-0.032) = -0.0288$.¹² According to this calculation, we can conclude that the rise of the debt-to-GDP ratio from zero to 90 per cent reduces the long-run output – via the classical crowding-out channel – by approximately 2–3 per cent. The distortionary taxation related to public debt does not change the picture. Contrary to the RCK model, it is impossible to calculate analytically the output loss related to distortionary taxes in the Blanchard model.¹³ Nevertheless, we can suppose that the magnitudes are similar in the two models. Thus, taking into account the results of the appendix, we can conclude that the total output loss of a 90 per cent debt-to-GDP ratio – including both the effects of distortionary taxes and classical crowding out –

¹¹Technically, the reason for the net wealth effect of public debt is the lower discount rate in the intertemporal budget constraint of the government compared to the case of households (Blanchard 1985).

¹²To provide a pessimistic estimation, we use the largest marginal impact on output in Table 1.

¹³The obstacle is that in the Blanchard model the interest rate cannot be expressed as the sole function of the model parameters, in sharp contrast to the RCK model (see the derivation of equation (A5) in the appendix).

Table 1. The crowding-out effect of public debt for different countries and country groups.

	LIC	LMIC	UMIC	HIC	OECD	EURO	USA	JPN	UK	GER	ITA	FRA	Source/Calculation
ρ	0.037	0.045	0.067	0.046	0.038	0.036	0.027	0.022	0.061	0.029	0.024	0.023	equation (24)
τ_A	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	calibration
$1/p$	32.8	45.0	52.0	58.0	58.6	58.4	57.2	61.8	58.6	58.5	60.5	59.7	WDI
p	0.030	0.022	0.019	0.017	0.017	0.017	0.017	0.016	0.017	0.017	0.017	0.017	
n	0.027	0.018	0.012	0.006	0.005	0.004	0.010	0.001	0.005	0.000	0.007	0.007	PWT 8.0
g	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	calibration
δ	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	calibration
\hat{w}^*/\hat{y}^*	0.617	0.532	0.462	0.561	0.592	0.589	0.650	0.529	0.633	0.638	0.539	0.629	PWT 8.0
\hat{k}^*/\hat{y}^*	2.563	3.066	2.973	3.013	3.046	3.156	2.999	4.221	2.251	3.081	4.002	3.347	PWT 8.0
\hat{b}^*/\hat{y}^*	0.800	0.614	0.448	0.441	0.477	0.519	0.611	1.751	0.470	0.649	1.055	0.637	HPDD
\hat{c}^*/\hat{y}^*	0.742	0.712	0.634	0.561	0.577	0.584	0.728	0.543	0.686	0.600	0.588	0.601	PWT 8.0
\hat{b}^*/\hat{c}^*	1.078	0.862	0.707	0.786	0.827	0.889	0.839	3.227	0.686	1.081	1.794	1.059	$(\hat{b}^*/\hat{y}^*)/(\hat{c}^*/\hat{y}^*)$
\hat{k}^*/\hat{c}^*	3.452	4.305	4.691	5.370	5.282	5.403	4.118	7.779	3.282	5.131	6.804	5.567	$(\hat{k}^*/\hat{y}^*)/(\hat{c}^*/\hat{y}^*)$
\hat{a}^*/\hat{c}^*	4.530	5.168	5.399	6.156	6.109	6.291	4.957	11.006	3.967	6.213	8.599	6.627	$(\hat{b}^*/\hat{c}^*) + (\hat{k}^*/\hat{c}^*)$
x	0.004	0.003	0.003	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	$(\rho + p)(\rho + n)$
$\partial f(\hat{k}^*)/\partial \hat{k}^*$	0.149	0.153	0.181	0.146	0.134	0.130	0.117	0.112	0.163	0.117	0.115	0.111	$\alpha(\hat{y}^*/\hat{k}^*)$
$1/\Phi$	-15.523	-18.280	-16.886	-25.961	-29.936	-32.115	-36.859	-40.031	-32.995	-44.921	-32.426	-37.082	equation (26)
Φ	-0.064	-0.055	-0.059	-0.039	-0.033	-0.031	-0.027	-0.025	-0.030	-0.022	-0.031	-0.027	
$\frac{\partial \hat{y}^*}{\partial \mu^*}$	-0.025	-0.026	-0.032	-0.017	-0.014	-0.013	-0.009	-0.012	-0.011	-0.008	-0.014	-0.010	equation (27)

