OPTIMAL RESOURCE SHARING AND SPECTRUM LEASING IN ENERGY EFFICIENT RELAY-ASSISTED NETWORKS

By

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To my dear wife and my lovely parents.

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List of Acronyms

3G	Third Generation
4G	Fourth Generation
CDMA	Code Division Multiple Access
HetNets	Heterogenous Networks
BS	Base Station
MWh	Mega Watt Hour
3GPP	Third Generation Partnership Project
ITU	International Telecommunication Union
MAC	Multiple Access
ATIS	Alliance Telecommunication Industry Solutions
ETSI	European Technical Standard Institute
MIMO	Multiple Input Multiple Output
AF	Amplify and Forward
DF	Decode and Forward
FF	Filter and Forward
CF	Compress and Forward
SR	Selective Relaying
TDBC	Time Division Broadcast
MABC	Multiple Access Broadcast
FCC	Federal Communication Commission
QoS	Quality of Service
AWGN	Additive White Gaussian Noise
CSI	Channel State Information
OFDMA	Orthogonal Frequency Division Multiple Access
PF	Proportional Fairness

AP	Access Point
SNR	Signal to Noise Ratio
SINR	Signal to Noise plus Interference Ratio
PU	Primary User
SU	Secondary User
JRAPA	Joint Relay Assignment and Power Allocation
MDP	Markov Decision Process
PTRX	Primary Transceivers
STRX	Secondary Transceivers
LP	Linear Programming
KKT	Karush-Kuhn-Tucker

Abstract

Assuming a bidirectional relay assisted network, we first study the problem of optimal resource sharing between two transceiver pairs. One of the pairs, referred to as the primary pair, owns the spectral resources while the other pair, called the secondary pair, is considered to own the relay infrastructure. Assuming amplify-and-forward relaying scheme and aiming to establish a cooperation between the two networks, we study three different design problems in a single carrier scenario. In the first approach we maximize the smaller of the secondary transceiver rates subject to two separate constraints on the total powers consumed in the primary and the secondary networks while providing a minimum data rate to the primary pair. In the second approach, we replace the per network power constraint by a constraint on the average total power consumed in both networks. The third approach combines the two aforementioned methods to materialize spectrum leasing and sharing for the case when the primary network is active with a certain probability. Then we investigate two different design approaches to the multi-carrier scenario. The first approach relies on maximizing the secondary network average sum-rate subject to two spectral power masks for the two networks while providing a minimum sum-rate to the primary pair in a multi-relay scenario. In the second approach, we replace the spectral power mask for each network by a constraint on the total power consumed in that network. Different from the previous studies, we further investigate the resource allocation problem between several energy harvesting relay nodes such that a unidirectional communication link is established between a pair of users and the harvested energy is optimally allocated between the relays such that the overall throughput of the network is maximized.

Assuming the availability of full knowledge of channel state information and that of the energy packets, we maximize the throughput of the network under two sets of constraints on the status of the battery. We then consider the problem of maximizing the average throughput of the system, for the case when only the statistics of the channels are available.

Acknowledgements

I would like to express my gratitude to my supervisor Dr. Shahram Shahbazpanahi for all his support, help, and motivation. I would also like to thank all my colleagues, friends and classmates whose support, reviews, insights, and company will always be remembered.

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Oshawa, Ontario   Adnan Gavili

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Chapter 1

Introduction

1.1 Overview

During the last decade, there has been an astronomical growth in wireless network market. The number of subscribers in all existing wireless technologies e.g., third generation (3G), fourth generation (4G), code division multiple access (CDMA), heterogenous networks (HetNets) and etc., as well as their data-rate demand have massively grown. In order to have a sense of the order of the increase in the wireless network market, we need to consider the number of smart phone devices that use their platform to run social networking applications, e.g., Facebook, Tweeter and etc., geo-location softwares, e.g., google maps, and networked games. Hence, mobile operators and service providers need to grow their networks to address and meet their costumers' demand, while keeping their costs minimum. One of the major aspects of the costs for service providers is energy. The growth in the size of networks and the number of subscribers result in higher amount of consumed energy in such industries. For instance, there existed 4 million base stations (BSs) in 2011, to cover areas where mobile users were connected to the cellular networks. Each BS consumes, on average, 25 Mega Watt hour (MWh) per year. Moreover, the number of BSs in developing

countries were doubled from 2007 to 2012 [4]. This shows how fast the size of the mobile networks and their corresponding consumed energy in such networks increase. Since the energy resources are limited and the trend of the increase in energy consumption becomes drastically important, international community is unified to take action. The reasons behind this action is to preserve the energy resources for future generations and to limit the environmental problems caused by energy usage, e.g., global warming, air pollution, forest destruction, etc. The rising energy costs, carbon footprint and the crisis about how to preserve energy for future generations led to an emerging trend to address the efficient ways to use energy amongst network operators and regulatory bodies, e.g., 3rd Generation Partnership Project (3GPP) and International Telecommunication Union (ITU) [1,2]. This trend has opened up a new direction for researchers called *Green Communications* which proposes an energy-efficient platform for both the user equipment devices as well as the backbone of the network. *Green Communications*, as a new research area, needs to address all issues regarding the protocol layers that are associated with a communication network, e.g., physical layer, multiple access (MAC) layer, network layer and etc. Figs 1.1 and 1.2, show the power consumption in different key parts of a typical cellular network and provide an insight into possible research areas to reduce the energy, or equivalently power consumption of the network. There are four key trade-offs in a design considering energy efficiency and performance of the network, namely deployment costs, bandwidth utilization, achievable rates, and end-to-end average delay. Among all promising energy aware technologies, e.g., efficient BS design, self-organizing network, opportunistic network access or cognitive radios, cooperative relay networks and HetNets, *cognitive radios* and *cooperative relay networks* have gain significant
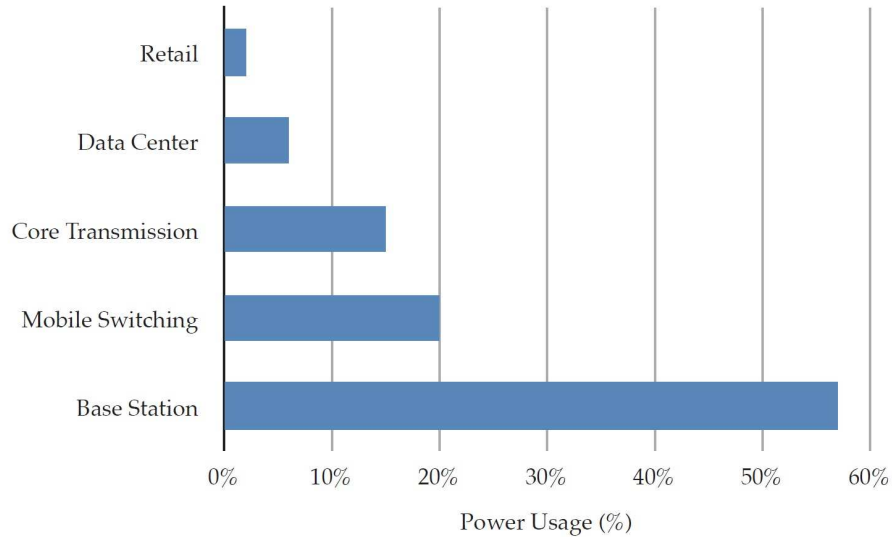
Figure 1.1: Power consumption of a typical wireless cellular network [1].

attentions among researchers. For instance, in most indoor or outdoor communications, where there is an obstacle between the BS and the mobile user, the signal can merely penetrate the obstacle and the quality of the signal at the end user will be degraded drastically. The conventional solution to such a problem is to increase the BS power in order to let the signal penetrate the obstacle and carry the information to the end user. However, it can be easily seen that this is a waste of energy at the BS. A well-known promising energy-efficient approach to tackle this problem is to use distributed wireless nodes as *relays* to provide an indirect link between the BS and the mobile user with overall lower power consumption and more reliable communication. We should note that many power and energy concerns about *cooperative relay networks* are still unanswered, and it needs more attentions to be addressed.
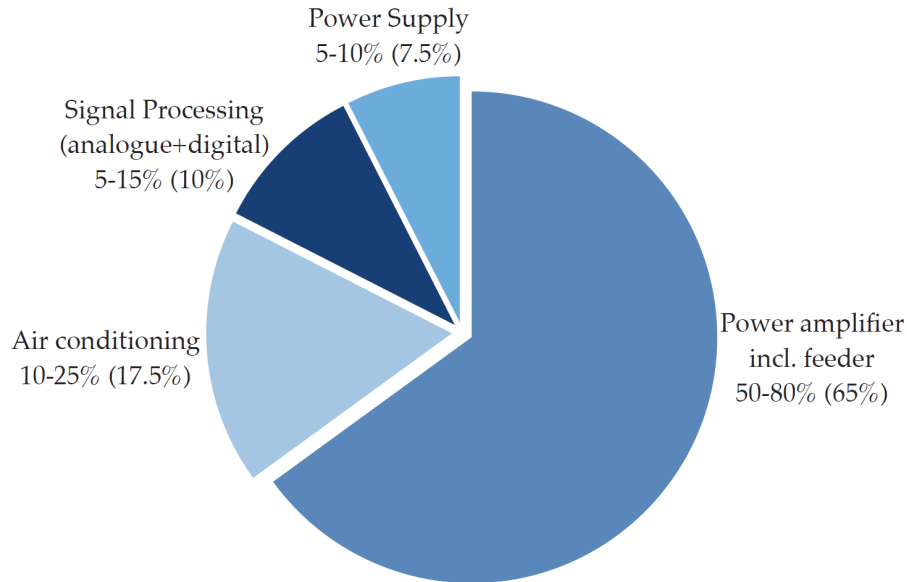
Figure 1.2: Power consumption distribution in radio base stations [2].

1.2   Measuring Greenness

One may ask *"what is the definition of green networks?"*. How can we measure the *"greenness"* of a network. The natural definition for *"greenness"* is to measure the amount of greenhouse gasses that is released into air by a specific technology [5]. However, as the amount of greenhouse gasses that are released by telecommunication technologies is less than 1% which can be assumed to be negligible, one may consider other factors in the definition, e.g., lower energy costs, increased battery life time for equipments, replacing conventional energy resources with renewable energies such as wind, solar and etc. Hence, it makes more sense if we use energy and power savings and performance of wireless networks as the measure of *"greenness"*. Standard organizations such as Alliance Telecommunications Industry Solutions (ATIS) and European Technical Standard Institute (ETSI) classified the energy efficiency of

communication networks in the following categories: *facility level* such as data centers and high level systems, *equipment level* such as cell phones and *network level* such as properties that assesses related issues such as capacity and coverage of the network [6, 7]. A common factor that appears in all of the above categories more frequently, $\frac{\text{Watt}}{\text{date-rate}}$. This factor can be interpreted in two different view points. In the first view point, one may fix the data-rate and minimize the power consumed in the network. The second view point suggests to fix the total power consumed in the network and then maximizes the data-rate. One may see these two approaches to be equivalent. The latter approach is mainly used in this dissertation as the benchmark for comparison between different design problems.

1.3   Cooperative Relay-Assisted Communications

The ever growing demand for wireless connectivity, anywhere, and providing such a service with better quality to the costumers have been the challenges for operators since the last decade. For instance, increasing demands from users that have smart phones with built-in applications such as Facebook, Youtube and etc, need huge amounts of resources to be assigned to the users of such technologies while the users of previous technologies are still needed to be serviced. As deploying more infrastructures to increase the resources that are needed in a network is expensive and sometimes impossible (due to limited spectral, temporal and technological resources), the operators need to redesign their networks in a way that the available resources are used more efficiently. Note that, the link between a transmitter and a receiver in wireless systems, called channel, has random quality. This means that the channel specifications change during time. This makes the communication link between the

the transmitter and the receiver less reliable since the channel quality can be degraded drastically over some transmission time intervals. To tackle this problem, different techniques such as time, space, frequency diversity have been proposed in the last decades. These techniques basically try to send a copy of the transmitter's message signal at different time slots, in different frequency bands, or through different spatial routes to the destination, respectively. It is less likely that all the copies of the transmitted signal that goes through different resources faces bad quality, hence the probability of successful delivery of the transmitted message becomes higher. Equivalently, the reliability of the link between the transmitter and the receiver becomes higher.

A more recent way to achieve spatial diversity is to use multiple-input-multiple-output (MIMO) technique where the transmitter and the receiver are equipped with multiple antennas. However, as the size and the cost of wireless modules are the main limits in designing mobile nodes, e.g., in sensor networks or in cellular phones, placing many antennas on a single device may not be practical. More recently, *Cooperative Relay-Assisted Communications* has opened up an opportunity to form a distributed antenna system. Such a distributed system benefits from the spatial diversity. In the simplest form of such systems, there is a pair of transmitter and receiver that are assisted with multiple distributed relays. The transmitter broadcasts its message signal to the relays. The relays receive a copy of the signal transmitter by the transmitted and rebroadcast it to the destination.

1.3.1 Unidirectional Communication

Unidirectional communications is a communication framework where the flow of information is from a specific node, called source, to another node (multiple nodes),
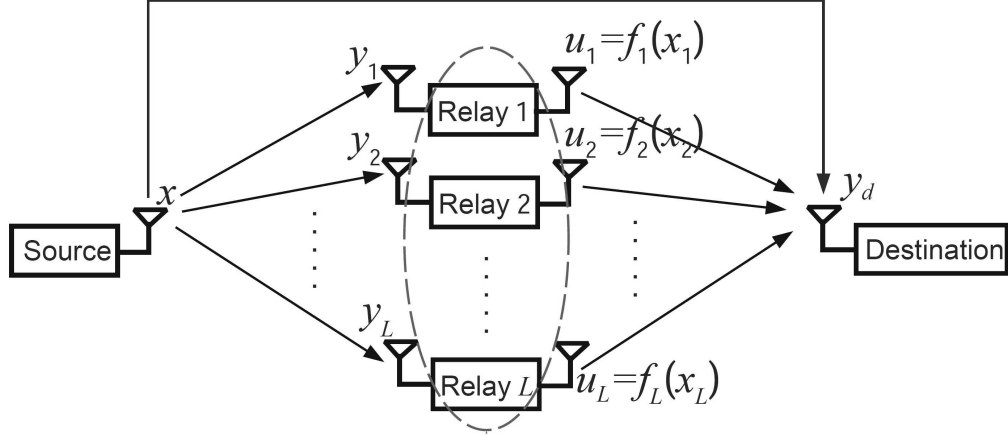
Figure 1.3: Unidirectional relay-assisted communications [3].

called destination (destinations). In other words, only the source can send the information symbols to the destination/destinations. In unidirectional relay-assisted communications, the source broadcast its message signal to the relays in a time slot. In the next time slot, the relays broadcast a function of the received signal, in the previous time slot, to the transceivers. Fig 1.3 shows the underlying communication framework [3].

Many cooperative relay-assisted strategies have been proposed in the literature based on different relaying protocols such as amplify-and-forward (AF), decode-and-forward (DF), selective-relaying (SR), filter-and-forward (FF) and compress-and-forward (CF). In AF relaying scheme, the relays simply scale the signal received from the transmitter by a factor and rebroadcast the so-obtained signal to the receiver. In DF relaying scheme, the relays decode the signal received from the transmitter and they broadcast the decoded signal to the receiver. Note that in this scheme there may occur error in decoding the transmitter signal. SR scheme selects a subset of

the distributed relays to establish the cooperation between the transmitter and the receiver. In FF method, each relay node is equipped with a finite impulse response filter that is used to *equalize* the channel between the transmitter and that relay as well as the channel between the relay and the receiver. In CF protocol, each relay quantizes the signal it receives from the source and encodes the samples into a new packet and rebroadcast it to the destination. In this scheme, the receiver can combine the two observations, the one that is directly delivered by the source and the other one that is delivered by the relays. More precisely, the relay employs source coding with side information at the destination.

1.3.2   Bidirectional Communication

The concept of bidirectional communication was proposed by Claude Shannon in 1961 where two transceivers are willing to communicate with each other in both directions at the same time [8]. In a bidirectional relay-assisted communication, two transceivers aim to establish a communication link to exchange information through the help of one or multiple relays. To realize simultaneous communication between two transceivers, three types of bidirectional relaying protocols have been proposed in the literature. The first and simplest approach is to consider four equal-length temporally orthogonal time slots, where each pair of these time slots are considered to establish a unidirectional communication, thereby allowing each user to convey its information symbols to the other user in its corresponding pair of time slots. The second approach is the so-called time division broadcast (TDBC) scheme, where three time slots are required to exchange two information symbols between the two transceivers. In the first and the second time slots, the two transceivers transmit their information symbols to the relays in their assigned time slots, and then, the relays

broadcast the processed version of the received signals to both transceivers. The last and the most bandwidth-efficient approach is the multiple access broadcast (MABC) bi-directional relaying scheme, where only two time slots are needed to establish a two-way communication between the two transceivers. In the first time slot, the two transceivers broadcast their information symbols to the relays, simultaneously. The relays then rebroadcast a modified version of the receives signals from the two transceivers over the next time frame. In next section, we describe different system designs which benefit from cooperation between the nodes in a network while assuring the *greenness* of the networks.

1.4 Cooperative Energy Efficient Technologies

Recently, research on energy efficient wireless networks gains a lot of attentions. Among all such research efforts, *cooperative relay networks* and *cognitive radio networks* are of significant importance since they employ the concept of *green communications* in designing new wireless technologies while implementing intelligent structure in their design. Beside energy efficiency, cognitive radios enable us to utilize the radio spectrum in a more efficient manner. Cooperative relays and cognitive radios can hence provide significant improvements in throughput, coverage and reliability of future wireless networks.

1.4.1 Green Communication via Cognitive Radio

Efficient bandwidth allocation has been always a crucial concern in wireless communications. Numerous studies have thus focused on this problem in the last decade [9–15].

However, most of these studies have not considered directly the power/energy efficiency in their design approaches. Based on the report released by Federal Communication Commission (FCC) in 2002, it has been realized that the conventional spectrum allocation is highly under-utilized [16]. Cognitive radio, as an enabling technology, appears to be the most efficient concept which aims to find the licensed spectral resources that are under-utilized and assign them to unlicensed users, intelligently [17]. The question is *why utilizing spectral resources efficiently is important and how it can reduce power consumption?* The answer lies under the work of Shannon on the tradeoff between the bandwidth and power. The capacity of a Gaussian channel increases linearly with bandwidth, but only logarithmically with power. This means that in order to reduce power, one should seek for more bandwidth [18]. Since the spectral resources are limited, allocating the spectral resources optimally and adaptively is the only choice to increase the capacity with a given power consumption limit. While this concept is the building block of cognitive radios, every possible parameter measurable by a wireless node or network is taken into account in the general definition of cognitive radios so that the network intelligently modifies its functionality to meet a certain objective [19]. One of these objectives can be power saving. It has been shown in recent works that the concept of cognitive radio can reduce the energy consumption, while maintaining the required quality-of-service (QoS), under various channel conditions [20].

1.4.2 Relay Cooperation to Deliver Green Communications

In infrastructure-based wireless networks, expanding the coverage of a BS to larger areas is a vital issue. Considering the channels random nature and their properties such as *small scale/large scale fading*, *path losses* and *shadowing effects*, providing

coverage for distant users via direct transmission becomes very expensive in terms of required power and energy consumption. Another drawback of expanding the coverage of a BS by increasing its transmission power, is high levels of interference to nearby users and BSs. Recently, cooperative communication techniques as an enabling technology for distributed MIMO systems in order to extend the coverage, capacity and reliability of a communication channel, have been considered in different studies [21]. Cooperative techniques also combat shadowing by covering *spatial holes.* This means that when there is no direct link between the transmitter and the receiver, the relay will establish another link to let the transmitter send its information to the receiver. Moreover, relaying techniques have been proposed to extend the battery life of user devices, which is the first step towards *green* wireless networks. The authors in [22] showed that relay assisted communication consumes less energy than direct communication. Delivering *green communication* via cooperative techniques can be achieved by two different approaches. One is to deploy fixed relays in different locations in the network to help users have access to the network constantly which results in less power consumption. The other technique is to exploit the users in the network as relays.

Fixed Relays

The path loss of a channel, as a known characteristic of the channel, is an interesting property since the transmitted signal from a BS fades away in a specific distance from the BS. This property makes the spectral resources reusable in different locations in a network. This leads to having the chance to deploy more BSs with lower transmitting powers as well as cover larger areas. The authors of [23] show that in a specific scenario in additive white Gaussian noise (AWGN) where the channel path loss exponent is

4, the number of BSs can be increased by a factor of 1.5 in the same area of the network while reducing the transmitting power of the BSs by a factor of 5. Indeed, by increasing the density of BSs, the overall energy consumption of the network becomes smaller. In fact, these features of fixed distributed relay network make it a good candidate for delivering green communication.

User Cooperation

User cooperation as a well-known promising technique to increases the overall data-rate and robustness of a network, i.e., the achievable rates are less sensitive to channel variations, is introduced in [24]. In [25], a game-theoretic approach is proposed to offer to each user an incentive to act as relays when they are idle, and it is shown that user cooperation has the potential of simultaneously improving users bits-per-energy efficiency under different channel conditions. Hence, this new approach can be a promising technique to increase the system performance in terms of energy efficiency in future wireless mobile networks.

1.5   Motivation and Problem Statement

Energy efficiency and costs of implementation have always been the challenges in deploying wireless networks. Although deploying more BSs in an area results in providing services for more users and can lead to consume less amount of energy in the whole network, it can be costly for the network operators, as deploying each BS is expensive. Moreover, it may lead to a more complex network structure such that the overhead of the interaction among the BSs can be significant. Efficiently allocating the available spectral resources such that more users can have access to the network is another solution to improve energy efficiency. Such an approach has been

considered in cognitive radio networks in the last few years and promising results have been obtained. Another possible solution, that is yet to be studied, is to deploy a number of distributed low power relays, instead of deploying a network of BSs which are expensive and consume more power than a similar relay network, and design the modified relay-assisted network to obtain an efficient spectral, temporal and power (or energy) allocation strategy. In this dissertation, we consider the problem of spectrum leasing and resource sharing between two bidirectional relay networks in both single- and multi-carrier schemes. Our motivation in considering this problem is to enable two networks to cooperate, thereby allowing the primary network (i.e., the network which owns the spectral resources) to extend its coverage and/or to consume less power. In return, the secondary users exploit the spectral resources of the primary network to serve its transceivers. We further study the problem of resource allocation for an energy harvesting cooperative relay network under different assumptions on the availability of channel state information (CSI) and on the knowledge of harvested energies. Our motivation in considering this problem is to enable a self-power supplied relay network to extend the range of a unidirectional communication between a pair of users and/or to increase the throughput of the network. More specifically, the problems that are studied in this dissertation are summarized below.

1. In the first and the second proposed approaches, we consider a secondary network and a primary network in a single-and a multi-carrier scenarios. The secondary network can help the users in the primary network to achieve higher data-rates by encouraging them to use the relay infrastructure, communicate with each other. In exchange for this cooperation, the secondary network will have the permission to efficiently utilize the spectral and temporal resources

of the primary network for its own users. Moreover, considering different constraints on the power consumption of different parts of each network (which is a measure of *greenness*), we aim to design the parameters of the secondary network as well as those of the primary network such that the users in the primary network are guaranteed to have a minimum achievable data-rate. Furthermore, the users in the secondary network can achieve maximum achievable data-rate as a reward for helping the users in the primary networks. We study these approaches for different system setups and, for each case, provide simple solutions to design the parameters of the two networks.

2. The third proposed approach is to consider two users in a primary network that are willing to communicate with each other using a distributed relay network. We assume that the relays are equipped with energy harvesting modules such as solar panels, wind turbines and etc. Hence, the relays can harvest and store energy from different sources in a specified battery for future use. Then we propose an energy-harvesting-power-efficient design approach such that the relays can maximize the throughput of the network. We note that this approach, considers the concept of harvesting renewable energies and designing a power-efficient network which both lie under the umbrella of *green wireless networks*.

1.6 Methodology

For both single-and multi-carrier bidirectional communication schemes, we develop two relay-assisted system models to tackle the problem of *inefficient resource allocation* in conventional wireless networks by employing the concept of *cooperative energy/power aware design in the next generation networks* in our design approaches.

We also develop a different system model for a single-carrier unidirectional relay-assisted network where the relays are equipped with energy harvesting modules to tackle the problem of optimal power allocation to the relays for different scenarios.

In our first design approach, we study and model the problem of resource sharing between two pairs of users, namely the primary pair and secondary pair, in two bidirectional relay-assisted networks in a single-carrier scenario. We formulate the corresponding resource sharing problems as the maximization of the smaller of the data-rate of the two secondary transceivers under individual per network power constraints or under a total power constraint to address power efficiency, while the smaller of the rates of the primary transceivers is guaranteed to be above a given threshold. Aiming to optimally calculate the optimal beamforming coefficients of each relay and their corresponding consumed power, we obtain a semi-closed form solution for the case of per network power constraints. A simple line search solution is also proposed for the case when there is a constraint on the total power consumed by the two networks. We further incorporate the concept of spectrum sensing in order to allocate the available spectral resources more efficiently.

In the second design approach, we study and model the problem of resource sharing between two pairs of transceivers, a primary pair and a secondary pair, in a bidirectional multi-carrier relay-assisted scenario. Aiming to optimally calculate the optimal beamforming coefficients of each relay over different subcarriers and their corresponding consumed power, we maximize the average sum-rate of the two secondary transceivers under per-network power spectral masks or under a per-network total power constraint. In the case of per-network power spectral masks, we simplify the optimization problem into a linear programming problem where the solution to that

problem can be easily obtained using convex optimization solvers. We also propose two alternating convex search solutions for the case where there exists a per-network total power constraint, for multi-relay and multi-relay scenarios. We emphasize that this is not a simple generalization of the single-carrier scenario and more challenges appear since the spectral resources are also needed to be allocated between the users in each network efficiently.

Finally, we study the resource allocation in a unidirectional relay-assisted network where there exists one pair of users that are willing to communicate with each other through the help of the relays. Assuming that the relays are equipped with energy harvesting modules, we formulate the problem of maximizing the data-rate of the two users over a specific number of time frames while optimally distributing the harvested energy among the relays. We prove that such an optimization problem is convex with respect to the total power consumed by all relays over each time frame. Assuming the knowledge of full CSI, we obtain the optimal powers that are to be allocated to the relays. We further solve the problem of maximizing the average data-rate of the two users for the case where the statistics of the channels are known. Last but not least, we obtain the total power that are to be allocated to the relays for the case where there is a temporal correlation between the coefficients of a channel using the proposed adaptive channel estimation and power allocation algorithm.

1.7 Contributions

We now summarize our contributions to the stated problems.

1. Assuming a bidirectional relay-assisted communication scheme, we develop a data model for optimal spectrum leasing and resource sharing between two

pairs of transceivers with different priorities, in a single-carrier scenario. To the best of our knowledge, this is the first attempt to consider such a scenario.

2. We obtain the optimal relay beamforming coefficients and the portion of the time that are to be assigned to each pair such that the transceivers in each network can simultaneously communicate with their peer user.

3. We further generalize the aforementioned data model for the multi-carrier scenario, under different sets of constraints, in order to obtain the optimal relay beamforming coefficients and the portions of the time, over different subcarrier, that are to be assigned to each pair.

4. Assuming a unidirectional communication scheme, we also develop a new model to consider the problem of optimal power allocation among a number of relay assisting nodes such that they can establish a reliable link between a pair of users for the case where the relays are equipped with energy harvesting modules.

1.8   Publications

The results of these studies have been published/submitted in several prestigious journals and conferences as we summarize below.

1. A. Gavili and S. Shahbaz Panahi, "Optimal Spectrum Leasing and Resource Sharing in Two-Way Relay Networks," *IEEE Trans. Signal Process.*, vol.62, Oct., 2014.

2. A. Gavili and S. Shahbaz Panahi, "Optimal Resource Sharing and Network Beamforming in Multi-Carrier Bidirectional Relay Networks," accepted for publication, *IEEE Trans. Signal Process.*, July 1st, 2015.

3. A. Gavili and S. Shahbaz Panahi, "Optimal Power Allocation and Network Beamforming in Collaborative Relay Networks With Centralized Energy Harvesting," to be submitted to, *IEEE Trans. Signal Process.*, July, 2015.

4. A. Gavili and S. Shahbaz Panahi, "Optimal Resource Sharing and Network Beamforming for Bidirectional Relay Networks," *International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, vol.62, Jun., 2014.

5. A. Gavili and S. Shahbaz Panahi, "Optimal Spectrum Leasing and Network Beamforming for Two-Way Relay Networks," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May, 2014.

1.9 Outline of The Dissertation

In this dissertation, we focus on the optimal resource allocation and the *greenness* of the proposed networks. The remainder of this dissertation is as follows: In Chapter 2, we provide a review on the literature of optimal resource allocation strategies in relay-assisted networks and *green* wireless networks. In Chapter 3, we study the problem of resource sharing between two bidirectional relay networks in a single-carrier scenario. For such a network model, we propose a semi-closed form solution in the case of individual per network power budget. For the case of imposing a constraint on the total power consumption, we propose a simple line-search solution with low complexity. We also incorporate the spectrum sensing concept in the case of total power constraint and show that how the overall rate of the secondary network would be increased. In Chapter 4, we study the problem of resource sharing between two bidirectional relay networks in a multi-carrier scenario. We propose a linear

programming solution in the case of imposing individual power spectral masks on each network. We further study the case where there are two power constraints on the total power consumed in each network. We show that this problem is not convex and may not be amenable to a low computationally complex solution. We also propose sub-optimal solutions for single-relay and multi-relay scenarios and provide two algorithms to obtain the parameters of the two networks. Chapter 5 is dedicated to the study on the optimal power allocation to a set of relay nodes, that are connected to a single energy harvesting battery module, to assist a pair of users to establish a communication link between them. We formulate the problem such that the overall throughput of the users is maximized under several constraints on the status of the battery and the assumption that full knowledge of the channel state information for all links are available. We prove that such a problem is a convex optimization problem with respect to the total power consumption of the relay nodes over each time frame. We further extend the study to consider the cases where partial information on the channel state information is known. In Chapters 6, we present the concluding remarks as well as the possible research directions.

