Essays on Horizontal Mergers in Private and Mixed Oligopoly: Strategic Managerial Delegation and Cooperative Game Approaches

（私的寡占および混合寡占市場における水平的企業間合併の研究 – 経営委任契約と協力ゲームの方法論の観点から –）

2010年7月

早稲田大学大学院 経済学研究科
理論経済学・経済史専攻 理論経済学専修

Yasuhiko Nakamura

Preface

In this thesis, we present the analyses of theoretical horizontal mergers in both the contexts of private oligopoly and mixed oligopoly. In chapter 1, we review the statements and discussions given in the previous literature, which are closely related to the result obtained in this thesis, and subsequently, from chapters 2 to 5, we provide our original results on theoretical horizontal mergers in both the private oligopoly and the mixed oligopoly. In Conclusion of the final chapter, we discuss the contributions of this thesis, and argue several future researches and possible extensions, which are left for us. The concrete contents of this thesis are given as follows:

In chapter 1, we first discuss the result obtained in the previous literature on the modern game theoretical horizontal mergers in the context of the private oligopoly where only profit-maximizing private firms exist in the market. More concretely, we argue that they are classifies with the following two categories: (1) *exogenous merger analysis*; (2) *endogenous merger analysis*. Furthermore, the respective results obtained in the works belonging to the two categories are treated in detail. Second, we review the discussion provided in the literatures on the influence of each firm's managerial delegation and the bargaining over its content between her/his owner and manager on her/his merger incentive. Finally, in the context of the mixed oligopoly where a welfare-maximizing public firm and profit-maximizing private firms coexist and compete, we state the results obtained in the literature on the horizontal mergers, and subsequently, we argue the relationship and different points between the results obtained in this thesis and them.

In chapter 2, in the context of the private oligopoly, we examines how managerial delegation contracts within each firm affect the correspondence between the equilibrium ownership structure and the most preferred ownership structure with respect to social welfare. We consider the problem of the disclosure of managerial delegation contracts prescribed in modern corporate

governance codes by studying the bargaining over the relative weightage of each firm's sales in a sales delegation contract between an owner and a manager with respect to a model of endogenous merger formation in a three-firm asymmetric Cournot industry. We show that within a large range of the relative bargaining power of each firm's manager, there exists a degree of cost asymmetry among firms in the industry, to have that the equilibrium ownership structure coincides with the most preferred ownership structure with respect to social welfare. Thus, the bargaining between an owner and a manager may reduce the requirement of an antitrust policy.

In chapter 3, we investigate the achievablity of the productivity-improving merger activities between a public firm and a private firm in mixed oligopoly. We assume that the merged firm has two plants (formerly, firms). We show that both the owners of a public firm and a private firm want to merge by coordinating their shareholding ratios in the merged firm, whenever the number of private firms is larger than a critical value, while the public firm does not want to merge without the effect of improving the productivity in the merged firm. In chapter 3, we further discuss the case wherein the public firm is less efficient than the private firms.

Furhermore, in chapter 4, we present the theoretical analysis of each firm's merger activity in a mixed oligopoly with respect to the stability of market structures. More precisely, by defining the *core of market structures* which describes market structures occurring after each firm's free merger activity on the basis of the core solution concept in the cooperative game, we consider the endogenous merger formation in the case wherein her/his technology is represented as the quadratic cost function in terms of her/his quantity. We analyze productivity-improving mergers in the mixed triopoly composed of one welfare-maximizing public firm and two profit-maximizing private firms and explore stable market structures. We find the only stable market structure contains a merged public-private firm and one private firm with about 57% of shares owned by the public firm. Moreover, we derive the core of market structures after each firm's merger activity in the case wherein the owner offers her/his managers the incentive contract that deviates from profit maximization.

In chapter 5, we examine endogenous merger formations in a mixed oligopoly. Applying the core as a solution concept, we analyze which market structure(s) remain(s) stable when three firms — two symmetric private firms and one inefficient public firm — are allowed to merge with each other in a mixed Cournot industry. We show that according to the value of the marginal cost of the public firm, there always exists a pair of share ratios of the owners of both the (pre-merged) public firm and the (pre-merged) private firm to have that the market structure with the merger between the public firm and one private firm belongs to the core. When the initial market

structure is a mixed triopoly, it can only be blocked when one public firm and one private firm merge. Furthermore, we conduct a similar analysis in a general mixed oligopoly with one public firm and n private firms.

In Conclusion of the final chapter of this thesis, in detail, we rearrange the contribution of the result obtained in this thesis, and we present several topics which should tackle as our future research.

Acknowledgements

More than six years have passed since I entered Graduate School of Economics, Waseda University. Although I must thank all the people who I met in my graduate student life, the names of all the people who I would thank are too many, and thus I can not raise the names of all of them here. However, I will not forget all of my memories that I have spent with them. It is a pleasant duty to express my gratitude for a couple of them in the rest of the spaces.

First of all, I would like to thank Professor Ryo Nagata. Throughout the life of my economic research, I have been learning the rigorous theory of economics as one of the members of his laboratory since joining his seminar in my undergraduate student times. Thus, I have done all of my researches in Economics under his kind guidance for more than eight years. I would like to dedicate my gratitude to his helpful and suggestive guidance with all my heart. Without his guidance, I would not have finished this thesis.

I am also grateful to Professor Koichi Suga. He was my sub-advisor in my doctoral course times. He read all of my papers in detail and gave me his helpful comments, and consequently, their qualities were substantially improved. Moreover, the discussion with him provided to me the way to survive the hard way of studying the theory of Economics. I also thank him for his appropriate guidance.

I also would like to express my gratitude to Professor Kazuhiko Kato for joining the thesis committee, and for giving me many detailed comments and suggestions.

Yoshimoto Honda, Norihisa Sato, and Koji Shirai, who are/were members with me in the laboratory of Professor Ryo Nagata, have given me many valuable and significant viewpoints throughout several discussions. I deeply express my gratitude for them and spending all of times with them.

I further would like to thank Yoshio Kamijo and Yoshihiro Tomaru, who are now a assistant professor at Waseda University and an assistant professor at Toyo University, respectively. They have been (probably will be) my teachers regarding of my researches on Industrial Organization (IO). From them, I learned how to write academic papers in theoretical IO, and how to describe

economic phenomena as theoretical models. Although their comments were frequently strict, they were actually indicative and suggestive. I express my special thanks for them.

To Tomohiro Inoue and Kohei Kamaga, I must also dedicate my extra gratitude. Tomohiro is my senior. With him, I first wrote an academic paper in English, and submitted it to an international refereed journal, Economics Bulletin (EB). I clearly remember my pleasure when the paper won the publication in the EB. Kohei is my colleague and friend. The discussion with him has been very helpful and suggestive.

Many people gave me many helpful comments and suggestions for my papers which compose chapters of this thesis, and this thesis itself. I wish to thank the following my teachers and colleagues: Tsuyoshi Adachi, Yukihiko Funaki, Kazuharu Kiyono, Takumi Kongo, Masuyuki Nishijima, Toshihiro Matsumura, Masayuki Saito, Daisuke Shimizu, and the participants at the 2007 Spring Meeting of JEA at Osaka Gakuin University, and the 2008 GLOPE-TCER Joint Junior Workshop on Political Economy at Waseda University.

The three chapters of this thesis are composed of the collaborative works with my co-authors, Tomohiro Inoue, Kohei Kamaga, and Yoshio Kamijo. I would like to thank all of them who are willing to admit to put them in this thesis. I will never forget the pleasures and pains which were shared between them and me when I did such joint researches with them. In addition, I would also thank Wiley-Blackwell, International Journal of Business and Economics office (IJBE office) at Feng Chia University, and Springer for admitting that I use my three papers as a part of this thesis, “Bargaining over managerial delegation contracts and merger incentives with asymmetric costs”, “The core and productivity-improving mergers in mixed oligopoly”, and “Stable market structures from merger activities in mixed oligopoly with asymmetric costs”, which were/will-be published in the Manchester School, International Journal of Business and Economics, and Journal of Economics, respectively.

Finally, I would like to thank my family, Yoshitsugu, Mutsumi, Shiori, and Nobuko for their encouragements and supports for a long time.

Yasuhiko Nakamura

June 2010

Tokyo

List of Figures

3.1 Illustration of Proposition 3.1 41
3.2 Illustration of Proposition 3.2 43
3.3 Illustration of Proposition 3.3 44

5.1 Relationship among the payoffs of the three owners in regimes (b), (c), and (d) . 84
5.2 The domain of s and t such that the state where the three firms merge is in the core 85
5.3 Relationship among the payoffs of the three owners in the regimes (c) and (d) . . 88
5.4 Relationship among the payoffs of the three owners in regimes (a), (c), and (d) . 88

Contents

1 Introduction and overview **1**

1.1 The general discussion on horizontal mergers in oligopoly – exogenous merger vs. endogenous merger – 1

1.2 Managerial delegation and merger incentives 3

1.3 Bargaining over managerial contracts and merger incentives 4

1.4 Horizontal mergers in mixed oligopoly 6

1.5 Comparison with vertical mergers 10

2 Bargaining over managerial delegation contracts and merger incentives with asymmetric costs **14**

2.1 Introduction 14

2.2 The model 17

2.3 Merger as a coalition formation and the core 20

2.4 Equilibrium ownership structure 21

2.5 Socially preferred ownership structure *versus* equilibrium ownership structure . . 25

2.6 Concluding Remarks 26

Appendix 28

3 Mixed oligopoly and productivity-improving mergers **36**

3.1 Introduction 36

3.2 The model 37

3.3 The decision by firms to merge 39

3.4 Concluding remarks 44

Appendix 45

4 The Core and productivity-improving mergers in mixed oligopoly **47**

4.1 Introduction 47

4.2 The model 50
4.2.1 Basic set-up of mixed oligopoly 50
4.2.2 Equilibrium outcomes in the regimes (a) to (d) 51
4.2.3 Market structures and the core 54
4.3 Results 56
4.4 Concluding remarks 61
Appendix A 62
Appendix B 65

5 Stable market structures from merger activities in mixed oligopoly with asymmetric costs 74
5.1 Introduction 74
5.2 The model 77
5.3 The core and stable market structures 81
5.4 Extension 92
5.5 Concluding remarks 96

6 Conclusion 100

Chapter 1

Introduction and overview

In this chapter, we review the results obtained in the existing literature on theoretical horizontal mergers from both the viewpoints of the private oligopoly and the mixed oligopoly, and we discuss the relationship between the result obtained in this thesis and them.

1.1 The general discussion on horizontal mergers in oligopoly – exogenous merger vs. endogenous merger –

Mergers are common practices in many markets. In order to provide economic explanations for many merger phenomena occuring in the real world, many theoretical literature on horizontal mergers have sought to reply the following two questions: (i) *why firms merge*; (ii) *how firms merge*.[1]

The modern game theoretical work on horizontal mergers which attempted to reply the question of (i) can be traced back to the seminal paper of Salant et al. (1983). In several literatures including Salant et al. (1983), the researchers exogenously set a group of firms which merges with each other in order to clarify under what the economic environment each firm would like to merge, which are referred to as the analysis of the *exogenous merger*, being regarded as belonging to a classic category. Their works basically compared the benefits of going together with the benefits of standing alone. On the basis of such an analytical method, Salant et al. (1983) showed that only the merger in which more than 80% of the firms participate can be profitable in the absence of cost savings. The result obtained Salant et al. (1983), which may be inconsistent with the fact that the merger among relatively small-numbered firms is frequently observed in

[1]In particular, during the past decades, not only national mergers but also cross-border mergers have become increasingly widespread. As in Hijzen et al. (2008), the number of $M\&A$s worldwide in excess of one million dollar from 2000 to 2001 is more than twice, relative to that from 1990 to 1991. Most recently, as firms grow larger and larger, mergers increasingly tend to be international. UNCTAD (2000) announced the increasing share of cross-border mergers in FDI, rising on their measure to 78% in the late 1990's.

the real world, is referred to as the *merger paradox*, and it becomes the driving force to expand the theoretical analyses of horizontal mergers on the basis of the exogenous merger approach under many economic situations. This extreme result is led from the fact that under Cournot competition, non-merged firms respond to the merger by producing more, which hurt the merged firms. Thus, if the result of the merger paradox can be resolved, the expansion of the production of non-merged firms should be restricted after any merger activity. Then, Perry and Porter (1985) and Qiu and Zhou (2007) tackled a resolution to the unprofitability of the merger involving the small-numbered firms, by taking into account decreasing returns to scale and product differentiation, respectively. Furthermore, Hennessy (2000) proposed the result that each firm's convex cost function can be an answer to resolve the merger paradox.[2] Daughety (1990) and Levin (1990) tackled such a problem by introducing a disadvantageous position for non-merged firms. Moreover, Deneckere and Davidson (1985) considered horizontal mergers under Bertrand competition, and showed that in contrast to the result under the quantity-setting competition, the mergers involving any-sized participants are beneficial, and the degree of its profitability increases, as its size becomes larger.

In order to predict a final market structure after each firm's merger when s/he can endogenously choose whether or not to merge and how to react a merger, many researchers recently adopted the other approach towards the problem of (ii) how firms merge, which is referred to as the *endogenous merger* analyses. To give an appropriate reply to the question of (ii) how firms merge, each firm's merger formation must be explicitly determined in this research category. Furthermore, the works on the endogenous merger formation are investigated on the basis of the two following different game theoretical approaches: (1) the *non-cooperative game* theoretical approach; (2) the *cooperative game* theoretical approach. In the endogenous merger formation, the works which adopt the non-cooperative game approach have been dominant since the pioneering paper of Kamien and Zang (1990). The studies which is based on the non-cooperative game approach including Kamien and Zang (1990) attempted to explain the dynamic merger process under several economic environments.[3] On the other hand, although the works adopting

[2]Subsequently, Heywood and Mcginty (2007a) returned to the contention that convex costs may provide a resolution to the merger paradox, which was considered in Hennessy (2000) and so on. Heywood and Mcginty (2007a) found that the relevance of convex costs as a resolution to the merger paradox may reduce by showing that for reasonable degrees of convexity, the minimum market share which is needed for a merger to be profitable remains close to that associated with constant marginal cost functions.

[3]Previous contributions which belong to this category are Faulí-Oller (2000), Gowrisankaran (1999), and Gowrisankaran and Holmes (2004). Most recently, Qiu and Zhou (2007) proposed two necessary conditions for mergers to occur; heterogeneity among firms and negative demand shocks, and further Fumagalli and Vasconcelos (2009) studied the effect of the trade policy on firms' choices between intra-national and cross-border mergers in an international Cournot oligopoly.

the cooperative game approach still have been few, their purposes are common. The literature which treated endogenous merger formation as a cooperative game did not describe the dynamic merger process in detail. However, they sought to check whether a market structure is stable since any firm has no incentive to change the market structure through her/his merger process. In the literature on the endogenous merger formation based on the cooperative game approach, two different equilibrium solutions were introduced by Barros (1998) and Horn and Persson (2001a), respectively, both of which were based on the *core* solution concept that is most familiar in the economic theory.[4] Due to the simplicity of their solution concept, in particular, in the context of the private oligopoly, many papers derived the final market structure after each firm's horizontal merger under several economic situations, following Horn and Persson (2001a). For example, Horn and Persson (2001b) considered cross-border mergers by focusing on each firm's trade and her/his production cost, and subsequently, Banal-Estañol et al. (2008) explored how each firm's investment decision and her/his internal organization influence the efficiency and stability of horizontal mergers. Except for chapter 3, in this thesis, from both the viewpoints of the private oligopoly and the mixed oligopoly, we will consider the endogenous merger formation under several economic situations by adopting the cooperative game approach.

1.2 Managerial delegation and merger incentives

Although the literature on horizontal mergers considered many economic contexts, one of their main streams has been examining the effect of each firm's internal structure with the separation between ownership and management on her/his merger incentive.[5] A theoretical analysis of the separation between ownership and management tackled the validity of the traditional assumption of a firm as the sole profit maximizing agent. As the development of a principal-agent model, Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) conducted to clarify the interaction between owner-manager contracts and market behaviors in the context of an oligopolistic industry with managerial delegation. More precisely, they considered the situation wherein each

[4] Brito and Gata (2006) discussed a theoretical difference between the two equilibrium notions which were introduced in Barros (1998) and Horn and Persson (2001a).

[5] As described in Footnote #1 at the front of this chapter, most recently. the works on cross-border mergers including DaimlerChrysler as one of the most famous examples have been extensively increasing, and thus, if we restrict our focus into the analyses of horizontal mergers, the participants in such a merger should be frequently composed of large enterprises with the separation between ownership and management. In this sense, when we tackle the problem of horizontal mergers, one may consider that it is inevitable duty to take into account the separation between ownership and management in order to sufficiently reflect the examples in the real world economy. Thus, in this thesis, we would like to emphasize the point that we regard considering such each firm's separation between ownership and management as our evident duty to capture the real world examples.

firm's owner offers her/his manager the type of delegation contract which is defined as the convex combination between her/his profit and revenue (the FJSV delegation contact), and they showed that in both the quantity- and price-setting oligopolistic competitions, profit-maximizing owners frequently choose to divert the preference of their managers away from profit maximization.

González-Maestre and López-Cuñat (2001) and Ziss (2001) first considered the interaction between the use of each firm's FJSV delegation contract and her/his merger incentive, and they showed that the FJSV delegation would generally increase the profitability of horizontal mergers in symmetric Cournot industries, since the decrease of the merged firm's profit with her/his output contraction is relaxed through the aggressive behavior led by the use of her/his FJSV delegation contract. Subsequently, based on the cooperative game approach presented in Horn and Persson (2001a), in an asymmetric Cournot industry with respect to each firm's marginal cost, Straume (2006) found that the use of each firm's FJSV delegation contract leads to wrong type of merger from a viewpoint of social welfare because of excess market concentration, while the use of each firm's FJSV delegation contract further increases her/his profitability with any merger through the rationalization effect that the merged firm can produce with her/his superior technology by regarding her/him as the agent operating the two pre-merged firms (plants). In chapter 2 of this thesis, in an asymmetric Cournot industry, we will tackle the non-coincidence between the market structure induced by each firm's merger incentive and the most preferred market structure with respect to social welfare.

1.3 Bargaining over managerial contracts and merger incentives

In chapter 2 of this thesis, by considering the situation wherein not only the owner but also the manager influence the content of her/his FJSV delegation contract, we attempt to resolve the negative result that in an asymmetric Cournot competition with each firm's FJSV delegation contract, the equilibrium market structure does not coincide with the most preferred market structure with respect to social welfare. Modern corporate governance codes include clauses that the disclosure of each firm's managerial delegation should be required in order to protect her/his owner from the opportunistic behavior of her/his manager. The presence of such a code is deeply related to the transparency in deciding the content of each firm's FJSV delegation contract. Without such a transparency, each firm's manager would boost her/his remuneration against the interest of her/his owner, and thus the managerial power between the owner and the manager within each

firm may determinately influence market outcomes under the economic situation that the disclosure of her/his FJSV delegation contract is required. Then, van Witteloostuijn et al. (2007) modeled the bargaining over the content of the FJSV delegation contract between the owner and the manager within each firm in a symmetric Cournot competition with each firm's constant marginal cost function. More precisely, van Witteloostuijn et al. (2007) considered the following two-stage game: In the first stage, the owner and the manager within each firm simultaneously and independently bargain over her/his FJSV delegation parameter, and in the second stage, each firm's manager simultaneously chooses her/his output level in terms of the maximization of the FJSV delegation contract offered by her/his owner. In their 2007 paper, van Witteloostuijn et al. showed that both the equilibrium consumer surplus and the equilibrium social welfare increases, while the equilibrium profit of each firm decreases, as the relative bargaining power of her/his manager to her/his owner increases.[6]

In chapter 2 of this thesis, taking into account the bargaining power between each firm's owner and manager, we consider the influence of the bargaining over the FJSV delegation parameter on each firm's merger incentive in an asymmetric Cournot competition.[7] More precisely, we

[6]van Witteloostuijn et al. (2007) also conducted the similar analysis to the case of the FJSV delegation contract under the situation wherein each firm adopts the relative performance delegation, which is introduced in Salas Fumas (1992) and Miller and Pazgal (2001) and is applied in Miller and Pazgal (2002), that the owner employs her/his manager who cares about other firms' profits as well as her/his own profit. In the relative performance delegation case, van Witteloostuijn et al. (2007) showed that as the relative bargaining power of each firm's manager increases, the equilibrium consumer surplus and social welfare decrease while her/his equilibrium profit increases, resulting from cartel-like behavior, in contrast to the result obtained in the FJSV delegation contract.

[7]In chapter 2, we rule out the monopoly from the merger among all the existing firms by the assumption that the authority corresponding to each firm's horizontal merger prohibits such a monopoly, whereas in chapters 4 and 5, in the context of mixed oligopoly, when the stability analysis on market structures before and after each firm's merger activity is conducted, we allow for the monopoly from such an all-firm-merger. This difference results from those of the welfare implications of horizontal mergers between the private oligopoly and the mixed oligopoly. More precisely, in the context of the private oligopoly, each firm's horizontal merger may yield the deterioration of social welfare because of the decrease of the equilibrium total output through decreasing the competition pressure (specifically, without the effect of the merged firm's productivity-improving), whereas in the context of the mixed oligopoly, since the payoff of the public firm's owner is assumed to be social welfare, the merger including the public firm is allowed by the merger authority, if the equilibrium social welfare improves after the merger. Thus, the works on theoretical horizontal mergers in the private oligopoly have been showing that the merger among all the firms must be strictly regulated by the merger authority, and therefore, many previous literature including Davidson and Ferret (2007), Lommerud et al. (2006), Qiu and Zhou (2006), Matsushima et al. (2008), and Matsushima et al. (2009) have assumed that the merger among all the firms is eliminated before their concrete analyses. On the other hand, in the context of the mixed oligopoly on theoretical horizontal mergers, the analyses conducted in chapters 4 and 5 are the first challenges against of the endogenous merger formation using our original cooperative game approach. In addition, in chapters 4 and 5, since we employ the assumption that the merger between arbitrary firms has an effect of increasing the productivity of the merged firm, relative to the pre-merger situation, the effect of the merger among all the firms on social welfare is not obvious. Thus, before our concrete stability analysis among market structures, we cannot straightforwardly obtain the result of whether or not the market structure with the merger among all the firms is stable, and hence, in the analysis of the mixed oligopoly on horizontal mergers in chapters 4 and 5, we take into account the monopoly from the merger between all the public and private firms, different from the analysis of the private oligopoly in chapter 2.

change the way of determining each firm's FJSV delegation parameter from the setting employed in Straume (2006), by taking into account explicitly the relative bargaining powers between both her/his owner and manager. As described above, Straume (2006) showed that each firm's merger incentive leads to the wrong type of market structure with respect to social welfare in an asymmetric Cournot competition, when each firm's delegation parameter is determined with respect to the maximization of her/his profit which is the objective function of her/his owner. Thus, Straume (2006) concluded that the active merger policy must be required in the asymmetric industry composed of firms with the separation between ownership and management. The analysis conducted in chapter 2 of this thesis provides the result that for a comparatively large area of the relative bargaining power of each firm's manager to her/his owner, the equilibrium market structure based on each firm's merger incentive coincides with the most socially preferred structure in the case wherein the degree of asymmetry among the existing firms is middle-leveled, if her/his FJSV delegation parameter is determined through the bargaining between her/his owner and manager.[8] Thus, if each firm's manager influences the determination of her/his FJSV delegation parameter, the necessity of an active merger policy may decrease. Moreover, we may be able to consider that the presence of the codes in modern corporate governance that the disclosure of each firm's managerial contract is needed contributes to the increase of the relative bargaining power of her/his owner. Therefore, from the combination of the result in Straume (2006) and that in chapter 2 of this thesis, the presence of the corporate governance codes that such a disclosure is excessively required can lead to the wrong type of market structure with respect to social welfare.

1.4 Horizontal mergers in mixed oligopoly

Although in chapter 2 of this thesis, we present a theoretical analysis of horizontal mergers in the private oligopoly which is composed of profit-maximizing firms, throughout the rest three chapters of this thesis, we will consider the merger between the public firm and the private firm in mixed oligopoly.[9] In not only the developing countries and former communist economies, in

[8] More precisely, this result holds as long as the relative bargaining power of each firm's manager to her/his owner is over about one fifth.

[9] In the european automobile industry, we frequently observe the merger and acquisition between the public enterprise and the private enterprise. As a typical example of the merger between the public firm and the private firm in the real world mixed oligopolistic industry, Bárcena-Ruiz and Garzón (2003) introduced the merger between SEAT which was a Spanish publicly owned automobile manufacturer and Volkswagen which is a German private enterprise in 1986. If the business collaboration can be allowed for discussing as one type of horizontal mergers, Renault which is a French partially privatized public-private firm enters into the business agreement with Nissan Motor and Nissan Diesel, both of which are Japanese private enterprises. Moreover, in Japanese telecommunication industry, in 1998, Teleway Japan which was a private enterprise is acquired by KDD (Kokusai Denshin Denwa) which was a public enterprise, and subsequently the merged firm, KDD corporation merged with IDO and DDI into

but also the developed countries, many public firms compete and coexist with private firms in many kinds of industries such as banking services, housing loans, life insurance, broadcasting services, and overnight deliveries.

The seminal paper which formulated the competition between the public firm and the private firm on the basis of the modern game theoretical approach can be traced back to De Fraja and Delbono (1989). De Fraja and Delbono (1989) first established the theoretical mixed oligopolistic model in which the public firm maximizes social welfare, while the private firm maximizes her/his own profit. Although many works on mixed oligopoly have dealt with several economic situations, one of them tackled the merger between the public firm and the private firm.[10] Bárcena-Ruiz and Garzón (2003) and Coloma (2006) presented the theoretical merger analyses between the public firm and the private firm, and subsequently Méndez-Naya (2008) extended the Bárcena-Ruiz and Garzón (2003)'s model in a mixed duopoly composed of a public firm and a private firm to the merger activity in a general mixed oligopoly wherein one public firm and n private firms exist. Although the above two papers of Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) considered the efficiency gains with any merger, both of them divert their focuses from such an effect of the productivity-improving of the merged firm.[11] In chapter 3 of this thesis, explicitly taking into account the productivity-improving effect with any merger, we will present a theoretical analysis of the merger between a public firm and a private firm in a mixed oligopoly. More precisely, in chapter 3 of this thesis, we assume the effect of the productivity-improving in the merged firm à la McAfee and Williams (1992). In chapter 3, the achievablity of the merger between a public firm and a private firm is explained through the following two effects: the *share effect* with in/decreasing a shareholding ratio of the public firm's

KDDI corporation in 2000.

[10]The recent literature on mixed oligopoly has become more increasing. For example, Ishibashi and Matsumura (2006), Nishimori and Ogawa (2002), and Poyago-Theotoky (1998) considered the competition of R&D investments between the public firm and the private firm. Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), and Matsumura and Matsushima (2006) investigated the international competition. Ohori (2006) and Bárcena-Ruiz and Garzón (2006) explored the role of optimal environmental tax. Furthermore, Pal (1998), Matsumura (2003a,b), Lu (2006), and Bárcena-Ruiz (2007) discussed the endogenous determination of the competition style when both the public firm and the private firm select when to choose their output levels.

[11]More precisely, regarding the achievablity of the horizontal mergers between the public firm and the private firm, Bárcena-Ruiz and Garzón (2003) focused his attention on the relationship between the degree of substitution of the goods produced by them and the share ratios of their owners in the merged firm, whereas Méndez-Naya (2008) noticed the relationship between the number of the existing private firm in the market and the share ratios of their owners in the merged firm. Thus, both of Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) did not pay their attentions into the effect of the efficiency gains with the merger. On the other hand, in the context of the analysis of horizontal mergers in private oligopoly, such an efficiency gains have been extensively investigated. The theoretical literature has showed that if the merger yields such efficiency gains, social welfare may increase. For the general discussion on the efficiency gains with any merger, see Farrell and Shapiro (1990) as a classic work and Banal-Estañol et al. (2008) and Motta and Vasconcelos (2005) and so on as more recent studies.

owner in the merged firm and the *competition effect* with in/decreasing the number of the private firms. When the relatively small-numbered private firms exist before any merger, the achievablity of such a merger is mainly explained by the share effect, since the share ratios of both the owners as the merger participants drastically influence their payoffs. As the number of the private firms increases, whether or not the merger is achieved is mainly determined by the competition effect, since the impact of the presence of the public firm on social welfare is sufficiently small. In addition, we will consider the achievablity of the merger between a public firm and a private firm in the following two cases: (1) there does not exist any productivity-improving effect with mergers; (2) the public firm is less efficient than the private firms.

Although the analyses conducted in chapter 3, in Bárcena-Ruiz and Garzón (2003), and in Méndez-Naya (2008) clarified under what conditions the merger between one public firm and one private firm is achieved, several theoretical restrictions still have been preserved. First, the above three analyses dealt with the merger between one public and one private firm, and thus, in the assumption of Bárcena-Ruiz and Garzón (2003) that the market structure occurring before any merger is a mixed duopoly, the market structure after the merger is under the monopoly composed of only the merged (partially privatized) firm. It is doubtful whether such an assumption is realistic or not, and whether such a merger is allowed for in the real world or not.[12] Second, although the general mixed oligopoly which is composed of one public firm and n private firms as the market structure before any merger was supposed in the analyses of both the chapter 3 of this thesis and Méndez-Naya (2008), only the merger between one public firm and one private firm was explored in their analyses. Thus, their analyses did not deal with the decision makings of whether or not to merger for all the firms' owners, although multiple symmetric private firms exist. Third, in all the above three analyses, the researchers took the merger participants and its size as given, and hence their analyses belong to the classic category of exogenous merger analyses. Fourth, in all the three analyses, the reactions of relevant merger participants against the change of several economic situations including other firms' mergers and their dissolutions were totally ignored, and thus, in their analyses, the dynamic change of such a market situation could not be considered.

In chapters 4 and 5 of this thesis, taking into account the restrictions preserved in the previous literature as described above, we will present the theoretical analyses of horizontal mergers in

[12] In addition, Bárcena-Ruiz and Garzón (2003) showed that when the ratio of the shares owned by the owner of the public firm takes an intermediate value, and the degree to which goods are substitutes is low enough, both the public firm and the private firm would like to merge. In particular, the latter condition on the substitutability of their production goods may be inconsistent with that in the merger observed in the real world.

mixed oligopoly on the basis of the cooperative game approach, which belong to the category of endogenous merger analyses. Thus, in chapters 4 and 5, we explicitly consider the decision-makings of whether or not to merge for the owners of all the existing firms. More precisely, in chapter 4, we will derive the core of market structures after each firm's free merger activity in the mixed triopoly which one public firm and two private firms with their production technologies, which are represented as the quadratic cost functions with respect to their output levels. Focusing our attention on the efficiency gains with mergers à la McAfee and Williams (1992), which was considered in chapter 3, in the analysis of chapter 4, we show that only the market structure including the merged firm involving one public firm and one private firm with about 57% of shares owned by the owner of the public firm can belong to the core. Thus, even though the decision-makings of whether or not to merge for the owners of all the existing firms are explicitly taken into account, the merger between one public firm and one private firm can be also supported on the basis of the stability of market structures occuring after each firm's merger activity. Moreover, in chapter 4 of this thesis, we discuss the figure of the core in the following two cases: (1) only the private firms exist in the market (after the full privatization of the public firm); (2) the public firm is less efficient than the private firms.[13]

However, throughout a series of the analyses in chapter 4 of this thesis, we assumed the mixed (private) triopoly as the market structure before any merger activity.[14] Then, in the final chapter 5 of this thesis, on the basis of the cooperative game approach which is the same as that adopted in chapter 4, we will derive the core of market structures after each firm's free merger activity in the general mixed oligopoly composed of one public firm and n private firms with their production technologies which are represented as the constant marginal cost functions with respect to their quantities.[15] In chapter 5, we provide a sufficient condition with respect to the level of the public

[13]In addition with the two cases (1) and (2), in chapter 4 of this thesis, we consider the case wherein each firm adopts her/his FJSV delegation contract à la Lambertini (2000) and Straume (2006). As described in detail in Introduction of chapters 4 and 5, similar to the private oligopoly, the horizontal merger phenomena in the real world mixed oligopoly involve large enterprises with the separation between ownership and management, like the mergers between Volkswagen and SEAT in the European automobile industry. Thus, it is inevitable for us to consider the separation between ownership and management within each firm by adopting her/his FJSV delegation contract in conducting our analysis on horizontal mergers in the context of mixed oligopoly. In all the three cases, we obtained tha result that the core of market structures can be empty. The (non-)emptiness of the core will be discussed in detail in Conclusion of the final chapter in this thesis.

[14]In a general mixed oligopoly with respect to the number of the existing private firms, because of computational problems, we can not obtain the general result on the figure of the core of market structures after each firm's merger activity, when the technology of each firm is represented as her/his quadratic cost function.

[15]Note that as the beginning of the analyses in chapter 5, we tackle the merger activity in the mixed triopoly wherein one public firm and two symmetric private firms exist. In such an analysis, we show that in accordance with the level of the public firm's marginal cost, the three types of market structures can belong to the core: (1) the mixed triopoly; (2) the merger between a public firm and a private firm; (3) the merger among all firms. Neverthless, we find that through the appropriate adjustment of the shareholding ratio of the public firm's owner in the merged firm,

firm's marginal cost to have that the market structure with the merger between one public firm and one private firm can belong to the core of market structures after each firm's merger activity. The analyses conducted in chapters 4 and 5 of this thesis are the pioneering studies applying the endogenous merger approach into the horizontal mergers in the (general) mixed oligopoly. We believe that these contributions are not small.

Finally, in Conclusion of the final chapter, we review our contributions obtained in this thesis and discuss future researches which are left for us.

1.5 Comparison with vertical mergers

Although in this paper, we exclusively deal with the problem of horizontal mergers, there exist the analyses of *vertical merger*s as other merger problem in the economic theory. The competitive effects of vertical mergers have long been a source of controversy in antitrust economic. Traditional literature on the vertical mergers claimed that a vertically integrated firm has no incentive to exclude her/his rivals, and if s/he attempted to exclude them, rivals could protect themselves by contracting with other unintegrated firms. However, this classical theory of the vertical mergers and vertical foreclosure has been criticized by the researchers including the Chicago School. More precisely, taking such a criticism into account, the new equilibrium approach on the basis of the modern industrial organization theory has been presented in Salop and Scheffman (1987) and Ordover et al. (1983) which were pioneering papers in this field: Both of them showed how vertical mergers can lead to anticompetitive effects under several economic environments.[16] More precisely, the works based on the equilibrium approach revealed that an vertically merged firm benefits from the higher costs imposed on her/his downstream opponent firms when s/he refrains from pricing aggressively in the input market. In particular, Chen (2001) showed that in the price-setting vertical oligopolistic model wherein $h \geq 2$ upstream firms produce a homogeneous input for the downstream industry and two downstream firms produce differentiated products, under more general conditions relative to those in Ordover et al. (1983) and Salop and Scheffman (1987), whether or not a vertical merger occurs strikingly depends on the magnitude between the productivity-improving effect of the vertically merged firm with some sort of vertical merger and the collusive effect that the vertically merged firm has a less incentive to decrease

the market structure with the merger between a public firm and a private firm can always belong to the core.

[16] As other important contributions in this area, there exist Salinger (1988) and Chen (2001). Furthermore, there exist the following two different approaches on the vertical merger. Frist, focusing their attentions on the notion of the incomplete contract as the organizational problem within each firm, Grossman and Hart (1986), Hart and Tirole (1990), and Williamson (1985) considered the vertical integration. Second, Arrow (1975) and Gal-Or (1999) paid their attentions to the problem of asymmetric information in analyzing the the vertical merger.

her/his downstream price. In addition, the 2001 paper of Chen revealed that vertical merger is achieved in equilibrium if and only if there exists any efficiency gain with vertical mergers.

The above findings in Chen (2001) are closely related with those in the literature on horizontal mergers in private oligopoly as shown in Farrell and Shapiro (1990) and so on. More precisely, similar to the case of the vertical mergers, in the horizontal merger case, each firm's horizontal merger yields the two effects: The first effect is that collusive behaviors of merger participants and the decrease of competitive pressure with decreasing the number of the existing firm leads to the decrease of the total output and the increase of the market price, while the second effect is the effect of the merged firm's productivity-improving, which leads to the increase of the total output and the decrease of the market price, for example, in the situation wherein the technology of each firm is represented by the quadratic cost function with respect to her/his output. Thus, analogous to the vertical mergers, in the horizontal merger, the tradeoff of the above two effects determines the competitive effect of each firm's horizontal merger.

Moreover, the result that the vertical merger occurs if and only if it yields the efficiency gain of the merged firm is similar to the key point to resolve the merger paradox in horizontal mergers. As in Farrell and Shapiro (1990) and Salant et al. (1983), the horizontal merger in which less than 80% of all the firms participate is not profitable because of the aggressive behavior of the non-merged firm. Then, Farrell and Shapiro (1990) and Salant et al. (1983) indicated that in order to increase the profitability of the horizontal mergers in which a few firms take part, relative to their individual profits before such a merger, the merged firm must enjoy the more advantageous position than that of the non-merged firm such as the situations wherein (i) s/he becomes a Stackerlberg leader in the market and (ii) the effect of productivity-improving effect with the merger is sufficiently strong (for example, in the case wherein each firm's cost function is quadratic with respect to her/his output level, the merged firm can most efficiently operate all the plants after the merger, which are regarded as the individual merger-participant firms before the merger).

As described above, in chapter 2 of this thesis, we investigate the problem of horizontal mergers in the private oligopolistic industry which is composed of managerial firms with their asymmetric constant marginal costs. In particular, we depart away from Straume (2006) which is the main reference in the analysis of chapter 2 by considering the situation wherein an owner and a manager within each firm bargain over the contents of her/his managerial delegation contract, and subsequently in the market, each firm's manager simultaneously sets her/his output level for the viewpoint of the maximization of her/his managerial delegation contract (her/his FJSV

delegation contract). In chapter 2, we obtain the result that the market structure with the merger including the least efficient firm can not only be observed in equilibrium but also become the most preferred market structure with respect to social welfare which is defined as the sum of consumer surplus and the profits of all the existing firms, different from the result obtained in Straume (2006). This result strictly depends on the assumption that any horizontal merger yields the productivity-improving of the merged firm in the asymmetric private oligopoly in the sense that the technology of a more efficient firm between merger participants can be adopted by the merged firm.[17] Therefore, we realize that the results between the vertical merger in price-setting private oligopoly which was considered in Chen (2001) and the horizontal merger considered in chapter 2 of this thesis are closely related in the sense that they strikingly depend on the fact of whether or not the merger leads to the cost-reducing effect in the merged firm.

In the context of the mixed oligopoly, the literature on the vertical merger with a downstream firms and a upstream firm includes Willner (2008) only. Willner (2008) found that when the vertical merged public firm whose objective function is a weighted sum of her/his profit and consumer surplus competes with the profit-maximizing private downstream firms, there is no weight for the consumer surplus in the objective function of the merged public firm such that the vertical merged public firm and the private firms coexist under the quantity-setting Cournot and Stackelberg competition within the same downstream market. More precisely, the output of the downstream private firm is zero because of the aggressive behavior of the vertically merged firm. Taking such an above result into account, Willner (2008) concluded that a market is less likely to remain a mixed oligopoly with vertical relations than without vertical relations. However, Willner (2008) did not consider the possibility of the merger between the public firm and the private firm in the mixed oligopoly with the vertical relationship.[18] Therefore, the merger between the public firm and the private firm which is our main remarkable point in the analyses of chapters 3, 4, and 5 of this thesis, has not been examined in the previous literature on the vertical merger/integration in the context of mixed oligopoly. The analyses of the horizontal mergers between the public firm and the private firm which are conducted in three chapters 3, 4, and 5 of this thesis should be regarded as one of footholds in order to reveal the linkages of the results and their implications

[17] If there exists no productivity-improving effect with any horizontal merger in the private oligopoly composed of managerial firms which are symmetric with respect to their marginal costs, this result cannot be obtained. More precisely, in such a case, the market structure(s) in equilibrium never coincide(s) with the most preferred market structure(s) with respect to social welfare.

