A Mathematical Analysis for Designing an Exception Reporting System

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Contrary to the numerous and varied studies on management information systems, there appears to be little organizationl rationale behind the development of most of today's management information systems. Critical evidence continues to accrue that "the information systems of today are not, in general, what they are made out to be" (Swanson, 1979, p. 237).

Ackoff (1967) argued that several assumptions commonly made by designers of management information systems were not justified in many cases and hence led to major deficiencies in the resulting systems. King and Cleland (1971) claimed that although modern management information systems were supposed to help the manager make better decisions, few were true management systems. According to their observations, information systems have been shaped by improvements in existing data processing functions, and have not significantly increased the decision making effectiveness of managers.

More recently, Wildavsky (1978) makes critique of today's information systems as "really made up of dumb data" and as "untheoretical, non-organizational, and a-historical". Hedberg and Jönsson (1978) express a similar view and state that:

> It appears that many modern information systems dysfunctionally add to organizations' inertia. Access to more information and more advanced decision aids does not necessarily make decision makers better informed or more able to decide (p. 48).

Where do these deficiencies result from? Designers of management information systems are inclined to believe the notion that to supply information to decision makers is a good thing in and out itself. This seemingly well accepted notion will be challenged by the following considerations:

(1) the problems of information overload,

(2) the problem of processing information by decision makers, and

(3) the fact that all information has a cost.

The viewpoint which has motivated this paper is the importance of shifting the emphasis of the information system design from supplying relevant information to eliminating irrelevant, unimportant information. The purpose of this paper is to develop evaluation methods for particular types of information and decision systems and to investigate solutions for designing systems which can reduce information overload and is economically more efficient than current systems. Several mathematical models will be examined on the basis of recent developments in the area of inquiry known as "information economics" to solve these problems.

I. Deficiencies in the Development of Information Systems

It is generally recognized that the amount of information inputs and the ways of their provision influence the utilization of information provided from an information system. Information is utilized to enable a decision maker to make informed judgments and decisions. For the information to be useful, it must bear upon or be usefully associated with the decision which it is designed to facilitate or with the result which it is desired to produce. The more closely information provided is attuned with a decision maker's needs, the more useful it will be in producing the desired result. Consequently, the information and the way of reporting it exert influence on the designated decisions. For these influences to be exerted effectively, relevant information must be available in a form and a time for it to be useful.

According to Ackoff (1967) and others, however, most managers suffer more from an overabundance of irrelevant information, while they lack a good deal of information that they should have. They receive more data than they can possibly absorb even if they spend all of their time trying to do so. In short, they suffer from a so-called information overload. As a result, they must spend a great deal of time separating the relevant from the irrelevant and searching for the kernels in the relevant documents.

Humans have limited capabilities for processing data. Beyond some point it becomes physically impossible to assimilate and react to incremented messages. Even modest increases in the amount of data provided could worsen decision effectiveness because of psychological effects. Increased information levels increase the perceived complexity of the environment. According to abundant psychological testing evidence, such changes in perceived environmental complexity induce changes in decision maker's cognitive processing capabilities (Schroder, Driver, and Straufert, 1967). These cognitive processing changes, in turn, can decrease the effectiveness of decision making by causing a decision maker to revert to a more concrete conceptual level in an attempt to cope with the new, more compex environment (Driver and Straufert, 1969).

That certain relevant reports are required to improve decision making effectiveness tends to be taken by many system designers as a "given". In real decision making situations, however, managers in an organization may not want to collect all the relevant data that are available or make use of the information they possess. According to Cyert and March (1965), managers in the typical firm do not scan all alternatives or have complete information about the alternatives selected. Time factor may be critical: the time limit of most decisions demands that they must be taken on less than complete information. Also, it has been suggested that search for data is initiated in response to a number of factors, for example, when existing decisions seem to be unsatisfactory or when a problem is looming.

An additional negative effect of the emphasis on supplying information is the problem of economic inefficiency. All information has a cost. Information should be treated as an economic commodity whose acquisition constitutes a problem of economic choice. This conception draws the inference that information should be obtained or supplied only if the benefits from its use can be justified as greater than or equal to its cost.

These considerations suggest that the notion that the informing-of-managers is a good thing in and out itself needs to be modified. The amount of information inputs and the way of providing them are important factors which affect not only information utilization and decision quality, but also resource allocation in an organization.

The problem of information overload is especially crucial to the development of management information systems. As implied by the term, information overload is caused by the amount of information inputs which are greater than those which can be processed adequately. One efficient solution is to alleviate the situation by reducing the information inputs without any critical loss of the benefits yielded if such reduction was not undertaken. It is important to note that this solution is consistent with the results of behavioral studies on human information processing which show that managers do indeed develop heuristics to reduce the amount of information processed.

II. An Exception Reporting System

This paper is based on the premise that in many modern organizations the problem of information system design should be primarily oriented toward the process of filtering, extracting and condensing information, rather than on the generation, storage, and retrieval of information. The need is to make decision makers aware of that portion of the total set of information which is relevant to their decisions and to focus their attention on situations requiring their judgments and interventions. We refer to this type of information systems as a management-by-exception reporting system, or simply an exception reporting system.

An exception reporting system is designed on the basis of the management-by-exception principle. Management-by exception is one of the basic principles of management which have been widely accepted by current classical theorists. Generally it states that decisions which recur frequently and are less important should be reduced to a routine and only those important issues or those which are non-recurring should be referred to focus decision maker's attention.

The idea behind the exception principle is the economic allocation of organizational resources; an organization must economize on its resources by dealing only with exceptional matters which have significant consequences on organizational effectiveness. Massie (1965) states that

> Generally, the exception principle has been important to the development of the process of delegation of authority. It is basic to the generalization that all decisions should be made at the lowest organizational level commensurate with personal ability and availability of information. It becomes useful in the development of systems for handling work. Of all the classical concepts, it probably comes closest to being a basic principle valid in many situations (pp. 397–8).