Notes: The data in the table are the medians for the individual panels and time series in the period of 2000–2007. The sources of the data are the WDI (World Development Indicators, downloaded: 24.06.2014), the PWT 8.0 (Penn World Table 8.0; Feenstra, Inklaar, and Timmer 2015) and the HPDD (Historical Public Debt Database (2013 Fall vintage) – IMF, Abbas et al. 2010). The subjective discount rate is calculated according to equation (24). The expected lifetime at birth adjusted for the inactive years is $1/p$ (expected lifetime – 20). The labour share ($\hat{w}^*/\hat{y}^* = 1 - \alpha$), the consumption share (\hat{c}^*/\hat{y}^*) and the physical capital per output (\hat{k}^*/\hat{y}^*) are measured by and calculated according to the 'labsh', the 'csh_c', the 'rkna' and the 'rgdpna' variables of PWT 8.0.

probably does not exceed 5–6 per cent in the Blanchard model. Consequently, the burden of public debt is not a serious concern in that model either.

Our conclusion is in line with Evans (1991), who was the first to prove that Ricardian equivalence is a good approximation in the Blanchard model. Nevertheless, the approximate neutrality of fiscal policy in the Blanchard model can hinge crucially upon the absence of some relevant life-cycle aspects of households' behaviour. In fact, the basic Blanchard model with age-independent mortality rate and wages can be considered rather as a model of dynasties with finite horizon than as a classical OLG model. Faruqee, Laxton, and Symansky (1997) and Faruqee and Laxton (2000) show that if wages follow a hump-shaped life-cycle pattern then the burden of public debt can be considerable in the Blanchard model as well. In another paper, Faruqee (2003) arrives at the same conclusion by introducing a death probability, which increases with the age. According to these papers the long-run output loss of a 90 per cent

debt-to-GDP ratio can be posited between 5 and 10 per cent. However, the exact value is very sensitive to the parameter calibration.¹⁴ So, we decided not to depart from the basic Blanchard framework. Moreover, in the next section, the Solow model also delivers output losses in the range of 5–10 per cent for developed countries at a 90 per cent debt-to-GDP ratio. Consequently, the main policy conclusion of the paper with regard to the upper boundary of the burden of public debt is unaffected by these potential modifications of the basic Blanchard framework.

IV. Government debt in the Solow model with human capital

The dynamic optimization of households and the underlying permanent income hypothesis can be heavily challenged on both theoretical and empirical grounds (Romer 2012). The central assumption of the permanent income hypothesis is that households base their consumption decision on the present value of their lifetime income instead

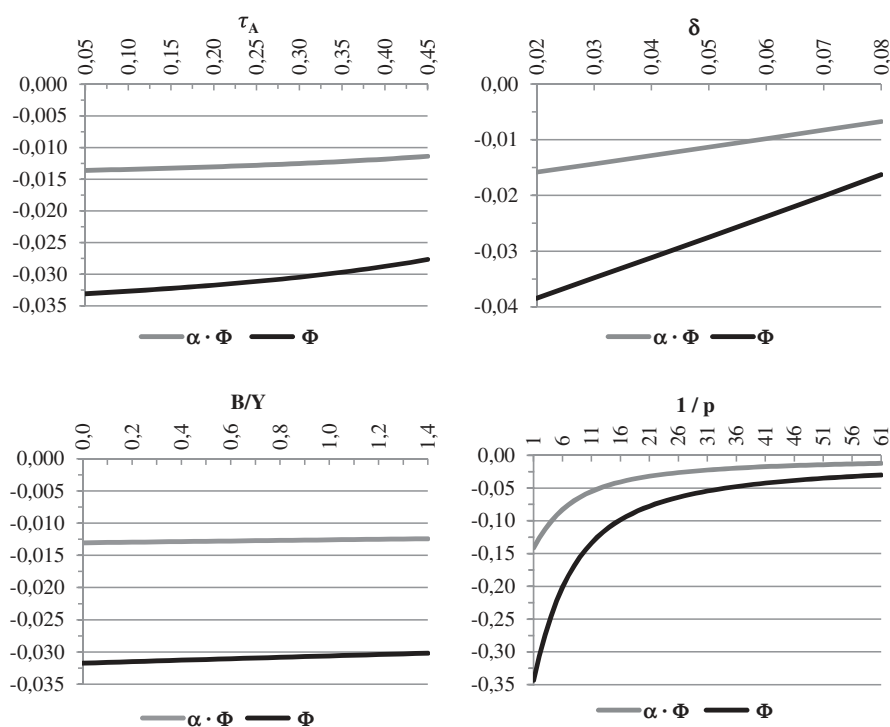


Figure 2. The sensitivity of the crowding-out effect and the resulting output loss in the Eurozone.

