Multi-Server Loss Systems with T-Limited Service for Traffic Control in Information Networks

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Abstract To handle more phone and personal computer users in such a natural disaster as terribly strong earthquake, a traffic control has been previously proposed by liming the individual call holding time. This traffic control mechanism leads to our T-limited service. By T-limited service, we mean that the service time is limited to a threshold T. The call whose service time reaches T is assumed to be lost. For evaluating the traffic control performance, we present multi-server loss systems with T-limited service. Without any retrial queues, we analyze a Poisson input and general service time loss system to derive the steady-state distribution of the number of calls in the system. With a retrial queue, assuming further that the call sojourn time at the retrial queue is exponentially distributed and that the T-limited service time is also exponentially distributed, we propose an approximation for the steady-state distribution of the number of calls in the system. Our approximation accuracy is validated by a simulation result.

Keywords: Traffic control, T-limited service, loss system, traffic analysis, retrials.

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1. Introduction

Hearing of a natural disaster like a terribly strong earthquake or severely wild typhoon, people use their phones or personal computers (PCs) to inquire after their close relative's and friend's safety via voice or data (e-mail, web) services. Voice and data traffic may exceed the system capacity to deteriorate the network performance. To handle more phone and PC users in such a disaster, or at least to avoid deteriorating the network performance; Okada [11] has previously proposed a traffic control by liming the individual call holding time during a traffic congestion period. Okada's traffic control mechanism is referred to as time-limited service in the queueing literature.

We introduce a concept and the notation for describing time-limited service. Let T be a threshold for the service time limitation. To be more exact, the service station will stop the service of a call whose service time reaches T. The service-stopped call will then leave the station and will not arrive at the station again (will not retry to enter the station). The call will be lost. Using the threshold T explicitly, we refer to our time-limited service as T-limited service.

A prime example of T-limited service is a cellular phone traffic control proposed in Okada [11] as mentioned above. Another example is a service time-out scheme which arises in a packetized communication system providing a recently-developed real-time service application. The necessity and importance of our T-limited service have been increasing in information networks. To the best of the author's knowledge, however, there is very little literature on the loss systems with T-limited service even for the simple Poisson input case. Our main goal of the paper is to present and analyze new teletraffic models, multi-server loss systems with T-limited service.

There has been much interest in exact and approximate approaches for service station systems with retrials; see Refs [6, 9, 12, 13, 16, 20, 22, 23]. In typical retrial queueing models, an arriving call that finds all servers busy will return and place another request for some later time. The call is assumed to enter the retrial queue, after some sojourn time at the retrial queue it will try to enter the service station. Retrial queueing models are of practical importance in the teletraffic filed of information networks. However, we are aware of no reported results on T-limited service. In the paper, we will consider a multi-server loss system with retrials as well as T-limited service.

The rest of the paper is organized as follows. Section 2 describes our queueing models in details and gives the notation and symbols. Statistical and stochastic assumptions are made in this section. Section 3 is devoted to analyzing the T-limited service time distribution. The coefficient of variation of the T-limited service time is derived. The statistical analysis of the T-limited service time provides a stepping stone for the subsequent queueing analysis. Section 4 develops our exact analysis for the Poisson input general service time multi-server loss system with T-limited service. It also develops our approximate analysis for the Poisson input exponential service time multi-server loss system with T-limited service and retrials. Our approximation accuracy is validated by a simulation result. Section 5 contains some concluding remarks and topics for further research.

2. Notation and Symbols

Our teletraffic model is characterized as follows. Calls arrive at a multi-server service station. An arriving call requests its service (holding a channel, transferring information, or processing a job in a network system) for a time-period duration. The time-period duration requested by a call is referred as to the service time (of a call). The inter-arrival times and the service times of calls are random variables and assumed to be statistically independent.

We denote by the Kendall notation A/B/c/K, a multi-server queueing model where the inter-arrival time distribution is A, the service time distribution is B, the number of servers is c, and the maximum number of calls allowed in the model (referred as to the system capacity) is K. The symbols introduced by Kendall [10] and traditionally used for A and B are

G: general inter-arrival or service time distribution

H_k: k-stage hyper-exponential inter-arrival or service time distribution

E_k: k-stage Erlang inter-arrival or service time distribution

M: exponential inter-arrival or service time distribution
(so that the arrivals or services form a Markov process)

D: deterministic inter-arrival or service time (unit distribution)

The shorter notation A/B/c is usually used if there is no limit to the length of queue (if the system capacity is infinite). When we say an M/G/1 queueing model, we mean that a single-server ($c = 1$) infinite-capacity ($K = \infty$) queueing system where the inter-arrival time distribution is exponential ($A = M$) while the service time distribution is general ($B = G$). As another Kendall notation example, when we say an M/G/c/c model, we mean that a multi-server loss system ($K = c$) where the inter-arrival time distribution is exponential ($A = M$) while the service time distribution is general ($B = G$). In the M/G/c/c model, an arriving call who finds the c servers all busy will be lost (will not be able to enter the service station).