1.10   Notation

We use $E\{\cdot\}$ to represent the statistical expectation. Matrices and vectors are represented by uppercase and lowercase boldface letters, respectively. Transpose and Hermitian (conjugate) transpose operations are represented by $(\cdot)^T$ and $(\cdot)^H$, respectively. The notation $\mathbf{I}$ stands for the identity matrix, diag($\mathbf{a}$) denotes a diagonal matrix with the elements of the vector $\mathbf{a}$ as its diagonal entries, and $\odot$ represents Schur-Hadamard (element-wise) vector product.

Chapter 2

Literature Review

In this chapter, the recent studies in relay-assisted and energy efficient wireless networks are discussed. Throughout this review, we concentrate on similar research results regarding resource sharing/leasing/allocation in conventional and cooperative cognitive relay-assisted networks, unidirectional and bidirectional communications in distributed relay networks and finally, designing green communications in relay-assisted networks. In our design proposal we assume that all channel state information (CSI) for all links are known perfectly[1], unless otherwise stated. Cooperative communication schemes have been proposed extensively in literature [26–28]. In some studies, the authors assume that the CSI in their design approaches is not known for any nodes in the network [29]. However, in some other studies it is assumed that the CSI is known only at the receiver. For instance, the authors of [30] and [31] studied non-coherent amplify-and-forward relaying scheme and distributed space-time coding, respectively. Finally, perfect knowledge of CSI into account in some studies is taken, such as the decode-and-forward in [30] and [32] and the coded cooperation in [28]. We will consider perfect channel state information knowledge in our proposed network

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[1]Channel estimation and estimation error are out of the scope of this proposal

models, unless otherwise stated, and discuss why this assumption is essential.

2.1 Resource Allocation in Relay Networks

Relay-assisted networks, as a promising technology to increase the bandwidth efficiency, reliability and decrease the total consumed power in the whole network, has gained significant attention in recent years. Allocating the available resources in such networks to different users is of interest for many researchers. Note that the available resources, that are needed to be allocated optimally in such networks, are spectral resources such as RF spectrum, temporal resources such as dedicated time slots to users to transmit their information symbols, spatial resources such as utilizing similar radio bands in different locations, physical resources such as relay infrastructures, and energy resources such as energy harvested from fuels or renewable energies resources. To address how a network can allocate these resources optimally, many studies have been done in the literature. In the following subsections, we summarize the results of such studies.

2.1.1 Conventional Relay-Assisted Networks

In conventional networks, the users in each network are authorized by the network operator to use its infrastructure in order to communicate with other users in many forms such as phone call, internet access and etc. In other words, the users are promised to have some services with different qualities anytime they need the services. Deploying relay-assisted wireless nodes in conventional networks is a new key idea that has attracted many attention in past few decades to extend the coverage, quality of

service and the reliability of the communication links as well as increase the number of users in such networks. Deploying a large number relays in a conventional network may cause more power consumption in the whole network, however, one can optimize the new structure of the network such that the power consumption of the whole network is minimized. In the following, we address the recent works on this topic.

Single Relay

In [33], the authors consider a single-relay AF scenario where several pairs of users wish to communicate with each other. The authors use game theory to analyze the relays power allocation among the signals of each user. The interaction among the users is modeled as a bargaining problem, where users negotiate with each other to set the relay powers. This problem has been solved for centralized and distributed cases. Also, a generalization of the proposed game-theoretic-based power allocation scheme and its distributed implementation has also been proposed to address the multi-user multi-relay scenario. The authors of [34] consider a multi-user scenario and design the joint power allocation, subcarrier allocation and coupling for orthogonal frequency-division multiple access (OFDMA) based uplink transmission using AF relaying scheme. The authors formulate sum-rate maximization problem and solve the problem in two steps. As the first step, a subcarrier allocation and coupling problem with a given power allocation for allocated subcarriers is considered. Then, the optimal power allocation problem is solved using a dual decomposition technique. Addressing subchannel allocation/pairing as well as joint source-relay power allocation, the authors of [35] propose an optimal adaptive resource allocation scheme for the

OFDM based multi-destination regenerative relay system. A more practical down-link OFDM-based cooperative network is considered in this study, where the relay is used to help the source (base station) to communicate with multiple destinations (users) rather than a single destination. In [36], the authors consider a cellular based multihop OFDMA broadcast network, where a source transmits signals to multiple destinations with the help of an AF relay node. The so-called proportional fairness (PF) concept is adopted in this paper to address the transmission rate fairness among the destinations. Considering PF scheme, subcarrier allocation problem is formulated as a mixed integer programming problem. The article [37] proposes a symmetric system model consisting of two user nodes and an access point (AP). It assumes that each user can act as a source as well as a potential relay. In this model, each user has the opportunity to share its resources (e.g., bandwidth and power) with other users and seek the other users' cooperation to relay its data to a specific destination. This scheme achieves cooperative diversity. The degree of cooperation depends on how much bandwidth the relay node is willing to contribute to the source to transmit its data. It is proved that such a problem can be modeled as a two-person bargaining problem. The authors then propose a cooperative Nash bargaining solution (NBS), in which if a certain condition is satisfied, users will cooperatively work, and each will share a certain fraction of its bandwidth for data relaying; otherwise, they will choose to independently operate. In [38], the authors consider power allocation problem in an AF relaying scheme and employ the concept of buyer and seller to jointly realize the benefit of source and relay node in a symmetric bargaining problem. In this two-person bargaining game, the source acts as a buyer who wants to buy data-rate from the seller, which is the relay, at some fixed price. Throughout this game, the net

utility of the source is the data-rate obtained by maximal ratio combining technique minus the data-rate that it buys from the relay. Such a problem has been proved to have a unique and Pareto optimal solution. In this paper, the authors consider one relay to cooperate with the source depending on the offered price and channel quality.

Multi-Relay

The authors of [39] consider the optimal power allocation between a source and multiple distributed relays. The authors aim to maximize the end-to-end achievable rate of such cooperative scenario using AF relaying protocol. This study considers the structural properties of the optimal power allocation in MIMO cooperative networks with per-node power constraints. In [40], a distributed ascending-clock auction-based algorithm is proposed for multi-source power allocation through the help of several cooperative relay nodes. More specifically, each source node sends its optimal power demand to each relay node in response to the prices announced by the relay nodes. It is proven that the proposed distributed algorithm enforces truthful power demands and converges quickly to the unique Walrasian Equilibrium (WE) allocation that maximizes the social welfare. In addition, the proposed algorithm is shown to maximize transmitters' sum-rate which coincides with centralized power allocation based on convex optimization problems. Article [41] studies joint bandwidth and power allocation for wireless multi-user networks with and without relaying. The joint bandwidth and power allocation is proposed to address three different design approaches, such as maximizing the sum-rate of all users, max-min rate balancing approach and minimizing the total power consumed by all users. The corresponding joint bandwidth

and power allocation problems are formulated and it is easy to show that such optimization problems are convex and that there are several efficient algorithm to solve convex problems. In [42], the problem of jointly optimizing the source power allocation and relay beamforming to improve the overall network performance. Two design problems are considered in this paper. The first problem is to maximize the minimum SINR among all users (max-min SINR balancing problem) subject to the constraints on individual and total transmitted powers. The second problem is to minimize the transmission power consumed by all sources and relays while guaranteeing SINR requirements of all users. In [43], the authors consider a relay-assisted network which consists of two single-antenna transceivers and several single-antenna relay nodes. The authors aim to minimize the total consumed power subject to two constraints on the transceivers' received signal-to-noise ratios (SNRs). In the second approach, they propose an SNR balancing technique where the smaller of the two transceiver SNRs is maximized while the total consumed power is kept below a certain power budget. The achievable rate-region under joint distributed beamforming and power allocation is studied in [44]. The authors obtain the achievable beamforming rate region for a two-way communication network consisting of two transceivers and several distributed relays. Assuming that the relay beamforming weights as well as the transceiver transmit powers are the design parameters, this region is characterized under a constraint on the total (network) transmit power consumption. Then, a sum-rate maximization approach to obtain jointly optimal relay beamforming weights and transceiver transmit powers is proposed. Using joint optimal power control and beamforming design, the authors of [45] study and compare the performance of two well-known

bidirectional network beamforming schemes, e.g., the multiple access broadcast channel (MABC) and the time division broadcast channel (TDBC) protocol. The authors design such a network through minimizing the total power consumed in the whole network subject to quality of service (QoS) constraints such as SNR and rate, for the two cases with and without a direct link between the two transceivers in TDBC model. The corresponding power minimization problems are carried out over the transceiver transmit powers as well as relay beamforming weights, thus resulting in a jointly optimal power allocation and beamforming criterion.

2.1.2 Cognitive Relay-Assisted Networks

Rapid growth of the number of users in wireless networks and limited spectral resources, inspired researchers to revise the conventional fixed resource allocation scheme into a more dynamic ones. Since the spectral resources in conventional networks are not allocated efficiently, thus many studies have been conducted to find vacant spectra in different time instances and locations, and assign them to new users that need to have access to the network. The users that have the licence to use the spectrum and they do not use their spectral resources all the time are called *primary users* (PU), while the new users that aim to utilize the spectral opportunities that are vacant are called *secondary users* (SU). The cognitive radio technology enables the users to [46]:

- Determine which portions of the spectrum are available and detect the presence of licensed users when a user operates in a licensed band (spectrum sensing).

- Select the best available channel (spectrum management).

- Coordinate access to this channel with other users (spectrum sharing).

- Vacate the channel when a licensed user is detected (spectrum mobility).

Among all, three different approaches have been studied recently. The first approach is *opportunistic spectrum access*, where the SUs search for spectral resources of the PUs in time and location, and whenever they find the spectrum vacant, they can start their communication with each other. However, if the PUs want to use their spectral resources, the SUs must vacate the spectrum that they use and try to find another opportunity to communicate. The second approach is *spectrum sharing*, where there is a cooperation between the PUs and SUs, meaning that the PUs allow the SUs to use their spectral resources for a specific amount of time. In exchange for this cooperation, the PUs have the authority to use the SUs as a relay to help them if they need to communicate with each other. The last approach is *spectrum leasing*, meaning that the PUs lease a part of their resources to the SUs for a fixed cost per use of resources. In what follows, we discuss relevant recent studies conducted on relay-assisted cognitive radio networks.

Resource Allocation

In [47], the authors propose the optimal power allocation between the users in a cognitive full-duplex relay assisted network. The optimal power allocation has been carried out to minimize the outage probability of the SUs, and then derive the outage probabilities of the SU in the noise- and interference-limited environments. Adding to this, they also propose an outage-constrained power allocation scheme to reduce the overhead of the feedback in the network. This means that the instantaneous channel state information (CSI) for the link between the PUs and SUs is not needed to be

known in advance. The authors of [48] study the problem of power allocation considering interference constraint for OFDM based cognitive relay network. The design problem in this paper is to maximize the average data rate for the SUs considering power constraint on all subcarriers. The conventional solution for such a set up is the so-called *water-filling* scheme where more power is allocated to the subcarrier with the best channel quality and less power is allocated to the subcarrier with worst channel quality. However, allocating more power to the best subcarriers may increase interference to the PUs in that subcarrier. Thus the power allocation problem is two fold: reduce interference to the PU as well as increase the data transmission rate of SU links. The authors in [49] consider a multiple relay scheme for MIMO two-way relay cognitive radio networks with AF strategy. The authors have formulated an optimization problem to maximize the secondary network sum-rate by taking into account the power budget of the system and the interference level tolerated by the PUs. They derive the optimal solution expression of the power allocation problem. Note that there are two general paradigms where the primary and secondary users coexist, namely the *underlay* and *overlay* paradigms. The underlay paradigm mandate that concurrent primary and secondary transmissions may occur only if the interference generated by the secondary transmitters at the primary receivers is below some acceptable threshold. Rather than determining the exact interference it causes, a secondary user can spread its signal over a very wide bandwidth such that the interference power spectral density is below the noise floor at any primary user location. These spread signals are then despread at each of their intended secondary receivers. The premise for overlay systems is that the secondary transmitter has knowledge of the primary users transmitted data sequence. Knowledge of a primary users data

sequence and/or codebook can be exploited in a variety of ways to either cancel or mitigate the interference seen at the secondary and primary receivers. The problem of spectrum allocation problem has been considered in [50]. This paper considers a cognitive relay network, in which multiple pairs of SUs share the unlicensed spectrum among themselves using overlay transmission mode and for each SU pair, the two ends exchange data via a common relay node that senses the spectrum and allocates the unused bandwidth. A multi-object auction problem has been formulated to allocate the spectrum between the users and a mechanism of sequential first price auction and sequential second price auction has been introduced to solve such problem. For each mechanism, the authors obtained the optimal bidding strategy and analyzed it. In [51], the authors analyze the performance of cooperative spectrum sharing in a single-carrier relay-assisted system using DF relaying protocol. Two relay selection schemes, namely a full CSI-based best relay selection and a partial CSI-based best relay selection, are proposed under two constraints on the users in the network, e.g., the peak interference power at the primary user and the maximum transmit power at the secondary user. The authors of [52] present an optimal power allocation scheme and relay selection strategy in a relay-assisted cognitive radio network where a pair of SUs communicate with each other with the help of the relays. The SUs are assumed to share the spectrum with a PU and each node is assumed to be equipped with a single transmit/receive antenna. Using an interference limited approach, they consider joint relay selection and optimal power allocation among the SUs achieving maximum throughput under transmit power and PU interference constraints. The problem of joint multiple-relay-assignment and power-allocation (JRAPA) has been investigated in [53]. The authors formulate a constrained optimization problem for

JRAPA in shared-band AF relaying in a cognitive radio set up. They proved that the proposed optimization problem is a non-convex-mixed-integer nonlinear optimization problem and it is generally NP-hard. An efficient greedy iterative algorithm has been proposed in order to jointly assigns the relays and assign the obtained powers to the SUs while satisfying the interference constraint on the primary network.

Resource Leasing

The problem of spectrum leasing where the SUs are used as relays for the primary users has been studied in [54]. Considering that the primary network is operating at a fixed target rate in a lease contract with the primary network, the problem of minimizing the outage probability of the primary network has been investigated. Meanwhile, a spectrum sharing policy that maximizes the outage capacity of the SUs has been proposed. Article [55] proposes an energy-efficient resource allocation algorithm to minimize the total average transmission power of SUs in a cognitive relay-assisted network. The proposed optimization problem in this work is a joint time slot scheduling, relay selection and power control algorithm to minimize the total average transmission power of SUs, while guaranteeing the minimum rate requirement of PUs and SUs. Authors in [56] studies a spectrum leasing scheme based on one-path alternate relaying and two-path successive relaying that is developed in the multi-user scenario. The primary system has the incentive to lease its licensed spectrum with the SUs. In exchange, the SUs will help the primary network whenever they want. In this paper, the SU that can guarantee the primary rate is selected before the data transmission and the outage probabilities of primary and secondary systems has been analyzed. Reference [57] studies a cooperation-based spectrum leasing model where

primary users grants some spectral resources to the secondary users in exchange for cooperation. In this study, SUs are assumed to act as relays to help the PUs, then the SUs may be given an access to use the spectrum bands of the PUs to transmit information symbols. Article [58] proposes a framework in which secondary terminals are granted the permission to use a given spectral resource by the incumbent primary users in exchange for cooperation. The rationale is that the primary nodes will be willing to lease their spectral resources for a fraction of time if, in exchange for this concession, they will benefit from SUs infrastructure, thanks to cooperation with the secondary nodes. In turn, the secondary nodes have the choice about whether to cooperate or not with the primary users on the basis of the amount of cooperation required by the primary users and the corresponding fraction of the time leased for secondary transmissions.

2.2 Energy/Power Efficiency in Relay-Assisted Networks

Energy consumption in wireless networks is closely related to their radio resource management schemes [5]. Allocating radio resources optimally and harvesting energy from variety of resources lead to a more energy efficient wireless network [59–65]. In [66], the authors consider the problem of optimal packet scheduling in a single-user energy harvesting wireless communication system, where both the data packets and the harvested energies are modeled to arrive at the source node randomly. The goal of this study is to adaptively allocate the rate of the user according to the traffic load and available energy, such that the time by which all packets are delivered is

minimized. Assuming that the energy arrival times and harvested energy amounts are known a-priori, two different scenarios have been proposed. As the first scenario, it is assumed that all bits at the transmitter are ready for transmission. In the second scenario, the case where packets arrive during the transmissions, with known arrival times and sizes is investigated. The authors of [67] consider a pair of transceivers where the transmitter has the ability to harvest energy from environment. The problem of throughput maximization in a specific amount of time as well as minimization of the time (or delay) by which the transmission of a specific number of bits is completed, is considered in this study. The authors solve the aforementioned problems under deterministic (offline) and stochastic (online) knowledge of energy arrivals and signal packets. The problem of maximizing the throughput of a single user energy harvester node via energy allocation over a finite horizon of time slots has been targeted in [68]. Considering the fact that the channel SNRs and the amount of the harvested energies change over different time slots, the authors study the problem of throughput maximization and the corresponding optimal energy allocation problem, using some properties such as concavity and monotonicity in their design problem. The authors in [69] study two-hop transmission in the case of energy harvesting nodes. The focus center of this study is on a two-hop network composed of an energy-harvesting source, an energy-harvesting relay and a destination. The energy harvesting process at each node is modeled as a packet arrival process, such that each energy packet of a random amount arrives at a random time instant. Their focus is on offline algorithms, that is, the instants and the amounts of random energy packet arrivals are assumed to be known. The problem of maximizing the total amount of data that can be transmitted

to the destination within a specific amount of time is considered in this paper. Article [70] addresses the problem of throughput maximization in an energy-harvesting two-hop amplify-and-forward relay network. The authors obtain optimal policies for transmission powers of the relay and the transmitter for two cases, namely non-causal knowledge of the harvested energy and that of the channel coefficients in a fading environment as well as causal knowledge of such parameters. An effective algorithm has been proposed to solve the power allocation problem in the non-causal (offline) case, while in the causal (online) case, a sub-optimal Markov decision process (MDP) has been applied to such a problem. The authors solve the resulting optimization problem using only causal knowledge of the fading and the harvested energy.

Chapter 3

Optimal Spectrum Leasing and Resource Sharing in Single-Carrier Setup

We study the problem of optimal resource sharing between a high-priority (called primary) transceiver pair and a low-priority (secondary) transceiver pair. The two transceivers in each pair wish to establish a two-way communication using the relay infrastructure. In our study, we assume that the primary pair owns the spectral resources, while the secondary pair owns the relay infrastructure. Moreover, we assume that there is no direct link between the primary transceivers, hence establishing a connection between the primary pair through the relay infrastructure is inevitable. As such, a cooperation scheme between the two networks is needed in order to allow the primary pair exploit the relays to guarantee a connection between its transceivers. In exchange for this cooperation, the primary pair allows the secondary pair to use the primary spectral resources for a specific amount of time. We thereby consider the problem of maximizing the smaller of secondary transceiver average rates, while preserving a minimum rate for the primary pair. Aiming to optimally calculate the

design parameters of the primary and secondary networks, we study three different approaches. In the first approach, we maximize the smaller of the secondary transceiver average rates subject to two separate constraints on the total power consumed in the two networks, while guaranteeing a minimum rate for each of the two primary transceivers. We prove that in this case the original optimization problem splits in two separate sub-problems. We simplify each sub-problem into a one dimensional optimization problem and show that it has a unique solution, which can be obtained in a semi-closed form. In the second approach, we consider a constraint on the total power consumed in both networks. In this case, we simplify the optimization problem to a simple line search with low complexity. In our third approach, we use the results obtained from the second approach to devise spectrum leasing technique for the case when the primary network is not active all the times, i,e., it is active with a certain probability. The contributions of this chapter can be summarized as listed below:

- We study and model the problem of resource sharing between two *bidirectional* relay networks under total and per network transmit power constraints.

- We formulate the corresponding resource sharing problems as the maximization of the smaller data rate of the two secondary transceivers under individual per network power budget or under a total power constraint, while the smaller rate of the primary transceivers is guaranteed to be above a given threshold.

- For such setups, we show how the corresponding resource sharing problems can be simplified such that parameters of each network can be obtained in a distributed manner.

3.1   System Model

We consider a cooperative communication scheme consisting of two transceiver pairs and n_r relay nodes. The two transceivers in each pair wish to exchange information with the help of the relays. The two pairs employ the relay nodes in temporally orthogonal time intervals. One of the transceiver pairs, referred to as the primary network (or primary pairs), owns the spectral resources, and a minimum rate between its transceivers has to be guaranteed. The other transceiver pair is assumed to be the secondary pair, meaning that they are allowed to communicate using the spectral and temporal resources of the primary network in exchange for helping the primary transceivers achieve a required minimum rate. It is assumed that the secondary network owns the relay infrastructure. Therefore, a cooperation scheme should be established between the two networks to guarantee the rate demand of the primary transceivers, while maximizing the date rate between the secondary pairs. In order to establish a cooperation between the primary and the secondary networks, the primary transceiver pair aims to let the secondary transceivers achieve the highest possible rate without violating their own minimum rate constraint. We assume that there is no direct link between the transceivers in each pair and that the only way for them to exchange data is to exploit the relays. Note that even if there is a direct link between the two transceivers, the transceivers cannot benefit from the direct link in an multiple access broadcast channel (MABC) based bidirectional relaying as they cannot receive and transmit at the same time. The only way to exploit such a direct link is to use a TDBC two-way relaying scheme. The time division broadcast channel (TDBC) scheme however requires three time slots to exchange two information symbols between the two transceivers, and as shown in [45], this

scheme can incur significant performance loss, in terms of the data rate, if the direct link is not strong enough. The primary transceivers $PTRX_1$ and $PTRX_2$, transmit their information symbols, s_1 and s_2, with transmit powers p_1 and p_2, respectively. Moreover, the secondary transceivers $STRX_1$ and $STRX_2$, transmit their symbols, s_3 and s_4, with powers p_3 and p_4, respectively. All the transmitted symbols are considered to be zero-mean, independent random variables with average powers equal to 1, i.e., $E\{|s_1|^2\} = E\{|s_2|^2\} = E\{|s_3|^2\} = E\{|s_4|^2\} = 1$.

The problem of joint distributed beamforming and power control for time-asynchronous two-way relay networks has been studied in [71–76]. In these studies, it is assumed that the propagation/relaying delay for each relay could be different from that of the other relays. As a result, the signal arrival time from one transceiver to the other one, corresponding to each relay, could be different from those for the other relays. As shown in these studies, in such time-asynchronous two-way relay networks, the end-to-end channel can be viewed as a multipath channel which produces inter-symbol interference at the two transceivers. Based on this model for a single-carrier two-way relay network, under a total power constraint, a min-max mean squared error (MSE) approach to design jointly optimal transceiver power control, post-channel equalization, and decentralized beamforming leads to a relay section scheme where only relays which correspond to one of the channel taps are turned on and the remainder of the relays are turned off [71, 75]. In such a relaying scheme, the data model is simplified to a time-synchronous two-way relay network, where the active relays induce time-delays which are within one symbol period, and thus, the network can be modeled as the relays are time synchronous. The same result can be achieved if a total MSE minimization approach or a max-min SNR technique is used to design jointly optimal

transceiver power control, post-channel equalization, and decentralized beamforming. In multi-carrier two-way time-asynchronous relay network, a max-min SNR approach to design jointly optimal subcarrier power control at the transceivers and distributed beamforming at the relays results in the very same aforementioned relay selection scheme, rendering the relay network time-synchronous [72,74,76]. Based on the above discussion, we assume that the relays are time-synchronous and this assumption will not result in any loss of generality when applied to time-asynchronous networks..

Furthermore, it is assumed that the channels are frequency flat. The $n_r \times 1$ vector of the complex channel coefficients corresponding to the links between PTRX_1 (PTRX_2) and the relays, is represented as $\mathbf{f}_1$ ($\mathbf{f}_2$). Similarly, the $n_r \times 1$ vector of the complex channel coefficients corresponding to the link between STRX_1 (STRX_2) and the relays, is represented as $\mathbf{g}_1$ ($\mathbf{g}_2$). Moreover, the transmission scheme is considered to be time-slotted. The total available time for both transceiver pairs is referred to as a time frame with T seconds duration. Without loss of generality, we assume that $T = 1$. Each time frame is divided into two time intervals called subframes. The primary transceivers communicate with each other in the first subframe, while the second subframe is considered for communication between the secondary transceivers. The parameter α is the portion of the time that the secondary transceivers communicate with each other using their own relaying infrastructure and the spectral resources of the primary network. The underlying communication scheme is shown in Fig. 3.1. The relays are assumed to operate in a half-duplex mode using an amplify-and-forward (AF) relaying scheme in both subframes. The AF relaying is of particular interest due to its implementation simplicity. The relays use an $n_r \times 1$ complex vector $\mathbf{w}_1$ to implement a network beamformer, thereby enabling communication between PTRX_1
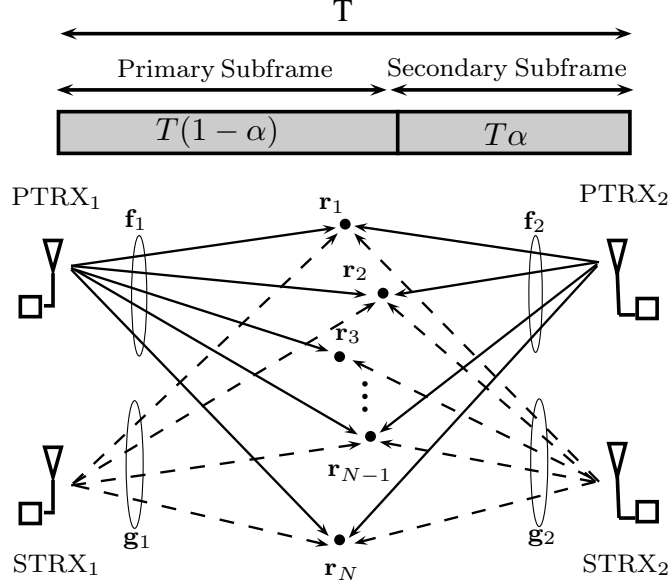
Figure 3.1: Sharing resources between two bidirectional relay networks.

and PTRX$_2$ in the first subframe. The relays also employ a second $n_\mathrm{r} \times 1$ complex vector $\mathbf{w}_2$ to materialize a second network beamformer, thereby establishing a two-way connection between STRX$_1$ and STRX$_2$ in the second subframe. The communication protocol for transmission and reception between the primary (secondary) transceivers and the relays is based on multiple access broadcast channel (MABC). In this protocol, each subframe is divided into two equal-length time-slots. In the first time slot, two transceivers transmit their information symbols simultaneously to the relays. In the next time slot, the relays re-transmit amplified- and phase-adjusted versions of the signals received in the previous time slot to the transceivers. The ith relay uses the ith entry of $\mathbf{w}_1$ to cooperate with other relays in establishing a two-way connection between PTRX$_1$ and PTRX$_2$ and uses the ith entry of $\mathbf{w}_2$ to participate in enabling a two-way connection between STRX$_1$ and STRX$_2$. Considering the MABC protocol,

the vector of signals received at the relays, corresponding to the two subframes, could be respectively written as

$$\mathbf{x}_{\mathrm{p}} = \sqrt{p_1}\,\mathbf{f}_1 s_1 + \sqrt{p_2}\,\mathbf{f}_2 s_2 + \boldsymbol{\nu}_1 \tag{3.1.1}$$

$$\mathbf{x}_{\mathrm{s}} = \sqrt{p_3}\,\mathbf{g}_1 s_3 + \sqrt{p_4}\,\mathbf{g}_2 s_4 + \boldsymbol{\nu}_2 \tag{3.1.2}$$

where the ith entries of $\mathbf{x}_{\mathrm{p}}$ and $\mathbf{x}_{\mathrm{s}}$ are the signals received at the ith relay in the first and second subframes, respectively, while $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are the corresponding relay noise vectors of size $n_{\mathrm{r}} \times 1$, which are assumed to be zero-mean complex Gaussian random vectors with $E\{\boldsymbol{\nu}_1 \boldsymbol{\nu}_1^H\} = E\{\boldsymbol{\nu}_2 \boldsymbol{\nu}_2^H\} = \mathbf{I}$, and $\mathbf{I}$ is the $n_{\mathrm{r}} \times n_{\mathrm{r}}$ identity matrix. The relays use an AF scheme to relay the signals they have already received. Hence the vector of the signals re-transmitted by the relays are given as

$$\mathbf{t}_{\mathrm{p}} = \mathbf{w}_1 \odot \mathbf{x}_{\mathrm{p}}, \quad \mathbf{t}_{\mathrm{s}} = \mathbf{w}_2 \odot \mathbf{x}_{\mathrm{s}}. \tag{3.1.3}$$

Here, the operator $\odot$ represents Schur-Hadamard (element-wise) vector product, whereas $\mathbf{t}_{\mathrm{p}}$ and $\mathbf{t}_{\mathrm{s}}$ are the $n_{\mathrm{r}} \times 1$ vectors of relay re-transmitted signals corresponding to the primary and secondary subframes, respectively. The ith entries of $\mathbf{t}_{\mathrm{p}}$ and $\mathbf{t}_{\mathrm{s}}$ are the amplified- and phase-adjusted versions of the signals received at the ith relay in the first and second subframes, respectively. The following definitions are used, $\mathbf{F}_1 \triangleq \mathrm{diag}(\mathbf{f}_1)$, $\mathbf{F}_2 \triangleq \mathrm{diag}(\mathbf{f}_2)$, $\mathbf{G}_1 \triangleq \mathrm{diag}(\mathbf{g}_1)$ and $\mathbf{G}_2 \triangleq \mathrm{diag}(\mathbf{g}_2)$, where $\mathrm{diag}(\cdot)$ is an operator producing a diagonal matrix whose diagonal elements are the input vector of the operator. Using the above definitions, the signals received at the primary transceivers are given as

$$y_{\mathrm{p}}^1 = \mathbf{f}_1^T \mathbf{t}_{\mathrm{p}} + n_1 = \sqrt{p_1}\mathbf{w}_1^H \mathbf{F}_1 \mathbf{f}_1 s_1 + \sqrt{p_2}\mathbf{w}_1^H \mathbf{F}_1 \mathbf{f}_2 s_2 + \mathbf{w}_1^H \mathbf{F}_1 \boldsymbol{\nu}_1 + n_1 \tag{3.1.4}$$

$$y_{\mathrm{p}}^2 = \mathbf{f}_2^T \mathbf{t}_{\mathrm{p}} + n_2 = \sqrt{p_1}\mathbf{w}_1^H \mathbf{F}_2 \mathbf{f}_1 s_1 + \sqrt{p_2}\mathbf{w}_1^H \mathbf{F}_2 \mathbf{f}_2 s_2 + \mathbf{w}_1^H \mathbf{F}_2 \boldsymbol{\nu}_1 + n_2 \tag{3.1.5}$$

where y_{p}^1 and y_{p}^2 are the signals received at PTRX$_1$ and PTRX$_2$, respectively, while n_1 and n_2 are the received noises at the corresponding primary transceivers. The signals received at the secondary transceivers are also given as

$$y_{\mathrm{s}}^1 = \mathbf{g}_1^T \mathbf{t}_{\mathrm{s}} + n_3 = \sqrt{p_3} \mathbf{w}_2^H \mathbf{G}_1 \mathbf{g}_1 s_3 + \sqrt{p_4} \mathbf{w}_2^H \mathbf{G}_1 \mathbf{g}_2 s_4 + \mathbf{w}_2^H \mathbf{G}_1 \boldsymbol{\nu}_2 + n_3 \qquad (3.1.6)$$

$$y_{\mathrm{s}}^2 = \mathbf{g}_2^T \mathbf{t}_{\mathrm{s}} + n_4 = \sqrt{p_3} \mathbf{w}_2^H \mathbf{G}_2 \mathbf{g}_1 s_3 + \sqrt{p_4} \mathbf{w}_2^H \mathbf{G}_2 \mathbf{g}_2 s_4 + \mathbf{w}_2^H \mathbf{G}_2 \boldsymbol{\nu}_2 + n_4 \qquad (3.1.7)$$

where y_{s}^1 and y_{s}^2 are the signals received at STRX$_1$ and STRX$_2$, respectively, while n_3 and n_4 are the noise components at the corresponding secondary transceivers. All the transceiver noise components in (3.1.4)-(3.1.7), are considered to be zero-mean complex Gaussian random variables with unit variance. Moreover, a control channel between the primary and secondary networks is considered for coordination.