Notes: The calculations are based on equations (26) and (27). The variables and the parameters take on the values of the Eurozone in each case (Table 1)

¹⁴For example, in Faruqee and Laxton (2000), the burden of public debt decreases with the intertemporal elasticity of substitution and becomes similar to the results of the basic Blanchard model as log-utility is reached.

of their current income. However, many empirical studies find a strong positive correlation between current income and consumption (e.g. Carroll and Summers 1991). Furthermore, some studies find that predictable changes in income lead to predictable changes in consumption at the time that they happen (Johnson, Parker, and Souleles 2006; Shapiro and Slemrod 2003).

Because of the objections raised against the permanent income hypothesis, we drop the assumption of dynamically optimizing households in this section and switch to the traditional consumption theory, which assumes that households base their consumption on their current disposable income and follow a rule-of-thumb decision. This implies practically that we study the impact of public debt on steady-state output in the framework of the Solow model with a constant and exogenous household saving rate. The constant, exogenous saving rate of households implies that government deficit reduces aggregate savings – and thereby investments as well – one-to-one in the extreme. Therefore, in this section, we consider the case of a complete crowding-out effect in the tradition of Elmendorf and Mankiw (1999).

In addition to the underlying consumption behaviour, a further departure from the RCK and the Blanchard models is that we take human capital into account. Our primary reason for the latter is that the presence of human capital affects the burden of public debt in a quantitatively important way in the Solow model.¹⁵

The discussion is based on the human capital augmented Solow model of Mankiw, Romer, and Weil (1992). The production function is

$$Y = K^\alpha H^\beta (EL)^{1-\alpha-\beta},$$

where H is human capital, $\alpha, \beta > 0$ and $\alpha + \beta < 1$. The steady-state output is

$$\begin{aligned} \ln(\hat{y}^*) &= \frac{\alpha}{1-\alpha-\beta} \ln(s_K) \\ &+ \frac{\beta}{1-\alpha-\beta} \ln(s_H) \\ &- \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta), \end{aligned} \quad (28)$$

where s_K and s_H are the saving (investment) rates in relation to physical and human capital, respectively.

To calculate the burden of public debt, we have to rearrange equation (28) in such a way that it already does contain the debt-to-GDP ratio (Dedák and Dombi 2018). In this derivation, the first step is to calculate the effect of the budget balance on the aggregate saving rate. The aggregate savings is the sum of private savings and government savings (i.e. the budget balance). For simplicity, it is assumed that government savings influences only the accumulation of physical capital:

$$s_K = s_{KP} - d \quad (29)$$

where s_{KP} is the savings of the private sector compared to the output. As we assume complete crowding out, private savings is unaffected by government savings. The budget deficit can be given as a constant z fraction of private savings: $d = z \cdot s_{KP}$. Parallel with the latter, equation (29) can be reformulated as $s_K = (1-z)s_{KP}$. Substituting this new formula for s_K in equation (28) and using the fact that $\ln(1-z) \approx -z$ if z is close to zero, the long-run output can be rewritten as follows:

$$\begin{aligned} \ln(\hat{y}^*) &= \Psi_1 \ln(s_{KP}) - \Psi_1 \frac{1}{s_{KP}} d + \Psi_2 \ln(s_H) \\ &+ \Psi_3 \ln(n+g+\delta), \end{aligned} \quad (30)$$

where $\Psi_1 = \frac{\alpha}{1-\alpha-\beta}$, $\Psi_2 = \frac{\beta}{1-\alpha-\beta}$ and $\Psi_3 = -\frac{\alpha+\beta}{1-\alpha-\beta}$. Substituting equation (9) for d in equation (30), we arrive at the formula of the *long-run output - debt* relationship:

