Cooperative and Non-cooperative R&D Policies Under Spillover Uncertainty

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Abstract

This paper first establishes a two-stage game model of a Cournot duopoly where two firms conduct R&D non-cooperatively or cooperatively under spillover uncertainty in the first stage while they choose competitively outputs in the second stage after the spillover uncertainty is resolved. Then, it examines the effects of spillover uncertainty on the equilibrium R&D level and the social welfare. As a result, the paper presents some interesting propositions that would modify the optimal R&D policies proposed by the deterministic models.

1. Introduction

In their pioneering paper, D'Aspremont and Jacquemin (1988) have exploited a two-stage game model of a Cournot duopoly with R&D spillovers, where the firms decide non-cooperatively or cooperatively the R&D levels in the first stage and choose competitively their outputs in the second stage. Then they examined the non-cooperative, cooperative and socially optimal R&D levels. They showed, as their main proposition, that the cooperative R&D level is always lower than the socially optimal R&D level, while it is lower (higher) than the non-cooperative R&D level when the spillover ratio is smaller (larger) than 1/2. Furthermore, Suzumura (1992), Kamien, Muller and Zang (1992) and others have established a number of more extended models considering R&D spillovers and presented some more generalized propositions.[1]

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[1] For example, among others, Suzumura (1992) has generalized a model of cooperative and non-cooperative R&D choices in a Cournot oligopoly with spillovers. Kamien, Muller and Zang (1992) have established a model of cooperative and non-cooperative R&D choices of a Bertrand oligopoly with spillovers. Coe and Helpman (1995) have extended

Although their propositions are quite interesting, these are derived from deterministic models adopting the assumption that firms do not face uncertainty. Without considering uncertainty, one could not fully understand the efficiency of R&D with respect to resource allocation in the actual industry, in which uncertainty is always a essential factor. Therefore, it is impossible to suggest whether cooperative R&D or non-cooperative R&D is preferable from a social point of view, and impossible to propose appropriate R&D policies.

In order to exploit a more generalized model, this paper introduces spillover uncertainty, because this kind of uncertainty is regarded as the most significant uncertainty in the case when firms choose their R&D levels cooperatively or non-cooperatively. Of course, there are many uncertainties in the actual world, and some of them might be asymmetrical among firms and governments. However, it is unnecessary for this paper to introduce further uncertainties and their asymmetry into its model, since it focuses on analysis of links among equilibrium R&D levels, social welfare and spillover uncertainty which is regarded as symmetrical among firms.

To emphasize the effects of spillover uncertainty on the equilibrium R&D level and the social welfare, this paper extends the model presented by D'Aspremont and Jacquemin (1988) so as to include spillover uncertainty, and presents a very simple two-stage game model of a Cournot duopoly under uncertainty. It is assumed that two firms conduct R&D non-cooperatively or cooperatively in the first stage under spillover uncertainty, while they choose competitively output levels in the second stage, at which the spillover uncertainty is resolved.

At first sight, it might seem that the model in this paper merely extends the calculus of the D'Aspremont and Jacquemin model. However, the two models are actually essentially different, from both practical and theoretical standpoints. The model in this paper considers uncertainty, a factor that a satisfactory micro-economic model should take into account, and consequently it is more realistic than the D'Aspremont and Jacquemin model and its results will have benefit that is more practical as suggestions for R&D policy. Therefore, it is obvious that the present model is more significant than just a calculus extension of the D'Aspre-

a model presented by Spencer and Brander (1983) so as to investigate the equilibrium R&D choices of an international duopoly with spillovers. And Anbarci, Lemke and Santanu (2002) have recently built a model considering complementarities of firm specific R&D inputs with spillovers. Thus, they have presented many distinguished propositions with respect to linkages among equilibrium R&D levels, socially optimal R&D levels, social welfare and R&D spillovers, and have given significant impacts to developments of the R&D theory.

mont and Jacquemin model.