We further use the following symbols throughout this paper..

λ : arrival rate of calls
μ : (originally requested) service rate
(reciprocal of the mean service time without T-limited service)
μ': effective service rate
(reciprocal of the mean service time with T-limited service)

3. T-limited Service Time Distribution

Let H be the originally requested service time by an arriving call that can enter the system. The service time is assumed to be independent, and identically distributed (i.i.d.) random variable. In this section we present a statistical analysis of the service time distribution with T-limited service.

Let H_{eff} be the effective service time with T-limited service for the original service time H, namely,

$$H_{eff} = \begin{cases} H & \text{if } H \leqq T \\ T & \text{if } H > T \end{cases} \tag{3.1}$$

The cumulative distribution function (c.d.f.) of the effective service time is given by

$$P(H_{eff} \leqq t) = \begin{cases} P(H \leqq t) & \text{if } t \leqq T \\ 1 & \text{if } t > T \end{cases} \tag{3.2}$$

If we further assume that the service time is exponentially distributed, we have

$$P(H \leqq t) = 1 - e^{-\mu t} \tag{3.3}$$

Substituting (3.3) into (3.2) yields

$$P(H_{eff} \leq t) = \begin{cases} 1 - e^{-\mu t} & \text{if } t \leqq T \\ 1 & \text{if } t > T \end{cases} \tag{3.4}$$

If we denote the first and second moments of the effective service time as

$$E(H_{eff}) \equiv \frac{1}{\mu'},$$

and

$$E(H_{eff}^2) \equiv h_{eff}^{(2)}$$

we have the first moment of the effective service time:

$$\begin{aligned} E(H_{eff}) &\equiv 1 / \mu' \\ &= \int_0^T t\mu e^{-\mu t} dt + Te^{-\mu T} \\ &= \frac{(1 - e^{-\mu T})}{\mu} \end{aligned} \tag{3.5}$$

and the second moment of the effective service time:

$$\begin{aligned} E(H_{eff}^2) &\equiv h_{eff}^{(2)} \\ &= \int_0^T t^2 \mu e^{-\mu t} dt + T^2 e^{-\mu T} \\ &= \frac{2(1 - e^{-\mu T} - \mu T e^{-\mu T})}{\mu^2} \end{aligned} \tag{3.6}$$

The variance and squared coefficient of variation of the effective service time are respectively given as

$$V(H_{eff}) = \frac{(1 - 2\mu T e^{-\mu T} - e^{-2\mu T})}{\mu^2} \tag{3.7}$$

$$C(H_{eff})^2 = \frac{(1 - 2\mu T e^{-\mu T} - e^{-2\mu T})}{(1 - 2e^{-\mu T} + e^{-2\mu T})} \tag{3.8}$$

It follows from Eq.(3.8) that the effective service time is not exponentially distributed since $C(H_{eff}) < 1$ while the original service time is exponentially distributed ($C(H) = 1$). Adopting T-limited service mechanism when a network congestion occurs is then seen to shorten not only the mean ($\mu < \mu'$) but also the coefficient of variation ($C(H_{eff}) < C(H)$). Our T-limited service control makes the original traffic smooth. Smooth traffic is known to be less congested than Poisson traffic; see e.g., Refs [4, 5] It also follows that the coefficient of variation of the effective service time converges to 1 (unity) as the threshold tends to infinity ($C(H_{eff}) \rightarrow 1$ as $T \rightarrow \infty$).

4. Multi-Server Loss Systems with T-Limited Service

As mentioned in Section 1, there is very little literature on multi-server loss systems with T-limited service. We start our traffic analysis with the simplest input but general service time model.

4.1 Poisson Input and General Service Time Model

We consider an M/G/c/c loss system with T-limited service. The inter-arrival time of calls is assumed to be exponentially distributed with rate λ (the call arrivals are assumed to form a Poisson process with rate λ), and the service time is assumed to be generally distributed with a mean of $1 / \mu$.