[18] In order to conduct such an analysis, for example, taking into account the situation that the vertically merged public firm can further-merge with the downstream private firm, in the context of the differentiated goods quantity – or price-setting competition, the achievablity of the merger should be investigated through the comparison between the payoffs of the owners of all the merger participants before and after such a merger.

between the merger between the public firm and the private firm in mixed oligopoly with the vertical relationship and the merger between the public firm and the private firm without the vertical relationship.[19]

[19]Most recently, in the context of private oligopoly, Ziss (1995, 2005) and Mizuno (2009) investigated the effect of horizontal mergers between upstream firms or between downstream firms in a vertical relationship. We must tackle such a problem as one of first steps to clarify the linkages of the results and their implications between the merger between the public firm and the private firm in the context of mixed oligopoly.

Chapter 2

Bargaining over managerial delegation contracts and merger incentives with asymmetric costs

2.1 Introduction

This chapter presents a theoretical analysis of horizontal mergers in a private oligopoly while considering the strategic managerial delegation contracts presented by Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985).[1] These works considered a situation in which each firm's owner delegates the production decision to a manager in order to improve the firm's strategic position in the market. In this chapter, we apply the managerial delegation contract à la Fershtman and Judd and Sklivas and Vickers—the so-called FJSV contract—to horizontal mergers in a Cournot game. In particular, we focus our attention on the bargaining over the FJSV contract between an owner and a manager within a single firm. The objective is to consider the merger problem in a private oligopoly in terms of the official announcement of managerial delegation contracts which is a significant issue from the perspective of modern corporate governance.

Although there are numerous works on horizontal mergers in a private oligopoly considered under several environments, recent literature on horizontal mergers focuses her/his attention on answering the question of how firms merge. The works in this category are referred to as analyses of *endogenous mergers*. Further, the works on endogenous mergers are classified according to two analytical tools: the *noncooperative game approach* and *cooperative game approach*. The noncooperative game approach is extensively adopted in the literature on endogenous mergers.[2] However, their results may depend on several assumptions that researchers arbitrarily postulate,

[1] All the analyses conducted in this chapter are based on those of Nakamura (2009a).

[2] For example, Kamien and Zang (1990), Gowrisankaran (1999), Gowrisankaran and Holmes (2004), Qiu and Zhou (2007), and Fumagalli and Vasconcelos (2009).

the order of moves and information structure among players. Thus, we follow the cooperative game approach presented by Horn and Persson (2001a) in order to comprehensively determine the market structure(s) that is/are stable after the horizontal mergers, even at the expense of describing the dynamic processes of merger formation.[3] Horn and Persson adopted the core, a newly defined concept in the context of horizontal mergers in an oligopolistic market. Based on the idea of the core in Horn and Persson (2001a), Straume (2006) compared the equilibrium ownership structure(s) with the most preferred ownership structure(s) for social welfare.

In recent works, in the context of managerial delegation contracts, van Witteloostuijn et al. (2007) studied the bargaining between a firm's owner and manager over their managerial contracts in order to explain the influence of the disclosure of managerial delegation contracts that are prescribed in modern corporate governance clauses on market outcomes.[4] They showed that when the FJSV contract is considered, the relative bargaining power of managers is positively associated with social welfare.[5] In this chapter, we examine the role of the bargaining between a firm's owner and manager over their managerial delegation contract during the merger process. In particular, similar to Straume (2006), this chapter considers the stability problem of market structures à la Horn and Persson in an asymmetric Cournot oligopoly and examines how the internal organization of firms affects the correspondence between private and social incentives for horizontal mergers. Through this process, we investigate the bargaining over the FJSV delegation parameter between an owner and a manager within a single firm.

The main purpose of this chapter is to confirm the scope of Straume's (2006) results by introducing the bargaining over the delegation parameter between an owner and a manager within a single firm in the case of FJSV delegation contracts. Similar to Straume (2006), we consider an asymmetric three-firm Cournot industry in which the merger process is controlled by the firms' owners.[6] In such a setting, we consider the following three-stage game. In the first stage, the

[3]Barros (1998) has already investigated the stability of horizontal mergers using the cooperative game approach. See Brito and Gata (2006) for a detailed discussion on the difference in the definition adopted by Barros (1998) and that considered by Horn and Persson (2001a).

[4]According to Aguilera and Cuervo-Cazurra (2004), such a corporate governance code has been codified since the early 1990s in the UK and US, and subsequently, 24 countries has introduced the corporate governance codes by the end of the 1990s. Furthermore, van Witteloostuijn et al. (2007) indicated that they have been prescribed in more than 50 countries by the end of 2005.

[5]Nakamura (2008a) and Kamaga and Nakamura (2008a) extended their scope to the cases of a differentiated good and a sequential competition, respectively. In both papers, they confirmed the robustness of the result in van Witteloostuijn et al. (2007). Subsequently, Nakamura (2008b) considered a different cost structure from those in van Witteloostuijn et al. (2007), Nakamura (2008a), and Kamaga and Nakamura (2008a). Thus, he obtained the result that if the managers' relative bargaining power is sufficiently high, its increase leads to the deterioration of social welfare because of the excessively high total cost in the market.

[6]Following the sizeable literature on horizontal mergers such as Barros (1998), Faulí-Oller (2002), Straume (2006), and Qiu and Zhou (2007), in this chapter, we assume that mergers can yield rationalization gains through

ownership structure(s) is/are cooperatively determined by the owner of each firm through each firm's merger activity. In the second stage, all the firms' owners negotiate with their managers over managerial delegation contracts. Subsequently, in the third stage, Cournot competition with homogeneous goods is observed.[7] We find that there exists a degree of cost asymmetry in the industry such that the equilibrium ownership structure coincides with the most preferred ownership structure with respect to social welfare. Our result fairly contrasts with that of Straume (2006) that each firm directly adopts the FJSV contract without the bargaining between the owner and manager within each firm. Hence, this implies that the importance of the antitrust policy decreases if each firm's manager has bargaining power relative to that of each owner, provided the degree of cost asymmetry among firms is in the relevant interval. We find that if each firm's manager can influence the decision of her/his delegation contract, the merger may lead to a socially preferred ownership structure.[8] Therefore, the result of Straume (2006) that the use of strategic delegation increases the scope for an antitrust policy crucially depends on the fact that the delegation parameter of the FJSV contract is determined by the sole profit maximization problem with respect to each firm's owner.

The rest of this chapter is organized as follows. Section 2.2 introduces the basic components of the model. In Section 2.3, we explain the assumptions of the endogenous merger process based on Horn and Persson (2001a) and Straume (2006). Section 2.4 presents the equilibrium payoff for each firm's owner for the four possible regimes: decentralized ownership structure, merger between low-cost and medium-cost firms, merger between low-cost and high-cost firms, and merger between high-cost and medium-cost firms. Furthermore, we derive the equilibrium ownership structure for managerial firms with the bargaining over managerial contracts between each firm's owner and manager. In Section 2.5, we discuss the relation between the equilibrium ownership structures and the social ranking of ownership structures. Section 2.6 presents the concluding remarks. The equilibrium delegation parameters and market outcomes in each ownership structure and the proofs of all the propositions are available in the Appendix.

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the reallocation of production from high-cost to low-cost plants.

[7] Note that we rule out the possibility of a merger to monopoly à la Barros (1998) and Straume (2006).

[8] In particular, whenever the bargaining power of each firm's manager exceeds one-fifth relative to that of the owner, there always exists a degree of cost asymmetry among the firms such that the equilibrium ownership structure coincides with the most preferred ownership structure with respect to social welfare.

2.2 The model

We consider the stable ownership structure in an industry to be comprised of three initial managerial firms producing homogeneous goods. The inverse demand function is linear in the total output $Q = \sum_{i=1}^{3} q_i$ as follows:

$$p(Q) = 1 - Q. \tag{2.1}$$

Each firm i's output is sold at the market-clearing price in (2.1) and competes à la Cournot $(i = 1, 2, 3)$. We assume that each firm i has an asymmetric constant marginal cost function c_i with each of the other firms, such that $c_1 < c_2 < c_3$. Similar to Barros (1998) and Straume (2006), we assume that the differences among the production costs of each firm are symmetric, and thus, each firm's cost function is represented as follows:

$$c_i = (i-1)\, c; \quad i = 1, 2, 3,$$

where $c > 0$ indicates a direct measure of production efficiency in the industry. In order to ensure that the most inefficient firm (firm 3) is active in any ownership structure, we assume that c is upper-bounded, *i.e.*, $\bar{c}$.[9] The profit of firm i is given by

$$\Pi_i = [p(Q) - (i-1)\, c]\, q_i; \quad i = 1, 2, 3.$$

Moreover, if owners i and j decide to merge into a newly merged firm ij, they earn the following combined profit of firm ij:

$$\Pi_{ij} = [p(Q) - \min\{(i-1)\, c, (j-1)\, c\}]\, q_{ij}; \quad i, j = 1, 2, 3\ ;\ i \neq j.$$

As usual, social welfare, denoted by W, is measured as the sum of the consumer surplus (CS) and producer surplus (PS):

$$W = CS + PS,$$

where $PS = \sum_i \Pi_i$, and consumer surplus is given by

$$CS = \frac{1}{2} Q^2.$$

Similar to Straume (2006), we exclude the possibility of a merger to monopoly because complete monopolization, *i.e.*, merger among all firms, may not be permitted by antitrust authori-

[9] Further analysis of this chapter shows that $c \in (0,\ \bar{c} = (1-\beta)\,/\,(11+7\beta))$.

ties.[10] Thus, with respect to whether a merger among the firms is realized, we have four possible ownership structures: (o) the decentralized structure, $M_o = \{\{1\},\{2\},\{3\}\}$; (a) merger between low-cost and medium-cost firms, $M_a = \{\{1,2\},\{3\}\}$; (b) merger between high-cost and low-cost firms, $M_b = \{\{1,3\},\{2\}\}$; and (c) merger between high-cost and medium-cost firms, $M_c = \{\{1\},\{2,3\}\}$.

In each ownership structure, the firm's owner hires a manager and delegates her/his output decision to the manager. Each manager sets the output to maximize her/his payoff denoted by an incentive contract presented by the firm's owner. Owners are able to assess the performance of their managers based on two easily observable indicators: profit (Π) and sales (q). In each firm i, the owner presents the following type of incentive contract U_i to the manager:

$$U_i(\Pi_i, q_i) = \Pi_i + \theta_i q_i. \tag{2.2}$$

The manager of firm i can maximize her/his payoff by setting the value of output q_i that maximizes U_i.[11] In this chapter, on the basis of the managerial delegation contract in (2.2), we

[10]Taking into account the definition of the decisive owners and the domination relation, similar to the model on horizontal mergers with international trades which was considered in Horn and Persson (2001b), our model has the property that whenever a monopoly with the merger among all the three firms makes a larger profit than their combined profits in more decentralized market structures, such a monopoly will occur if it is permitted. We can realize that the foundation of the monopoly with the merger among all the firms results from the facts that each firm's is free to communicate on the possibility of any merger and to write binding contracts on the distribution way of the merged firm's profit among the merger participants. However, similar to many previous literature on theoretical horizontal mergers including Horn and Persson (2001b), we are not interested in the case wherein the monopoly for the three-firm merged firm is formed, since in the analysis of this chapter, we focus our attention on replying the question of whether the market structure(s) with either the merger between two efficient firms or the merger between two relatively inefficient firm, or the merger between an efficient firm and an inefficient firm can be stable in the economic environment wherein an owner and a manager bargain over the the content of her/his delegation contract in an asymmetric private oligopoly. As described in Horn and Persson (2001b), there are two ways of ruling out the monopoly market structure resulting from all-firm-merger. The first way is that we adopt the assumption that the monopoly makes a smaller profit than the combined profit in more decentralized market structure. As the economic situation which such an assumption implies, Porter (1985) indicated the following situation: The increases of both the absolute size of the merged firm that all the firms are participants and the lack of competitive pressure lead to her/his organizational inefficiency, and consequently, her/his profit becomes sufficiently low. Athough we can succeed in being absence of the monopoly for all-firm-merged firm endogenously by taking into account such a situation, relative to the present model, we must newly adopt further several (probably, somewhat opaque) assumptions such as the assumption that there is no production-improvement with any merger irrespective of the facts that the cost function of each firm is asymmetric and constant-marginal-cost typed, and/or the assumption that there exists some sort of fitting cost between any merger participants which represents the cost incurring in order to transfer technological know-how between them. Under the above assumptions, we will consider an extremely restrictive economic situation. Then, as the second alternative way, following Horn and Persson (2001b), we assume that the monopoly is not permitted for the reason why the monopoly is prohibited in most countries.

[11]This fact can be supported by the assumption that the payoff to the manager of firm i is defined as $\lambda_i + \mu_i U_i$ for some real number λ_i and some positive real number μ_i. Moreover, this type of delegation contract has been employed by Lambertini (2000), Lambertini and Trombetta (2002), Nakamura (2008c), Nakamura and Inoue (2009), van Witteloostuijn et al. (2007). Throughout this chapter, we adopt this type of managerial delegation contract used in Lambertini (2000), Lambertini and Trombetta (2002), and van Witteloostuijn et al. (2007), which is slightly changed from that introduced in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985), *i.e.*,

consider a situation in which the owner and manager bargain over the delegation parameter θ_i, where the owner wants to maximize her/his profit, while the manager wants to maximize her/his payoff $(i = 1, 2, 3)$. The outcome of the bargaining process in firm i is the incentive parameter θ_i that maximizes the following Nash product:[12]

$$\mathfrak{B}_i = U_i^{\beta} \Pi_i^{(1-\beta)}.$$

Note that $\beta \in [0, 1)$ represents a measure for the bargaining power of each firm's manager.[13]

FJSV delegation contract. More precisely, the original FJSV delegation contract in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) is the weighted sum of each firm's profit and her/his revenue, and thus, as the level of the delegation parameter becomes lower in the original FJSV delegation contract, each firm's manager more aggressively attempts to produce in the market, while in the delegation contract of the model in this chapter, the higher level of the delegation parameter yields the more aggressive behavior in the market. Thus, although the delegation parameters reversely work on market outcomes between the the original FJSV model and the model in this chapter, the similar results on each firm's merger incentive and its welfare implications in the two models are obtained. We have the following two reasons why we employ the delegation contract à la van Witteloostuijn et al. (2007) in eq. (2.2), which is slightly changed from the original FJSV delegation contract: First, this is so because we take into account the firm without any cost owing to the normalization with respect to the cost asymmetry among existing firms. More precisely, in this case, since the revenue of the firm which can produce without any cost is the same as her/his profit, if we adopt the approach regarding the original FJSV delegation contract, the objective function of the manager of the firm becomes her/his revenue/profit, and thus, her/his delegation parameter does not function in her/his quantity-setting. Thus, we use the approach of the delegation contract à la Straume (2006) and van Witteloostuijn et al. (2007) in order to avoid such a somewhat problematic situation. Second, this is so because we would like to investigate the influence of the bargaining over the delegation parameter between an owner and a manager on each firm's merger incentive and its welfare implications under the economic situation wherein the settings of both Straume (2006) and van Witteloostuijn et al. (2007) are faithfully and simultaneously reflected. Thus, in this chapter, the settings of both the delegation contract type and its bargaining style correspond to those in Straume (2006) and van Witteloostuijn et al. (2007), respectively.

[12]Throughout this chapter, the bargaining power of each firm's manager relative to her/his owner, β is exogenously given. Note that both the concrete value of β and the in/decrease of its value depend on the business quality of the existing firms in the industry, each firm's adhesive terms and conditions, the degree of progress of the owner-shareholder's sovereignty in the industry or the country, and so on. The industry in which the level of β becomes lower corresponds to the one that the owner-shareholder's sovereignty more deeply proceeds, since its situation implies the fact that the relative bargaining power of each firm's owner is sufficiently high. For example, in most of the developing countries, since the owner-shareholder's sovereignty is not likely to prevail, the bargaining power of the owner relative to that of the manager may stay low. In addition, even in the developed countries, if the development of the legal system with respect to corporate governances does not proceed, the relative bargaining power of the owner similarly becomes low. Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) considered the situation wherein each firm's owner can choose the content of her/his delegation which is represented as her/his delegation parameter from a viewpoint of her/his objective function, *i.e.*, her/his profit. However, one may consider that such a modelling does not reflect the industry where each firm's manager opportunistically behaves against her/his owner's interest within each firm. In this sense, we can regard the economic situation considered in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) as the situation in which the right of each firm's owner-shareholder is extremely guaranteed. Most recently, as described in Introduction of this chapter, taking into account that the clauses in the modern governance codes such that the disclosure of each firm's managerial delegation contract is required are prescribed to protect the right of her/his owner from the opportunistic behavior of her/his manager, the presence of such a clauses in the modern governance codes should lead to increasing the relative bargaining power of the owner within each firm, *i.e.*, decreasing the level of β. In this chapter, we consider the influence of the presence of such a clauses in the modern governance codes which has an effect of increasing the bargaining power of each firm's owner relative to that of the manager on her/his managerial incentive and welfare implications.

[13]The bargaining between the owner and the manager is modeled on the generalized Nash bargaining solution in Binmore et al. (1986). Note that we adopt the generalized Nash bargaining in order to reflect the influence of the

In this chapter, we assume that all the firms' managers have the same bargaining power. The disagreement point of both the owner and the manager is zero: This fact is supported by assuming that all the owners do not have the ability to operate their own firms and all the managers cannot obtain employment elsewhere.

2.3 Merger as a coalition formation and the core

In this section, in order to analyze the owners' decision-making on merger activities, we will use an approach developed by Horn and Persson (2001a) and applied by Straume (2006). In the rest of the chapter, we use the notations 1, 2, and 3 to denote firms 1, 2, and 3, respectively, as well as to denote their respective owners.

First, we define the domination relations among the ownership structures to introduce the formal definition of the core of ownership structures. An ownership structure M_i is defined as a partition of the set $\{1, 2, 3\}$ of owners into coalitions. An ownership structure M_j is said to be **dominated by M_i via a decisive group of owners** if the combined profit of the decisive group of owners is larger in M_i than in M_j. Here, the decisive owners indicate the smallest group of owners that can induce a new market structure M_i from the present markets structure M_j. Similar to Straume (2006), we do not permit any side-payment between the coalitions of owners. Thus, when M_i is formed from M_j, the decisive owners are all the remaining owners, except the owners that belong to the same coalitions in M_i and M_j.[14]

We illustrate this notion of decisive owners with the help of some examples. First, let us consider ownership structures M_o and M_i $(i = a, b, c)$. In this case, the decisive group of owners is the subset of $\{1, 2, 3\}$ to which only the independent owner in M_i does not belong $(i = a, b, c)$. For example, in the case of M_o and M_a, the decisive group of owners is $\{1, 2\}$. Second, we consider M_i and M_j, $(i, j = a, b, c\ ;\ i \neq j)$, *i.e.*, a pair of ownership structures with merger. In all three cases, the decisive group of owners is equal to $\{1, 2, 3\}$. For example, in the case of M_a and M_b, if both owners 1 and 2 prefer M_a to M_b, while owner 3 disagrees on the foundation of the ownership structure M_a from M_b, it would be natural that owner 1, who has jointly operated

manager within each firm in deciding her/his delegation parameter which denotes the content of her/his managerial delegation contract with respect to her/his salary, different from the determination type of the FJSV delegation in the original setting of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985), *i.e.*, the maximization of each firm's profit. This is so because we would like to attempt to consider the effect of the disclosure of managerial delegation contract which increases the influence of the owner within each firm on her/his merger incentive and its welfare implication by taking into account the situation that the influence of the manager within her/his firm is high, when her/his delegation parameter is determined through the bargaining between the owner and the manager.

[14] See Horn and Persson (2001a) for the formal definition of the decisive group.

firm 13 with owner 3 in M_b must seek owner 3's cooperation in order to create a new ownership structure M_a. Thus, all the three owners are decisive.

Finally, in accordance with the above domination relation, we specify the concept of the core as our solution concept in order to predict the outcome of the merger formation based on the ranking of any pair of ownership structures. As usual, the core is defined as a set composed of ownership structures that are not dominated by any other ownership structure. Following Horn and Persson (2001a) and Straume (2006), we refer to ownership structures belonging to the core as *equilibrium ownership structures (EOSs)*.

2.4 Equilibrium ownership structure

We now discuss the equilibrium outcomes in each ownership structure. Let $\theta_i(M_j)$ denote the equilibrium outcome of the delegation parameter of owner i in the ownership structure M_j.[15]

We find that $\theta_i(M_j) > 0$ for all $\beta \in [0,1)$, i, and j. Thus, similar to a standard delegation game such as that in Straume (2006), the bargaining between a firm's owner and manager over the delegation parameter leads to more competitive equilibrium in each ownership structure. Further, we observe the same result as that in Straume (2006) that $\theta_i < \theta_j$ if $c_j < c_i$ for all i and j. Thus, we also obtain the result that the use of the sales delegation reinforces the asymmetry of each firm's market outcome caused by the initial difference of their marginal costs, which is similar to the results obtained in Fershtman and Judd (1987), Das (1997), Saracho (2002), and Straume (2006) under various environments.

Moreover, the increase of the relative bargaining power of the manager in each firm yields further differences in the outputs of a strong firm and a weak firm in all the market structures, *i.e.*, given that $c \in (0, \bar{c})$,

$$\begin{aligned}
&\frac{\partial\theta_1(M_o)}{\partial\beta} > \frac{\partial\theta_2(M_o)}{\partial\beta} > \frac{\partial\theta_3(M_o)}{\partial\beta}, \quad \forall\beta \in [0,1), \\
&\frac{\partial\theta_{12}(M_a)}{\partial\beta} > \frac{\partial\theta_3(M_a)}{\partial\beta}, \quad \forall\beta \in [0,1), \\
&\frac{\partial\theta_{13}(M_b)}{\partial\beta} > \frac{\partial\theta_2(M_b)}{\partial\beta}, \quad \forall\beta \in [0,1),
\end{aligned}$$

and

$$\frac{\partial\theta_1(M_c)}{\partial\beta} > \frac{\partial\theta_{23}(M_c)}{\partial\beta}, \quad \forall\beta \in [0,1).$$

[15] In each ownership structure, the equilibrium delegation parameter of each firm and the equilibrium market outcomes are represented in the Appendix.

Consequently, in all the ownership structures, the bargaining within each firm over the delegation parameter between an owner and a manager leads to a further increase in the market share of the strong firm, as compared to the case when the sales delegation is undertaken by firms' owners for profit-maximizing.[16]

Next, we present the payoff of each firm's owner in all the four ownership structures.[17] Let $V_i(M_j)$ denote an payoff of each firm i's owner in ownership j $(j = o, a, b, c)$.

(o) Decentralized ownership structure $\{\{1\},\{2\},\{3\}\}$**:** In this case, the three firms with independent operations, and the payoffs of the three firms' owners are respectively given as follows:

$$V_1(M_o) = \Pi_1(M_o) = \frac{3(1+\beta)[1-\beta+9c(1+\beta)]^2}{4(1-\beta)(5+4\beta)^2}, \quad V_2(M_o) = \Pi_2(M_o) = \frac{3(1-c)^2(1-\beta^2)}{4(5+4\beta)^2},$$
$$\text{and} \quad V_3(M_o) = \Pi_3(M_o) = \frac{3(1+\beta)[1-\beta-c(11+7\beta)]^2}{4(1-\beta)(5+4\beta)^2}.$$

(a) Merger between the low-cost and medium-cost firms $\{\{1,2\},\{3\}\}$**:** In this structure, firms 1 and 2 merge into a new firm 12, while firm 3 operates independently. Then, the sum of the payoffs of the owners of the pre-merged firms 1 and 2 obtained in firm 12, and the payoff of firm 3's owner are respectively given as follows:

$$V_{12}(M_a) = \Pi_{12}(M_a) = \frac{2(1+\beta)[1-\beta+4c(1+\beta)]^2}{(1-\beta)(5+3\beta)^2},$$
$$\text{and} \quad V_3(M_a) = \Pi_3(M_a) = \frac{2(1+\beta)[1-\beta-2c(3+\beta)]^2}{(1-\beta)(5+3\beta)^2}.$$

(b) Merger between the high-cost and low-cost firms $\{\{1,3\},\{2\}\}$**:** In this case, firms 1 and 3 merge into a new firm 13, while firm 2 operates independently. The sum of the payoffs of the

[16] In addition, the value of the partial differentiation of the delegation parameter of the relative weak firm with respect to β in each of the ownership structures may be negative when the degree of cost asymmetry among firms is sufficiently large, as is illustrated below:

$$\begin{cases} \partial\theta_3(M_o)/\partial\beta = \frac{6\left[(1-\beta)^2 - c\left(26+38\beta+17\beta^2\right)\right]}{(5-\beta-4\beta^2)^2} \gtreqless 0 \Leftrightarrow c \lesseqgtr \frac{(1-\beta)^2}{26+38\beta+17\beta^2} \left(< \bar{c}, \quad \forall\beta \in [0,1)\right), \\ \partial\theta_3(M_a)/\partial\beta = \frac{4\left[3(1-\beta)^2 - 4c\left(7+6\beta+3\beta^2\right)\right]}{(5-2\beta-3\beta^2)^2} \gtreqless 0 \Leftrightarrow c \lesseqgtr \frac{3(1-\beta)^2}{4(7+6\beta+3\beta^2)} \left(< \bar{c}, \quad \forall\beta \in (0.124647,1)\right), \\ \partial\theta_2(M_b)/\partial\beta = \frac{4\left[3(1-\beta)^2 + 2c\left(7+6\beta+3\beta^2\right)\right]}{(5-2\beta-3\beta^2)^2} \gtreqless 0 \Leftrightarrow c \lesseqgtr \frac{3(1-\beta)^2}{2(7+6\beta+3\beta^2)} \left(< \bar{c}, \quad \forall\beta \in (0.504889,1)\right), \\ \partial\theta_{23}(M_c)/\partial\beta = \frac{4\left[3(1-\beta)^2 - 2c\left(7+6\beta+3\beta^2\right)\right]}{(5-2\beta-3\beta^2)^2} \gtreqless 0 \Leftrightarrow c \lesseqgtr \frac{3(1-\beta)^2}{2(7+6\beta+3\beta^2)} \left(< \bar{c}, \quad \forall\beta \in (0.504889,1)\right). \end{cases}$$

[17] The case where $\beta = 0$ on the equilibrium payoffs of the firms' owners in each ownership structure as described below corresponds to that without the bargaining between an owner and a manager, which was obtained in Straume (2006). Note that the equilibrium outcomes obtained in this chapter do not completely coincide with those in Straume (2006) because he adopted a slightly different contract regime from that in this chapter; $U_i'(\Pi_i, q_i) = \theta_i\Pi_i + (1-\theta_i)q_i$.

owners of the pre-merged firms 1 and 3 obtained in firm 13 and the payoff of firm 2's owner are respectively given as follows:

$$V_{13}(M_b) = \Pi_{13}(M_b) = \frac{2(1+\beta)[1-\beta+2c(1+\beta)]^2}{(1-\beta)(5+3\beta)^2},$$
$$\text{and} \quad V_2(M_b) = \Pi_2(M_b) = \frac{2(1+\beta)[1-\beta-c(3+\beta)]^2}{(1-\beta)(5+3\beta)^2}.$$

(c) Merger between the high-cost and medium-cost firms $\{\{1\},\{2,3\}\}$**:** In this structure, firms 2 and 3 merge into a new firm 23. The sum of the payoffs of the owners of the pre-merged firms 2 and 3 obtained in firm 23 and the payoff of the owner of firm 1 are respectively given as follows:

$$V_1(M_c) = \Pi_1(M_c) = \frac{2(1+\beta)[1-\beta+2c(1+\beta)]^2}{(1-\beta)(5+3\beta)^2},$$
$$\text{and} \quad V_{23}(M_c) = \Pi_{23}(M_c) = \frac{2(1+\beta)(1-\beta-c(3+\beta)]^2}{(1-\beta)(5+3\beta)^2}.$$

The game is characterized by the following order of moves: In the first stage, the equilibrium ownership structure(s), *i.e.*, the $EOS(s)$ is/are determined by negotiation among the firms' owners. In the second stage, each firm's delegation parameter is determined through the bargaining between the owner and the manager within each firm, and subsequently, in the third and final stage, each firm's manager simultaneously chooses the output level with respect to the maximization of her/his FJSV delegation contract. The following illustrates each firm's incentive for a horizontal merger on the basis of the firm's owner, determined by the bargaining between the firm's owner and manager over the delegation parameter.

Proposition 2.1. *The $EOSs$ are classified into the following two cases in accordance with a direct measure of cost asymmetry in the industry, denoted below as c:*

$$\begin{cases} M_b \text{ or } M_c & \text{if} \quad c \in \left(0, \frac{2(1-\beta)^2}{3(13+14\beta+5\beta^2)}\right], \\ M_a & \text{if} \quad c \in \left[\frac{2(1-\beta)^2}{3(13+14\beta+5\beta^2)}, \bar{c}\right). \end{cases}$$

Proof. See Appendix. □

The statement of Proposition 2.1 coincides with that of Proposition 2 in Straume (2006). Thus, as indicated in Straume (2006), our result is analogous to those in González-Maestre and López-Cuñat (2001) and Ziss (2001). These papers showed that the optimal managerial delegation contract à la Fershtman and Judd (1987) and Sklivas (1987) will generally increase the profitability of horizontal mergers in a symmetric Cournot system. Moreover, in the case where

the original firms are cost-asymmetric, it is well-known that the use of the managerial delegation contract increases the profitability of horizontal mergers as compared to its non-use. Thus, similar to Straume (2006), in the case where the owner and manager within each firm bargain over the delegation parameter, an ownership structure(s) with some type of merger is/are $EOS(s)$ for all $c \in (0, \bar{c})$. The increase in the relative bargaining power of each firm's manager, β implies aggressive behavior of each firm in market competition, and the low-cost firm becomes a stronger competitor in the ownership structure with some merger.[18] Further, the effect of the increase of cost asymmetry among the initial three firms plays the same role as that of the increase of β.[19] Thus, in particular, when the degree of cost asymmetry of the industry is sufficiently high, the efficient merged entity, as a result of the merger between firms 1 and 2, enjoys its near-monopoly position, wherein the industry profit in M_a is higher than those in M_b and M_c. Hence, ownership structure M_a is a unique EOS. On the other hand, in the case where the value of c is sufficiently low, the other two types of ownership structures with different mergers are $EOSs$, and thus, are in the core.

[18] In ownership structures with some type of merger, the market shares of the stronger firms in each structure are as follows: for any $\beta \in [0, 1)$,

$$\frac{q_{12}(M_a)}{Q(M_a)} = \frac{1-\beta+4c(1+\beta)}{2(1-c)(1-\beta)} > \frac{1-\beta+2c(1+\beta)}{(2-c)(1-\beta)} = \frac{q_{13}(M_b)}{Q(M_b)} = \frac{q_1(M_c)}{Q(M_c)} > 0, \quad \forall c \in (0, \bar{c}).$$

Thus, the market share of the stronger firm in M_a is the highest among those in the three ownership structures with some type of merger. Moreover, the partial differentiation of the market share of the stronger firm in each regime with some merger with respect to β is as follows: for any $\beta \in [0, 1)$,

$$\partial\left(\frac{q_{12}(M_a)}{Q(M_a)}\right)/\partial\beta = \frac{4c}{(1-c)(1-\beta)^2} > \frac{4c}{(2-c)(1-\beta)^2} = \partial\left(\frac{q_{13}(M_b)}{Q(M_b)}\right)/\partial\beta = \partial\left(\frac{q_1(M_c)}{Q(M_c)}\right)/\partial\beta > 0,$$
$$\forall c \in (0, \bar{c}).$$

Thus, as the value of the relative bargaining power of each firm's manager, β increases, the market share of the low-cost firm in each structure with some form of merger increases. Consequently, the stronger firm in each ownership structure with some type of merger enjoys a more advantageous position in this model with $\beta \in [0, 1)$, relative to Straume (2006)'s model, which corresponds to $\beta = 0$.

[19] The partial differentiation of the market share of the stronger firm in each regime with some merger with respect to c is as follows: for any $\beta \in [0, 1)$,

$$\partial\left(\frac{q_{12}(M_a)}{Q(M_a)}\right)/\partial c = \frac{5+3\beta}{2(1-c)^2(1-\beta)} > \frac{5+3\beta}{(2-c)^2(1-\beta)} = \partial\left(\frac{q_{13}(M_b)}{Q(M_b)}\right)/\partial c = \partial\left(\frac{q_1(M_c)}{Q(M_c)}\right)/\partial c > 0,$$
$$\forall c \in (0, \bar{c}).$$

Therefore, similar to the increase in β, the increase in c provides the stronger firm in each ownership structure with some type of merger with more advantage.

2.5 Socially preferred ownership structure *versus* equilibrium ownership structure

Our next interest is to examine whether the most preferred ownership structure with respect to social welfare coincides with the EOS. If $W(M_i) > W(M_j)$, M_i is considered to be *socially preferred to* M_j and is represented as $M_i \succ M_j$. In this section, since $W(M_b) = W(M_c)$, the two ownership structures M_b and M_c are together referred to as M_b. By comparing the social welfare in different ownership structures, we obtain the following proposition.

Proposition 2.2. *In accordance with the value of* c*, the most preferred ownership structure with respect to social welfare is classified according to the following two cases:*

$$\begin{cases} M_b & \textit{if } c \in (c^*, c^{**}), \ \textit{when } \beta \in (0.174504, 1), \ \textit{and} \\ M_o & \textit{otherwise}, \end{cases}$$

where

$$c^* = \frac{\begin{bmatrix} 1275 + 950\beta - 852\beta^2 - 1062\beta^3 - 311\beta^4 \\ -4\left(25 + 10\beta - 23\beta^2 - 12\beta^3\right)\sqrt{-39 + 194\beta + 169\beta^2} \end{bmatrix}}{26875 + 71510\beta + 71692\beta^2 + 31692\beta^3 + 5321\beta^4}$$

and

$$c^{**} = \frac{\begin{bmatrix} 1275 + 950\beta - 852\beta^2 - 1062\beta^3 - 311\beta^4 \\ +4\left(25 + 10\beta - 23\beta^2 - 12\beta^3\right)\sqrt{-39 + 194\beta + 169\beta^2} \end{bmatrix}}{26875 + 71510\beta + 71692\beta^2 + 31692\beta^3 + 5321\beta^4}.$$

Proof. See Appendix. □

The merger among relatively low-cost firms yields a higher market concentration, resulting in an increase in the market price, and thus, distorting the market allocation. In ownership structure M_a, the large cost difference between firms 12 and 3 increases the market price. Consequently, its negative effect toward social welfare outweighs the positive effect of cost saving arising from such a merger. Moreover, by applying the discussion in Straume (2006), the intuition behind the fact that M_o is the most socially preferred ownership structure of the four, in the large range of direct measures of cost asymmetry in the industry c is explained as follows. With an increase in the relative bargaining power of the manager, β, vis-à-vis that of the owner, production becomes aggressive. This deters the output from rival firms and results in a greater market share of the low-cost firm in this case than in the case of the managerial delegation à la Straume. Thus, the increase in the value of β enhances the efficiency of production allocation in M_o. As a result, the social welfare in ownership structures with some type of merger is lower than that of the

decentralized ownership structure in the large range of the value of parameter c. On the other hand, at the medium level of c, either ownership structure — M_b or M_c — becomes the most preferred ownership structure with respect to social welfare within a large range of parameter β. At the medium level of c, the positive effect toward social welfare, whereby the merger involving the highest cost firm leads to the saving of the total cost in the industry, outweighs the negative effect of underproduction arising from such a merger. In contrast to Proposition 4 in Straume (2006), in the case where each firm's owner and manager bargain over the delegation parameter, we find that there exist certain values of parameter c such that the social welfare in both ownership structures M_b and M_c is higher than that in M_o. Considering the above two propositions, we obtain the following result.

Proposition 2.3. *In the case where an owner and a manager bargain over the delegation parameter, the EOS, M_b or M_c, coincides with the most preferred ownership structure with respect to social welfare for any $c \in \left(c^*, 2\left(1-\beta\right)^2/3\left(13+14\beta+5\beta^2\right)\right]$ when $\beta \in (0.193340, 1)$.*

Proof. See Appendix. □

Unlike Proposition 5 in Straume (2006), we find that the bargaining between a firm's owner and manager over the delegation parameter decreases the necessity for regulating merger policies by antitrust authorities in the medium level of a direct measure of cost asymmetry in the industry, c, when the relative bargaining power of the manager of each firm exceeds one-fifth. As described in Introduction of this chapter, modern corporate governance codes include clauses requiring the disclosure of managerial delegation contract in order to protect the owner-shareholders from the opportunistic behaviors of the managers. Thus, since the presence of such clauses in corporate governance lead to increasing the influence of the owner within each firm, it results in decreasing the level of β. From Proposition 2.3, we realize that the revelation of each firm's delegation contract may increase the necessity of a more active merger policy, if the value of β is around $1/5$ and the direct measure of cost asymmetry in the industry, c is intermediate-level.

2.6 Concluding Remarks

This chapter examined horizontal mergers in a private oligopoly composed of asymmetric firms with respect to productivity, where owners and managers bargain over FJSV delegation contracts. We found that there exists a range of values of the direct measure of cost asymmetry in the industry, c, such that the social welfare in the ownership structure with some type of merger is

higher than that in the decentralized ownership structure, when the bargaining over the delegation parameter between each firm's owner and manager occurs. Thus, in our setting, the disclosure of managerial delegation contracts required by modern corporate governance codes may lead to an appropriate type of merger with respect to social welfare, and consequently, the scope for an antitrust policy may decrease. It is worth mentioning that a socially preferred merger occurs as an equilibrium where the relative bargaining power of each firm's manager influences her/his managerial contract through the bargaining between an owner and a manager, in contrast to the result in Straume (2006) where the relative bargaining power of the manager is not considered.

An interesting extension of our model would be to investigate the robustness of our result the sequential-movement of each firm in the market, for example, the two types of Stackelberg competitions for the merged firm's leader and follower. As another plausible next step, it would be fruitful to examine the implications of international trade in an international oligopoly. Note that the extension of the analysis in this chapter towards such a direction has been already attempted in Nakamura (2010). Although Nakamura (2010) considered the influence of the bargaining over the delegation parameter between an owner and a manger within each firm on her/his merger incentive and its welfare implications in the context of the international oligopoly, he restricted his attention into the situation wherein each firm's cost function is symmetric. Thus, the research on the horizontal mergers in the international oligopoly with asymmetric firms which was investigated in this chapter is still left.