¹⁵The inclusion of human capital into the RCK model and the Blanchard model would have complicated the discussion of the previous sections considerably, without improving our understanding. First, the net wealth effect of government bonds continues to be zero in the RCK model with human capital as well. Second, the tiny magnitudes of the crowding-out effect calculated in section 3 suggest that the inclusion of human capital into the Blanchard model is quantitatively not an important issue.

$$\ln(\hat{y}^*) = \Psi_1 \ln(s_{KP}) - \Psi_4 \mu^* + \Psi_2 \ln(s_H) + \Psi_3 \ln(n + g + \delta),$$

$$\text{where } \Psi_4 = \Psi_1 \frac{g + n}{s_{KP}} \quad (31)$$

The most important message of equation (31) is that the coefficient of μ^* is not constant but changes with the private sector's saving rate, the population growth rate, the technology growth rate and the parameters of the production function.¹⁶ Note that the coefficient is negative, so public debt reduces long-run output. The magnitude of output loss decreases with the private sector's saving rate and increases with the population growth rate. The intuitive explanation is the following (Dedák and Dombi 2018). If the saving rate of the private sector is high, then the budget deficit necessary to maintain a given debt-to-GDP ratio decreases the aggregate saving rate only modestly in percentage terms and hence leads to small changes in steady-state output. Furthermore, according to equation (9), a higher population growth rate allows a higher government deficit for a given debt-to-GDP ratio, thereby reducing the aggregate saving rate and output.

In the following, we calculate the effect of a 90 per cent debt-to-GDP ratio on long-run output according to equation (31). Figures 3 and 4 present the results based on standard parameter calibration. As Figure 3 shows, the marginal effect of public debt decreases (in absolute value) with the saving rate of the private sector and so does the burden of public debt as well. For example, in the case of a 15 per cent private saving rate, $-\psi_4 = -0.167$, which means that a one percentage point increase in the debt-to-GDP ratio reduces the long-run output by 0.167 per cent. In this case, the total loss of output emanating from a 90 per cent debt-to-GDP ratio is 15 per cent. In contrast, at higher saving rates, the output loss is much smaller.

The results in Figure 4 show that the marginal effect of public debt increases (in absolute value) with the population growth rate and so does the burden of public debt as well. For example, if the private saving rate is 20 per cent, $-\psi_4 = -0.2$ at $n = 0.02$. In this case, the total loss of output emanating from a 90 per cent debt-to-GDP ratio is 18 per

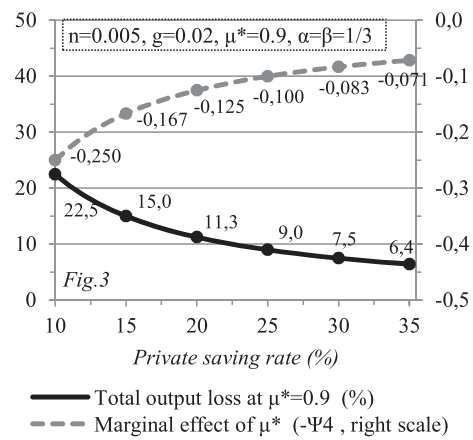


Figure 3. The burden of a 90% debt-to-GDP ratio as a function of the private saving rate.

Notes: The g , α , and β parameters are calibrated according to Mankiw, Romer, and Weil (1992). The total output loss is calculated as follows: $-100 \cdot 0.9 \cdot (-\psi_4)$

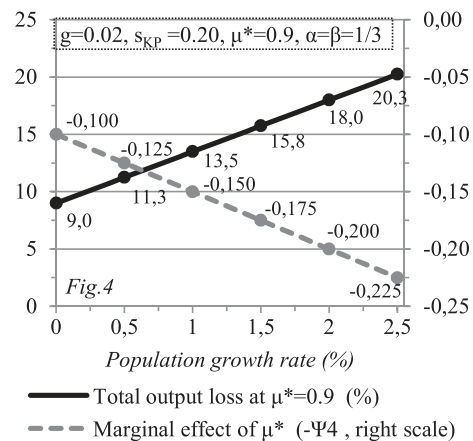


Figure 4. The burden of a 90% debt-to-GDP ratio as a function of the population growth rate.

Notes: The g , α , and β parameters are calibrated according to Mankiw, Romer, and Weil (1992). The total output loss is calculated as follows: $-100 \cdot 0.9 \cdot (-\psi_4)$

cent. In contrast, at lower population growth rates this value is much lower.