The primary transceivers are assumed to be responsible to calculate network parameters and broadcast these parameters to the relays and to the secondary transceivers. We assume that each transceiver knows the channel coefficients corresponding to the links between itself and the relays as well as those corresponding to the links between its peer transceiver and the relays, i.e., PTRX$_1$ and PTRX$_2$ know both $\mathbf{f}_1$ and $\mathbf{f}_2$, while STRX$_1$ and STRX$_2$ know both $\mathbf{g}_1$ and $\mathbf{g}_2$. This assumption, is frequently used in the literature [43, 45, 77–96], and it is reasonable as it implies that channel state information (CSI) is available at the receiving nodes. We will show in next section that this assumption on the availability of CSI in the two networks is essential for all forthcoming algorithms. Each algorithm may however require additional exchange of CSI between the two networks. We explain those additional exchanges as we present each algorithm.

Each relay can use the control channel to transmit to the primary (secondary)

transceivers, the complex estimate[1] of its *local* channel coefficients corresponding to its link to the primary (secondary)[2] transceivers. Taking into account that the perfect knowledge of $\mathbf{f}_1$ and $\mathbf{f}_2$ as well as that of $\mathbf{g}_1$ and $\mathbf{g}_2$ are available at the primary and secondary transceivers, respectively, each pair can obtain the optimal beamforming vector corresponding to its subframe. Therefore, the first term in (3.1.4) ((3.1.6)) and the second term in (3.1.5) ((3.1.7)) are respectively known to the PTRX$_1$ (STRX$_1$) and PTRX$_2$ (STRX$_2$). These terms, often called self-interference, can hence be canceled out from (3.1.4), (3.1.5), (3.1.6) and (3.1.7). After the self-interferences cancelation, the residual signals at the primary and secondary transceivers are respectively given by

$$\tilde{y}_{\mathrm{p}}^1 = y_{\mathrm{p}}^1 - \sqrt{p_1}\mathbf{w}_1^H\mathbf{F}_1\mathbf{f}_1 s_1 = \underbrace{\sqrt{p_2}\mathbf{w}_1^H\mathbf{F}_1\mathbf{f}_2 s_2}_{\text{desired signal for P}TRX_1} + \underbrace{\mathbf{w}_1^H\mathbf{F}_1\boldsymbol{\nu}_1 + n_1}_{\text{noise at P}TRX_1} \tag{3.1.8}$$

$$\tilde{y}_{\mathrm{p}}^2 = y_{\mathrm{p}}^2 - \sqrt{p_2}\mathbf{w}_1^H\mathbf{F}_2\mathbf{f}_2 s_2 = \underbrace{\sqrt{p_1}\mathbf{w}_1^H\mathbf{F}_2\mathbf{f}_1 s_1}_{\text{desired signal for P}TRX_2} + \underbrace{\mathbf{w}_1^H\mathbf{F}_2\boldsymbol{\nu}_1 + n_2}_{\text{noise at P}TRX_2} \tag{3.1.9}$$

$$\tilde{y}_{\mathrm{s}}^1 = y_{\mathrm{s}}^1 - \sqrt{p_3}\mathbf{w}_2^H\mathbf{G}_1\mathbf{g}_1 s_3 = \underbrace{\sqrt{p_4}\mathbf{w}_2^H\mathbf{G}_1\mathbf{g}_2 s_4}_{\text{desired signal for S}TRX_1} + \underbrace{\mathbf{w}_2^H\mathbf{G}_1\boldsymbol{\nu}_2 + n_3}_{\text{noise at S}TRX_1} \tag{3.1.10}$$

$$\tilde{y}_{\mathrm{s}}^2 = y_{\mathrm{s}}^2 - \sqrt{p_4}\mathbf{w}_2^H\mathbf{G}_2\mathbf{g}_2 s_4 = \underbrace{\sqrt{p_3}\mathbf{w}_2^H\mathbf{G}_2\mathbf{g}_1 s_3}_{\text{desired signal for S}TRX_2} + \underbrace{\mathbf{w}_2^H\mathbf{G}_2\boldsymbol{\nu}_2 + n_4}_{\text{noise at S}TRX_2} \tag{3.1.11}$$

where $\tilde{y}_{\mathrm{p}}^1$ and $\tilde{y}_{\mathrm{p}}^2$ as well as $\tilde{y}_{\mathrm{s}}^1$ and $\tilde{y}_{\mathrm{s}}^2$ are the residual signals at the primary and secondary transceivers, respectively. Using the residual signals derived in (3.1.8)-(3.1.11), the primary and the secondary transceivers can extract their corresponding desired signals. In the remaining of this section, we derive the total consumed power in each subframe and the signal-to-noise-ratios (SNRs) for the primary and secondary

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[1]Considering the estimation error does not fit in the scope of this study.

[2]Alternatively, these channel coefficients can be estimated at the transceiver through channel training [97,98].

transceivers in terms of p_1, p_2, p_3, p_4, $\mathbf{w}_1$, and $\mathbf{w}_2$. Using (3.1.1)-(3.1.3), the total relay transmit powers corresponding to the primary and secondary subframes are given by

$$p_{\mathrm{p}}^{\mathrm{r}}(p_1, p_2, \mathbf{w}_1) = E\{\mathbf{t}_{\mathrm{p}}^H \mathbf{t}_{\mathrm{p}}\} = \mathbf{w}_1^H(p_1\mathbf{D}_1 + p_2\mathbf{D}_2 + \mathbf{I})\mathbf{w}_1 \qquad (3.1.12)$$

$$p_{\mathrm{s}}^{\mathrm{r}}(p_3, p_4, \mathbf{w}_2) = E\{\mathbf{t}_{\mathrm{s}}^H \mathbf{t}_{\mathrm{s}}\} = \mathbf{w}_2^H(p_3\mathbf{E}_1 + p_4\mathbf{E}_2 + \mathbf{I})\mathbf{w}_2 \qquad (3.1.13)$$

where we have used the following definitions:

$$\mathbf{D}_1 \triangleq \mathbf{F}_1\mathbf{F}_1^H, \mathbf{D}_2 \triangleq \mathbf{F}_2\mathbf{F}_2^H, \mathbf{E}_1 \triangleq \mathbf{G}_1\mathbf{G}_1^H, \mathbf{E}_2 \triangleq \mathbf{G}_2\mathbf{G}_2^H. \qquad (3.1.14)$$

Note that $\mathbf{D}_1$ and $\mathbf{D}_2$ as well as $\mathbf{E}_1$ and $\mathbf{E}_2$ are diagonal matrices. Furthermore, using (3.1.12) and (3.1.13), the total power consumed in the primary and secondary subframes are, respectively given as

$$\begin{aligned} p_{\mathrm{p}}(p_1, p_2, \mathbf{w}_1) &= p_1 + p_2 + p_{\mathrm{p}}^{\mathrm{r}}(p_1, p_2, \mathbf{w}_1) \\ &= p_1(1 + \mathbf{w}_1^H\mathbf{D}_1\mathbf{w}_1) + p_2(1 + \mathbf{w}_1^H\mathbf{D}_2\mathbf{w}_1) + \mathbf{w}_1^H\mathbf{w}_1 \qquad (3.1.15) \end{aligned}$$

$$\begin{aligned} p_{\mathrm{s}}(p_3, p_4, \mathbf{w}_2) &= p_3 + p_4 + p_{\mathrm{s}}^{\mathrm{r}}(p_3, p_4, \mathbf{w}_2) \\ &= p_3(1 + \mathbf{w}_2^H\mathbf{E}_1\mathbf{w}_2) + p_4(1 + \mathbf{w}_2^H\mathbf{E}_2\mathbf{w}_2) + \mathbf{w}_2^H\mathbf{w}_2. \qquad (3.1.16) \end{aligned}$$

Moreover, using (3.1.8) and (3.1.9) as well as (3.1.10) and (3.1.11), the received SNRs corresponding to the signals received at PTRX$_1$, PTRX$_2$, STRX$_1$ and STRX$_2$ can be respectively, expressed as

$$\mathrm{SNR}_1(p_2, \mathbf{w}_1) = \frac{p_2\mathbf{w}_1^H\mathbf{h}_{\mathrm{p}}\,\mathbf{h}_{\mathrm{p}}^H\mathbf{w}_1}{\mathbf{w}_1^H\mathbf{D}_1\mathbf{w}_1 + 1} \qquad (3.1.17)$$

$$\mathrm{SNR}_2(p_1, \mathbf{w}_1) = \frac{p_1\mathbf{w}_1^H\mathbf{h}_{\mathrm{p}}\,\mathbf{h}_{\mathrm{p}}^H\mathbf{w}_1}{\mathbf{w}_1^H\mathbf{D}_2\mathbf{w}_1 + 1} \qquad (3.1.18)$$

$$\mathrm{SNR}_3(p_4, \mathbf{w}_2) = \frac{p_4\mathbf{w}_2^H\mathbf{h}_{\mathrm{s}}\,\mathbf{h}_{\mathrm{s}}^H\mathbf{w}_2}{\mathbf{w}_2^H\mathbf{E}_1\mathbf{w}_2 + 1} \qquad (3.1.19)$$

$$\mathrm{SNR}_4(p_3, \mathbf{w}_2) = \frac{p_3\mathbf{w}_2^H\mathbf{h}_{\mathrm{s}}\,\mathbf{h}_{\mathrm{s}}^H\mathbf{w}_2}{\mathbf{w}_2^H\mathbf{E}_2\mathbf{w}_2 + 1} \qquad (3.1.20)$$

where

$$\mathbf{h}_{\mathrm{p}} \triangleq \mathbf{F}_1\mathbf{f}_2, \ \ \mathbf{h}_{\mathrm{s}} \triangleq \mathbf{G}_1\mathbf{g}_2. \tag{3.1.21}$$

We also define the information theoretic rates of the primary transceivers as well as those of the secondary transceivers as

$$R_1(p_2, \mathbf{w}_1) = \frac{1}{2}\log_2(1 + \mathrm{SNR}_1(p_2, \mathbf{w}_1)) \tag{3.1.22}$$

$$R_2(p_1, \mathbf{w}_1) = \frac{1}{2}\log_2(1 + \mathrm{SNR}_2(p_1, \mathbf{w}_1)) \tag{3.1.23}$$

$$R_3(p_4, \mathbf{w}_2) = \frac{1}{2}\log_2(1 + \mathrm{SNR}_3(p_4, \mathbf{w}_2)) \tag{3.1.24}$$

$$R_4(p_3, \mathbf{w}_2) = \frac{1}{2}\log_2(1 + \mathrm{SNR}_4(p_3, \mathbf{w}_2)). \tag{3.1.25}$$

In the following sections, we use our data model to design our spectrum leasing scheme, thereby allowing the primary transceivers calculate both network parameters and broadcast them to the relays and the secondary transceivers using the control channel.

3.2   Spectrum sharing system design

In our system model, obtaining the optimal values of the network parameters (i.e., the beamforming vectors $\mathbf{w}_1$ and $\mathbf{w}_2$, the transceiver transmit powers p_1, p_2, p_3 and p_4, and the time sharing factor α) is of interest. Depending on the objective and requirements of the design, different approaches can be proposed. Since it is assumed that there is no direct link between the primary pairs, cooperation between the primary and secondary networks is essential in order to provide a minimum quality of service (QoS) for the primary transceivers. As a measure of QoS, the minimum average rate of the two primary transceivers is constrained to be larger than a predefined

threshold. In exchange for this cooperation, the primary network assigns a portion of its spectral or temporal resources to the secondary network so that the secondary transceivers can exchange their information using the relay infrastructure. In order to establish this cooperation scheme, it is rational that the primary network offers the highest attainable average rate to the secondary transceivers, while preserving a minimum QoS for its own transceivers. Therefore, the goal of our design approaches is to maximize the secondary network average rate under a QoS constraint on the primary transceivers.

In this design approach, one has to ensure a minimum QoS for the primary transceivers, meaning that the minimum average rates for the two primary transceivers is constrained to be larger than a predefined threshold η. Other limiting constraints on the network parameters, are the transmit powers consumed in the two networks in their corresponding subframes. Two different types of power constraints are considered in this section. First, two constraints are considered to restrict the total transmit powers consumed by each network in the corresponding subframes. Second, we consider a constraint on the total power consumed in both subframes. In the following subsections, two optimization problems are presented and the corresponding optimal solutions are developed.

3.2.1 Separate power constraint per network

In order to determine the primary and secondary network parameters, let us consider the following optimization problem:

$$\underset{\{p_i\}_{i=1}^4,\alpha,\mathbf{w}_1,\mathbf{w}_2}{\text{maximize}} \quad \alpha \min(R_3(p_4, \mathbf{w}_2), R_4(p_3, \mathbf{w}_2))$$

$$\text{subject to} \quad p_{\mathrm{p}}(p_1, p_2, \mathbf{w}_1) \leq P_{\mathrm{p}}, \quad p_{\mathrm{s}}(p_3, p_4, \mathbf{w}_2) \leq P_{\mathrm{s}}$$

$$(1 - \alpha) \; \min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)) \geq \eta$$

$$0 \leq \alpha \leq 1. \tag{3.2.1}$$

In the objective function of (3.2.1), we use a max-min rate fair design approach, thereby maximizing the smaller of the *average* rates of the two transceivers in the secondary network. Indeed, the parameter α takes into account that the secondary transceivers utilize the channel only α fraction of the time. The first and the second constraints limit the total amounts of power consumed in the primary and secondary subframes, to P_{p} and P_{s}, respectively. In fact, P_{p} and P_{s} are the peak powers for the primary and secondary networks in their corresponding subframes, respectively.

From each network point of view, a total power constraint is of great importance, since it provides a flexible environment to control or optimize the total consumed power in that network by allowing each element of the network to have its own optimal power relative to the total power constraint. In addition, such a constraint provides a guideline for how to set individual relay powers. For example, as was shown in [43, 44, 78], when applying a max-min SNR (or rate) design approach to the case of a single two-way relay network with n_{r} relay nodes, the relays will collectively consume half of the available total transmit power. In such a network, it is reasonable to assume that each relay, in average, consumes $\frac{1}{n_{\mathrm{r}}}$ fraction of half of the total power

budget. This argument is particularly correct when the relays are moving randomly in the environment. In such a scenario, different relay channels appear to be drawn from the same probability distribution. For all these reasons, total power constraints have been adopted in the literature for performance analysis and optimal design [43, 44, 78, 80, 86, 89].

Finally, the third constraint in (3.2.1) ensures that the smallest *average* rate of the two primary transceivers are above a given threshold η, while taking into account that these two transceivers are communicating $(1-\alpha)$ fraction of the time. It is worth mentioning that η is measured in bit per channel use (b/cu) unit.

We now show how the optimization problem (3.2.1) can be solved. To do so, note that in the optimization problem (3.2.1), the primary network QoS inequality constraint becomes equality. In order to prove this claim, assume that at the optimal solution, the third constraint in (3.2.1) is satisfied with inequality. One can then choose a value for α to be larger than its optimal value such that this constraint is satisfied with equality. However, the new value of α will further increase the objective function, thereby contradicting optimality. Hence, the third constraint holds with equality, leading us to the following expression for α at the optimum:

$$\alpha = 1 - \frac{\eta}{\min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1))}. \tag{3.2.2}$$

It is worth mentioning that since the condition $0 \leq \alpha \leq 1$ must be satisfied, it follows from (3.2.2) that $\min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)) \geq \eta$ must hold true. Using (3.2.2) in the objective function of the optimization problem (3.2.1) leads us to the following

maximization problem:

$$\underset{\{p_i\}_{i=1}^4, \mathbf{w}_1, \mathbf{w}_2}{\text{maximize}} \quad (1 - \frac{\eta}{\min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1))}) \min(R_3(p_4, \mathbf{w}_2), R_4(p_3, \mathbf{w}_2))$$

$$\text{subject to} \quad p_{\mathrm{p}}(p_1, p_2, \mathbf{w}_1) \leq P_{\mathrm{p}}, \ p_{\mathrm{s}}(p_3, p_4, \mathbf{w}_2) \leq P_{\mathrm{s}}$$

$$\min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)) \geq \eta. \tag{3.2.3}$$

Since the constraints as well as the objective function in (3.2.3) depend on two mutually exclusive sets of parameters, the optimization problem (3.2.3) could be split into the following two separate sub-problems:

$$\text{Sub-Problem 1} \quad \underset{p_1, p_2, \mathbf{w}_1}{\text{maximize}} \quad \min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1))$$

$$\text{subject to} \quad p_{\mathrm{p}}(p_1, p_2, \mathbf{w}_1) \leq P_{\mathrm{p}}$$

$$\min(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)) \geq \eta \tag{3.2.4}$$

$$\text{Sub-Problem 2} \quad \underset{p_3, p_4, \mathbf{w}_2}{\text{maximize}} \quad \min(R_3(p_4, \mathbf{w}_2), R_4(p_3, \mathbf{w}_2))$$

$$\text{subject to} \quad p_{\mathrm{s}}(p_3, p_4, \mathbf{w}_2) \leq P_{\mathrm{s}} \tag{3.2.5}$$

The last constraint in Sub-Problem 1 is only a feasibility condition. We can ignore this constraint, solve the remaining problem, and then check if this constraint holds true or not. The two sub-problems in (3.2.4) and (3.2.5) are similar in formulation, therefore, we concentrate on how to solve one of them. It has been proven in [43] that a max-min SNR (or rate) fair approach to design a bidirectional network beamformer, under a total power budget, leads to SNR balancing. The results of [44] can be applied directly to solve (3.2.4) and (3.2.5). Using the following definitions, $\mathbf{\Phi}(x, y) = 2x\mathbf{D}_1 + (y - 2x)\mathbf{D}_2 + \mathbf{I}$ and $\mathbf{\Psi}(x, y) = 2x\mathbf{E}_1 + (y - 2x)\mathbf{E}_2 + \mathbf{I}$, the optimal values of the primary and the secondary network parameters are given as

Solution to Sub-Problem 1:

$$p_1^\circ(P_{\mathrm{p}}) = \arg \max_{0 \le p_1 \le \frac{P_{\mathrm{p}}}{2}} p_1(P_{\mathrm{p}} - 2p_1)\mathbf{h}_{\mathrm{p}}^H \mathbf{\Phi}^{-1}(p_1, P_{\mathrm{p}})\mathbf{h}_{\mathrm{p}} \tag{3.2.6}$$

$$p_2^\circ(P_{\mathrm{p}}) = \frac{1}{2}P_{\mathrm{p}} - p_1^\circ(P_{\mathrm{p}}) \tag{3.2.7}$$

$$\mathbf{w}_1^\circ(P_{\mathrm{p}}) = \kappa(P_{\mathrm{p}})\sqrt{p_2^\circ(P_{\mathrm{p}})}\,\mathbf{\Phi}^{-1}\left(p_1^\circ(P_{\mathrm{p}}), P_{\mathrm{p}}\right)\mathbf{h}_{\mathrm{p}} \tag{3.2.8}$$

$$\kappa(P_{\mathrm{p}}) = \left(\mathbf{h}_{\mathrm{p}}^H (2p_1^\circ(P_{\mathrm{p}})\mathbf{D}_1 + \mathbf{I})\mathbf{\Phi}^{-2}\left(p_1^\circ(P_{\mathrm{p}}), P_{\mathrm{p}}\right)\mathbf{h}_{\mathrm{p}}\right)^{-\frac{1}{2}} \tag{3.2.9}$$

Solution to Sub-Problem 2:

$$p_3^\circ(P_{\mathrm{s}}) = \arg \max_{0 \le p_3 \le \frac{P_{\mathrm{s}}}{2}} p_3(P_{\mathrm{s}} - 2p_3)\mathbf{h}_{\mathrm{s}}^H \mathbf{\Psi}^{-1}(p_3, P_{\mathrm{s}})\mathbf{h}_{\mathrm{s}} \tag{3.2.10}$$

$$p_4^\circ(P_{\mathrm{s}}) = \frac{1}{2}P_{\mathrm{s}} - p_3^\circ(P_{\mathrm{s}}) \tag{3.2.11}$$

$$\mathbf{w}_2^\circ(P_{\mathrm{s}}) = \mu(P_{\mathrm{s}})\sqrt{p_4^\circ(P_{\mathrm{s}})}\,\mathbf{\Psi}^{-1}\left(p_3^\circ(P_{\mathrm{s}}), P_{\mathrm{s}}\right)\mathbf{h}_{\mathrm{s}} \tag{3.2.12}$$

$$\mu(P_{\mathrm{s}}) = \left(\mathbf{h}_{\mathrm{s}}^H (2p_3^\circ(P_{\mathrm{s}})\mathbf{E}_1 + \mathbf{I})\mathbf{\Psi}^{-2}\left(p_3^\circ(P_{\mathrm{s}}), P_{\mathrm{s}}\right)\mathbf{h}_{\mathrm{s}}\right)^{-\frac{1}{2}}. \tag{3.2.13}$$

It has been proven in [43] and [44] that the maximization problem (3.2.6) and (3.2.10) are convex in terms of p_1 and p_3, respectively. We can thus use efficient algorithms such as interior point methods in order to solve (3.2.6) and (3.2.10). Furthermore, the optimal primary and secondary transceivers' instantaneous rates as well as the optimal value of the time sharing factor are, respectively, given by

$$f(P_{\mathrm{p}}) \triangleq R_1\left(p_2^\circ(P_{\mathrm{p}}), \mathbf{w}_1^\circ(P_{\mathrm{p}})\right) = R_2\left(p_1^\circ(P_{\mathrm{p}}), \mathbf{w}_1^\circ(P_{\mathrm{p}})\right) \tag{3.2.14}$$

$$g(P_{\mathrm{s}}) \triangleq R_3\left(p_4^\circ(P_{\mathrm{s}}), \mathbf{w}_2^\circ(P_{\mathrm{s}})\right) = R_4\left(p_3^\circ(P_{\mathrm{s}}), \mathbf{w}_2^\circ(P_{\mathrm{s}})\right) \tag{3.2.15}$$

$$\alpha^\circ(P_{\mathrm{p}}) = 1 - \frac{\eta}{f(P_{\mathrm{p}})}. \tag{3.2.16}$$

As mentioned earlier, in order to cancel the self-interference, each transceiver should know its corresponding beamforming vector. To calculate $\mathbf{w}_1^\circ(P_{\mathrm{p}})$ $(\mathbf{w}_2^\circ(P_{\mathrm{s}}))$,

the two primary (secondary) transceivers should know the channel coefficients corresponding to the links between themselves and the relays. In the case of separate total power constraint for each subframe, the design problem, as shown above, can be split into two disjoint sub-problems, hence the primary transceivers do not need to know the secondary CSI. This is the advantage of this approach as each transceiver pair can design the transmission parameters of their own network knowing only the channel coefficients of the link between themselves and the relays.

To summarize our contribution in this subsection, we showed that the optimization problem (3.2.1) can be separated into two optimization problems (3.2.4) and (3.2.5), and that time sharing factor can obtained as in (3.2.16) using only the channel state information of the primary network and the corresponding required minimum data rate.

3.2.2   Total power constraint for both networks

In this subsection, we propose another optimization framework, where individual per-network constraints on the total transmit powers are replaced with a constraint on the average total power consumed by the two networks in a time frame. Instead of separate power constraint per network, it is rational that using a constraint on the average total power, consumed in the whole time frame, leads to a more flexible power allocation between the primary and secondary networks. This is due to the fact that the available average power in each time frame could be allocated optimally between the primary and the secondary networks. Hence, when using an average total power constraint, it is easier to satisfy the primary QoS constraint compared to the case of separate power allocation to the two networks in the first approach. This

is the advantage of this approach compared to the first approach[3]. One application, where using a constraint on the total power consumed by both networks is useful, is a scenario where the relays are not moving and they are powered up from the power grid. In this case, as will be shown, the proposed solution results in a total relay power consumption equal to half of the total available power for the two networks. Hence, using a total power constraint indeed limits the total power the relays collectively consume from the power grid. Such a relay power control is indeed a desired feature for stationary relays, which are powered up from the power grid. Based on this discussion, our second optimization problem is presented as

$$\underset{\{p_i\}_{i=1}^4,\alpha,\mathbf{w}_1,\mathbf{w}_2}{\text{maximize}} \quad \alpha \min(R_3(p_4,\mathbf{w}_2), R_4(p_3,\mathbf{w}_2))$$

$$\text{subject to} \quad (1-\alpha)\, p_{\mathrm{p}}(p_1,p_2,\mathbf{w}_1) + \alpha\, p_{\mathrm{s}}(p_3,p_4,\mathbf{w}_2) \leq P_T$$

$$(1-\alpha)\min(R_1(p_2,\mathbf{w}_1), R_2(p_1,\mathbf{w}_1)) \geq \eta$$

$$0 \leq \alpha \leq 1. \tag{3.2.17}$$

Although, the solution to the optimization problem (3.2.17) does not appear to be straightforward, we show how this problem can be simplified. To do so, we first prove that $R_3(p_4,\mathbf{w}_2) = R_4(p_3,\mathbf{w}_2)$ and $R_1(p_2,\mathbf{w}_1) = R_2(p_1,\mathbf{w}_1)$ hold true at the optimum. In order to prove these rate balancing properties, let us assume, without loss of generality, that at the optimum, $R_1(p_2,\mathbf{w}_1) < R_2(p_1,\mathbf{w}_1)$ holds true. Without loss of optimality, we can decrease the optimal value of p_1 in order to decrease $R_2(p_1,\mathbf{w}_1)$ and make it equal to $R_1(p_2,\mathbf{w}_1)$. This reduction of the optimal p_1 will not violate the total consumed power[4], neither will it affect the primary network QoS constraints. Now, assume without loss of generality that at any optimal solution to (3.2.17),

---

[3]This approach has its own requirements as will be pointed out later.
[4]Note that according to (3.1.15), when p_1 is decreased, $p_{\mathrm{p}}(p_1,p_2,\mathbf{w}_1)$ is also decreased.

$R_3(p_4, \mathbf{w}_2) < R_4(p_3, \mathbf{w}_2)$ holds true. We can then decrease the optimal value of p_3 to reduce $R_4(p_3, \mathbf{w}_2)$ and make it equal to $R_3(p_4, \mathbf{w}_2)$, without violating the total power constraint[5] and without changing the value of the objective function. We hence conclude that at each transceiver pair, the equality of rates can be assumed without loss of optimality. Using an auxiliary parameter $\tilde{P}_\mathrm{s}$, we can rewrite the optimization problem (3.2.17) as

$$
\begin{aligned}
\underset{\tilde{P}_\mathrm{s}, p_1, p_2, \alpha, \mathbf{w}_1}{\text{maximize}} \quad & \alpha \max_{p_3, p_4, \mathbf{w}_2} \quad \min\big(R_3(p_4, \mathbf{w}_2), R_4(p_3, \mathbf{w}_2)\big) \\
\text{subject to} \quad & p_\mathrm{s}(p_3, p_4, \mathbf{w}_2) \leq \tilde{P}_\mathrm{s} \\
& (1 - \alpha)\, p_\mathrm{p}(p_1, p_2, \mathbf{w}_1) + \alpha\, \tilde{P}_\mathrm{s} \leq P_T \\
& (1 - \alpha)\, \min\big(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)\big) \geq \eta \\
& 0 \leq \alpha \leq 1.
\end{aligned}
\tag{3.2.18}
$$

For any fixed $\tilde{P}_\mathrm{s}$, the inner maximization problem is the same as the problem stated in (3.2.5), when P_s is replaced with $\tilde{P}_\mathrm{s}$. Hence, the secondary transceiver powers as well as the optimal secondary beamforming vector are, respectively, the same as (3.2.10), (3.2.11) and (3.2.12), when P_s is replaced with $\tilde{P}_\mathrm{s}$. Using the fact that at the optimum, the secondary rates are equal when P_s is replaced with $\tilde{P}_\mathrm{s}$, we simplify the optimization problem (3.2.17) as

$$
\begin{aligned}
\underset{\tilde{P}_\mathrm{s}, p_1, p_2, \alpha, \mathbf{w}_1}{\text{maximize}} \quad & \alpha\, g(\tilde{P}_\mathrm{s}) \\
\text{subject to} \quad & (1 - \alpha)\, p_\mathrm{p}(p_1, p_2, \mathbf{w}_1) + \alpha\, \tilde{P}_\mathrm{s} \leq P_T \\
& (1 - \alpha)\, \min\big(R_1(p_2, \mathbf{w}_1), R_2(p_1, \mathbf{w}_1)\big) \geq \eta \\
& 0 \leq \alpha \leq 1
\end{aligned}
\tag{3.2.19}
$$

---

[5]Note that according to (3.1.16), when p_3 is decreased, $p_\mathrm{s}(p_3, p_4, \mathbf{w}_2)$ is also decreased.

where $g(\tilde{P}_s)$ is given in (3.2.15), when P_s is replaced with $\tilde{P}_s$. At the optimum, the first constraint in the optimization problem (3.2.19) is satisfied with equality. To show this, assume that at the optimum, this constraint is satisfied with inequality. One can then increase the optimal value of $\tilde{P}_s$ to turn this inequality into equality. This increase of the optimal $\tilde{P}_s$ will lead to a higher value for the objective function as $g(\tilde{P}_s)$ is increasing in $\tilde{P}_s$ (see appendix). This is a contradiction, and hence, the total consumed power should be satisfied with equality. Moreover, the second constraint in the optimization problem (3.2.19) is also satisfied with equality. To show this, assume that at the optimum, this constraint is satisfied with inequality. One can decrease the optimal value of p_1, thereby decreasing $R_2(p_1, \mathbf{w}_1)$, to turn the second constraint in the optimization problem (3.2.19) into equality[6]. However, this decrease of the optimal value of p_1 results in lower value for $p_p(p_1, p_2, \mathbf{w}_1)$, thereby turning the first constraint in (3.2.19) into inequality. Hence, one can increase the optimal value of $\tilde{P}_s$ until the first constraint in (3.2.19) turns into equality. This increase of the optimal value of $\tilde{P}_s$, will lead to higher value for the objective function as $g(\tilde{P}_s)$ is an increasing function of $\tilde{P}_s$, thereby contradicting optimality. Hence, the second constraint in (3.2.19) must be satisfied with equality.

