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An L^1-theory for stochastic conservation laws with boundary conditions

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1 Introduction

In this paper we study the first order stochastic conservation law of the following type

$$du + \mathrm{div}(A(u))dt = \Phi dW(t) \quad \text{in } \Omega \times Q, \tag{1}$$

with the initial condition

$$u(0,\cdot) = u_0(\cdot) \quad \text{in } \Omega \times D, \tag{2}$$

and the formal boundary condition

$$\text{``}u = u_b\text{''} \quad \text{on } \Omega \times \Sigma. \tag{3}$$

Here $D \subset \mathbb{R}^d$ is a bounded domain with a Lipschitz boundary ∂D, $T > 0$, $Q = (0,T) \times D$, $\Sigma = (0,T) \times \partial D$ and W is a cylindrical Wiener process defined on a stochastic basis $(\Omega, \mathscr{F}, (\mathscr{F}_t), P)$. More precisely, $(\mathscr{F}_t)$ is a complete right-continuous filtration and $W(t) = \sum_{k=1}^{\infty} \beta_k(t) e_k$ with $(\beta_k)_{k \geq 1}$ being mutually independent real-valued standard Wiener processes relative to $(\mathscr{F}_t)$ and $(e_k)_{k \geq 1}$ a complete orthonormal system in a sepa-rable Hilbert space H (cf. [4] for example). Our purpose of this paper is to present the definitions of an L^1-kinetic solution to the initial-boundary value problem (1)–(3) and to prove a result of uniqueness and existence of such a solution.

In the case of $\Phi = 0$, Eq.(1) becomes a deterministic scalar conservation law. In this case, a well known difficulty for the boundary condition (3) is that if (3) were assumed in the classical sense the problem (1)–(3) would be overdetermined. In the BV setting Bardos, Le Roux and Nédélec [2] first gave an interpretation of the boundary condition (3) as an "entropy" inequality on Σ. This condition is known as the BLN condition. However the BLN condition makes sense only if there exists a trace of solutions on ∂D. Otto [13] extended it to the L^∞ setting by introducing the notion of boundary entropyflux pairs. Imbert and Vovelle [7] gave a kinetic formulation of weak entropy solutions of the initial-boundary value problem and proved the equivalence between such kinetic solutions and weak entropy solutions. Concerning deterministic degenerate parabolic equations, see [9] and [11]. Bénilan, Carrillo and Wittbold [3] developed the L^1-theory for the Cauchy problem in the deterministic framework. In [3], a notion of “renormalized entropy solution” is introduced.

To add a stochastic forcing $\Phi(u)dW(t)$ is natural for applications, which appears in a wide variety of fields as physics, engineering and others. There are a few paper concerning the Dirichlet boundary value problem for stochastic conservation laws. See Kim [8], Vallet and Wittbold [14] and the references therein. Using the notion of entropy solutions, Bauzet, Vallet and Wittbold [1] studied the Dirichlet problem in the case of multiplicative noise under the restricted assumption that the flux function A is globally Lipschitz. On the other hand, using the notion of kinetic solutions, Kobayasi and No-boriguchi [10] extended the result of Debussche and Vovelle [5] to the multidimensional Dirichlet problem with multiplicative noise without the assumption that A is global Lipschitz. Moreover, Noboriguchi [12] proved equivalence between such kinetic solutions and weak entropy solutions. These results treat the equation (1) in the framework of L^p-spaces for p larger than the degree of polynomial growth of A. However, to discuss the invariant measure for the stochastic conservation law (1) we need to develop a theory of well-posedness for small $p \geq 1$. This is the motivation for considering an L^1-theory for the problem (1)–(3). In fact, our main result is a counterpart of the result of [6, Appendix] in the case of initial-boundary value problems. In [6] the authors treat a periodic stochastic conservation laws.

We consider here the case of an additive noise and assume that the derivative of the flux function is bounded. Although the basic idea of the proof is analogous to that of [6] and [10], our purpose of this paper is to give the complete proof.

We now give the precise assumptions under which the problem (1)–(3) is considered:

(H_1) The flux function $A: \mathbb{R} \to \mathbb{R}^d$ is of class C^2 and its derivative $a = A'$ is bounded.