For the M/G/c/c loss system with T-limited service, we denote by πi the steady-state probability that there are i calls in the system ($i = 0, 1, ..., c$). The supplementary variable technique [17] is applied to get the following flow-balance equations:

$$\lambda \pi_i = (i + 1)\mu' \pi_{i+1} \quad (i = 0, 1, 2, \ldots, c - 1) \tag{4.1}$$

where μ' is the effective service rate. The left-hand side of Eq.(4.1) represents the flow-up speed from state i to state i+1, while the right-hand side of Eq.(4.1) represents the flow-down speed from state i+1 to state i. It should be noted that the flow-balance Eq.(4.1) is still valid even if the effective service time is no longer exponentially distributed; see Refs.[4, 17] for this validity.

Eq.(4.1) enables us to recursively obtain the steady-state probability distribution $\{ \pi_i : i = 0, 1, \ldots, c \}$:

$$\pi_i = \left(\frac{\lambda}{\mu'}\right)^i \frac{1}{i!} \pi_0 \quad (i = 0, 1, 2, \ldots, c) \tag{4.2}$$

The total probability law leads to the normalization condition:

$$\sum_{i=0}^{c} \pi_i = 1 \tag{4.3}$$

Substituting Eq.(4.2) into (4.3), we have the system idle probability, π_0, as

$$\pi_0 = \frac{1}{\sum_{i=0}^{c} \left(\frac{\lambda}{\mu'}\right)^i \frac{1}{i!}} \tag{4.4}$$

Thus, Eqs (4.2) and (4.4) completely determine the steady-state probabilities for the M/G/c/c loss system with T-limited service time.

We are now in a position to derive the system performance measures. The time congestion $B_c(\lambda, \mu')$, which is defined as the probability that a virtual (test) call finds all c servers busy, is straightforward:

$$B_c(\lambda, \mu') = \pi_c$$

$$= \frac{\left(\frac{\lambda}{\mu'}\right)^c \frac{1}{c!}}{\sum_{i=0}^{c} \left(\frac{\lambda}{\mu'}\right)^i \frac{1}{i!}} \tag{4.5}$$

where the effective service rate μ' is obtained from Eq.(3.5) in the preceding section.

Since we assume Poisson arrivals in this subsection, the PASTA (Poisson arrivals see time averages [4, 8, 18]) property leads to the fact that the time congestion $B_c(\lambda, \mu')$ is identical to the loss probability P_{loss}. Here, is defied as the probability that an actual arriving call finds the all c servers busy.

$$P_{loss} = B_c(\lambda, \mu').$$

4.2 Poisson Input and Exponential Service Time Model with Retrials

We next consider an M/M/c/c loss system with T-limited service and retrials. The inter-arrival time of calls is assumed to be exponentially distributed with rate λ (the call arrivals are assumed to form a Poisson process with rate λ), and the service time is assumed to be exponentially distributed with a mean of $1 / \mu$. An arriving call that finds all c serves busy will enter the retrial queue with probability α and it will leave the system with probability $1 - \alpha$ (it will be lost with probability $1 - \alpha$). Except for the service time distribution, our model described here reduces to the model in the preceding subsection 4.1 for $\alpha = 0$. We assume that the retrial queueing capacity is infinite and that the sojourn time of a call at the retrial queue is exponentially distributed with a mean of $1 / \zeta$.

The total arrival rate Λ is composed of the original arrival rate λ and the arrival rate γ from the retrial queue:

$$\Lambda = \lambda + \gamma \tag{4.6}$$

We develop our approximation taking Hashida and Kawashima's [6] approach. Hashida and Kawashima [6] actually treated a single-server M/M/1/K finite-capacity queueing system with retrials, but their approach enables us to treat our multi-server M/M/c/c loss system with retrials and T-limited service.

As in Hashida and Kawashima [6], we consider a situation where the mean retrial interval is infinitely large ($\zeta = 0$). In this extreme situation, the superposition process of the original arrival and retrial input streams can be regarded as a Poisson process with rate Λ. Hence, we have the time congestion $B_c(\Lambda, \mu')$ as follows:

$$B_c(\Lambda, \mu') = \pi_c$$

$$= \frac{\left(\frac{\lambda-\gamma}{\mu'}\right)^c \frac{1}{c!}}{\sum_{i=0}^{c}\left(\frac{\lambda+\mu}{\mu'}\right)^i \frac{1}{i!}} \tag{4.7}$$

Since we approximate the superposition process by a Poisson process with rate Λ, we have

$$(\lambda + \gamma)\, B_c(\lambda + \gamma, \mu')\, \alpha = \gamma \tag{4.8}$$

The two factors in the left-hand side of Eq.(4.8) represent the mean number of calls (per time unity) that find all c servers busy. With probability α those calls enter the retrial queue, from which Eq.(4.8) follows.