Filtering or extracting is the essencial function of the exception principle, applied to a management information system. Only those items which deserve attention or require action are singled out. This essentially involves a search for selected information, or the scanning of a given set of data in order to identify a subset with predetermind attributes.

The term exception reporting systems covers certain organizational information-decision systems whereby the decision about a given action variable is normally made as the routine, but may be made on the basis of more information if the original information variables take on *exceptional* values.

Before giving a precise definition of an exception reporting system, the following description may be helpful. Suppose that the range of possible values of uncertain variable x is divided into two parts, "ordinary" values and "exceptional" values. Let R denote the set of exceptional values. If, in a particular instance, an information evaluator who designs and selects the information system observes x to be ordinary, that is, not in R, then he does not provide a decision maker with that information, and the decision maker deals with the case as a routine. On the other hand, if the information evaluator observes x to be exceptional, that is, in R, then he reports that value to the decision maker. The decision maker then makes his decision on the basis of the exceptional observations.

More precisely, the structure of an exception reporting system, denoted by η_e, is defined as follows:

$$(2.1)\qquad \eta_e=\begin{cases} x & \text{if } x\in R \\ \text{constant (independent of x)} & \text{if } x\in \tilde{R} \end{cases}$$

where x is the state of uncertain environment and R is the given subset of the real line (the set of exceptional values).

From the previous discussion about information structures, we notice that the structure of an exception reporting system defines a class of incomplete information structures which contain the finest and coarsest information structures.

The finest information structure, called complete information structure generally, is defined mathematically by

$$(2.2)\qquad \eta_c=\{x\}\qquad \text{for all x in X.}$$

On the other hand, the coarsest information structure, called null information structure generally, is given by

$$(2.3)\qquad \eta_n=\text{constant (independent of x)}\qquad \text{for all x in X.}$$

A decision rule based on the null information structure becomes a routine. However, if this structure is incorporated into the exception reporting system, this part informs the decision maker that the observed value of x is not exceptional.

Using *Lemma* 2.1 we can readily prove the following theorem. *Lemma* 2.1 is the part of the Fineness Corollary of Blackwell's Theorem.

Lemma 2.1: If η' is at least a fine as η'', then

$$\Omega(\eta'', \delta; \omega, \phi) \leq \Omega(\eta', \delta; \omega, \phi).$$

Proof: Let y′ and y″ be information signals from η' and η'', respectively. Since every signal from η' is fully contained in a signal from η'', we have that

$$\pi(y''|\eta'') = \sum_{y' \subset y''} \pi(y'|\eta').$$

Upon receipt of signal y under η, the best decision function δ^* maximizes the following expected payoff:

$$E(U|\eta, \delta^*) = \max_{\delta} \sum_{s \in y} \omega[s, \delta(y)]\phi(s|y, \eta).$$

The expected payoff obtained under information structure η is therefore

$$\Omega(\eta, \delta^*, \omega, \phi) = \sum_{y} \pi(y|\eta) E(U|\eta, \delta^*).$$

Some algebraic manipulation of these results establishes the desired result:

$$\begin{aligned} \Omega(\eta'', \delta^*; \omega, \phi) &= \sum_{y''} \pi(y''|\eta'') E(U|\eta'', \delta^*) \\ &= \sum_{y''} \sum_{y' \subset y''} \pi(y'|\eta') E(U|\eta'', \delta^*) \\ &\leq \sum_{y''} \sum_{y' \subset y''} \pi(y'|\eta') E(U|\eta', \delta^*) \\ &\leq \sum_{y'} \pi(y'|\eta') E(U|\eta', \delta^*) \\ &= \Omega(\eta', \delta^*; \omega, \phi). \end{aligned}$$

Theorem 2.1: Let η_c, η_e and η_n be the three information structures defined above and $\Omega(\eta_c)$, $\Omega(\eta_e)$ and $\Omega(\eta_n)$ be the gross values of these

Table 2. 1. Outcome Function

Actions	States 1	2	3	4
1	10	6	−10	−20
2	6	8	−6	−10
3	2	0	5	−8
4	−5	−3	−2	6
5	0	0	0	0
Probability	.25	.25	.25	.25

information structures. Then we have that

$$\Omega(\eta_c) \geq \Omega(\eta_e) \geq \Omega(\eta_n).$$

Proof: Since the three information structures are comparable with respect to fineness, *Lemma* 2. 1 applies to this Theorem.

A simple numerical example will serve to illustrate the concept of value of information structures and to show the usefulness of exception reporting systems.

Let there be four equally likely states and five alternative actions, with a numerical outcome function as shown in Table 2. 1.

We shall consider the three types of information structures defined in this section. We suppose now, for the time being, that the information systems have zero cost.

(i) $\eta_n(s) = \text{constant}$ for all s (no information)

(ii) $\eta_c(s) = s$ for all s (complete information)

(iii) $\eta_e(s) = \begin{cases} s & \text{if } s \in R \\ \text{constant} & \text{if } s \in \tilde{R} \end{cases}$ (exception information)

(i) Null Information Structure

It is easily verified that the best action under the null information structure is $a^* = 5$, and that the expected outcome for this action is zero, that is,

$$\Omega(\eta_n) = \max E\omega(s, a) = 0.$$

(ii) Complete Information Structure

If the decision maker learns the value of s under the complete information structure, he will choose the action that maximizes the outcome for that value of s. For example, when s=1, the best action is a*=1 with the outcome 10. Hence, the optimal decision function is written as $\delta(s)=s$, and the maximum expected outcome yielded under η_c is given by

$$\Omega(\eta_c)=(.25)(10)+(.25)(8)+(.25)(5)+(.25)(6)=29/4.$$

(iii) Exception Information Structure

We have eight alternative structures, depending upon how we define the exceptional vales of s:

$\eta_e=1$: $R=\{1\}$ and $\tilde{R}=\{2, 3, 4\}$
$\eta_e=2$: $R=\{2\}$ and $\tilde{R}=\{1, 3, 4\}$
$\eta_e=3$: $R=\{3\}$ and $\tilde{R}=\{1, 2, 4\}$
$\eta_e=4$: $R=\{4\}$ and $\tilde{R}=\{1, 2, 3\}$
$\eta_e=5$: $R=\{1, 2\}$ and $\tilde{R}=\{3, 4\}$
$\eta_e=6$: $R=\{2, 3\}$ and $\tilde{R}=\{1, 4\}$
$\eta_e=7$: $R=\{3, 4\}$ and $\tilde{R}=\{1, 2\}$
$\eta_e=8$: $R=\{1, 4\}$ and $\tilde{R}=\{2, 3\}$

The first four structures partition S into the two sets while the latter four into the three sets. Note that these partiotions are not necessarily comparable with respect to fineness. What are the maximum expected outcomes under each of the alternative structures, not counting the information costs? We can apply the computational procedure on maximizing conditional expectations.