(H_2) The map $\Phi: H \to L^2(D)$ is defined by $\Phi e_k = g_k$, where $g_k \in C(D)$ satisfies the following conditions:

$$G^2(x) = \sum_{k=1}^{\infty} |g_k(x)|^2 \leq L, \tag{4}$$

$$\sum_{k=1}^{\infty} |g_k(x) - g_k(y)|^2 \leq L\,|x-y|^{22} \tag{5}$$

for every x , $y \in D$. Here, L is a constant.

(H_3) $u_0 \in L^2(\Omega; L^1(D))$ and is $\mathscr{F}_0$-measurable. $u_b \in L^1(\Sigma)$.

(H_3') $u_0 \in L^\infty(\Omega; L^\infty(D))$ and is $\mathscr{F}_0$-measurable. $u_b \in L^\infty(\Sigma)$.

This paper is organized as follows. In Section 2 we introduce the notion of kinetic solutions to (1)–(3) and state our result. In Section 3 we give a proof of it.

2 Statement of the result

To begin with, we give the definition of kinetic solution. The motivation for this notion is given by the non-existence of a strong solution and by the non-uniqueness of weak solutions.

Definition 2.1 (Kinetic measure). A map m from Ω to $\mathcal{M}^+([0,T) \times D \times \mathbb{R})$ is said to be a kinetic measure if

(i) m is weakly measurable,

(ii) m vanishes for large ξ in the sense that

$$\lim_{N\to\infty} \mathbb{E}m(A_N) = 0, \tag{6}$$

where $A_N = [0,T) \times D \times \{\xi \in \mathbb{R}; N \leq |\xi| \leq N+1\}$

(iii) for all $\phi \in C_c^\infty(D \times \mathbb{R})$, the process

$$t \mapsto \int_{[0,t]\times D\times\mathbb{R}} \phi(x,\xi)\, dm(s,x,\xi) \tag{7}$$

is predictable,

where $\mathcal{M}^+([0,T)\times D\times\mathbb{R})$ is the set of non-negative Radon measures over $[0,T)\times D\times\mathbb{R}$.

Definition 2.2 (Kinetic solution). Let u_0 and u_b satisfy (H_3). A measurable function $u : \Omega\times Q \to \mathbb{R}$ is said to be a kinetic solution of (1)–(3) if the following conditions (i)–(iii) hold:

(i) $\{u(t)\}$ is predictable,

(ii) there exists a constant $C \geq 0$ such that for a.e. $t \in [0,T]$,

$$\|u(t)\|_{L^1(\Omega\times D)} \leq C, \tag{8}$$

(iii) there exist a kinetic measure m and nonnegative functions $\bar{m}^\pm \in L^\infty_{loc}(\Omega\times\Sigma\times\mathbb{R})$ such that $\{\bar{m}^\pm(t)\}$ are predictable, lim are predictable, $\lim_{\xi\to\infty} \bar{m}^+(\xi) = \lim_{\xi\to-\infty} \bar{m}^-(\xi) = 0$ and $f_+ := \mathbf{1}_{u>\xi}$, $f_- := f_+ - 1 = -\mathbf{1}_{u\leq\xi}$ satisfy: for all $\varphi \in C_c^\infty([0,T) \times \overline{D} \times \mathbb{R})$,

$$\int_Q \int_{\mathbb{R}} f_\pm(\partial_t + a(\xi)\cdot\nabla)\varphi \; d\xi dx dt$$

$$+ \int_D \int_{\mathbb{R}} f_\pm^0 \varphi(0) \; d\xi dx + M \int_\Sigma \int_{\mathbb{R}} f_\pm^b \varphi \; d\xi d\sigma dt$$

$$= -\sum_{k=1}^{\infty}\int_0^T\int_D g_k(x,u(t,x))\,\varphi(t,x,u(t,x))\ dxd\beta_k(t)$$

$$-\frac{1}{2}\int_Q G^2(x,u(t,x))\partial_\xi\varphi(x,t,u(t,x))\ dxdt$$

$$+\int_{[0,T)\times D\times\mathbb{R}}\partial_\xi\varphi\ dm(t,x,\xi)+\int_\Sigma\int_\mathbb{R}\partial_\xi\varphi\,\bar{m}^{\pm}\ d\xi d\sigma dt \quad \text{a.s.,} \tag{9}$$

where $M=\max_{\xi\in\mathbb{R}}|a(\xi)|$. In (9), $f_+^0=\mathbf{1}_{u_0>\xi}$, $f_+^b=\mathbf{1}_{u_b>\xi}$, $f_-^0=f_+^0-1$, and $f_-^b=f_+^b-1$.