The unknown parameter γ is numerically obtained. In fact, solving the transcendental Eq.(4.8) for γ via e.g., Newton's method, we can numerically determine the arrival rate from the retrial queue γ.

The loss probability Ploss is then calculated from

$$P_{loss} = B_c(\lambda + \gamma, \mu') \cdot (1 - \alpha) \tag{4.9}$$

As in Hashida and Kawashima [6], we consider alternative situation where the mean retrial interval is almost zero ($\zeta = \infty$). In this extreme situation, we can regard our loss system with prompt retrials as a queueing system.

In Section 3, we see that the effective service time is no longer exponentially distributed even if the original service time is exponentially distributed, since

$C(H_{eff}) < 1 = C(H)$.

Nevertheless, to simplify our analysis, we assume that the effective service time is also exponentially distributed with a mean of $1 / \mu'$.

Assuming that our multi-server loss system with prompt retrials is approximated by an M/M/c system, we have for the steady-state probability distribution $\{ \pi_i: i = 0, 1, ...\}$:

$$\lambda \pi_i = (i + 1) \mu' \pi_{i+1} \qquad (i = 0, 1, 2, ..., c-1) \tag{4.10a}$$

$$\alpha \lambda \pi_i = c\mu' \pi_{i+1} \qquad (i = c, c+1, c+2, ...) \tag{4.10b}$$

The left-hand side of Eq.(4.10) represents the flow-up speed from state i to state i+1, while the right-hand side of Eq.(4.10) represents the flow-down speed from state i+1 to state i. Arriving call that finds all c servers busy will enter the retrial queue with probability α, the flow-up rate is nothing but $\alpha \lambda$, as in the left-hand side of Eq.(4.10b).

It should be noted that the flow-balance equation (4.10) is valid only for an exponentially distributed effective service time. We can then improve the accuracy of our approximation if we replace Eq.(4.10) by the flow-balance equations for the M/G/c queueing model. The diffusion approximation technique [5, 19, 21] may lead to the flow-balance equation for the M/G/c queueing model.

Eq.(4.10) enables us to recursively obtain the steady-state probability distribution $\{ \pi_i: i = 0, 1, ...\}$:

$$\pi_i = \left(\frac{\lambda}{\mu'} \right)^i \frac{1}{i!} \pi_0 \quad (i = 0, 1, 2, ..., c) \tag{4.11a}$$

$$\pi_{c+i} = \left(\frac{\lambda}{\mu'} \right)^c \frac{1}{c!} \frac{\alpha \lambda}{c\mu'} \pi_0 \quad (i = 0, 1, 2, ...) \tag{4.11b}$$

The total probability law leads to the normalization condition:

$$\sum_{i=0}^{\infty} \pi_i = 1 \tag{4.12}$$

Substituting Eq.(4.11) into (4.12), we have the system idle probability, π_0

$$\pi_0 = \frac{1}{\sum_{i=0}^{c} \left(\frac{\lambda}{\mu'}\right)^i \frac{1}{i!} + \left(\frac{\lambda}{\mu'}\right)^c \frac{1}{c!} \frac{\alpha\lambda}{c\mu' - \alpha\lambda}} \tag{4.13}$$

Here, we assume stability condition:

$$\frac{\alpha\lambda}{\mu'} < c \tag{4.14}$$

Thus, Eqs (4.11) and (4.13) completely determine the steady-state probabilities for the M/M/c queueing system with T-limited service time.

Note that the arrival rate of calls from the retrial queue, γ, is approximately equal to the sojourn time rate at the retrial queue multiplied by the mean number of calls in the retrial queue. We then have:

$$\gamma = \zeta \cdot \sum_{i=0}^{\infty} (i - c)\, \pi_i \tag{4.15}$$

With the infinite summation result on the geometric series:

$$\sum_{i=0}^{\infty} i\, r^i = \frac{r}{(1-r)^2} \quad (|r| < 1) \tag{4.16}$$

and Eq.(4.11), Eq.(4.15) yields to:

$$\gamma = \zeta \cdot \{ (\lambda / \mu')^c / c! \}\, \pi_0\, [\, \alpha\lambda / (c\mu')\,] \,/\, [\, 1 - \alpha\lambda / (c\mu')\,]^2 \tag{4.17}$$

Recall that the system idle probability, π_0, in Eq.(4.17) is given by Eq.(4.13).