Furthermore, as one of our future researches, we should consider the other managerial delegation regimes except for the sales delegation, for example, the market share delegation presented in Jansen et al. (2007) and Ritz (2008), and the relative-performance delegation case in Miller and Pazgal (2001, 2002) and Salas Fumas (1992), but in the above two delegation regimes, the market share delegation regime that each firm i's delegation contract is the weighted sum of her/his profit (Π_i) and market share (q_i/Q) and the relative performance delegation regime that each firm i's delegation contract is the weighted sum of her/his profit (Π_i) and the other firm j's profit (Π_j), $(i \neq j)$, we cannot obtain the result on each firm's merger incentive and its welfare implication from the calculational problem occuring in solving the subgame perfect Nash equilibrium of each ownership structure.[20] Therefore, if in the other managerial delegation regimes except for the sales delegation, we would like to attempt to clarify the influence of the bargain-

[20]In both the market share delegation and the relative performance delegation, the subgame perfect Nash equilibrium in each ownership structure cannot be obtained owing to the asymmetry of her/his marginal cost among firms, although they can be explicitly obtained in the main previous works regarding the market share delegation of Jansen et al. (2007) and Ritz (2008), and regarding the relative performance delegation of Miller and Pazgal (2001, 2002) and Salas Fumas (1992), respectively, in the case wherein the cost function of each firm is symmetric.

ing over the delegation parameter between an owner and a manager within each firm on her/his merger incentive and its welfare implications under the same setting as that in this chapter which the existing firm's productivity is different with each other, we must engage in simulation and numerical analyses using computer softwares in order to obtain some sort of tractable results. Extending our model into these directions remains for our future researches.

Appendix

Equilbrium outcomes in four types of market outcomes

(o) Decentralized ownership structure $\{\{1\},\{2\},\{3\}\}$**:** In this structure, we consider that the equilibrium outcome of the bargaining between the owners and managers over the delegation parameter within each of the three firms. In the third stage, the three firms' managers simultaneously choose their output levels with respect to the maximization of their FJSV delegation contract. The delegation contract of the firm's manager is

$$U_i\left(q_j; j=1,2,3\right) = \left(1-\sum_{i=1}^{3} q_i\right) q_i - \left[\left(i-1\right)c - \theta_i\right] q_i, \quad i=1,2,3.$$

Then, as a consequence of the simultaneous quantity-setting competition among the three firm's managers in the third stage, the output level of each firm is obtained as follows:

$$q_i\left(\theta_j; j=1,2,3\right) = \frac{1}{4}\left[1+\left(7-4i\right)c+3\theta_i - \sum_{j=1,j\neq i}^{3}\theta_j\right], \quad i=1,2,3.$$

Further, the bargaining function of each firm in the second stage is as follows:

$$\mathfrak{B}_i\left(\theta_j; j=1,2,3\right) = U_i^{\beta}\Pi_i^{(1-\beta)}, \quad i=1,2,3,$$

where

$$U_i\left(\theta_j; j=1,2,3\right) = \frac{1}{16}\left[1+\left(7-4i\right)c+3\theta_i - \sum_{j=1,j\neq i}^{3}\theta_j\right]^2,$$

$$\Pi_i\left(\theta_j; j=1,2,3\right) = \frac{1}{16}\left[1+\left(7-4i\right)c+3\theta_i - \sum_{j=1,j\neq i}^{3}\theta_j\right]\left[1+\left(7-4i\right)c - \sum_{i=1}^{3}\theta_i\right],$$

$$i=1,2,3.$$

The solution to the bargaining problem of each firm in the second stage is

$$\theta_i\left(\theta_j; i\neq j\right) = arg\max_{\theta_i}\mathfrak{B}_i\left(\theta_j; j=1,2,3\right)$$

$$\Leftrightarrow 1+\left(1+2\beta\right)\left(7-4i\right)c - 3\theta_i - \sum_{j=1,j\neq i}^{3}\theta_j + 2\beta\left(1-\sum_{j=1,j\neq i}^{3}\theta_j\right) = 0, \quad i=1,2,3,$$

yielding

$$\theta_1(M_o) = \frac{(1+2\beta)[1-\beta+9c(1+\beta)]}{(1-\beta)(5+4\beta)}, \quad \theta_2(M_o) = \frac{(1+2\beta)(1-c)}{5+4\beta},$$
$$\theta_3(M_o) = \frac{(1+2\beta)[1-\beta-c(11+7\beta)]}{(1-\beta)(5+4\beta)}.$$

The equilibrium market outcomes of each firm's output, consumer surplus, and social welfare are obtained as follows:

$$q_1(M_o) = \frac{3(1+\beta)[1-\beta+9c(1+\beta)]}{2(1-\beta)(5+4\beta)}, \quad q_2(M_o) = \frac{3(1-c)(1+\beta)}{2(5+4\beta)},$$
$$q_3(M_o) = \frac{3(1+\beta)[1-\beta-c(11+7\beta)]}{2(1-\beta)(5+4\beta)}, \quad Q(M_o) = \frac{9(1-c)(1+\beta)}{2(5+4\beta)},$$
$$CS(M_o) = \frac{81(1-c)^2(1+\beta)^2}{8(5+4\beta)^2},$$
$$W(M_o) = \frac{3(1+\beta)[3(1-2c)(1-\beta)(11+7\beta)+c^2(433+628\beta+235\beta^2)]}{8(1-\beta)(5+4\beta)^2}.$$

(a) Merger between low-cost and medium-cost firms $\{\{1,2\},\{3\}\}$**:** Next, we consider the case where the two relatively low-cost firms merge into a new firm denoted by 12. Let q_{12} and Π_{12} be the output level and profit of firm 12, respectively. In this structure, in the third stage, the delegation contracts of the managers of both firm 12 and firm 3 are

$$\begin{cases} U_{12}(q_{12},q_3) = [1-(q_{12}+q_3)+\theta_{12}]q_{12}, & \text{and} \\ U_3(q_{12},q_3) = [1-(q_{12}+q_3)]q_3 - [2c-\theta_3]q_3, \end{cases}$$

respectively. As a consequence of the market competition in the third stage, the output level of each firm is obtained as follows:

$$q_{12}(\theta_{12},\theta_3) = \frac{1}{3}(1+2c+2\theta_{12}-\theta_3), \quad q_3(\theta_{12},\theta_3) = \frac{1}{3}(1-4c-\theta_{12}+2\theta_3).$$

Then, the bargaining function of firm 12 in the second stage is as follows:

$$\mathfrak{B}_{12}(\theta_{12},\theta_3) = \frac{1}{9}\left[(1+2c+2\theta_{12}-\theta_3)^2\right]^{\beta}\left[(1+2c-\theta_{12}-\theta_3)(1+2c+2\theta_{12}-\theta_3)\right]^{(1-\beta)}.$$

Moreover, the bargaining function of firm 3 in the second stage is as follows:

$$\mathfrak{B}_3(\theta_{12},\theta_3) = \frac{1}{9}\left[(1-4c-\theta_{12}+2\theta_3)^2\right]^{\beta}\left[(1-4c-\theta_{12}+2\theta_3)(1-4c-\theta_{12}-\theta_3)\right]^{(1-\beta)}.$$

The solution to the bargaining problem of each firm in the second stage is obtained as follows:

$$\begin{cases} \theta_{12}(\theta_3) = arg\max_{\theta_{12}} \mathfrak{B}_{12}(\theta_{12},\theta_3) \Leftrightarrow 1+c(2+6\beta)-4\theta_{12}+3\beta(1-\theta_3)-\theta_3 = 0, \\ \theta_3(\theta_{12}) = arg\max_{\theta_3} \mathfrak{B}_3(\theta_{12},\theta_3) \Leftrightarrow 1-4c(1+3\beta)+3\beta(1-\theta_{12})-\theta_{12}-4\theta_3 = 0, \end{cases}$$

yielding

$$\theta_{12}(M_a) = \frac{(1+3\beta)[1-\beta+4c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad \theta_3(M_a) = \frac{(1+3\beta)[1-\beta-2c(3+\beta)]}{(1-\beta)(5+3\beta)}.$$

In the equilibrium, we obtain the following market outcomes:

$$q_{12}(M_a) = \frac{2(1+\beta)[1-\beta+4c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad q_3(M_a) = \frac{2(1+\beta)[1-\beta-2c(3+\beta)]}{(1-\beta)(5+3\beta)},$$
$$Q(M_a) = \frac{4(1-c)(1+\beta)}{5+3\beta}, \quad CS(M_a) = \frac{8(1-c)^2(1+\beta)^2}{(5+3\beta)^2},$$
$$W(M_a) = \frac{4(1+\beta)[(1-2c)(1-\beta)(3+\beta)+4c^2(7+7\beta+2\beta^2)]}{(1-\beta)(5+3\beta)^2}.$$

(b) Merger between low-cost and high-cost firms $\{\{1,3\},\{2\}\}$**:** In this structure, the most efficient and inefficient firms—firms 1 and 3, respectively—merge into a new firm 13. Let q_{13} and Π_{13} denote the output and profit, respectively, of the merged firm 13. In this structure, the delegation contracts of both firm 13 and firm 2 in the third stage are obtained as follows:

$$\begin{cases} U_{13}(q_{13},q_2) = [1-(q_{13}+q_2)+\theta_{13}]\,q_{13}, \\ U_2(q_{13},q_2) = [1-(q_{13}+q_2)]\,q_2-(c-\theta_2)\,q_2. \end{cases}$$

In the market competition, the following result on each firm's output level in the third stage is

$$q_{13}(\theta_{13},\theta_2) = \frac{1}{3}(1+c+2\theta_{13}-\theta_2), \quad q_2(\theta_{13},\theta_2) = \frac{1}{3}(1-2c-\theta_{13}+2\theta_2).$$

Then, the bargaining function of firm 13 in the second stage is obtained as follows:

$$\mathfrak{B}_{13}(\theta_{13},\theta_2) = \frac{1}{9}\left[(1+c+2\theta_{13}-\theta_2)^2\right]^{\beta}\left[(1+c-\theta_{13}-\theta_2)(1+c+2\theta_{13}-\theta_2)\right]^{(1-\beta)}.$$

Furthermore, the bargaining function of firm 2 in the second stage is as follows:

$$\mathfrak{B}_2(\theta_{13},\theta_2) = \frac{1}{9}\left[(1-2c-\theta_{13}+2\theta_2)^2\right]^{\beta}\left[(1-2c-\theta_{13}+2\theta_2)(1-2c-\theta_{13}-\theta_2)\right]^{(1-\beta)}.$$

The solution to the bargaining problem of each firm in the second stage is obtained as follows:

$$\begin{cases} \theta_{13}(\theta_2) = arg\max_{\theta_{13}} \mathfrak{B}_{13}(\theta_{13},\theta_2) \Leftrightarrow 1+c+3c\beta-4\theta_{13}+3\beta(1-\theta_2)-\theta_2 = 0, \\ \theta_2(\theta_{13}) = arg\max_{\theta_2} \mathfrak{B}_2(\theta_{13},\theta_2) \Leftrightarrow 1-c(2+6\beta)+3\beta(1-\theta_{13})-\theta_{13}-4\theta_2 = 0, \end{cases}$$

yielding

$$\theta_{13}(M_b) = \frac{(1+3\beta)[1-\beta+2c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad \theta_2(M_b) = \frac{(1+3\beta)[1-\beta-c(3+\beta)]}{(1-\beta)(5+3\beta)}.$$

Moreover, the equilibrium market outcomes are obtained as follows:

$$q_{13}(M_b) = \frac{2(1+\beta)[1-\beta+2c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad q_2(M_b) = \frac{2(1+\beta)[1-\beta-c(3+\beta)]}{(1-\beta)(5+3\beta)};$$
$$Q(M_b) = \frac{2(2-c)(1+\beta)}{5+3\beta}, \quad CS(M_b) = \frac{2(2-c)^2(1+\beta)^2}{(5+3\beta)^2},$$
$$W(M_b) = \frac{4(1+\beta)[(1-c)(1-\beta)(3+\beta)+c^2(7+7\beta+2\beta^2)]}{(1-\beta)(5+3\beta)^2}.$$

(c) Merger between high-cost and medium-cost firms $\{\{1\},\{2,3\}\}$**:** Finally, we examine the case where the two relatively inefficient firms—firms 2 and 3—merge into one firm denoted by 23. Let q_{23} and Π_{23} be firm 23's output level and profit, respectively. In the third stage, the delegation contracts of both the firms 1 and 23 are obtained as follows:

$$\begin{cases} U_1(q_1,q_{23}) = [1-(q_1+q_{23})+\theta_1]q_1, \\ U_{23}(q_1,q_{23}) = [1-(q_1+q_{23})]q_{23}-(c-\theta_{23})q_{23}. \end{cases}$$

The following output level of each firm is obtained as a result of market competition:

$$q_1(\theta_1,\theta_{23}) = \frac{1}{3}(1+c+2\theta_1-\theta_{23}), \quad q_{23}(\theta_1,\theta_{23}) = \frac{1}{3}(1-2c-\theta_1+2\theta_{23}).$$

Then, the bargaining function of firm 1 in the second stage is obtained as follows:

$$\mathfrak{B}_1(\theta_1,\theta_{23}) = \frac{1}{9}\left[(1+c+2\theta_1-\theta_{23})^2\right]^{\beta}\left[(1+c-\theta_1-\theta_{23})(1+c+2\theta_1-\theta_{23})\right]^{(1-\beta)}.$$

Moreover, the bargaining function of firm 23 is obtained as follows:

$$\mathfrak{B}_{23}(\theta_1,\theta_{23}) = \frac{1}{9}\left[(1-2c-\theta_1+2\theta_{23})^2\right]^{\beta}\left[(1-2c-\theta_1+2\theta_{23})(1-2c-\theta_1-\theta_{23})\right]^{(1-\beta)}.$$

The solution to the bargaining problem of each firm is

$$\begin{cases} \theta_1(\theta_{23}) = arg\max_{\theta_1}\mathfrak{B}_1(\theta_1,\theta_{23}) \Leftrightarrow 1+c+3c\beta-4\theta_1+3\beta(1-\theta_{23})-\theta_{23}=0, \\ \theta_{23}(\theta_1) = arg\max_{\theta_{23}}\mathfrak{B}_{23}(\theta_1,\theta_{23}) \Leftrightarrow 1-c(2+6\beta)+3\beta(1-\theta_1)-\theta_1-4\theta_{23}=0, \end{cases}$$

yielding

$$\theta_1(M_c) = \frac{(1+3\beta)[1-\beta+2c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad \theta_{23}(M_c) = \frac{(1+3\beta)[1-\beta-c(3+\beta)]}{(1-\beta)(5+3\beta)}.$$

Furthermore, we obtain the equilibrium market outcomes as follows:

$$q_1(M_c) = \frac{2(1+\beta)[1-\beta+2c(1+\beta)]}{(1-\beta)(5+3\beta)}, \quad q_{23}(M_c) = \frac{2(1+\beta)[1-\beta-c(3+\beta)]}{(1-\beta)(5+3\beta)},$$
$$Q(M_c) = \frac{2(2-c)(1+\beta)}{5+3\beta}, \quad CS(M_c) = \frac{2(2-c)^2(1+\beta)^2}{(5+3\beta)^2},$$
$$W(M_c) = \frac{4(1+\beta)[(1-c)(1-\beta)(3+\beta)+c^2(7+7\beta+2\beta^2)]}{(1-\beta)(5+3\beta)^2}.$$

Proof of Proposition 2.1

We compare the three ownership structures with some type of merger involving the decentralized ownership structure. Thus, we show that the ownership structure M_o is dominated by any of the three ownership structures with a merger. First, we obtain the following result regarding the two ownership structures M_o and M_a. Considering that

$$\frac{d\left(V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right)}{dc}$$
$$=\frac{\left(1-\beta\right)\left[100+205\beta+33\beta^2-217\beta^3-121\beta^4-c\left(1475+3930\beta+3638\beta^2+1242\beta^3+83\beta^4\right)\right]}{\left(1-\beta\right)\left(25+35\beta+12\beta^2\right)^2}$$
$$\gtreqless 0 \Leftrightarrow c \lesseqgtr \frac{\left(1-\beta\right)\left(100+305\beta+338\beta^2+121\beta^3\right)}{1475+3930\beta+3638\beta^2+1242\beta^3+83\beta^4},$$

and

$$\bar{c}-\frac{\left(1-\beta\right)\left(100+305\beta+338\beta^2+121\beta^3\right)}{1475+3930\beta+3638\beta^2+1242\beta^3+83\beta^4}$$
$$=\frac{\left(1-\beta\right)\left(5+4\beta\right)\left(75-85\beta-375\beta^2-191\beta^3\right)}{\left(11+7\beta\right)\left(1475+3930\beta+3638\beta^2+1242\beta^3+83\beta^4\right)} \gtreqless 0 \Leftrightarrow \beta \gtreqless 0.328092,$$

we will analyze the following two cases: (a) $\beta \geq 0.328092$ and (b) $\beta < 0.328092$.

$Case\,(a):\ \beta \geq 0.328092$

In this case, we obtain the result that

$$\min_{c\in[0,\bar{c})}\left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]=\left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]\Big|_{c=0}$$
$$=\frac{\left(1-\beta\right)\left(1+\beta\right)\left(25+70\beta+37\beta^2\right)}{2\left(25+35\beta+12\beta^2\right)^2}>0, \qquad \forall\beta\in[0,1).$$

$Case\,(b):\ \beta < 0.328092$

In this case, we obtain the result that

$$\min_{c\in[0,\bar{c}]}\left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]$$
$$=\min\left\{\left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]\Big|_{c=0},\ \left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]\Big|_{c=\bar{c}}\right\}$$
$$=\left[V_{12}\left(M_a\right)-\left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)\right]\Big|_{c=0}$$
$$=\frac{\left(1-\beta\right)\left(1+\beta\right)\left(25+70\beta+37\beta^2\right)}{2\left(25+35\beta+12\beta^2\right)^2}>0, \qquad \forall\beta\in[0,1).$$

Thus, from the analysis of the above two cases, we obtain that $V_{12}\left(M_a\right) > \left(V_1\left(M_o\right)+V_2\left(M_o\right)\right)$ for any $c \in (0, \bar{c})$.

Second, we compare both the values of $V_{13}(M_b)$ and $(V_1(M_o)+V_3(M_o))$. Since

$$\begin{aligned}&\frac{d\left(V_{13}(M_b)-(V_1(M_o)+V_3(M_o))\right)}{dc}\\&=\frac{(1+\beta)\left[275+260\beta-150\beta^2-284\beta^3-101\beta^4-c\left(7175+19500\beta+19886\beta^2+8964\beta^3+1499\beta^4\right)\right]}{(1-\beta)\left(25+35\beta+12\beta^2\right)^2}\\&\gtreqless 0\Leftrightarrow c\lesseqgtr\frac{(1-\beta)\left(275+535\beta+385\beta^2+101\beta^3\right)}{7175+19500\beta+19886\beta^2+8964\beta^3+1499\beta^4},\end{aligned}$$

and

$$\begin{aligned}&\bar{c}-\frac{(1-\beta)\left(275+535\beta+385\beta^2+101\beta^3\right)}{7175+19500\beta+19886\beta^2+8964\beta^3+1499\beta^4}\\&=\frac{2(1-\beta)(1+\beta)(5+4\beta)\left(415+422\beta+99\beta^2\right)}{(11+7\beta)\left(7175+19500\beta+19886\beta^2+8964\beta^3+1499\beta^4\right)}>0,\qquad\forall\beta\in[0,1),\end{aligned}$$

we get

$$\begin{aligned}&\min_{c\in[0,\bar{c}]}\left[V_{13}(M_b)-(V_1(M_o)+V_3(M_o))\right]\\&=\min\left\{\left[V_{13}(M_b)-(V_1(M_o)+V_3(M_o))\right]\Big|_{c=0},\ \left[V_{13}(M_b)-\left(V_1^A(M_o)+V_3^A(M_o)\right)\right]\Big|_{c=\bar{c}}\right\}\\&=\left[V_{13}(M_b)-(V_1(M_o)+V_3(M_o))\right]\Big|_{c=\bar{c}}\\&=\frac{2(1-\beta)(1+\beta)\left(19+54\beta+27\beta^2\right)}{\left(55+68\beta+21\beta^2\right)^2}>0,\qquad\forall\beta\in[0,1).\end{aligned}$$

Thus, we obtain the result that $V_{13}(M_b)>(V_1(M_o)+V_3(M_o))$ for any $c\in(0,\bar{c})$.

Third, we compare $V_{23}(M_c)$ with $(V_2(M_o)+V_3(M_o))$. Since

$$\begin{aligned}&\frac{d\left(V_{23}(M_c)-(V_2(M_o)+V_3(M_o))\right)}{dc}\\&=\frac{(1+\beta)\left[150+35\beta-105\beta^2-63\beta^3-17\beta^4-c\left(3675+9150\beta+8726\beta^2+3758\beta^3+611\beta^4\right)\right]}{(1-\beta)\left(25+35\beta+12\beta^2\right)^2}\\&\gtreqless 0\Leftrightarrow c\lesseqgtr\frac{(1-\beta)\left(150+185\beta+80\beta^2+17\beta^3\right)}{3675+9150\beta+8726\beta^2+3758\beta^3+611\beta^4},\end{aligned}$$

and

$$\begin{aligned}&\bar{c}-\frac{(1-\beta)\left(150+185\beta+80\beta^2+17\beta^3\right)}{3675+9150\beta+8726\beta^2+3758\beta^3+611\beta^4}\\&=\frac{(1-\beta)(5+4\beta)\left(405+889\beta+599\beta^2+123\beta^3\right)}{(11+7\beta)\left(3675+9150\beta+8726\beta^2+3758\beta^3+611\beta^4\right)}>0,\qquad\forall\beta\in[0,1),\end{aligned}$$

we find that

$$\begin{aligned}
&\min_{c\in[0,\bar{c}]}\left[V_{23}\left(M_c\right)-\left(V_2\left(M_o\right)+V_3\left(M_o\right)\right)\right]\\
&=\min\left\{\left[V_{23}\left(M_c\right)-\left(V_2\left(M_o\right)+V_3\left(M_o\right)\right)\right]\Big|_{c=0},\ \left[V_{23}\left(M_c\right)-\left(V_2\left(M_o\right)+V_3\left(M_o\right)\right)\right]\Big|_{c=\bar{c}}\right\}\\
&=\left[V_{23}\left(M_c\right)-\left(V_2\left(M_o\right)+V_3\left(M_o\right)\right)\right]\Big|_{c=\bar{c}}\\
&=\frac{\left(1-\beta\right)\left(1+\beta\right)\left(53+102\beta+45\beta^2\right)}{\left(55+68\beta+21\beta^2\right)^2}>0, \qquad \forall\beta\in[0,1).
\end{aligned}$$

Thus, we find that $V_{23}\left(M_c\right)>\left(V_2\left(M_o\right)+V_3\left(M_o\right)\right)$ for any $c\in(0,\bar{c})$. The above three results show that the decentralized ownership structure is dominated by all three ownership structures characterized by the presence of some form of merger.

Next, by comparing two different types of structures with any merger, we obtain the following two results

$$\begin{aligned}
V_{13}\left(M_b\right)+V_2\left(M_b\right)&=\frac{2\left(1+\beta\right)\left[2\left(1-\beta\right)^2-2c\left(1-\beta\right)^2+c^2\left(13+14\beta+5\beta^2\right)\right]}{\left(1-\beta\right)\left(5+3\beta\right)^2}\\
&=V_1\left(M_c\right)+V_{23}\left(M_c\right);\\
V_{12}\left(M_a\right)+V_3\left(M_a\right)-\left(V_{13}\left(M_b\right)+V_2\left(M_b\right)\right)&=\frac{2c\left(1+\beta\right)\left[3c\left(13+14\beta+5\beta^2\right)-2\left(1-\beta\right)^2\right]}{\left(1-\beta\right)\left(5+3\beta\right)^2}\\
>0\Leftrightarrow c\in\left(\frac{2\left(1-\beta\right)^2}{3\left(13+14\beta+5\beta^2\right)},\bar{c}\right)&\subsetneq(0,\bar{c}), \quad \forall\beta\in[0,1).
\end{aligned}$$

Thus, M_a dominates both M_b and M_c if $c>2\left(1-\beta\right)^2/3\left(13+14\beta+5\beta^2\right)$. Consequently, we obtain the result that both M_b and M_c are not dominated by any other ownership structure and are also not dominated by each other for any $c\in\left(0,2\left(1-\beta\right)^2/3\left(13+14\beta+5\beta^2\right)\right]$, whereas M_a is never dominated by any other ownership structure for any $c\in\left[2\left(1-\beta\right)^2/3\left(13+14\beta+15\beta^2\right),\bar{c}\right)$.

□

Proof of Proposition 2.2

First, we compare the values of both $W\left(M_o\right)$ and $W\left(M_b\right)$.

$$\begin{aligned}
&W\left(M_b\right)-W\left(M_o\right)\\
&=-\frac{\left(1+\beta\right)\left[\begin{array}{c}5\left(1-\beta\right)^3\left(15+11\beta\right)-2c\left(1275+950\beta-852\beta^2-1062\beta^3-311\beta^4\right)\\+c^2\left(26875+71510\beta+71692\beta^2+31692\beta^3+5321\beta^4\right)\end{array}\right]}{8\left(1-\beta\right)\left(25+35\beta+12\beta^2\right)^2}>0\\
&\Leftrightarrow c^*<c<c^{**}, \qquad \forall\beta\in(0.174504,1).
\end{aligned}$$

Note that the condition on the value of β for both c^* and c^{**} to become real roots is that $\beta \in [0.174504, 1)$ and $\bar{c} > c^{**}$ for any $\beta \in [0.174504, 1)$.

Second, by comparing $W(M_a)$ with $W(M_b)$, we obtain the following result:

$$W(M_b) - W(M_a) = \frac{4c(1+\beta)\left[3 - 2\beta - \beta^2 - 3c(7 + 7\beta + 2\beta^2)\right]}{(1-\beta)(5+3\beta)^2} > 0,$$
$$\Leftrightarrow c \in \left(0, \frac{3 - 2\beta - \beta^2}{3(7 + 7\beta + 2\beta^2)}\right) (\supsetneq (0, \bar{c})), \quad \forall \beta \in [0, 1).$$

Thus, we find that $M_b \succ M_a$ for any value of $c \in (0, \bar{c})$. Therefore, we have the desired result in the statement of this proposition. □

Proof of Proposition 2.3

Let us recall that both the ownership structures, M_b and M_c, are in the core if $c \in \left(0, 2(1-\beta)^2 / (13 + 14\beta + 5\beta^2)\right)$ for any $\beta \in [0, 1)$. Then, we derive the condition of β, such that $c^* < 2(1-\beta)^2/3(13 + 14\beta + 5\beta^2) < c^{**}$. By simple calculations, we obtain the following results: for any $\beta \in (0.174504, 1)$,

$$\frac{2(1-\beta)^2}{3(13 + 14\beta + 5\beta^2)} - c^*$$
$$= \frac{(1-\beta)\begin{bmatrix} 4025 - 51055\beta - 165758\beta^2 - 182630\beta^3 - 86939\beta^4 - 15307\beta^5 \\ +12\sqrt{-39 + 194\beta + 169\beta^2}\,(325 + 805\beta + 771\beta^2 + 343\beta^3 + 60\beta^4) \end{bmatrix}}{3(13 + 14\beta + 5\beta^2)(26875 + 71510\beta + 71692\beta^2 + 31962\beta^3 + 5321\beta^4)} \gtreqless 0$$
$$\Leftrightarrow \beta \gtreqless 0.193340,$$

and

$$c^{**} - \frac{2(1-\beta)^2}{3(13 + 14\beta + 5\beta^2)}$$
$$= \frac{(1-\beta)\begin{bmatrix} -4025 + 51055\beta + 165758\beta^2 + 182630\beta^3 + 86939\beta^4 + 15307\beta^5 \\ +12\sqrt{-39 + 194\beta + 169\beta^2}\,(325 + 805\beta + 771\beta^2 + 343\beta^3 + 60\beta^4) \end{bmatrix}}{3(13 + 14\beta + 5\beta^2)(26875 + 71510\beta + 71692\beta^2 + 31962\beta^3 + 5321\beta^4)}$$
$$> 0, \qquad \forall \beta \in (0.174504, 1).$$

Thus, we find that the EOS, M_b or M_c, coincides with the most preferred ownership structure with respect to social welfare for any $c \in \left(c^*, 2(1-\beta)^2/3(13 + 14\beta + 5\beta^2)\right]$ when $\beta \in (0.193340, 1)$. □

Chapter 3

Mixed oligopoly and productivity-improving mergers

3.1 Introduction

In this chapter, on the basis of the non-cooperative game approach, we present the theoretical analysis of the merger between a public firm and a private firm in mixed oligopoly.[1] The literature on horizontal mergers is roughly divided into two categories. The first deals with the profit effects of mergers. Salant et al. (1983) and Deneckere and Davidson (1985) examine whether mergers are beneficial with regard to the profits of the participants in a quantity and price setting game, respectively. The second category deals with the welfare effects of mergers. In particular, Farrell and Shapiro (1990) indicate that mergers may have welfare-improving effects by redistributing production from less efficient to more efficient firms.

Except for Bárcena-Ruiz and Garzón (2003), there exist few studies on the decision to merge by public and private firms in a mixed oligopoly. They explore the case in which a public and a private firm merge into a multiproduct firm and show that both firms want to merge when the shareholding ratio of the owner of the public firm takes an intermediate value and the substitutability of the goods produced by both the public and private firms is sufficiently low.

Although, Bárcena-Ruiz and Garzón (2003) considered the case where mergers improve production efficiency, they hardly paid their attentions into such an efficiency gains effect of the merged firm.[2] In general, several reasons exist regarding why mergers may lead to an improvement of productivity. One is the learning effect, in which a partner to the merger learns from the other partner's patents, management expertise, *etc*. Furthermore, if firms combine some form of

[1] All the analyses conducted in this chapter are based on those of Nakamura and Inoue (2007).

[2] They assume that firms have identical technologies represented by the quadratic cost function, and after the merger between the public firm and the private firm, the merged firm utilize the scale of economics by having two plants (formerly, firms).

"capital" between their facilities after a merger, it certainly results in improving productivity of the merged firm when economies of scale exist. However, there are no studies on the horizontal mergers, focusing their attentions on *productivity-improving mergers* in the context of a mixed oligopoly. This study aims to fill this gap and have an impact on the subject. For this purpose, we investigate the productivity-improving merger as considered in McAfee and Williams (1992) under the assumption that firms have identical technologies represented by the quadratic cost function.

In our model, there exist one public firm and n identical private firms in a homogeneous goods market; this is in contrast to Bárcena-Ruiz and Garzón (2003), who explore a mixed duopoly in a differentiated goods market. We show that if a merger improves productivity, both a public and a private firm want to merge when the shareholding ratio of the owner of the public firm takes an intermediate value after the merger, even though there exist only a few private firms in the market. In addition, we find that if the number of private firms is sufficiently large, the owner of the public firm is always willing to merge whenever its shareholding ratio in the merged firm is lower than a critical value.

One example in the real world as our subject of research is presented by European automobile industry. In particularly, the German public firm Volkswagen acquired the Spanish firm SEAT in 1986. Similarly, Renault, which was privatized in 1986, owned parts of equities in both Nissan Motor and Nissan Diesel in 1999.

This chapter has four sections and an appendix. Section 3.2 sets up the model. We refer to McAfee and Williams (1992) for the cost function of the merged firm.[3] In Section 3.3, we explore the problem of a merger between a public firm and a single private firm. Our purpose here is to analyze whether the public and the private firm want to merge, when the merger has an effect of productivity improvement. Section 3.4 provides the concluding remarks. In Appendix, we investigate in detail the case of a merger without any improvement in productivity.

3.2 The model

We consider a mixed market in which $(n+1)$ firms produce a homogeneous good. One of the firms is a welfare-maximizing public firm (denoted by firm 0), and the others are symmetric profit-maximizing private firms (denoted by firm 1, firm 2, $\cdots$, and firm n). We assume the

[3]They assume that the cost of firm i $(i = 1, \ldots, n)$ is equal to $(q_i)^2/2k_i$, where k_i is the firm's capital stock. In addition, we assume that the capital stock of each firm is normalized to 1, *i.e.*, $k_0 = k_1 = \cdots = k_n = 1$.

following linear inverse demand function:

$$P(Q) = a - Q \qquad a > 0,$$

where Q is the total output of the good. Each firm produces the good using identical technology, and the cost function of firm i is given by

$$C_i(q_i) = (q_i)^2 \qquad i = 0, 1, \ldots, n,$$

where q_i $(i = 0, 1, \ldots, n)$ is the output of each firm. The profit of firm i is expressed as

$$\pi_i = P(Q) - C_i(q_i) = (a - Q)q_i - (q_i)^2 \qquad i = 0, 1, \ldots, n. \tag{3.1}$$

Each private firm chooses its output level in order to maximize (3.1). On the other hand, the public firm chooses its output to maximize social welfare. Social welfare is represented by the sum of consumer surplus (denoted by CS) and profits of all firms as follows:

$$W = CS + \sum_{i=0}^{n} \pi_i, \tag{3.2}$$

$$\text{where} \quad CS = \int_0^Q P(z)dz - P(Q)Q = \frac{1}{2}Q^2.$$

We assume that the public firm and one of the private firms decide whether to merge and set up a multiplant firm whose ownership is shared by the owners of the public and private firms. For simplicity, we describe the owner of the public firm after the merger as *the public sector* and the owner of the private firm as *the private sector*. Since the private firms are symmetric, we assume that firm 1 can merge with the public firm without loss of generality. We consider that the merged firm (denoted by firm m) has two plants, one of which is owned by the public firm and the other by the private firm before the merger. Thus, the merged firm can produce the good at lower cost than the other firms. The cost function of the merged firm is given by[4]

$$C_m(q_m) = \frac{1}{2}(q_m)^2,$$

where q_m is the output of the merged firm. The profit of the firm is expressed as

$$\pi_m = (a - Q)q_m - \frac{1}{2}(q_m)^2.$$

[4]The merged firm may be regarded as a multiplant firm, operating the two former firms as "plants." In this chapter, we assume that a multiplant merged firm operates under a situation in which both plants perform most efficiently (see McAfee and Williams, 1992). We assume that the productivity of the public and private firms is symmetric, *i.e.*, the cost function of each firm is represented by the quadratic form of its own output. Therefore, a merged firm has technology that is twice as efficient as that of the two pre-merger firms.

Note that the total number of firms is reduced from $(n+1)$ to n by the merger.

The public and private sectors share the ownership of the merged firm. Let $\alpha \in [0,1]$ denote the shareholding ratio of the public sector and let the merged firm choose its output q_m to maximize the weighted average of social welfare and its own profit as in Matsumura (1998). This objective function is given by

$$V = \alpha W + (1-\alpha)\pi_m. \tag{3.3}$$

Since the total number of the firms is reduced by the merger, social welfare is as follows:

$$W = CS + \sum_{k=2}^{n} \pi_k + \pi_m.$$

The profit of the merged firm is distributed according to the shareholding ratio. Thus, we assume that the private sector receives profit at the rate of $(1-\alpha)$.

Assumption 3.1. *The payoff of the private sector that partially owns the merged firm is* $(1-\alpha)\pi_m$.

When social welfare improves and the profit received by the private sector increases as the result of the merger, the public and the private firm merge.

We consider a two-stage game: In the first stage, both the public and the private firm decide whether to merge. In the second stage, all firms choose their own output levels.

3.3 The decision by firms to merge

We consider the following two cases: First, the firms do not merge, resulting in the competition between one public and n private firms. We denote this case as N (No merger). Second, the firms merge; this case is denoted as M (Merger).

We first examine the second stage of the game in case N. As stated in the previous section, the public firm chooses q_0 to maximize (3.2), while the private firm j chooses q_j to maximize (3.1) ($j = 1, \ldots, n$). Solving these maximization problems simultaneously, we obtain the Nash equilibrium in the second stage:

$$q_0^N = \frac{3a}{9+2n}, \quad q_j^N = \frac{2a}{9+2n}, \quad \pi_0^N = \frac{9a^2}{(9+2n)^2}, \quad \pi_j^N = \frac{8a^2}{(9+2n)^2},$$
$$CS^N = \frac{a^2(3+2n)^2}{2(9+2n)^2}, \quad W^N = \frac{a^2(27+28n+4n^2)}{2(9+2n)^2}, \qquad j = 1, \ldots, n.$$

The output of the public firm is larger than that of each private firm regardless of the number of private firms, n. Consumer surplus and social welfare are increasing functions of n, while the profit of each firm is a decreasing function of n.

When the public firm (firm 0) and the private firm (firm 1) merge, they set up a multiplant firm that chooses q_m to maximize (3.3). The other firms choose their output level to maximize (3.1). As a result, we obtain the Nash equilibrium in the second stage:

$$
\begin{aligned}
q_k^M &= \frac{a(2-\alpha)}{7+2n-\alpha(2+n)}, \quad q_m^M = \frac{3a}{7+2n-\alpha(2+n)}, \\
\pi_k^M &= \frac{2a^2(2-\alpha)^2}{[7+2n-\alpha(2+n)]^2}, \quad \pi_m^M = \frac{9a^2(3-2\alpha)}{[7+2n-\alpha(2+n)]^2}, \\
CS^M &= \frac{a^2[1+2n-\alpha(n-1)]^2}{2[7+2n-\alpha(2+n)]^2}, \\
W^M &= \frac{a^2(2-\alpha)(6+10n+2n^2+3\alpha-2n\alpha-n^2\alpha)}{2[7+2n-\alpha(2+n)]^2}, \qquad k=2,\ldots,n.
\end{aligned}
$$

The output of the merged firm is larger than that of each private firm irrespective of n and α. In addition, consumer surplus and social welfare are increasing functions of n, while social welfare decreases as α increases when the value of α is sufficiently high.[5] The rise of α widens the output gap between each private firm and the merged firm. Although the productivity-improving merger enhances social welfare within the bounds of low α, the widening gap reduces social welfare because of the convexity of the cost function when α is sufficiently high. In addition, when the market is a monopoly after the merger ($n = 1$), social welfare is maximized at $\alpha = 1$.

Next, we analyze both the public and private firm's incentives to merge in the first stage of the game. In order to explain these incentives, we employ two effects: *share effect* and *competition effect*. The share effect, which is represented by α, affects the firms in case M through the weight of social welfare in objective function of the merged firm (see Bárcena-Ruiz and Garzón, 2003). The competition effect, which is represented by n, affects them in both cases N and M. When parameter n increases, for a given of parameter α, consumer surplus and social welfare increase, while profits of all the firms decrease.

First, we examine whether the public firm wishes to merge with the private firm. Since the public firm aims at maximizing social welfare, it has an incentive to merge if $W^M > W^N$. Let α_0^* and α_0^{**} denote the values of α such that $W^M = W^N$:

$$
\begin{aligned}
\alpha_0^* &= \frac{378+122n+4n^2+3(9+2n)\sqrt{27-2n+2n^2}}{351+166n+14n^2}, \\
\alpha_0^{**} &= \frac{378+122n+4n^2-3(9+2n)\sqrt{27-2n+2n^2}}{351+166n+14n^2}.
\end{aligned}
$$

We obtain the following proposition using α_0^* and α_0^{**}.

[5] Since $\frac{\partial W^M}{\partial \alpha} = \frac{3a^2[8+n-\alpha(7+2n)]}{[7+2n-\alpha(2+n)]^3}$, W^M decreases as α rises when $\alpha > \frac{8+n}{7+2n}$. In addition, $\frac{8+n}{7+2n}$ is a decreasing function of n.