Our purpose of the paper is to prove the following

Theorem 2.3 *Assume (H_1)–(H_3). Then, there is at most one kinetic solution to (1)–(3) in the sense of Definition 2.2 with data (u_0, u_b). Moreover, given data (u_0^1, u_b^1) and (u_0^2, u_b^2), we have the following L^1-contraction property:*

$$\mathbb{E}\left\|u^1(t)-u^2(t)\right\|_{L^1(D)}\le\mathbb{E}\left\|u_0^1-u_0^2\right\|_{L^1(D)}+M\mathbb{E}\int_0^t\left\|u_b^1(s)-u_b^2(s)\right\|_{L^1(\partial D)}ds, \tag{10}$$

As for the uniqueness of kinetic solutions and the $L1$-contraction property (10), we can proceed as in the manner of the proof of [10, Theorem 3.1 and Corollary 3.2]. So, we focus on the proof of the existence in next section.

Remark 2.4 By [10, Theorem 3.1, Corollary 3.2 and Theorem 4.1] the above theorem has been proved under the strong assumption (H_3') instead of (H_3).

3 Existence

Let $u_0\in L^1(D)$ and $u_b\in L^1(\Sigma)$. We approximate u_0, u_b by $u_0^n:=T_n(u_0)$, $u_b^n:=T_n(u_b)$ where the truncation operator T_n is defined $T_n(v)=\max(-n,\min(v,n))$. Then by Remark 2.4 this defines a sequence of kinetic solutions (u_n) with data (u_0^n, u_b^n). We will show that the limit of this sequence is a kinetic solution with data (u_0, u_b). With the purpose to prepare the proof of the existence of kinetic solutions, we begin with the following proposition.

Proposition 3.1 (Decay of the kinetic measure). *Let u_0 and ub satisfy* (H_3). *Let measurable functions un $u_n:\Omega\times Q\to\mathbb{R}$ be kinetic solutions to (1)–(3) with data $(u_0^n, u_b^n)=(T_n(u_0),T_n(u_b))$ and m^n, $\bar{m}^{\pm,n}$ be kinetic measures and non-negative functions associated with u_n, respectively. Then, there exists a decreasing function $\varepsilon:\mathbb{R}_+\to\mathbb{R}_+$ with $\lim_{k\to\infty}\varepsilon(k)=0$ depending on the functions*

$$k \mapsto \left\|(u_0 \mp k)^{\pm}\right\|_{L^2(\Omega;L^1(D))}, \quad k \mapsto \left\|(u_b \mp k)^{\pm}\right\|_{L^1(\Sigma)}$$

only such that, for all $k \geq 1$ *,*

$$\mathbb{E}\left(\text{ess sup}_{t\in[0,T]} \left\|(u_n(t) \mp k)^{\pm}\right\|_{L^1(D)}\right) + \mathbb{E}m^n(A_k) + \mathbb{E}\int_{B_k^{\pm}} \bar{m}^{\pm,n} \, dtd\sigma d\xi \leq \varepsilon(k), \tag{11}$$

where $A_k^+ = [0,T) \times D \times \{\xi \in \mathbb{R}; k \leq \xi \leq k+1\}$, $A_k^- = [0,T) \times D \times \{\xi \in \mathbb{R}; -\xi \in A_k^+\}$, $A_k = A_k^+ \cup A_k^-$. $B_k^+ = [0,T) \times \partial D \times \{\xi \in \mathbb{R}; k \leq \xi \leq k+1\}$, $B_k = [0,T) \times \partial D \times \{\xi \in \mathbb{R}; -\xi \in B_k^+\}$, $B_k = B_k^+ \cup B_k^-$.

Proof. Step 1. For $k \geq 0$, set

$$\theta_k(\xi) = \mathbf{1}_{k\leq\xi\leq k+1}, \quad \Theta_k(\xi) = \int_0^{\xi}\int_0^{\zeta} \theta_k(r) \, drd\zeta.$$