We are finally in a position to derive the system performance measures. The time congestion B_{time} is obtained as

$$B_{time} = B_c(\lambda + \gamma, \mu') \tag{4.18}$$

As for a safer-side approximation, the time congestion corresponds to the probability of delay for the M/M/c queueing system with T-limited service time. We then have.

$$B_{time} = \sum_{i=c}^{\infty} \pi_i = \frac{\left(\frac{\lambda}{\mu'}\right)^c}{c!} \pi_0 \frac{1}{1 - \frac{\alpha\lambda}{c\mu'}} \tag{4.19}$$

The system idle probability, π_0, in Eq.(4.19) is again given by Eq.(4.13).

The call congestion, B_{call}, which is defined as the probability that an arriving call finds the all c servers busy, is obtained from Eq.(4.8):

$$B_{call} = \frac{\gamma}{\alpha(\lambda + \gamma)} \tag{4.20}$$

The arrival rate from the retrial queue, γ, in Eq.(4.20) is given by Eq.(4.17).

The loss probability P_{loss} is then calculated from

$$P_{loss} = B_{call} \cdot (1 - \alpha) \tag{4.21}$$

The retrial rate, R, is defined as in Hashida and Kawashima [6]:

$$R = \frac{\Lambda - \lambda}{\Lambda}$$

$$= \frac{\gamma}{\Lambda} \tag{4.22}$$

Figure 1 shows our approximation and simulation results on the retrial rate R as a function of call arrival rate λ, where we assume $c = 50$, $1/\mu = 20$, $1/\zeta = 2$, and $\alpha = 1$. The solid line and closed circles (black spots) respectively denote the approximated and simulated results for $T = 30$, while the broken line and × denote the approximated and simulated results for $T = 40$.

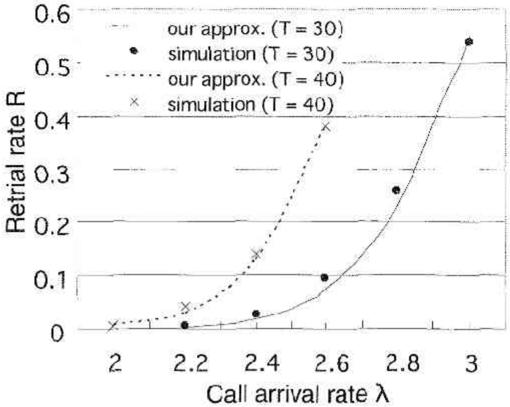

Figure 1: A performance comparison between our approximation and simulations.

5. Conclusion

At least to avoid deteriorating the network performance in such a natural disaster as severely wild typhoon, Okada [11] proposed a traffic control by liming the individual call holding time, leading to our T-limited service. By T-limited service, we have meant that the service time is limited to a threshold T. The service station will stop the service of a call whose service time reaches T. The service-stopped call will then leave the station and will not arrive at the station again (will not retry to enter the station). The call will be lost. So far, we have seen no literature on studying multi-server loss systems with T-limited service, which have a potential applicability to Okada's traffic control in information networks.

We have modeled and analyzed multi-server loss systems with T-limited service.

Without retrial queue, we have considered the Poisson input and general service time M/G/c/c loss system. We have derived the steady-state distribution of the number of calls in the system, by using the supplementary variable approach [4, 17]. With retrial queue, we have considered the Poisson input and exponential service time M/M/c/c loss system. By taking Hashida and Kawashima's technique [6], we have approximately derived the steady-state distribution of the number of calls in the system. With the retrial queue, our approximation accuracy has been confirmed by a simulation result.

For the Poisson input T-limited general service model without retrials, we have almost completely solved the problem to obtain the system performance measures in the M/G/c/c loss system. For the retrial queueing model, however, we have not been able to treat general service time systems We have restricted ourselves to the case of exponential service time. It is then left for future work to extend our exponentially distributed M/M/c/c loss system to a general service time M/G/c/c loss system. It is also worthwhile to improve our approximation by taking other approaches including the diffusion approximation technique [5, 19, 21].

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