We now introduce a new auxiliary variable $\tilde{P}_p$ in (3.2.19) to denote the total power consumed in the primary subframe. As such, the optimization problem (3.2.19) can

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[6]Note that for the new value of p_1, $R_1(p_2, \mathbf{w}_1) > R_2(p_1, \mathbf{w}_1)$

be rewritten as

$$\underset{\tilde{P}_\mathrm{s},\tilde{P}_\mathrm{p},p_1,p_2,\alpha,\mathbf{w}_1}{\text{maximize}} \quad \alpha\, g(\tilde{P}_\mathrm{s})$$

$$\text{subject to} \quad p_\mathrm{p}(p_1,p_2,\mathbf{w}_1) = \tilde{P}_\mathrm{p}$$

$$(1-\alpha)\tilde{P}_\mathrm{p} + \alpha\,\tilde{P}_\mathrm{s} = P_T$$

$$(1-\alpha)\min(R_1(p_2,\mathbf{w}_1), R_2(p_1,\mathbf{w}_1)) = \eta$$

$$0 \le \alpha \le 1. \tag{3.2.20}$$

In order to further simplify the optimization problem (3.2.20), let us assume that at the optimum, the total consumed power in a time frame as well as those consumed in the primary and secondary subframes are given by P_T^o, P_p^o and P_s^o, respectively. Furthermore, assume that the optimal time sharing factor is given by α^o. The primary network QoS constraint in (3.2.20) forces that the condition $R_1(p_2,\mathbf{w}_1) = R_2(p_1,\mathbf{w}_1) = \frac{\eta}{(1-\alpha^\mathrm{o})}$ must hold true. This rate balancing property holds at point a in the (R_1, R_2) plane, as shown in Fig. 3.2. In addition, for any given $\tilde{P}_\mathrm{p}$, the rate region for the two primary transceivers is denoted as the set $\mathcal{C}(\tilde{P}_\mathrm{p})$, and it is given by [44]

$$\mathcal{C}(\tilde{P}_\mathrm{p}) = \{(R_1(p_2,\mathbf{w}_1), R_2(p_1,\mathbf{w}_1)) | R_1(p_2,\mathbf{w}_1) \ge 0, R_2(p_1,\mathbf{w}_1) \ge 0,$$

$$R_2(p_1,\mathbf{w}_1) \le \frac{1}{2}\log_2(2 + 2\gamma^\mathrm{max}(\tilde{P}_\mathrm{p}) - 2^{2R_1(p_2,\mathbf{w}_1)})\}. \tag{3.2.21}$$

In (3.2.21), $\gamma^\mathrm{max}(\tilde{P}_\mathrm{p})$ is the maximum balanced SNR for each of the two primary transceivers for any given $\tilde{P}_\mathrm{p}$. For the optimal value P_p^o, any achievable rate pair for the primary pair is located inside or on the boundary of the set $\mathcal{C}(P_\mathrm{p}^\mathrm{o})$ (see Fig. 3.2) [44]. We now prove that the optimal values for the two primary transceiver rates, $R_1(p_2,\mathbf{w}_1)$ and $R_2(p_1,\mathbf{w}_1)$, must be on the boundary of $\mathcal{C}(P_\mathrm{p}^\mathrm{o})$. To show this,

let us assume that the two primary transceiver rates are located inside $\mathcal{C}(P_\mathrm{p}^\mathrm{o})$ (see Fig. 3.2). One can then decrease the optimal total power consumed in the primary subframe to $\hat{P}_\mathrm{p}$ ($\hat{P}_\mathrm{p} < P_\mathrm{p}^\mathrm{o}$) (for example by scaling down the magnitude of $\mathbf{w}_1$), such that the optimal point a is still inside $\mathcal{C}(\hat{P}_\mathrm{p})$. This decrease in the optimal value of the total power consumed in the primary subframe turns the first constraint in the optimization problem (3.2.20) into inequality. Hence, $\exists\, \varepsilon > 0$ such that $P_\mathrm{s}^\mathrm{o} + \varepsilon$ is still feasible. As $g(P_\mathrm{s}^\mathrm{o} + \varepsilon) > g(P_\mathrm{s}^\mathrm{o})$, this is contradiction. We hence conclude that the optimal point a in Fig. 3.2, must be on the boundary of $\mathcal{C}(P_\mathrm{p}^\mathrm{o})$. Using the above discussion, we can simplify the optimization problem (3.2.20) as:

$$\underset{\tilde{P}_\mathrm{s},\tilde{P}_\mathrm{p},\alpha}{\text{maximize}}\ \alpha\, g(\tilde{P}_\mathrm{s})$$

$$\text{subject to}\ (1-\alpha)\, \tilde{P}_\mathrm{p} + \alpha\, \tilde{P}_\mathrm{s} = P_T$$

$$(1-\alpha)\, f(\tilde{P}_\mathrm{p}) = \eta,\ \ 0 \le \alpha \le 1 \tag{3.2.22}$$

where $f(\tilde{P}_\mathrm{p})$, as defined in (3.2.14), is given by

$$f(\tilde{P}_\mathrm{p}) = \underset{p_1,p_2,\mathbf{w}_1}{\max}\qquad \min\big(R_1(p_2,\mathbf{w}_1), R_2(p_1,\mathbf{w}_1)\big)$$

$$\text{subject to}\ p_\mathrm{p}(p_1,p_2,\mathbf{w}_1) \le \tilde{P}_\mathrm{p}. \tag{3.2.23}$$

Note that $\tilde{P}_\mathrm{p}$ is the total consumed power in the primary subframe and is to be optimized by solving (3.2.22). If the optimal value of $\tilde{P}_\mathrm{p}$ is given, then the primary transceiver powers as well as the optimal primary beamforming vector are, respectively, the same as (3.2.6), (3.2.7), and (3.2.8), when P_p is replaced with the optimal value of $\tilde{P}_\mathrm{p}$. Furthermore, using the first and the second constraints in (3.2.22), we can express $\tilde{P}_\mathrm{p}$ and $\tilde{P}_\mathrm{s}$ in terms of α. Hence, the optimization problem (3.2.22) can be easily turned into a one-dimensional problem. Indeed, using the first two constraints
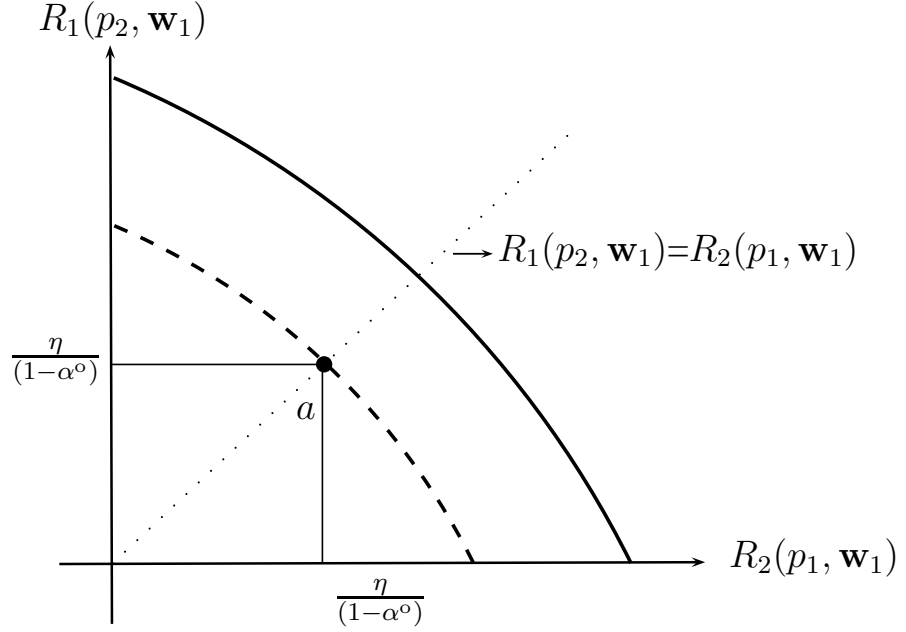
Figure 3.2: Rate region graph.

in (3.2.22), we can eliminate $\tilde{P}_{\mathrm{p}}$ and $\tilde{P}_{\mathrm{s}}$ and can rewrite (3.2.22) as

$$\underset{0 \leq \alpha \leq 1}{\text{maximize}} \; \alpha \, g \left(\frac{P_T - (1-\alpha)f^{-1}(\frac{\eta}{1-\alpha})}{\alpha} \right)$$

$$\text{subject to } (1-\alpha)f(\frac{P_T}{1-\alpha}) \geq \eta \tag{3.2.24}$$

where $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$. Since $f(\cdot)$ is a monotonically increasing function of its argument, its inverse function exists. We can see that the optimization problem (3.2.24) is a line search in the following interval, $0 \leq \alpha \leq 1$. To employ the line search scheme, we can turn this interval into a grid with a small enough step size, then for each point in the interval, we check if the constraint in (3.2.24) is feasible or not. Among all feasible points in the interval $0 \leq \alpha \leq 1$, we introduce the one which results in the highest value for the objective function, as the optimal

solution. Note that, since there is no closed-form solution for $f(\cdot)$, deriving a closed-form solution for the inverse function $f^{-1}(\cdot)$ is not possible. To tackle this issue, note that $f(\tilde{P}_\mathrm{p})$, as defined in (3.2.23), is the maximum achievable balanced rates of the two primary transceivers when $p_\mathrm{p}(p_1, p_2, \mathbf{w}_1) = \tilde{P}_\mathrm{p}$. Finding $f^{-1}(\frac{\eta}{1-\alpha})$ means that we want to obtain the value of the total power consumed in the primary subframe, when the achieved balanced rates satisfy $R_1(p_2, \mathbf{w}_1) = R_2(p_1, \mathbf{w}_1) = \frac{\eta}{1-\alpha}$. Now, we argue that this total consumed power should be as small as possible, thereby allowing $\tilde{P}_\mathrm{s}$ to be as large as possible[7]. Hence, to obtain the smallest value of $f^{-1}(\frac{\eta}{1-\alpha})$, we need to solve the following optimization problem:

$$f^{-1}(\tfrac{\eta}{1-\alpha}) = \min_{p_1, p_2, \mathbf{w}_1} \; p_\mathrm{p}(p_1, p_2, \mathbf{w}_1)$$
$$\text{subject to } R_1(p_2, \mathbf{w}_1) = R_2(p_1, \mathbf{w}_1) = \tfrac{\eta}{1-\alpha}. \qquad (3.2.25)$$

It has been proved in [78], the problem (3.2.25) has a unique minimizer, hence we can solve it using the efficient algorithm proposed in [78].

Note that the uniqueness of the solution to the optimization problem (3.2.24) may not be claimed as objective function does not have an explicit form and that is exactly why we are proposing a line search to solve (3.2.24).

We now discuss how the control channel can be utilized to broadcast the information needed by the two networks nodes to calculate their design parameters of interest. Let us assume that the optimal values of the time sharing factor and the total powers consumed in the primary and secondary subframes are, respectively, given by α^o, $P_\mathrm{p}^\mathrm{o}(\alpha^\mathrm{o})$, and $P_\mathrm{s}^\mathrm{o}(\alpha^\mathrm{o})$. As we assumed earlier, each transceiver in the two pairs knows the channel coefficients corresponding to the link between itself and the relays to cancel its self-interference. In addition, the optimization problem (3.2.24)

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[7]The largest possible value of $\tilde{P}_\mathrm{s}$, for a given α, results in the largest value of $g(\tilde{P}_\mathrm{s})$.

is a joint optimization problem, therefore, further assumption on the availability of the secondary network CSI to the primary transceivers is needed. To calculate $g\left(P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})\right)$ in the optimization problem (3.2.24), the two primary transceivers need to know only the magnitude of the channel coefficients corresponding to the link between the secondary transceivers and the relays, i.e., the absolute values of g_{1i} and g_{2i} for $i = 1, 2, ..., n_{\mathrm{r}}$, where g_{1i} and g_{2i} are the ith elements of $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively. Hence, we assume that the two primary transceivers know the magnitude of the channel coefficients between each of the two secondary transceivers and the relays. These channel coefficient magnitudes can be sent to the primary transceivers, either by the secondary transceivers or by the relays, one-by-one.

Now, we explain what parameters are needed to be broadcasted in the control channel after the optimization problem is solved at the primary transceivers. Once the optimal values α^{o}, $P_{\mathrm{p}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$, and, $P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$ are obtained, the primary transceivers can then calculate their own optimal transmitting power as well as the corresponding optimal beamforming vector in the first subframe, by substituting P_{p} in (3.2.6), (3.2.7), and (3.2.8) with $P_{\mathrm{p}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$. The two primary transceivers then broadcast α^{o} and $P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$ in the control channel to the secondary transceivers[8] in order to allow them to calculate their design parameters, by replacing P_{s} in (3.2.10), (3.2.11), and (3.2.12) with $P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$. According to (3.2.8) (or (3.2.12)), each relay beamforming coefficient depends only on the *local* CSI corresponding to the links between the relay and the two primary (or secondary) transceivers. Moreover, considering the optimal beamforming vector provided in (3.2.8), the primary transceivers only need to broadcast the parameters $P_{\mathrm{p}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$, $\kappa\left(P_{\mathrm{p}}^{\mathrm{o}}(\alpha^{\mathrm{o}})\right)$, and $p_1^{\mathrm{o}}\left(P_{\mathrm{p}}^{\mathrm{o}}(\alpha^{\mathrm{o}})\right)$, to allow each relay to calculate its own

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[8]The relays also receive these two parameters in the control channel.

beamforming coefficient corresponding to the primary subframe. Similarly, the secondary transceivers broadcast the following parameters, $\mu\left(P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})\right)$ and $p_3^{\mathrm{o}}\left(P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})\right)$, in the control channel to allow each relay calculate its own beamforming coefficient[9].

To end up this subsection, we provide a comparison between the two approaches presented in this section. On one hand, the advantage of the first approach is its inherent separability in which both networks can simultaneously calculate their own parameters without the need for too much information exchange between the two networks. Indeed, in the first approach, the only parameter that the primary network should transmit to the secondary transceivers is $\alpha^{\mathrm{o}}(P_{\mathrm{p}})$, where P_{p} is total available power in the primary subframe. However, if the two network consumed powers are fixed, the first approach may not be feasible for some primary network QoS demand. This is the disadvantage of this approach. On the other hand, the advantage of the second approach is its flexibility with respect to the primary network QoS demand. Whenever, the primary network QoS demand is high, more power is assigned to the primary network, and vice versa. However, this approach has its own disadvantages. First, the optimization problem in this approach is a line search problem which may not be convex, hence efficient algorithms may not be applicable in this case. Second, the two transceivers in the primary network need to know the magnitudes of the channel coefficients between the two secondary transceivers and the relays. This means that the second approach requires more information exchange between the two networks through the control channel. This is the price to be paid to guarantee relatively high rate demand for the primary network.

To summarize our contribution in this subsection, we simplified the optimization

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[9]$P_{\mathrm{s}}^{\mathrm{o}}(\alpha^{\mathrm{o}})$ is already broadcasted by the primary transceivers and there is no need to be retransmitted by the secondary transceivers.

problem (3.2.17) as in (3.2.24), first by proving that the two constraints in (3.2.18) are satisfied with equality, and then, by proving that the optimal values for the two primary transceiver rates, $R_1(p_2, \mathbf{w}_1)$ and $R_2(p_1, \mathbf{w}_1)$, must be on the boundary of $\mathcal{C}(P_{\mathrm{p}}^{\mathrm{o}})$. To show the latter, we rely on the monotonicity of $g(\cdot)$, as shown in the appendix. Part of the novelty of our work resides in this rigorous proof. Another novel aspect of our work is to prove that to solve (3.2.24), one can use the optimization problem (3.2.25) to calculate the value of $f^{-1}(\frac{\eta}{1-\alpha})$ for any feasible value of α. Without such a proof, one may not be able to solve (3.2.24) as the function $f^{-1}(\cdot)$ does not have a closed form.

3.3   Spectrum sharing-leasing system design

In the previous section, we implicitly assumed that the primary transceivers always have information to exchange. If the traffic of this network is bursty, i.e., if the primary transceivers have data to exchange with a certain probability, then the two methods proposed earlier cannot benefit from those instances where the primary network is inactive. In this section, the idea of spectrum sharing-leasing is proposed in order to increase the secondary network throughput. This aim is accomplished by considering that the secondary transceivers can use the spectrum during the whole time frame, when the primary transceivers have no data to transmit. In this case, the primary network leases out the whole bandwidth to the secondary network. We refer to this mode as leasing mode, in order to emphasize that the primary network allows the secondary transceivers to exploit almost the whole time frame when the primary transceivers are inactive. In exchange for providing this opportunity to the secondary

transceivers, the primary network is also able to increase its minimum rate threshold, thereby offering higher balanced rates to its transceivers whenever they have data to exchange. In order to further explain this idea, we consider the same network structure as the one introduced in the previous section. The only difference is the structure of the time frame. Let us assume that at the beginning of each time frame, a small portion of the time is reserved for the secondary network to sense and detect the presence of the transceivers in the primary network. This reserved time interval is called sensing time and is denoted as T_s. If the secondary transceivers does not detect the presence of the primary network, they switch to the leasing mode and utilize the remainder of the time frame. However, if the presence of the primary network is detected, the secondary transceivers switch to the sharing mode as described in the previous subsections, see Fig. 3.3 for more details.
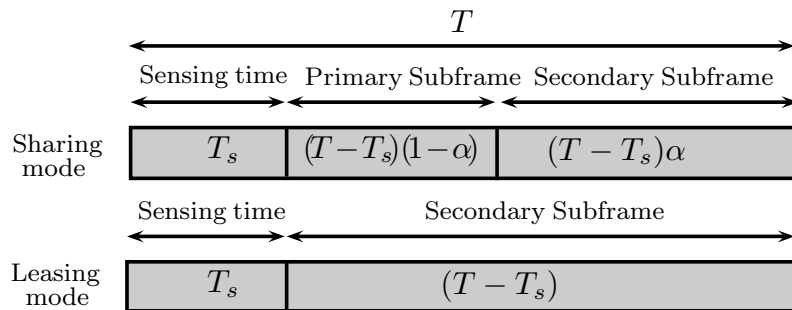
Figure 3.3: Cognitive leasing-sharing scheme.

In the sharing mode, the primary and secondary transceivers establish a two-way communication in their corresponding time intervals but, in the leasing mode, only the secondary transceivers communicate with each other. We should mention that all of the notations defined in the previous section for the primary and secondary network parameters are still valid. Depending on the status of the primary transceivers, two

optimization problems are considered as

Leasing mode

$$\underset{P_{\text{s}} \geq 0}{\text{maximize}} \ (1 - \frac{T_s}{T}) \, g(P_{\text{s}})$$

subject to $P_{\text{s}} \leq P_T$ (3.3.1)

Sharing mode

$$\underset{0 \leq \alpha \leq 1}{\text{maximize}} \ (1 - \frac{T_s}{T})\alpha \, g\left(\frac{P_T - (1-\alpha)f^{-1}(\frac{\eta}{1-\alpha})}{\alpha}\right)$$

subject to $(1-\alpha)(1 - \frac{T_s}{T})f(\frac{P_T}{1-\alpha}) \geq \eta.$ (3.3.2)

The solutions to both problems in (3.3.1) and (3.3.2) were presented in the previous sections. Using the results of the first approach, the optimal values of the average secondary transceiver maximum balanced rates, when the primary transceivers are passive, is $g(P_T)$. Furthermore, when the primary transceivers are active the optimal average secondary transceiver maximum balanced rates and the time sharing factor are denoted by $\alpha^{\text{o}}g(P_{\text{s}}^{\text{o}}(\alpha^{\text{o}}))$ and α^{o}, respectively, where $P_{\text{s}}^{\text{o}}(\alpha^{\text{o}})$ is the corresponding optimal total power consumed in the secondary subframe obtained by solving (3.3.2). We refer to the former case as the leasing mode of cooperation while the latter is called as the sharing mode.

Let us assume that the probability of the primary transceivers being active in each time frame is q. To consider the problem of detecting the signal of the primary transceiver, where the detection is performed at one of the secondary transceivers, we denote the probability of detection and the probability of false alarm as P_d and P_f, respectively. Assuming an energy detection scheme, the relationship between these

two probabilities, for a given sensing time T_s, is shown to be [99]

$$P_d = Q(\frac{Q^{-1}(P_f) - \sqrt{N_s}\,\theta}{\sqrt{2\,\theta + 1}}) \tag{3.3.3}$$

where, $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{t^2}{2})dt$, $N_s = T_s\,f_s$ is the number of samples taken during the sensing time, f_s is the sampling frequency and θ is the SNR received at the secondary transceiver which is in charge of detecting the presence of the primary transceivers signals. One can write the average secondary transceiver maximum balanced rates in the sharing-leasing and the sharing modes, respectively, as

$$\text{Sharing-Leasing rate} = (1 - q)(1 - P_f)g(P_T) + P_d\,q\alpha^\circ g(P_s^\circ) \tag{3.3.4}$$

$$\text{Sharing rate} = \alpha^\circ g(P_s^\circ(\alpha^\circ)). \tag{3.3.5}$$

Compared to the sharing mode, the gain obtained by using the sharing-leasing approach is then given by

$$\text{gain} = (1-q)(1-P_f)\Big(g(P_T) - \alpha^\circ g(P_s^\circ)\Big) + \Big((P_d - 1)q + P_f(q - 1)\Big)\alpha^\circ g(P_s^\circ). \tag{3.3.6}$$

where (3.3.6) is obtained by subtracting (3.3.5) from (3.3.4). One can see that, for given P_T, θ, N_s and a given channel realization, there are a pairs of (P_d, P_f) which results in a positive value for the gain in (3.3.6). In a special case, where the received SNR from the primary transceivers θ during the sensing time, or the frequency of sampling f_s is large enough, then it follows from (3.3.3) that $P_d = 1$ and $P_f = 0$. Replacing $P_d = 1$ and $P_f = 0$ in (3.3.6), will lead to the following equation for the gain obtained from sharing-leasing method when compared to the sharing mode

$$\text{gain} = (1 - q)\Big(g(P_T) - g(P_s^\circ)\Big). \tag{3.3.7}$$

This gain is always greater than zero since the average secondary transceiver maximum balanced rates, when all the resources are allocated to the secondary transceivers,

i.e., $g(P_T)$, is larger than that of the secondary transceivers when the resources are shared between the primary and secondary transceivers, i.e., $\alpha^o g(P_s^o(\alpha^o))$. Therefore, using sharing-leasing approach leads to a higher value for the average secondary transceiver maximum balanced rates, compared to the sharing approach, thereby resulting in utilizing the available resources more effectively. The increase in the average secondary transceiver maximum balanced rates in the sharing-leasing approach for any feasible values of η in (3.2.24), depends on the probability of the primary transceivers being active. This means that the lower the probability of the primary network being active is, the higher the average secondary transceiver maximum balanced rates will be. In general, the received SNR, θ from the primary transceiver during the sensing time, or the number of samples N_s may not be large enough, hence we need to evaluate the gain obtained in (3.3.6) for different values of θ and N_s. To do so, we use numerical simulations to study the effect of choosing different values for θ and N_s on the average gain.

Remark: We wrap up this section by emphasizing that since in all three resource allocation schemes presented, the underlying two-way relay beamforming is the one presented in [78], [100], [44], in each sub-frame, the relays collectively consume half of the total transmit power corresponding to that subframe. It is thus reasonable to assume that each relay, in average, consumes $\frac{1}{nr}$ fraction of half of the total power budget. Hence the proposed schemes provide a guideline for choosing the maximum average total transmit power for each relay.

3.4 Simulation Results

We consider a network of $n_{\text{r}} = 10$ relays and two transceiver pairs. We assume that the distance between the two transceivers in each pair is 2 units of distance[10]. We also assume that the relays are randomly located between the two transceivers. Indeed, we assume that the x-coordinate of each relay is uniformly distributed between the two transceivers and its y-coordinate has a Gaussian probability density with the variance equal to 1 unit distance. Hence, the channel coefficient between each transceiver and each relay is drawn from a complex Gaussian random distribution where its variance is proportional to l^{ρ}, where l is the distance between the transceiver and that relay and ρ is the path-loss factor. Moreover, the noises at the transceivers as well as those at the relays are assumed to be i.i.d with unit variance.

3.4.1 Separate power constraint per network

Fig. 3.4 illustrates the average maximum balanced rate of the secondary transceivers versus η, for different values of P_{s}, when $P_{\text{p}} = 30$ (dBW) is chosen. We can see that the average maximum balanced rates of the secondary transceivers decrease linearly when the primary demand η increases. This effect can be explained using (3.2.3), where the relationship between the average secondary transceivers balanced rates and η is linear. This figure confirms that when P_{s} and P_{p} are fixed, if the primary pair rate demand η is too large such that the design problem becomes infeasible, none of the two pairs can exploit the spectral and temporal resources to transmit their information symbols. Moreover, for sufficiently small values of η, the design problem

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[10]The chosen unit of the distance does not affect our discussion on the performance of the proposed schemes.
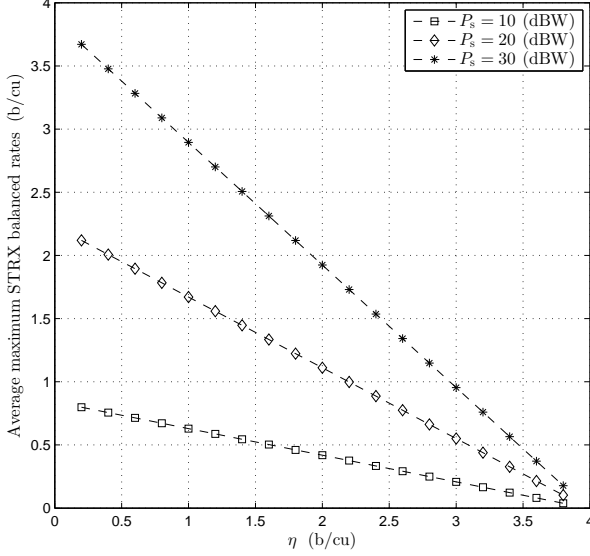
Figure 3.4: Average maximum balanced rates of the secondary transceivers versus η for different values of P_s, when $P_p = 30$ (dBW)
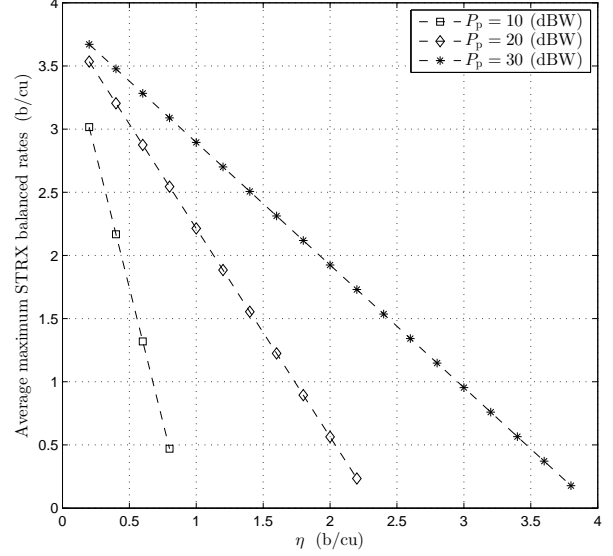
Figure 3.5: Average maximum balanced rates of the secondary transceivers versus η for different values of P_p, when $P_s = 30$ (dBW).

is always feasible, meaning that the primary pair is served with its rate demand equal to η and the secondary transceivers communicate with the largest possible average balanced rates. Furthermore, we can see form Fig. 3.4 that as P_s is increased, the secondary transceivers achieve higher balanced rates.