(1) Compute the maximal expected outcome conditional upon each of the two possible signals obtained: $s\in R$ and $s\in\tilde{R}$.

(2) Then, compute the weighted average of the two conditional expectations.

For case $\eta_e=1$, the decision maker identifies the state precisely and takes the action a=1 when s=1. This is the optimal action which yields the maximum outcome 10. When $s\in\tilde{R}$, the decision maker understands that $s \neq 1$. The optimal action for $s \neq 1$ is the one which gives the largest of the following expected outcomes:

$$(1/3)(6)+(1/3)(-10)+(1/3)(-20)=-8 \quad \text{(when } a=1)$$
$$(1/3)(8)+(1/3)(-6)+(1/3)(-10)=-8/3 \quad \text{(when } a=2)$$
$$(1/3)(0)+(1/3)(5)+(1/3)(-8)=-1 \quad \text{(when } a=3)$$
$$(1/3)(-3)+(1/3)(-2)+(1/3)(6)=1/3 \quad \text{(when } a=4)$$
$$(1/3)(0)+(1/3)(0)+(1/3)(0)=0 \quad \text{(when } a=5).$$

Hence, the best action is $a=4$, yielding the maximal expected outcome 1/3. Since $s \in R$ occurs with probability 1/4 and $s \in \tilde{R}$ with 3/4, we compute the conditional expected outcome given that $\eta_e=1$ as follows:

$$\Omega(\eta_e=1)=E(U|\eta_e=1)=(1/4)(10)+(3/4)(1/3)=11/4.$$

For case $\eta_e=5$, the decision maker is informed of the value of s when $s=1$ or $s=2$. The optimal actions for $s=1$ and $s=2$ yield the maximum outcomes 10 and 8, respectively. When $s \in \tilde{R}$, the best action is given by $a=4$, with the expected outcome 2,

$$E\omega(s, a=4|s \in \tilde{R})=(1/2)(-2)+(1/2)(6)=2.$$

Then, the conditional expected outcome obtained by using $\eta_e=5$ is

$$\Omega(\eta_e=5)=E(U|\eta_e=5)=(1/4)(10)+(1/4)(8)+(1/2)(2)=22/4$$

For the rest of the information structures of exception reporting systems, the computations are quite similar. We summarize the results in Table 2.2 for the purpose of comparing the gross values of alternative structures.

The ranking in maximum expected outcomes shown in Table 2.2 agrees with the result of *Theorem* 2.1 in this section. Complete information structure η_c is finer than any other structures and has the largest value 29/4. This structure, however, may incur information costs which are so high as to make exception information structures more preferable. The structure $\eta_e=7$, among others, results in the second highest expected value 27/4. If differencial information costs between η_c and $\eta_e=7$ are larger than 2/4, $\eta_e=7$ will be selected.

For this particular problem, we notice that it is important to

Table 2. 2.

Information Structure		Maximum Expected Outcome	Ranking
η_n		0	10
η_c		29/4	1
	R		
$\eta_e=1$	{1}	11/4	7
2	{2}	8/4	9
3	{3}	9/8	8
4	{4}	14/4	5
5	{1, 2}	22/4	3
6	{2, 3}	14/4	5
7	{3, 4}	27/4	2
8	{1, 4}	21/4	4

discriminate s=3 and s=4 because eroneous actions result in higher losses when s=3 and s=4. As shown in Table 2. 2, the best selection of exceptional states are s=3 and s=4. In general, this selection depends upon outcome function ω and probability distribution function ϕ. We will investigate this problem in further details in this study.

Another interesting type of information structure is the one which cannot inform the exact value of uncertain states, but can discriminate one group of states from another. We may refer to this type as discriminatry information structures. In our example we find three structures which belong to this type.

Denote by η_d the discriminary information structure. Then we have

$$\eta_d=1: Y=(\{1, 2\}, \{3, 4\})$$
$$\eta_d=2: Y=(\{1, 3\}, \{2, 4\})$$
$$\eta_d=3: Y=(\{1, 4\}, \{2, 3\}).$$

Consider, for example, the case of $\eta_d=1$. The decision maker is imformed that state 1 or 2 will occur when he receives the first signal. But he does not know which state will actually occur. We compute the maximal expected outcome conditional upon receipt of the first signal. The optimal action is a=1 with the outcome 8,

$$E\omega(s, a_1|y_1)=(1/2)(10)+(1/2)(6)=8.$$

Similarly, upon receipt of the second signal, the decision maker obtains the maximal expected outcome 2 by using action a_4,

$$E\omega(s, a_4|y_2)=(1/2)(-2)+(1/2)(6)=2.$$

Thus, we have the following conditional expected outcome for $\eta_d=1$,

$$\Omega(\eta_d=1)=(1/2)(8)+(1/2)(2)=5.$$

Applying the same computational procedure for other two structures, we obtain the results as shown below,

$$\Omega(\eta_d=2)=(1/2)(7/2)+(1/2)(3/2)=5/2$$
$$\Omega(\eta_d=3)=(1/2)(1/2)+(1/2)(5/2)=3/2.$$

From the comparison of these results, we realize that discrimination of {1, 2} and {3, 4} in our particular example is more valuable than the other two.