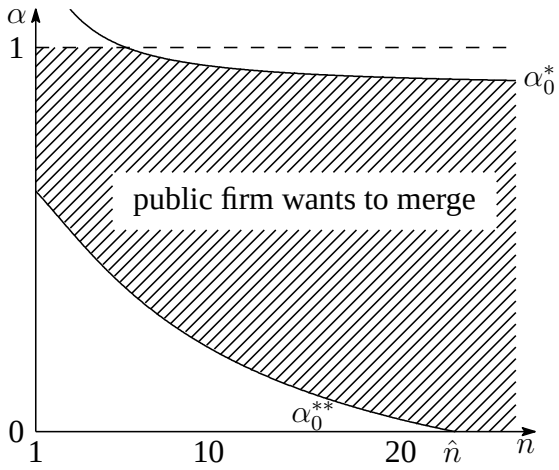

Figure 3.1: Illustration of Proposition 3.1

Proposition 3.1. *$W^M > W^N$ if and only if $\alpha_0^{**} < \alpha < \alpha_0^*$.*

Proof. Subtracting W^N from W^M, we obtain the following equation:

$$W^M - W^N = \frac{-a^2[(351 + 166n + 14n^2)\alpha^2 - (756 + 244n + 8n^2)\alpha + 351 + 76n - 4n^2]}{2(9 + 2n)^2[7 + 2n - \alpha(2 + n)]^2}.$$

The sign of RHS depends on that of its numerator. Since this numerator is a quadratic concave function of α and is equal to zero when $\alpha = \alpha_0^*$ or $\alpha = \alpha_0^{**}$, $W^M > W^N$ if and only if $\alpha_0^{**} < \alpha < \alpha_0^*$. □

This proposition shows that if the number of private firms is greater than or equal to 6 ($n \geq 6$), the public firm does not want to merge at $\alpha = 1$, since $\alpha_0^*|_{n\,=\,6} < 1$ and α_0^* is a decreasing function of n. In addition, when the number is greater than or equal to 23 ($n \geq 23$), the public firm wants to merge at $\alpha = 0$, because $\alpha_0^{**}|_{n\,=\,23} < 0$ and α_0^{**} is a decreasing function of n. In other words, even if the public sector does not have a share of the merged firm, the public firm has an incentive to merge in $n \geq 23$. Figure 3.1 illustrates this incentive in relation with parameters n and α. The shaded area represents the range in which the public firm wants to merge. This range broadens as n increases until $n = \hat{n}$, but when $n > \hat{n}$, it narrows conversely.[6]

The increase in the number of private firms reduces the public firm's contribution to consumer surplus, but the output gap between the public and private firms remains. Since the gap decreases social welfare, the increase enhances the public firm's incentive to merge. Thus, the shaded area widens as n increases. This logic coincides with that of De Fraja and Delbono (1989), who show that the privatization of a public firm can improve social welfare.

[6]The critical value is $\hat{n} = (19 + 2\sqrt{178})/2 \approx 22.8417$.

However, Proposition 3.1 depends heavily on our assumption that the merger improves the productivity of the firm. If we do not assume this effect, the public firm will not wish to merge with the private firm regardless of the number of private firms (see Appendix).

Next, we consider whether the private firm (firm 1) decides to merge with the public firm. By Assumption 3.1, the private firm decides to merge if $(1-\alpha)\pi_m^M > \pi_1^N$. Let α_1^* and α_1^{**} denote the values of α such that $(1-\alpha)\pi_m^M = \pi_1^N$:

$$\alpha_1^* = \frac{3197 + 1268n + 116n^2 + 3(9+2n)\sqrt{3289 + 1156n + 100n^2}}{2(1394 + 584n + 56n^2)},$$
$$\alpha_1^{**} = \frac{3197 + 1268n + 116n^2 - 3(9+2n)\sqrt{3289 + 1156n + 100n^2}}{2(1394 + 584n + 56n^2)}.$$

We obtain the following proposition using these equations.

Proposition 3.2. *$(1-\alpha)\pi_m^M > \pi_1^N$ if and only if $\alpha < \alpha_1^{**}$.*

Proof. Subtracting π_1^N from $(1-\alpha)\pi_m^M$, we obtain the following equation:

$$(1-\alpha)\pi_m^M - \pi_1^N = \frac{a^2[(1394 + 584n + 56n^2)\alpha^2 - (3197 + 1268n + 116n^2)\alpha + 1403 + 524n + 44n^2]}{2(9+2n)^2[7 + 2n - \alpha(2+n)]^2}.$$

The sign of RHS depends on that of its numerator. Since this numerator is a quadratic convex function of α and is equal to zero when $\alpha = \alpha_1^*$ or $\alpha = \alpha_1^{**}$, $(1-\alpha)\pi_m^M > \pi_1^N$ if $\alpha > \alpha_1^*$ or $\alpha < \alpha_1^{**}$. However, $\alpha_1^* > 1$ for all n, and thus the constraint of $\alpha \in [0,1]$ is violated. Therefore, $(1-\alpha)\pi_m^M > \pi_1^N$ if and only if $\alpha < \alpha_1^{**}$. □

Proposition 3.2 is illustrated in Figure 3.2. Since the increase in the number of private firms reduces the market price and the increment of profit by the merger, the private firm demands a higher profit distribution ratio to compensate the profit reduction. Therefore, α_1^{**} is a decreasing function of n (in other words, $(1-\alpha_1^{**})$ is an increasing function of n). Note that $\lim_{n\to\infty} \alpha_1^{**} = 1/2$; thus, the private firm always decides to merge irrespective of n when the shareholding ratio of the private sector is more than $1/2$.

We present the following lemma in which we compare α_1^{**} with α_0^* and α_0^{**} to determine whether the public and private firms merge.

Lemma 3.1. *$\alpha_0^* > \alpha_1^{**}$ for $n \in [1,\infty)$ and $\alpha_0^{**} > \alpha_1^{**}$ at $n = 1$, but there exists $\tilde{n} \in (1,\infty)$ such that $\alpha_1^{**} \geq \alpha_0^{**}$ for $n \geq \tilde{n}$.*

Proof. See Appendix.

When the number of private firms is sufficiently small, α_0^{**} is greater than α_1^{**}. However, when the number exceeds the critical value $\tilde{n}$, this relation is reversed ($\alpha_1^{**} \geq \alpha_0^{**}$). We obtain

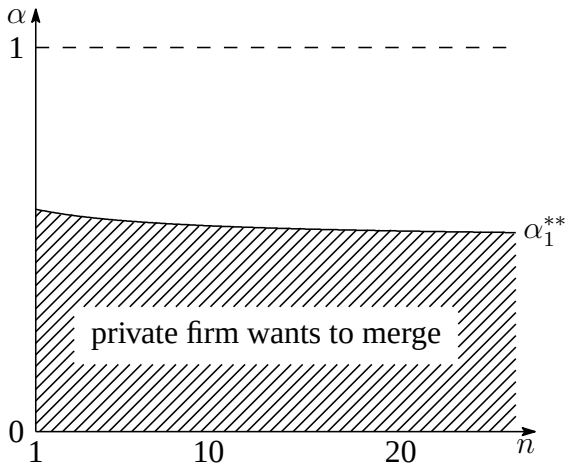

Figure 3.2: Illustration of Proposition 3.2

an approximate value of n such that $\alpha_0^{**} = \alpha_1^{**}$ is 1.9907, *i.e.*, the firms do not merge in mixed "duopoly." This coincides with the result of Bárcena-Ruiz and Garzón (2003).[7] By Propositions 3.1 and 3.2 and Lemma 3.1,[8] we obtain the following proposition:

Proposition 3.3. *The public firm 0 and the private firm 1 will merge when* $\alpha_0^{**} < \alpha < \alpha_1^{**}$.

Figure 3.3 illustrates Proposition 3.3. If $n \in (\tilde{n}, \hat{n})$, the area in which both the public and private firm want to merge broadens as n increases.[9] In addition, when n is larger than $\hat{n}$, both firms want to merge even if the merged firm is owned only by the private sector (*viz.*, $\alpha = 0$). This is because the welfare loss due to the excess production of the public firm is larger than the welfare improvement as a result of increasing consumer surplus as stated above.

The intuition behind Proposition 3.3 is as follows: The shareholding ratio is important for both the public and private sector of the merged firm because it directly affects the behavior of the firm. Thus, when parameter n is sufficiently small, the merger does not achieve around either $\alpha = 0$ or $\alpha = 1$. However, since an increase of n decreases q_0/Q and thus reduces the impact of the public firm on social welfare, the share effect decreases as n increases (interaction between the share effect and the competition effect). In addition to this, the existence of more efficient firm reduces total output cost. Therefore, the share effect for the public sector decreases with n and α_0^{**} is severely reduced by increase of n. On the other hand, the share effect for the private sector is not affected by the competition effect so much because the payoff of the private sector, $(1-\alpha)\pi^M$, is directly affected by the shareholding ratio. Thus, the share effect has a more crucial

[7] Bárcena-Ruiz and Garzón (2003) do not consider the productivity-improving merger. However, even if the merger improves the productivity of the merged firm, the firms do not merge in a mixed duopoly with a homogeneous good.

[8] Lemma 3.1 guarantees the existence of the range in which $\alpha \in (\alpha_0^{**}, \alpha_1^{**})$.

[9] However, in $n \geq \hat{n}$, α_0^{**} is less than 0, and the area narrows as n increases by the constraint of $\alpha \geq 0$.

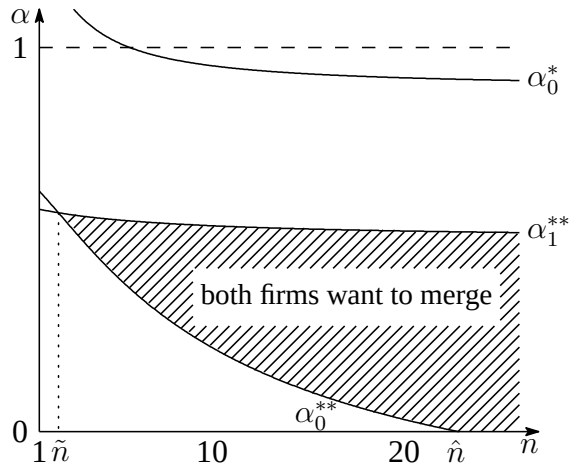

Figure 3.3: Illustration of Proposition 3.3

effect on the decision of the private sector than that of the public sector and α_1^{**} is almost stable against an increase of n. Accordingly, the area in which both the public and private firms want to merge expands with the number of private firms.

3.4 Concluding remarks

This chapter investigated how a public and private firm's decision whether to merge depends on the shareholding ratio and the number of private firms. We showed that when the shareholding ratio of the public sector is $\alpha \in (\alpha_0^{**}, \alpha_1^{**})$, which is achieved in $n > \tilde{n}$, both firms decide to merge. Note that, in a mixed duopoly, the merger is not achieved.

Bárcena-Ruiz and Garzón (2003) demonstrate that the firms do not merge in a mixed duopoly with a homogeneous good. However, we proved that if mergers improve the efficiency of the firms and the number of private firms is sufficiently large, the result is not necessarily the same as theirs. In particular, the productivity-improving merger is critical to the result. If we do not assume this effect, the public firm does not choose to merge regardless of the number of private firms.

We assumed that each firm produces a homogeneous good using identical technology. We also briefly mention the case that they have asymmetric technologies, in particular, a public firm is less efficient than a private firm. If a public firm is inefficient compared to a private one, the public one wants to merge with the private one more aggressively in pursuit of enhancing social welfare with the improvement of productivity in the merged firm. Thus, in the case that both firm's cost functions are asymmetric, a curve corresponding to α_0^{**} in Figure 3.1 and 3.3 is more rapidly decreasing with n around 1. On the other hand, the private firm is reluctant to

merge with the public firm, since the productivity improvement rate is lower than that in the case where the firms have identical technologies. Thus, a curve corresponding to α_1^{**} in Figure 3.2 and 3.3 shifts down below. Therefore, when a public firm is less efficient than a private firm, it is ambiguous whether the area in which both the public and private firms want to merge expands or not compared to the case where their cost functions are symmetric. However, the area still exists in (n, α)-plane.

Our analysis contributes to the literature on mixed oligopoly by showing that a public firm may have an incentive to merge with a private firm in a homogeneous goods market. However, two interesting extensions of our model still remain. One is the situation in which the public firm merges with multiple private firms, and another is where there exist foreign shareholders of the private firms. Since it would appear that these situations would have an impact on the firms' decision to merge, the investigation of these situations is important for future studies of mergers in mixed markets.

Appendix

The public firm's decision without productivity improvement

We show that the public firm does not have an incentive to merge with the private firm in the case where the merger does not improve the productivity of the merged firm. In this case, social welfare before and after the merger is as follows:

$$W^N = \frac{a^2(27+28n+4n^2)}{2(9+2n)^2},$$
$$W^M = \frac{a^2[36n+9n^2+(6-18n-6n^2)\alpha-(3-2n-n^2)\alpha^2]}{2[9+3n-(2+n)\alpha]^2}.$$

Subtracting W^N from W^M, we examine the public firm's incentive to merge with the private firm.

$$W^M - W^N = \frac{-a^2[(351+166n+14n^2)\alpha^2-(1458+576n+36n^2)\alpha+2187+810n+54n^2]}{2(9+2n)^2[9+3n-(2+n)\alpha]^2}.$$

The sign of RHS depends on that of its numerator. This numerator is a quadratic concave function of α and the discriminant of this quadratic equation, D, is[10]

$$D = -432(9+2n)^2(27+14n+n^2) < 0.$$

Thus, for all n, W^N is larger than W^M and the public firm does not want to merge.

[10] a^2 is omitted for simplicity.

Proof of Lemma 3.1

We divide this proof into three steps.

First, we prove that $\alpha_0^* > \alpha_1^{**}$ for $n \in [1, \infty)$. Evaluating α_0^* and α_1^{**} at $n = 1$, then

$$\alpha_0^*|_{n\,=\,1} = \frac{56 + 11\sqrt{3}}{59} \approx 1.2721 > 0.5792 \approx \frac{509 - 11\sqrt{505}}{452} = \alpha_1^{**}|_{n\,=\,1}.$$

Using computer software, we obtain

$$\lim_{n\to\infty} \alpha_0^* = \frac{2 + 3\sqrt{2}}{7} \approx 0.8918 > \alpha_1^{**}|_{n\,=\,1}.$$

Since, in addition to this, α_0^* and α_1^{**} are decreasing functions of n, we obtain $\alpha_0^* > \alpha_1^{**}$ for $n \in [1, \infty)$.

Second, we prove that $\alpha_0^{**} > \alpha_1^{**}$ at $n = 1$. Evaluating α_0^{**} at $n = 1$, we obtain

$$\alpha_0^{**}|_{n\,=\,1} = \frac{56 - 11\sqrt{3}}{59} \approx 0.6262.$$

Therefore, $\alpha_0^{**}|_{n\,=\,1} > \alpha_1^{**}|_{n\,=\,1}$.

Finally, we prove that there exists $\tilde{n} \in (1, \infty)$ such that $\alpha_1^{**} \geq \alpha_0^{**}$ for $n \geq \tilde{n}$. As mentioned in Section 3.3, both α_0^{**} and α_1^{**} are decreasing functions of n. In addition, we obtain the following limit relations:

$$\lim_{n\to\infty} \alpha_0^{**} = \frac{2 - 3\sqrt{2}}{7} \approx -0.3204 < 0.5 = \lim_{n\to\infty} \alpha_1^{**}.$$

Considering $\alpha_0^{**}|_{n\,=\,1} > \alpha_1^{**}|_{n\,=\,1}$, there exists $\tilde{n} > 1$ such that $\alpha_1^{**} \geq \alpha_0^{**}$ for $n \geq \tilde{n}$. □

Chapter 4

The Core and productivity-improving mergers in mixed oligopoly

4.1 Introduction

The purpose of this chapter is to provide a theoretical analysis on merger activities in the industry composed of one public firm and two private firms.[1] Such an industry is usually referred to as *mixed oligopoly*, or more specifically *mixed triopoly*. The literature on mixed oligopoly can be traced back to the paper of De Fraja and Delbono (1989). The mixed oligopoly is distinguished from the oligopoly composed only of private firms especially in the objective of a public firm. In many existing works on mixed oligopoly, it is assumed that the objective of a public firm is social welfare maximization, whereas private firms aims to maximize their own profits. Since, in the real world, public firms are financed by tax revenues, it seems quite reasonable to assume that a public is devoted to improving social welfare. Although there have been many analyses of a merger in private oligopoly (e.g. Salant et al. (1983), Deneckere and Davidson (1985), and Farrell and Shapiro (1990)), not so many efforts have been carried out in studying merger activities in mixed oligopoly. Exceptions are Bárcena-Ruiz and Garzón (2003), and Coloma (2006). Both of the papers analyzed a merger in mixed duopoly, *i.e.*, a merger in the industry composed of a public firm and a private firm. In the paper of Bárcena-Ruiz and Garzón, the two firms were assumed to produce heterogeneous products and the decision to merge by the firms was analyzed. On the other hand, in his paper, Coloma considered the case where the two firms produce homogeneous products and made welfare comparisons among possible market structures.

There are two respects in which this chapter contributes to the literature on mergers in mixed oligopoly. First, although neither of the papers of Bárcena-Ruiz and Garzón nor of Coloma considered a synergy effect of a merger, we assume that a merger yields a synergy effect to the

[1] All the analyses conducted in this chapter are based on those of Kamaga and Nakamura (2007).

technology of the merged firm and entails the improvement on productivity. Without assuming any synergy effects of a merger, Bárcena-Ruiz and Garzón obtained the result that, in their setting, both of the private firm and the public firm want to merge only when the degree to which the two heterogeneous products are substitutes is sufficiently low and, moreover, the merger does not take place when the two products are perfectly substitutable. Since, in the real world, there are many examples of mergers among firms which produce highly substitutable products, this result is counterintuitive to what we would expect. In this chapter, we re-examine the mergers among the firms producing homogeneous, *i.e.*, perfectly substitutable, products in mixed triopoly under the assumption that a merger yields the improvement on productivity. It seems very natural to assume that the merger between the firms that produce highly substitutable heterogeneous products entails a synergy effect because merger participants may easily learn a strong point of each firm's production skill and/or their patents from one another. In the study of the horizontal mergers in private oligopoly, Farrell and Shapiro (1990) showed that the merger could improve social welfare if the merged firm exploits economies of scale well. In order to analyze mergers that entail the improvement on productivity, we follow McAfee and Williams (1992). In this chapter, the technology of each of the three firms is identically represented by the quadratic cost function $C(q_i) = q_i^2$, where q_i is the amount of the production of the firm i, and, as considered in the paper of McAfee and Williams, the merged firm operates the plants which were previously owned by the pre-merged firms most efficiently and thus the technology is represented by $C(q) = q^2/n$, where q is the amount of the output of the merged firm and n $(= 2, 3)$ is the number of the merger participants. Such a cost function of the merged firm clearly shows that a merger entails the improvement on productivity.

The other respect in which our analysis is clearly distinguished from the earlier ones is that we especially focus our attention on the stability of market structures. We extend the usual way of analysis of mergers where solely the decision to merge by the firms is discussed. In this chapter, we treat merger activities as coalition formations among the firms that are allowed to freely merge and freely break off the merger . For example, the merger between firms, say 0 and 1, with leaving a firm, say 2, standing alone can be considered as the coalition formation of $\{\{0, 1\}, \{2\}\}$. Viewing merger activities as coalition formations among the firms, to find the stable coalition formations, *i.e.*, stable market structures, is of our interest. In order to analyze the stability of market structures, we adopt the *core*, the well-established solution concept in cooperative game theory and examine which of all possible market structures is/are stable in the sense that once any of such market structures is actually realized none of the owners of the firms

wants to change this present market structure by merging with other firm or breaking off the merger.

The motivation to analyze the stability problem of merger activities perhaps needs some elaboration. In this chapter, we consider the industry of mixed triopoly. In the mixed triopoly market, the variation of possible forms of a merger among the firms increases and becomes more complicated than in mixed duopoly. Consequently, it might be the case that, while the owners of some two firms, say 0 and 1, have an incentive to merge into one firm by comparing their payoffs obtained in each of the initial market structure, *i.e.*, the coalition structure $\{\{0\},\{1\},\{2\}\}$, and the one realized after the merger, *i.e.*, $\{\ \{0,1\}\ ,\{\ 2\}\ \}$, the owner of the firm 0 could receive higher payoff if s/he breaks off the merger with the firm 1 and alternatively merges with the firm 2, *i.e.*, in the structure $\{\{0,2\},\{1\}\}$, than in the case of the merger with the firm 1. In this case, if the owner of the firm 2 also has an incentive to merge with the firm 0, the merger between the firms 0 and 2 will be realized, and the merger between the firms 0 and 1 can never be realized. Therefore, in the presence of more than two firms, it is not sufficient to analyze the decision to merge in each particular case, and we should examine merger activities in terms of stable coalition formations. In the literature on mergers in private oligopoly, Barros (1998), Horn and Persson (2001a), and Straume (2006) adopted the same approach. However, with the only exception of Kamijo and Nakamura (2009), there has not been any works that analyze mergers in mixed oligopoly along the approach using the core property. Among these existing works, there is a slight difference in the definitions of core property. The core property considered in this chapter is the same as the one considered in Barros (1998) and Kamijo and Nakamura (2009). We refer the reader to Brito and Gata (2006) for the detailed discussion about the difference between the core property adopted by Barros (1998) and the one considered in Horn and Persson (2001a) and Straume (2006). Using the core property à la Barros (1998) and Kamijo and Nakamura (2009), this chapter shows that, in our mixed triopoly model, the core of market structures is non-empty and the core consists solely of the market structures derived by the merger between a public firm and one of the two private firms with about 0.57 share ratio by the public firm.

This chapter is organized as follows. The next section introduces our model and presents the Cournot-Nash equilibrium for each of four regimes; mixed triopoly; merger between private firms; merger between a public firm and a private firm; and merger among all the three firms. Our results are provided in Section 4.3. Section 4.4 concludes with some remarks. In the Appendix A, we present the proof of Proposition 4.1, and subsequently in the Appendix B, we consider the core of market structures after each firm's merger activity in the case wherein s/he adopts the

FJSV delegation contract.

4.2 The model

4.2.1 Basic set-up of mixed oligopoly

We analyze stable market structures in the industry composed of one public firm, denoted by 0, and two private firms, 1 and 2. Each firm produces a single homogeneous good and is assumed to be entrepreneurial one, *i.e.*, the owners themselves make every managerial decision making. The public firm (resp. each of the private firms) is owned by the government (resp. a single private shareholder). In accordance with whether a merger among the firms is realized or not, we have four possible market regimes: (a) *mixed triopoly* $\{\{0\},\{1\},\{2\}\}$, (b) *merger between private firms* $\{\{0\},\{1,2\}\}$, (c) *merger between a public firm and a private firm* $\{\{0,i\},\{j\}\}$ $(i,j=1,2,\ i\neq j)$, and (d) *merger among all the three firms* $\{\{0,1,2\}\}$. Although the details of the formal descriptions of the four regimes are slightly different, we mainly introduce the set-up of the mixed triopoly. The other regimes are easily understand as an extension of the mixed triopoly.

As have been usually considered in the literature on mixed oligopoly, the inverse demand function is given as a linear function of the total output Q,

$$P(Q) = a - Q, \tag{4.1}$$

where a is sufficiently large positive number. As assumed in Bárcena-Ruiz and Garzón (2003), each firm i $(=0,1,2)$ has an identical technology represented by the quadratic cost function

$$C(q_i) = q_i^2, \tag{4.2}$$

where q_i is the quantity of the good produced by the firm i. The profit function of the firm i $(=0,1,2)$ is given as:

$$\Pi_i = (a-Q)q_i - q_i^2. \tag{4.3}$$

As usual, social welfare W is measured by the sum of consumer surplus $CS = Q^2/2$, and firms' profits.

In their papers, Bárcena-Ruiz and Garzón have not discussed the case where a merger yields the improvement on productivity. The productivity-improving merger has been analyzed in McAfee and Williams (1992). As in the paper of McAfee and Williams and also of Nakamura

and Inoue (2007) and Heywood and McGinty (2007a; 2007b), we consider that a merged firm shows the improvement on productivity. The market regimes derived by mergers, *i.e.*, (b), (c), and (d), show the differences particularly in the forms of cost functions. If n $(= 2, 3)$ firms merge into one firm, the total cost of the merged firm C_m is represented as:

$$C_m(q_m) = \frac{q_m^2}{n}, \tag{4.4}$$

where q_m is the output of the merged firm m. Such a cost function is supported by the assumption that the merged firm adopts the most efficient operation plan of the plants previously owned by the pre-merged firms. More precisely, the cost function in (4.4) corresponds to the case of the most efficient operation rates $(\lambda_1^*, \ldots, \lambda_n^*) = (1/n, \ldots, 1/n)$ of the plants derived from the total cost minimization problem:

$$\min_{(\lambda_1,\ldots,\lambda_n)} \sum_{i=1}^{n} (\lambda_i q_m)^2, \quad \text{subject to } \sum_{i=1}^{n} \lambda_i = 1 \text{ and } \lambda_i \geq 0 \text{ for all } i. \tag{4.5}$$

The profit of the merged firm is given by replacing q_i^2 with q_m^2/n in (3).

4.2.2 Equilibrium outcomes in the regimes (a) to (d)

We now examine the Cournot equilibrium for each regimes. Let U_i^r denote an objective function that the firm i maximizes in the regime r $(= a, b, c, d)$. In the rest of the chapter, functions and variables with superscript r $(= a, b, c, d)$ denote those considered in the regime r.

(a) Mixed triopoly $\{\{\mathbf{0}\}, \{\mathbf{1}\}, \{\mathbf{2}\}\}$: In this regime, each of the three firms has the following objective function, respectively:

$$U_0^a(q_0^a; q_1^a, q_2^a) = W^a = \frac{1}{2}\left(q_0^a + \sum_{i=1}^{2} q_i^a\right)^2 + \Pi_0 + \sum_{i=1}^{2} \Pi_i, \tag{4.6}$$

$$U_i^a(q_i^a; q_0^a, q_j^a) = \Pi_i, \quad (i, j = 1, 2 \text{ and } i \neq j). \tag{4.7}$$

The first order conditions of the maximization problems give the following Cournot equilibrium:

$$q_0^{a*} = \frac{3}{13}a \text{ and } q_i^{a*} = \frac{2}{13}a, \quad (i = 1, 2). \tag{4.8}$$

Therefore, in the Cournot equilibrium, we obtain the following equilibrium profits Π_i^a, consumer surplus, and social welfare:

$$\Pi_0^a = \frac{9}{169}a^2,\ \Pi_i^a = \frac{8}{169}a^2,\ (i = 1, 2),\ CS^a = \frac{49}{338}a^2, \text{ and } W^a = \frac{99}{338}a^2.$$

Notice that, in this regime, the equilibrium profit of the public firm 0 is larger than those of the private firms 1 and 2. As has been shown in extensive literature on mixed oligopoly, in the case of quantity competition, a public firm whose objective is welfare maximization chooses the output larger than the one by a private firm, profit maximizer, because the choice of the output by the public firm is largely affected by the level of consumer surplus. Consequently, this leads to the larger market share and higher profit of the public firm. The payoffs to the owners of the firms, denoted by V_i^a $(i = 0, 1, 2)$, are

$$V_0^a = W^a = \frac{99}{338}a^2, \text{ and } V_i^a = \Pi_i^a = \frac{8}{169}a^2 \ (i = 1, 2).$$

(b) Merger between private firms $\{\{\mathbf{0}\}, \{\mathbf{1}, \mathbf{2}\}\}$: Next, we consider the case where the two private firms merge into a new private firm denoted by 12. Let q_{12}^b be the amount of the output of the merged firm 12. The objective of the merged firm 12 is to maximize its profit:

$$U_{12}^b(q_{12}^b; q_0^b) = \Pi_{12} = \left[a - \left(q_0^b + q_{12}^b\right)\right] q_{12}^b - \frac{1}{2}\left(q_{12}^b\right)^2. \tag{4.9}$$

The objective function of the public firm is:

$$\begin{aligned} U_0^b(q_0^b; q_{12}^b) &= W^b = CS^b + \Pi_0 + \Pi_{12} && \text{(4.10a)} \\ &= \frac{1}{2}\left(q_0^b + q_{12}^b\right)^2 + \left[a - \left(q_0^b + q_{12}^b\right)\right] q_0^b - \left(q_0^b\right)^2 + \left[a - \left(q_0^b + q_{12}^b\right)\right] q_{12}^b \\ &\quad - \frac{1}{2}\left(q_{12}^b\right)^2. && \text{(4.10b)} \end{aligned}$$

Note that, in the last term of its profit function, the merged firm 12 shows the improvement on productivity. In the Cournot equilibrium, we obtain the following:

$$q_0^{b*} = \frac{1}{4}a, \quad q_{12}^{b*} = \frac{1}{4}a, \ \Pi_0^b = \frac{1}{16}a^2, \ \Pi_{12}^b = \frac{3}{32}a^2, \ CS^b = \frac{1}{8}a^2, \text{ and } W^b = \frac{9}{32}a^2.$$

Let $\alpha \in [0, 1]$ be a ratio of shareholding by the owner of the firm 1 in the merged firm 12. Then, the payoff to the owner of the public firm 0, V_0^b, and those to the pre-merged private firms 1 and 2, V_1^b and V_2^b, are

$$V_0^b = W^b = \frac{9}{32}a^2, \ V_1^b = \alpha\Pi_{12}^b = \frac{3}{32}\alpha a^2, \text{ and } V_2^b = (1-\alpha)\Pi_{12}^b = \frac{3}{32}(1-\alpha)a^2.$$

(c) Merger between a public firm and a private firm $\{\{\mathbf{0}, \mathbf{i}\}, \{\mathbf{j}\}\}$: In this regime the public firm 0 and one of the private firms i $(= 1 \text{ or } 2)$ merge into a new firm $0i$. Let q_{0i}^c and Π_{0i}^c denote the output and profit of the merged firm $0i$. As the objective function of the public-private

merged firm $0i$, we consider the weighted average of social welfare and the profit of the merged firm:

$$U^c_{0i}(q^c_{0i}; q^c_j) = \beta W^c + (1-\beta)\Pi_{0i} \tag{4.11a}$$

$$= \beta\left[\frac{1}{2}\left(q^c_{0i}+q^c_j\right)^2 + \Pi_{0i} + \Pi_j\right] + (1-\beta)\Pi_{0i}, \quad (i,j=1,2 \text{ and } i\neq j) \tag{4.11b}$$

where $\beta \in [0,1]$ is a ratio of shareholding by the government in the merged firm $0i$ and Π_{0i} and Π_j are the profit functions of the firms $0i$ and j, respectively, given as:

$$\Pi_{0i} = \left[a - \left(q^c_{0i}+q^c_j\right)\right] q^c_{0i} - \frac{1}{2}\left(q^c_{0i}\right)^2, \tag{4.12}$$

$$\left(U^c_j\left(q^c_j; q^c_{0i}\right) \equiv\right)\ \Pi_j = \left[a - \left(q^c_{0i}+q^c_j\right)\right] q^c_j - \left(q^c_j\right)^2. \tag{4.13}$$

The weighted average of social welfare and the profit in the objective of a public-private merged firm has first been suggested in Matsumura (1998) and also been adopted in Bárcena-Ruiz and Garzón (2003). In the Cournot equilibrium of this regime, we get:

$$q^{c*}_{0i} = \frac{3}{11-4\beta}a,\ q^{c*}_j = \frac{(2-\beta)}{11-4\beta}a,\ \Pi^c_{0i} = \frac{9(3-2\beta)}{2(11-4\beta)^2}a^2,\ \Pi^c_j = \frac{2(2-\beta)^2}{(11-4\beta)^2}a^2,$$

$$CS^c = \frac{(5-\beta)^2}{2(11-4\beta)^2}a^2, \text{ and } W^c = \frac{(68-44\beta+5\beta^2)}{2(11-4\beta)^2}a^2.$$

The payoffs to the owners of the pre-merged public firm 0 and pre-merged private firm i, V^c_0 and V^c_i, are

$$V^c_0 = W^c = \frac{(68-44\beta+5\beta^2)}{2(11-4\beta)^2}a^2, \text{ and } V^c_i = (1-\beta)\Pi^c_{0i} = \frac{9(3-2\beta)(1-\beta)}{2(11-4\beta)^2}a^2,$$

and the one to the owner of the non-merged private firm $j \neq i$, V^c_j, is

$$V^c_j = \Pi^c_j = \frac{2(2-\beta)^2}{(11-4\beta)^2}a^2.$$

(d) Merger among all the three firms $\{\{\mathbf{0},\mathbf{1},\mathbf{2}\}$: Finally, we examine the case where all of the three firms, 0, 1, and 2, merge into one firm denoted by 012. In the similar way to the regime (c), the objective function of the merged firm is defined as follows:

$$U^d_{012}(q^d_{012}) = \gamma W^d + (1-\gamma)\Pi_{012} \tag{4.14a}$$

$$= \gamma\left[\frac{1}{2}\left(q^d_{012}\right)^2 + \Pi_{012}\right] + (1-\gamma)\Pi_{012}, \tag{4.14b}$$

where $\gamma \in [0,1]$ is a ratio of shareholding by the government in the merged firm 012 and Π_{012} is the profit function of the merged firm given as:

$$\Pi_{012} = \left(a - q^d_{012}\right) q^d_{012} - \frac{1}{3}\left(q^d_{012}\right)^2. \tag{4.15}$$

Note that the merged firm in this regime shows further improvement on productivity than in the regimes (b) and (c). In the Cournot equilibrium, we obtain the following:

$$q_{012}^{d*} = \frac{3}{8-3\gamma}a,\ \Pi_{012}^{d} = \frac{3(4-3\gamma)}{(8-3\gamma)^2}a^2,\ CS^d = \frac{9}{2(8-3\gamma)^2}a^2,\ \text{and}\ W^d = \frac{3(11-6\gamma)}{2(8-3\gamma)^2}a^2.$$

The payoffs to the owners of the pre-merged firms, V_i^d $(i = 0, 1, 2)$, are

$$V_0^d = W^d = \frac{3(11-6\gamma)}{2(8-3\gamma)^2}a^2,\ V_1^d = (1-\gamma)\delta\Pi_{012}^d = \frac{3(4-3\gamma)(1-\gamma)\delta}{(8-3\gamma)^2}a^2,\ \text{and}$$
$$V_2^d = (1-\gamma)(1-\delta)\Pi_{012}^d = \frac{3(4-3\gamma)(1-\gamma)(1-\delta)}{(8-3\gamma)^2}a^2,$$

where $\delta \in [0, 1]$ measures a ratio of profit distribution among the private sector, *i.e.*, $(1-\gamma)\delta$ is a ratio of shareholding by the owner of the firm 1 in the merged firm 012.

Table 4.1 summarizes objective functions and payoffs of the firms in each of the four regimes.

Table 4.1: Firms' objectives and owners' payoffs

regime (r)	firms' objectives: U_i^r	owners' payoffs: V_i^r
regime (a)	$U_0^a = W^a$	$V_0^a = W^a(q^{a*})$
	$U_1^a = \Pi_1$	$V_1^a = \Pi_1(q^{a*})$
	$U_2^a = \Pi_2$	$V_2^a = \Pi_2(q^{a*})$
regime (b)	$U_0^b = W^b$	$V_0^b = W^b(q^{b*})$
	$U_{12}^b = \Pi_{12}$	$V_1^b = \alpha\Pi_{12}(q^{b*})$
		$V_2^b = (1-\alpha)\Pi_{12}(q^{b*})$
regime (c)	$U_{0i}^c = \beta W^b + (1-\beta)\Pi_{0i}$ $(i = 1, 2)$	$V_0^c = W^c(q^{c*})$
	$U_j^c = \Pi_j$ $(j \neq i)$	$V_i^c = (1-\beta)\Pi_{0i}(q^{c*})$
		$V_j^c = \Pi_j(q^{c*})$
regime (d)	$U_{012}^d = \gamma W^d + (1-\gamma)\Pi_{012}$	$V_0^d = W^d(q^{d*})$
		$V_1^d = (1-\gamma)\delta\Pi_{012}(q^{d*})$
		$V_2^d = (1-\gamma)(1-\delta)\Pi_{012}(q^{d*})$

4.2.3 Market structures and the core

Each of the four regimes, except for the mixed triopoly which itself represents the market structure, includes more than one market structure, each of which can be identified in terms of shareholding ratio in the merged firm and of merger participants, *i.e.*, a coalition formation. For example, in the regime (c), we can find one particular market structure that is composed of the merged firm 01 with the government's shareholding $\beta = 0.5$ and the private firm 2. Which of the

possible market structures will actually occur fairly depends on the managerial decision making of the three owners of the firms 0, 1, and 2: *merge*, *not to merge*, or *break off the merger*.

In the preceding subsection, given the market structure that will actually occur as a result of coalition formation among the owners, we have examined the Cournot equilibrium for each market structure. Now, a natural question to ask is which of the market structures will occur as a consequence of the owners coalition formation. This problem can be analyzed in terms of the game of coalition formation among the owners. As discussed in the introduction, in the case of more than two firms' owners, it is not sufficient to analyze the decision by the owners for each particular case, and the stability problem will be of importance. Thus, we especially focus on which market structure, or coalition formation among the owners, will be stable in the sense that once a market structure in question is realized, it never shift into any other market structure. To analyze this stability problem we invoke the *core*, the well-established solution concept in cooperative game theory. We assume that each of the owners determines the managerial decision on a merger to maximize her/his own payoff V_i. The reader may notice that the market structures and the payoffs in our framework corresponds to the feasible allocations and the utilities (or preferences) in the market game in exchange economy which is well-established topic in microeconomic theory (see for example, Varian (1992) pp.387-388).

To define the core of the market structures, we should start with the definition of a *blocking* market structure. A market structure M is said to *block* another market structure M' if there exists a deviant coalition of the owner(s) of the pre-marged firm(s) such that:

> (i) M can be constructed from M' by solely the decision by the owner(s) in the deviant coalition, and
> (ii) every owner in the coalition achieves strictly higher payoff in M than in M'.

An example will help understanding the definition of blocking. Let $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$ be the market structure composed of the merged firm 01 with $\beta = 0.5$ and the private firm 2. In this case, for example, the coalition of the owners of the firms 0 and 2, $\{0, 2\}$, can construct, if they want, the new market structure that consists of the merged firm 02 with $\beta = 0.45$ and the private firm 1, denoted by $M_{\beta=0.45}^{\{\{0,2\},\{1\}\}}$. If the owner of the firm 2 gains more payoff, *i.e.*, the distributed profit, and the owner of the firm 0, *i.e.*, the government, also achieves higher payoff, *i.e.*, social welfare, in the new structure $M_{\beta=0.45}^{\{\{0,2\},\{1\}\}}$ than in $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$, then the structure $M_{\beta=0.45}^{\{\{0,2\},\{1\}\}}$ blocks $M_{\beta=0.5}^{\{\{0,1\},\{2\}\}}$. Note that it is also possible that a deviant coalition consists of a single owner of a pre-merged firm. In the above example, it is possible for each of the owners of the pre-merged firms 0 and

1 to deviate from the structure by breaking off the merger and to operate their own pre-merged firms respectively, *i.e.*, to shift into the mixed triopoly, as well. The core of the market structures is defined as:

the set of market structures that are *never blocked by any other market structure*.

We denote the core of the market structures by $\mathfrak{Co}$. If a market structure is in the core, all of the three owners of the pre-merged firms have no incentive to construct a new market structure. In this sense, the market structure in the core can be regarded as the stable one. In the next section, we examine which of the market structures is/are in the core.

4.3 Results

We now explore the core of the market structures, *i.e.*, stable structures. Our argument proceeds through some lemmata, each of which points out the market structures which are blocked by some other market structure. Our first lemma shows that the market structure of the merged firm 012 is not in the core no matter what a ratio of shareholding by the pre-merged firms is adopted.

Lemma 4.1. *For any ratio of shareholding by the three owners of the pre-merged firms, the market structure of the merged firm* 012*,* $M^{\{\{0,1,2\}\}}_{\gamma,\delta}$*, can not belong to the core, i.e.,* $M^{\{\{0,1,2\}\}}_{\gamma,\delta} \notin \mathfrak{Co}$*, for any* $\gamma \in [0,1]$ *and any* $\delta \in [0,1]$*.*

Proof. The proof proceeds in two steps.