In Fig. 3.5, we plot the average maximum balanced rates of the secondary pair versus η for different values of P_p, when $P_s = 30$ (dBW). One can see from this figure that as P_p is increased, the design problem can satisfy higher primary pair demand η. It is worth mentioning that, when P_s is fixed, as η approaches zero the average maximum balanced rates of the secondary transceivers becomes equal. This is because of the fact that when η approaches zero, the optimal value of the time sharing

factor becomes equal to 1. This means that all the spectral and temporal resources are allocated to the secondary transceivers, thereby increasing the average maximum secondary transceiver balanced rates to a value which is independent of P_p. Fig. 3.6 shows the probability of feasibility of the resource sharing problem under individual per network power constraint for different values of η and P_p. As expected, this figure shows that as the minimum required data rate of the primary network is increased, the power allocated to this network has to be increased to ensure the problem is feasible. Fig. 3.7 illustrates the average maximum balanced rates of the secondary transceivers versus P_s, for different values of P_s, when $\eta = 3$ (b/cu). It is obvious that when the design problem is feasible, increasing P_s leads to a higher value for the average maximum balanced rates of the secondary transceivers. Furthermore, considering the optimal values of the design parameters throughout the simulations, we have observed that each transceiver in the primary (secondary) network consumes, in *average*, a quarter of the total power consumed in the primary (secondary) subframe. Furthermore, for *any channel realization*, the relays collectively consume half of the total power consumed in the primary subframe and half of the total power consumed in the secondary subframe. These results are in agreement with, and can be explained by the studies of [43, 78] and [44].

3.4.2 Total power constraint for both networks

Considering a total power constraint for the two networks, Fig. 3.8 shows the average maximum balanced rates of the secondary transceivers is increasing in P_T, meaning that for a fixed value of η, the power allocated to the secondary pair increases, when P_T increases. However, as η is increased, the maximum achievable balanced rates
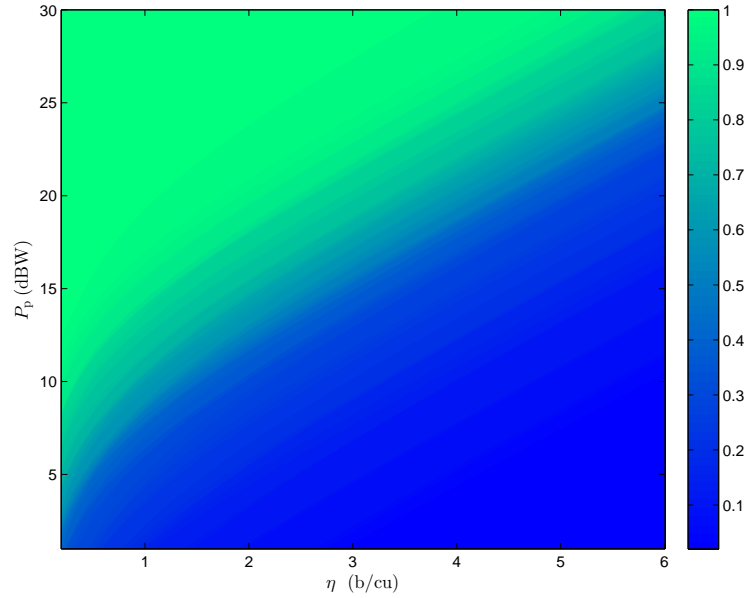
Figure 3.6: Probability of feasibility of the resource sharing problem under individual per network power constraints

of the two secondary transceivers is decreased. This means that as η is increased, the primary transceivers need to exploit the spectral and temporal resources further, hence less and less resources remain for information exchange between the secondary transceivers.

In Fig. 3.9, we plot the average maximum achievable balanced rates of the secondary transceivers versus η, for different values of P_T. We observe that the average maximum balanced rates of the secondary transceivers decreases as the rate demand of the primary pair, η increases. However, increasing P_T can compensate the effect of increasing η by providing more power to the secondary transceivers. This means that when the amount of the total available power increases, the primary transceivers rate demand can be satisfied easier. For instance, increasing η from 3 (b/cu) to
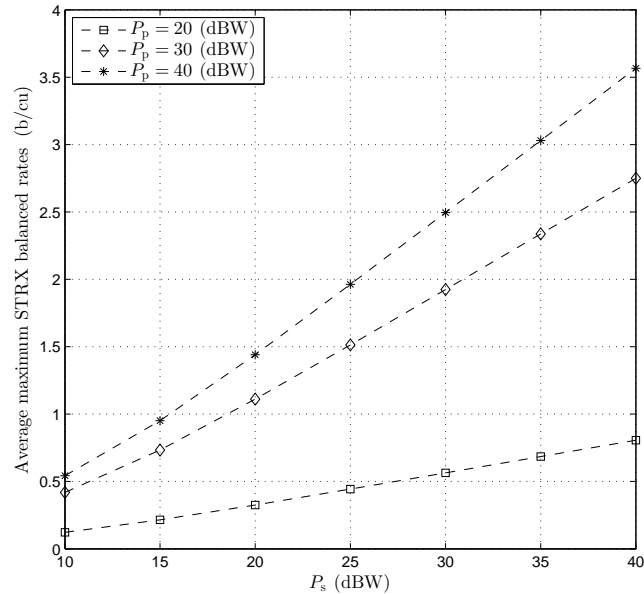
Figure 3.7: Average maximum balanced rates of the secondary transceivers versus P_{s} for different values of P_{p}, when $\eta = 3$ (b/cu).

4 (b/cu), when $P_T = 25$ (dBW), decreases the secondary pair rates roughly by 0.2 (b/cu), however this decrease of the secondary transceivers rates can be recovered by adding roughly 5 (dBW) to P_T.

3.4.3 Spectrum sharing-leasing system design

Figs. 3.10 and 3.11 show the average gain of the sharing-leasing scheme, as an image, versus θ and N_s, for two different values of q, when $P_T=40$ (dBW). We can see from these figures that this gain can indeed be positive if N_s is properly chosen. In other words, given q, we can design and use a detector at one of the two secondary transceivers such that the gain achieved by using the sharing-leasing scheme, compared to the sharing scheme, is positive.
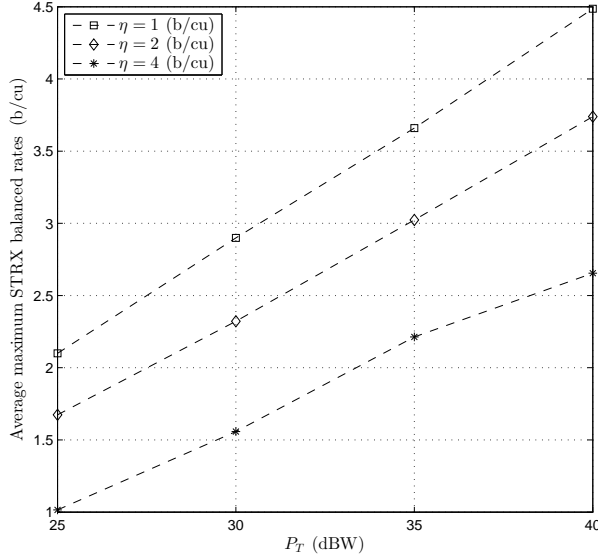
Figure 3.8: Average maximum balanced rates of the secondary transceivers versus P_T, for different values of η.

Figure 3.9: Average maximum balanced rates of the secondary transceivers versus η, for different values of P_T.

Fig. 3.12 demonstrates the effect of the probability q of the primary pair being active, on the maximum achievable balanced rates of the secondary transceivers. In this figure, we plot the maximum achievable balanced rates of the secondary transceivers versus q, for different values of η, when P_T=30 (dBW). This figure shows that when the primary pair utilizes the shared resources more frequently, i.e., when q is large, the average maximum balanced rates of the secondary transceivers are lower compared to the case when the primary pair is less active, i.e., when q is small. This means that increasing q reduces the average maximum balanced rates of the secondary transceivers. Furthermore, the slope of the curve of the average maximum balanced rates of the secondary transceivers versus $1 - q$ increases, when the value of η is increased.
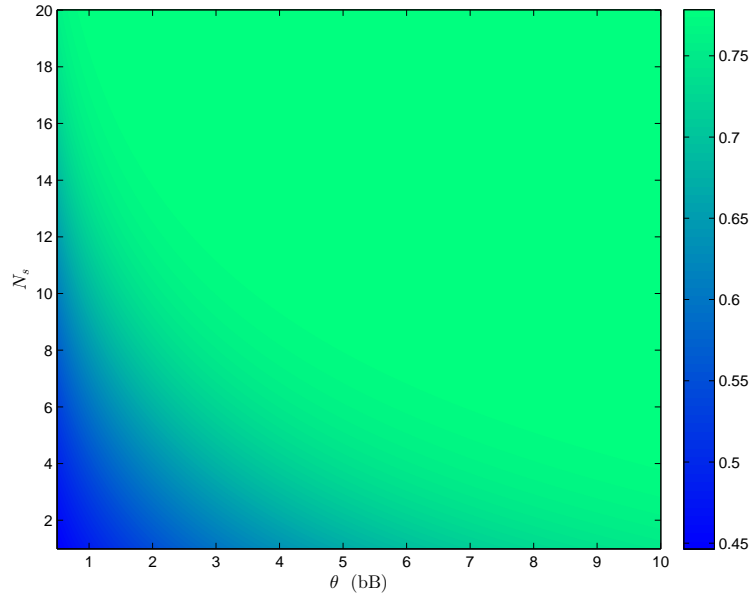
Figure 3.10: Average gain versus θ and N_s for given $P_T = 40\,(\text{dBW})$, $P_f = 0.01$ and $q=0.2$

Fig. 3.13 depicts the average maximum balanced rates of the secondary transceivers versus $1 - q$, for $\eta = 4$ (b/cu) and for different values of P_T. We can see that increasing P_T, increases the highest achievable balanced rates of the secondary pair. This is the common result of both the second and the third approaches as increasing the total available power helps to increase the average maximum balanced rates of the secondary transceivers. These discussions imply that the sharing-leasing approach outperforms the sharing approach when $q > 0$.

3.5 Conclusions

We studied optimal resource sharing between two pairs of transceivers which exploit a network of n_r relays. We considered a communication framework in which the primary
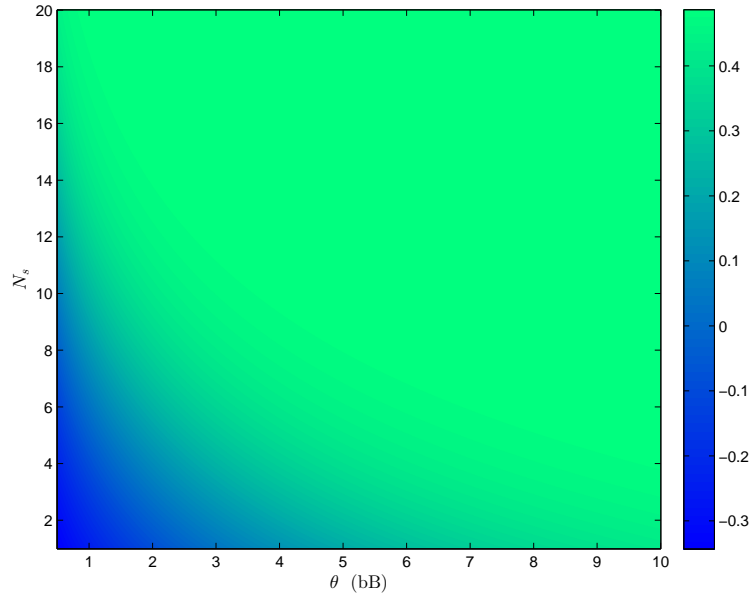
Figure 3.11: Average gain versus θ and N_s for given $P_T = 40\,(\text{dBW})$, $P_f = 0.01$ and $q=0.5$

pair leases out a portion of its spectral and temporal resources to the secondary pair in exchange for using the relays to guarantee a minimum data rate for the primary transceivers. We proposed three approaches with different pros and cons. In each approach, we formulated an optimization problem in order to optimally calculate the corresponding design parameters.

As the first approach, we maximize the secondary transceivers rates while guaranteeing a minimum data rate for the primary transceivers and limiting the total powers consumed in the primary and the secondary network to be less than predefined thresholds. We showed that for the primary and the secondary transceivers, the design problem can be simplified into two SNR balancing problems, each with its own semi-closed-form solution.

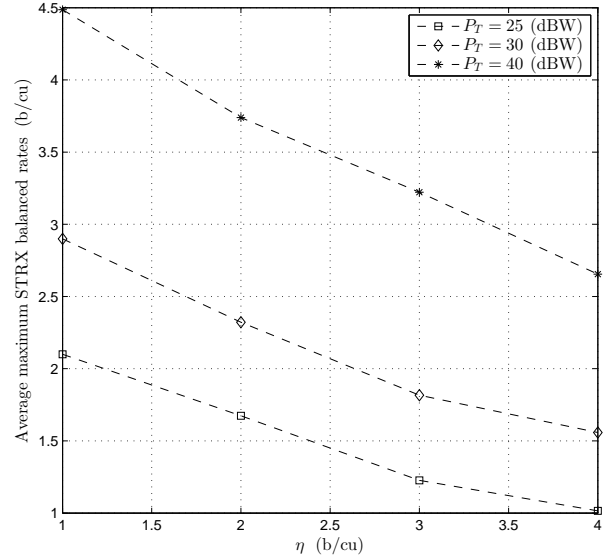Figure 3.12: Average maximum balanced rates of the secondary transceivers versus $1 - q$, for different values of η, when $P_T = 30$ (dBW).
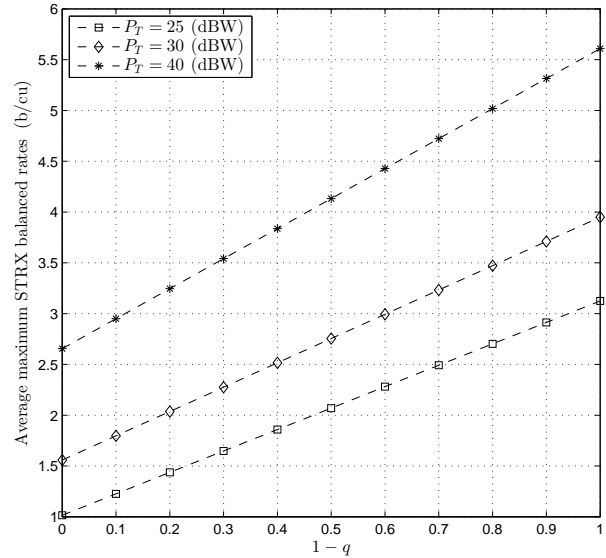
Figure 3.13: Average maximum balanced rates of the secondary transceivers versus $1 - q$, for different values of P_T, when $\eta = 4$ (b/cu).

In the second approach, we replaced the two separate constraints on the total power consumed in the primary and secondary networks used in the first approach, with a constraint on the total power consumed in the whole time frame. We proved that the optimization problem in this approach can be simplified to a simple line search problem with low complexity. Furthermore, we showed that the second approach is superior to the first approach as in the latter approach one can optimally allocate the available power between the primary and the secondary transceivers.

The third approach combines the two aforementioned methods to materialize spectrum leasing and sharing for the case when the primary network is active with a certain probability.

3.6 Proving that $g(P_s)$ is increasing in P_s

Let us define

$$\tilde{g}(p_3, P_s) \triangleq p_3(P_s - 2p_3) \times$$

$$\mathbf{h}_s^H(2p_3\mathbf{E}_1 + (P_s - 2p_3)\mathbf{E}_2 + \mathbf{I})^{-1}\mathbf{h}_s. \tag{3.6.1}$$

Note that as follows from (3.2.10), for a given P_s, the maximum balanced SNR of the secondary network is obtained by finding the maximum value of $\tilde{g}(p_3, P_s)$, when $p_3 \in [0 \quad \frac{P_s}{2}]$. We herein prove that the function $g(P_s)$, which is given by

$$\begin{aligned} g(P_s) \quad &= \frac{1}{2}\log_2(1 + \tilde{g}(p_3^o(P_s), P_s)) \\ &= \frac{1}{2}\log_2(1 + \max_{0 \le p_3 \le \frac{P_s}{2}} \tilde{g}(p_3, P_s)), \end{aligned} \tag{3.6.2}$$

is a monotonically increasing function of P_s, meaning that if $P_s^1 > P_s^2$ then, $g(P_s^1) > g(P_s^2)$. In order to show this, we first prove that, for any given $p_3 > 0$, the first derivative of $\tilde{g}(p_3, P_s)$ with respect to P_s is positive. To do so, we recall that

$$\mathbf{\Psi}(p_3, P_s) = (2p_3\mathbf{E}_1 + (P_s - 2p_3)\mathbf{E}_2 + \mathbf{I})^{-1}, \tag{3.6.3}$$

and write

$$\begin{aligned} \frac{\partial \tilde{g}(p_3, P_s)}{\partial P_s} &= p_3\mathbf{h}_s^H\mathbf{\Psi}(p_3, P_s)\mathbf{h}_s \\ &\quad + p_3(P_s - 2p_3)\mathbf{h}_s^H\frac{\partial \mathbf{\Psi}(p_3, P_s)}{\partial P_s}\mathbf{h}_s. \end{aligned} \tag{3.6.4}$$

As $\mathbf{\Psi}(p_3, P_s)$ is a diagonal matrix, we can write

$$\frac{\partial \mathbf{\Psi}(p_3, P_s)}{\partial P_s} = -\mathbf{\Psi}^2(p_3, P_s)\,\mathbf{E}_2. \tag{3.6.5}$$
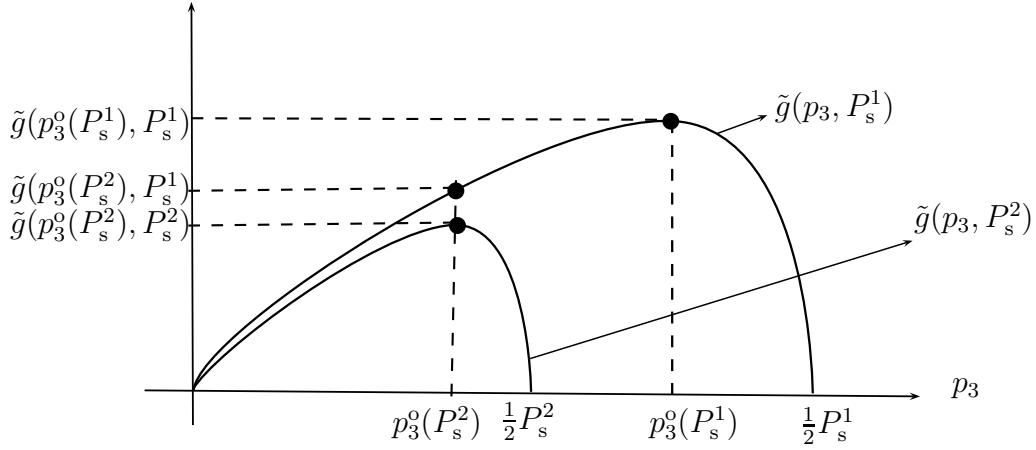
Figure 3.14: Descriptive diagram of the two functions $\tilde{g}(p_3, P_{\mathrm{s}}^1)$ and $\tilde{g}(p_3, P_{\mathrm{s}}^2)$ versus p_3, when $P_{\mathrm{s}}^1 \geq P_{\mathrm{s}}^2$.

Using (3.6.5) and the auxiliary variable $\tilde{p}_3 = P_{\mathrm{s}} - 2p_3$, we can write (3.6.4) as

$$
\begin{aligned}
\frac{\partial \tilde{g}(p_3, P_{\mathrm{s}})}{\partial P_{\mathrm{s}}} &= p_3 \mathbf{h}_{\mathrm{s}}^H \mathbf{\Psi}(p_3, P_{\mathrm{s}}) \mathbf{h}_{\mathrm{s}} - p_3 \tilde{p}_3 \mathbf{h}_{\mathrm{s}}^H \mathbf{\Psi}^2(p_3, P_{\mathrm{s}}) \, \mathbf{E}_2 \mathbf{h}_{\mathrm{s}} \\
&= p_3 \mathbf{h}_{\mathrm{s}}^H \mathbf{\Psi}^2(p_3, P_{\mathrm{s}}) \left(\mathbf{\Psi}^{-1}(p_3, P_{\mathrm{s}}) - \tilde{p}_3 \mathbf{E}_2 \right) \mathbf{h}_{\mathrm{s}} \\
&= p_3 \mathbf{h}_{\mathrm{s}}^H \mathbf{\Psi}^2(p_3, P_{\mathrm{s}})(2p_3 \mathbf{E}_1 + \mathbf{I}) \mathbf{h}_{\mathrm{s}} \, .
\end{aligned}
\tag{3.6.6}
$$

It follows from (3.6.6), that $\frac{\partial \tilde{g}(p_3, P_{\mathrm{s}})}{\partial P_{\mathrm{s}}} > 0$, for $p_3 > 0$. Hence, $\tilde{g}(p_3, P_{\mathrm{s}})$ is an increasing function of P_{s}, when $p_3 > 0$. Moreover, it has been proven in [43], that for any given P_{s} when $p_3 \in [0 \ \frac{P_{\mathrm{s}}^1}{2}]$, the function $\tilde{g}(p_3, P_{\mathrm{s}})$ is a concave function of p_3, with a unique maximum. Fig. 3.14 is a descriptive diagram of the two functions $\tilde{g}(p_3, P_{\mathrm{s}}^1)$ and $\tilde{g}(p_3, P_{\mathrm{s}}^2)$ versus p_3, when $P_{\mathrm{s}}^1 \geq P_{\mathrm{s}}^2$.

The optimal value of p_3, as an implicit function of P_{s}, is given by

$$
p_3^{\mathrm{o}}(P_{\mathrm{s}}) = \arg \max_{0 \leq p_3 \leq \frac{P_{\mathrm{s}}}{2}} \tilde{g}(p_3, P_{\mathrm{s}}).
\tag{3.6.7}
$$

Using (3.6.7), the following inequality holds true:

$$g(P_{\rm s}^1) = \frac{1}{2}\log_2(1 + \tilde{g}(p_3^{\rm o}(P_{\rm s}^1), P_{\rm s}^1)) \geq$$

$$\frac{1}{2}\log_2(1 + \tilde{g}(p_3, P_{\rm s}^1)) \quad \text{for any } p_3 \in [0 \quad \frac{P_{\rm s}^1}{2}] \tag{3.6.8}$$

As $p_3^{\rm o}(P_{\rm s}^2) \leq \frac{P_{\rm s}^2}{2} \leq \frac{P_{\rm s}^1}{2}$, we can use (3.6.8) to write

$$
\begin{aligned}
g(P_{\rm s}^1) &= \frac{1}{2}\log_2(1 + \tilde{g}(p_3^{\rm o}(P_{\rm s}^1), P_{\rm s}^1)) \\
&\geq \frac{1}{2}\log_2(1 + \tilde{g}(p_3^{\rm o}(P_{\rm s}^2), P_{\rm s}^1)) \\
&\geq \frac{1}{2}\log_2(1 + \tilde{g}(p_3^{\rm o}(P_{\rm s}^2), P_{\rm s}^2)) = g(P_{\rm s}^2)
\end{aligned}
\tag{3.6.9}
$$

where the second inequality follows from the fact that for any given $p_3 > 0$, $\tilde{g}(p_3, P_{\rm s})$ is monotonically increasing in $P_{\rm s}$. This completes the proof that $g(P_{\rm s})$ is an increasing function of $P_{\rm s}$.

Chapter 4

Optimal Spectrum Leasing and Resource Sharing in Multi-Carrier Setup

In the previous chapter, we studied and modeled the problem of resource sharing between two bidirectional relay networks under total and per network transmit power constraints in a single carrier setup. We herein study the problem of resource sharing between a high-priority (called primary) transceiver pair and a low-priority (secondary) transceiver pair in a multi-carrier scenario. Aiming to optimally calculate the design parameters of the primary and secondary networks, we study two different approaches.

In the first approach, we consider a multi-relay scenario where we maximize the average sum-rate of the secondary transceivers subject to two spectral power masks to limit the total power consumed in each network over each subchannel while guaranteeing the primary transceivers' rate demand. We show that the design problem turned into a linear programming problem.

In the second approach, we consider the secondary network sum-rate maximization subject to two constraints on the total power consumed in each network over all

subchannles while guaranteeing a certain rate demand for the primary transceivers. In this approach, we considered a single-relay and a multi-relay scenario and proposed iterative based convex search solutions to each scenario to obtain the design parameters.

For the single relay case, we use a high SNR approximation and develop an iterative convex search algorithm which exploits the biconcavity of the approximated objective function in terms of the design parameters, namely the vector of the total powers allocated to each network over different subchannels and the vector of rates of the primary network over all subchannels.

In the case of a multi-relay scenario, we expand the design parameters to five vectors and showed that the problem is concave in any parameter vector, given the other four vectors are fixed. As such, we propose an iterative convex search algorithm to introduce a solution to the underlying problem.

The contributions of this chapter can be summarized as listed below:

- We study and model the problem of resource sharing between two *bidirectional multi-carrier* relay networks. We maximize the average sum-rate of the two secondary transceivers under power spectral masks for each networks or under a per-network total power constraint, while the average rate of the primary transceivers is guaranteed to be above a given threshold.

- In the case where we apply the two power mask constraints on the design problem, we prove that the proposed optimization problem turns into a *linear programming* for which the optimal solution to the problem can be obtained by almost all optimization tool boxes.

- In the case where there are two constrains on the total power consumed in each network, we show that the design problem is not convex. Then, we provide two alternate convex search solutions, one for the single-relay case and one for the multi-relay case.

4.1 System Model

We consider a communication framework where two transceiver pairs exchange their information symbols via sharing their spectral, temporal, and physical resources. We assume that the frequency resources are composed of k orthogonal subbands, that the temporal resources are time-slots during which each pair is active, and that the physical resources consist of n_r relay nodes. The two transceivers in each pair wish to communicate with each other by exploiting the available resources through cooperation with the other pair. Assuming that there is no direct link between the two transceivers in each pair, they need to use the relay infrastructure to exchange their information. One of the transceiver pairs, which owns the spectral resources, is called the primary pair. In our scheme, it is required to serve the primary transceiver pair with minimum average sum-rate between its transceivers. The secondary pair owns the relay infrastructure and is allowed to use the remaining resources to maximize the sum-rate of its transceivers. The assumption that the primary network owns only spectrum and the secondary network owns only the relays, is a common assumption in the literature on spectrum leasing based cognitive radio networks, see for example [11]. This resource sharing scheme is implemented using a cooperation scheme between the two pairs as explained in the sequel. The primary pair can establish a link between

its transceivers using the secondary network relay resources. In exchange for this cooperation, the primary pair will lease out the remaining resources to the transceivers in the secondary pair in order to exchange their information symbols. Moreover, the two pairs communicate in one time frame with length T. In the mth subchannel, each time frame is divided into four non-overlapping time intervals, called time slots. In the first and second time slots, the primary and the secondary transceivers transmit their information symbols to the relays, respectively. In the third time slot, the relays broadcast the amplitude- and phase-adjusted versions of the signals they received in the first time slot to the primary transceivers. Similarly, in the fourth time slot, the relays broadcast the amplitude- and phase-adjusted versions of the signals they received in the second time slot to the secondary transceivers. We note that in the MABC relaying protocol, the length of the first and the third time slots must be the same. Similarly, the second and the fourth time slots have the same duration. We refer to the pair of the first and the third time slots (and the second and the fourth time slots), the primary (secondary) subframe. Since we assume that the relays operate in half-duplex mode and thus, the relays cannot receive and transmit data at the same time, our proposed model guarantee that none of the nodes is required to transmit and received, simultaneously. As shown in Fig. 4.1, in the mth subchannel, the fraction of the time that the primary pair leases its RF spectrum to the secondary transceivers is denoted as α_m, for $m = 1, 2, ..., k$. We note that in our scheme, when a node is required to stop transmission in a certain subchannel, it can replace the baseband signal corresponding to that subchannel to zero without affecting the signal of its other active subchannels. Hence, any node will stop transmission only if all of its subchannels are deactivated.

$$T$$

Relays receiving data		Relays transmitting data		
$T(1-\alpha_1)/2$ PTRXs $\to$ relays	$T\alpha_1/2$ STRXs $\to$ relays	$T(1-\alpha_1)/2$ relays $\to$ PTRXs	$T\alpha_1/2$ relays $\to$ STRXs	subchannel 1
$T(1-\alpha_2)/2$ PTRXs $\to$ relays	$T\alpha_2/2$ STRXs $\to$ relays	$T(1-\alpha_2)/2$ relays $\to$ PTRXs	$T\alpha_2/2$ relays $\to$ STRXs	subchannel 2
$\vdots$		$\vdots$		
$T(1-\alpha_k)/2$ PTRXs $\to$ relays	$T\alpha_k/2$ STRXs $\to$ relays	$T(1-\alpha_k)/2$ relays $\to$ PTRXs	$T\alpha_k/2$ relays $\to$ STRXs	subchannel k
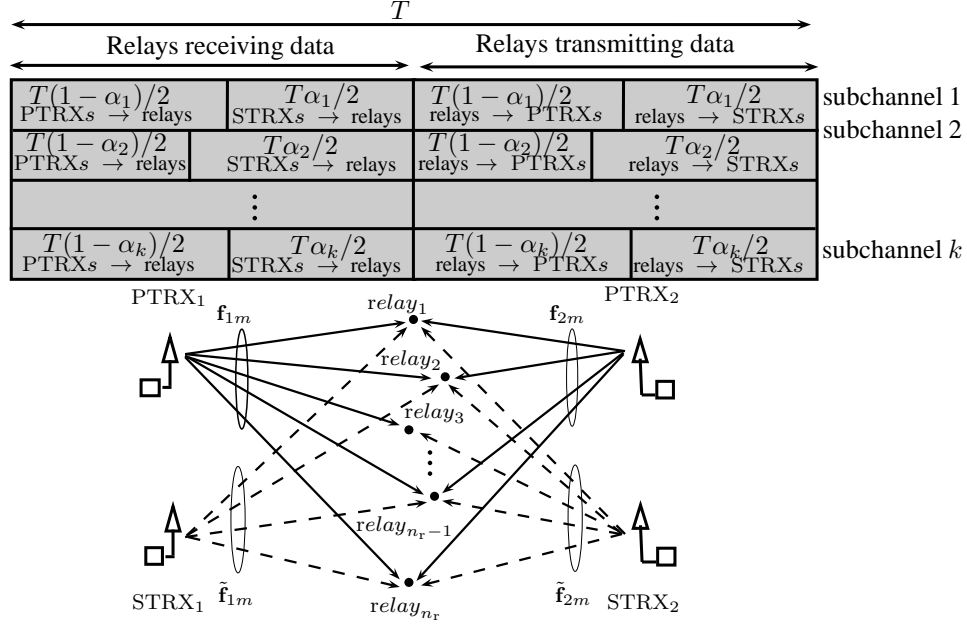
Figure 4.1: Sharing resources between two bidirectional relay networks.