III. An Exception Reporting System for Production: A Uniform Distribution Case

Consider a problem to select among alternative information structures which support the production manager's decision to determine an optimal output a* when product demand x is uncertain. Suppose that x is distributed uniformly over the interval [0, 100], as shown in Figure 3.1. The payoff function is defined as follows:

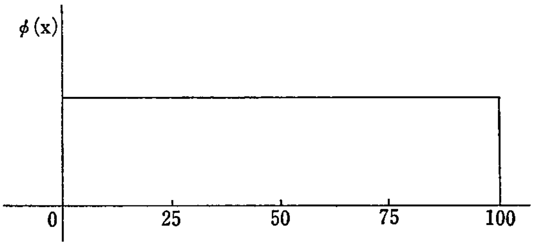
(3.1) $u=\omega(x, a)=ka-g(x, a),$

where

$$g(x, a)=\begin{cases}\alpha(x-a) & \text{if } x-a\geq 0\\ \beta(x-a) & \text{if } x-a<0.\end{cases}$$

In the above functions, parameter k is marginal revenue per unit of product and parameter α represents the penalty of underproduction whereas β represents that of overproduction.

Alternative information structures may be characterized by a different number n of subintervals of equal length into which the



$$\phi(x)=\begin{cases}\frac{1}{100} & \text{if } 0\leq x\leq 100\\ 0 & \text{otherwise}\end{cases}$$

Figure 3. 1. Uniform Distribution

whole interval [0, 100] is partitioned. For example, let $n=1$ and it represents null information structure. The manager will not be informed at all of the demand for the product. If $n=\infty$, it means that he has the complete information structure which enables him to predict the demand precisely.

Under the null information structure η_n, the manager seeks the optimal action which yields the maximum expected payoff, denoted by $\Omega(\eta_n)$,

$$(3.2)\qquad \Omega(\eta_n)=\max_a E\omega(x, a)=\max_a\left[\int_0^{100}\{ka-g(x, a)\}\phi(x)dx\right].$$

The first term inside the bracket is simply

$$(3.3)\qquad \int_0^{100}ka\phi(x)dx=ka.$$

The second term is computed as follows:

$$(3.4)\qquad \int_0^{100}g(x, a)\phi(x)dx=\int_0^a\beta(a-x)\phi(x)dx+\int_a^{100}\alpha(x-a)\phi(x)dx$$

$$=\frac{1}{100}\int_0^a\beta adx-\frac{\beta}{100}\int_0^a xdx+\frac{\alpha}{100}\int_a^{100}xdx-\frac{\alpha a}{100}\int_a^{100}dx$$

$$=\frac{\beta}{100}a^2-\frac{\beta}{200}a^2+50\alpha-\frac{\alpha}{200}a^2-\alpha a+\frac{\alpha}{100}a^2$$

$$=\frac{\alpha+\beta}{200}a^2-\alpha a+50\alpha.$$

Hence, we have

$$(3.5)\qquad E\omega(x,\ a)=-\frac{\alpha+\beta}{200}a^2+(k+\alpha)a-50\alpha.$$

Differentiating the above equation with respect to a and setting the result equal to zero, we obtain the optimal action as

$$(3.6)\qquad a^*=100\frac{k+\alpha}{\alpha+\beta}.$$

Since the second derivative is negative, that is, $-(\alpha+\beta)/100<0$, a sufficient condition is satisfied. Substituting a* given by (3.6) into the payoff function, we find the gross value of the null information structure,

$$(3.7)\qquad \Omega(\eta_n)=\frac{50}{\alpha+\beta}(k+\alpha)^2-50\alpha.$$

Next, we consider the complete information structure in which the manager is informed about the demand for the product. Obviously, the best production decision is $a^*=x$ under this structure. The maximum expected value obtained by using this structure, denoted by $\Omega(\eta_c)$, is calculated as follows:

$$(3.8)\qquad \Omega(\eta_c)=E\max_a\omega(x,\ a)=\int_0^{100}kx\phi(x)dx$$

$$=\frac{k}{100}\left[\frac{x^2}{2}\right]_0^{100}$$

$$=50k.$$

In order to compute the gross value of η_c, we evaluate the difference between $\Omega(\eta_c)$ and $\Omega(\eta_n)$. It may be reasonable to define the gross value of η_c as the incremental value over the maximum payoff obtained under the null information structure. Denote by $V(\eta_c)$ the gross value of η_c, and we have

$$(3.9)\qquad V(\eta_c)=\Omega(\eta_c)-\Omega(\eta_n)$$

$$=50k-\frac{50}{\alpha+\beta}(k+\alpha)^2+50\alpha$$

$$= \frac{50}{\alpha+\beta}(k+\alpha)(\beta-k).$$

Without loss of generality, we may suppose that $\beta \geq k$. It is interesting to note that $V(\eta_c)$ becomes positive if the marginal revenue is less than the penalty of overproduction. In general, overproduction causes the manager to incur the costs of inventory and expenses of disposing the quantities overproduced. If the marginal revenue were larger than the penalty cost of overproduction, he would be always better off by producing as much as possible.