Using such a model, this paper analyzes the effects of spillover uncertainty on the equilibrium R&D levels and the social welfare, and presents some interesting propositions. First, it finds that even though the (expected) spillover ratio is small enough (smaller than 1/2), the equilibrium R&D level is higher in the cooperative game than in the non-cooperative game when the degree of spillover uncertainty is large enough. This result runs contrary to the deterministic models which propose that if the spillover ratio is small enough the equilibrium R&D level is lower in the cooperative game than in the non-cooperative game. Second, it is shown that the effect of a change in the spillover uncertainty on the equilibrium R&D level in the non-cooperative game is opposite to that in the cooperative game. That is, while a rise in the spillover uncertainty reduces the equilibrium R&D level in the non-cooperative game, it raises the equilibrium R&D level in the cooperative game. Finally, it demonstrates that when there is the spillover uncertainty the cooperative R&D game is better than the non-cooperative R&D game from the point of view of social welfare, since the resource allocation is more efficient and stable in the former game than in the latter.

The rest of this paper is organized as follows. Section 2 establishes a basic model and presents the equilibrium R&D levels in the non-cooperative and cooperative games. Section 3 analyzes the effects of a change in the spillover uncertainty on the equilibrium R&D levels in the two games. Section 4 examines how the expected social welfare depends on the spillover uncertainty in order to show the essential roles of spillover uncertainty in the firms' optimal R&D choices. Finally, section 5 presents some concluding remarks which should be considered for extending the model in this paper.

2. Basic Model and Equilibrium R&D Levels

Consider a Cournot duopoly where two firms conduct R&D non-cooperatively or cooperatively in the first stage under spillover uncertainty, and determine outputs independently in the second stage, at which the spillover uncertainty is resolved. Following D'Aspremont and Jacquemin (1988), let me define the demand function and the cost functions as:

(a) the inverse demand function: $D^{-1} = a - b\ (q_1 + q_2)$,

(b) the production cost function: $C_i = [A - x_i - \beta x_j]q_i,\ i = 1, 2, i \neq j$

$$\text{with } 0 < A < a,\ 0 < \beta < 1;\ x_i + \beta x_j \leq A;\ Q = q_1 + q_2 \leq a / b,$$

(c) the R&D cost function: $V_i = \frac{\gamma x_i^2}{2}$, $i = 1, 2$,

where q_i and x_i are output and R&D levels of a firm i (= 1, 2) respectively, and other elements of notation are all positive constants. Therefore, the profit of a firm i is defined as

$$\Pi_i = (a - bQ)q_i - (A - x_i - \beta x_j)q_i - \frac{\gamma x_i^2}{2}, \; i = 1, 2, \; i \neq j$$

This is the same as the firm's profit presented by D'Aspremont and Jacquemin (1988), when there is no spillover uncertainty.[2]

However, the firm's profit is uncertain in this model, since the spillover ratio β is a random variable in the first stage when the optimal R&D level is chosen by the firms. It is assumed that the firms' belief concerning the uncertain spillover ratio β is given by a density function $\Phi(\beta)$. Furthermore, since the profit maximization hypothesis is meaningless when the firm's profit is uncertain, this paper assumes that the firm behaves so as to maximize expected profit under spillover uncertainty.[3]

Now, taking into account the profit Π_i of the firm i and the density function $\Phi(\beta)$ defined above, the expected profit $E[\Pi_i]$ of the firm i is defined as:

$$\text{(1)} \quad E[\Pi_i] = \int_0^1 [(a - bQ)q_i - (A - x_i - \beta x_j)q_i - \frac{\gamma x_i^2}{2}]\Phi(\beta)d\beta$$

$$= E[(a - bQ)q_i - (A - x_i - \beta x_j)q_i - \frac{\gamma x_i^2}{2}], \; i = 1, 2, \; i \neq j.$$

where $E[\Pi_i]$ is an expectation operator with respect to β. Then, the firm i decides the R&D level x_i and output q_i so as to maximize its expected profit, given its rival's choices. In what follows this problem of the expected profit maximization will be solved backward.

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[2] As far as I know, though Marjit (1991) and others have considered R&D success uncertainty, and Ishii (2000) has introduced demand uncertainty into a model of an international duopoly considering R&D spillovers, there are no papers introducing the spillover uncertainty. Furthermore, though the number of papers that adopts asymmetric information has been recently increasing, the assumption that some firms have perfect information while the others have no information seems to be very restrictive in considering spillover uncertainty. No firm gets perfect information about R&D spillovers. Restrictive assumptions would make models more unrealistic.

[3] The other plausible hypothesis under uncertainty might be the expected utility maximization hypothesis. Though this is regarded as the more general hypothesis, which includes the expected profit maximization hypothesis as a special case, the latter is not as restrictive in analyzing the effects of the spillover uncertainty on the equilibrium R&D decisions in the cooperative and non-cooperative R&D games.