The primary (secondary) transceivers PTRX$_1$ and PTRX$_2$ (STRX$_1$ and STRX$_2$), transmit their mth information symbols denoted as s_{1m} and s_{2m} ($\tilde{s}_{1m}$ and $\tilde{s}_{2m}$), respectively, in the mth subchannel, to the relays with transmit powers p_{1m} and p_{2m} ($\tilde{p}_{1m}$ and $\tilde{p}_{2m}$). The transmitted symbols of all transceiver pairs in all subchannels are considered to be zero-mean, independent random variables with variance equal to 1, i.e., $E\{|s_{1m}|^2\} = E\{|s_{2m}|^2\} = E\{|\tilde{s}_{1m}|^2\} = E\{|\tilde{s}_{2m}|^2\} = 1$. We assume that the frame length is much smaller than the coherence time of the channel, implying that during each transmission frame, the channel specifications do not change. In the mth subchannel, the $n_{\mathrm{r}} \times 1$ vector of the complex channel coefficients corresponding to the links between PTRX$_1$ (PTRX$_2$) and the relays is represented as $\mathbf{f}_{1m}$ ($\mathbf{f}_{2m}$). Similarly, in the mth subchannel, the $n_{\mathrm{r}} \times 1$ vector of the complex channel coefficients corresponding to the links between STRX$_1$ (STRX$_2$) and the relays, is denoted as

$\tilde{\mathbf{f}}_{1m}$ ($\tilde{\mathbf{f}}_{2m}$). We herein assume that both transceivers in the two pairs are synchronized both at symbol level and at carrier level. We do not address time or frequency synchronization as this topic does not fit in the scope of this work. The underlying communication scheme is shown in Fig. 4.1.

As a bandwidth efficient transmission scheme, we utilize the MABC protocol to establish a bidirectional communication between the transceivers in each pair. In this protocol, each subframe consists of two equal-length time-slots as explained earlier. Assuming that the transceivers operate in a half-duplex mode, in the first time slot, the two transceivers in each pair simultaneously transmit their information symbols to the relays. In the second time slot, the relays broadcast, to the transceivers, amplified and phase-adjusted versions of the signals they received in the previous time slot. Note that even if there is a direct link between the two transceivers in each pair, the transceivers cannot benefit form such a direct link in an MABC based bidirectional relaying scheme as they operate in half-duplex mode (i.e., they cannot receive and transmit at the same time). We assume that the ith relay, $i \in \{1, 2 \cdots, n_{\mathrm{r}}\}$, uses the ith element of the $n_{\mathrm{r}} \times 1$ complex vector $\mathbf{w}_m$ ($\tilde{\mathbf{w}}_m$) to participate in building a network beamformer over the mth subchannel, thereby enabling communication between PTRX$_1$ and PTRX$_2$ (STRX$_1$ and STRX$_2$). In the mth subchannel, the $n_{\mathrm{r}} \times 1$ vectors $\mathbf{x}_m$ and $\tilde{\mathbf{x}}_m$ of the signals received at the relays corresponding to the primary and secondary subframes can be written as

$$\mathbf{x}_m = \sqrt{p_{1m}}\, \mathbf{f}_{1m} s_{1m} + \sqrt{p_{2m}} \mathbf{f}_{2m} s_{2m} + \boldsymbol{\nu}_m, \qquad (4.1.1)$$

$$\tilde{\mathbf{x}}_m = \sqrt{\tilde{p}_{1m}}\, \tilde{\mathbf{f}}_{1m} \tilde{s}_{1m} + \sqrt{\tilde{p}_{2m}}\, \tilde{\mathbf{f}}_{2m} \tilde{s}_{2m} + \tilde{\boldsymbol{\nu}}_m \qquad (4.1.2)$$

Here, $\boldsymbol{\nu}_m$ ($\tilde{\boldsymbol{\nu}}_m$) is an $n_{\mathrm{r}} \times 1$ zero-mean complex Gaussian random vector, which represents the relay noise vector in the primary (in the secondary) subframe over the mth

subchannel. We assume that $E\{\boldsymbol{\nu}_m \boldsymbol{\nu}_m^H\} = E\{\tilde{\boldsymbol{\nu}}_m \tilde{\boldsymbol{\nu}}_m^H\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of size $n_r \times n_r$. We also note that p_{1m} and p_{2m} are the transmit powers of the transceivers in the primary network and $\tilde{p}_{1m}$ and $\tilde{p}_{2m}$ are the transmit powers of the transceivers in the secondary network. The relays use the AF relaying protocol, hence the $n_r \times 1$ vectors of the signals re-transmitted by the relays over the mth subchannel, corresponding to the primary and the secondary subframes, are given as $\mathbf{t}_m \triangleq \mathbf{w}_m \odot \mathbf{x}_m$ and $\tilde{\mathbf{t}}_m \triangleq \tilde{\mathbf{w}}_m \odot \tilde{\mathbf{x}}_m$, respectively. Let us define $\bar{q} = 1$, if $q = 2$ and, $\bar{q} = 2$, if $q = 1$. Using the following definitions, $\mathbf{F}_{1m} \triangleq \mathrm{diag}(\mathbf{f}_{1m})$, $\mathbf{F}_{2m} \triangleq \mathrm{diag}(\mathbf{f}_{2m})$, $\tilde{\mathbf{F}}_{1m} \triangleq \mathrm{diag}(\tilde{\mathbf{f}}_{1m})$, and $\tilde{\mathbf{F}}_{2m} \triangleq \mathrm{diag}(\tilde{\mathbf{f}}_{2m})$, we express signals received at the primary transceiver pairs over the mth subchannel as

$$y_{qm} = \mathbf{f}_{qm}^T \mathbf{t}_m + n_{qm} = \sqrt{p_{qm}}\mathbf{w}_m^H \mathbf{F}_{qm} \mathbf{f}_{qm} s_{qm} + \sqrt{p_{\bar{q}m}}\mathbf{w}_m^H \mathbf{F}_{qm} \mathbf{f}_{\bar{q}m} s_{\bar{q}m}$$
$$+ \mathbf{w}_m^H \mathbf{F}_{qm} \boldsymbol{\nu}_m + n_{qm} \qquad (4.1.3)$$

where $q \in \{1,2\}$ and y_{qm} is the signal received by the qth primary transceiver over the mth subchannel, n_{qm} is the received noise measured at the corresponding transceiver over the mth subchannel. We can also express signals received at the secondary transceiver pairs over the mth subchannel as

$$\tilde{y}_{qm} = \tilde{\mathbf{f}}_{qm}^T \tilde{\mathbf{t}}_m + \tilde{n}_{qm} = \sqrt{\tilde{p}_{qm}}\tilde{\mathbf{w}}_m^H \tilde{\mathbf{F}}_{qm} \tilde{\mathbf{f}}_{qm} \tilde{s}_{qm} +$$
$$\sqrt{\tilde{p}_{\bar{q}m}}\tilde{\mathbf{w}}_m^H \tilde{\mathbf{F}}_{qm} \tilde{\mathbf{f}}_{\bar{q}m} \tilde{s}_{\bar{q}m} + \tilde{\mathbf{w}}_m^H \tilde{\mathbf{F}}_{qm} \tilde{\boldsymbol{\nu}}_m + \tilde{n}_{qm} \qquad (4.1.4)$$

where $q \in \{1,2\}$ and $\tilde{y}_{qm}$ is the signal received at the qth secondary transceiver over the mth subchannel, $\tilde{n}_{qm}$ is the received noise measured at the corresponding transceiver over the mth subchannel. All the received noises in all subchannels are considered to be zero-mean complex Gaussian random processes with unit variance.

We consider a control channel between the primary and secondary networks for co-ordination. We also assume that both PTRX$_1$ and PTRX$_2$ have full knowledge of the channel vectors $\mathbf{f}_{1m}$, $\mathbf{f}_{2m}$ and the knowledge of the amplitude of different entries of $\tilde{\mathbf{f}}_{1m}$, and $\tilde{\mathbf{f}}_{2m}$, while STRX$_1$ and STRX$_2$ both know only their local channel state information (CSI) $\tilde{\mathbf{f}}_{1m}$ and $\tilde{\mathbf{f}}_{2m}$. The global CSI knowledge at the primary transceiver is an essential assumption in the design approaches provided in this chapter. Discussing methods of obtaining the CSI for all transceivers does not fit in the scope of this study. Partial or global CSI knowledge assumption is frequently used in the literature [43, 78, 80, 88, 89, 101]. Based on these assumptions, we show that each pair can obtain its corresponding optimal beamforming vector. Hence, the two transceivers in each pair can cancel out, from their received signals, the so-called self-interference signal (i.e., the first terms in (4.1.3) and (4.1.4)). After self-interference cancelation in the mth subchannel, the primary and the secondary transceivers can decode their corresponding desired information symbols in each subchannel. After self-interference cancelation at the primary and secondary transceivers, the remaining signals can then be written as

$$z_{qm} = \sqrt{p_{\bar{q}m}}\mathbf{w}_m^H\mathbf{F}_{qm}\mathbf{f}_{\bar{q}m}s_{\bar{q}m} + \mathbf{w}_m^H\mathbf{F}_{qm}\boldsymbol{\nu}_m + n_{qm} \qquad (4.1.5)$$

$$\tilde{z}_{qm} = \sqrt{\tilde{p}_{\bar{q}m}}\tilde{\mathbf{w}}_m^H\tilde{\mathbf{F}}_{qm}\tilde{\mathbf{f}}_{\bar{q}m}\tilde{s}_{\bar{q}m} + \tilde{\mathbf{w}}_m^H\tilde{\mathbf{F}}_{qm}\tilde{\boldsymbol{\nu}}_m + \tilde{n}_{qm}. \qquad (4.1.6)$$

where $q \in \{1, 2\}$. Note that z_{qm} ($\tilde{z}_{qm}$) is the residual signal at the qth primary (secondary) transceiver over the mth subchannel. Moreover, in the mth subchannel, the total relay transmit power corresponding to the primary and secondary subframes

are, respectively, given by

$$p_m^{\mathrm{r}}(p_{1m}, p_{2m}, \mathbf{w}_m) \triangleq E\{\mathbf{t}_m^H \mathbf{t}_m\} = \mathbf{w}_m^H(p_{1m}\mathbf{D}_{1m} + p_{2m}\mathbf{D}_{2m} + \mathbf{I})\mathbf{w}_m \qquad (4.1.7)$$

$$\tilde{p}_m^{\mathrm{r}}(\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m) \triangleq E\{\tilde{\mathbf{t}}_m^H \tilde{\mathbf{t}}_m\} = \tilde{\mathbf{w}}_m^H(\tilde{p}_{1m}\tilde{\mathbf{D}}_{1m} + \tilde{p}_{2m}\tilde{\mathbf{D}}_{2m} + \mathbf{I})\tilde{\mathbf{w}}_m \qquad (4.1.8)$$

where the following definitions are used: $\mathbf{D}_{1m} \triangleq \mathbf{F}_{1m}\mathbf{F}_{1m}^H$, $\mathbf{D}_{2m} \triangleq \mathbf{F}_{2m}\mathbf{F}_{2m}^H$, $\tilde{\mathbf{D}}_{1m} \triangleq$ $\tilde{\mathbf{F}}_{1m}\tilde{\mathbf{F}}_{1m}^H$, and $\tilde{\mathbf{D}}_{2m} \triangleq \tilde{\mathbf{F}}_{2m}\tilde{\mathbf{f}}_{2m}^H$. Note that $\mathbf{D}_{1m}, \mathbf{D}_{2m}, \tilde{\mathbf{D}}_{1m}$, and $\tilde{\mathbf{D}}_{2m}$ are diagonal matrices. Using (4.1.7) and (4.1.8), we can express the total power consumed in the primary and secondary subframes over the mth subchannel as

$$P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \triangleq \sum_{q=1}^{2} p_{qm} + p_m^{\mathrm{r}}(p_{1m}, p_{2m}, \mathbf{w}_m)$$

$$= \sum_{q=1}^{2} p_{qm}(1 + \mathbf{w}_m^H \mathbf{D}_{qm}\mathbf{w}_m) + \mathbf{w}_m^H\mathbf{w}_m \qquad (4.1.9)$$

$$\tilde{P}_m(\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m) \triangleq \sum_{q=1}^{2} \tilde{p}_{qm} + \tilde{p}_m^{\mathrm{r}}(\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m)$$

$$= \sum_{q=1}^{2} \tilde{p}_{qm}(1 + \tilde{\mathbf{w}}_m^H \tilde{\mathbf{D}}_{qm}\tilde{\mathbf{w}}_m) + \tilde{\mathbf{w}}_m^H\tilde{\mathbf{w}}_m. \qquad (4.1.10)$$

Also, we use (4.1.5) and (4.1.6) to express the received SNRs corresponding to the signals received at the primary and secondary receivers over the mth subchannel, respectively, as

$$\mathrm{SNR}_{qm}^{\mathrm{p}}(p_{\bar{q}m}, \mathbf{w}_m) = \frac{p_{\bar{q}m}\mathbf{w}_m^H\mathbf{h}_m\mathbf{h}_m^H\mathbf{w}_m}{\mathbf{w}_m^H\mathbf{D}_{qm}\mathbf{w}_m + 1},$$

$$\mathrm{SNR}_{qm}^{\mathrm{s}}(\tilde{p}_{\bar{q}m}, \tilde{\mathbf{w}}_m) = \frac{\tilde{p}_{\bar{q}m}\tilde{\mathbf{w}}_m^H\tilde{\mathbf{h}}_m\tilde{\mathbf{h}}_m^H\tilde{\mathbf{w}}_m}{\tilde{\mathbf{w}}_m^H\tilde{\mathbf{D}}_{qm}\tilde{\mathbf{w}}_m + 1} \qquad (4.1.11)$$

where the following definitions are used: $\mathbf{h}_m \triangleq \mathbf{F}_{1m}\mathbf{f}_{2m}$ and $\tilde{\mathbf{h}}_m \triangleq \tilde{\mathbf{F}}_{1m}\tilde{\mathbf{f}}_{2m}$. In (4.1.11), the superscripts p and s stand for primary and secondary networks, respectively. The information theoretic rates of the two transceivers over the mth subchannel in the

primary and secondary pairs are also defined as

$$R_{qm}(p_{\bar{q}m}, \mathbf{w}_m) = \frac{1}{2} \log_2 \left(1 + \text{SNR}^{\text{p}}_{qm}(p_{\bar{q}m}, \mathbf{w}_m) \right) \tag{4.1.12}$$

$$\tilde{R}_{qm}(\tilde{p}_{\bar{q}m}, \tilde{\mathbf{w}}_m) = \frac{1}{2} \log_2 \left(1 + \text{SNR}^{\text{s}}_{qm}(\tilde{p}_{\bar{q}m}, \tilde{\mathbf{w}}_m) \right). \tag{4.1.13}$$

In the next two sections, we use the proposed data model to design our spectrum leasing schemes.

4.2   Resource Sharing with Power Spectral Masks

In order to optimally obtain the transceiver transmit powers over different subchannels, the corresponding beamforming vectors, and the time sharing factors in all subchannels, we resort to a sum-rate maximization approach whereby the sum of the *average* of the two secondary transceivers rates over all subchannels is maximized by solving the following optimization problem:

$$\underset{\mathcal{P}_{\text{p}}, \mathcal{P}_{\text{s}}, \boldsymbol{\alpha}, \mathcal{W}_{\text{p}}, \mathcal{W}_{\text{s}}}{\text{maximize}} \sum_{m=1}^{k} \alpha_m \left(\tilde{R}_{1m}(\tilde{p}_{2m}, \tilde{\mathbf{w}}_m) + \tilde{R}_{2m}(\tilde{p}_{1m}, \tilde{\mathbf{w}}_m) \right) \tag{4.2.1a}$$

$$\text{subject to } \tilde{P}_m(\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m) \le \tilde{\gamma}_m, \ p_{1m} \ge 0, \ p_{2m} \ge 0 \ \text{for} \ m = 1, 2, ..., k \tag{4.2.1b}$$

$$P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \le \gamma_m, \ \tilde{p}_{1m} \ge 0, \ \tilde{p}_{2m} \ge 0 \ \text{for} \ m = 1, 2, ..., k \tag{4.2.1c}$$

$$\sum_{m=1}^{k} (1 - \alpha_m) \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m) \right) \ge \eta \tag{4.2.1d}$$

$$0 \le \alpha_m \le 1, \ \text{for} \ m = 1, 2, \cdots, k. \tag{4.2.1e}$$

In the objective function in (4.2.1), we take into account that the secondary transceivers utilize the mth subchannel only α_m fraction of the time. Also, we define $\mathcal{P}_{\text{p}} \triangleq \{\mathbf{p}_1, \mathbf{p}_2\}$ and $\mathcal{P}_{\text{s}} \triangleq \{\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2\}$ where $\mathbf{p}_i \triangleq [p_{i1} \ p_{i2} \ \cdots \ p_{ik}]^T$ and $\tilde{\mathbf{p}}_i \triangleq [\tilde{p}_{i1} \ \tilde{p}_{i2} \ \cdots \ \tilde{p}_{ik}]^T$ are the vectors of subchannel transmit powers of the ith primary and secondary transceivers,

respectively. We also define $\mathcal{W}_\mathrm{p} \triangleq \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_k\}$ and $\mathcal{W}_\mathrm{s} \triangleq \{\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \cdots, \tilde{\mathbf{w}}_k\}$ as the sets of the beamforming vectors for the primary and secondary transceivers, respectively, and $\boldsymbol{\alpha} \triangleq [\alpha_1 \ \ \alpha_2 \ \ \cdots \ \ \alpha_k]^T$ represents the vector of time sharing factors over different subcarriers. The constraints in (4.2.1b) and (4.2.1c) are used to limit the *peak* total power consumed in the secondary and the primary subframes, respectively, over all channels. Indeed, to ensure that our design meets a given spectral power mask requirement, it is often desired that a network meets a certain set of requirements for the amount of the power it transmits in each frequency band allocated to it. Spectral power masking[1] is a well-known approach to limit the co-channel interference between different transceivers in a shared bandwidth and it has been widely used in literature [102, 103]. Hence, we herein incorporate the use of a spectral power masks into our design by using (4.2.1b) and (4.2.1c). It is worth mentioning that a constraint on the total power consumed in the mth subchannel can lead to a significant performance improvement compared to the per-node power constraints. Indeed, such a constraint provides a flexible environment to optimize the network performance by allowing each element of the network to have its own optimal power. Such a constraint also provides a guideline to set individual relay powers . Considering a total power constraint in a design problem is important in network planning as it provides the flexibility required to control the amount of the power that each node consumes in the network. Furthermore, using total power constraint provides a guideline for allocating the optimal powers for each relay. For instance, when SNR balancing approach is applied to the case of synchronous/asynchronous two-way relay networks with n_r relay nodes, half of the total available power will be allocated to

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[1]A spectral power mask prescribes the maximum transmit power that one network may use at different frequency bands within the available RF spectrum.

the relays [43, 104]. In such networks, it is rational to assume that, on average, each relay consumes $\frac{1}{n_{\mathrm{r}}}$ fraction of the half of the total available power. This assumption is particularly correct when the relays are moving randomly in the environment and thus their channel coefficients appear to be drawn from the same probabilistic distribution. In the case where all the nodes are powered up from the electric grid (implying that the locations of the nodes are fixed), using a total power constraint is a must to ensure that the total power consumed by the network is restricted. We also note that considering per-node power constraints in the design of two-way relaying schemes is challenging and computationally prohibitive. The results of [77] show that even in the case of synchronous single-carrier two-way relay networks, finding the optimal AF relaying scheme leads to a two-dimensional search over a sufficiently fine grid of transceiver powers, and solving a convex feasibility programming problem on each vertex of this grid. The multi-carrier nature of the scheme considered in this study will only add to the level of difficulty associated with using per-node power constraints in the context of two-way relay networks. Furthermore, in the sum-rate maximization problem under per-node power constraints for any given channel realization, the relays may not consume all the power available to them. As a result, the average power consumed by the relays for different channel realizations will be less than the power available to each of them. It is also worth mentioning that sum-rate maximization under a total power constraint provides an upper bound to the same problem when per-node power constraints are considered. For all these reasons, total power constraint has been adopted in the literature for performance analysis and optimal design. The extension to individual power constraints is an interesting problem especially in relay networks where each relay is expected to have a limited amount of

power. But this extension may not have a computationally affordable solution. These reasons have inspired us to benefit from this well-known constraint on the total power budget in our design approaches. The constraint in (4.2.1d) guarantees the primary transceivers' rate demand by ensuring that the sum of the *average* sum-rate of the two primary transceivers over all subchannels are larger than a predefined threshold. The factor $(1 - \alpha_m)$ in (4.2.1d) represents the fraction of the time that the secondary transceivers occupy the mth subchannel. The constraint (4.2.1e) ensures that the time sharing factor of the mth subchannel must be in the interval $[0\ 1]$. Note that η, the primary network rate demand, is measured in bits per use of the parallel channel (b/cu). Note that our approach in (4.2.1) is essentially different from our approach in [101]. Indeed, in (4.2.1), the objective function and the rate demand constraint is of the form of *sum-rate* of transceiver pairs over all subchannels in a multi-carrier scenario, while the objective function and the rate demand constraint in [101] is of the form of max-min of the rate of the transceiver pairs in a single-carrier scenario. To simplify (4.2.1), we note that the constraints on the total power consumed in the secondary subframes for each subchannel are disjoint. Hence we can rewrite the optimization problem (4.2.1) as

$$\underset{\mathcal{P}_{\mathrm{p}}, \boldsymbol{\alpha}, \mathcal{W}_{\mathrm{p}}}{\text{maximize}} \sum_{m=1}^{k} \alpha_m \underset{\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m}{\max} \left(\tilde{R}_{1m}(\tilde{p}_{2m}, \tilde{\mathbf{w}}_m) + \tilde{R}_{2m}(\tilde{p}_{1m}, \tilde{\mathbf{w}}_m) \right) \tag{4.2.2a}$$

$$\text{subject to } \tilde{P}_m(\tilde{p}_{1m}, \tilde{p}_{2m}, \tilde{\mathbf{w}}_m) \leq \tilde{\gamma}_m, \tilde{p}_{1m} \geq 0, \tilde{p}_{2m} \geq 0, \text{ for } m = 1, 2, \cdots, k \tag{4.2.2b}$$

$$P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \leq \gamma_m, \ p_{1m} \geq 0, \ p_{2m} \geq 0 \text{ for } m = 1, 2, \cdots, k \tag{4.2.2c}$$

$$\sum_{m=1}^{k} (1 - \alpha_m) \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m) \right) \geq \eta \tag{4.2.2d}$$

$$0 \leq \alpha_m \leq 1, \text{ for } m = 1, 2, \cdots, k. \tag{4.2.2e}$$

For any cubchannel index m, given the total available power $\tilde{\gamma}_m$ in the mth subchannel, the inner maximization in (4.2.2), given as

$$\tilde{\psi}_m\left(\tilde{\gamma}_m\right) \triangleq \max_{\tilde{p}_{1m},\tilde{p}_{2m},\tilde{\mathbf{w}}_m} \left(\tilde{R}_{1m}(\tilde{p}_{2m},\tilde{\mathbf{w}}_m) + \tilde{R}_{2m}(\tilde{p}_{1m},\tilde{\mathbf{w}}_m)\right)$$

$$\text{subject to} \quad \tilde{P}(\tilde{p}_{1m},\tilde{p}_{2m},\tilde{\mathbf{w}}_m) \leq \tilde{\gamma}_m, \tilde{p}_{1m} \geq 0, \tilde{p}_{2m} \geq 0 \qquad (4.2.3)$$

can be shown to be a rate balancing problem[2] and the optimal values of the $\tilde{p}_{1m}$, $\tilde{p}_{2m}$ and $\tilde{\mathbf{w}}_m$ (denoted as $\tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m)$, $\tilde{p}^{\text{o}}_{2m}(\tilde{\gamma}_m)$ and $\tilde{\mathbf{w}}^{\text{o}}_m(\tilde{\gamma}_m)$) are given by [44]

$$\tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m) = \arg \max_{0 \leq \tilde{p}_{1m} \leq \frac{\tilde{\gamma}_m}{2}} \tilde{p}_{1m}(\tilde{\gamma}_m - 2\tilde{p}_{1m})\tilde{\mathbf{h}}_m^H \tilde{\mathbf{Q}}_m^{-1}(\tilde{p}_{1m},\tilde{\gamma}_m)\tilde{\mathbf{h}}_m \qquad (4.2.4a)$$

$$\tilde{p}^{\text{o}}_{2m}(\tilde{\gamma}_m) = \frac{1}{2}\tilde{\gamma}_m - \tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m) \qquad (4.2.4b)$$

$$\tilde{\mathbf{w}}^{\text{o}}_m(\tilde{\gamma}_m) = \tilde{\kappa}_m(\tilde{\gamma}_m)\sqrt{\tilde{p}^{\text{o}}_{2m}(\tilde{\gamma}_m)}\tilde{\mathbf{Q}}_m^{-1}\left(\tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m),\tilde{\gamma}_m\right)\tilde{\mathbf{h}}_m \qquad (4.2.4c)$$

$$\tilde{\kappa}_m(\tilde{\gamma}_m) = \left(\tilde{\mathbf{h}}_m^H(2\tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m)\tilde{\mathbf{D}}_{1m} + \mathbf{I})\mathbf{Q}_m^{-2}\left(\tilde{p}^{\text{o}}_{1m}(\tilde{\gamma}_m),\tilde{\gamma}_m\right)\tilde{\mathbf{h}}_m\right)^{-\frac{1}{2}} \qquad (4.2.4d)$$

$$\tilde{\mathbf{Q}}_m(\tilde{p}_{1m},\tilde{\gamma}_m) = (2\tilde{p}_{1m}\tilde{\mathbf{D}}_{1m} + (\tilde{\gamma}_m - 2\tilde{p}_{1m})\tilde{\mathbf{D}}_{2m} + \mathbf{I})^{-1}. \qquad (4.2.4e)$$

As proven in [43, 44], the optimization problem (4.2.4) is convex and it can be efficiently solved using Newton-Raphson method. Using (4.2.3), we can rewrite the optimization problem (4.2.2) as

$$\underset{\mathcal{P}_{\text{p}},\boldsymbol{\alpha},\mathcal{W}_{\text{p}}}{\text{maximize}} \quad \sum_{m=1}^{k} \alpha_m \tilde{\psi}_m\left(\tilde{\gamma}_m\right) \qquad (4.2.5a)$$

$$\text{subject to} \quad P_m(p_{1m},p_{2m},\mathbf{w}_m) \leq \gamma_m, \; p_{1m} \geq 0, \; p_{2m} \geq 0, \; \text{for } m = 1,2,...,k \qquad (4.2.5b)$$

$$\sum_{m=1}^{k} (1 - \alpha_m)\left(R_{1m}(p_{2m},\mathbf{w}_m) + R_{2m}(p_{1m} + \mathbf{w}_m)\right) \geq \eta \qquad (4.2.5c)$$

$$0 \leq \alpha_m \leq 1, \; \text{for } m = 1,2,\cdots,k. \qquad (4.2.5d)$$

Note that at the optimum, we can show that the inequality constraint in (4.2.5c) on the primary network rate demand is satisfied with equality. To do so, assume

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[2]We note that under total power constraint, max-min SNR balancing problem and the sum-rate maximization problem ina two-way relay network are equivalent, see [44] for a proof.

that at the optimum, this constraint is satisfied with inequality. One can increase the optimal values of α_m's to turn the primary network QoS constraint into equality, however this increase of the values of α_m's leads to a value for the objective function which is larger than its optimum value, thereby contradicting optimality. Hence, at the optimum, the primary network QoS constraint has to be satisfied with equality. One can rewrite the optimization problem (4.2.5) as

$$\underset{\mathcal{P}_{\mathrm{p}},\boldsymbol{\alpha},\mathcal{W}_{\mathrm{p}}}{\operatorname{maximize}} \quad \sum_{m=1}^{k} \alpha_m \tilde{\psi}_m\left(\tilde{\gamma}_m\right) \tag{4.2.6a}$$

$$\text{subject to } P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \le \gamma_m, \ p_{1m} \ge 0, \ p_{2m} \ge 0, \text{ for } m = 1, 2, \cdots, k \tag{4.2.6b}$$

$$\sum_{m=1}^{k} (1 - \alpha_m)\left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m} + \mathbf{w}_m)\right) = \eta \tag{4.2.6c}$$

$$0 \le \alpha_m \le 1, \text{ for } m = 1, 2, \cdots, k. \tag{4.2.6d}$$

To further simplify (4.2.6), let us define the auxiliary variables ζ_m as

$$\zeta_m \triangleq (1 - \alpha_m)\left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right), \text{ for } m = 1, 2, \cdots, k, \tag{4.2.7}$$

where ζ_m is the *average* primary sum-rate in the mth subchannel and it must satisfy the constraint $\zeta_m \ge 0$, for $m = 1, \cdots, k$. Using (4.2.7), one can rewrite α_m in terms of ζ_m as

$$\alpha_m = 1 - \frac{\zeta_m}{\left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right)}, \text{ for } m = 1, 2, \cdots, k. \tag{4.2.8}$$

Using (4.2.8), we can rewrite the constraint in (4.2.6d) as

$$0 \le \zeta_m \le \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right). \tag{4.2.9}$$