Now, consider a discriminatry information structure. For example, let $n=2$ and then the whole interval [0, 100] is divided equally into the two subintervals. Under this structure, the manager will be told only whether the demand will be larger than 50 or not. The maximal expected payoff in this case, denoted by $\Omega(\eta_d)$, is calculated in the following way:

$$(3.10) \quad \Omega(\eta_d) = \left(\frac{1}{2}\right)\max_a \int_0^{50} \{ka - g(x, a)\}\phi(x|\eta_d)dx$$

$$+ \left(\frac{1}{2}\right)\max_a \int_{50}^{100} \{ka - g(x, a)\}\phi(x|\eta_d)dx.$$

First, we calculate $g(x, a)$ for $0 \leq x \leq 50$ and $50 \leq x \leq 100$,

$$(3.11) \quad \int_0^{50} g(x, a)\phi(x|\eta_d) = \int_0^a \beta(a-x)\frac{1}{50}dx + \int_0^{50} \alpha(x-a)\frac{1}{50}dx$$

$$= \frac{\alpha+\beta}{100}a^2 - \alpha a + 25\alpha,$$

$$(3.12) \quad \int_{50}^{100} g(x, a)\phi(x|\eta_d) = \int_{50}^a \beta(a-x)\frac{1}{50}dx + \int_a^{100} \alpha(x-a)\frac{1}{50}dx$$

$$= \frac{\alpha+\beta}{100}a^2 - (2\alpha+\beta)a + 100\alpha + 25\beta.$$

Our problem is to find a solution that maximizes the expected value of the payoff function, that is,

$$(3.13) \quad \operatorname*{Max}_a E\omega(x, a) = ka - \frac{\alpha+\beta}{100}a^2 + \alpha a - 25\alpha,$$

$$(3.14)\quad \underset{a}{\text{Max}}\, E\omega(x,\ a) = ka - \frac{\alpha+\beta}{100}a^2 + (2\alpha+\beta)a - 100\alpha - 25\beta.$$

Using the classical method of calculus, we get the optimal solutions,

$$(3.15)\quad 0 \leq a^* = 50\frac{\alpha+k}{\alpha+\beta} \leq 50$$

$$(3.16)\quad 50 \leq a^* = 50\frac{k+2\alpha+\beta}{\alpha+\beta} \leq 100.$$

After substituting the best actions into the payoff function and rearranging the terms, we finally have the maximum expected value yielded under the discriminatry information structure,

$$(3.17)\quad \Omega(\eta_d) = \frac{25}{2(\alpha+\beta)}\{(k+\alpha)^2 + (k+2\alpha+\beta)^2\} - \frac{125}{2}\alpha - \frac{25}{2}\beta.$$

To calculate the gross values of this information structure, we again evaluate the difference between $\Omega(\eta_d)$ and $\Omega(\eta_n)$,

$$(3.18)\quad \begin{aligned} V(\eta_d) &= \Omega(\eta_d) - \Omega(\eta_n) \\ &= \frac{25}{2(\alpha+\beta)}\{(k+\alpha)^2 + (k+2\alpha+\beta)^2\} - \frac{125}{2}\alpha - \frac{25}{2}\beta \\ &\quad - \left\{\frac{50}{\alpha+\beta}(k+\alpha)^2 - 50\alpha\right\} \\ &= \frac{25}{\alpha+\beta}(\alpha+k)(\beta-k). \end{aligned}$$

Comparison of $V(\eta_d)$ with $V(\eta_c)$ in (3.9) reveals that complete information is twice as valuable as discriminatory information in terms of the gross value for this particular problem. This differencial value gives the decision maker a guideline to evaluate these information structures when he takes into consideration information costs.

Finally, we investigate the information structure of an exception reporting system, which is defined as

$$(3.19)\quad \eta_e(x) = \begin{cases} x & \text{if } 0 \leq x \leq 25 \text{ or } 75 \leq x \leq 100 \\ \text{constant} & \text{otherwise.} \end{cases}$$

The exceptional range R in this structure includes the lower interval

[0, 25] and the upper interval [75, 100]. The manager will be informed of the uncertain, future demand of the product only when $x \in R$.

If x falls in the exceptional range, the informed optimal decision is written as $a^* = x$. The maximal expected payoffs may be calculated separately for each of the ranges,

(i) $0 \leq x \leq 25$

$$(3.20)\quad \Omega_1 = E \max_a \omega(x,\ a) = \int_0^{25} kx\phi(x \mid 0 \leq x \leq 25)dx = \frac{25}{2}k$$

(ii) $70 \leq x \leq 100$

$$(3.21)\quad \Omega_2 = E \max_a \omega(x,\ a) = \int_{75}^{100} kx\phi(x \mid 75 \leq x \leq 100)dx = \frac{175}{2}k.$$

If x is not in R, that is $25 < x < 75$, the manager may receive some information signal which is independent of x. So he cannot identify the exact quantities of demand, but can recognize that x will not be exceptional values. The best decision he should make is to produce the quantities that maximize the expected value of his payoff function given that $25 < x < 75$,

$$(3.22)\quad \Omega_3 = \max_a \int_{25}^{75} \{ka - g(x,\ a)\}\phi(x \mid 25 < x < 75)dx.$$

For the second term inside the bracket, we obtain

$$(3.23)\quad \int_{25}^{75} g(x,\ a)\phi(x \mid 25 < x < 75)dx$$

$$= \int_{25}^{a} \beta(a-x)\frac{1}{50}dx + \int_{a}^{75} \alpha(x-a)\frac{1}{50}dx$$

$$= \frac{\beta a}{50}(a-25) - \frac{\beta}{50}\left(\frac{a^2}{2} - \frac{25^2}{2}\right)$$

$$+ \frac{\alpha}{50}\left(\frac{75^2}{2} - \frac{a^2}{2}\right) - \frac{\alpha a}{50}(75-a)$$

$$= \frac{\alpha+\beta}{100}a^2 - \frac{3\alpha+\beta}{2}a + \frac{225}{4}\alpha + \frac{25}{4}\beta.$$

Then, Ω_3 may be rewritten in the following form:

$$(3.24)\quad \Omega_3=\max_a\left\{ka-\frac{\alpha+\beta}{100}a^2+\frac{3\alpha+\beta}{2}a-\frac{225}{4}\alpha-\frac{25}{4}\beta\right\}.$$

Taking the first derivative of Ω_3 with respect to a and setting it equal to zero, we get

$$(3.25)\quad a^*=\frac{25}{\alpha+\beta}(3\alpha+2k+\beta).$$

Since the second derivative is negative and it is readily proved that $25<a^*<75$, a^* derived above represents the optimal decision in the case of $25<x<75$. Substituting a^* into the payoff function, we find

$$(3.26)\quad \Omega_3=\frac{25}{4(\alpha+\beta)}(3\alpha+2k+\beta)^2-\frac{25}{4}(9\alpha+\beta).$$