Since the spillover uncertainty is resolved in the second stage, the optimal outputs chosen by the firms that make the production decisions non-cooperatively in the second stage are obtained through solving simultaneously the firms' output reaction functions given by

$$(2)\quad a-2bq_i-bq_j-(A-x_i-\beta x_j)=0,\ i=1,2,\ i\neq j.$$ [4]

Consequently, the Nash-Cournot equilibrium output of the firm i ($i=1, 2$) in the second stage is:

$$(3)\quad q_i=\frac{(a-A)+(2-\beta)x_i+(2\beta-1)x_j}{3b},\ i=1,2,\ i\neq j.$$

Clearly, the firms' optimal output levels in this paper given by (3) are the same as those derived by D'Aspremont and Jacquemin (1988) if the realized spillover ratios are equivalent to each other.

By contrast, as will be shown soon, the firms' optimal R&D choices in this paper are quite different from those proposed by D'Aspremont and Jacquemin (1988), since the former model considers spillover uncertainty while the latter does not. Moreover, the firms' optimal R&D choices in this paper are also different from each other, depending on whether the firms conduct R&D non-cooperatively (say the non-cooperative game) or cooperatively (say the cooperative game). Hence, this paper will investigate these two games separately and compare the results.

In the non-cooperative game where the firms choose R&D independently, the expected profit $E[\Pi_i]$ of the firm i is rewritten as:

$$(4)\quad E[\Pi_i]=\{(a-A)^2+(4-4\beta_e+\sigma_\beta^2+\beta_e^2-4.5b\gamma)x_i^2 +(1-4\beta_e+4\sigma_\beta^2+4\beta_e^2)x_j^2+2(a-A)(2-\beta_e)x_i +2(a-A)(2\beta_e-1)x_j-2(2-5\beta_e+2\sigma_\beta^2+2\beta_e^2)x_ix_j\}/9b,\quad j\neq i,\ i=1,2,$$

where β_e and σ_β^2 are respectively the expectation and the variance of the uncertain spillover ratio β. Taking into consideration that $0<\beta<1$ holds, β_e and σ_β^2 satisfy the conditions expressed as $0<\beta_e<1$ and $0<\sigma_\beta^2<1$, respectively. Moreover, it is quite natural that the variance σ_β^2 measures the degree of spillover uncertainty.

Since the firm i chooses x_i which maximizes its expected profit $E[\Pi_i]$ defined as (4) in the first stage, the R&D reaction function of the firm i is given by

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4 The second-order condition in output choice is always satisfied.

$$(5)\quad (4-4\beta_e+\sigma_\beta^2+\beta_e^2-4.5b\gamma)x_i-(2-5\beta_e+2\sigma_\beta^2+2\beta_e^2)x_j +(a-A)(2-\beta_e)=0, \qquad j\neq i,\ i=1,2,$$

Then, solving (5) simultaneously, one obtains the Cournot-Nash equilibrium R&D level x^* $(=x_1^*=x_2^*)$ of the firm in the non-cooperative game under spillover uncertainty:

$$(6)\quad x^*=\frac{(a-A)(2-\beta_e)}{4.5b\gamma-(2+\beta_e-\sigma_\beta^2-\beta_e^2)},\ i=1,2.$$[5]

In the cooperative game where the firms determine the R&D levels cooperatively, they act together so as to maximize the expected joint profits $E[\Pi_1]+E[\Pi_2]$, where $E[\Pi_i]$ $(i=1, 2)$ are given by (4), respectively. Therefore, the equilibrium R&D level $x^c(=x_1^c=x_2^c)$ of the firm in the cooperative game is given by

$$(7)\quad x^c=\frac{(a-A)(1+\beta_e)}{4.5b\gamma-(1+2\beta_e+\sigma_\beta^2+\beta_e^2)},\ i=1,2.$$[6]

It is immediately obvious, from (6) and (7), that the equilibrium R&D levels in both the cooperative and non-cooperative games are functions of not only the expectation β_e of spillover ratio but also the spillover uncertainty σ_β^2.

3. Spillover Uncertainty and Equilibrium R&D Levels

In the model which captures the spillover uncertainty by randomness of the spillover ratio β, it is quite natural that the variance σ_β^2 of β is regarded as the degree of spillover uncertainty. Therefore, one can derive the effects of a change in the spillover uncertainty on the equilibrium R&D levels in the non-cooperative and cooperative cases by differentiating both sides of (6) and (7) with respect to σ_β^2, respectively.