By substituting (4.2.8) in the objective function in (4.2.6) and replacing (4.2.6d)

with (4.2.9), we can rewrite the optimization problem (4.2.6) as

$$\underset{\mathcal{P}_p, \boldsymbol{\zeta}, \mathcal{W}_p}{\text{maximize}} \sum_{m=1}^{k} \tilde{\psi}_m\left(\tilde{\gamma}_m\right)\left(1 - \frac{\zeta_m}{\left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right)}\right)$$

subject to $P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \leq \gamma_m, \; p_{1m} \geq 0, \; p_{2m} \geq 0, \;$ for $\; m = 1, 2, \cdots, k$

$$0 \leq \zeta_m \leq \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right), \;\text{ for } \; m = 1, 2, \cdots, k$$

$$\sum_{m=1}^{k} \zeta_m = \eta \tag{4.2.10}$$

where $\boldsymbol{\zeta} \triangleq [\zeta_1 \quad \zeta_2 \quad \cdots \quad \zeta_k]^T$ is the vector of the primary transceivers sum-rate in all subchannels. Moreover, since the constraints on the total power consumed in the primary subframes are disjoint, one can rewrite (4.2.10) as

$$\underset{\boldsymbol{\zeta}}{\text{maximize}} \; \sum_{m=1}^{k} \frac{1}{2}\tilde{\psi}_m\left(\tilde{\gamma}_m\right) \times$$

$$\underset{p_{1m}, p_{2m}, \mathbf{w}_m}{\max}\left(1 - \frac{\zeta_m}{\left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right)}\right) \tag{4.2.11a}$$

subject to $P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \leq \gamma_m, p_{1m} \geq 0, p_{2m} \geq 0$, for $m = 1, 2, \cdots, k$ (4.2.11b)

$$\zeta_m \leq \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right), \text{ for } m = 1, 2, \cdots, k \tag{4.2.11c}$$

$$\sum_{m=1}^{k} \zeta_m = \eta \;\text{ and }\; \zeta_m \geq 0, \;\text{ for } m = 1, 2, \cdots, k \tag{4.2.11d}$$

For any m, given the total available power γ_m in the mth subchannel for the primary transceivers, and given $\zeta_m \geq 0$, the inner maximization in (4.2.11) can be written as

$$\underset{p_{1m}, p_{2m}\mathbf{w}_m}{\max} \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right)$$

subject to $P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \leq \gamma_m, p_{1m} \geq 0, p_{2m} \geq 0$

$$\zeta_m \leq \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m)\right). \tag{4.2.12}$$

Note that, for any m and fixed value of ζ_m, the last constraint in (4.2.12) is only a feasibility condition. We first ignore this constraint, and solve the following problem

$$\psi_m(\gamma_m) \triangleq \max_{p_{1m}, p_{2m} \mathbf{w}_m} \left(R_{1m}(p_{2m}, \mathbf{w}_m) + R_{2m}(p_{1m}, \mathbf{w}_m) \right)$$
$$\text{subject to} \quad P_m(p_{1m}, p_{2m}, \mathbf{w}_m) \le \gamma_m, p_{1m} \ge 0, p_{2m} \ge 0. \tag{4.2.13}$$

Note that the optimization problem (4.2.12) is feasible if and only if $\zeta_m \le \psi_m(\gamma_m)$. The solution to (4.2.13) (denoted as $p_{1m}^o(\gamma_m)$, $p_{2m}^o(\gamma_m)$ and $\mathbf{w}_m^o(\gamma_m)$), is given by [44]

$$p_{1m}^o(\gamma_m) = \arg \max_{0 \le p_{1m} \le \frac{\gamma_m}{2}} p_{1m}(\gamma_m - 2p_{1m}) \mathbf{h}_m^H \mathbf{Q}_m^{-1}(p_{1m}, \gamma_m) \mathbf{h}_m \tag{4.2.14a}$$

$$p_{2m}^o(\gamma_m) = \frac{1}{2}\gamma_m - p_{1m}^o(\gamma_m) \tag{4.2.14b}$$

$$\mathbf{w}_m^o(\gamma_m) = \kappa_m(\gamma_m) \sqrt{p_{2m}^o(\gamma_m)} \mathbf{Q}_m^{-1}\left(p_{1m}^o(\gamma_m), \gamma_m\right) \mathbf{h}_m \tag{4.2.14c}$$

$$\kappa_m(\gamma_m) = \left(\mathbf{h}_m^H (2p_{1m}^o(\gamma_m)\mathbf{D}_{1m} + \mathbf{I}) \mathbf{Q}_m^{-2}\left(p_{1m}^o(\gamma_m), \gamma_m\right) \mathbf{h}_m \right)^{-\frac{1}{2}} \tag{4.2.14d}$$

$$\mathbf{Q}_m(p_{1m}, \gamma_m) = (2p_{1m}\mathbf{D}_{1m} + (\gamma_m - 2p_{1m})\mathbf{D}_{2m} + \mathbf{I})^{-1}. \tag{4.2.14e}$$

Using (4.2.12), one can rewrite the optimization problem (4.2.11) as

$$\underset{\zeta}{\text{maximize}} \quad \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{\psi_m(\gamma_m)} \right) \tilde{\psi}_m(\tilde{\gamma}_m)$$

$$\text{subject to} \quad \sum_{m=1}^{k} \zeta_m = \eta, \ \ 0 \le \zeta_m \le \psi_m(\gamma_m), \ \text{for } m = 1, 2, \cdots, k. \tag{4.2.15}$$

To solve (4.2.15), one has to obtain $\tilde{\psi}_m(\tilde{\gamma}_m)$ and $\psi_m(\gamma_m)$ as defined in (4.2.3) and (4.2.12). The optimal solutions to (4.2.3) and (4.2.12) are similar and are amenable to semi-closed-form solutions, as given in (4.2.4) and (4.2.14), respectively. Note that ζ_m is the amount of the primary rate that is to be assigned to the mth sub-channel, hence in order to ensure feasibility, the total rate demand η of the primary transceivers must satisfy the following condition, $\sum_{m=1}^{k} \zeta_m = \eta \le \sum_{m=1}^{k} \psi_m(\gamma_m)$. Otherwise, the optimization problem in (4.2.1) is not feasible. Let us define $a \triangleq$

$\sum_{m=1}^{k} \tilde{\psi}_m (\tilde{\gamma}_m)$, $\mathbf{1} \triangleq \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$, $\mathbf{b} \triangleq \begin{bmatrix} \frac{\tilde{\psi}_1(\tilde{\gamma}_1)}{\psi_1(\gamma_1)} & \frac{\tilde{\psi}_2(\tilde{\gamma}_2)}{\psi_2(\gamma_2)} & \cdots & \frac{\tilde{\psi}_k(\tilde{\gamma}_k)}{\psi_k(\gamma_k)} \end{bmatrix}^T$ and

$\mathbf{c} \triangleq \begin{bmatrix} \psi_1(\gamma_1) & \psi_2(\gamma_2) & \cdots & \psi_k(\gamma_k) \end{bmatrix}^T$. Using these definitions, the optimization

problem (4.2.15) can be written as

$$\underset{\boldsymbol{\zeta}}{\text{maximize}} \quad a - \mathbf{b}^T \boldsymbol{\zeta}$$

$$\text{subject to} \quad \mathbf{1}^T \boldsymbol{\zeta} = \eta, \ 0 \preccurlyeq \boldsymbol{\zeta} \preccurlyeq \mathbf{c}, \tag{4.2.16}$$

where $\preccurlyeq$ is elementwise inequality operator. The optimization problem (4.2.16) is a linear programming (LP) problem and it can be easily solved using any LP solver. We summarize the proposed solution in Algorithm 1.

Algorithm 1 : LP Based Problem Solution to (4.2.16)

Step 1. Given the values of $\tilde{\gamma}_m$ and γ_m, the secondary and the primary users can calculate

$\tilde{\psi}_m (\tilde{\gamma}_m)$ and $\psi_m (\gamma_m)$ using (4.2.3) and (4.2.13), respectively, for $m = 1, 2, \cdots, k$.

Step 2. Obtain $a = \sum_{m=1}^{k} \tilde{\psi}_m (\tilde{\gamma}_m)$, $\mathbf{b} = \begin{bmatrix} \frac{\tilde{\psi}_1(\tilde{\gamma}_1)}{\psi_1(\gamma_1)} & \frac{\tilde{\psi}_2(\tilde{\gamma}_2)}{\psi_2(\gamma_2)} & \cdots & \frac{\tilde{\psi}_k(\tilde{\gamma}_k)}{\psi_k(\gamma_k)} \end{bmatrix}^T$ and $\mathbf{c} =$

$\begin{bmatrix} \psi_1(\gamma_1) & \psi_2(\gamma_2) & \cdots & \psi_k(\gamma_k) \end{bmatrix}^T$.

Step 3. Solve the optimization problem (4.2.16) and obtain the optimal value for $\boldsymbol{\zeta} \triangleq$

$\begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_k \end{bmatrix}^T$.

Step 4. Use the so-obtained optimal $\{\zeta_m\}_{m=1}^{k}$ in (4.2.8) to calculate the optimal value

of $\boldsymbol{\alpha}$.

We now discuss what parameters are needed to be broadcasted to the transceivers to enable them to calculate their transmission parameters. Given the maximum total power γ_m ($\tilde{\gamma}_m$) available in the mth primary (secondary) subchannel, the two primary (secondary) transceivers can calculate the values of $\psi_m (\gamma_m)$ ($\tilde{\psi}_m (\tilde{\gamma}_m)$), for $m = 1, 2, \cdots, k$. The primary transceivers need the values of $\tilde{\psi}_m (\tilde{\gamma}_m)$ in order to

calculate ζ_m in (4.2.16) (or equivalently the optimal value of the time sharing factors α_m's in (4.2.8)). Hence for every m, the secondary transceivers need to broadcast, to the primary transceivers, the values of $\tilde{\psi}_m\left(\tilde{\gamma}_m\right)$, for $m = 1, 2, \cdots, k$. The primary transceivers then solve the LP problem (4.2.16) and broadcast to the relays and the secondary transceivers, the optimal value of $\boldsymbol{\zeta}$ (or equivalently the optimal values of time sharing factors α_m, $m = 1, \cdots, k$). Using such a parameter distribution scheme, one can alleviate the need to provide the information about the channel coefficients in the secondary network to the primary network. We further note that, a control channel between the primary and the secondary networks is also a commnly-used assumption in spectrum leasing based cognitive radio networks [11]. A control channel is needed in order to send crucial paramaters between the two networks.

4.3 Resource Sharing with A Total Power Constraint

In the previous section, we considered a resource sharing problem where there was a power mask that limits the total transmitted power in each subcarrier. If there is no such a restriction on the spectral resources, we can optimize the allocation of the total power budget and the spectral resources in a joint design approach. Let us consider the underlying communication architecture in Fig. 1. To obtain the design parameters of the two networks, we now consider the following optimization problem:

$$\underset{\mathcal{P}_{\mathrm{p}},\mathcal{P}_{\mathrm{s}},\boldsymbol{\alpha},\mathcal{W}_{\mathrm{p}},\mathcal{W}_{\mathrm{s}}}{\operatorname{maximize}} \sum_{m=1}^{k} \alpha_m \left(\tilde{R}_{1m}(\tilde{p}_{2m},\tilde{\mathbf{w}}_m) + \tilde{R}_{2m}(\tilde{p}_{2m},\tilde{\mathbf{w}}_m) \right) \tag{4.3.1a}$$

$$\text{subject to} \ \sum_{m=1}^{k} \tilde{P}_m(\tilde{p}_{1m},\tilde{p}_{2m},\tilde{\mathbf{w}}_m) \leq \tilde{\gamma}, \ \tilde{p}_{1m} \geq 0, \ \tilde{p}_{2m} \geq 0 \tag{4.3.1b}$$

$$\sum_{m=1}^{k} P_m(p_{1m},p_{2m},\mathbf{w}_m) \leq \gamma, \ \tilde{p}_{1m} \geq 0, \ \tilde{p}_{2m} \geq 0 \tag{4.3.1c}$$

$$\sum_{m=1}^{k} (1-\alpha_m) \left(R_{1m}(p_{2m},\mathbf{w}_m) + R_{2m}(p_{1m},\mathbf{w}_m) \right) \geq \eta \tag{4.3.1d}$$

$$0 \leq \alpha_m \leq 1, \ \text{for } m = 1,2,\cdots,k \tag{4.3.1e}$$

where $\mathcal{P}_{\mathrm{p}},\mathcal{P}_{\mathrm{s}},\boldsymbol{\alpha},\mathcal{W}_{\mathrm{p}}$, and $\mathcal{W}_{\mathrm{s}}$ are defined in the previous section. Different from (4.2.1), in (4.3.1) we assume that there is a constraint on the total power consumed over all subchannels in each network. Hence, we aim to allocate the total available power to each subchannel in each pair, while optimally determining the time sharing factors for all subchannels. Although this approach does not allow spectral power mask, but it limits the total power consumed in the whole available RF spectrum. Assuming distributed beamforming at the relays, the objective function in (4.3.1) is the sum-rate for the secondary network over all subchannels. The constraints in (4.3.1b) and (4.3.1c) are used to limit the *peak* total power consumed in the secondary and the primary subframes by γ and $\tilde{\gamma}$, respectively, over all subchannels[3]. The constraint in (4.3.1d) is used to guarantee a minimum sum-rate for the primary transceivers. Using the approach we used in the previous section, we can reformulate

---

[3]In order to take into account the constraint on the average power, one can can replace (4.3.1b) and (4.3.1c) with the following two constraints, $\sum_{m=1}^{k}(1-\alpha_m)\gamma_m \leq \gamma$ and $\sum_{m=1}^{k}\alpha_m\tilde{\gamma}_m \leq \tilde{\gamma}$. We will later show that how the corresponding design problem will be solved under these two average power constraints.

the problem in (4.3.1) as

$$\underset{\zeta,\gamma,\tilde{\gamma}\succcurlyeq 0}{\text{maximize}} \quad \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{\psi_m\left(\gamma_m\right)} \right) \tilde{\psi}_m\left(\tilde{\gamma}_m\right) \tag{4.3.2a}$$

$$\text{subject to} \quad \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}, \ \sum_{m=1}^{k} \gamma_m = \gamma \tag{4.3.2b}$$

$$\sum_{m=1}^{k} \zeta_m = \eta, \ 0 \le \zeta_m \le \psi_m\left(\gamma_m\right), \ \text{for } m = 1, 2, \cdots, k, \tag{4.3.2c}$$

where the following definitions are used: $\boldsymbol{\gamma} \triangleq \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_k \end{bmatrix}^T$ and $\tilde{\boldsymbol{\gamma}} \triangleq \begin{bmatrix} \tilde{\gamma}_1 & \tilde{\gamma}_2 & \cdots & \tilde{\gamma}_k \end{bmatrix}^T$. In the next two subsections, we provide a solution to (4.3.2) for a single-relay and multi-relay scenarios.

4.3.1 Single-Relay Scenario

For the single-relay case, one can write $\psi_m\left(\gamma_m\right)$ and $\tilde{\psi}_m\left(\tilde{\gamma}_m\right)$, respectively, as [104]

$$\psi_m\left(\gamma_m\right) = \log_2\left(1 + \frac{\gamma_m^2 |h_m|^2}{2\left(\sqrt{1 + \gamma_m d_{1m}} + \sqrt{1 + \gamma_m d_{2m}} \right)^2} \right) \tag{4.3.3}$$

$$\tilde{\psi}_m\left(\tilde{\gamma}_m\right) = \log_2\left(1 + \frac{\tilde{\gamma}_m^2 |\tilde{h}_m|^2}{2\left(\sqrt{1 + \tilde{\gamma}_m \tilde{d}_{1m}} + \sqrt{1 + \tilde{\gamma}_m \tilde{d}_{2m}} \right)^2} \right). \tag{4.3.4}$$

Here, we define $d_{1m} \triangleq |f_{1m}|^2$ ($d_{2m} \triangleq |f_{2m}|^2$), where f_{1m} (f_{2m}) is the channel coefficient between PTRX$_1$ (PTRX$_2$) and the relay in the mth subchannel. Similarly, we define $\tilde{d}_{1m} \triangleq |\tilde{f}_{1m}|^2$ ($\tilde{d}_{2m} \triangleq |\tilde{f}_{2m}|^2$), where f_{1m} (f_{2m}) is the channel coefficient between STRX$_1$ (STRX$_2$) and the relay in the mth subchannel. The optimization problem (4.3.2) is not convex and it may not be amenable to a computationally efficient solution. To tackle this optimization problem, we aim to maximize an upper-bound for the objective function in (4.3.2), thereby providing a sub-optimal solution for this

problem. More specifically, to simplify the problem, we use an upper bound for each of the two functions $\psi_m(\gamma_m)$ and $\tilde{\psi}_m(\tilde{\gamma}_m)$. If we consider a high SNR approximation where, $1 \ll \gamma_m d_{1m}$, $1 \ll \gamma_m d_{2m}$, $1 \ll \tilde{\gamma}_m \tilde{d}_{1m}$, and $1 \ll \tilde{\gamma}_m \tilde{d}_{2m}$, the functions $\psi_m(\gamma_m)$ and $\tilde{\psi}_m(\tilde{\gamma}_m)$ can be, respectively, upper-bounded by $\varphi_m(\gamma_m)$ and $\tilde{\varphi}_m(\tilde{\gamma}_m)$ which are given as

$$\varphi_m(\gamma_m) \triangleq \log_2(1 + \gamma_m |h_m^e|^2), \quad \tilde{\varphi}_m(\tilde{\gamma}_m) \triangleq \log_2(1 + \tilde{\gamma}_m |\tilde{h}_m^e|^2), \tag{4.3.5}$$

where we have used the following definitions:

$$h_m^e \triangleq \frac{|h_m|}{\sqrt{2}(\sqrt{d_{1m}} + \sqrt{d_{2m}})}, \quad \tilde{h}_m^e \triangleq \frac{|\tilde{h}_m|}{\sqrt{2}(\sqrt{\tilde{d}_{1m}} + \sqrt{\tilde{d}_{2m}})}. \tag{4.3.6}$$

Since, for any given pair of γ_m and $\tilde{\gamma}_m$, $\varphi_m(\gamma_m) > \psi_m(\gamma_m) \geq 0$ and $\tilde{\varphi}_m(\tilde{\gamma}_m) > \tilde{\psi}_m(\tilde{\gamma}_m) \geq 0$, then $1 - \frac{\zeta_m}{\varphi_m(\gamma_m)} > 1 - \frac{\zeta_m}{\psi_m(\gamma_m)} \geq 0$ holds true due to the constraints in (4.3.2c). Hence, we can write $\left(1 - \frac{\zeta_m}{\varphi_m(\gamma_m)}\right)\tilde{\varphi}_m(\tilde{\gamma}_m) > \left(1 - \frac{\zeta_m}{\psi_m(\gamma_m)}\right)\tilde{\psi}_m(\tilde{\gamma}_m) \geq 0$. As a result, we now replace $\left(1 - \frac{\zeta_m}{\psi_m(\gamma_m)}\right)\tilde{\psi}_m(\tilde{\gamma}_m)$ in (4.3.2a) with its upper bound $\left(1 - \frac{\zeta_m}{\varphi_m(\gamma_m)}\right)\tilde{\varphi}_m(\tilde{\gamma}_m)$. Based on these discussions, we rewrite the maximization problem of the upper bound (4.3.2) as

$$\underset{\zeta, \gamma, \tilde{\gamma} \succcurlyeq 0}{\text{maximize}} \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{\log_2(1 + \gamma_m |h_m^e|^2)}\right) \log_2(1 + \tilde{\gamma}_m |\tilde{h}_m^e|^2) \tag{4.3.7a}$$

$$\text{subject to} \sum_{m=1}^{k} \zeta_m = \eta, \quad \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma} \tag{4.3.7b}$$

$$\sum_{m=1}^{k} \gamma_m = \gamma, \quad 0 \leq \zeta_m \leq \psi_m(\gamma_m), \quad \text{for } m = 1, 2, \cdots, k. \tag{4.3.7c}$$

We now provide an approximate solution to the design problem. One can see that for any given pair of vectors ζ, γ and $\tilde{\gamma}$, the optimization problem (4.3.7) is convex with respect to the third vector. Hence, using an alternating convex search algorithm

(Algorithm 2), it is guaranteed that at each iteration, the value of the objective function will increase until it reaches a maximum. Algorithm 2, summarizes our proposed solution to the resource sharing problem (4.3.7). This technique will help us to obtain the design parameters iteratively. We note that the alternating convex search algorithm converges to at least a local optimum. However, global optimality may not be claimed at this point. To do so, at each iteration, we fix two of the vectors $\boldsymbol{\zeta}, \boldsymbol{\gamma}$ and $\tilde{\boldsymbol{\gamma}}$, and obtain the third vector. Note that since $\psi_m(\gamma_m)$ is an increasing function of γ_m [101], its inverse $\psi_m^{-1}(\cdot)$ exists. Hence, when $\boldsymbol{\zeta}$ and $\tilde{\boldsymbol{\gamma}}$ are fixed, the constraint $\zeta_m \le \psi_m(\gamma_m)$ is equivalent to $\psi_m^{-1}(\zeta_m^j) \le \gamma_m$ which is a linear constraint. Furthermore, in Step 5 of Algorithm 2, we need to calculate $\psi_m^{-1}(\zeta_m^j)$. It can be shown that $\psi_m^{-1}(\zeta_m^j)$ is given by

$$\psi_m^{-1}(\zeta_m^j) = 2\mu_m^j \left(\frac{2\sqrt{1 + \frac{1}{2\mu_m^j}}}{|f_{1m}||f_{2m}|} + \frac{d_{1m} + d_{2m}}{|h_m|^2} \right), \qquad (4.3.8)$$

where $\mu_m^j = 2^{\zeta_m^j} - 1$. To initialize the search algorithm, one good candidate is the solution to the optimization problem (4.2.1), where $\gamma_m = \frac{\gamma}{k}$ and $\tilde{\gamma}_m = \frac{\tilde{\gamma}}{k}$. The reason for this choice is that for such a setup in (4.2.1), the optimal solution can be easily obtained by solving a simple LP problem[4].

We now provide a semi-closed-form solution to the powers allocated in the secondary network over all subchannels in Step 4 of the Algorithm 2. Let us assume that ζ_m^j and γ_m^j are, respectively, the values of the primary transceivers' sum-rate and their total consumed power over the mth subchannel in the jth iteration. Also, we

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[4]Obtaining a feasible initial point for the optimization problem (4.3.7) is not trivial. In order to satisfy the primary network rate demand η, one needs to choose the total power available for the primary network γ to be sufficiently large. If the above mentioned initial point is not feasible, we may resort to a random selection of a feasible initial point. However, this random selection process could be time consuming, and thus, computationally infeasible, meaning that one cannot afford to solve this problem.

Algorithm 2 : Alternating Convex Search Solution to (4.3.7) - Single-Relay Scenario

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Step 1. Choose ε as the stopping criterion of the algorithm.

Step 2. Assume j to be the number of iterations and set $j = 0$.

Step 3. Initialize the problem (4.3.7) with $\gamma_m^0 = \frac{\gamma}{k}, \tilde{\gamma}_m^0 = \frac{\tilde{\gamma}}{k}$ and obtain the vector $\tilde{\boldsymbol{\zeta}}^0$ as the solution to the optimization problem (4.2.16), when γ_m and $\tilde{\gamma}_m$ are replaced with γ_m^0 and $\tilde{\gamma}_m^0$, respectively. Define the value of the objective function in (4.3.7) in the jth iteration as $\xi(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \tilde{\boldsymbol{\gamma}}^j) \triangleq \sum_{m=1}^{k} \left(1 - \frac{\zeta_m^j}{\varphi_m(\gamma_m^j)} \right) \tilde{\varphi}_m(\gamma_m^j)$.

Step 4. Solve the following convex optimization problem to find the optimal $\tilde{\boldsymbol{\gamma}}^{j+1}$ as

$$\tilde{\boldsymbol{\gamma}}^{j+1} = \arg\max_{\tilde{\boldsymbol{\gamma}} \succcurlyeq 0} \ \xi(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \tilde{\boldsymbol{\gamma}}), \ \text{ s.t. } \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}$$

Step 5. Solve the following convex optimization problem to find the optimal $\boldsymbol{\gamma}^{j+1}$ as

$$\boldsymbol{\gamma}^{j+1} = \arg\max_{\tilde{\boldsymbol{\gamma}} \succcurlyeq 0} \ \xi(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}, \tilde{\boldsymbol{\gamma}}^{j+1})$$
$$\text{s.t. } \sum_{m=1}^{k} \gamma_m = \gamma, \ \ 0 \le \psi_m^{-1}(\zeta_m^j) \le \gamma_m, \ \text{ for } m = 1, 2, \cdots, k.$$

Step 6. Find the optimal $\boldsymbol{\zeta}^{j+1}$ by solving the following convex optimization problem

$$\boldsymbol{\zeta}^{j+1} = \arg\max_{\tilde{\boldsymbol{\gamma}} \succcurlyeq 0} \ \xi(\boldsymbol{\zeta}, \boldsymbol{\gamma}^{j+1}, \tilde{\boldsymbol{\gamma}}^{j+1})$$
$$\text{s.t. } \sum_{m=1}^{k} \zeta_m = \eta, \ \ 0 \le \zeta_m \le \psi_m\left(\gamma_m^{j+1}\right), \ \text{ for } m = 1, 2, \cdots, k.$$

Step 7. If $|\xi(\boldsymbol{\zeta}^{j+1}, \boldsymbol{\gamma}^{j+1}, \tilde{\boldsymbol{\gamma}}^{j+1}) - \xi(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \tilde{\boldsymbol{\gamma}}^j)| < \varepsilon$, go to Step 8, otherwise set $j = j + 1$ and go to Step 4.

Step 8. Stop the algorithm.

---

define $t_m^j \triangleq \left(1 - \frac{\zeta_m^j}{\log_2(1+\gamma_m^j|h_m^e|^2)}\right)$. To obtain the values of γ_m^{j+1}, one has to solve the following optimization problem:

$$\tilde{\boldsymbol{\gamma}}^{j+1} = \arg \max_{\tilde{\boldsymbol{\gamma}} \succcurlyeq 0} \sum_{m=1}^{k} t_m^j \log_2(1 + \tilde{\gamma}_m|\tilde{h}_m^e|^2)$$

$$\text{subject to } \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}. \qquad (4.3.9)$$

The dual Lagrangian of the convex optimization problem (4.3.9) can be defined as

$$\mathcal{L}(\tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \tilde{\boldsymbol{\lambda}}) \triangleq \sum_{m=1}^{k} t_m^j \log_2(1 + \tilde{\gamma}_m|\tilde{h}_m^e|^2) + \tilde{\rho}(\tilde{\gamma} - \sum_{m=1}^{k} \tilde{\gamma}_m) - \sum_{m=1}^{k} \tilde{\lambda}_m \tilde{\gamma}_m, \qquad (4.3.10)$$

where $\tilde{\rho}$ and $\tilde{\lambda}_m$ are Lagrange multipliers or dual variables. The KKT optimality conditions for the convex optimization problem (4.3.10) implies that

$$\frac{\partial \mathcal{L}(\tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \tilde{\boldsymbol{\lambda}})}{\partial \tilde{\gamma}_m} = \frac{t_m^j |\tilde{h}_m^e|^2}{\ln 2(1 + \tilde{\gamma}_m|\tilde{h}_m^e|^2)} - \tilde{\rho} - \tilde{\lambda}_m = 0$$

$$\sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}, \quad \tilde{\lambda}_m \tilde{\gamma}_m = 0, \quad \tilde{\gamma}_m \geq 0, \quad \tilde{\lambda}_m \geq 0. \qquad (4.3.11)$$

The solution to the KKT conditions in (4.3.11) is given by $\tilde{\gamma}_m^{j+1} = \max\{0, \frac{t_m^j}{\ln 2}\left(\frac{1}{\tilde{\rho}^*} - \frac{\ln 2}{t_m^j|\tilde{h}_m^e|^2}\right)\}$, where $\tilde{\rho}^*$ is found such that $\sum_{m=1}^{k} \tilde{\gamma}_m^{j+1} = \tilde{\gamma}$.