Through these calculations, we finally get the maximum expected value yielded under the exception information structure,

$$(3.27)\quad V(\eta_e)=\frac{1}{4}\Omega_1+\frac{1}{4}\Omega_2+\frac{1}{2}\Omega_3-\Omega(\eta_n)$$

$$=25k+\frac{25}{8(\alpha+\beta)}(3\alpha+2k+\beta)^2-\frac{25}{8}(9\alpha+\beta)$$

$$-\left\{\frac{50}{\alpha+\beta}(\alpha+k)^2-50\alpha\right\}$$

$$=\frac{75}{2(\alpha+\beta)}(\alpha+k)(\beta-k).$$

How efficient the exception information structure is in comparison with the complete information structure may be an interesting question to ask. The former gains 3/4 of the gross value yielded by the latter and 3/2 of that yielded by the discriminatry information structure. We compute the differencial value between $V(\eta_c)$ and $V(\eta_e)$ and denote it by $V(\eta_c/\eta_e)$,

$$(3.28)\quad V(\eta_c/\eta_e)=\frac{25}{2(\alpha+\beta)}(\alpha+k)(\beta-k).$$

As we may expect, the value increases as penalty costs of under-and overproduction become larger. This suggests that we should enlarge or shorten the exceptional ranges, depending upon the values of α and β. If we take α and β as given, the difference is a quadratic, concave function of k. This function takes the maximum if $k=(\beta-\alpha)/2$ and becomes smaller and smaller as k approaches β.

From the comparison of Ω_1 with Ω_2, it follows that complete information brings much higher benefits when $75\leq x\leq 100$ than when $0\leq x\leq 25$. This fact, of course, results from particular forms of payoff function as well as probability distribution function. As for as this example is concerned, we may correctly conjecture that we receive higher expected benefits by setting the exceptional ranges as [50, 75] and [75, 100] rather than [0, 25] and [75, 100].

IV. A Quadratic Function of a Single Decision Variable

The models used in this and next sections are adapted from those developed by Marschak and Radner (1972). They describe the properties of decreasing returns with a quadratic approximation.

Suppose that the profit function is written as

(4.1) $u=\omega(x, a)=-a^2-ax+v^*-b^*x,$

where v^* and b^* are constant. The payoff depends upon the environment variable x (input price measured from its mean level) and the decision variable a (input quantity measured from that level which is the best one at the mean input price). Since the term (v^*-b^*x) is of little interest, the profit can be measured as

(4.2) $u=\omega(x, a)=-a^2-ax.$

For the sake of computational convenience, but without loss of the essence, we assume that $Ex=0$. We now proceed to find the optimal decision rules and evaluate the resulting expected payoffs under each of the alternative information structures considered.

First, under the complete information structure, the decision maker is kept informed of the price variable x. Making the optimal decision, (4.3) $\delta(x) = -(x/2)$,

he obtains the maximum expected payoff

(4.4) $\Omega(\eta_c) = Ex^2/4 = (1/4)\{s^2 + (Ex)^2\} = s^2/4$,

where s^2 is the variance of x.

Second, on the other hand, under the null information structure the decision maker receives no information about the price. The best action is found by maximizing, with respect to a, the expected profit

(4.5) $u = -a^2 - aEx$.

Since the optimal action a* must satisfy the condition

(4.6) $-2a - Ex = 0$,

we have $a^* = 0$ and therefore

(4.7) $\Omega(\eta_n) = 0$.

Because of (4.7), $V(\eta)$ which is defined as $\Omega(\eta) - \Omega(\eta_n)$ will be simply be equal to the expected payoff $\Omega(\eta)$.

Next, we consider the information structure of an exception reporting system which is specified as

(4.8) $\eta_e(x) = \begin{cases} x & \text{if } x \in R \\ \text{constant} & \text{if } x \in \tilde{R}, \end{cases}$

where R denotes the set of exceptional values and $\tilde{R}$ its complementary set, that is, the set of ordinary values.

If, in a particular instance, an information evaluator observes x to be exceptional, he will inform the decision maker of its exceptional value. The decision maker will choose the best action for the state which the information signal represents. On the other hand, if x is observed to be not exceptional, that is, $x \in \tilde{R}$, the decision maker will make his decision based upon *a priori* information. It is not necessary to investigate the exact value of x in this instance. Under this structure it is sufficient that the information evaluator observes x to be not exceptional.

Since the decision maker will learn the value of $x \in R$, he will choose the decision $\delta(x)$ that maximizes u in (4.2) for the informed value of x. The optimal decision, as in (4.3), is

(4.9) $\delta(x) = -(x/2) \quad \text{for } x \in R.$

The maximum expected payoff would be

(4.10) $$\Omega(\eta_e | x \in R) = \frac{1}{4} E(x^2 | x \in R) = \frac{1}{4} [s_R^2 + \{E(x | x \in R)\}^2],$$

where s_R^2 is the conditional variance of x, given that x is exceptional,

(4.11) $s_R^2 \equiv \text{var}(x | x \in R).$

When $x \in \tilde{R}$, the decision maker will learn that x is not exceptional. The best action is obtained by maximizing, with respect to a, the conditional expected profit

(4.12) $E(u | x \in \tilde{R}) = -a^2 - aE(x | x \in \tilde{R}).$

The optimal output a* must satisfy the condition

(4.13) $-2a^* - E(x | x \in \tilde{R}) = 0.$

Although x is assumed to have zero mean, it does not necessarily follow that $E(x | x \in \tilde{R}) = 0$. Hence, we have

(4.14) $$a^* = -\frac{1}{2} E(x | x \in \tilde{R}),$$

(4.15) $$\Omega(\eta_e | x \in \tilde{R}) = \frac{1}{4} \{E(x | x \in \tilde{R})\}^2.$$