Step 1. Let $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta=1/2}$ be the market structure of the merged firm 012 with $\gamma \in [0,1]$ and $\delta = 1/2$, and $M^{\{\{0,1\},\{2\}\}}_{\beta\in[0,1]}$ be that of the public-private merged firm 01 and the private firm 2 with a ratio of shareholding $\beta \in [0,1]$ in the merged firm 01. We will show that the owner of the private firm 2 wants to deviate from the merger among the three firms. Since

$$\frac{dV_2^c(\beta)}{d\beta} = \frac{12(2-\beta)}{(-11+4\beta)^3}a^2 < 0, \quad \forall \beta \in [0,1], \tag{4.16}$$

we have

$$\min_{\beta\in[0,1]} V_2^c(\beta) = V_2^c(\beta)\big|_{\beta=1} = \frac{2}{49}a^2. \tag{4.17}$$

Then, solving the following equation:

$$V_2^d(\gamma,\delta)\big|_{\delta=1/2} = \frac{2}{49}a^2, \tag{4.18}$$

we obtain the result

$$V_2^d(\gamma,\delta)\big|_{\delta=1/2} = \min_{\beta\in[0,1]} V_2^c(\beta) \quad \text{if} \quad \gamma = \frac{93-7\sqrt{41}}{90} \approx 0.5353. \tag{4.19}$$

Since $d\left(V_2^d(\gamma,\delta)|_{\delta=1/2}\right)/d\gamma = -3a^2\left(32-27\gamma\right)/2\left(8-3\gamma\right)^3 < 0, \forall\gamma\in[0,1]$, we obtain

$$V_2^d(\gamma,\delta)\big|_{\delta=\frac{1}{2}} < \min_{\beta\in[0,1]} V_2^c(\beta), \ \ \forall\gamma\in\left(\frac{93-7\sqrt{41}}{90},1\right]. \tag{4.20}$$

Therefore, if $\gamma > \left(93-7\sqrt{41}\right)/90$, the owner of the pre-merged firm 2 deviates from $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta=1/2}$ and operates her/his own firm regardless of what a ratio β is, *i.e.*, $M^{\{\{0,1\},\{2\}\}}_{\beta\in[0,1]}$ blocks $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta=1/2}$. In cases where $\delta\neq 1/2$, the same conclusion also follows for one of the owners of the pre-merged private firms, 1 or 2, because one of them inevitably receives strictly less payoff than in the case of $\delta = 1/2$.

Step 2. Let $\mathcal{I}$ be the interval $\left[0,\left(93-7\sqrt{41}\right)/90\right]$. To complete the proof, we have to show that $M^{\{\{0,1,2\}\}}_{\gamma\in\mathcal{I},\delta\in[0,1]}$ is blocked by some other market structure. Consider the market structure of the public-private merged firm 01 and the private firm 2 with a ratio of shareholding $\beta=\gamma$, *i.e.*, $M^{\{\{0,1\},\{2\}\}}_{\beta=\gamma}$. We show that the coalition $\{0,1\}$ has an incentive to deviate from the merger among the three firms if $\delta = 1/2$. Let $\beta:\mathbb{R}\to\mathbb{R}$ be such that $\beta(t)=t$. When $\gamma=\left(93-7\sqrt{41}\right)/90$, the difference between the payoffs to the owner of the firm 0 across the two market structures is

$$\left(V_0^c(\beta(\gamma)) - V_0^d(\gamma)\right)\big|_{\gamma=\left(93-7\sqrt{41}\right)/90} = \frac{25(1024237+79947\sqrt{41})}{98(32396969+4258989\sqrt{41})}a^2 > 0. \tag{4.21}$$

Moreover, we have

$$\begin{aligned}\frac{d\left(V_0^c(\beta(\gamma)) - V_0^d(\gamma)\right)}{d\gamma} &= -\frac{3(6859-13655\gamma+9324\gamma^2-2682\gamma^3+279\gamma^4)}{(8-3\gamma)^3(11-4\gamma)^3}a^2 \\ &< 0, \quad \forall\gamma\in[0,1]\,.\end{aligned} \tag{4.22}$$

Therefore, the government can achieve higher payoff, *i.e.*, higher social welfare, in $M^{\{\{0,1\},\{2\}\}}_{\beta=\gamma}$ than in $M^{\{\{0,1,2\}\}}_{\gamma\in\mathcal{I},\delta=1/2}$. Similarly, we obtain the following results on the payoff to the owner of the pre-merged firm 1,

$$\left(V_1^c(\beta(\gamma)) - V_1^d(\gamma,\delta)\right)\big|_{\gamma=\left(93-7\sqrt{41}\right)/90,\delta=1/2} = \frac{(2267+51177\sqrt{41})}{196(309+14\sqrt{41})^2}a^2 > 0, \tag{4.23}$$

and, for all $\gamma\in[0,1]$,

$$\frac{d\left(V_1^c(\beta(\gamma)) - V_1^d(\gamma,\delta)|_{\delta=1/2}\right)}{d\gamma} = -\frac{3(5024-8031\gamma+5460\gamma^2-1759\gamma^3+216\gamma^4)}{2(8-3\gamma)^3(11-4\gamma)^3}a^2 < 0. \tag{4.24}$$

Thus, the owner of the pre-merged firm 1 can gain more payoff in $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}}$ than in $M_{\gamma\in\mathcal{I},\delta=1/2}^{\{\{0,1,2\}\}}$. Thus, the joint deviation by $\{0,1\}$ is beneficial to each of the owners of the firms 0 and 1. The same argument as in the step 1 can be directly applied to any case of $\delta \neq 1/2$ to show that the market structures of the merger among the three firms is blocked through the joint deviation of the government and one of the owners of the pre-merged private firms. □

The intuition behind the lemma is explained as follows. In the cases of high values of γ, the merged firm 012 sets relatively high output because of the considerable influence of the owner of the pre-merged public firm 0, and this hurts the payoffs to the owners of the pre-merged private firms. On the other hand, for low values of γ, the merged firm 012 attaches relatively high importance to its profit, then the owner of the pre-merged public firm 0 can do better by breaking off the merger. This trade-off in the owners' interests in the merged firm 012 makes the merger unstable. Indeed, as shown in the proof of the lemma, in the case of high values of $\gamma \in \left[\left(93-7\sqrt{41}\right)/90, 1\right]$, either of the two owners of pre-merged private firms, say i, has an incentive to deviate from the merged firm 012. On the other hand, in the case of $\gamma \in \left[0, \left(93-7\sqrt{41}\right)/90\right]$, the owners of public firm 0 has an incentive to break off the merger and to make an offer of organizing new merged firm $0i$ to the one of the two private owners 1 and 2. In both cases, the key is that the positive effect of the improvement on productivity in the merger among the three firms is relatively small to the merger between two firms.

Next, we provide our second lemma which tells that at least one of the two owners of the pre-merged private firms prefers the mixed triopoly rather than the merger between these two private firms regardless of what a ratio of the shareholding between them is adopted, *i.e.*, the market structure of the merger between the private firms is not in the core no matter what a ratio of shareholding is in the merged firm.

Lemma 4.2. *For any ratio of shareholding $\alpha \in [0,1]$, the market structure of the merger between the private firms, $M_{\alpha}^{\{\{0\},\{1,2\}\}}$, is blocked by the mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$.*

Proof. Since we have $\sum_{i=1}^{2}\Pi_i^a = 16a^2/169 > 3a^2/32 = \Pi_{12}^b$, it is obvious that there exists no $\alpha \in [0,1]$ such that $\alpha\Pi_{12}^b \geq \Pi_1^a$ and $(1-\alpha)\Pi_{12}^b \geq \Pi_2^a$. □

This result is due to the strengthened market share of the public firm. It is known that two-firm mergers in the Cournot oligopoly tend often to be unprofitable, due to the aggressive response from the firms not participating in the merger (see for example, Salant et al. (1983)). Although

the private merged firm 12 gets an advantage of the improvement on productivity in the current framework, the subsequent expansion of the market share of the non-merged firm, the public firm 0 which aims to maximize not profit but social welfare, becomes larger than in the case of the private Cournot oligopoly. Consequently, the profit of the merged firm 12 can not exceed the sum of the profits gained by the pre-merged private firms, and the merger between the private firms will never be beneficial to the owners of the pre-merged private firms in the current framework, either.

We now move to our third lemma. While the mixed triopoly, as stated in Lemma 4.2, blocks the market structures of the regime (b) and, consequently, excludes them from the core, the following lemma shows that the mixed triopoly can not belong to the core, either. To state the lemma, we let

$$\underline{\beta} = \frac{638 - 39\sqrt{31}}{739} \approx 0.56950 \quad \text{and} \quad \bar{\beta} = \frac{6197 - 39\sqrt{6001}}{5572} \approx 0.56996. \tag{4.25}$$

Lemma 4.3. *The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$, is blocked by the market structure of the public-private merged firm $0i$ and the private firm $j \neq i$, $M_{\beta}^{\{\{0,i\},\{j\}\}}$, if the ratio of shareholding β in the merged firm $0i$ is in the interval $\big(\underline{\beta},\ \bar{\beta}\big)$.*

Proof. In the Cournot equilibrium of each of the regimes (a) and (c), we have

$$V_0^a = \frac{99}{338}a^2,\ V_0^c = \frac{(68 - 44\beta + 5\beta^2)}{2(11 - 4\beta)^2}a^2,\ V_i^a = \frac{8}{169}a^2, \text{ and } V_i^c = \frac{9(3 - 2\beta)(1 - \beta)}{2(11 - 4\beta)^2}a^2.$$

Thus, we obtain the following:

$$\begin{cases} 0 \leq \beta \leq \underline{\beta} \Rightarrow V_0^a \geq V_0^c \\ \underline{\beta} < \beta \leq 1 \Rightarrow V_0^a < V_0^c \end{cases} \quad \text{and} \quad \begin{cases} 0 \leq \beta < \bar{\beta} \Rightarrow V_i^c > V_i^a \\ \bar{\beta} \leq \beta \leq 1 \Rightarrow V_i^c \leq V_i^a. \end{cases}$$

Thus, each of the owners of 0 and i have an incentive to jointly found the merged firm $0i$ if the shareholding ratio β is in $\big[0, \bar{\beta}\big) \cap \big(\underline{\beta}, 1\big] = \big(\underline{\beta},\ \bar{\beta}\big)$. □

As we have just shown in the proof of Lemma 4.3, if the ratio of shareholding by the government is more than $\underline{\beta}$, *i.e.*, $\beta > \underline{\beta}$, the government will agree to the merger with a private firm i since she can achieve higher social welfare by the positive effect of productivity improvement. On the other hand, the owner of the pre-merged private firm i can gain more payoff in the merged firm $0i$ than in the mixed triopoly whenever $\beta < \bar{\beta}$. Therefore, for any $\beta \in \big(\underline{\beta},\ \bar{\beta}\big)$, both of the two owners have an incentive to merge into a new public-private firm $0i$. From this observation, we immediately obtain the following lemma.

Lemma 4.4. *The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$, blocks the market structure of the public-private merged firm $0i$ and the private firm $j \neq i$, $M_{\beta}^{\{\{0,i\},\{j\}\}}$, whenever the ratio of shareholding β in the merged firm $0i$ is in $\left[0, \underline{\beta}\right)$ or $\left(\bar{\beta}, 1\right]$, i.e., $\beta \in \left[0, \underline{\beta}\right) \cup \left(\bar{\beta}, 1\right]$.*

Proof. This lemma immediately follows from the proof of Lemma 4.3 where we have shown that if $\beta \in \left(\bar{\beta}, 1\right]$ (resp. $\left[0, \underline{\beta}\right)$) then the owner of the private firm i (resp. the owner of the public firm 0) has an incentive to deviate and change the present market structure into the mixed triopoly. □

From Lemmata 4.1 to 4.4, we now know that almost all market structures can not be in the core. The market structures that belong to any of the regimes (a), (b), and (d) are not in the core. Moreover, in the regime (c), the market structures with $\beta \in \left[0, \underline{\beta}\right) \cup \left(\bar{\beta}, 1\right]$ can not belong to the core, either. As a consequence, the remaining candidates that could belong to the core are the market structures of the public-private merged firm $0i$ and the private firm $j \neq i$, with the ratio of shareholding by the government $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$. We now state our main result, which shows that any of these market structures is in the core.

Proposition 4.1. *The market structure of the public-private merged firm $0i$ and the private firm $j \neq i$, $M_{\beta}^{\{\{0,i\},\{j\}\}}$, is in the core whenever the ratio of shareholding in the merged firm $0i$, β, is in the closed interval $\left[\underline{\beta}, \bar{\beta}\right]$, i.e., $M_{\beta}^{\{\{0,i\},\{j\}\}} \in \mathfrak{Co}, \forall \beta \in \left[\underline{\beta}, \bar{\beta}\right]$.*

From this proposition, it can be concluded that the market structures of the public-private merged firm $0i$ and the private firm $j \neq i$ with $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$ are stable in the sense that any of these market structures is never blocked by the other market structures. In other words, once any of these structures is realized, it will never be replaced by any of the other market structures. It should be emphasized that the interval of the admissible ratio β in the core $\left[\underline{\beta}, \bar{\beta}\right]$ is very short, $\bar{\beta} - \underline{\beta} \approx 0.00047$. This result is fairly remarkable in that it shows a considerable contrast to the result obtained in Kamijo and Nakamura (2009). In their paper, Kamijo and Nakamura analyzed the industry composed by two symmetric private firms and a less efficient public firm. Assuming that each of the three firms has constant marginal cost of production, Kamijo and Nakamura showed that all of the four regimes, except for the regime (b), have the market structures that belong to the core. Therefore, it can be said that the stable mergers in mixed oligopoly crucially depend on the assumptions of firms' technology.

Finally, we briefly examine the case where the industry is composed only of private firms, *i.e.*, private oligopoly, and compare the results between mixed oligopoly and private oligopoly.

In the case of private oligopoly, we need to change the model summarized in Table 4.1 as follows: $U_0^a = \Pi_0$; $V_0^a = \Pi_0(q^{a*})$; $U_0^b = \Pi_0$; $V_0^b = \Pi_0(q^{b*})$; $U_{0i}^c = \Pi_{0i}$; $V_0^c = \beta\Pi_{0i}(q^{c*})$; $U_{012}^d = \Pi_{012}$; and $V_0^d = \gamma\Pi_{012}(q^{d*})$. Consequently, the regimes (b) and (c) become the same ones, and we let the regime (b) represent them. The Cournot equilibria of the regimes (a), (b) and (d) are obtained as:

$$(q_i^{a*}, q_1^{a*}, q_2^{a*}) = \left(\frac{a}{6}, \frac{a}{6}, \frac{a}{6}\right),\ (q_0^{b*}, q_{12}^{b*}) = \left(\frac{2}{11}a, \frac{3}{11}a\right),\ \text{and}\ q_{012}^{d*} = \frac{3}{8}a.$$

Then, the payoffs to the owners are determined as follows:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{a^2}{18}, \frac{a^2}{18}, \frac{a^2}{18}\right),\ (V_0^b, V_1^b, V_2^b) = \left(\frac{8}{121}a^2, \frac{27}{242}\alpha a^2, \frac{27}{242}(1-\alpha)a^2\right),\ \text{and}$$
$$(V_0^d, V_1^d, V_2^d) = \left(\frac{3}{16}\gamma a^2, \frac{3}{16}(1-\gamma)\delta a^2, \frac{3}{16}(1-\gamma)(1-\delta)a^2\right).$$

Given the above payoffs, we obtain the following result.

Proposition 4.2. *In the case where the industry is composed only of private firms, none of the market structures belongs to the core, i.e.,* $\mathfrak{Co} = \varnothing$.

Proof. The proof is similar to those of Lemmata 4.1 to 4.4. Thus, we limit ourselves to providing the examples of blocking market structures for each market structure. For the regime (a), $M^{\{\{0\},\{1\},\{2\}\}}$ is blocked by $M_\alpha^{\{\{0\},\{1,2\}\}}$ whenever $\alpha \in (121/243, 122/243)$. For the regime (b), $M_\alpha^{\{\{0\},\{1,2\}\}}$ is blocked (i) by $M^{\{\{0\},\{1\},\{2\}\}}$ if $\alpha \in [0, 121/243)$ or $\alpha \in (122/243, 1]$; and (ii) by $M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ with $\gamma = 9/25$ and $\delta = 1/2$ if $\alpha \in [121/243, 122/243]$. For the regime (d), $M_{\gamma=1/3,\delta=1/2}^{\{\{0,1,2\}\}}$ is blocked by $M_\alpha^{\{\{0\},\{1,2\}\}}$ with $\alpha \in [0, 1]$. By the same argument as in the proof of Lemma 4.1, the case of $(\gamma, \delta) = (1/3, 1/2)$ is sufficient to complete the proof of the regime (d). □

The never-ending coalition formation increases transaction costs unboundedly. Furthermore, it eliminates our ability to predict which of the market structure will actually occur, which also means that it is hardly possible to prescribe economic policies in a effective way. Comparing Propositions 4.1 and 4.2, we can conclude that the presence of the public firm has a stabilizing effect in the current framework and allows us to avoid such undesirable costs.

4.4 Concluding remarks

This chapter explored the stable market structures in mixed oligopoly when a single public firm and two symmetric private firms in the homogeneous good market are allowed to freely merge

and freely break off the merger. We adopted the core as the solution concept to analyze the stability of market structures. We showed that the core consists solely of the market structures derived by the merger between a public firm and one of the two private firms with the shareholding ratio by the public firm, β, which is greater than $\underline{\beta} \approx 0.56950$ and less than $\bar{\beta} \approx 0.56996$. These market structures are stable in the sense that, by the definition of the core, once any of these market structures is actually realized, it never be replaced by any of the other market structures. The admissible interval of β that ensures the stability of market structures is very short. This strong result fairly relies on the assumption that a merger yields the improvement on productivity. Without such a positive effect of a merger, our result would change and the mixed triopoly would be a unique stable market structure.[2]

Two interesting extensions of our model remain. The first is to consider the model in which the foreign shareholders are taken into account. In the real world, some firms are foreign-owned. In this case, social welfare that the government is to maximize should not include the profits of the foreign-owned firms. Thus, the existence of the foreign shareholders will change the public firm's decision making and, consequently, the equilibrium outcomes as well. The other possible extension is to introduce the asymmetricity among the production technologies of firms in a way like $C(q) = k_i q_i^2$. In this chapter, we assumed that all the three firms have identical technologies $(k_0, k_1, k_2) = (1, 1, 1)$. It seems more natural to assume that a public firm shows inefficient performance relatively to private firms, e.g. X-inefficiency in a public firm. In the case of $(k_0, k_1, k_2) = (3, 1, 1)$, the reader may easily check that the core becomes empty in the similar method to the proofs of Lemmata 4.1 to 4.4 and Proposition 4.2. The analysis of more general cases of $k = (k_0, k_1, k_2)$ and the comparison among different values of the weight k is left for future research.

Appendix A: Proof of Proposition 4.1

Let $M^{\{\{0,i\},\{j\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ be the market structure of the merged firm $0i$ with a ratio of shareholding $\beta \in [\underline{\beta}, \bar{\beta}]$ and the private firm j ($\neq i$). In a series of claims below, we will show that $M^{\{\{0,i\},\{j\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is never blocked by any other market structure. We assume, without loss of generality, $i = 1$ and $j = 2$.

Claim 4.1. *$M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is never blocked by the mixed triopoly in any case of $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$.*

[2] In the Appendix B, we present a theoretical analysis of the stable market structures after each firm's merger activity in the case wherein s/he adopts the FJSV delegation contract à la Lambertini (2000) and Straume (2006).

By Lemma 4.3, $M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is not blocked by the mixed triopoly if $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$. Moreover, in the proof of Lemma 4.3, we have shown that, in the case of $\beta = \underline{\beta}$, the government in $M^{\{\{0,1\},\{2\}\}}_{\beta=\underline{\beta}}$ can achieve the same level of social welfare as in the mixed triopoly, and thus the government has no incentive to deviate from $M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$, and also that the owner of the pre-merged firm 1 in $M^{\{\{0,1\},\{2\}\}}_{\beta=\underline{\beta}}$ gains more payoff than in the mixed triopoly. Thus, neither of these two owners want to break off the merger. The case of $\beta = \bar{\beta}$ can be proved by the symmetric argument to the case of $\beta = \underline{\beta}$.

Claim 4.2. *$M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is never blocked by the market structure of the public firm 0 and the private merged firm 12 with $\alpha \in [0,1]$, $M^{\{\{0\},\{1,2\}\}}_{\alpha\in[0,1]}$, in any case of $\alpha \in [0,1]$.*

By (4.16), $V_2^c(\beta)$ is decreasing on $[0,1]$, and thus we have

$$\min_{\beta\in[\underline{\beta},\bar{\beta}]} V_2^c(\beta) = V_2^c(\bar{\beta}) = \frac{(1649+13\sqrt{6001})^2}{1352(234+\sqrt{6001})^2}a^2. \tag{4.26}$$

When $\beta = \bar{\beta}$ in $M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$, *i.e.*, in $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$, the owner of the private firm 2 will agree with the merger between the two private firms if and only if

$$V_2^b(\alpha) > V_2^c(\bar{\beta}) \Leftrightarrow \frac{3}{32}(1-\alpha)a^2 > \frac{(1649+13\sqrt{6001})^2}{1352(234+\sqrt{6001})^2}a^2 \tag{4.27a}$$

$$\Leftrightarrow \alpha < \frac{401707-1768\sqrt{6001}}{621075} \approx 0.4263. \tag{4.27b}$$

On the other hand, we obtain the following result on the payoffs to the firm 1: for any $\alpha < \left(401707-1768\sqrt{6001}\right)/621075$,

$$V_1^b(\alpha) - V_1^c(\bar{\beta}) = \frac{3}{32}\alpha a^2 - \frac{8}{169}a^2 \tag{4.28a}$$

$$< \frac{3}{32}\alpha a^2\Big|_{\alpha=\frac{401707-1768\sqrt{6001}}{621075}} - \frac{8}{169}a^2 \tag{4.28b}$$

$$= \frac{88107-1768\sqrt{6001}}{6624800}a^2 \approx -0.0074a^2 < 0. \tag{4.28c}$$

Thus, by (4.27b) and (4.28c), the joint deviation by the owners of the firms 1 and 2 can not be realized if $\beta = \bar{\beta}$. Since V_2^c is decreasing with respect to β, by (4.27a) to (4.28c) altogether, the joint deviation by the owners of the private firms is still impossible in any case of $\beta \in \left[\underline{\beta}, \bar{\beta}\right)$.

Claim 4.3. *$M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is never blocked by $M^{\{\{0,2\},\{1\}\}}_{\beta'\in[0,1]}$ in any case of $\beta' \in [0,1]$.*

We start with the case of $\beta = \bar{\beta}$ in $M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$. In this case, the owner of the firm 2 prefers $M^{\{\{0,2\},\{1\}\}}_{\beta'\in[0,1]}$ rather than $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$ if and only if the payoff, *i.e.*, the distributed profit, in $M^{\{\{0,2\},\{1\}\}}_{\beta'\in[0,1]}$

is strictly greater than the payoff, *i.e.*, the stand-alone profit, gained in $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$, *i.e.*, the following value, $\Delta(\beta')$, must be positive:

$$\Delta(\beta') := (1-\beta')\Pi^c_{02}(\beta') - \Pi^c_2(\bar{\beta}) \tag{4.29a}$$

$$= \frac{9(3-2\beta')(1-\beta')}{2(11-4\beta')^2}a^2 - \frac{(1649+13\sqrt{6001})^2}{1352(234+\sqrt{6001})^2}a^2 \tag{4.29b}$$

$$= a^2 \cdot \Xi(\beta'), \tag{4.29c}$$

where $\Xi(\beta') = \Big[28(12134951+89440\sqrt{6001})(\beta')^2 - 2(379922845+2615912\sqrt{6001})\beta' + 328599497 + 1677091\sqrt{6001}\Big] \,/\, \Big[676(234+\sqrt{6001})^2(11-4\beta')^2\Big]$. Solving the equation $\Delta(\beta') = 0$ subject to $\beta' \in [0,1]$, we obtain

$$\beta'^* = \frac{\left(379922845 + 2615912\sqrt{6001} - 39\sqrt{31920488675573 + 391144962052\sqrt{6001}}\right)}{339778628 + 2504320\sqrt{6001}}$$

$$\approx 0.5151. \tag{4.30}$$

Since

$$\frac{d\Delta(\beta')}{d\beta'} = \frac{d((1-\beta')\Pi^c_{02}(\beta'))}{d\beta'} = -\frac{9(31-24\beta')}{2(11-4\beta')^3}a^2 < 0, \quad \forall \beta' \in [0,1], \tag{4.31}$$

we obtain the intermediate result that the owner of the firm 2 prefers $M^{\{\{0,2\},\{1\}\}}_{\beta' \in [0,1]}$ to $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$ if and only if

$$\beta' \in [0, \beta'^*)\,. \tag{4.32}$$

On the other hand, since we have

$$\frac{dV^c_0(\beta)}{d\beta} = \frac{3(10-11\beta)}{(11-4\beta)^3}a^2 > 0, \;\; \forall \beta \in [0,\bar{\beta}] \;\big(\supsetneq [0,\beta'^*]\big), \tag{4.33}$$

the owner of the public firm 0 strictly prefers $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$ rather than $M^{\{\{0,2\},\{1\}\}}_{\beta'}$ if $\beta' < \beta'^*$ $(< \bar{\beta})$, and thus the joint deviation by the owners of the firms 0 and 2 from $M^{\{\{0,1\},\{2\}\}}_{\beta=\bar{\beta}}$ can never be realized. Note that, from (4.16), the profit (or payoff to the owner) of the firm 2 in $M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is decreasing with respect to β, which in turn implies that, by (4.29a) to (4.29c) and (4.31), a decrease in β leads to a decrease in β'^*. Thus, from the fact that $\beta'^* < \underline{\beta}$ and (4.33), the owner of the firm 0 never agrees with the joint deviation with the owner of the firm 2 in any case of $\beta \in [\underline{\beta}, \bar{\beta})$.

Claim 4.4. *$M^{\{\{0,1\},\{2\}\}}_{\beta\in[\underline{\beta},\bar{\beta}]}$ is never blocked by the merger among the three firms $M^{\{\{0,1,2\}\}}_{\gamma\in[0,1],\delta\in[0,1]}$ regardless of what the ratios $\gamma \in [0,1]$ and $\delta \in [0,1]$ are.*

We start with the case of $\beta = \underline{\beta}$. In this case, we have $V_0^c(\underline{\beta}) = 99a^2/338$. Since $dV_0^d(\gamma)/d\gamma = 27a^2(1-\gamma)/(8-3\gamma)^3 \geq 0$ for all $\gamma \in [0,1]$ (equality holds only in the case of $\gamma = 1$),

$$V_0^d(\gamma) - V_0^c(\underline{\beta}) = \frac{3(11-9\gamma)(-23+33\gamma)}{338(8-3\gamma)^2}a^2 > 0, \quad \forall \gamma \in \left(\frac{23}{33}, 1\right]. \tag{4.34}$$

On the other hand, we obtain the following result on the payoff to the owner of the pre-merged private firm 2:

$$\frac{d(V_2^d(\gamma,\delta)|_{\delta=0})}{d\gamma} = \frac{3(32-27\gamma)}{(-8+3\gamma)^3}a^2 < 0, \quad \forall \gamma \in [0,1] \tag{4.35}$$

and

$$V_2^d(\gamma,\delta)\big|_{\gamma=23/33,\delta=0} - V_2^c(\underline{\beta}) = \frac{4(817-260\sqrt{31})}{616005}a^2 \approx -0.0041a^2 < 0. \tag{4.36}$$

Note that the case of $\delta = 0$ is the most favorable case of δ for the owner of the pre-merged private firm 2. Hence, in the case of $\beta = \underline{\beta}$, by (4.34) to (4.36), the owners of the pre-merged public firm 0 and pre-merged private firm 2 can never reach an agreement about the shareholding in the merged firm 012, and thus the merger among the three firms can never be realized. Now, we examine the other cases of $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$, *i.e.*, $\beta \in \left(\underline{\beta}, \bar{\beta}\right]$. By (4.33), the value of γ^* which solves the equation $V_0^d(\gamma) - V_0^c(\beta) = 0$ increases in any case of $\beta \in \left(\underline{\beta}, \bar{\beta}\right]$ than in the case of $\beta = \underline{\beta}$, *i.e.*, $\gamma^* > 23/33$. From (4.16) and (4.35), we have

$$V_2^d(\gamma^*,\delta)|_{\delta=0} - V_2^c(\beta)\big|_{\beta\in(\underline{\beta},\bar{\beta}]} < V_2^d(\gamma,\delta)\big|_{\gamma=23/33,\delta=0} - V_2^c(\bar{\beta}) \tag{4.37a}$$

$$= \frac{13739-221\sqrt{6001}}{828100}a^2 \approx -0.0041a^2 < 0. \tag{4.37b}$$

Therefore, by the same argument as in the case of $\beta = \underline{\beta}$, the merger among the three firms is impossible in any case of $\beta \in \left(\underline{\beta}, \bar{\beta}\right]$.

From (4.33) and the fact that $dV_1^c(\beta)/d\beta = -9a^2(31-24\beta)/2(11-4\beta)^3 < 0, \forall \beta \in [0,1]$, any alteration on the ratio β never improves the payoffs to both owners of the firms 0 and 1 simultaneously. Therefore, combining the assertions of the claims, we have successfully shown that $M_\beta^{\{\{0,1\},\{2\}\}}$ is in the core whenever β is in the closed interval $\left[\underline{\beta}, \bar{\beta}\right]$. □

Appendix B: Exention to a managerial delegation case

To formalize managerial delegation, we mainly follow Straume (2006).[3] In each firm, an owner delegates the output decision to a manager. Each manager sets the output to maximize her/his

[3]The analyses conducted in this Appendix is base od those provided by Kamaga and Nakamura (2008b).

payoff defined by an incentive contract provided by the owner of the firm. Let q_i and Π_i denote the output and profit of a firm i. In each firm i, an owner provides the following type of incentive contract ϕ_i to a manager:

$$\phi_i(\Pi_i(q_i), q_i; \theta_i) = \theta_i \Pi_i(q_i) + (1 - \theta_i) q_i. \tag{4.38}$$

where θ_i is a contract parameter chosen by the owner (or owners in a merged firm). A manager of a firm i can maximize her/his payoff by choosing the output q_i which maximizes ϕ_i. This can be supported by the assumption that the payoff to a manager of a firm i is represented as $\lambda_i + \mu_i \phi_i$ for some real number λ_i and some positive real number μ_i.

In our delegation model, owners and managers play the following two-stage game: in the first stage, owners simultaneously choose incentive contracts for managers, then in the second stage, the firms' managers simultaneously set the outputs. The equilibrium outcomes of each regime are those derived by the subgame perfect Nash equilibrium.

From the routine backward calculation, for a given list of contract parameters, managers' equilibrium outputs, each denoted q_i^{mr}, are determined as follows. In regime (a), the equilibrium outputs set by the managers of the public firm 0 and private firms i ($i = 1, 2$) are

$$q_0^{ma} = \frac{1}{18}\left(3(a-1) + \frac{5}{\theta_0^a} - \frac{1}{\theta_1^a} - \frac{1}{\theta_2^a}\right) \text{ and } q_i^{ma} = \frac{1}{18}\left(3(a-1) - \frac{1}{\theta_0^a} + \frac{5}{\theta_i^a} - \frac{1}{\theta_j^a}\right), \tag{4.39}$$

where $j = 1, 2$ and $j \neq i$. In regime (b), the equilibrium outputs of the managers of the public firm 0 and the private merged firm 12 are given as

$$q_0^{mb} = \frac{1}{11}\left(2(a-1) + \frac{3}{\theta_0^b} - \frac{1}{\theta_{12}^b}\right) \text{ and } q_{12}^{mb} = \frac{1}{11}\left(3(a-1) - \frac{1}{\theta_0^b} + \frac{4}{\theta_{12}^b}\right). \tag{4.40}$$

In regime (c), the equilibrium outputs set by the managers of the public-private merged firm $0i$ and private firm j ($i, j = 1, 2$ with $i \neq j$) are

$$q_{0i}^{mc} = \frac{1}{11}\left(3(a-1) + \frac{4}{\theta_{0i}^c} - \frac{1}{\theta_j^c}\right) \text{ and } q_j^{mc} = \frac{1}{11}\left(2(a-1) - \frac{1}{\theta_{0i}^c} + \frac{3}{\theta_j^c}\right). \tag{4.41}$$

Finally, in regime (d), the manager of the merged firm 012 sets the following output

$$q_{012}^{md} = \frac{1}{8}\left(3(a-1) + \frac{3}{\theta_{012}^d}\right). \tag{4.42}$$

We are ready to present the equilibrium outcomes for each of the four market regimes. Taking into account of the outputs subsequently realized in the second stage: (4.39), (4.40), (4.41), and (4.42), the owners choose their optimal incentive contracts to maximize their (constituted) objectives. Tables 4.2 and 4.3 summarize the equilibrium incentive contracts θ_i^{r*} and the equilibrium outputs q^{r*}, profits Π_i^{r*}, and social welfare W^{r*} in each regime r.

Table 4.4 summarizes the payoffs to the owners for each regime (r).

Table 4.2: Equilibrium incentive contracts in each regime (r)

(r)	incentive contract θ_i^{r*}		
(a)	$\theta_0^{a*} = \frac{289}{289+47a}$	$\theta_i^{a*} = \frac{289}{289+20a}$	$(i = 1, 2)$
(b)	$\theta_0^{b*} = \frac{7}{7+a}$	$\theta_{12}^{b*} = \frac{14}{14+a}$	
(c)	$\theta_{0i}^{c*} = \frac{109-35\beta}{109-35\beta+a(8+20\beta)}$	$\theta_j^{c*} = \frac{109-35\beta}{109-35\beta+a(7-3\beta)}$	$(i, j = 1, 2; i \neq j)$
(d)	$\theta_{012}^{d*} = \frac{8-3\gamma}{8-3\gamma+3a\gamma}$		

Table 4.3: Equilibrium outputs, profits, and social welfare in each regime (r)

(r)	equilibrium outcomes
(a)	$q_0^{a*} = \frac{59}{289}a, \quad q_i^{a*} = \frac{50}{289}a \quad (i = 1, 2)\,;$ $\Pi_0^{a*} = \frac{4189}{83521}a^2, \quad \Pi_i^{a*} = \frac{4000}{83521}a^2 \quad (i = 1, 2)\,;$ $W^{a*} = \frac{49659}{167042}a^2$
(b)	$q_0^{b*} = \frac{3}{14}a, \quad q_{12}^{b*} = \frac{2}{7}a;$ $\Pi_0^{b*} = \frac{3}{49}a^2, \quad \Pi_{12}^{b*} = \frac{5}{49}a^2;$ $W^{b*} = \frac{113}{392}a^2$
(c)	$q_{0i}^{c*} = \frac{2a(16-\beta)}{109-35\beta}, \quad q_j^{c*} = \frac{3a(7-3\beta)}{109-35\beta}, \quad (i, j = 1, 2; i \neq j)\,;$ $\Pi_{0i}^{c*} = \frac{2a^2\left(640-408\beta+23\beta^2\right)}{(109-35\beta)^2}, \quad \Pi_j^{c*} = \frac{15a^2(7-3\beta)^2}{(109-35\beta)^2} \quad (i, j = 1, 2; i \neq j)\,;$ $W^{c*} = \frac{a^2\left(6839-4058\beta+483\beta^2\right)}{2(109-35\beta)^2}$
(d)	$q_{012}^{d*} = \frac{3a}{8-3\gamma};$ $\Pi_{012}^{d*} = \frac{3a^2(4-3\gamma)}{(8-3\gamma)^2};$ $W^{d*} = \frac{3a^2(11-6\gamma)}{2(8-3\gamma)^2}$

From the equilibrium outcomes presented in Table 4.3, the payoffs to the owners, (V_0^r, V_1^r, V_2^r),

Table 4.4: Owners' payoffs V_i^r in each regime (r)

(r)	payoffs (V_0^r, V_1^r, V_2^r)
(a)	$\left(V_0^a, V_1^a, V_2^a\right) = \left(W^{a*},\ \Pi_1^{a*},\ \Pi_2^{a*}\right)$
(b)	$\left(V_0^b, V_1^b, V_2^b\right) = \left(W^{b*},\ \alpha\Pi_{12}^{b*},\ (1-\alpha)\Pi_{12}^{b*}\right)$
(c)	$\left(V_0^c, V_i^c, V_j^c\right) = \left(W^{c*},\ (1-\beta)\Pi_{0i}^{c*},\ \Pi_j^{c*}\right), \quad (i,j=1,2,\ i\neq j)$
(d)	$\left(V_0^d, V_1^d, V_2^d\right) = \left(W^{d*},\ (1-\gamma)\delta\Pi_{012}^{d*},\ (1-\gamma)(1-\delta)\Pi_{012}^{d*}\right)$

are explicitly given as follows:

$$(V_0^a, V_1^a, V_2^a) = \left(\frac{49659}{167042}a^2,\ \frac{4000}{83521}a^2,\ \frac{4000}{83521}a^2\right); \tag{4.43a}$$

$$(V_0^b, V_1^b, V_2^b) = \left(\frac{113}{392}a^2,\ \frac{5\alpha}{49}a^2,\ \frac{5\,(1-\alpha)}{49}a^2\right); \tag{4.43b}$$

$$(V_0^c, V_i^c, V_j^c) = \left(\frac{(6839-4058\beta+483\beta^2)}{2(109-35\beta)^2}a^2,\ \frac{2(1-\beta)(640-408\beta+23\beta^2)}{(109-35\beta)^2}a^2,\ \frac{15(7-3\beta)^2}{(109-35\beta)^2}a^2\right); \tag{4.43c}$$

$$(V_0^d, V_1^d, V_2^d) = \left(\frac{3\,(11-6\gamma)}{2\,(8-3\gamma)^2}a^2,\ \frac{3\,(1-\gamma)\,(4-3\gamma)\,\delta}{(8-3\gamma)^2}a^2,\ \frac{3\,(1-\gamma)\,(4-3\gamma)\,(1-\delta)}{(8-3\gamma)^2}a^2\right). \tag{4.43d}$$

The main body of this chapter has shown that the core is non-empty in the case of entrepreneurial firms, and that the core consists solely of the market structures of the merger between the public firm 0 and one of the two private firms with the share ratio near around 0.57. In contrast to their result, we obtain the following striking but serious impossibility result in our delegation model.

Proposition 4.3. *None of the market structures belongs to the core, i.e.,* $\mathfrak{Co} = \varnothing$.

Proof. The proof proceeds through a series of claims (a) to (d). In each claim (r) (= a, b, c, d), it will be shown that, for any market structure M in the regime (r), we can find some other market structure which blocks M via some coalition $S \subseteq \{0,1,2\}$.

Claim (a): The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$ is blocked by the market structure of the merger between the private firms, $M_\alpha^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in (\underline{\alpha}, \bar{\alpha})$, where $\underline{\alpha} = 39200/83521$ and

$\bar{\alpha} = 44321/83521$.

This claim is easily checked as follows:

$$V_1^b(\alpha) - V_1^a = \frac{-5a^2(39200 - 83521\alpha)}{4092529} > 0 \;\Leftrightarrow\; \alpha > \frac{39200}{83521}, \tag{4.44}$$

$$V_2^b(\alpha) - V_2^a = \frac{5a^2(44321 - 83521\alpha)}{4092529} > 0 \;\Leftrightarrow\; \alpha < \frac{44321}{83521}. \tag{4.45}$$

Thus, the joint deviation by $\{0, 1\}$ will take place if $\alpha \in (39200/83521, 44321/83521)$.