Let $\gamma \in C_c^1([0,T))$ be non-negative and satisfy $\gamma(0) = 1, \gamma' \leq 0$. After a preliminary step of approximation, we take $\varphi(t,x,\xi) = \gamma(t)\Theta_k'(\xi)$ in (9) to obtain

$$\begin{aligned}
&\int_Q \Theta_k(u^n)\gamma'(t) \, dxdt + \int_D \Theta_k(u_0^n) \, dx + M\int_{\Sigma} \Theta_k(u_b^n)\gamma(t) \, dtd\sigma \\
&= -\sum_{j=1}^{\infty}\int_0^T\int_D g_j(x)\Theta_k'(u^n)\gamma(t) \, dxd\beta_j(t) - \frac{1}{2}\int_Q G^2(x)\theta_k(u^n)\gamma(t) \, dxdt \\
&+ \int_{A_k^+} \gamma(t) \, dm(t,x,\xi) + \int_{B_k^+} \gamma(t)\bar{m}^+(t,x,\xi) \, dtd\sigma d\xi.
\end{aligned} \tag{12}$$

Taking then expectation, we deduce

$$\begin{aligned}
&\mathbb{E}\int_Q \Theta_k(u^n)|\gamma'(t)| \, dxdt + \mathbb{E}\int_{A_k^+} \gamma(t) \, dm + \mathbb{E}\int_{B_k^+} \gamma(t)\bar{m}^+ \, dtd\sigma d\xi \\
&= \frac{1}{2}\mathbb{E}\int_Q G^2(x)\theta_k(u^n)\gamma(t) \, dxdt + \mathbb{E}\int_D \Theta_k(u_0^n) \, dx + M\int_{\Sigma} \Theta_k(u_b^n)\gamma(t) \, dtd\sigma
\end{aligned} \tag{13}$$

Note that

$$(\xi - (k+1))^+ \leq \Theta_k(\xi) \leq (\xi - k)^+, \quad k \geq 0, \ \xi \in \mathbb{R}, \tag{14}$$

$$\theta_{k_2}(\xi) \leq \mathbf{1}_{k_2\leq\xi} \leq \frac{(\xi - k_1)^+}{k_2 - k_1}, \quad 0 \leq k_1 \leq k_2, \tag{15}$$

$$\mathbb{E}\int_D (u_0^n - k)^+ \, dx \leq \mathbb{E}\int_D (u_0 - k)^+ \, dx, \quad k \geq 0, \ n \in \mathbb{N}, \tag{16}$$

$$\int_{\Sigma} (u_b^n - k)^+ \, d\sigma dt \leq \int_{\Sigma} (u_b - k)^+ \, d\sigma dt, \quad k \geq 0, \ n \in \mathbb{N}. \tag{17}$$

In particular, using $m \geq 0$, $\bar{m}^+ \geq 0$, (4), (14)–(17) and taking $k_1 = k$, $k_2 = k + \theta - 1$ where $\theta > 1$ in (13) give

$$\mathbb{E}\int_Q (u^n - (k+\theta))^+ |\gamma'(t)| \ dxdt$$
$$\leq \frac{L}{2(\theta-1)}\mathbb{E}\int_Q (u^n - k)^+ \gamma(t) \ dxdt + \mathbb{E}\int_D (u_0 - k)^+ \ dx + \int_\Sigma (u_b - k)^+ \ d\sigma dt. \tag{18}$$

Choose θ large enough so that $\frac{L}{2(\theta-1)} = \alpha < 1$. Denote by ψ_m, β_m

$$\psi_m(t) = \mathbb{E}\int_D (u^n - m\theta)^+ \ dx,$$

$$\beta_m = \mathbb{E}\int_D (u_0 - m\theta)^+ \ dx + \int_\Sigma (u_b - m\theta)^+ \ d\sigma dt.$$

and let $I_m \subset [0,T)$ be the set of Lebesgue points of ψ_m. Then $I = \cap_{n\in\mathbb{N}} I_m$ is of full measure. For $t \in I$, we take $k = m\theta$, $\gamma(s) = \min(1, \frac{1}{\varepsilon}(s-(t+\varepsilon))^-)$ for $\varepsilon < T - t$ in (18) and let $\varepsilon \downarrow 0$. This yields the following inequality:

$$\psi_{m+1}(t) \leq \alpha \int_0^t \psi_m(s) \ ds + \beta_m \tag{19}$$

Since $\lim_{m\to+\infty} \beta_m = 0 = 0$, by standard argument, we deduce

$$\text{ess sup}_{t\in[0,T]} \psi_m(t) \leq \delta_m, \tag{20}$$

where $\{\delta_m\}$ is a sequence with $\lim_{m\to\infty} \delta_m = 0$. Set $\varepsilon^+(\xi) = \delta([\frac{k}{\theta}])$, hand side below is a decreasing function of k,

$$\text{ess sup}_{t\in[0,T]} \mathbb{E}\int_D (u^n - k)^+ \ dx \leq \varepsilon^+(k), \tag{21}$$

and $\varepsilon^+ : \mathbb{R}_+ \to \mathbb{R}_+$ is a function depending on the functions $k \mapsto \|(u_0 - k)^+\|_{L^2(\Omega;L^1(D))}$, $k \mapsto \|(u_b - k)^+\|_{L^1(\Sigma)}$ only such that $\lim_{k\to+\infty} \varepsilon^+(k) = 0$