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5 In the non-cooperative game the second-order condition of firm's R&D choice, the stability condition of a Cournot-Nash R&D equilibrium, and the condition of positive equilibrium R&D level are respectively given by $(4-4\beta_e+\sigma_\beta^2+\beta_e^2-4.5b\gamma)<0$, $(4-4\beta_e+\sigma_\beta^2+\beta_e^2-4.5b\gamma)^2-(2-5\beta_e+2\sigma_\beta^2+2\beta_e^2)>0$, and $\{4.5b\gamma-(2+\beta_e-\sigma_\beta^2-\beta_e^2)\}>0$. Thus, considering $0\leq\beta_e\leq1$ and $0\leq\sigma_\beta^2\leq1$, when $b\gamma$ is bigger than 10/9 these conditions are all satisfied, regardless of the values of β_e and σ_β^2. Since it is necessary, as is indicated by Henriques (1990), to set the proper parameter restriction, this paper will add the other restriction of $b\gamma>10/9$ in order to satisfy simultaneously all the conditions mentioned above. Of course, it is clear that this additional restriction also satisfies the second-order condition of firm's R&D choice and the condition of positive equilibrium R&D level in the cooperative game. This restriction, however, does not undermine the essential theses of this paper.

6 $\{4.5b\gamma-(1+2\beta_e+\sigma_\beta^2+\beta_e^2)\}>0$ holds when the second-order conditions are satisfied. Further, contrary to D,Aspremont and Jacquemin (1988), the variables with a superscript c in this paper denote the variables belonging to the cooperative game.

Differentiating both sides of (6) with respect to σ_β^2, one obtains:

$$(8)\quad \frac{\partial x^*}{\partial \sigma_\beta^2} = -\frac{(a-A)(2-\beta_e)}{\{4.5b\gamma-(2+\beta_e-\sigma_\beta^2-\beta_e^2)\}^2} < 0,$$

which implies in turn that the equilibrium R&D level x^* in the non-cooperative game declines as the degree of spillover uncertainty increases, and *vice versa*. On the other hand, differentiating both sides of (7) with respect to σ_β^2, one gets:

$$(9)\quad \frac{\partial x^c}{\partial \sigma_\beta^2} = \frac{(a-A)(1+\beta_e)}{\{4.5b\gamma-(1+2\beta_e+\sigma_\beta^2+\beta_e^2)\}^2} > 0,$$

which shows that the equilibrium R&D level x^c in the cooperative game is an increasing function of the degree of spillover uncertainty.

Although (8) and (9) combine to demonstrate that the effects of a change in the spillover uncertainty on the equilibrium R&D levels are quite different form each other in non-cooperative and cooperative games, this is not a surprising result because the R&D spillover has quite different effects in these two games. In the non-cooperative game the R&D spillover is just an external variable that literally means a leakage of the R&D to each firm. Thus, it would be regarded as a detestable phenomenon by each firm. By contrast, in the cooperative game the spillover is not a leakage of the R&D since it is internalized. Hence the firms would not consider the R&D spillover as detestable. The results shown by (8) and (9) reflect such differences between the effects of the R&D in the two games.

Next, as regards the difference between the equilibrium R&D levels in the non-cooperative and cooperative games, (6) and (7) combine to present:

$$(10)\quad x^* - x^c = \frac{(a-A)\{4.5b\gamma(1-2\beta_e)-3\sigma_\beta^2\}}{H_1H_2},$$

where H_1 and H_2 are given by $H_1 = 4.5b\gamma - (2+\beta_e-\sigma_\beta^2-\beta_e^2)$ and $H_2 = 4.5b\gamma - (1+2\beta_e+\sigma_\beta^2+\beta_e^2)$, respectively. Clearly, as the right side of (10) is positive (negative), the equilibrium R&D level is higher (lower) in the non-cooperative game than in the cooperative game.

Taking into consideration that $(a-A)$, H_1 and H_2 are all positive, the sign of the right side of (10) is equal to that of $\{4.5b\gamma(1-2\beta_e)-3\sigma_\beta^2)\}$. Hence, if $\sigma_\beta^2 = 0$ holds, (10) gives:

$$(11)\quad x^* < (>) x^c \text{ as } 0.5 < (>) \beta_e.$$

This implies that if there is no spillover uncertainty, the equilibrium R&D level is higher in the cooperative R&D game than in the non-cooperative R&D game when the expected spillover ratio is larger than 0.5, and *vice versa*. Clearly, this is the same as the result derived by D'Aspremont and Jacquemin (1988).