Claim (b): (i) The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$ blocks the the market structure of the merger between the private firms, $M_\alpha^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in [0, 39200/83521)$;

(ii) When $\beta = 1/3$, the market structure of the public-private merged firm $0i$ and the private firm $j \neq i$ $M_\beta^{\{\{0,i\},\{j\}\}}$ blocks the the market structure of the merger between the private firms, $M_\alpha^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in [39200/83521, 44321/83521]$;

(iii) The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$ blocks the market structure of the merger between the private firms, $M_\alpha^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in (44321/83521, 1]$.

The statements (i) and (iii) are straightforward from the fact that the equivalence assertions in (4.44) and (4.45) still hold when we reverse the inequality signs. We provide the proof of (ii). Let, without loss of generality, $i = 1$. For the owner 0, we have

$$V_0^c(\beta) - V_0^b = \frac{-a^2(2109 - 66822\beta + 43757\beta^2)}{392(109 - 35\beta)^2} > 0 \Leftrightarrow \beta > \frac{4773 - 164\sqrt{777}}{6251} \approx 0.0322422. \tag{4.46}$$

On the other hand, for the owner of the firm 1, we have

$$V_1^c(\beta)|_{\beta=\frac{1}{3}} - V_1^b(\alpha)|_{\alpha=\frac{44321}{83521}} = \frac{4486643171a^2}{261709044492} > 0. \tag{4.47}$$

Note that $44321/83521 = \arg\max_{\alpha \in [39200/83521, 44321/83521]} V_1^b(\alpha)$. Thus, by (4.46) and (4.47), $M_{\beta=1/3}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_\alpha^{\{\{0\},\{1,2\}\}}$ in any case of $\alpha \in [39200/83521, 44321/83521]$.

Claim (c): (i) The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$ blocks the market structure of the public-private merged firm $0i$ and the private firm, $j \neq i$ $M_\beta^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in (3/5, 1]$;

(ii) The mixed triopoly, $M^{\{\{0\},\{1\},\{2\}\}}$ blocks the market structure of the public-private merged firm $0i$ and the private firm $j \neq i$, $M_\beta^{\{\{0,i\},\{j\}\}}$ in any case of $\beta \in [0, 3/5]$.

Let, without loss of generality, $i = 1$. For the owner 1, we have

$$V_1^a - V_1^c(\beta) = \frac{-2a^2(29691440 - 72270008\beta + 33547551\beta^2 - 1920983\beta^3)}{83521(109 - 35\beta)^2} > 0,$$
$$\forall \beta \in \left(0.543873,\ 1\right] \supseteq \left(\frac{3}{5},\ 1\right]. \tag{4.48}$$

On the other hand, taking into account that $d\left(V_0^a - V_0^c(\beta)\right)/d\beta = -169a^2\left(111 - 112\beta\right)/\left(109 - 35\beta\right)^3 < 0$ for all $\beta \in [0, 3/5]$, we have

$$V_0^a - V_0^c(\beta) = \frac{2a^2(4699615 - 9992488\beta + 5122908\beta^2)}{83521(109 - 35\beta)^2} > 0 \Leftrightarrow \forall \beta \in \left[0,\ \frac{3}{5}\right], \tag{4.49}$$

since $V_0^a - V_0^c(\beta)|_{\beta=3/5} = 13709227a^2/8084832800 > 0$. Thus, the statement (i) follows from (4.48), and (ii) does from (4.49), respectively.

Claim (d): (i) When $\beta = \gamma$, the market structure of the public-private merged firm 01 and the private firm 2, $M_\beta^{\{\{0,1\},\{2\}\}}$ blocks the market structure with the merger among all the three firms, $M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in [0, 4/5)$ and $\delta \in [0, 1]$;

(ii) When $\beta = \gamma/\left[\gamma + (1-\gamma)\delta\right]$, the market structure of the public-private merged firm 01 and the private firm 2, $M_\beta^{\{\{0,1\},\{2\}\}}$ blocks the market structure with the merger among all the three firms, $M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ in any case of $\gamma \in [4/5, 1]$ and $\delta \in [0, 1]$.

We begin with the proof of (i). For the payoff to the owner of the public firm 0, we have

$$V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma) > 0, \quad \forall \gamma < \frac{4}{5}, \tag{4.50}$$

from the fact that $d\left(V_0^c(\beta)|_{\beta=\gamma} - V_0^d(\gamma)\right)/d\gamma = -a^2\big(25645335 - 48758498\gamma + 29986128\gamma^2 - 7514154\gamma^3 + 661689\gamma^4\big)/\left(8 - 3\gamma\right)^3\left(109 - 35\gamma\right)^3 < 0$ for all $\gamma \in [0, 1]$ and $V_0^c(\beta)|_{\beta=4/5} - V_0^d(\gamma)|_{\gamma=4/5} = 202087a^2/257191200 > 0$. For the payoff to the owner of the private firm 1, we also obtain,

$$V_1^c(\beta)|_{\beta=\gamma} - V_1^d(\gamma,\delta)|_{\delta=\frac{1}{2}} > 0, \quad \forall \gamma < \frac{4}{5}, \tag{4.51}$$

because we have

$$\frac{d\big(V_1^c(\beta)|_{\beta=\gamma} - V_1^d(\gamma,\delta)|_{\delta=1/2}\big)}{d\gamma}$$
$$= \frac{-a^2\begin{bmatrix}17873952 - 52618843\gamma + 67857663\gamma^2 - 39401297\gamma^3 \\ +11066013\gamma^4 - 1507788\gamma^5 + 86940\gamma^6\end{bmatrix}}{2(8-3\gamma)^3(109-35\gamma)^3} < 0, \quad \forall \gamma \in [0,1], \tag{4.52}$$

and $V_1^c(\beta)|_{\beta=4/5} - V_1^d(\gamma,\delta)|_{\gamma=4/5,\delta=1/2} = 28043a^2/5953500 > 0$. Thus, by (4.50) and (4.51), we obtain $M_{\beta=\gamma}^{\{\{0,1\},\{2\}\}} \succ_{\{0,1\}} M_{\gamma<4/5,\delta=1/2}^{\{\{0,1,2\}\}}$. Since in the cases of $\delta \neq 1/2$ either of the two owners 1 and 2 will receive smaller payoff than in the case of $\delta = 1/2$, we can apply the above argument to the owner with the smaller payoff and complete the proof of (i).

Next, we prove (ii). Because $\delta = 0$ is the most favorable case of δ for the owner 2 and $dV_2^c(\beta)/d\beta = -2460a^2(7-3\beta)/(109-35\beta)^3 < 0$ for all $\beta \in [0,1]$, it is sufficient to show that $M_{\gamma\geq 4/5,\delta=0}^{\{\{0,1,2\}\}}$ is blocked by $M_{\beta=\gamma/[\gamma+(1-\gamma)\delta]}^{\{\{0,1\},\{2\}\}}$, *i.e.*, by $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, via $\{2\}$. In $M_{\gamma\geq 4/5,\delta=0}^{\{\{0,1,2\}\}}$, the payoff to the owner of the private firm 2 is given as:

$$V_2^d(\gamma,\delta)|_{\delta=0} = \frac{3a^2(1-\gamma)(4-3\gamma)}{(8-3\gamma)^2}. \tag{4.53}$$

From the fact that $dV_2^d(\gamma,\delta)|_{\delta=0}/d\gamma = -3a^2(32-27\gamma)/(8-3\gamma)^3 < 0$ for all $\gamma \in [4/5,1]$,

$$\max_{\gamma\in[4/5,1]} V_2^d(\gamma,\delta)|_{\delta=0} = V^d\left(\frac{4}{5},0\right) = \frac{3a^2}{98}. \tag{4.54}$$

On the other hand, in the market structure $M_{\beta=1}^{\{\{0,1\},\{2\}\}}$, the payoff to the owner of the private firm 2 is

$$V_2^c(\beta)|_{\beta=1} = \frac{15a^2(7-3\beta)^2}{(109-35\beta)^2}\Big|_{\beta=1} = \frac{60a^2}{1369} > \frac{3a^2}{98} = \max_{\gamma\in[4/5,1]} V_2^d(\gamma,\delta)|_{\delta=0}, \tag{4.55}$$

Thus, $M_{\beta=1}^{\{\{0,1\},\{2\}\}} \succ_{\{2\}} M_{\gamma,\delta=0}^{\{\{0,1,2\}\}}$ for any case of $\gamma \in [4/5,1]$. □

Similar to the case wherein only the private firms exist in the market, we obtained the result that the core is empty.

By the definition of the core, none of the market structures is stable in the sense that there always exists at least one owner who wants to deviate and induce a new market structure: in the regime (a), the private owners 1 and 2; in (b), the coalition of the owners 0 and i $(= 1,2)$ or a single private owner j $(= 1,2)$; in (c), either of the owners 0 and i $(= 1,2)$; and in (d), the two owners 0 and i $(= 1,2)$, or a single private owner j $(= 1,2)$. In what follows, we explain the reasoning behind the results we stated as claims in the proof of proposition 4.3.

Our claim (a) tells that the market structure of the mixed triopoly is blocked by those structures of the regime (b) with $\alpha \in (\underline{\alpha},\bar{\alpha})$. In the regime (b), the private merged firm 12 enjoys improved production technology represented by the cost function $C_{12}(q_{12}) = (q_{12})^2/2$ and competes with the public firm 0 whose production technology is less efficient than the firm 12, while every firm in the regime (a) has a symmetric production technology. Faced

with this asymmetry of technologies in the regime (b), the owner of the public firm, a welfare maximizer, chooses the contract parameter θ_0^b larger than the one in the regime (a) (note that $\theta_0^b - \theta_0^a = 40a/(7+a)(289+47a) > 0$) to induce more of the production by the merged firm 12 which is now operating more efficient production technology. Consequently, both of the two owners of the merged firm 12 can achieve higher payoffs in the regime (b) than in the regime (a).

In the model of mixed triopoly of entrepreneurial firms, which was considered in the main body in this chapter, it was shown that the market structures of the regime (c) with $\beta \in [0.56950, 0.56996]$ belong to the core, where 0.56996 (resp. 0.56950) is the highest (resp. lowest) value of β that the owner of the private firm i (resp. the public firm 0) agrees on the merger. Our claim (c), however, tells that, in the case of managerial firms, any market structure of the regime (c) is blocked by the market structure of the mixed triopoly. To explain the reasoning behind this result, we compare the equilibrium outcomes obtained in the entrepreneurial model which was considered in the main body of this chapter and those in the managerial model here. We use a superscript e to denote equilibrium outcomes in the entrepreneurial model. In the entrepreneurial model of the main body of this chapter, the equilibrium outputs of the public firm 0 and private firm i and the equilibrium profit of the private firm i in the regime (a) are obtained as follows:

$$q_0^{ae} = \frac{3}{13}a, \quad q_i^{ae} = \frac{2}{13}a, \quad \Pi_i^{ae} = \frac{8}{169}a^2;$$

and the equilibrium outputs of the merged firm $0i$ and private firm j and the equilibrium profit of the merged firm $0i$ in the regime (c) are given as:

$$q_{0i}^{ce} = \frac{3}{11-4\beta}a, \quad q_j^{ce} = \frac{2-\beta}{11-4\beta}a, \quad \Pi_{0i}^{ae} = \frac{9(3-2\beta)}{2(11-4\beta)^2}a^2.$$

Comparing these outcomes with those obtained in the managerial model, we will observe the following changes brought by managerial delegation: (i) in the regime (a), the output of the private firm i increases ($q_i^{a*} - q_i^{ae} = 72a/3757$) while the output of the public firm, now operated by a manager whose objective is not welfare maximization, decreases ($q_0^{a*} - q_0^{ae} = -100a/3757$); (ii) consequently, the profit of the private firm i in the regime (a) becomes higher in the managerial case than in the entrepreneurial case ($\Pi_i^{a*} - \Pi_i^{ae} = 7832a^2/14115049$); (iii) but in the regime (c) where the merged firm $0i$ increases the output for the case of relatively small β ($q_{0i}^{c*} - q_{0i}^{ce} = a(5-\beta)(5-8\beta)/(11-4\beta)(109-35\beta) > 0$ for all $\beta \in [0, 5/8)$), the competitor of the merged firm $0i$ is not a public firm but solely the private firm j which also increases the output ($q_j^{c*} - q_j^{ce} = a(13-4\beta+\beta^2)/(11-4\beta)(109-35\beta) > 0$ for all $\beta \in [0,1]$), and such an expansion of the output by the private firm j will decrease the profit of the merged firm $0i$ for almost all cases of

β $(\Pi_{0i}^{c*} - \Pi_{0i}^{ae} = -[a^2(11027 + 2884\beta - 25293\beta^2 + 12158\beta^3 - 1472\beta^4)/2(11 - 4\beta)^2(109 - 35\beta)^2] < 0$ for all $\beta \in [0, 0.959536))$. As a consequence, the shareholding ratio by the owner i, i.e. $(1 - \beta)$, need to become higher in the managerial case than in the entrepreneurial case to induce an agreement of the owner i on the merger between the public firm 0 and private firm i. In fact, as show in (4.48), the value of $1 - \beta$ must be larger than 0.456127, or equivalently β smaller than 0.543873 in our managerial model, whereas β must be smaller than 0.56996 in the entrepreneurial case. However, as shown in (4.49), the owner of the public firm 0 never agrees on the merger as long as β is lower than 0.543873.

Although the market structures of the regime (b) with $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ blocks the mixed triopoly, these market structures, as stated in (ii) of Claim (b), are blocked by those of the regime (c) with $\beta = 1/3$. By (4.46), (4.47), and the derivative of $V_1^c(\beta)$, it is easily checked that this result also holds for any $\beta \in \big((4773 - 164\sqrt{777})/6251, 1/3\big]$. The reasoning behind this result is explained as follows. Because of the regime (b) is less competitive than the regime (a), social welfare which the owner of the public firm 0 wants to maximize becomes lower than the level in regime (a). Consequently, while the owners of the public firm 0 and private firm i in the regime (a) never reach an agreement on the merger between these two firms, the owner of the public firm 0 in the regime (b) now has an incentive to make a conciliatory offer to the owner of the private firm i on the merger of these two firms, and they can reach an agreement on the merger.

Finally, Claim (d) shows that the market structures in the regime (d) are blocked by those in the regime (c). This result is due to the fact that the positive effect of the improvement on productivity in a merged firm is relatively smaller in the case of the shift from the regime (c) (or (b)) into (d) than in the cases of the shift from (a) into (c) (or (b)).

Chapter 5

Stable market structures from merger activities in mixed oligopoly with asymmetric costs

5.1 Introduction

This chapter presents a theoretical analysis of the merger activities in an industry composed of one state-owned welfare-maximizing public firm and multiple profit-maximizing private firms with constant marginal cost functions.[1,2] We often observe mergers between public and private firms in the real world, for example, in the European automobile industry; fewer efforts have been made to study merger activities in a mixed oligopoly.[3] Some exceptions include Bárcena-Ruiz and Garzón (2003), Coloma (2006) and Méndez-Naya (2008).[4] However, these three papers assume a fixed number of merger participants. Applying a cooperative game theoretical approach, the present chapter analyzes mergers in a mixed oligopoly without assuming who the merger participants are. and takes into account of the fact that each firm is freely allowed to merge and break off mergers.[5]

[1] All the analyses conducted in this chapter are based on those of Kamijo and Nakamura (2009).

[2] The literature on mixed oligopoly have tackled several economic issues. For example, Bárcena-Ruiz and Garzón (2006), Kato (2006) and Ohori (2006) analyzed environmental policies. Ishibashi and Matsumura (2006), Nishimori and Ogawa (2002) and Poyago-Theotoky (1998) investigated R&D competition between the public firm and the private firm. Moreover, Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), and Matsumura and Matsushima (2006) explored the international competition.

[3] Bárcena-Ruiz and Garzón (2003) discusses the $M\&A$ of SEAT, a Spanish publicly-owned automobile enterprise, with Volkswagen in 1986 and the business cooperation between Renault, a French partially state-owned firm, and Nissan, a Japanese private firm, as examples of mergers between public and private firms.

[4] Both Bárcena-Ruiz and Garzón (2003) and Coloma (2006) tackle horizontal mergers in a mixed duopoly composed of a public firm and a private firm. Méndez-Naya (2008) extends the scope of these papers to the analysis of mergers in a more general mixed market in which one public firm and n private firms exist in the same market.

[5] We treat merger formation as a cooperative game for two reasons. First, we would like to emphasize that the results in this chapter more generally predict the final outcomes of market structures after merger activities than the previous literature on horizontal mergers in a mixed oligopoly. Second, by adopting the cooperative game approach,

There are many papers on merger activities in a private oligopoly, and they take two different analytical approaches. The first one – the classic analysis – is called the *exogenous merger*. The works on exogenous mergers attempt to answer the question of why firms merge. In this literature, researchers fix a group of firms whose members compare the benefits of merging with the benefits of standing alone. Salant et al. (1983), the pioneering paper in this field, obtained a result that contradicted several empirical examples that the formation of a merged entity will be an equilibrium phenomenon only if numerous firms participate in the merge; this is called the merger paradox. More recent literature on exogenous mergers theoretically tackles the resolution of the phenomenon of the merger paradox and works with the merger profitability under several economic environments, such as strategic delegation and strategic trade policy.[6]

The other type of analysis on merger activities is called the *endogenous merger*, which is our approach in this chapter. The literature in this area focuses her/his attention on answering the question of how firms merge, and to this end, it assumes that all firms are allowed to choose whether or not to merge and how to react to a merger. Moreover, we can observe two strands with respect to the manner of analysis in the literature on modeling endogenous merger formation. One strand comprises the works that use a non-cooperative game theoretical approach, and these constitute the majority of the literature on endogenous mergers.[7] The other strand is the cooperative game theoretical approach to analyzing endogenous mergers. The literature in this field analyzes whether a particular market structure can be the outcome of a merger process by studying the situation where no firm wants to induce a new market structure; hence, it focuses its attention on the stability of market structures. Although Barros (1998) and Horn and Persson (2001a) similarly consider the outcome of merger formations using a cooperative game approach, they each adopt slightly different solution concepts.[8] Straume (2006) and Banal-Estañol et al.

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we can generalize the traditional criterion that determines the success of a merger on the basis of whether or not the payoffs of the preliminary fixed merger participants improve after the merger.

[6]Perry and Porter (1985) proposes that convex cost functions provide a resolution to the merger paradox. However, most recently, Heywood and Mcginty (2007a) reconsiders this issue in several environments and show that the convex costs are not as relevant as the resolution to the merger paradox. Moreover, in the literature on strategic delegation, González-Maestre and López-Cuñat (2001) and Ziss (2001) examine how managerial delegation affects the profitability of horizontal mergers, and in the context of strategic trade policy, Huck and Konrad (2004) shows that the presence of a strategic trade policy can make otherwise unprofitable mergers profitable.

[7]Deneckere and Davidson (1983) was the first a multi-stage non-cooperative model of endogenous merger formation. Nilssen and Sørgard (1998) and Fauli-Oller (2000) deal with the dynamics of merger processes, taking into account that a merger might trigger other mergers. Gowrisankaran (1999) and Gowrisankaran and Holmes (2004) examine the dynamic model of endogenous mergers to explain the extent to which an industry in which mergers are feasible will tend toward monopoly.

[8]Barros (1998) does not name the solution concept in which the conditions that are usually required with respect to the core should be satisfied, while Horn and Persson (2001a) explicitly calls its solution concept the core. See Brito and Gata (2006) for a detailed discussion about the difference between the definition adopted by Barros (1998) and the one considered in Horn and Persson (2001a).

(2008) adopt the core concept presented by Horn and Persson (2001a) as their solution concept.[9]

In this chapter, in order to consider the stability of market structures in a mixed triopoly, we invoke the core concept considered in Barros (1998) in the context of a private oligopoly and in Kamaga and Nakamura (2007) in the context of a mixed oligopoly.[10] In our framework, coalitions of firms (including singletons or stand-alone firms) compete in the market so that the payoff of a firm relies on the coalitions composed of the other firms.[11] Furthermore, in order to achieve a merger, the merger participants have to agree with the shareholding ratios in the merged firm, similar to Barros (1998) and Kamaga and Nakamura (2007), and when a public firm and one private firm merge, the objective of the merged firm is to maximize the weighted average of social welfare and the profit of the merged firm with respect to the share ratio of each the pre-merged firm's owner, similarly to Matsumura (1998). Then, the characteristic difficulty of the case of a mixed oligopoly occurs in the analysis of endogenous mergers, which differs from that of the private oligopoly in Barros (1998) and Horn and Persson (2001a). Along with the merger between public and private firms, the market outcomes themselves, such as the price and the output level of each firm, are influenced by the share ratios of the merged firm. Thus, the analysis of the stability problem of merger activities in the context of a mixed oligopoly certainly needs more elaboration than that of a private oligopoly.

Using the above theoretical approach, in this chapter, we reveal that the market structures that belong to the core, *i.e.*, *stable market structures*, change according to the value of the marginal cost of the public firm; however, we also show that according to the value of the marginal cost of the public firm, there always exists a pair of share ratios of the owners of both the (pre-merged) public firm, *i.e.*, the government, and the (pre-merged) private firm such that the market structure with the merger between the public firm and one private firm belongs to the core.[12] Moreover,

[9] Straume (2006) compares the equilibrium market structure with the socially preferred market structure in order to establish the correspondence between the social and private incentives for horizontal mergers, taking both the cases of entrepreneurial and managerial firms into consideration. Banal-Estañol et al. (2008) analyzes the effects of investment decisions and a firm's internal organization on the efficiency and stability of horizontal mergers.

[10] More precisely, Kamaga and Nakamura (2007) considers the stability of horizontal merger activities under economic environments where a technology of each firm is represented as a quadratic cost function in a mixed triopoly. In their paper, Kamaga and Nakamura assume that a merger improves productivity on the basis of the hypothesis that the merged firm adopts the most efficient operation plan among the merger participants, described in McAfee and Williams (1992), Nakamura and Inoue (2007), and Heywood and Mcginty (2007a), and show that only the stable market structure contains a merged public-private firm with about 57% of its shares owned by the public firm. Furthermore, Kamaga and Nakamura (2007) finds that without any productivity-improving effect with a merger, the mixed oligopoly is a unique stable market structure.

[11] Yi (1997) provides two cases in accordance with the influence of the formation of coalitions on the payoffs of players who belong to other coalitions. Yi defines the case in which the formation of coalitions *increases* (*decreases*) the payoffs of players within other coalitions as the one of *positive* (*negative*) effects.

[12] Bárcena-Ruiz and Garzón (2003) considers the merger between a public firm and a private firm in the mixed duopolistic industry (thus, the endogenous merger analysis is conducted), and it shows that the merger between a

we newly define an *achievable* market structure as follows: When the initial market structure is a mixed triopoly, the market structure blocks the mixed triopoly, and it is never blocked by other market structures. This answers the question of which market structure(s) that belong(s) to the core is/are most likely to occur. Then, we find that the stable and achievable market structure exists only when there is a market one with the merger between one public firm and one private firm. Furthermore, we consider a general mixed oligopolistic model to confirm the robustness of the result in the mixed triopolistic model, and then we obtain the result that there exists a pair of share ratios of the owners of both the (pre-merged) public and private firms such that the market structure with the merger between one public firm and one public firms is stable and achievable.

The remainder of this chapter is organized as follows. In Section 5.2, we formulate our model and describe four regimes: (a) no mergers, (b) merger between two private firms, (c) merger between a private firm and a public firm, and (d) merger among all firms. Furthermore, we obtain the Cournot-Nash equilibrium values of various variables and the equilibrium payoffs of the owners in each regime. In Section 5.3, we discuss the stability problem for mergers among firms in a mixed oligopoly by adopting the core as the solution concept. Moreover, we consider the achievablity of the stable market structures, *i.e.*, which of the structures is/are likely to emerge from the mixed oligopoly. In Section 5.4, we present the analysis of the general model with respect to the number of firms existing in the mixed market and confirm the robustness of the result obtained in the mixed triopolistic model. Section 5.5 concludes the chapter.

5.2 The model

We consider an industry consisting of two identical private firms and one public firm, all with a single homogeneous output. Let q_0 and q_i denote the quantities of the public firm and private firm i, respectively $(i = 1, 2)$ and let $Q = q_0 + \sum_{i=1}^{2} q_i$ denote the aggregate quantity of the market. The market price is determined by the inverse demand function $P = a - Q$, where we assume that a is sufficiently large. Let $c_j(q_j), j = 0, 1, 2$ denote the cost function of firm j. While the two private firms have constant and identical marginal costs of production normalized to 0, the public firm is assumed to be less efficient than they are and its marginal cost is denoted by $c > 0$

public firm and a private firm is observed in equilibrium, depending on the values of the following two parameters: the share ratio of the (pre-merged) public firm's owner and the degree of differentiation of the goods produced by both firms. Therefore, from the viewpoint that the merger between one public firm and one private firm is sustainable, our result is similar to that in Bárcena-Ruiz and Garzón (2003).

and $c \leq a/3$.[13,14] Thus, $c_0(q_0) = cq_0$ and $c_i(q_i) = 0$ for $i = 1, 2$.[15] Since entry decisions are not considered, we assume that there are no fixed costs.

The profit of firm i is given by

$$\pi_0 = q_0(a - Q) - cq_0, \text{ and } \pi_i = q_i(a - Q), \ i = 1, 2,$$

and social welfare W is defined as the sum of consumer surplus ($CS = Q^2/2$) and the profits of the firms (π_i) in the market.

Let the owner of firm i be referred to as owner i. By definition, owner 0 is from the public sector and owners 1 and 2 are from the private sector. Following the literature, it is assumed that the public sector is concerned with social welfare; thus public firm 0 is a welfare maximizer. On the other hand, the private firms are assumed to be profit maximizers. When the public firm and private firms decide to merge, the merged entity, m, is owned by two parties that have different objectives. Thus, merged firm m must consider both profit as well as social welfare. Let $s \in [0, 1]$ denote the public sector's shareholding proportion in the merged firm and $(1 - s) \in [0, 1]$ denote the private sector's proportion. Then, following Matsumura (1998) and other studies related to partial privatization,[16] we assume that firm m maximizes

$$V(s, 1 - s) = sW + (1 - s)\pi_m,$$

where q_m and $\pi_m = (a - Q)q_m - c_m(q_m)$ are the amount of output and the profit of firm m, respectively.[17] Note that in this model, the production cost of any merged firm is always 0.[18]

[13]There are studies on a mixed oligopoly with a constant marginal cost setting, such as Mujumdar and Pal (1998), Pal (1998), Matsumura (2003a), and Lu (2006).

[14]This implies that a is sufficiently large to the extent that $a \geq 3c > 0$. This assumption guarantees that each of the equilibrium outputs of the public firm in all four regimes is positive or equal to 0.

[15]It is assumed that the technologies of all the firms are represented by constant marginal cost functions; however, the public firm is less efficient than the two private firms. Some theoretical works show that in mixed markets, public firms are more inefficient than private firms. For example, see Matsumura and Matsushima (2004), which shows that the private firm's cost is lower than that of the public firm, because the private firm engages in excessive strategic cost-reducing activities in a Hotelling-type spatial model. In the real world, it is well-known that the experiences of some countries, such as New Zealand and Great Britain provide an adequate explanation for the assumption that public firms have higher costs of production. See Mizutani and Uranishi (2003) for empirical studies.

[16]For example, Tomaru (2006) and Fujiwara (2006).

[17]The objective function of the merged public-private firm is supported by the fact that in European countries, various types of coordination of public and private power interests are observed, such as regulation up and active interference through state ownership and participation in mixed companies, as indicated in Staudinger (1937).

[18]This form of the cost function is supported by the assumption that the merged firm adopts the most efficient operation plan among the merger participants. More precisely, the cost function of the merged firm is defined by the following total cost minimization problem:

$$\min \sum_{i \in S} c_i(q_i) \ \text{ s.t.} \sum_{i \in S} q_i = q_m \text{ and } q_i \geq 0 \text{ for } i \in S,$$

where $S \subseteq \{0, 1, 2\}$ is the set of merger participants.

In what follows, we consider four types of regimes classified by the type of merger: (a) no merger, *i.e.*, in the case where three firms compete in the market, (b) merger between two private firms, (c) merger between a private firm and a public firm, and (d) merger among all firms. Let us denote the merged entity by m_S, where S is the set of the participants in the merger, and denote the shares of owner i in the merged entity by $s_i \in [0,1]$, $i \in S$, $\sum_{i\in S} s_i = 1$.

(a) Mixed triopoly $\{\{\mathbf{0}\},\{\mathbf{1}\},\{\mathbf{2}\}\}$

First, we consider a mixed triopoly with one public firm and two private firms, assuming that there is no merger (or that the merger has not yet occurred). In this case, private firm i chooses q_i to maximize its own profit π_i while the public firm chooses q_0 to maximize social welfare $W = (q_0+q_1+q_2)^2/2 + \sum_{i=0}^{2}\pi_i$. Solving these three problems simultaneously, we obtain the Cournot-Nash equilibrium outcome for case (a) as follows:

$$q_0^a = a-3c,\ \ q_i^a = c,\ \ Q^a = a-c,\ \ P^a = c,$$
$$\pi_0^a = 0,\ \ \pi_i^a = c^2,\ \ W^a = \frac{a^2-2ac+5c^2}{2}.$$

Here, we use the symbol a to denote the equilibrium corresponding to regime (a). In addition, in this case, the payoffs of owner 0 and private owner i $(=1,2)$ are the equilibrium welfare and equilibrium profit of firm i, respectively. Thus, the payoffs of owner 0 and private owner i $(=1,2)$ are given by

$$u_0^a = W^a = \frac{a^2-2ac+5c^2}{2},\ \text{ and }\ u_i^a = \pi_i^a = c^2,\ \ (i=1,2).$$

(b) Merger between two private firms $\{\{\mathbf{0}\},\{\mathbf{1},\mathbf{2}\}\}$

Second, we consider the case where two private firms (two private owners) decide to merge. The market structure after the merger is a mixed duopoly consisting of the public firm and the new "merged" private firm 12, where we use 12 instead of $m_{\{1,2\}}$ for convenience. Social welfare, *i.e.*, the objective function of the public firm, is defined by $W = (q_0+q_{12})^2/2 + \pi_0 + \pi_{12}$. According to the ownership of the firm, the objective of firm 12 is defined as the profit of the merged firm, π_{12}. In the equilibrium outcomes for regime (b), we obtain the following:

$$q_0^b = a-2c,\ \ q_{12}^b = c,\ \ Q^b = a-c,\ \ P^b = c,$$
$$\pi_0^b = 0,\ \ \pi_{12}^b = c^2,\ \ W^b = \frac{a^2-2ac+3c^2}{2}.$$

In addition, the payoff of owner 0 is given by

$$u_0^b(s_1,s_2) = W^b = \frac{a^2-2ac+3c^2}{2}.$$

Let $s_1 = t$ and $s_2 = 1 - t$. Then, the payoffs of private owners 1 and 2 are given by

$$u_1^b(s_1, s_2) = s_1\pi_{12}^b = tc^2, \text{ and } u_2^b(s_1, s_2) = s_2\pi_{12}^b = (1-t)c^2.$$

(c) Merger between a private firm and a public firm $\{\{\mathbf{0}, \mathbf{i}\}, \{\mathbf{j}\}\}$

Next, we consider the case in which a private firm and a public firm decide to merge. We assume that public firm 0 merges with private firm i. Thus, firm $j = 1, 2$, $j \neq i$ is outside the merger. Social welfare is given by $W = (q_{0i} + q_j)^2/2 + \pi_{0i} + \pi_j$. While firm j maximizes π_j, the merged firm, m_{0i}, maximizes $V(s_0, s_i) = V(s, 1-s)$, where $s_0 = s$ and $s_i = 1 - s$ denote the ratios of public owner 0 and private owner i, respectively, in the merged firm's shareholding.

Solving the maximization problem given shares s, we now obtain the Cournot-Nash equilibrium in regime (c) as follows:

$$q_{0i}^c = \frac{a}{3-2s}, \quad q_j^c = \frac{a(1-s)}{3-2s}, \quad Q^c = \frac{a(2-s)}{3-2s}, \quad P^c = \frac{a(1-s)}{3-2s},$$
$$\pi_{0i}^c = \frac{a^2(1-s)}{(3-2s)^2}, \quad \pi_j^c = \frac{a^2(1-s)^2}{(3-2s)^2}, \quad W^c = \frac{a^2(2-s)(4-3s)}{2(3-2s)^2},$$

The payoffs of owner 0 and private owner i, who is inside the merger, are given by

$$u_0^c(s_0, s_i) = W^c = \frac{a^2(2-s)(4-3s)}{2(3-2s)^2}, \text{ and } u_i^c(s_0, s_i) = (1-s_0)\pi_{0i}^c = \frac{a^2(1-s)^2}{(3-2s)^2}.$$

In addition, since the payoff of private owner j, who is outside the merger, is the equilibrium profit of firm j, it is given by

$$u_j^c(s_0, s_i) = \pi_j^c = \frac{a^2(1-s)^2}{(3-2s)^2}.$$

Here, we find an interesting property. The payoff of the private owner i, who is a participant in the merger, coincides with the payoff of another private owner, j, who is outside the merger. This property plays an important role when the stability problem is considered.[19]

(d) Merger among all firms $\{\{\mathbf{0}, \mathbf{1}, \mathbf{2}\}\}$

Finally, we consider the case where all firms merge. Thus, the market becomes a public monopoly.[20] In this regime, social welfare is defined by $W = (q_{012})^2/2 + \pi_{012}$. The merged firm 012 maximizes $V(s_0, s_1 + s_2) = V(s, 1-s)$, where s_i denotes the shares of owner i and $s = s_0$. Solving

[19] This property holds as long as all private firms have symmetric constant marginal costs of production even if there are more than two private firms. To be more precise, when there are n private firms and one public firm, and the public firm and some of the private firms agree to merge, then, the distributed profits of the private owners of the merged firm coincide with the profits of each outsider of this merger. (See Section 5.4)

[20] Barros (1998) excludes merger for monopoly by assuming that the antitrust authority would not give clearance to such a merger. However, we do not eliminate this case, because our focus is on stable states resulting from merger activities.

the maximization problem, we obtain the Cournot-Nash equilibrium in regime (d) as follows:

$$q_{012}^d = \frac{a}{2-s},\ \ P^d = \frac{a\,(1-s)}{2-s},\ \ \pi_{012}^d = \frac{a^2\,(1-s)}{(2-s)^2},\ \ W^d = \frac{a^2\,(3-2s)}{2\,(2-s)^2}.$$

Given shares s, the payoff of owner 0 is defined as

$$u_0^d(s_0, s_1, s_2) = W_{012}^d = \frac{a^2\,(3-2s)}{2\,(2-s)^2}.$$

On the other hand, let $t = s_1/(1-s)$. Then, the payoffs of private owners 1 and 2, are given by

$$u_1^d(s_0, s_1, s_2) = (1-s)t\pi_{012}^d = \frac{a^2\,(1-s)^2\,t}{(2-s)^2},$$

and

$$u_2^d(s_0, s_1, s_2) = (1-s)(1-t)\pi_{012}^d = \frac{a^2\,(1-s)^2\,(1-t)}{(2-s)^2},$$

respectively.

5.3 The core and stable market structures

Except for the mixed triopoly, each regime of (b), (c), and (d) includes more than one market structure. Each market structure can be identified in terms of the merger participants and share ratios. For example, in (c), we observe a situation composed of merged public-private firm 01 with the government's shareholding ratio $s = 0.5$ and private firm 2 as one market structure.

In the following analysis, we consider the stability problem that exists between market structures, using the concept of the core, since the persistence of the market structures is important in the study of horizontal mergers in the industry that comprises more than two firms.[21] To define the cores of the market structures, we start with the definition of a blocking market structure, M, if there exists a deviant coalition of owners (firms) such that

1. M can be constructed from M' solely based on the decisions made by the owners in the deviant coalition, and

2. every owner in the coalition achieves a strictly higher payoff in M than in M'.

An example will help to explain blocking. Let M_s^{ij} be the market structure composed of the merged firm ij with i's shareholding ratio s and j's shareholding ratio $1-s$. Consider $M_{0.5}^{01}$, *i.e.*,

[21] In the literature on applied economics, the core concept is broadly used, e.g., in Riezman (1985) in the context of customs unions in international economics, and in Okada (2003), which analyzes international CO_2 emissions trading in environmental economics.

the market structure composed of merged firm 01 with shareholding ratio $s = 0.5$ and firm 2. In this case, for example, the coalition of owners 0 and 2 can choose to construct a new market structure that consists of merged firm 02 with shareholding ratio $s = 0.6$ and firm 1, denoted as $M^{02}_{0.6}$. If both owners 0 and 2 achieve higher payoffs in $M^{02}_{0.6}$ than in $M^{01}_{0.5}$, then $M^{02}_{0.6}$ blocks $M^{01}_{0.5}$. Note that it is also possible for a deviant coalition to have a single owner. In the example, it is possible for owner 0 (or owner 2) to deviate from $M^{02}_{0.6}$ by breaking off the merger and to operate her/his own firm (*i.e.*, to shift into the mixed triopoly, which is denoted by $M^{\emptyset}$) as well. Note that each firm's owner freely merges her/his firm with (an)other firm(s) and can break off the merger without any cost.[22] Note also that only the reallocation of their shareholding in the merged firm is considered by the deviation of the merger participants themselves.

The core of the market structure is defined as the set of market structures that are never blocked by any other market structure. We denote the core of the market structure as $\mathfrak{Co}$. If a market structure is in the core, no participant has the incentive to shift to a different market structure. In this sense, the market structure(s) in the core can be regarded as stable.

We now explore the stable market structures, *i.e.*, the core $\mathfrak{Co}$. Our arguments proceed by proving several lemmas. The first lemma shows that there are no shares of two private owners so that the state where two private firms merge is in the core, $\mathfrak{Co}$.

Lemma 5.1. *For any $t \in [0, 1]$, market structure M^{12}_t does not belong to the core, $\mathfrak{Co}$.*

Proof. From the results of regime (b) in Section 5.2, private owners 1 and 2 obtain tc^2 and $(1-t)c^2$, respectively, in M^{12}_t. On the other hand, each of the two private owners receives c^2 in market structure $M^{\emptyset}$. Thus, either of them certainly prefers $M^{\emptyset}$ to M^{12}_t, and is able to induce $M^{\emptyset}$ from M^{12}_t. Therefore, market structure M^{12}_t is blocked by $M^{\emptyset}$ through the coalition of the two private owners. □

Lemma 5.1 shows that the merger of the two private firms brings no advantage to their owners, because it only leads to the expansion of the market shares of the public firm.[23]

[22] Even though breaking off a merger is generally costly, this assumption is quite reasonable if we interpret the merger activity problem as a negotiation that can be terminated without any cost and that is carried out before realizing the merger. For example, if the present market structure is a mixed triopoly, each firm's owner negotiates (with no cost) whether her/his firm will be part of the merger, taking her/his payoff after the merger into account. In our model, the *stable market structure* can be regarded as a consequence of some sort of thought experiment between each firm's owner. Thus, in our model, it is not unnatural that breaking off a merger does not yield any positive cost. Moreover, in the following analysis, we additionally assume that the initial market structure is a mixed triopoly and consider which of stable market structure(s) belonging to the core will most likely be realized. This point is analyzed through the notion of "achievablity" after Proposition 5.1.

[23] This result is closely related to that in Salant et al. (1983) that a merger tends to be unprofitable, unless many firms take part in the merger. Moreover, Kamaga and Nakamura (2007) shows that in the context of a mixed

Next, we determine whether the market structure where all the three firms merge is in the core, $\mathfrak{Co}$. Let $M_{s,t}^{012}$ denote the market structure wherein firms 0, 1, and 2 merge, and the share ratios in the merged firm of the government and the owners of the two private firms are s, $(1-s)t$, and $(1-s)(1-t)$, respectively. The next lemma shows that there exist some shareholding proportions of the three owners such that the market structure with the merger among the three firms is in the core, $\mathfrak{Co}$.