Step 2. By (13) we have the estimates on $\mathbb{E}m^n(A_k^+)$ and $\mathbb{E}\int_{B_k^+} \bar{m}^{\pm,+}$ on the left hand of (11). Therefore, to conclude, we need to show an estimate on $\mathbb{E}(\text{ess sup}_t \int_D (u^n - k)^+)$. This is the classical argument for semi-martingales that we will use. We will merely focus on the martingale term in (12). We first let γ approach $\mathbf{1}_{(0,t)}$ as in Step 1. Note that

$$|\Theta_k'(\xi)|^2 \leq \mathbf{1}_{k\leq\xi} \leq (\xi-(k-1))^+. \tag{22}$$

Then, using the Burkholder-Davis-Gundy inequality, the Cauchy-Schwartz inequality, (4), (22) and the Jensen inequality we have,

$$\begin{aligned}
&\mathbb{E}\left(\text{ess sup}_{t\in[0,T]}\left|\sum_{j=1}^{\infty}\int_0^t\int_D g_j(x)\Theta_k'(u^n)\ dxd\beta_j(s)\right|\right)\\
&\leq C\mathbb{E}\left(\int_0^T\sum_{j=1}^{\infty}\left(\int_D |g_j(x)\Theta_k'(u)|\ dx\right)^2\right)^{1/2}\\
&\leq C\mathbb{E}\left(\int_0^T\int_D|\Theta_k'(u)|^2\ dx\right)^{1/2}\\
&\leq C\left(\varepsilon^+(k+1)\right)^{1/2}.
\end{aligned}$$

This concludes the proof of the Proposition. □

Lemma 3.2. *Let (X,μ) be a finite measure space. Let u, $u_n\in L^1(X)$. Assume that $u_n\to u$ in $L^1(X)$-strong. Then, there exists a subsequence still denoted by (u_n) such that $\mathbf{1}_{u_n>\xi}\rightharpoonup\mathbf{1}_{u>\xi}$ in $L^\infty(X\times\mathbb{R})$-weak*.*

Proof. There exists a subsequence still denoted by (u_n) such that $u_n(z)\to u(z)$ a.e. $z\in X$. We infer that

$$\mathbf{1}_{u_n(z)>\xi}\to\mathbf{1}_{u(z)>\xi},\qquad \text{a.e. } z\in X$$

for fixed ξ with $\mu(u=\xi)=0$. However, the set $\{\xi\in\mathbb{R};\mu(u=\xi)>0\}$ is at most countable since we deal with finite measure μ. Therefore, we obtain that for a.e. $\xi\in\mathbb{R}$,

$$\mathbf{1}_{u_n(z)>\xi}\to\mathbf{1}_{u(z)>\xi},\qquad \text{a.e. } z\in X.$$

Take $\psi_1\in L^1(X)$ and $\psi_2\in L^1(\mathbb{R})$, and set $\varphi=\psi_1\psi_2$. We obtain by the dominated convergence theorem

$$\int_{X\times\mathbb{R}}\mathbf{1}_{u_n>\xi}\ \varphi\ d\mu d\xi\to\int_{X\times\mathbb{R}}\mathbf{1}_{u>\xi}\ \varphi\ d\mu d\xi.$$

Since tensor functions are dense in $L^1(X\times\mathbb{R})$ we conclude the weak* convergence $\mathbf{1}_{u_n>\xi}\rightharpoonup\mathbf{1}_{u>\xi}$ in $L^\infty(X\times\mathbb{R})$. □

Theorem 3.3 (Existence). *Assume* (H_1)–(H_3). *Then, there exists a kinetic solution to* (1)–(3) *with data* (u_0, u_b).