Note that Figures 3 and 4 are partial analyses because they only investigate the effect of one variable on the burden of public debt. However, in reality, the *ceteris paribus* condition does not hold. It is well known that output correlates positively with the saving rate and negatively with the population growth rate (Durlauf, Jonhson, and Temple

¹⁶If human capital is absent ($\beta = 0$), Ψ_1 is lower, implying a smaller debt coefficient in absolute value. The difference can be considerable. For example, if $\alpha = \beta = 1/3$, Ψ_4 is twice as high with human capital than without human capital.

2005). Consequently, the least developed countries are usually characterized by low saving rates and high population growth rates. This implies that the burden of public debt is under double pressure in these countries and can be significantly higher than Figures 3 and 4 might suggest. For example, if the private saving rate is only 15 per cent, while the population growth rate is 2 per cent, the coefficient of public debt is -0.267 in equation (31), implying a total output loss of 24 per cent at a 90 per cent debt-to-GDP ratio.

To sum up, we can conclude that in the Solow model with a constant saving rate of households and with complete crowding out of physical capital, the burden of public debt can be remarkably different across countries. In developed countries with high saving rates and low population growth rates, public debt has only a minor impact on the steady-state output. However, in the least developed countries, which are typically characterized by low saving rates and high population growth rates, the burden of public debt is vast and reducing government indebtedness could improve steady-state output significantly. These results are in line with the most recent findings of the empirical literature (e.g. Eberhardt and Presbitero 2015; Ramos-Herrera and Sosvilla-Rivero 2017).

V. A general framework for the burden of public debt in neoclassical growth models

The specific framework used in the previous section can easily be extended to a general one embracing all kinds of neoclassical growth models in terms of households' saving behaviour. The key insight is to incorporate a reaction function into the original Solow framework that reflects the responsiveness of households' saving to the budget deficit through a moveable parameter (Dedák and Dombi 2018). Let's denote this responsiveness parameter by q with the following content: $q = -\partial I / \partial D$, where I is investments and D is the budget deficit in absolute term. Note, that q represents the extent to which budget deficit crowds out investments and is limited to the interval of $[0; 1]$. Conversely, $(1-q)$ reflects how private savings react to the budget deficit.

Based on the latter, a possible reaction function of private savings to the budget deficit can be constructed as follows:

$$s_{KP} = \bar{s}_{KP} + (1 - q)d, \quad (32)$$

where \bar{s}_{KP} is the autonomous savings of households independent of the budget balance compared to output, and the second part of the function expresses the impact of the budget deficit on private savings.

Equation (32) is capable of mimicking the whole spectrum of households' reactions on the budget deficit by calibrating the q parameter accordingly. For instance, if $q = 0$, the budget deficit increases private savings by its own amount and therefore leaves aggregate savings (equation.29.) unaffected. This is the case of Ricardian equivalence. In contrast, if $q = 1$, private savings do not react to the budget deficit at all, and therefore aggregate savings are decreased by the budget deficit one in one. Complete crowding out of physical capital holds in the latter case. Setting the value of q between these two extremes accordingly, the full array of households' saving behaviour concerning the budget deficit can be mapped.

Substituting equation (32) into equation (29), and assuming that the budget deficit can be given as a constant z fraction of the *autonomous* private savings now ($d = z \cdot \bar{s}_{KP}$), we can go through all the steps outlined between equation (29) and (31) to arrive at the general equation of the burden of public debt:

$$\ln(\hat{y}^*) = \Psi_1 \ln(\bar{s}_{KP}) - \Psi_4 \mu^* + \Psi_2 \ln(s_H) + \Psi_3 \ln(n + g + \delta), \quad (33)$$

where $\Psi_4 = \Psi_1 \frac{g+n}{\bar{s}_{KP}} q$. Equation (33) represents the general relationship between steady-state output and public debt independent of whatever assumptions on households' consumption behaviour are made. Depending on the calibration of q , this equation is capable of delivering the results of all neoclassical models on the burden of public debt. If the responsiveness parameter is set to zero, the coefficient of the debt/GDP ratio in equation (33) becomes zero too, corresponding to the case of the RCK model in which public debt does not imply any output loss. On the other hand, if q is set to one we arrive at