Lemma 5.2. *When $a\left(3-\sqrt{3}\right)/9 \leq c \leq a/3$, there exist $s \in [0,1]$ and $t \in [0,1]$ such that the market structure $M_{s,t}^{012}$ belongs to the core, $\mathfrak{Co}$. Thus, s is less than or equal to $\bar{s} = (5-3\sqrt{2})/7 \approx 0.11$ and the shareholding proportion of private owner 1 is negligibly different from that of private owner 2.*[24] *On the other hand, when $c < a\left(3-\sqrt{3}\right)/9$, for any $s,t \in [0,1]$, market structure $M_{s,t}^{012}$ does not belong to the core.*

Proof. The following analyses are performed with the aid of Figure 5.1. Let curve AD in Figure 5.1 denote the locus between the payoff of owner 0 u_0^c (horizontal axis) and the payoff of private owner 1 u_1^c (vertical axis) in market structure M_s^{01} when s varies in the interval $[0,1]$ (A and D represent the states where $s=0$ and $s=1$, respectively). In addition, let curve BD express the locus between the payoff of the public owner u_0^d (horizontal axis) and $u_{12}^d/2 := (u_1^d + u_2^d)/2$ (vertical axis), *i.e.*, the value when private owners 1 and 2 equally share $(1-s)\pi_{012}^d$ and s varies in the interval $[0,1]$ (B and D indicate the states where $s=0$ and $s=1$, respectively). Moreover, the share ratio of owner 0, *i.e.*, s, at point E is $\bar{s} = (5-3\sqrt{2})/7 \approx 0.11$.

Step 1. First we show that when $s > \bar{s}$, market structure $M_{s,t}^{012}$ is blocked by the coalition of owner 0 and one of the private owners.

In Figure 5.1, market structure $M_{s,t}^{012}$ with $s > \bar{s}$ corresponds to the region on curve ED, except for point E (note that in curve ED, the two private owners obtain equal payoffs). From the figure, for any point on ED, there exist points on curve AD, which is located in the region upper right from these points. Thus, in the case where the proportions of two private owners' shareholdings are equal, both public owner 0 and private owner 1 (or 2) can obtain better payoffs by establishing merged firm 01 and adjusting the proportions of their shareholdings in the firm after deviating from the market structure $M_{s,t}^{012}$. Hence, when $s > \bar{s}$, market structure $M_{s=0.5}^{012}$ is

oligopoly, a merger between private firms is never beneficial, even if there exists a relevant productivity-improving effect in the merged firm.

[24] The concrete region of s and t in which market structure $M_{s,t}^{012}$ is present in the core, $\mathfrak{Co}$, is given in Figure 5.2. More precisely, as indicated behind the proof of Lemma 5.2, the range of the share ratios in the merged firm of all the pre-merged firms' owners such that $M_{s,t}^{012}$ belongs to the core is larger, as the value of the marginal cost of the public firm increases.

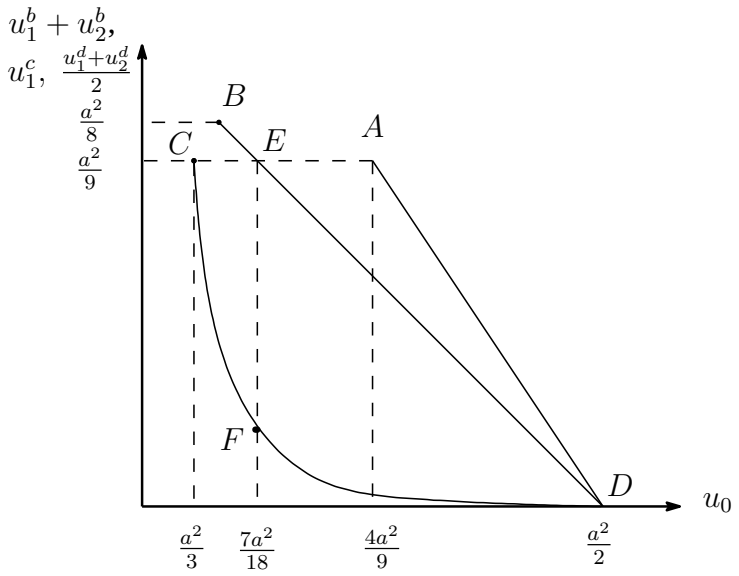

Figure 5.1: Relationship among the payoffs of the three owners in regimes (b), (c), and (d)

blocked by market structure M_s^{0i} through the coalition of the government and private owner i, (i = 1 or 2). In addition, in the case where the shareholding proportions of the two private owners are different, we select the coalition of the public owner and a private owner who has fewer shares in the firm. Then, the locus between the payoffs of both the public owner and the private owner is located under curve ED. After all, the market structure with an all-firms merger is blocked by the public owner and the private owner with lower shareholdings. Therefore, market structure $M_{s,t}^{012}$ is blocked if $s > \bar{s}$ and $t \in [0, 1]$.

Step 2. We will show that there exist $s \leq \bar{s}$ and $t \in [0, 1]$ such that the market structure $M_{s,t}^{012}$ is not blocked by any coalition of owners, if $c \in [a(3 - \sqrt{3})/9,\ a/3]$. To show this, we consider the following five cases.

Case 1: The deviation of all the owners. The three owners of firm 012 can change their shareholding ratios in the merged firm. It is easily verified that u_0^d is increasing in s and u_i^d $(i = 1, 2)$ is decreasing in s. Moreover, u_1^d is increasing in t and u_2^d is decreasing in t. Thus, such a deviation does not succeed.

Case 2: The deviation of the public owner and one of the two private owners. If $s \leq \bar{s}$, we find from Figure 5.1 that public owner 0 prefers $M_{s'}^{01}$ (or $M_{s'}^{02}$) to $M_{s,t}^{012}$, irrespective of the values of s, t, and s'. Therefore, we investigate only whether there is an incentive for private owner 1 (or 2) to merge her/his firm with the public firm by deviating with the public owner. The payoff of private owner 1 (or 2) is $a^2/9$ at the maximum when private firm 1 (or 2) merges with the public

firm. Thus, for $s \leq \bar{s}$, if either

$$u_1^d = (1-s)t\pi_{012}^d < \frac{1}{9}a^2 \quad \text{or} \quad u_2^d = (1-s)(1-t)\pi_{012}^d < \frac{1}{9}a^2$$

holds, $M_{s,t}^{012}$ is blocked by the deviation of the public owner and one of the private owners, and thus, the state is not in the core. On the other hand, if the following two conditions hold:

$$u_1^d \geq \frac{1}{9}a^2 \quad \text{and} \quad u_2^d \geq \frac{1}{9}a^2,$$

then $M_{s,t}^{012}$ is not blocked by the deviation of the two.[25] The pairs of s and t that satisfy the above conditions are found in the shaded area of Figure 5.2.

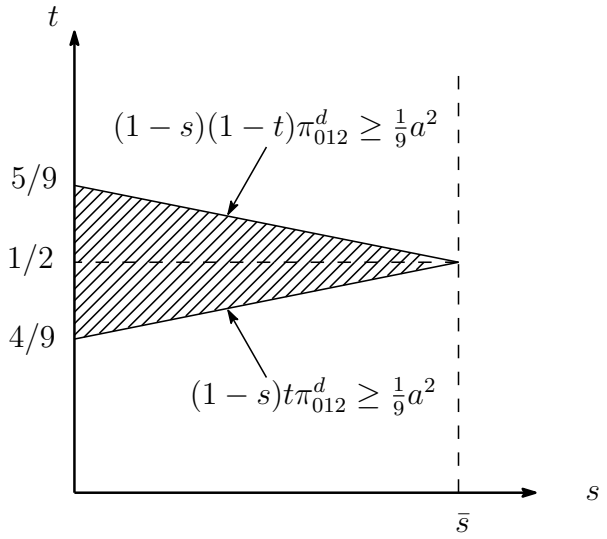

Figure 5.2: The domain of s and t such that the state where the three firms merge is in the core

In the following three cases, we assume that the pair (s,t) is selected from the shaded area of Figure 5.2.

Case 3: The deviation of two private owners. Since both private owners 1 and 2 obtain c^2 in $M^{\emptyset}$ and the payoffs are less than or equal to c^2 in M_t^{12}, it blocks the market structure of the merger among all three firms only if $u_1^d < c^2$ and $u_2^d < c^2$. However, in market structure $M_{s,t}^{012}$, both private owners 1 and 2 can obtain payoffs that are greater than or equal to $a^2/9$ $(\geq c^2)$ by the assumption of the marginal cost of the public firm, $c \in [0, a/3]$. Thus, $M_{s,t}^{012}$ is not blocked if (s,t) are chosen in the shaded area of Figure 5.2.

[25] These two conditions also hold since each of the two private owners cannot obtain the payoff more than $a^2/9$ by assuming the marginal cost of the public firm, $0 < c \leq a/3$, even if the public owner 0 and private owner 1 (or 2) induce the state M after assuming from $M_{s,t}^{012}$, taking into account that $u_i^a = c^2$, $(i = 1, 2)$, as in the following analysis.

Case 4: The deviation of one private owner. Suppose private owner 2 deviates and induces a new market structure, M_s^{01}. Her/His payoff in M_s^{02} is $u_2^c = a^2(1-s)^2/(3-2s)^2$, which is less than or equal to $a^2/9$. Thus, M^{012} is not blocked by the deviation of one private owner.

Case 5: The deviation of one public owner. After the deviation of the public owner from $M_{s,t}^{012}$, the new market becomes M_t^{12}. Social welfare depends on the marginal cost of public firm c in market structure M_t^{12}. Let curve CD in Figure 5.1 denote the locus between the payoff of public owner 0, u_0^b, and the sum of those of two private owners $u_{12}^b := u_1^b + u_2^b = c^2$ in market structure M_t^{12}, when the marginal cost of public firm c changes in the interval $(0, a/3]$ (C and D represent the states where $c = a/3$ and $c = 0$, respectively). When the value of c varies on curve FD (note that F is not included), we find that the state in the shaded area in Figure 5.2 is blocked by market structure M_t^{12}, which is induced by the public owner. On the contrary, when the value of c changes on curve CF, the state where the public owner's shareholding proportion, s, is near $\bar{s}$ (e.g., $s = \bar{s}, t = 0.5$) is not blocked and is in $\mathfrak{Co}$. We obtain $c = a(3-\sqrt{3})/9$ as the value of c on F by solving $(a^2 - 2ac + 3c^2)/2 = 7a^2/18$. That is, if $c \geq a(3-\sqrt{3})/9$, there exists a pair of s and t such that $M_{s,t}^{012}$ is not blocked by any other market structures.

Step 3. Finally, we show that if $c \in (0, a(3-\sqrt{3}))$, for any $s \in [0,1]$ and $t \in [0,1]$, market structure $M_{s,t}^{012}$ is blocked.

This is easily proved as follows. From Step 1, when $s > \bar{s}$, $M_{s,t}^{012}$ is blocked by the deviation of one public owner and one private owner. On the other hand, when $s \leq \bar{s}$, from the proof of Case 5, $M_{s,t}^{012}$ is blocked by the deviation of one public owner because $c < a(3-\sqrt{3})$. This step concludes the proof. □

Lemma 5.2 outlines the fact that the shareholding proportions of the three owners in firm 012 such that $M_{s,t}^{012}$ is in the core vary depending on the value of the marginal cost of the public firm, c. The larger the value of $c \in [a(3-\sqrt{3})/9,\ a/3]$ is, the weaker the incentive for deviation of the public owner is; thus, as the value of c approaches $a/3$, the domain of the pairs of s and t in which market structure $M_{s,t}^{012}$ is in the core is broader than the one in which c is relatively small as compared to $a/3$. Note that the public owner prefers the merger between a single private firm and a public firm to the merger of three firms because relative to the merger with one private firm, an all-firms merger only results in further deteriorating the competitive pressure without improving the production efficiency. However, once the market structure $M_{s,t}^{012}$ with the values of s and t in Figure 5.2 is established, neither of the private owners has an incentive to accept the proposal of the deviation by the public owner, since they obtain relatively high payoffs. On

the other hand, the deviation of the public owner from $M_{s,t}^{012}$ does not increase social welfare when the values of s and t are in the shaded area of Figure 5.2 and s is sufficiently near to $\bar{s}$, if $c \in \left[a(3-\sqrt{3})/9, a/3\right]$. This is so because the negative effect of the inefficiency of the public firm overshadows the positive effect of the increase of the competitive pressure in the market.

Next, we explore whether the market structure where a single private firm and a public firm merge is in the core, $\mathfrak{Co}$. The following lemma holds, and we obtain a positive result.

Lemma 5.3. *When $0 < c \leq a/3$, there exists $s \in [0,1]$ such that market structure $M_s^{0i}(i = 1, 2)$ belongs to the core, $\mathfrak{Co}$.*

Proof. Without loss of generality, we assume that $i = 1$ in advance of the proof of this lemma. We prove this lemma by showing that the deviation of any coalition does not succeed if s is appropriately chosen.

Case 1: The deviation of the three owners. Let us consider a point on curve AD in Figure 5.3 and call it F. Note that F corresponds to a pair of payoffs of public owner 0 and private owner 1 in regime (c) for some of their shares. By the definition of blocking, the payoff of the public owner has to be strictly improved by the merger of the three firms. Note again that curve BD denotes the locus between the payoff of public owner u_0^d and $u_{12}^d := (u_1^d + u_2^d)/2$ in the market structure of the merger of the three firms. Thus, to enhance the payoff of the public owner through the merger of the three firms, its shares and payoff in the merged firm must correspond to the point on curve BD, which is lower right to point F (see Figure 5.3). Let us consider such a point and call it point G. Here, we find that the value of the vertical axis of point G is always lower than that of point F. This implies that the payoff of the private owner 1 in the initial situation (on point F) is greater than the average of the payoffs of private owners 1 and 2 in the market structure with the three-firms merger corresponding to point G. In market structure M_s^{0i}, since private owner 2 obtains the same payoff as that of private owner 1 (see regime (c) in Section 5.2), the payoff of one of the two private owners certainly decreases by the deviation from F to G. Therefore, M_s^{0i} is not blocked by the deviation of the three owners.

Case 2: The deviation of public owner 0 or private owner 1. The arguments presented below proceed with the aid of Figure 5.4. Let the upper right curve connecting A and D in Figure 5.4 denote the locus between the payoff of the public owner and that of private owner 1 in regime (c) when s varies from 0 to 1. On the other hand, let the left under curve connecting A and D express the locus between the payoff of the public owner and that of the private owners in regime (a) when marginal cost c varies from 0 to $a/3$ (A represents the state wherein $c = a/3$ and D is

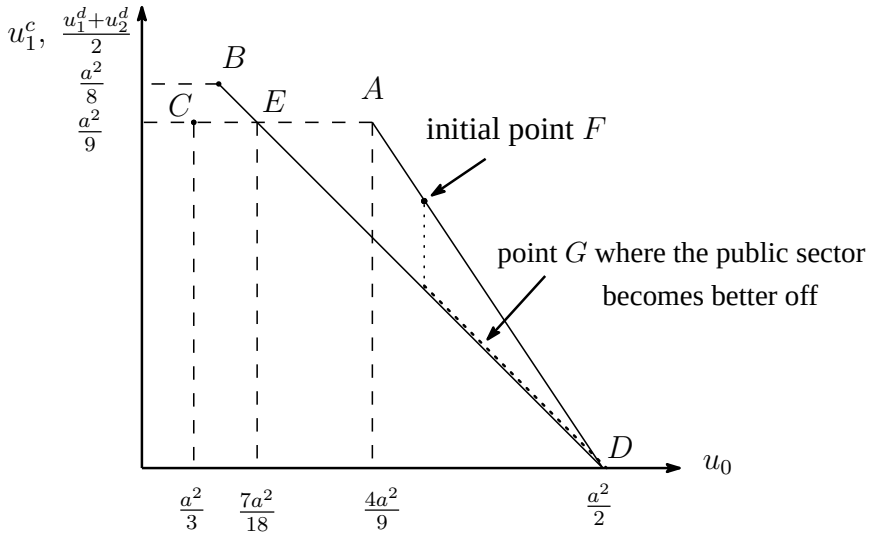

Figure 5.3: Relationship among the payoffs of the three owners in the regimes (c) and (d)

the one where $c = 0$). Here, note that $c^2 \leq a^2/9$ by the assumption of the marginal cost of the public firm.

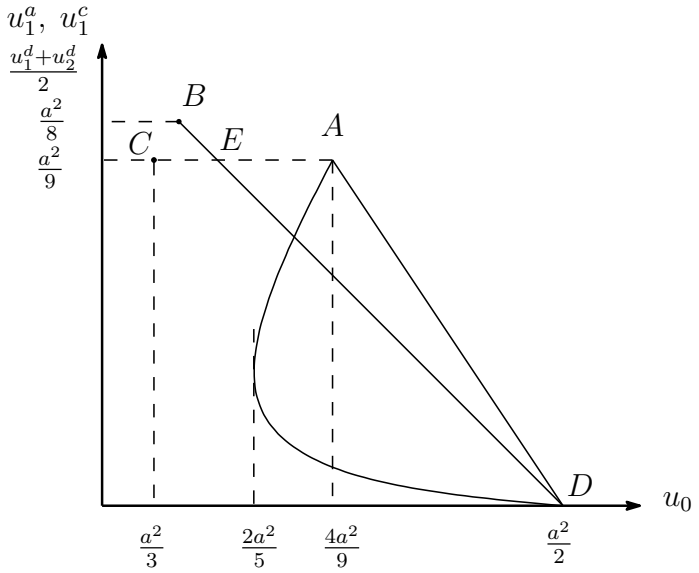

Figure 5.4: Relationship among the payoffs of the three owners in regimes (a), (c), and (d)

From Figure 5.4, we find that for any point on the left under curve AD, there exist points on the upper right of curve AD, which is located upper right to or at the same point. This means that for any value of c, there exists some $s \in [0, 1]$ such that the market structure M_s^{0i} is not blocked by a single player deviation of public owner 0 or private owner 1.

Case 3: The deviation of two private owners. It is obvious that the market structure with the merger between a private firm and a public firm with the shareholding proportions defined in Case 2 is also unblocked by the deviation of the two private owners, because the payoff of a

private owner in M_t^{12} is less than or equal to that in $M^{\emptyset}$.

Case 4: The deviation of public owner 0 *and private owner* 1. We investigate the deviation such that the public and private owners in the merged firm simply changes their shareholding ratios. From the results in regime (c) in the previous section, it is easily shown that u_0^c is increasing in s and u_1^c is decreasing in s. Thus, the deviation of the two owners in the merged firm does not succeed.

Case 5: The deviation of public owner 0 *and private owner* 2. Note that by observing regime (c) in the previous section, private owner 2 obtains the same payoff as that of private owner 1 in the market structure M_s^{0i}. By the proof of Case 4, such a deviation does not succeed. This case concludes the proof. □

From Lemma 5.3, we can infer that for any $c \in (0, a/3)$, there exist pairs of share proportions of public owner 0 and private owner i such that market structure M_s^{0i} $(i = 1, 2)$ belongs to the core. When the marginal cost of public firm c is higher, the market structures, which is advantageous to a private owner in terms of the share ratio after the merger, will be in the core, $\mathfrak{Co}$. On the other hand, as c decreases, the structures where the two owners' shareholding proportions in the merged firm are favorable to the government are introduced in the core $\mathfrak{Co}$. The payoff of the public owner (or social welfare) in regime (c) is higher than or equal to the one in regime (d), which is the same as the value of s.[26] Therefore, to improve the payoff of the public owner by inducing the structure wherein all three firms merge, the value of s must be higher than that of the original state, *i.e.*, the structure where a private firm and a public firm merge. However, this results in a decrease in the payoff of at least one private owner. This is the reason why market structure $M_s^{0i}(i = 1, 2)$ is unblocked through the structure wherein the three firms merge. On the other hand, in a mixed triopoly, the existence of a public firm whose productivity is worse as compared to that of the two private firms will be a structure wherein social welfare is relatively low. In addition, in the structure where a private and a public firm merge, both the pre-merged private owners can acquire the payoffs that are equal to and higher than the structures in regime (a) by coordinating the shareholding proportions of the public owner and the private owner in the merged firm. Thus, market structure $M_s^{0i}(i = 1, 2)$ is not blocked by the deviation of a single player, *i.e.*, the public owner or the private owner.

[26] This property is confirmed by the following easy calculation.

$$u_0^c - u_0^d = \frac{a^2(1-s)^3(5-3s)}{2(6-7s+2s^2)^2} \geq 0, \ \forall s \in [0, 1].$$

Note that $u_0^c - u_0^d$ is equal to zero only when $s = 1$.

Finally, we confirm whether the market structure $M^{\emptyset}$ belongs to the core, $\mathfrak{Co}$.

Lemma 5.4. *When $c = a/3$, market structure $M^{\emptyset}$ belongs to the core, $\mathfrak{Co}$. On the other hand, when $c < a/3$, $M^{\emptyset}$ does not belong to the core.*

Proof. When $0 < c < a/3$, from Figure 5.4, we find that market structure $M^{\emptyset}$ is blocked by M_s^{0i} through the coalition of public owner 0 and private owner i $(i = 1, 2)$. Point A in Figure 5.4 denotes each of the owners' payoffs in the mixed triopoly when $c = a/3$. Thus, the payoff of the public owner is $4a^2/9$, whereas those of both the private owners are $a^2/9$. It is easily verified that the mixed triopoly is unblocked by any other market structure induced by all deviations of owners. □

In the mixed triopoly, when $c = a/3$, each of the two private owners obtains the highest payoff equal to $a^2/9$, because the public firm exits the market. Thus, to enhance the payoffs of the two private owners, it is necessary that the shares of the public owner in the three-firm merger are low enough. However, it is inevitable that the decrease of s results in a deterioration in social welfare, *i.e.*, the payoff of the public owner. Therefore, the mixed triopoly is not blocked by any other market structure.

To state the following proposition, we note the following. If a market structure is in the core, we call it a stable market structure. In addition, merger $\{i, j\}$ is stable if there exists some $s \in [0, 1]$ such that $M_s^{ij} \in \mathfrak{Co}$, merger $\{0, 1, 2\}$ is stable if there exist some $s \in [0, 1]$ and $t \in [0, 1]$ such that $M_{s,t}^{012} \in \mathfrak{Co}$, and $M^{\emptyset}$ is stable if $M^{\emptyset} \in \mathfrak{Co}$.

We obtain the next proposition according to the abovementioned analyses.

Proposition 5.1. *Stable mergers are classified into three cases depending on the value of the marginal cost of the public firm c.*

1. *When $c = a/3$, four states of $M^{\emptyset}$, merger $\{0, i\}$, $(i = 1, 2)$ and $\{0, 1, 2\}$ are stable.*
2. *When $a(3 - \sqrt{3})/9 \leq c < a/3$, three states of merger $\{0, i\}$ $(i = 1, 2)$ and $\{0, 1, 2\}$ are stable.*
3. *When $0 < c < a(3 - \sqrt{3})/9$, two states of merger $\{0, i\}$ $(i = 1, 2)$ are stable.*

We explain the intuitions behind Proposition 5.1 as follows. First, market structure $M^{\emptyset}$ is stable only if the marginal cost of the public firm is the highest, *i.e.*, $c = a/3$ to the degree that the public firm exits the market. In this state, the two private owners obtain relatively high payoffs because of the existence of the public firm with low productivity. Thus, the public owner

desires a merger with a private firm to improve its productivity, however, when $c = a/3$, both private owners disagree on the merger, no matter how much of a concession the public owner makes to the shareholding proportion in the merged firm. As the marginal cost of the public firm is lower than $c = a/3$, market structure $M^{\emptyset}$ is blocked by M_s^{0i} through a coalition of two players, *i.e.*, public owner 0 and private owner i $(i = 1, 2)$, and is not in the core. In addition, from the viewpoint of the public owner, the productivity of the public firm is higher when it merges with a single private firm, because the merger with two private firms deteriorates social welfare by reducing the competitive pressure. Considering this fact, we can determine why market structure $M_{s,t}^{012}$ is in the core when the marginal cost of the public firm is under the high and middle levels. If the sum of the two private owners' shareholding proportions is high, a single private owner can share a relatively high payoff with another private owner. Thus, there are no incentives for both private owners to deviate with the public owner and induce the market structure wherein a private firm and a public firm merge, although the public owner wants to merge with a single private firm. Moreover, when the productivity of the public firm is low, social welfare deteriorates even if the public owner deviates on her/his loan from the market structure wherein the three firms merge. In other words, when c is relatively high and the shares of the public owner in the three-firm merger are low, it is a clever choice for the public owner to leave the situation as it stands, because the public owner is too inefficient to deviate on her/his own from the state wherein all three firms merge. However, market structure $M_{s,t}^{012}$ does not belong to the core when the marginal cost of the public firm is below the low level, because the public owner does not have any incentive to merge with the two private firms due to the cutdown on the improvement of its productivity.

Finally, we check which of the market structures in the core is/are most likely to occur. Although this is quite a difficult question theoretically, we can provide a reasonable answer because we obtain additional information on the initial situation by viewing a merger as a form of "privatization." Since the initial situation is a mixed triopoly composed of one public firm and two private firms, we focus our attention on the elements of the core that block the initial mixed triopoly and then consider the market structure that blocks the mixed triopoly and is never blocked by other market structures. We call such a market structure *achievable*. The following proposition shows that only the two-firm merger between a public firm and a private firm is likely to occur as a result of the merger activity from the mixed triopoly.

Corollary 5.1. *For any $c \in (0, a/3)$, only the two-firm merger $\{0, i\}$, $i = 1, 2$, is stable and achievable from an initial mixed triopoly.*

Proof. By the proof of Lemmas 5.3 and 5.4, if M_s^{0i} belongs to the core, it blocks $M^{\emptyset}$ for any $c \in (0, a/3)$.

Next, we show that any market structure except for M_s^{0i} in the core, $\mathfrak{Co}$, is not achievable from $M^{\emptyset}$. By Lemma 5.2, the three-firms merger in the core, $M_{s,t}^{012} \in \mathfrak{Co}$, is on curved segment BE of Figure 5.4. Because the pair of payoffs of the public and private owners in the initial mixed triopoly $M^{\emptyset}$ is expressed by the under left curve AD, the public owner always prefers $M^{\emptyset}$ to $M_{s,t}^{012}$ on curved segment BE. □

5.4 Extension

In this section, we consider a general mixed oligopoly with respect to the number of (private) firms, a mixed oligopoly with one public firm and n ($n \geq 3$), private firms, and analyze the stable market structure in this setting in order to confirm the validity of the results in the previous sections. All of the settings except for the number of private firms are the same as in the previous sections. Here, we assume that $a > (n+1)c$.

In the Cournot equilibrium of the mixed market of one public firm and n private firms, we have

$$\begin{aligned}
\Pi_u^*(n) &= 0, \\
\Pi_r^*(n) &= c^2, \\
W^*(n) &= \frac{1}{2}\left[a^2 - 2ac + (2n+1)c^2\right],
\end{aligned}$$

where $\Pi_u^*(n)$ and $\Pi_r^*(n)$ denote the profits of a public firm and n private firms, respectively.

To investigate the market outcome after firms merge, it is convenient to consider a market with one partially privatized firm (a merged public-private firm) with the government's share s and m $(= 1, 2, \ldots, n-1)$ private firms. Then, the equilibrium profits of the partially privatized firm and private firms and the equilibrium social welfare are given as follows:

$$\begin{aligned}
\Pi_u^{**}(s,m) &= \frac{a^2(1-s)}{\left[m(1-s)+2-s\right]^2}, \\
\Pi_r^{**}(s,m) &= \frac{a^2(1-s)^2}{\left[m(1-s)+2-s\right]^2}, \\
W^{**}(s,m) &= \frac{a^2(1+m-ms)(3+m-2s-ms)}{2\left[2+m-s(1-m)\right]^2},
\end{aligned}$$

where $\Pi_u^{**}(s,m)$ and $\Pi_r^{**}(s,m)$ denote the profits of a partially privatized firm and a private firm when there are m private firms and one partially privatized firm, respectively, in the case

where the government's shareholding ratio in the partially privatized firm is s. Note that $(1-s)\Pi_u^{**}(s,m) = \Pi_r^{**}(s,m)$, and it is easy to verify that $\partial\Pi_u^{**}/\partial m < 0$, $\partial\Pi_r^{**}/\partial m < 0$, $\partial\Pi_r^{**}/\partial s < 0$, $\partial W^{**}/\partial m > 0$, and $\partial W^{**}/\partial s > 0$.

We consider the value of s that equalizes the profit of a private firm in the mixed oligopoly with one public firm and n private firm and the profit of the private owner in the partially privatized firm with the government's share s. This value is obtained as a solution of the following equation:

$$(1-s)\Pi_u^{**}(s,n-1) = \Pi_r^{*}(n) = c^2.$$

Then, the solution of this equation is

$$s = s^* := \frac{a-c(n+1)}{a-cn} \in (0,1).$$

Moreover, we know that $(1-s)\Pi_u^{**}(s,n-1) \geq \Pi_r^{*}(n)$ if and only if $s \leq s^*$. This implies that private owner i does not have an incentive for single deviation from the merger between firms 0 and i if and only if $s \leq s^*$.

Proposition 5.2. *For any c less than some critical value $\bar{c}$, a state where the public firm merges with one private firm and other private firms remain single becomes a stable market structure.*

Proof. We conduct the proof by showing that *when one public firm merges with one private firm with the government's share s^* and other private firms remain single, such a market structure is not blocked by any coalition of firms, if c is less than some value $\bar{c}$.* Let private firm i be the one that merges with the public firm throughout the rest of this proof.

Step 1. We will show that any coalition composed of private owners other than i does not have an incentive to merge if c is less than some value $\bar{c}_1$. Because a merger is conducted only when it benefits all of the participants, k $(= 2, 3, \ldots, n-1)$, private firms do not merge with each other if

$$\Pi_r^{**}(s,n-1) \geq \frac{\Pi_r^{**}(s,n-k)}{k}$$

Here, note that $f(k) = \Pi_r^{**}(s,n-k)/k$ is a U-shape function in the range from 1 to $n-1$.[27] Thus, if $\Pi_r^{**}(s,n-1) \geq \Pi_r^{**}(s,1)/(n-1)$, the above condition holds for any k. After performing

[27] This is proved as follows: Differentiate $f(k)$ with k and we have

$$f'(k) = -\frac{a^2(1-s)^2(2-3k+n-s+3ks-ns)}{k^2(2-k+n-s+ks-ns)^3}.$$

From the above fact, we understand that $f(k)$ is decreasing in the range from 1 to $\frac{n+2-s-sn}{3(1-s)}$ with respect to the value of k and increasing from $\frac{n+2-s-sn}{3(1-s)}$ to $\frac{n+2-s-sn}{1-s} > n$ with respect to the value of k.

some calculations, we obtain the following result:

$$s > \hat{s} \;\; \Rightarrow \;\; \Pi_r^{**}(s, n-1) > \frac{\Pi_r^{**}(s,1)}{n-1},$$

where

$$\hat{s} = \frac{n-3-\sqrt{n-1}}{n-2}.$$

Let $\bar{c}_1$ be defined by

$$\bar{c}_1 = \frac{a(1-\hat{s})}{n(1-\hat{s})+1}.$$

Then, $c < \bar{c}_1$ guarantees that $s^* > \hat{s}$. Thus, if $c < \bar{c}_1$, any coalition composed of private firms (owners) other than i does not deviate and the firms merge with each other when the public firm and private firm i found the merged public-private firm $0i$ with share s^*.[28]

Step 2. We will show that any coalition composed of private owners including i does not have an incentive to merge. The deviation of private owner i and other k private owners induces the new market structure, which consists of one public firm and $(n-k)$ private firms including the new merged (private firm) firm. The profit of the merged firm in the new market structure is c^2. On the other hand, in the initial situation where only public owner 0 and private owner i found the merged public-private firm with share s^*, the payoffs of private owner i are

$$(1-s^*)\Pi_u^{**}(s^*, n-1) = c^2,$$

and the profits of the other private owners are

$$\Pi_r^{**}(s^*, n-1) = c^2.$$

Since $(k+1)c^2$ is strictly greater than c^2 when $c > 0$, taking into account that the merged firm's profit must be distributed between private owners whose firms participate in the merger, such a merger does not occur.

Step 3. We will show that the public owner her/himself does not have an incentive to deviate from the merger and induce an mixed oligopoly with one public firm and n private firms. It is easily to verify that

$$W^{**}(s^*, n-1) > W^*(n).$$

[28] More precisely, from an easy calculation, we obtain the following result:

$$\hat{s} < 0, \quad \text{if} \quad n \in [3, 5)\,.$$

Thus, since $s^* > \hat{s}$ holds for any $c < a\,(n-1)$, if $n \in [3,5)$, we find that for any value of c, any coalition composed of private firms (owners) other than i does not deviate and merge when the public firm and private firm i found the merged public-private firm $0i$ with share s^*.

Thus, the public owner does not have an incentive to deviate and remain single.[29]

Step 4. We will show that any coalition composed of one public owner and multiple private owners does not have an incentive to (re-)merge with each other if $c < \bar{c}_2$. As shown in Step 2, in the initial market structure composed of the merged public-private firm and $(n-1)$ private firms, all the private owners obtain c^2. This implies that k private owners accept the merger with the public firm only if

$$(1-s)\Pi_u^{**}(s, n-k) > kc^2.$$

The above inequality is satisfied if

$$s > \tilde{s}(k) = \frac{a - c\sqrt{k}(2-k+n)}{a - c\sqrt{k}(1-k+n)}$$

Here, we have

$$s^* - \tilde{s}(k) = \frac{c(\sqrt{k}-1)(a - c\sqrt{k} - ck)}{(a-cn)(a - c\sqrt{k}(1-k+n))}.$$

Therefore, for sufficiently small $c > 0$, $s^* - \tilde{s}(k)$ becomes non-negative. Thus, there exist some $\bar{c}_2$ such that for any $c < \bar{c}_2$, $s^* - \tilde{s}(k)$ is non-negative for any $k = 1, \ldots, n$. On the other hand, this implies that

$$W^{**}(s^*, n-1) \geq W^{**}(s^*, n-k) \geq W^{**}(\tilde{s}\,(k)\,, n-k),$$

where the first and second inequalities follow from $\partial W^{**}/\partial m > 0$ and $\partial W^{**}/\partial s > 0$, respectively. Thus, the public owner does not become better off as a result of the merger, even if the merger is accepted by the private owners. Therefore, when $c < \bar{c}_2$, any coalition composed of one public owner and multiple private owners does not deviate from the initial situation where one public firm merges with one private firm with the government's share s^* and other private firms remain single.

From Steps 1 to 4, we conclude that when $c < \bar{c} = \min\{\bar{c}_1, \bar{c}_2\}$, a state where the public firm merges with one private firm with share s^* and the other private firms remain single is not blocked by other market structures through any coalition. □

From the proof of the above proposition, the continuity of both the equilibrium profit of each firm and the equilibrium welfare with respect to share s means that a state where the public firm

[29] More precisely, we find that

$$W^{**}\,(s^*, n-1) - W^*\,(n) = c\,[a - c\,(n+1)] > 0.$$

merges with one private firm with share s', ($s' < s^*$), and other private firms remain single is not blocked by any coalition, if s' is sufficiently close to s^*. When a public firm merges with a private firm with share s', the profit of this private owner after the merger is strictly greater than c^2, which means that the private owner accepts the merger proposal of the public firm if the initial market structure is composed of one public firm and n private firms. From this observation, we obtain the following statement as the corollary of Proposition 5.2.

Corollary 5.2. *For any c less than some critical value $\bar{c}$, a state where the public firm merges with one private firm and the other private firms remain single is a stable and achievable market structure from the initial mixed oligopoly with one public and n private firms.*

5.5 Concluding remarks

In this chapter, we used the core concept to investigate which market structures are stable when firms are freely allowed to merge with each other and break off mergers in a mixed oligopoly consisting of one public firm and two symmetric private firms. First, we classified that the stable market structures in the core into three cases depending on the degree of inefficiency of the public firm. Then, although multiple types of mergers are stable for some range of the marginal cost of the public firm ($a(3-\sqrt{3})/9 \leq c \leq a/3$), it is only the market structure with the merger between one public firm and one private firm that blocks the mixed triopoly and is never blocked by other market structures when it is additionally assumed that the initial market structure is a mixed triopoly. Thus, the structure with the merger between one public firm and one private firm is stable and achievable. Furthermore, in the general mixed oligopolistic model where multiple private firms exist in the market, we also obtained a similar result to the one described above.

We discussed three features of stable market structures that belong to the core. First, the market structure wherein a single private firm and a public firm merge belonged to the core in all three cases, classified on the basis of the value of the marginal cost of the public firm. According to this fact, we were able to recognize the market structure wherein the public firm is merged and acquired by the private firm at a constant rate as stable. Therefore, we succeeded in showing that "the (partial) acquisition of an inefficient public firm by a private firm" is certainly adequate. Second, the market structure wherein all three firms merge belonged to the core when the marginal cost of the public firm was under the high and middle levels. This is so because the government, whose payoff is social welfare, wishes to enhance the productivity of the public firm through the merger at the expense of deteriorating consumer surplus by declining the aggregate

quantity, resulting in the monopoly of the merged public-private firm. This clearly demonstrates that the enhancement of the productivity of the public firm through the merger with private firms is more important for the government than the disadvantage of the monopoly, *i.e.*, relatively low consumer surplus, when the marginal cost of the public firm is relatively high. Finally, the mixed triopoly belonged to the core only if the marginal cost of the public firm was the highest, *i.e.*, $c = a/3$, to the extent that the public firm exited the market. In such a case, the market structure that is actually observed is a private duopoly. We find that each owner of the two private firms does not assent to all deviations from the mixed triopoly since they both obtain comparatively high payoffs due to the disparity between productivity of the private and public firms.

Three interesting extensions remain. The first extension, which has been already described in Concluding remarks of chapter 4, is examining the model in which the foreign shareholders are taken into account. In the real world economy, some firms are foreign-owned.[30] In this case, the social welfare that the public owner (*i.e.*, the government) maximizes may not include the profits of foreign firms, if the market is confined to the domestic country.[31] Thus, the presence of foreign shareholders will change not only the public firm's decision-making but also the equilibrium market outcomes. Taking into account the above facts, under the assumption that each firm has the quadratic cost function with respect to her/his quantity, Nakamura (2009b) dealt with the problem of horizontal mergers including firms existing in different countries with each other in an international mixed oligopoly. He showed that ruling out the monopoly with the merger among all the in the market, in the entrepreneurial case wherein every managerial decision-making is done by each firm's owner, both the market structure with a merger between the public firm and the domestic private firm and the market structure with a merger between the public firm and the foreign private firm can belong to the core, whereas in the managerial case, only the market structure with the merger between the public firm and the foreign private firm can belong to the core. Thus, the latter result obtained in the managerial case between his two findings corresponds to the example in the real world economy that in the mixed oligopolistic industry, the large enterprises with the separation between ownership and management, which exist in the different country with each other, participate in such a merger. However, as described above, Nakamura (2009b) supposed the following two assumptions: (1) the monopoly with the merger among all the existing firms is eliminated; (2) each firm can export her/his output without any physical

[30] In the real world example of (partial) horizontal mergers in mixed oligopolistic industries, as indicated in Bárcena-Ruiz and Garzón (2003), Renault, which is a French (parastatal) public firm, holds a 36.8% equity stake in Nissan Motor and a 22.5% stake in Nissan Diesel.