However, when $\sigma_\beta^2 \neq 0$ holds, (10) yields

$$(12) \quad x^* < (>) x^c \text{ as } 0.5 < (>) \frac{\sigma_\beta^2}{3b\gamma} + \beta_e.$$

Therefore, when the firms face spillover uncertainty the difference between the equilibrium R&D levels in the cooperative and non-cooperative games depends on not only the expected spillover ratio β_e but also the degree of spillover uncertainty σ_β^2. This is quite different from the proposition presented by D'Aspremont and Jacquemin (1988). Furthermore, it is obvious from (10) that the larger is the spillover uncertainty σ_β^2, the larger is the range that $x^* < x^c$ holds, and *vice versa.* This is one of the essential differences between the propositions presented by the deterministic models and the present model. This will be also explained more closely by using Fig. 1.

In Fig. 1, the cc line depicts the set of σ_β^2 and β_e on which x^c is equal to x^* since $\frac{\sigma_\beta^2}{3b\gamma} + \beta_e = 0.5$ holds. Hence, while the area A (shadowed by vertical lines) on the right side of the cc line expresses the set of σ_β^2 and β_e on which x^c is larger than x^*, the area B (shadowed by horizontal lines) on the left side of the cc line corresponds to the set of σ_β^2 and β_e where x^c is smaller than x^*. The deterministic model presented by D'Aspremont and Jacquemin (1988) has judged the sign of $(x^* - x^c)$

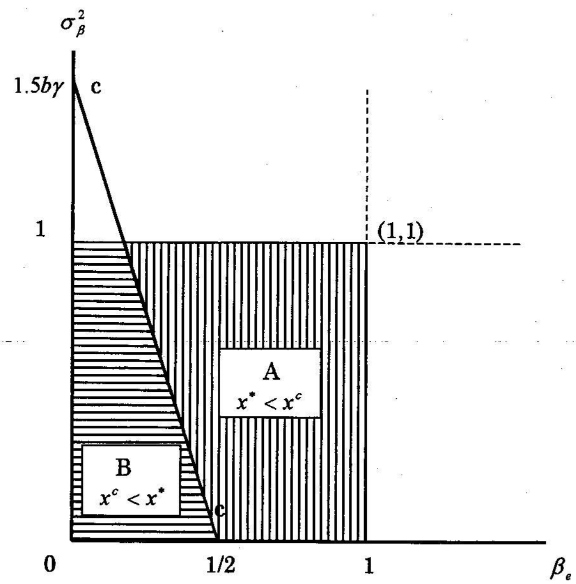

Fig. 1.

only by the (expected) spillover ratio along the horizontal axis, and completely neglected the degree of spillover uncertainty on the vertical axis. Indeed, if $\sigma_\beta^2 = 0$ holds, (10) reduces to (11) which gives the same proposition as presented in their deterministic model.

However, when the spillover uncertainty is considered, the sign of $(x^* - x^c)$ depends on both the expected spillover ratio along the horizontal axis and the degree of spillover uncertainty along the vertical axis. Clearly the area of the expected spillover ratio in which x^c is larger than x^* becomes bigger in this model than in the D'Aspremont and Jacquemin's model, as is illustrated in Fig. 1. Therefore, even if the expected spillover ratio is small enough ($\beta_e < 0.5$), the equilibrium R&D level is higher in the cooperative game than in the non-cooperative game when the degree of spillover uncertainty is large enough ($1.5b\gamma - 3b\gamma\beta_e < \sigma_\beta^2$).

4. Spillover Uncertainty and Social Welfare

Denoting the firm's expected profits in the cooperative and non-cooperative games as $E[\Pi^c]$ and $E[\Pi^*]$ respectively, one has

$$(13)\quad E[\Pi^c] - E[\Pi^*] = \frac{(a-A)\{4.5b\gamma(1-2\beta_e) - 3\sigma_\beta^2\}^2}{H_1^2 H_2}.$$

Consequently, the expected profit is larger in the cooperative R&D game than in the non-cooperative R&D game, except for a special case where $\{4.5b\gamma\,(1-2\beta_e) - 3\sigma_\beta^2)\} = 0$ holds with $x^c = x^*$. This is one of the reasons why some firms want to conduct R&D cooperatively, regardless of the degree of spillover uncertainty. If one justification for participating in cooperative R&D is the achievement of higher expected profits, (13) demonstrates that all firms choose the cooperative R&D game if possible. However, in the actual world all firms do not always have the opportunity to negotiate a contract for cooperative R&D. Consequently, while some firms choose to conduct R&D cooperatively, others conduct R&D independently.