Proof. By the L^1-contraction property (10), the sequence of solutions (u_n) is a Cauchy sequence, hence it converges to a $u \in L^1(\Omega \times (0,T) \times D)$. This u is predictable. Let now m^n be the kinetic measure associated to u^n. By (12), we have

$$\sup_{n \in \mathbb{N}} \mathbb{E}\,|m^n(K_r)|^2 \le C_r, \tag{23}$$

for $r \in \mathbb{N}_0$, where $K_r := [0,T) \times D \times [-r,r]$. Let $\mathcal{M}_r$ denote the space of bounded measures over K_r. It is the topological dual of $C_b(K_r)$. Since $\mathcal{M}_r$ is separable, the space $L^2(\Omega; \mathcal{M}_r)$ is the topological dual space of $L^2(\Omega; C_b(K_r))$. The estimate (23) gives a uniform bound on (m^n) in $L^2(\Omega; \mathcal{M}_r)$: there exists $m_r \in L^2(\Omega; \mathcal{M}_r)$ such that up to subsequence, $m^n \rightharpoonup m_r$ in $L^2(\Omega; \mathcal{M}_r)$-weak*. By a diagonal process, we obtain, for $r \in \mathbb{N}_0$, $m_r = m_{r+1}$ in $L^2(\Omega; \mathcal{M}_r)$ and the convergence in all the spaces $L^2(\Omega; \mathcal{M}_r)$-weak* of a single subsequence still denoted (m^n). Let us then set $m := m_r$ on K_r, a.s. The conditions (i) and (iii) in Definition 2.1 are stable by weak convergence, hence satisfied by m. We deduce that condition (ii) is satisfied thanks to the uniform estimate of Proposition 3.1. This shows that u is a solution to (1)–(3) with data (u_0, u_b). □

References

C. Bauzet, G. Vallet, P. Wittbolt, The Dirichlet problem for a conservation law with a multiplicative stochastic perturbation, J. Funct. Anal. 266 (2014) 2503–2545.

C. Bardos, A. Y. Le Roux, J.-C. Nédélec, First order quasilinear equations with boundary condition, Comm. Partial Differential Equations 4 (1979) 1017–1034.

P. Bénilan, J. Carrillo, and P. Wittobold, Renormalized entropy solutions of scalar conservation laws, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 29 (2000), no. 2, 313–327.

G. Da Prato, J. Zabczyk, Stochastic Equations in Infinite Dimensions, Encyclopedia Math. Appl., vol. 44, Cambridge University Press, Cambridge, 1992.

A. Debussche, J. Vovelle, Scalar conservation laws with stochastic forcing, J. Funct. Anal. 259 (4) (2010) 1014–1042.

A. Debussche, J. Vovelle, Imvariant measure of scalar first-order conservation laws with stochastic forcing, http://math.univ-lyon1.fr/vovelle/InvariantMeasure- DebusscheVovelle.pdf.

C. Imbert, J. Vovelle, A kinetic formulation for multidimensional scalar conservation laws with boundary conditions and applications, SIAM J. Math. Anal. 36 (2004) 214–232.

J. U. Kim, On a stochastic scalar conservation law, Indiana Univ. Math. J. 52 (1) (2003) 227–256.

K. Kobayasi, A kinetic approach to comparison properties for degenerate parabolic- hyperbolic equations with boundary conditions, J. Differential Equations 230 (2006) 682–701.

K. Kobayasi, D. Noboriguchi, A stochastic conservation law with nonhomogeneous Dirichlet boundary conditions, preprint.

A. Michel, J. Vovelle, Entropy formulation for parabolic degenerate equations with general Dirichlet boundary conditions and application to the convergence of FV methods, SIAM J. Numer. Anal. 41 (2003) 2262–2293.

D. Noboriguchi, The Equivalence Theorem of Kinetic solutions and Entropy solutions for Stochastic Scalar Conservation Laws, preprint.

F. Otto, Initial-boundary value problem for a scalar conservation law, C. R. Acad. Sci. Paris Sér. I Math., 322 (1996), 729–734.

G. Vallet, P. Wittbold, On a stochastic first-order hyperbolic equation in a bounded domain, Infin. Dimens. Anal. Quantum Probab. 12 (4) (2009) 1–39.

ABSTRACT

An L^1-theory for stochastic conservation laws with boundary conditions

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In this paper, we develop the $L1$-theory for stochastic scalar conservation laws with boundary conditions. We adopt the notion of $L1$ -kinetic solutions which sup- plies a good technical framework to prove the $L1$-contraction property. In such an $L1$-kinetic solution, we obtain a result of uniqueness and existence.

Key words: Conservation laws; Initial-boundary value problem; Kinetic solution MSC. 35L04, 60H15