[31] This assumption is often supposed in the literature on mixed oligopoly in the context of open economy, such as Fjell and Pal (1996), Pal and White (1998), and Fjell and Heywood (2002).

trade cost.[32] In conducting the theoretical analyses on horizontal mergers between the public firm and the private firm in an international mixed oligopoly, although we must consider both the possibility of the monopoly with the merger among all the existing firms and the physical trade cost to export their output to their foreign countries in order to sufficiently reflect the examples in the real world economy, it is difficult for us to obtain the general result on what market structure(s) is/are stable in such analyses. Thus, when we investigate the cross-border merger between the public firm and the private firm under the such a setting, we must conduct the simulation analysis regarding the difference between the demand scale and the physical trade cost by using relevant computer softwares. In addition, similar to the analysis in chapter 4, Nakamura (2009b) considered the situation wherein each firm's production cost is quadratic with respect to her/his quantity. Thus, of course, taking into account the presence of foreign firms, we should also tackle the problem of what market structure(s) is/are stable under the same setting as that in this chapter that the technologies of both the public firm and the private firm are represented as the constant marginal cost functions with respect to their output levels and the public firm is less efficient that the private firm.

The second remaining extension is determining how the cost function of a merged firm should be treated. In our analysis, we assumed that when a private firm and a public firm merge, the merged firm can utilize the technology of the private firm, whose productivity is higher, independently of the public owner's shareholding proportion in the merged firm. This assumption on the cost function of the merged firm may be adequate when it represents the production technology of the firm itself (for example, the performance of machines in the plant, etc.). However, we also assumed that a public firm is more inefficient than a private one. If the inefficiency of a public firm is intrinsic, such as X-inefficiency, the assumption that the merged firm can use the more efficient technology may not be appropriate. In this case, one may consider that the assumption that the productivity of the merged firm has deteriorated in response to the increase in the proportion of the public owner's shareholding is adequate.

[32] Nakamura (2009b) supposed that the market exists in the domestic country only, and thus foreign private firms always engage in exporting their outputs to the market. Hence, under such a setting, it may be natural that foreign firms can export their output without any physical trade, analogous to Fjell and Pal (1996), Pal and White (1998), and Fjell and Heywood (2002). However, most recently, the works on cross-border mergers including Fumagalli and Vasconcelos (2009) have been significantly emphasizing the so-called *tariff-jumping* effect that the cross-border mergers enables the merged firm to serve her/his output to both the markets without incurring any physical trade cost. In addition, like Horn and Persson (2001b) and Fumagalli and Vasconcelos (2009), if the segmented market hypothesis is adopted and thus there exists one market in each country, the tariff-jumping effect plays further important role on each firm's merger incentive and its welfare implications. Especially, there does not exist any literature on horizontal mergers in the context of mixed oligopoly under the segmented market hypothesis. Therefore, as a plausible next step, we should consider such a tariff-jumping effect on social welfare in each country and total social welfare.

More interestingly, we should tackle the problem of what figures of the core of market structures are characterized in the case wherein each firm adopts her/his FJSV delegation contract. In the entrepreneurial case considered in this chapter wherein each firm's owner chooses her/his output level with respect to her/his objective function, we derived the core by using the several particular properties, *e.g.*, the payoff of the private owner who participates in the merger with the public firm coincides with the payoff of another private owner who is outside the merger. Thus, in the case wherein each firm's adopts the FJSV delegation contract, it is difficult to derive the core of market structures after each firm allows for her/his free mergers on the bases of the cooperative game approach which was introduced in this chapter. However, in particular, in the general mixed oligopoly wherein one public firm and n private firms exist, it is very interesting whether or not the market structure with the merger between one public firm and one private firm belong the core, *i.e.*, whether or not the result obtained in the entrepreneurial case is robust against the use of each firm's FJSV delegation contract. As our future research, in the case wherein each firm adopts her/his FJSV delegation contract, we must consider whether or not the market structure with the merger between one public firm and one private firm becomes stable and achievable.[33] Extending our model in these directions remains a subject for future research.

[33]On the figures of the core of market structures before and after each firm's merger activity when s/he adopts her/his FJSV delegation contract, we would like to state the following predictions: White (2001) showed the effect of the use of the FJSV delegation contracts within both the public firm and the private firm on the equilibrium market outcomes under the same assumption as that in this chapter, which their technologies are represented as their constant marginal cost functions and the public firm is less efficient than the private firm. More precisely, in his 2001 paper, White found that in a market with linear costs and Cournot competition between a public firm and n private firms, both the private firm's profit and social welfare become higher with using their FJSV delegation contracts than without using them. Therefore, since the use of the FJSV delegation contracts within the public firm and the private firm increases the payoffs of the owners of both the firms, the figure of the core of market structures is not likely to drastically change against the introduction of the FJSV delegation contract within each firm, as compared to that in the case wherein every managerial decision makings are carried out by the owners of both the firms. Note that as described in chapter 4, the use of the FJSV delegation contracts within both the firms may increase the merger profitability with the merger between private firms, and thus, in such a case, we can consider that it is difficult for the status quo mixed oligopoly belongs to the core of market structures, since the market structure with the merger between the private firms tends to dominate its mixed oligopoly.

Chapter 6

Conclusion

This thesis tackled several topics in horizontal mergers in not only a private oligopoly where only profit-maximizing private firms exist, in but also a mixed oligopoly where a welfare-maximizing public firm and profit-maximizing private firms coexist and compete in the same market. More concretely, in the context of the private oligopoly, we examined how managerial delegation contracts within each firm affect the correspondence between the equilibrium ownership structure and the most preferred ownership structure with respect to social welfare. In the context of the mixed oligopoly, on the basis of the non-cooperative game approach adopted in the existing literature in this area such as Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008), we scrutinized the effect of the productivity-improving with some sort of merger, which was not considered in their previous works, on the achievablity of the merger between a public firm and a private firm. Furthermore, taking into account the above-mentioned productivity-improving with mergers, we derived the market structure which finally occurs after each firm's free merger activity by focusing the stability of market structures on the basis of the cooperative game approach. Speficically, in the last two chapters of this thesis, we derived the core of market structures in the cases wherein the technology of each firm is represented as the quadratic cost function and the constant marginal cost function, respectively.

More precisely, after in chapter 1 we discussed the relationship between the results obtained in the previous literature and those in this thesis, in chapter 2, in the context of the private oligopoly which is composed of asymmetric firms with respect to their marginal costs, taking into account the managerial delegation contract à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) (the FJSV delegation contract) which is entered into between the owner and the manager within each firm, we analyzed the mergers from the viewpoint of the stability of market structures which take place after each firm's free merger activity. In particular, we focused our attention on the fact that modern corporate governance codes include clauses requiring the disclosure of man-

agerial delegation contract to protect shareholders' objectives from the opportunistic behaviors of their managers. In order to model such a situation, we considered the influence of the bargaining process over the content of each firm's FJSV delegation contract between her/his owner and manager à la van Witteloostuijn et al. (2007) on the equilibrium ownership structure founded after her/his merger activity. In chapter 2, as the equilibrium solution concept which describes the equilibrium ownership structure(s) (EOS(s)) after each firm's horizontal merger, we adopted the core concept introduced in Horn and Persson (2001a) by regarding each firm's merger activity as the application of the coalition formation in the cooperative game. Although Straume (2006), which considered the situation wherein the content of each firm's FJSV delegation contract is determined for the viewpoint of her/his owner objective, *i.e.*, her/his profit, showed that the EOSs do not coincide with the most preferred ownership structures with respect to the equilibrium social welfare for any value of the degree of asymmetry among the existing firms, in chapter 2 of this thesis, we contrastingly found that for a comparatively large area of the relative bargaining power of the maneger to her/his owner within each firm, the EOSs can coincide with the most preferred structures with respect to the equilibrium social welfare. This result indicates that the bargaining over the content of the FJSV delegation contract between each firm's owner and manager is likely to lead the most socially preferred ownership structure, in contrast to the case wherein it is determinated for the viewpoint of her/his profit which is her/his owner's objective function. Consequently, the result obtained in chapter 2 implies that the necessity of an active merger policy can decrease through the bargaining of each firm's FJSV delegation contract, different from thoses shown in González-Maestre and López-Cuñat (2001) and in Straume (2006) which the owner directly determines the content of her/his FJSV delegation contract. On the other hand, the disclosure of each firm's managerial delegation contract, which is prescribed in modern corporate governance code, can be regarded as increasing the relative bargaining power of her/his owner. Therefore, from a couple of the result obtained in Straume (2006) and that in chapter 2, it is found that if the authority must excessively seek to adhere such a governance code to each firm, the more active merger policy should be newly needed. Furtheremore, we here must add several comments on the solution concept to measure the stability on each firm's merger incentive among ownership structures. As described above, in order to measure the stability on each firm's merger incentive among ownership structures, in the analysis of chapter 2, we adopted the core of ownership structures introduced in Horn and Persson (2001a) as the solution concept, different from those in chapters 4 and 5. More concretely, in the case wherein each firm's owner deviates from any merger, we assumed that in the new founded ownership structure,

s/he must take into account the payoff(s) of the other owner(s) who is deviated by her/him, and hence the decisive owners are composed of the owners of all the three firms, when the ownership structure with some sort of merger is induced from the ownership structure with another merger.[1] Then, when in the analysis of chapter 2, the domination relation adopted in chapters 4 and 5 is employed, it is our duty to reply the question of what figures of the core of ownership structure are. However, since in such an analysis, we must newly add one parameter which represents the shareholding ratio of each pre-merger firm's owner in the merged firm in each ownership structure, it is too difficult to characterize the core of ownership structures owing to the computational problem. Note that we fortunately realize that in the comparison between the status quo private triopoly and the private duopoly with some sort of merger, the domination relation between such two ownership structures is determined on the basis of the payoffs of the owners whose firms participate in the merger, and further, since the shareholding ratio of each firm's owner works in order to only distribute the merged firm's profit between the owners whose firms participate in the merger without influencing the value of each firm's profit directly, the numerical way in such a comparison is in essence unchanged between the two different domination relations. Therefore, the result that the status quo private oligopoly never belongs to the core of ownership structures for the arbitrary values of both the direct measure of production efficiency in the industry and the bargaining power of the manager within each firm relative to her/his owner is not changed

[1]On the other hand, in chapters 4 and 5, the domination relation was defined as the fact of whether only the payoff(s) of the owner(s) who deviate(s) from the present market structure increase(s) in the new founded market structure or not, relative to the past market structure.

between the two domination relations.[2] Consequently, in definition of the core of ownership structures adopted in chapters 4 and 5, we obtain the similar result to that in the definition in chapter 2 that the ownership structure with the merger participating in the least efficient firm 3 not only become the equilibrium ownership structure belonging to the core, but also achieves the highest social welfare among the four market structures.[3]

In chapter 3, in the context of mixed oligopoly, on the basis of the non-cooperative game approach which is similar to those adopted in Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) as the seminal works in this field, we considered the achievablity of the merger between a public firm and a private firm in equilibrium.[4] In particular, in chapter 3 of this thesis, we focused

[2]When we introduce three parameters, α, γ, and δ to denote the shareholding ratio of the owner of firm 1 in pre-merged firm 12 in (a) merger between the low-cost and medium-cost firms, the shareholding ratio of the owner of pre-merged firm 1 in firm 13 in (b) merger between the high-cost and low-cost firms, and the shareholding ratio of the owner of pre-merged firm 2 in firm 23 in (c) merger between the high-cost and medium-cost firms, respectively, we obtain the following result on the domination relations between the status quo private triopoly and the private duopoly with some sort of merger: For any $c \in (0, \bar{c})$, if $\beta \in [0, 1)$,

$$\text{Comparison between } M_a \text{ and } M_o \begin{cases} \alpha\Pi_{12}(M_a) - \Pi_1(M_o) > 0 \Leftrightarrow \alpha > \underline{\alpha}, \\ (1-\alpha)\Pi_{12}(M_a) - \Pi_2(M_o) > 0 \Leftrightarrow \alpha < \bar{\alpha}, \end{cases}$$

$$\text{Comparison between } M_b \text{ and } M_o \begin{cases} \gamma\Pi_{13}(M_b) - \Pi_1(M_o) > 0 \Leftrightarrow \gamma > \underline{\gamma}, \\ (1-\gamma)\Pi_{13}(M_b) - \Pi_3(M_o) > 0 \Leftrightarrow \gamma < \bar{\gamma}, \end{cases}$$

$$\text{Comparison between } M_c \text{ and } M_o \begin{cases} \delta\Pi_{23}(M_c) - \Pi_2(M_o) > 0 \Leftrightarrow \delta > \underline{\delta}, \\ (1-\delta)\Pi_{23}(M_c) - \Pi_3(M_o) > 0 \Leftrightarrow \delta < \bar{\delta}, \end{cases}$$

where $\underline{\alpha} = 3(5+3\beta)^2[1-\beta+9c(1+\beta)]^2/8(5+4\beta)^2[1-\beta+4c(1+\beta)]^2$, $\bar{\alpha} = \big[(1-\beta)^2(125+230\beta+101\beta^2) + 2c(875+1220\beta-366\beta^2-1244\beta^3-485\beta^4) + c^2(3125+11580\beta+15566\beta^2+9180\beta^3+2021\beta^4)\big]/8(5+4\beta)^2[1-\beta+4c(1+\beta)]^2$, $\underline{\gamma} = 3(5+3\beta)^2[1-\beta+9c(1+\beta)]^2/8(5+4\beta)^2[1-\beta+2c(1+\beta)]^2$, $\bar{\gamma} = \big[(1-\beta)^2(125+230\beta+101\beta^2) + 2c(1225+1330\beta-732\beta^2-1378\beta^3-445\beta^4) - c^2(8275+19560\beta+16930\beta^2+6264\beta^3+811\beta^4)\big]/8(5+4\beta)^2[1-\beta+2c(1+\beta)]^2$, $\underline{\delta} = 3(1-c)^2(1-\beta)^2(5+3\beta)^2/8(5+4\beta)^2[1-\beta-c(3+\beta)]^2$, and $\bar{\delta} = \big[(1-\beta)^2(125+230\beta+101\beta^2) + 2c(225+130\beta-132\beta^2-162\beta^3-61\beta^4) - 5c^2(1455+3672\beta+3506\beta^2+1496\beta^3+239\beta^4)\big]/8(5+4\beta)^2[1-\beta-c(3+\beta)]^2$. Thus, for any $\beta \in [0,1)$ and $c \in (0, \bar{c})$, the status quo private triopoly $M_o = \{\{1\},\{2\},\{3\}\}$ is blocked by $M_a = \{\{1,2\},\{3\}\}$, $M_b = \{\{1,3\},\{2\}\}$, and $M_c = \{\{1\},\{2,3\}\}$ for any $\alpha \in (\underline{\alpha}, \bar{\alpha})$, $\gamma \in (\underline{\gamma}, \bar{\gamma})$, and $\delta \in (\underline{\delta}, \bar{\delta})$, respectively.

[3]For example, in the case of $\gamma = \underline{\gamma}$, we obtain the following result: $M_b = \{\{1,3\},\{2\}\}$ belongs to the core of ownership structures for any $c \leq c^{\underline{\gamma}} := \big[725+1030\beta-312\beta^2-1046\beta^3-397\beta^4-2\sqrt{2}(5-\beta-4\beta^2)\sqrt{875+4720\beta+7734\beta^2+5080\beta^3+1175\beta^4}\big]/(4675+12000\beta+10626\beta^2+3536\beta^3+267\beta^4)$, when $\beta \in [0,1)$. Moreover, M_b not only becomes the equilibrium ownership structure belonging to the core, but also achieves the highest social welfare (i) for any $c \in (c^*, c^{**})$, when $\beta \in (0.174504, 0.211923]$ and (ii) for any $c \in (c^*, c^{\underline{\gamma}}]$, when $\beta \in (0.211923, 1)$, where $c^* = \big[1275+950\beta-852\beta^2-1062\beta^3-311\beta^4-4\sqrt{-39+194\beta+169\beta^2}(25+10\beta-23\beta^2-12\beta^3)\big]/(26875+71510\beta+71692\beta^2+31962\beta^3+5321\beta^4)$ and $c^{**} = \big[1275+950\beta-852\beta^2-1062\beta^3-311\beta^4+4\sqrt{-39+194\beta+169\beta^2}(25+10\beta-23\beta^2-12\beta^3)\big]/(26875+71510\beta+71692\beta^2+31962\beta^3+5321\beta^4)$.

[4]Although Bárcena-Ruiz and Garzón (2003) considered the decision to merge by a public firm and a private firm in a mixed duopolistic market that they produce the differentiated goods, Méndez-Naya (2008) subsequently analyzed the achievablity of the merger between a public firm and a private firm in the mixed oligopoly where the

our attention on the productivity-improving effect after some sort of merger, which was supposed in McAfee and Williams (1992), when the technology of each firm is represented as the quadratic cost function with respect to her/his output level.[5] Although the productivity-improving effect of the merged firm à la McAfee and Williams (1992) is supported by the assumption that s/he adopts the most efficient operation plan among the merger participants, we derived the relationship between the ratio of shareholding of the public sector in the merged firm and the number of the existing private firms, when the merger between one public firm and one private firm is achieved. In chapter 3, by explaining the merger incentives for the owners of both the public firm and the private firm through the two following effects; the *share effect* with in/decreasing the ratio of shareholding of the pre-merged public firm's owner in the merged firm, α; the *competition effect* with in/decreasing the number of the existing private firms, n, we found that if any merger yields the productivity-improving of the merged firm à la McAfee and Williams (1992), there exists an area of the (n, α)-plane to have that the owners of both the public firm and the private firm would like to merge in the case wherein more than one private firm exist. Furthermore, we showed that the area of the (n, α)-plane to have that the owners of both the firms would like to merge broadens, as n increases. In addition, in chapter 3, we discussed the two situations: (1) the merger between the public firm and the private firm yields no effect of the productivity-improving in the merged firm; (2) their cost functions are asymmetric, *i.e.*, the public firm is less efficient than the private firm.

In chapter 4 of this thesis, taking into account the productivity-improving effect with any merger considered in the previous chapter, which was introduced in McAfee and Williams (1992) and applied in Nakamura and Inoue (2007), we attempted to derive the stable market structure(s) belonging to the core after each firm's free merger activities in the mixed triopolistic industry composed of one welfare-maximizing public firm and two profit-maximizing private firms with the production technology which is represented as their quadratic cost functions. Then, we focused our attention on the ratio of shareholding in the merged firm among merger participants. In chapter 4, we showed that only the market structure including the merger between one public firm and one private firm with about 57% of shares in the merged firm owned by the owner of the public firm belongs to the core and becomes the stable market structure in the case wherein all the three firms have their symmetric quadratic cost functions. Although we realize that from

public firm and n private firms exist in a quantity-setting mixed oligopolistic market with a homogeneous goods.

[5] Although similar to our setting, Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) considered the economic situation wherein each firm has a quadratic cost function with respect to her/his quantity, they did not take into account the efficiency gains after the merger between the public firm and the private firm.

the above result, the market structure with the merger between one public firm and one private firm wherein the owner of the public firm shares about 57% of the stocks in the merged (partially privatized) firm is strongly stable against free deviations of firms' owners, in chapter 4, it was simultaneously shown that none of market structures belongs to the core in the case wherein only private firms exist in the market (one may regard its market situation as the market structure occurring after the full privatization of the public firm). The fact that the core of market structures is empty is of course problematic, since the never-ending coalition formation yields transaction costs up to infinity, which was described in Myerson (1991). However, the reason why the core of market structures is empty is that we assume that the deviating firms can freely induce a new market structure without compensating to the deviated firms. One may consider that in the real world economy, the owner of the deviated firm proposes some counter offer to such a deviation, unless at least, the payoff obtained in the previous market structure is also achieved in the new market structure. Reflecting the above idea that the deviating firm must compensate for the payoff of the deviated firm when s/he deviates from a market structure, if we attempt to derive the core after each firm's merger activity, such a hypothesis leads to the result that the core can be larger with such a compensation than without it, since such a compensation straightforwardly means that the deviation of each firm becomes somewhat difficult. In addition, in chapter 4, we found that without an improvement in productivity with any merger à la McAfee and Williams (1992), the mixed triopoly would be the unique market structure, and further if the public firm is less efficient than the private firms, the core of market structures can be become empty.[6]

In chapter 5, on the basis of the same cooperative game approach as that employed in chapter 4, we presented a theoretical analysis of the horizontal merger activity in the mixed oligopoly wherein the technology of each firm is represented as the constant marginal cost function. First, in the mixed triopoly that one welfare-maximizing public firm and two symmetric profit-maximizing private firms exist, we derived the core after each firm's merger activity. Diffrerent from the result obtained in the case wherein the cost function of each firm is quadratic with respect to her/his quantity, we showed that in accordance with the value of the marginal cost of the public firm, several market structures can belong to the core. Especially, in chapter 5, we obtained the result that there certainly exists a pair of the shareholding ratios of the owners of both the public firm and private firm in the merged (partially privatized) firm such that irrespective of the level of the marginal cost of the public firm, the market structure with the merger between one public

[6]Similar to the case wherein only the private firms exist, in the Appndix B of chapter 4, we obtained the result that the core of market structures can be empty, when each firm adopts her/his FJSV delegation contract.

firm and one private firm can belong to the core. Second, although in the case wherein each firm has her/his quadratic cost function, which was considered in chapter 4, we can not conduct the stability analysis of market structures after each firm's merger activity in the general mixed oligopoly with respect to the number of the existing private firms, in chapter 5, we extended our attention on the stability problem of horizontal mergers in the mixed triopoly into the one in the mixed oligopoly that the generalized-numbered private firms exist in the market. Even if there exist multiple private firms in a mixed oligopolistic industry, we obtained the result that as long as the marginal cost of the public firm is sufficiently low, the market structure with the merger between one public firm and one private firm can belong to the core, which becomes the stable market structure. Although in the general mixed oligopoly wherein one public firms and n private firms exist, we can not characterize the figure of the core of market structures after each firm's merger activity, we believe that it is very significant that we found that the merger between one public firm and one private firm can be supported not only on the basis of the non-cooperative game approach as in Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008) but also on the basis of our cooperative game approach. In addition, by defining the new notion, the *achievablity* which means the market structure not only belonging to the core and but also blocking a mixed oligopolistic industry, when the initial state is the mixed oligopoly, we attempted to provide an appropriate reply to the question of which of the market structure(s) in the core is/are likely to occur. In chapter 5, we showed that only the market structure with the merger between one public firm and one private firm can be stable and achievable in both the contexts of the mixed triopoly and oligopoly. Furtheremore, in chapter 5, among the analyses in both the contexts of the private oligopoly and the mixed oligopoly, we first presented the theoretical analysis of horizontal mergers in the general (mixed) oligopoly that the number of firms existing in the market is not restricted, which belongs to the analysis of the endogenous merger formation on the basis of the cooperative game approach.

Similar to the discussion in chapter 2, we must provide several discussions on the definitions of both the domination relation and the core of market structures to describe the stable market structures before and after each firm's merger activity. As described above, in chapters 4 and 5, we derived the core of market structures by the domination relation based on the fact of whether only the payoff(s) of the owner(s) who deviate(s) from the present market structure and found(s) the new market structure increase(s) or not, which is different from that adopted in chapter 2. If we conduct the stability analysis on each firm's merger incentive in mixed oligopoly which is composed of the public firm and the private firms by adopting the definition of the domination

relation in chapter 2, we must judge each firm's merger incentive on the basis of the sum of social welfare which is the objective function of the public firm's owner and the payoffs of the private firms' owners, in the comparison of the market structures with the merger including the public firm.[7] However, in the context of mixed oligopoly, it is somewhat unnatural that each firm's merger incentive is evaluated on in the basis of the ranking order of the sum of social welfare and the payoffs of the private firms' owners, whereas in the context of private oligopoly, it is natural that the sum of each firm's profit is divided between the owners of the firms. This is so because it is likely to be opaque that the value including social welfare is divided among firms' owners. If we adopt the domination relation in chapter 2 in deriving the core of market structures on the problem of horizontal mergers in the mixed oligopoly, we obtain the following result: (i) *Merger between a private firm and a public firm* $M_{\beta}^{\{\{0,i\},\{j\}\}}$ can belong to the core of market structures for any $\beta \leq \left(4921 - 117\sqrt{489}\right)/4094 \approx 0.570039$, $(i, j = 1, 2; i \neq j)$, and (ii) *Mixed triopoly* $M^{\{\{0\},\{1\},\{2\}\}}$ can belong to the core of market structures for arbitrary values of $\beta \geq \left(4921 - 117\sqrt{489}\right)/4094 \approx 0.570039$ and $\gamma \geq \left(214 - 13\sqrt{101}\right)/207 \approx 0.402665$. Note that the parameters of β and γ denote the shareholding ratios of the public firm's owner in the merged firm 12 and the merged firm 012, respectively.[8] Different from the result obtained in main body of chapter 4 of this thesis, from the above result, we find that by using the domination relation adopted in chapter 2, the mixed triopoly can belong to the core of market structures, subject to the values of the shareholding ratios of the public firm's owner in both the merged firm $0i$ and the merged firm 012, β and γ, respectively, since $W^c + (1-\beta)\,\Pi_{0i}^c$ and $W^d + (1-\gamma)\,\Pi_{012}^d$ stay low in the case wherein both the values of β and γ are also low, $(i = 1, 2)$.[9] We must pay our attention to the facts that whether the mixed triopoly becomes the equilibrium market structure belonging to the core depends on the values of β and γ and that whether the market structure with the merger between a public firm and a private firm becomes the equilibrium market structure

[7]For example, in the comparison between (a) $M^{\{\{0\},\{1\},\{2\}\}}$ and (c) $M_{\beta}^{\{\{0,1\},\{2\}\}}$, the domination relation is evaluated through the ranking order of the values of $W^a + \Pi_1^a$ and $W^c + (1-\beta)\,\Pi_{01}^c$. Note that the notations here are the same as those in chapter 4.

[8]On the other hand, the two types of market structures, *Merger between two private firms* $M_{\alpha}^{\{\{0\},\{1,2\}\}}$ and *Merger among all firms* $M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ do not belong to the core of market structures, since $M_{\alpha}^{\{\{0\},\{1,2\}\}}$ is blocked by $M^{\{\{0\},\{1\},\{2\}\}}$ and $M_{\gamma,\delta}^{\{\{0,1,2\}\}}$ is blocked by $M_{\beta}^{\{\{0,i\},\{j\}\}}$, *i.e.*, $\Pi_1^a + \Pi_2^a > \alpha\Pi_{12}^b + (1-\alpha)\,\Pi_{12}^b = \Pi_{12}^b$ for any $\alpha \in [0,1]$ and $\left[W^c + (1-\beta)\,\Pi_{0i}^c + \Pi_j^c\right]|_{\beta=\gamma} > W^d + (1-\gamma)\,\delta\Pi_{012}^d + (1-\gamma)\,(1-\delta)\,\Pi_{012}^d = W^d + (1-\gamma)\,\Pi_{012}^d$ for any $\gamma \in [0,1]$, respectively, $(i, j = 1, 2; i \neq j)$.

[9]From easy calculations, we find that $W^c + (1-\beta)\,\Pi_{0i}^c$ and $W^d + (1-\gamma)\,\Pi_{012}^d$ decrease in β and γ, respectively, as follows:

$$\begin{cases} \partial\left[W^c + (1-\beta)\,\Pi_{0i}^c\right]/\partial\beta = -3a^2\,(73 - 50\beta)\,/2\,(11 - 4\beta)^3 < 0, & \forall\beta \in [0,1]\,, \\ \partial\left[W^d + (1-\gamma)\,\Pi_{012}^d\right]/\partial\gamma = -3a^2\,(23 - 18\gamma)\,/\,(8 - 3\gamma)^3 < 0, & \forall\gamma \in [0,1]\,, \quad (i = 1, 2)\,. \end{cases}$$

depends on β only. Thus, in the sense that the market structure with the merger between public firm 0 and the private firm i can belong to the core by coordinating the shareholding ratio of the public firm's owner in the merged firm $0i$ founded in its market structure, the market structure with the firm $0i$ can belong to the core easier than the mixed triopoly, $(i = 1, 2)$.

Finally, we finish this thesis by discussing several issues which we should tackle as our future researches in the context of horizontal mergers in both the private oligopoly and the mixed oligopoly. First, we discuss our future researches on the theoretical analysis of the horizontal mergers in the context of the private oligopoly. In chapter 2 of this thesis, on the basis of the cooperative game approach, in the asymmetric private oligopoly with respect to each firm's marginal cost, we presented the relationship between the equilibrium ownership structure and the most preferred ownership structure with respect to the equilibrium social welfare for the viewpoint of the stability of ownership structures that occur after her/his horizontal merger activity in a closed economy wherein all the firms exist in the same and single region and country. We must extend our attention to the horizontal merger activity in the international private oligopoly that the existing firms have different nationalities. Nakamura (2010) has already investigated the effect of the bargaining over each firm's FJSV delegation contract between her/his owner and manager on the possibility of cross-border mergers and its welfare implications on total social welfare which is defined as the sum of each country's social welfare. However, he did not reveal the relationship between the equilibrium ownership structure and the most preferred structure with respect to social welfare in each country.[10] Then, we should scrutinize both the domestic and cross-border mergers not only with respect to total social welfare which is defined as the sum of social welfare in both the countries but also with respect to social welfare in the relevant country.[11] In addition, Nakamura (2010) assumed that all the existing firms can produce at zero costs in order to restrict our attention into clarifying the effect of saving trade costs with the cross-border mergers, and

[10] By considering the relationship between the equilibrium ownership structure and the most preferred structure with respect to total social welfare, Nakamura (2010) showed that in sufficiently large areas of the physical trade cost to export each firm's output and the manager's bargaining power relative to her/his owner, the duopoly structure with two international mergers that the highest total social welfare is achieved is observed in equilibrium. On the other hand, if both of them are sufficiently large, only the duopoly with two national mergers becomes the equilibrium market structure, though it simultaneously cannot become the most preferred market structure with respect to the total social welfare.

[11] In the private oligopoly, cross-border mergers are increasingly important phenomena. For example, by conducting the largest cross-national comparison of the effects of mergers to date, Gugler et al. (2003) found that there is an upward trend in the percentage of mergers which are cross-border, and specifically, this upward trend is pronounced for EU countries, where the percentage of all mergers in the sample which were cross-border rose from 24.2% in 1991–92 to 39.8% in 1997–98. Therefore, in the context of the international private oligopoly, we consider that it is very important to disclose the relationship between the equilibrium ownership structure and the most preferred ownership structure with respect to each country's social welfare in order for not only the supra-national authority like EU but also each country's government to prescribe an more appropriate merger policy.

thus, his analysis did not reflect the influence of the asymmetry of each firm's marginal cost on the equilibrium ownership structure and its welfare implications, different from that in chapter 2. Thus, in particular, as our next step, taking into account the asymmetry of their marginal costs, we should investigate the problem of cross-border mergers in the context of the international private oligopoly. Note that we consider that the general result on the figure of the core of ownership structures may not be obtained easily, and thus, in such a problematic case, the simulation analysis regarding the difference between each firm's productivity relative to her/his physical trade cost must be conducted. Even from the simulation analysis, we may be able to obtain the tendency of the change of the figure of the core of ownership structures with respect to that in the the difference between each firm's productivity relative to her/his physical trade cost.[12] Moreover, although we must confirm the result on the stable ownership structure(s) belonging to the core of ownership structures in the case wherein n asymmetric firms exist in the market, we consider that the analysis in such a case is difficult because of the computational problem.[13] Thus, we should conduct the simulation and numerical analysis on the number of the firms existing in the market, n. Second, we discuss our future researches in the context of the mixed oligopoly. We must take into account the features of the horizontal mergers between a public firm and a private firm in the real world, when we theoretically model their merger activities in the mixed oligopolistic industry: (i) such a merger in the real world is frequently cross-bordered; (ii) sufficiently large enterprises are involved in such a merger. From (i), we should consider the theoretical model of the merger between the public firm and the private firm in the context of the international mixed oligopoly wherein they exist in different countries with each other. Once we extend the theoretical model of the horizontal mergers between the public firm and the private firm in a closed economic situation into the model in the context of the international mixed oligopoly, we can consider many topics which have not been considered in the existing literature of this field yet. First, as supposed in Fjell and Pal (1996), Pal and White (1998), and Fjell and Heywood (2002), we must consider the mergers between the public firm and the private firm in the economic environment wherein only private firms export to a country or region containing one public firm and several private firms.[14] After conducting such analyses, in the context of an international mixed

[12]From the same reason on the computational problem, we may have to conduct the simulation and numerical analysis using computer softwares, if we would like to reveal the figure of the core of ownership structures before and after each firm's merger activity, when s/he adopts the other delegation regimes such as the market share delegation regime à la Jansen et al. (2007) and Ritz (2008) and the relative performance delegation regime à la Miller and Pazgal (2001, 2002) and Salas Fumas (1992).

[13]If all the n firms are asymmetric with respect to their marginal costs, taking into account all the ownership structures, it is too difficult for us to derive the figure of the core of ownership structures.

[14]Nakamura (2009b) considered the problem of horizontal mergers in the economic environment wherein a public

oligopoly with the strategic interaction between public firms existing in different countries, which was investigated in Bárcena-Ruiz and Garzón (2005a), Bárcena-Ruiz and Garzón (2005b), and Dadpay and Heywood (2006), we should pay our attention to investigating multiple merger phenomena between the public firm and the private firm that have different nationalities with each other, which are observed in the real world automobile industry.[15] In particular, the latter analysis can give us an adequate reply to the question of whether or not the merger between the different-nationalityed public and private firms is appropriate not only with respect to social welfare in the single country but also with respect to the total social welfare. This analysis is very important and relevant to the case wherein the supra-national authority, for examples, EU, judges the merger between the public firm and the private firm which have different nationality with each other from diversified standpoints.[16] Moreover, from (ii), we must consider the internal structure of both the public firm and the private firm, *i.e.*, the separation of ownership and management within them. As one of theoretical ways in order to focus their attention on each firm's internal structure in an oligopolistic industry, one may consider the *strategic delegation* à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985), so-called the FJSV delegation contract, which was explicitly taken into account in chapter 2 of this thesis. Although it is difficult to deal with large enterprises

firm and a private firm exist in the domestic country and a private firm exists the foreign country, and the market is in the domestic country. Furthermore, he assumed that all the three firms have the quadratic cost functions with respect to their output levels. Under such a setting, Nakamura (2009b) derived the stable market structures in both the entrepreneurial case wherein every managerial decision-making is done by each firm's owner and the managerial case wherein each firm with the separation between ownership and management adopts her/his FJSV delegation contract between her/his owner and manager, and in particular, he obtained the results that in the managerial case, only the market structure with the merger between the public firm and the foreign private firm can belong to the core of market structures, which is relevant to the example of the merger among large enterprises observed in the real world mixed oligopoly. However, as described in Concluding remarks of chapter 5 in this thesis, Nakamura (2009b) did not consider the economic situation wherein each firm export her/his output with any physical trade cost. Of course, the setting in Nakamura (2009b) is same as those in Fjell and Pal (1996), Pal and White (1998), and Fjell and Heywood (2002), and thus we do not consider that it is unnatural. However, the most recent works on horizontal mergers emphasize the tariff-jumping effect that the cross-bordered merged firm can export her/his output without any physical trade cost, since in the real world economy, each firm has an incentive to merged with the different-nationalityed firm in order to avoid paying the trade physical cost. In order to consider such a tariff-jumping effect in the mixed oligopoly, as our future research, we should suppose the physical trade cost to export each firm's output to her/his foreign country.

[15] In the real world automobile industry, we can observe multiple mergers involving the public firms and the private firms that exist in different country with each other. For example, in 1986, SEAT which is a Spanish public enterprise was merged with Volkswagen which is a German enterprises, and further, as described in Bárcena-Ruiz and Garzón (2003), the French firm, Renault acquired a 36.8% equity stake in Nissan Motor and a 22.5% stake in Nissan Diesel.

[16] Although Bárcena-Ruiz and Garzón (2005a), Bárcena-Ruiz and Garzón (2005b), and Dadpay and Heywood (2006) supposed one market comprising two countries which is regarded as a world market, the literature on international horizontal mergers in the private oligopoly including Horn and Persson (2001b) and Fumagalli and Vasconcelos (2009) adopted the segmented market hypothesis in order to reveal the effect of tariff-jumping that the merged firms with the cross-border mergers can avoid paying the trade physical cost. As our future research, we should also tackle the problem of international horizontal mergers in the mixed oligopoly under the segmented market assumption, supposing the physical trade cost to export each firm's output to her/his foreign country and/or the tariff-jumping effect.

in the theoretical economic analysis including theoretical horizontal mergers, as our first step to explicitly investigate the merger between large public and private enterprises in mixed oligopoly, we would like to begin with taking into the effect of the FJSV delegation contract within each firm on the equilibrium market structure after her/his merger activity. In particular, as one of our future researches, we must derive the core of market structures occurring after each firm's merger activity in the general mixed oligopoly with the use of her/his FJSV delegation contract wherein her/his technology is represented as her/his constant marginal cost function, which was considered in chapter 5 of this thesis.[17] In the private oligopoly, although in the previous literature on horizontal mergers, it is known that the use of each firm's FJSV delegation contract yields the increase of merger profitability and the decrease of social welfare owing to sufficiently high market concentration, the analysis of chapter 2 in this thesis revealed that the introduction of the bargaining over the content of each firm's delegation contract between her/his owner and manager can lead to the result that the equilibrium market structure with the merger including the least efficient firm coincides with the most preferred structure with respect to social welfare, and thus, the merger authority should pay her/his attention to the degree of the power balance between each firm's owner and her/his manager in order to prescribe an appropriate merger policy. Therefore, in the analysis on cross-border mergers in international oligopoly as our future researches, we must focus our attention on whether any welfare-improving merger occurs or not, and if it occurs, what conditions should be satisfied regarding the degree of asymmetry among the existing firms and the levels of their physical trade costs, and so on. In the mixed oligopoly on horizontal mergers between the public firm and the private firm, the analyses of chapters 4 and 5 were the first challenges in the endogenous merger formation that the decision-makings of whether to merger for all the firms' owners are endogenously considered within the model, and thus, as our further meaningful step, we should examine horizontal mergers between the public firm and the private firm, taking into account the merger examples of in the real world economy that large enterprises exist in the different countries with each other. Thus, in order to reflect

[17] Under the setting that the technologies of both the public firm and the private firm are represented as the constant marginal cost functions with respect to their output levels and the public firm is less efficient than the private firm, White (2001) investigated the effect of the use of their FJSV delegation contracts on the equilibrium market outcomes, and he showed that the use of each firm's delegation contract yields her/his higher profit and higher social welfare, relative to the non-use of such a delegation contract. Thus, we consider that the figure of the core of market structures on the basis of each firm's merger incentive will be unchanged since the payoffs of the owners of both the public firm and the private firm increase through the use of each firm's FJSV delegation contract. Note that as shown in Appendix B of chapter 4 in this thesis, the use of each firm's FJSV delegation contract yields the increase of the merger profitability between private firms. Thus, we consider that the status quo mixed oligopoly is likely not to become the stable market structure, since it may be dominated by the market structure with the merger between the private firms.

most adequately the examples of horizontal mergers in the real world mixed oligopoly, we must simultaneously take into account the two factors: (1) the possibility of cross-border mergers and (2) the use of each firm's FJSV delegation contract within the separation between ownership and management in the more general settings, for example, the number of the private firms and the existence of the physical trade cost. Such a theoretical analysis become the unswerving evidence in order both the supra-national authority including EU and each country's government to prescribe more appropriate merger policies in the mixed oligopolistic industry.[18] Extending our analyses in this thesis into these directions is left for our future researches.

[18]If such an analysis is difficult owing to the computational problem, we must conduct the computational analysis through the relevant PC software.

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