While (8) indicates that a rise in the degree of spillover uncertainty σ_β^2 reduces the equilibrium R&D level x^* in the non-cooperative game, (9) shows that a rise in the degree of spillover uncertainty increases the equilibrium R&D level x^c in the cooperative game. Thus, since $\partial(x^c - x^*) / \partial\sigma_\beta^2 > 0$ is easily shown, the difference between the equilibrium R&D levels in the cooperative and non-cooperative games becomes larger as the degree of spillover uncertainty increases. Then, considering that the firm's expected profit is an increasing function of its own R&D level, as is well known, the difference between the expected profits in the two games also rises with the spillover uncertainty. It follows that the firm's incentive to participate into the cooperative R&D game is reinforced by a rise in the degree of spillover uncertainty.

In the range where the equilibrium R&D level is higher in the cooperative game than in the non-cooperative game (that is, $x^{*} < x^{c}$ holds), the total output $Q (= q_1 + q_2)$ produced by two firms is also bigger in the cooperative game than in the non-cooperative game, i.e., $Q^{*} < Q^{c}$. Consequently, considering that the expected social welfare is the sum of the expected consumer surplus and the expected producer surplus(= the firm's expected profit), the expected social welfare $E[W^{c}]$ in the cooperative game is always larger than the expected social welfare $E[W^{*}]$ in the non-cooperative game, i.e., $E[W^{*}] < E[W^{c}]$, because $E[\Pi^{*}] \leq E[\Pi^{c}]$ holds from (13).

On the other hand, in the case of $x^{*} > x^{c}$ it is ambiguous whether $E[W^{*}]$ is larger or smaller than $E[W^{c}]$, since Q^{c} is smaller than Q^{*} though $E[\Pi^{*}] \leq E[\Pi^{c}]$ still holds. Hence, in this case there is a possibility that the expected social welfare is larger in the non-cooperative case than in the cooperative case, that is, $E[W^{c}] < E[W^{*}]$ holds. As a result, one can not judge definitely which of the cooperative and non-cooperative games is better from the point of view of expected social welfare.

However, as is obvious from the arguments with respect to Fig. 1, a rise in the degree of spillover uncertainty σ_β^2 raises the possibility that the expected social welfare in the cooperative game becomes bigger than that in the non-cooperative game, *ceteris paribus*. This is one of the reasons why the cooperative game is recommended when the degree of spillover uncertainty is large enough. In addition, in such a case the larger spillover uncertainty is regarded as a better phenomenon from the point of view of expected social welfare.

Assuming $x_1 = x_2 = x$, the expected social welfare $E[W]$ is expressed as:

$$E[W] = E[bQ^2/2 + (a - bQ)Q - AQ + (1+\beta)xQ - \gamma x^2].$$

Then, the socially optimal R&D level x^{**} which maximizes the expected social welfare $E[W]$ is given by

$$(14) \quad x^{**} = \frac{(a-A)(b+\beta_e)}{2b\gamma - (1 + 2\beta_e + \sigma_\beta^2 + \beta_e^2)}.$$

Then, (6), (7) and (14) combine to indicate that while the sign of $(x^{*} - x^{c})$ is ambiguous form the arguments of (10), the signs of $(x^{**} - x^{c})$ and $(x^{**} - x^{*})$ are always positive, i.e., $(x^{**} - x^{c}) > 0$ and $(x^{**} - x^{*}) > 0$. It follows that although all of the equilibrium R&D levels and the socially optimal R&D level depend on the spillover uncertainty, the firms do not commit over-investments but always choose under-investments in R&D, regardless of the degree of spillover uncertainty in either of the cooperative or non-cooperative games.