Let p be the frequency with which x turns out to be exceptional,

(4.16) $p \equiv \text{prob}[x \in R].$

Then the expected value of this structure is derived as

(4.17) $$V(\eta_e) = \frac{1}{4} pE(x^2 | x \in R) + \frac{1}{4} (1-p) \{E(x | x \in \tilde{R})\}^2.$$

Suppose that x has a continuous distribution with the probability desity function f(x). Then, $V(\eta_e)$ in (4.17) may be rewritten as follows:

(4.18) $$V(\eta_e) = \frac{1-p}{4} \left\{ \int_m^n x \frac{f(x)}{1-p} dx \right\}^2 + \frac{p}{4} \left\{ \int_{-\infty}^m x^2 \frac{f(x)}{p} dx \right.$$

$$+\int_{n}^{\infty} x^2 \frac{f(x)}{p} dx\Big\}$$

$$= \frac{1}{4\int_{m}^{n} f(x)dx}\left\{\int_{m}^{n} xf(x)dx\right\}^2 + \frac{1}{4}\left\{\int_{-\infty}^{m} x^2 f(x)dx\right.$$

$$\left. + \int_{n}^{\infty} x^2 f(x)dx\right\}.$$

Since each term in (4.18) is expressed by a definite integral, variable x will vanish in the process of calculation. Accordingly, $V(\eta_e)$ is specified as a function of lower and upper limits of R. In (4.18) m and n represent lower and upper limits of R.

Given that a few conditions hold, the value of the exception structure varies depending upon the values of m and n. It is a job of the information evaluator to determine the exceptional range R. What are the optimal values of m and n which maximize $V(\eta_e)$?

For the sake of convenience, we again rewrite (4.18) as follows:

$$(4.19)\quad V(\eta_e) = \frac{1}{4\{F(n)-F(m)\}}\{G(n)-G(m)\}^2 + \frac{1}{4}\{H(m)-H(-\infty)+H(\infty)-H(n)\}$$

where: $F(x) \equiv \int f(x)dx - C$
$G(x) \equiv \int xf(x)dx - C$
$H(x) \equiv \int x^2 f(x)dx - C.$

Partially differentiating $V(\eta_e)$ with respect to m and n, we get

$$(4.20)\quad \frac{\partial V(\eta_e)}{\partial m} = \frac{f(m)\{G(n)-G(m)\}}{4\{F(n)-F(m)\}^2}[-2m\{F(n)-F(m)\} + G(n)-G(m)] + \frac{1}{4}m^2 f(m) = 0,$$

$$(4.21)\quad \frac{\partial V(\eta_e)}{\partial n} = \frac{f(n)\{G(n)-G(m)\}}{4\{F(n)-F(m)\}^2}[2n\{F(n)-F(m)\} - \{G(n)-G(m)\}] - \frac{1}{4}n^2 f(n) = 0.$$

From the two equations above, we have

$$(4.22)\quad (n-m)\left[\frac{2\{G(n)-G(m)\}}{F(n)-F(m)}-(n+m)\right]=0.$$

Because $n \neq m$, the following condition is obtained as the necessary condition for a maximum:

$$(4.23)\quad m+n=2\frac{G(n)-G(m)}{F(n)-F(m)}.$$

This result suggests that for the concave function of (4.18), given the probability p and one exceptional limit, the other limit should be determined as it satisfies (4.23).

Finally, suppose that x has a symmetrical distribution about its mean, zero. It is readily shown that the condition is simply specified as

$$(4.24)\quad n=-m,$$

since $G(n)-G(m)=0$. Hence, given the probability p, the optimal choice of R is obtained by taking it to be the complement of an interval symmetric around zero.

V. A Quadratic Function of Two Decision Variables

In this section we extend the previous analysis to the case in which two decision variables have to be determined by using alternative information structures. This case brings out a problem of organizational structures: decentralization vs. centralization.

In the centralized organization, a single decision maker may decide about two physically distinct action variables: he may choose simultaneously the values of two variables on the basis of some information about uncertain states of environment. On the other hand, in the decentralized organization each decision maker decides upon only one of the action variables on the basis of his individual information. If a communication system is established between the two decision makers, it results in providing them with the same information on which their decisions are based. When there may exist complementarity between the two actions, error-free communication will never decrease the benefits.

Suppose that the output is a quadratic function of the two decision variables (inputs). According to Marschak ang Radner (1972), the profit in this decision setting may be expressed as

$$(5.1) \quad u=\omega(x_1, x_2, a_1, a_2)=-a_1^2-a_2^2-2qa_1a_2+2a_1x_1+2a_2x_2,$$

where $x_i (i=1$ and $2)$ denotes the price variable of i-th input and q measures the degree of interaction between a_1 and a_2.(1)

The previous definition of the structure of an exception reporting system can be extended to the case where exist more than one environmental variables. For each variable i, the exceptional set R_i may be specified. We define this information structure as

$$(5.2) \quad \eta_e(x)=\begin{cases} x_i & \text{if } x_i \in R_i \\ \tilde{R}_j & \text{if } x_j \in \tilde{R}_j. \end{cases}$$

Since we have two environmental variables in our example, this information structure provides four kinds of information signals,

$$(5.3) \quad \eta_e(x)=\begin{cases} x_1 \text{ and } x_2 & \text{if } x_1 \in R_1 \text{ and } x_2 \in R_2 \\ x_1 \text{ and } \tilde{R}_2 & \text{if } x_1 \in R_1 \text{ and } x_2 \in \tilde{R}_2 \\ \tilde{R}_1 \text{ and } x_2 & \text{if } x_1 \in \tilde{R}_1 \text{ and } x_2 \in R_2 \\ \tilde{R}_1 \text{ and } \tilde{R}_2 & \text{if } x_1 \in \tilde{R}_1 \text{ and } x_2 \in \tilde{R}_2. \end{cases}$$

Suppose that each exceptional set R_i is specified as symmetrical around the mean of x_i. In addition, we assume that x_i which have symmetrical distributions with means zero and variances s_i are statistically independent.