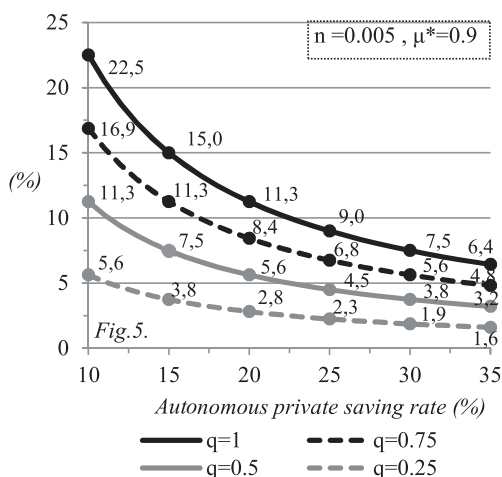


Figure 5. The total output loss at a 90% debt-to-GDP ratio as a function of households' consumption behaviour and the autonomous private saving rate

Notes: Total output loss is calculated as follows: $(100 \cdot 0.9 \cdot \psi_4)$. The g , α , and β parameters are set as in Figures 3 and 4. In Figure 5, $n = 0.005$. In Figure 6, $\bar{s}_{KP} = 0.20$.

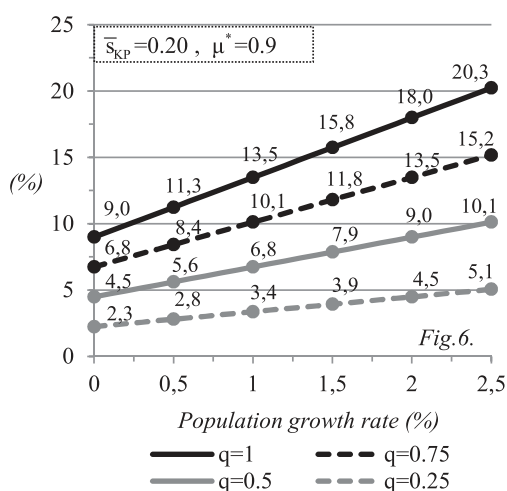


Figure 6. The total output loss at a 90% debt-to-GDP ratio as a function of households' consumption behaviour and the population growth rate

Notes: Total output loss is calculated as follows: $(100 \cdot 0.9 \cdot \psi_4)$. The g , α , and β parameters are set as in Figures 3 and 4. In Figure 5, $n = 0.005$. In Figure 6, $\bar{s}_{KP} = 0.20$.

equation (31), which is the case of the Solow model.¹⁷

Beyond these two extreme cases, the closer q is set to zero the more intergenerational links and dynamic optimization characterize households' behaviour and the smaller the burden of debt is.¹⁸

Figures 5 and 6 illustrate the decrease in the burden

of public debt at a debt-to-GDP ratio of 90 per cent in parallel with the transmission of the responsiveness parameter (crowding out intensity) from one to zero. It is impressive how remarkably the long-run output loss of being indebted decreases with q .

Finally, we note that equation (33) has a special appeal. It can serve as a basis for structural regressions in empirical estimations. To date, namely, the empirical literature has mostly operated with arbitrarily constructed growth–debt regressions lacking any thorough underpinning by growth theory.¹⁹

VI. Conclusions

This paper investigated the impact of public debt on capital accumulation and long-run output in the framework of neoclassical growth models with exogenous technological change. Regarding the effect of fiscal policy, the principal question is how the different methods of financing government expenditures affect the consumption decisions of households. The more dynamic optimization and intergenerational links characterize consumer behaviour, the less public debt affects long-run output. Because the empirics do not support any consumption theory unambiguously, we considered three different cases: two extremes and an intermediate one. The RCK model is one of the well-known extreme cases in which Ricardian equivalence holds, at least when taxes are lump-sum. Although, if taxes are distortionary, public debt decreases long-run output in the RCK model too, but the burden remains rather small, amounting to a few percentage points only.

The Blanchard model represents the intermediate case in which no intergenerational links are present but consumption behaviour is still governed by dynamic optimization. Because of the lack of intergenerational links, public debt affects the consumption of households directly and results in lower steady-state physical capital and output. However, the magnitudes of the crowding-out effect and the related output loss are negligible. For example, the increase in the debt-to-GDP ratio from zero to 90 per cent lowers the steady-state output only by 2–3 per cent via the classical

¹⁷Note, that if $q = 1$, $s_{KP} = \bar{s}_{KP}$.