Although this paper has just shown that $(x^{**}-x^{c})>0$ and $(x^{**}-x^{*})>0$ always hold, it is not yet obvious how $(x^{**}-x^{c})$ and $(x^{**}-x^{*})$ depend on the R&D spillover uncertainty. So, I finally analyze the effects of the R&D spillover uncertainty on $(x^{**}-x^{c})$ and $(x^{**}-x^{*})$ in more detail. Differentiating (14) with respect to σ_β^2, one gets:

$$(15)\quad \frac{\partial x^{**}}{\partial \sigma_\beta^2}=\frac{(a-A)(1+\beta_e)}{\{2b\gamma-(1+2\beta_e+\sigma_\beta^2+\beta_\beta^2)\}^2}>0,$$

which implies in turn that the socially optimal R&D level x^{**} increases as the degree of spillover uncertainty σ_β^2 raises, and *vice versa*. Then, (8), (9) and (15) combine to give

$$(16)\quad \frac{\partial(x^{**}-x^{c})}{\partial \sigma_\beta^2}>0,\ \text{and}\ \frac{\partial(x^{**}-x^{*})}{\partial \sigma_\beta^2}>0.$$

Generally speaking, the difference between the socially optimal R&D level x^{**} and the equilibrium R&D level x^{*} or x^{c} is taken as a measure of the discrepancy between the socially efficient resource allocation and the market resource allocation in the cooperative game or in the non-cooperative game. Then, (16) indicates that since the discrepancy rises (reduces) as the spillover uncertainty increases (decreases) in both the non-cooperative and cooperative games, the smaller the spillover uncertainty becomes, the closer the market resource allocation approaches to the socially optimal resource allocation, and *vice versa*. Therefore, the smaller spillover uncertainty is more desirable from the standpoint of efficient discrepancy of resource allocation.

Furthermore, one gets from (8), (9) and (15):

$$(17)\quad \frac{\partial(x^{**}-x^{*})}{\partial \sigma_\beta^2}>\frac{\partial(x^{**}-x^{c})}{\partial \sigma_\beta^2}>0.$$

It is obvious from (17) that the effect of a change in the degree of spillover uncertainty on the efficient discrepancy of resource allocation is always smaller in the cooperative game than in the non-cooperative game. Therefore, the efficiency of resource allocation is more stable to a change in the spillover uncertainty in the cooperative game than in the non-cooperative game. This is another reason why the cooperative game is preferable to the non-cooperative game in choosing the equilibrium R&D level under spillover uncertainty.

5. Concluding Remarks

Extending the D'Aspremont and Jacquemin model so as to include R&D

spillover uncertainty, this paper has examined the effects of spillover uncertainty on equilibrium R&D levels and social welfare. As a result, it has presented some interesting propositions which are not shown in the deterministic models. These propositions presented above have demonstrated that it is essential to consider the R&D spillover uncertainty in analyzing the linkages among the equilibrium R&D levels, the social welfare and the R&D spillovers. Therefore, all the propositions derived from the deterministic models may be regarded as special cases of those derived from the model considering R&D spillover uncertainty.

One important economic implication of the arguments in this paper is that one could not propose any appropriate political suggestions without considering the spillover uncertainty. This paper has found, among other things, that the social welfare levels in both the cooperative and non-cooperative games depend on the spillover uncertainty. Hence, one could not judge which of these two games it would be desirable to implement in the actual world without considering spillover uncertainty. Furthermore, it has also been shown that not only the equilibrium R&D levels in the cooperative and non-cooperative games, but also the socially optimal R&D level, depend on the spillover uncertainty. Thus, it is impossible for us to suggest the appropriate policies, the aim of which is to achieve the socially optimal R&D level, without introducing spillover uncertainty.

Of course, since this paper has concentrated on the analysis of the effects of spillover uncertainty on equilibrium R&D levels and social welfare, it has included some aspects that should be modified. First, though this paper has adopted the hypothesis that firms maximize the expected profits, the expected utility maximization might be regarded as a more reasonable hypothesis under spillover uncertainty. Moreover, this paper has introduced only spillover uncertainty, but some other uncertainties, i.e., demand uncertainty and success uncertainty, should be also considered. And, though the spillover uncertainty in this paper is assumed to be constant and symmetry, its precise status might depend on whether firms conduct R&D non-cooperatively or cooperatively and might be asymmetrical among firms. An appropriately extended model would present more general propositions, which reinforce those in this paper.[7]

References

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[7] See also D, Aspremont and Jacquemin (1990) and Katz (1986) with respect to a number of other extensions.

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