In the first case of $x_1 \in R_1$ and $x_2 \in R_2$, both variables belong to the exceptional ranges. The exact values of them are informed before the decisions are made. The payoff function to be maximized is the same as in the case of complete information structure. The

(1) To guarantee that the maximum profit is achieved at input levels other than the boundaries, the absolute value of q must be bounded as $|q|<1$.

best decision function are given by

$$(5.4)\quad \begin{aligned} \hat{a}_1 &= \hat{\delta}_1(x_1 \in R_1,\ x_2 \in R_2) = \frac{x_1 - qx_2}{1-q^2} \\ \hat{a}_2 &= \hat{\delta}_2(x_1 \in R_1,\ x_2 \in R_2) = \frac{x_2 - qx_1}{1-q^2}. \end{aligned}$$

For given $x_1 \in R_1$ and $x_2 \in R_2$, we have the maximum profit

$$(5.5)\quad u = \omega(x_1,\ x_2,\ \hat{a}_1,\ \hat{a}_2) = \frac{x_1^2 - 2qx_1x_2 + x_2^2}{1-q^2}.$$

Second, when $x_1 \in R_1$ and $x_2 \in \tilde{R}_2$, the exact value o x_1, but not x_2 is provided before the decisions are made. In this case x_2 is known to be not exceptional and only the range of x_2 can be determined. We maximize, with respect to a_1 and a_2, the payoff function

$$(5.6)\quad E(u \mid x_1 \in R_1,\ x_2 \in \tilde{R}_2) = -a_1^2 - a_2^2 - 2qa_1a_2 + 2a_1x_1 + 2a_2E(x_2 \in \tilde{R}_2).$$

From the assumption of symmetry, we have that $E(x_i \in R_i) = 0$. Equating the partial derivatives of (5.6) with respect to a_1 and a_2 to zero, we get

$$(5.7)\quad \begin{aligned} a_1 + qa_2 &= x_1 \\ a_2 + qa_1 &= 0. \end{aligned}$$

The solution (5.7) gives the optimal decision rules

$$(5.8)\quad \begin{aligned} \hat{\delta}_1(x_1,\ \tilde{R}_2) &= \frac{x_1}{1-q^2} \\ \hat{\delta}_2(x_1,\ \tilde{R}_2) &= \frac{-qx_1}{1-q^2}. \end{aligned}$$

The maximum profit yielded by using the optimal decision rules is calculated as

$$(5.9)\quad E(u \mid x_1,\ \tilde{R}_2) = \frac{x_1^2}{1-q^2}.$$

For the third case in which $x_1 \in \tilde{R}_1$ and $x_2 \in R_2$, x_2 is correctly informed, but x_1 is known to be ordinary. The situation is just opposite to the second case. Therefore, we find $\hat{\delta}_1$ and $\hat{\delta}_2$ simply by interchanging the subscripts 1 and 2 in (5.8),

$$\hat{\delta}_1(x_2, \tilde{R}_1) = \frac{-qx_2}{1-q^2}$$
(5.10)
$$\hat{\delta}_2(x_2, \tilde{R}_1) = \frac{x_2}{1-q^2}$$

and the maximum profit by a similar interchange of subscripts in (5.9)

(5.11) $$E(u|x_2, R_1) = \frac{x_2^2}{1-q^2}.$$

Finally, when both variables are known to be not excetional, the actions become "routine." The expected profit is equal to

(5.12) $$E(u|\tilde{R}_1, \tilde{R}_2) = -a_1^2 - a_2^2 - 2qa_1a_2 + 2a_1E(x_1 \in \tilde{R}_1) + 2a_2E(x_2 \in \tilde{R}_2)$$

which is to be maximized with respect to a_1 and a_2. The optimal actions in this case are constant,

(5.13) $$\hat{a}_1 = \hat{a}_2 = 0.$$

Hence, the maximum profit is zero,

(5.14) $$E(u|\tilde{R}_1, \tilde{R}_2) = 0.$$

Let p_i be the frequency with which x_i turns out to be exceptional. Combining the maximum profit obtained in each of the four cases, we derive the value of the exception information structure as follows:

(5.15) $$\begin{aligned} V(\eta_e) &= p_1p_2\frac{s_{R_1}^2 + s_{R_2}^2}{1-q^2} + p_1(1-p_2)\frac{s_{R_1}^2}{1-q^2} \\ &\quad + (1-p_1)p_2\frac{s_{R_2}^2}{1-q^2} + (1-p_1)(1-p_2)(0) \\ &= \frac{1}{1-q^2}(p_1s_{R_1}^2 + p_2s_{R_2}^2), \end{aligned}$$

where $s_{R_i}^2$ is the conditional variance of x_i, given that it is exceptional.

It appears from (5.15) that the larger the conitional variances $s_{R_i}^2$, the larger the gross value of the exception information structure. Given the probabilities p_i, the optimal choice of R_i is that which maximizes $s_{R_i}^2$. Under the assumption that x_i are statistically independent, we can apply the same rule derived in the previous section.

To set the optimal exceptional ranges of x_i, we take R_i to be the complement of an interval symmetric around the mean of x_i.

VI. Concluding Remarks

This paper has been based on the premise that the major problem in the design of information systems is the filtration of relevant information from irrelevant information and elimination of the latter.

In the management literature there have been several articles and fragmentary statements dealing with this subject. None of them, however, contain more than simple rules of thumb. In addition, the literature on the design of information systems seldom considers explicitly the function of filtration or extraction. This lack of the literature may be attributed to the fact that the exception principle has not attempted to define which activities and issues are routine and which are exceptional. As a result, it has been stated as a matter of degree depending on circumstances and subjective evaluation.

This paper has proposed conceptual models of an exception reporting system which incorporates some filtration mechanism. Since information economics is concerned with the trade-off between the cost of information and the value derived from that information, it can provide some criteria to decide which information sets to be eliminated. In short, the paper has sought a useful method for designing information systems which operationalizes the exception principle in management and integrates the principle with information economics precepts.

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