¹⁸Note, that a q close to zero corresponds to the Blanchard model.

¹⁹See Panizza and Presbitero (2013) for a survey.

crowding-out channel. Taking the effect of distortionary taxes also into account does not change the picture: the burden of debt remains low, probably under 5–6 per cent.

The other extreme case of consumption behaviour is represented by the Solow model, in which households save a constant fraction of their income. Under such a condition, the crowding-out effect of public debt is complete. This implies that the Solow model serves as a means to calculate the upper boundary of the burden of public debt. Our results show that in countries with a high saving rate and with a low population growth rate, conditions typical for developed countries, the impact of public debt on steady-state output is modest. For example, if the private sector's saving rate is 25 per cent and the population growth rate is 0.5 per cent, the increase in the debt-to-GDP ratio from zero to 90 per cent lowers the steady-state output only by 9 per cent. In contrast, the burden of public debt is much higher under the conditions of the least developed countries with low saving rates and high population growth rates.

Finally, by developing a general framework for assessing the burden of public debt, the message of neoclassical growth theory becomes even more apparent: if Ricardian equivalence fails, the enormous differences in the saving and population growth rates observed across the world lead to huge differences in output loss caused by public debt. The greater the departure from Ricardian equivalence the wider the scale on which the burden of debt may vary.

Our analysis relies on a neoclassical framework with exogenous technological change. Lifting the restriction of exogenous technological development, the quantitative results on the burden of public debt may alter to some extent. In this respect, the extension of our general framework to incorporate endogenous technological change seems to be especially promising and is the subject of future research. Despite this caveat, our results are already established enough to draw some cautious conclusions for European economic policies. Since the burden of public debt in developed countries seems to be rather small – even in the case of significant departure from Ricardian equivalence – the paying down

of debt to pre-crisis levels would probably not improve much the growth performance of European economies. Therefore, regarding the lingering and fragile recovery of the Eurozone, economic policies striving to slash government indebtedness significantly in the years to come seems to be neither advisable nor desirable. This conclusion holds despite the fact that debt-sustainability is certainly a major source of concern in some highly-indebted European countries. However, in the recent years budget deficits have become moderate all over Europe, so currently the primary risk concerning debt-sustainability is not loose fiscal policies but sluggish economic growth.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix

The effect of public debt on steady-state output through distortionary taxes

In order to quantify the effect of public debt on long-run output through distortionary taxation, we use the framework of the RCK model presented in section 2. The differences compared to section 2 are twofold. First, the tax rates of wage income and capital income are set to be equal for mathematical convenience. Second, the production function takes the Cobb–Douglas form. Considering the above, the steady state and the equilibrium of the economy are described partly by the following equations (asterisks are neglected):

$$\hat{y} = (\hat{k})^\alpha \quad (\text{A.1})$$

$$(1 - \tau)r = \rho + g \rightarrow r = \frac{\rho + g}{(1 - \tau)} \quad (\text{A.2})$$

$$r = \alpha \frac{\hat{y}}{\hat{k}} - \delta \rightarrow \frac{\hat{y}}{\hat{k}} = \frac{r + \delta}{\alpha} \quad (\text{A.3})$$

$$\begin{aligned} \mu &= \frac{(\tau r B + \tau W + \tau r K - G)/Y}{r - (g + n)} \\ &= \frac{\tau r \mu + \tau - \tau \delta \hat{k} / \hat{y} - \phi}{r - (g + n)} \end{aligned} \quad (\text{A.4})$$

Equations (A.2), (A.3) and (A.4) are equivalent to equations (7), (3) and (9), respectively. The last term in equation (A.4) takes into account that $Y = W + rK + \delta K$.

Substituting equation (A.2) and equation (A.3) for r and \hat{k}/\hat{y} in equation (A.4), we arrive at a quadratic equation in the tax rate after some manipulation:

$$\begin{aligned} &(\delta - \delta\alpha)\tau^2 + ((\alpha - \mu\rho + \mu n - \phi - 1)\delta - \rho - g)\tau + \\ &+ (\mu\rho - \mu n + \phi + \mu g + \mu\delta)\rho + (g + \delta)(\phi - \mu n) = 0. \end{aligned} \quad (\text{A.5})$$

The solution of equation (A.5) delivers the tax rate, which is consistent with the targeted μ debt-to-GDP ratio. After the determination of the tax

rate, the calculations of the steady-state interest rate, capital-output ratio and output per effective labour are straightforward, according to equations (A.1), (A.2) and (A.3). Appendix A1 presents the results of the calculations for different debt-to-GDP ratios based on the typical parameter settings for developed countries.²⁰

As can be seen in Appendix A1, the tax rate increases with the long-run debt-to-GDP ratio. The underlying reason is the need to achieve a higher primary budget surplus in order to maintain a higher debt-to-GDP ratio. The increment in the tax rate compared to the $\mu = 0$ case can be interpreted as the tax burden of public debt. The consequences of the higher tax rate are the higher interest rate and, thus, the lower capital and output. Although public debt reduces the long-run output due to tax distortions, our results show that this effect is not important from a quantitative point of view. For example, if the long-run debt-to-GDP ratio jumps from 0 to 90 per cent, the tax rate and the interest rate increase only by 2.8 and 0.7 percentage points, respectively. In accordance with these rather small movements, the steady-state output decreases only by 2.75 per cent.

We performed an extended robustness analysis and found that the output loss of public debt triggered by distortionary taxation is sensitive to the n , α , ϕ and ρ parameters. The results show that the output loss decreases with the population growth rate and increases with the subjective discount rate, the capital share and the government expenditures relative to GDP. However, as Appendix A2 demonstrates, the magnitudes remain small. Another tendency to be observed is that the output loss is larger for those parameter combinations which are more typical for less developed countries (grey-coloured fields).²¹

²⁰The tax rate corresponds to the smaller root of equation (A.5), because the larger root is above one in each case.

²¹The results are similar when human capital and consumption tax are included in the model.

Appendix A1. The effect of public debt on steady-state output through distortionary taxes.

$n = 0, \alpha = 0.4, \delta = 0.04, g = 0.02, \phi = 0.45, \rho = 0.03$											
μ^* (%)	0	10	20	30	40	50	60	70	80	90	100
τ^* (%)	50.7	51	51.4	51.7	52	52.3	52.6	52.9	53.2	53.5	53.8
r^* (%)	10.1	10.2	10.3	10.3	10.4	10.5	10.5	10.6	10.7	10.8	10.8
\dot{k}^*/\dot{y}^*	2.827	2.814	2.801	2.789	2.776	2.763	2.750	2.737	2.724	2.711	2.698
\dot{y}^*	1.999	1.993	1.987	1.981	1.975	1.969	1.963	1.957	1.951	1.944	1.938
Tax burden of public debt (ppt)	0.0	0.3	0.7	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1
Output loss (%)	0.00	0.30	0.60	0.91	1.21	1.51	1.82	2.13	2.44	2.75	3.06

Notes: 'ppt' stands for percentage points. The tax burden is calculated as follows: $(\tau^* - \tau_{\mu=0}^*)$. The output loss is calculated as follows: $-100(\dot{y}^* - \dot{y}_{\mu=0}^*)/\dot{y}_{\mu=0}^*$

Appendix A2. The sensitivity of the output loss related to the tax burden of $\mu^* = 90$ (%).

		n		α		φ		ρ				
<i>Partial Sensitivity</i>		0.01	0.02	0.35	0.45	0.50	0.35	0.40	0.50	0.02	0.04	0.05
Output loss (%)		1.82	0.91	2.18	3.42	4.24	2.23	2.46	3.09	1.76	3.77	4.82
<i>Joint Sensitivity</i>	α = 0.35 ρ = 0.02	α = 0.45 ρ = 0.05	n = 0.01 ρ = 0.02	n = 0.02 ρ = 0.05	φ = 0.5 ρ = 0.02	φ = 0.35 ρ = 0.05	φ = 0.5 ρ = 0.02	φ = 0.35 ρ = 0.05	φ = 0.5 ρ = 0.02	α = 0.35 n = 0.01	φ = 0.35 ρ = 0.05	α = 0.45 n = 0.02
Output loss (%)	1.39	5.97	0.88	2.86	1.99	3.95	0.78	2.92				

Notes: The results of the joint sensitivity analysis are coloured grey if the underlying parameter combination is more typical for less developed countries than for developed ones.