4.3.2 Multi-Relay Scenario

We now consider a multi-relay scenario, where n_r relays are assisting the two transceivers in each pair to exchange their information symbols. In this case, in the optimization problem (4.3.2), there is no closed-form solution for $\psi_m(\gamma_m)$ and $\tilde{\psi}_m(\tilde{\gamma}_m)$. It is worth mentioning that this problem is not convex hence it may not be amenable to a solution with low computational complexity. We now discuss some of the properties of the optimization problem (4.3.2) and present a solution to (4.3.2) with relatively low

computational complexity. To do so, using (4.2.3) and (4.2.4a) as well as (4.2.12) and (4.2.14a), we can write $\tilde{\psi}_m(\tilde{\gamma}_m)$ and $\psi_m(\gamma_m)$ as

$$\tilde{\psi}_m(\tilde{\gamma}_m) = \max_{0 \leq \tilde{p}_{1m} \leq \frac{\tilde{\gamma}_m}{2}} \tilde{v}_m(\tilde{p}_{1m}, \tilde{\gamma}_m), \quad \psi_m(\gamma_m) = \max_{0 \leq p_{1m} \leq \frac{\gamma_m}{2}} v_m(p_{1m}, \gamma_m). \qquad (4.3.12)$$

where the following definitions are used:

$$\tilde{v}_m(\tilde{p}_{1m}, \tilde{\gamma}_m) \triangleq \log_2\left(1 + \tilde{p}_{1m}(\tilde{\gamma}_m - 2\tilde{p}_{1m})\tilde{\mathbf{h}}_m^H \tilde{\mathbf{Q}}_m(\tilde{p}_{1m}, \tilde{\gamma}_m)\tilde{\mathbf{h}}_m\right) \qquad (4.3.13\text{a})$$

$$\tilde{\mathbf{Q}}_m(\tilde{p}_{1m}, \tilde{\gamma}_m) \triangleq (2\tilde{p}_{1m}\tilde{\mathbf{D}}_{1m} + (\tilde{\gamma}_m - 2\tilde{p}_{1m})\tilde{\mathbf{D}}_{2m} + \mathbf{I})^{-1} \qquad (4.3.13\text{b})$$

$$v_m(p_{1m}, \gamma_m) \triangleq \log_2\left(1 + p_{1m}(\gamma_m - 2p_{1m})\mathbf{h}_m^H \mathbf{Q}_m(p_{1m}, \gamma_m)\mathbf{h}_m\right) \qquad (4.3.13\text{c})$$

$$\mathbf{Q}_m(p_{1m}, \gamma_m) \triangleq (2p_{1m}\mathbf{D}_{1m} + (\gamma_m - 2p_{1m})\mathbf{D}_{2m} + \mathbf{I})^{-1}. \qquad (4.3.13\text{d})$$

Using (4.3.12)-(4.3.13), we rewrite the optimization problem in (4.3.2) as

$$\underset{\boldsymbol{\zeta},\boldsymbol{\gamma},\tilde{\boldsymbol{\gamma}},\mathbf{p}_1,\tilde{\mathbf{p}}_1 \succcurlyeq 0}{\text{maximize}} \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{v_m(p_{1m}, \gamma_m)}\right) \tilde{v}_m(\tilde{p}_{1m}, \tilde{\gamma}_m)$$

$$\text{subject to} \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}, \quad \sum_{m=1}^{k} \gamma_m = \gamma$$

$$\sum_{m=1}^{k} \zeta_m = \eta, \quad 0 \leq \zeta_m \leq \psi_m(\gamma_m).$$

$$0 \leq \tilde{p}_{1m} \leq \frac{\tilde{\gamma}_m}{2}, 0 \leq p_{1m} \leq \frac{\gamma_m}{2}, \quad m = 1, 2, \cdots, k, \qquad (4.3.14)$$

where we have used the following definitions: $\mathbf{p}_1 \triangleq [p_{11} \quad p_{12} \quad \cdots \quad p_{1k}]^T$ and $\tilde{\mathbf{p}}_1 \triangleq [\tilde{p}_{11} \quad \tilde{p}_{12} \quad \cdots \quad \tilde{p}_{1k}]^T$. It can be shown that $\tilde{v}(\tilde{p}_{1m}, \tilde{\gamma}_m)$ $(v(p_{1m}, \gamma_m))$ is a biconcave[5] function of $\tilde{p}_{1m}$ and $\tilde{\gamma}_m$ $(p_{1m}$ and $\gamma_m)$, where $0 \leq \tilde{p}_{1m} \leq \frac{\tilde{\gamma}_m}{2}$ $(0 \leq p_{1m} \leq \frac{\gamma_m}{2})$. If we fix any four vectors out of the five vectors $\boldsymbol{\zeta}$, $\boldsymbol{\gamma}$, $\tilde{\boldsymbol{\gamma}}$, $\mathbf{p}_1$, and $\tilde{\mathbf{p}}_1$, the objective function (4.3.14) is concave with respect to the fifth vector. This property inspires

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[5]We note that $r(x, y)$ is said to be a biconcave function if, for fixed x, $r(x, y)$ is concave with respect to y, and for fixed y, $r(x, y)$ is concave with respect to x.

us to derive an alternating convex search technique to the optimization problem (4.3.14). In each iteration, we fix four out of five of the above variable vectors, and solve the optimization problem (4.3.14) for the fifth vector. At each iteration, we can use efficient algorithms (e.g., interior point) to solve the problem. This iterative algorithm will be repeated until the objective function reaches a maximum. Algorithm 3 summarizes our proposed solution to the problem (4.3.14).

It is worth mentioning that since there is no closed-form-solution for $\psi_m(\cdot)$ in the multi-relay case, deriving a closed-form-solution for the inverse function $\psi_m^{-1}(\cdot)$ is not possible. To tackle this issue, note that $\psi_m(\gamma_m)$, as defined in (4.2.12), is the maximum achievable balanced rates of the two primary transceivers in mth subchannel, when $P_m(p_{1m}, p_{2m}, \mathbf{w}_m) = \gamma_m$. Finding $\psi_m^{-1}(\zeta_m^j)$ means that we want to obtain the value of the total power consumed in the primary subframe, when the achieved balanced sum-rate satisfies $R_{1m}(p_{2m}, \mathbf{w}_{1m}) + R_{2m}(p_{1m}, \mathbf{w}_{1m}) = 2\,R_{1m}(p_{2m}, \mathbf{w}_{1m}) = \zeta_m^j$. Now, we argue that this total consumed power should be as small as possible, thereby allowing γ_m to be as large as possible. Hence, to obtain the smallest value of $\psi_m^{-1}(\zeta_m^j)$, we need to solve the following optimization problem:

$$
\psi_m^{-1}(\zeta_m^j) = \min_{p_{1m}, p_{2m}, \mathbf{w}_{1m}} \quad P_{\mathrm{m}}(p_{1m}, p_{2m}, \mathbf{w}_{1m})
$$
$$
\text{subject to } R_{1m}(p_{2m}, \mathbf{w}_{1m}) + R_{2m}(p_{1m}, \mathbf{w}_{1m}) = \zeta_m^j
$$
$$
p_{1m} \geq 0, p_{2m} \geq 0. \tag{4.3.15}
$$

It has been proven in [78], the problem (4.3.15) has a unique minimizer, hence it can be solved it using the efficient algorithm proposed in [78].

Remark 1: If we consider the *average* power constraints, i.e., $\sum_{m=1}^{k}(1-\alpha_m)\gamma_m \leq$

Algorithm 3 : Alternating Convex Search Solution to (4.3.14) - Multi-Relay Scenario

Step 1. Choose ε as the stopping criterion of the algorithm.

Step 2. Assume j to be the number of iterations and set $j = 0$.

Step 3. Set $\gamma_m^0 = \frac{\gamma}{k}, \tilde{\gamma}_m^0 = \frac{\tilde{\gamma}}{k}$, and obtain the vectors of $\boldsymbol{\zeta}^0, \mathbf{p}^0$ and, $\tilde{\mathbf{p}}^0$ as the solution to the optimization problems (4.2.16), (4.2.14a) and (4.2.4a), when γ_m and $\tilde{\gamma}_m$ are replaced with γ_m^0 and $\tilde{\gamma}_m^0$, respectively. Initialize the problem (4.3.7) with the above values. Define the value of the objective function in (4.3.14) in the jth iteration as $\delta(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \mathbf{p}^j, \tilde{\boldsymbol{\gamma}}^j, \tilde{\mathbf{p}}^j) \triangleq \sum_{m=1}^{k} \left(1 - \frac{\zeta_m^j}{v_m(p_{1m}^j, \gamma_m^j)} \right) \tilde{v}_m(\tilde{p}_{1m}^j, \tilde{\gamma}_m^j)$.

Step 4. For each m, $m = 1, 2, \cdots, k$, solve the following convex optimization problem to find the optimal $\tilde{\boldsymbol{\gamma}}^{j+1}$ as

$$\tilde{\boldsymbol{\gamma}}^{j+1} = \arg\max_{\tilde{\gamma} \geq 0} \ \delta(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \mathbf{p}^j, \tilde{\gamma}, \tilde{\mathbf{p}}^j) \sum_{m=1}^{k} \tilde{\gamma}_m = \tilde{\gamma}, \ 0 \leq \tilde{p}_{1m}^j \leq \frac{\tilde{\gamma}_m}{2},$$

Step 5. For $m = 1, 2, \cdots, k$, obtain $\tilde{p}_{1m}^{j+1}$ by solving the following convex optimization problem:

$$\tilde{p}_{1m}^{j+1}(\tilde{\gamma}_m) = \arg\max_{0 \leq \tilde{p}_{1m} \leq \frac{\tilde{\gamma}_m^{j+1}}{2}} \tilde{p}_{1m}(\tilde{\gamma}_m^{j+1} - 2\tilde{p}_{1m})\tilde{\mathbf{h}}_m^H \tilde{\mathbf{Q}}_m^{-1}(\tilde{p}_{1m}, \tilde{\gamma}_m^{j+1})\tilde{\mathbf{h}}_m.$$

Step 6. Solve the following convex optimization problem, ror $m = 1, 2, \cdots, k$, to find the optimal $\boldsymbol{\gamma}^{j+1}$:

$$\boldsymbol{\gamma}^{j+1} = \arg\max_{\gamma \geq 0} \ \delta(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}, \mathbf{p}^j, \tilde{\boldsymbol{\gamma}}^{j+1}, \tilde{\mathbf{p}}^{j+1}) \sum_{m=1}^{k} \gamma_m = \gamma, \ 0 \leq \psi_m^{-1}(\zeta_m^j) \leq \gamma_m, \ 0 \leq p_{1m}^j \leq \frac{\gamma_m}{2},$$

Step 7. For $m = 1, 2, \cdots, k$, obtain p_{1m}^{j+1} by solving the following convex optimization problem:

$$p_{1m}^{j+1}(\gamma_m) = \arg\max_{0 \leq p_{1m} \leq \frac{\gamma_m^{j+1}}{2}} p_{1m}(\gamma_m^{j+1} - 2p_{1m})\mathbf{h}_m^H \mathbf{Q}_m^{-1}(p_{1m}, \gamma_m^{j+1})\mathbf{h}_m.$$

Step 8. For each m, $m = 1, 2, \cdots, k,$, solve the following convex optimization problem to find the optimal $\boldsymbol{\zeta}^{j+1}$:

$$\boldsymbol{\zeta}^{j+1} = \arg\max_{\zeta \geq 0} \ \delta(\boldsymbol{\zeta}, \boldsymbol{\gamma}^{j+1}, \mathbf{p}^{j+1}, \tilde{\boldsymbol{\gamma}}^{j+1}, \tilde{\mathbf{p}}^{j+1}) \sum_{m=1}^{k} \zeta_m = \eta, \ 0 \leq \zeta_m \leq \psi_m(\gamma_m^{j+1}),$$

Step 9. If $|\delta(\boldsymbol{\zeta}^{j+1}, \boldsymbol{\gamma}^{j+1}, \mathbf{p}^{j+1}, \tilde{\boldsymbol{\gamma}}^{j+1}, \tilde{\mathbf{p}}^{j+1}) - \delta(\boldsymbol{\zeta}^j, \boldsymbol{\gamma}^j, \mathbf{p}^j, \tilde{\boldsymbol{\gamma}}^j, \tilde{\mathbf{p}}^j)| < \varepsilon$, go to Step 10, otherwise set $j = j + 1$ and go to Step 4.

Step 10. Stop the algorithm.

γ and $\sum_{m=1}^{k} \alpha_m \tilde{\gamma}_m \leq \tilde{\gamma}$, the optimization problem (4.3.1) can be simplified as

$$\max_{\zeta, \gamma, \tilde{\gamma} \succcurlyeq 0} \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{\psi_m(\gamma_m)} \right) \tilde{\psi}_m(\tilde{\gamma}_m)$$

$$\text{s.t.} \quad \sum_{m=1}^{k} \left(1 - \frac{\zeta_m}{\psi_m(\gamma_m)} \right) \tilde{\gamma}_m = \tilde{\gamma}, \quad \sum_{m=1}^{k} \frac{\zeta_m}{\psi_m(\gamma_m)} \gamma_m = \gamma$$

$$\sum_{m=1}^{k} \zeta_m = \eta, \quad 0 \leq \zeta_m \leq \psi_m(\gamma_m), \quad \text{for } m = 1, 2, \cdots, k.$$

This problem is also biconcave and can be solved using an algorithm similar to Algorithm 3.

Remark 2: With respect to the assumption of global CSI, note that the global CSI is not needed, since

- In the first approach, the secondary transceivers need to send only the values of $\{\tilde{\psi}_m(\tilde{\gamma}_m)\}_{m=1}^{k}$ to the primary transceivers in order to solve the optimization problem (4.2.16).

- In the second and third approaches, the secondary transceivers need to transmit the magnitude of their CSI to the primary transceivers, i.e., $\{|\tilde{f}_{1m}|\}_{m=1}^{k}$ and $\{|\tilde{f}_{2m}|\}_{m=1}^{k}$, since in (4.3.3), $\tilde{\psi}_m(\tilde{\gamma}_m)$ is a function of the amplitudes of the channel coefficients not their phases [43].

Remark 3: Note that in all our design problems, we observe that half of the total available power is consumed by the relays [43, 101, 104]. Hence if the relay-transceiver coefficients are drawn from the same distribution, then each relay will consume, in average, $\frac{1}{2n_r}$ fraction of the total power consumed by either network in their corresponding time frames.

4.4 Simulation Results

We consider a network with $n_{\mathrm{r}} = 10$ relays, $k = 10$ subchannels, and two transceiver pairs. We assume that the relays are randomly located between the two transceivers. Indeed, we assume that the channel coefficient between each transceiver and each relay is drawn from a complex Gaussian random distribution with zero mean and unit variance. Moreover, the noises at the transceivers as well as those at the relays are assumed to be i.i.d Gaussian with unit variance.

4.4.1 Multi-relay scenario with two spectral power mask constraints

In our simulations, we consider a flat power spectral mask where the maximum total power available to the primary (secondary) network over all subchannels are equal. In other words, we assume that $\gamma_m = \frac{\gamma}{k}$ ($\tilde{\gamma}_m = \frac{\tilde{\gamma}}{k}$), where γ ($\tilde{\gamma}$) is the total power consumed in the primary (secondary) network over all subchannels. Fig. 4.2 illustrates the average sum-rate of the secondary transceivers versus γ, for different values of η, when $\tilde{\gamma} = 25$ dBW. We observe that the average secondary sum-rate increases when the total power consumed in the secondary network increases. Given γ and $\tilde{\gamma}$ are fixed, this figure demonstrates that when the primary network rate demand increases, the secondary network sum-rate will decreases, implying that the higher is the rate demand of the primary network, the less resources will remain for the secondary network users. Fig. 4.3 shows the secondary network sum-rate versus η, for different values of $\tilde{\gamma}$, when $\gamma = 25$ dBW. This figure illustrates that the secondary network sum-rate is a decreasing function of the primary network rate demand. For
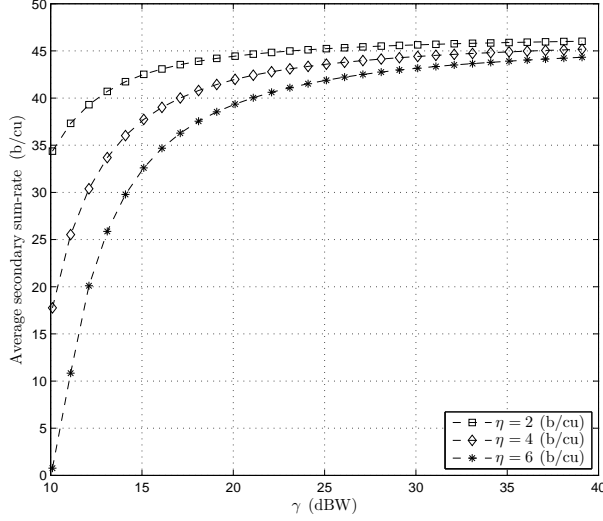
Figure 4.2: Average secondary sum-rate versus γ, in the multi-relay scenario with spectral power mask constraints, for different values of η, when $\tilde{\gamma} = 25$ dBW.
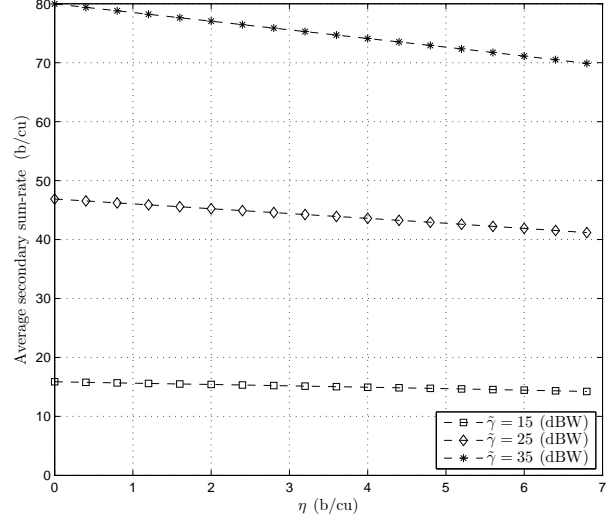
Figure 4.3: Average secondary sum-rate versus η, in the multi-relay scenario with spectral power mask constraints, for different values of $\tilde{\gamma}$, when $\gamma = 25$ dBW.

different values of $\tilde{\gamma}$, Fig. 4.3 depicts that the higher the value of $\tilde{\gamma}$ is, the larger the value of secondary network sum-rate will be. This effect can be justified based on (4.2.15), noting that the value of the objective function is increasing in $\tilde{\psi}_m(\tilde{\gamma}_m)$. Hence, when $\tilde{\gamma}$ increases, the value of the objective function also increases.

4.4.2 Single-relay scenario, with per-network total power constraints

Fig. 4.4 shows the value of the average sum-rate of the secondary transceivers achieved by our solution to (4.3.7) and the value of the objective function of (4.3.7) for the

same solution versus γ, for different values of η, when $\tilde{\gamma} = 35\,\mathrm{dBW}$. This figure shows that when γ is increased, the value of the secondary network sum-rate increases. However, this sum-rate becomes saturated for large values of γ. This saturation behaviour can be explained using (4.2.8) and (4.3.7), where the contribution of γ_m (and equivalently that of γ) to the secondary network sum-rate is in the time-sharing factor α_m. Since the maximum value for each time sharing factor is 1, increasing the primary network power budget, can not increase the secondary network sum-rate substantially, when α_m in (4.2.8) is close to 1. Note that Fig. 4.4 reveals that there is a large gap between the graphs for low γ, and this gap becomes smaller for higher values of γ. This effect is in agreement with the high-SNR approximation that we used in our proposed method, where the gap between the value of the objective function and its upperbound becomes smaller for higher values of γ. Interestingly, the gap is relatively small even for low values of γ. Fig. 4.5 shows the value of the secondary network sum-rate achieved by our solution to (4.3.7) and the value of the objective function of (4.3.7) for the same solution, versus η, for different values of $\tilde{\gamma}$. This figure illustrates that the secondary network sum-rate is a decreasing function of the primary network rate demand.

Fig. 4.6 shows the convergence behaviour of Algorithm 2, when $\epsilon = 10^{-4}$ is chosen, for single-relay multi-carrier scenario. In this figure, we have plotted the values of the secondary network sum-rate normalized to its final value for different channel realizations and the average value of the secondary network sum-rate, normalized to its final value, averaged over 100 channel realizations. As can be seen from this figure, the proposed algorithm converges very fast to a stationary point.
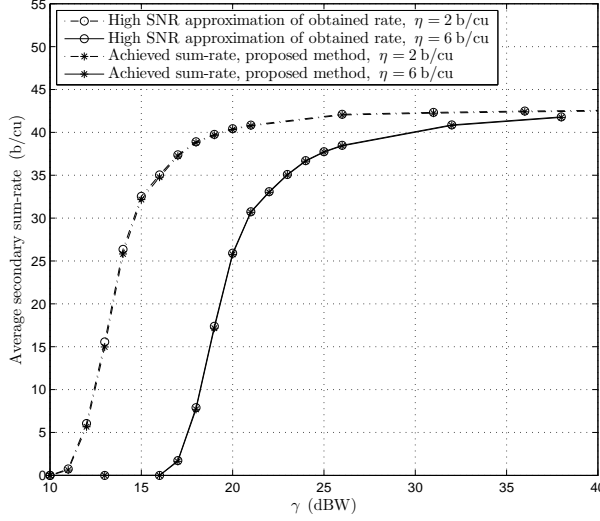
Figure 4.4: Average secondary sum-rate versus γ, in the single-relay scenario with per-network total power constraint, for different values of η, when $\tilde{\gamma} = 35$ dBW.
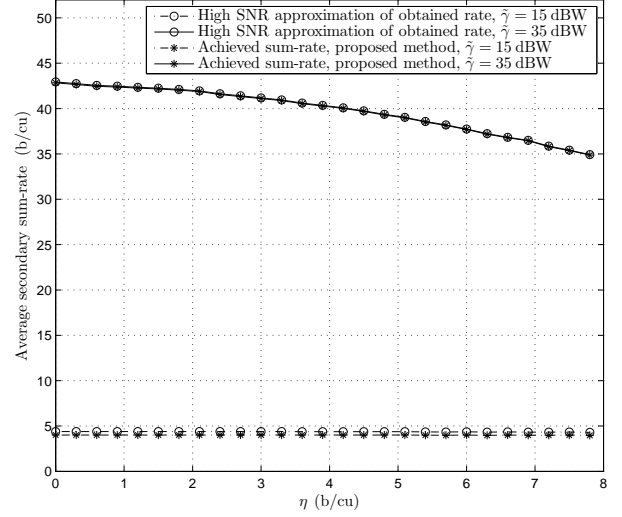
Figure 4.5: Average secondary sum-rate versus η, in the single-relay scenario with per-network total power constraint, for different values of $\tilde{\gamma}$, when $\gamma = 25$ dBW.

4.4.3 Per-network total power constraint in multi-relay scenario

Fig. 4.7 illustrates the values of the average sum-rate of the secondary transceivers versus γ and $\tilde{\gamma}$, for different values of η. This figure shows that when γ or $\tilde{\gamma}$ is increased, the value of the secondary network sum-rate increases. Fig. 4.8 shows the effect of η on the sum-rate of the secondary users. We observe that increasing the value of $\tilde{\gamma}$ leads to a higher value for the secondary network sum-rate. We also observe that the slope of reduction in the sum-rate of the secondary users in terms of η is larger as $\tilde{\gamma}$ is increased, while γ is fixed. The reason is that, these curves have to
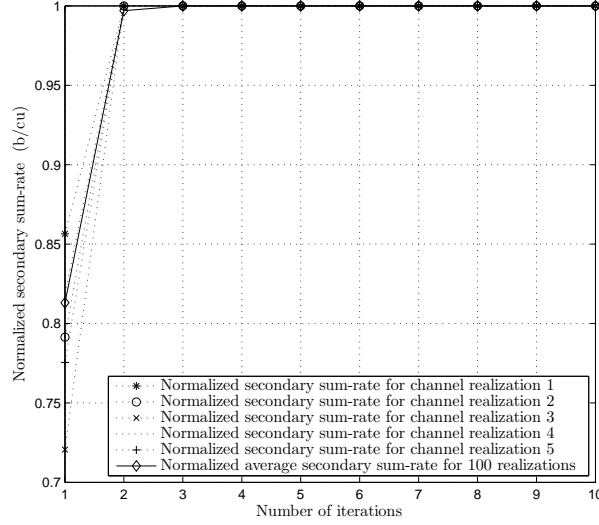
Figure 4.6: Normalized secondary sum-rate versus the number of iterations in the multi-relay scenario, when $\eta = 4$, $\gamma = 25\,\mathrm{dBW}$ and $\tilde{\gamma} = 25\,\mathrm{dBW}$.

be monotonically decreasing in terms of η and they all have to approach zero as η approaches a certain value. This value of η, say η^{max}, is the same for different values of $\tilde{\gamma}$ and when γ is fixed. Indeed, η^{max} depends only on γ. As for higher values of $\tilde{\gamma}$, the secondary network sum-rate is higher compared to the same rate for lower values of $\tilde{\gamma}$. This sum-rate has to decay faster compared to the same curve for lower values of $\tilde{\gamma}$, as η is increased, to ensure that this sum-rate approaches zero as η approaches η^{max}.

Fig. 4.9 shows the convergence behaviour of Algorithm 3, when $\epsilon = 10^{-4}$ is chosen, for a multi-relay scenario. In this figure, we have plotted the values of the secondary network sum-rates normalized to its corresponding final value for different channel realizations and the average value of the secondary network sum-rate, normalized to its final value, averaged over 100 channel realizations. We observe that the proposed algorithm reaches its final value in 5-8 iterations. This figure shows that the
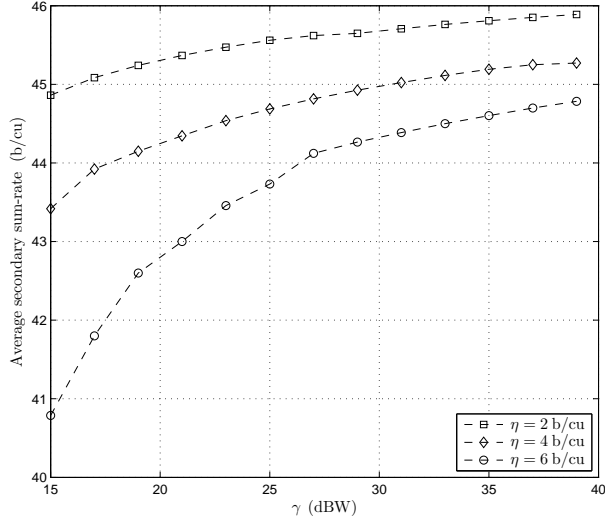
Figure 4.7: Average secondary sum-rate versus $\tilde{\gamma}$, in the multi-relay scenario with per-network total power constraint, for different values of η, when $\tilde{\gamma} = 25$ dBW.
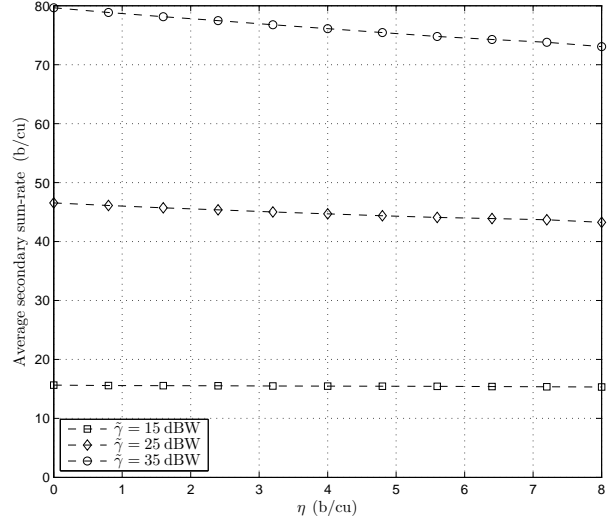
Figure 4.8: Average secondary sum-rate versus η, in the multi-relay scenario with per-network total power constraint, for different values of $\tilde{\gamma}$, when $\gamma = 25$ dBW.

convergence of the proposed algorithm is relatively fast.

4.5   Conclusions

We studied the problem of the temporal, spectral and relay resource sharing between two pairs of transceivers which exploit a network of one or multiple relays. We proposed two spectrum leasing and resource sharing approaches, each of which has its own application. In each approach, we formulated an optimization problem in order to calculate the corresponding design parameters.

In the first approach, we considered a multi-relay scenario and maximized the

Figure 4.9: Normalized secondary sum-rate versus the number of iterations in the multi-relay scenario, when $\eta = 4$, $\gamma = 25$ dBW and $\tilde{\gamma} = 25$ dBW.
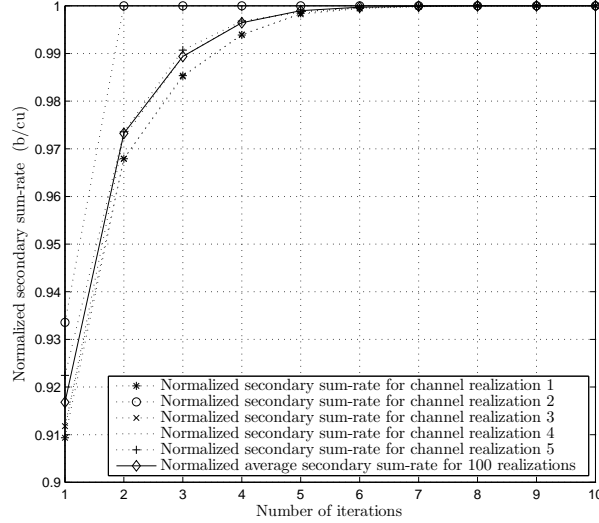
average sum-rate of the secondary transceivers, while guaranteeing an average sum-rate for the primary transceivers, under two spectral power masks to limit the total power consumed in each network over each subchannel. We showed that this design problem can be turned into a linear programming problem.

In the second approach, we considered the maximization of the secondary network sum-rate subject to per-network total power constraints, while guaranteeing a minimum sum-rate for the primary transceivers. In this approach, we considered two different scenarios: a single-relay scenario and a multi-relay scenario. For the single-relay case, we used a high-SNR approximation and developed an alternating convex search algorithm which exploits the biconcavity of the approximated objective function in terms of the design parameters, namely the vector of the total powers allocated to each network over different subchannels and the vector of rate of the primary network over all subchannels. In the case of a multi-relay scenario, we expanded

the design parameters to five vectors and showed that the problem is concave in any parameter vector, given the other four vectors are fixed. Doing so, we proposed an alternating convex search algorithm, to tackle the underlying problem.

Chapter 5

Optimal Power Allocation in Unidirectional Collaborative Relay Networks: Centralized Energy Harvesting Case

In this chapter, we consider a network consisting of several distributed relay nodes and a pair of users. The relays are connected to a central energy harvesting module, that is equipped with a battery to store the harvested energy, via a wire network. The underlying transmission scheme consists of k temporally orthogonal time frames. We assume that energy packets arrive at the beginning of each time frame. In each time frame, a specific amount of the harvested energy will be allocated to each relay and the remaining energy remains in the battery for future possible relay transmission. We assume that there is no direct link between the users, hence, a communication link will be established through the help of the relays. The design problem in this chapter is to obtain the optimal transmission policy for the relays in a unidirectional communication scheme, under several constraints on the total power consumed by the relays over each time interval, such that the overall throughput of the network is maximized. The contributions of this chapter is summarized as:

- We first study the problem of obtaining optimal transmission policy under the assumptions that full CSI is available and the amount of energies, that are to be harvested in future, are known. We then formulate the problem of maximizing the *throughput* of the network restricted by several constraints on the total power consumed by the relays over each time frame. We prove that the design problem is convex with respect to the design variables, i.e., the total power consumed by the relays over each time frame.

- We then study the case where only the statistics of the channels are known. In this case, we resort to optimizing the *average throughput* of the users under the same constrains of the first optimization problem. We obtain a semi-closed form expression for the average throughput as a function of the the total consumed power by the relays over the different time frames. Furthermore, we obtain the optimal power allocation in each time frame using the concept of the so-called energy-tunnel.

We discuss the proposed model in detail in the sequel.

5.1   System Model for Relay Energy Harvesting

We consider a network which consists of a pair of transmitter-receiver and n_r relay nodes. Assuming that there is no direct link between the transmitter and the receiver, the relays cooperate to establish a communication link between the transmitter and the receiver. We assume that the transmitter and the receiver are connected to a power grid, meaning that they have access to a constant source of electricity to power up their electronic modules. In other words, the transmitter and the receiver have

their own source of power. However, the relays distributed between the transmitter and the receiver are not connected to the power grid. We assume that renewable energies, i.e., energy that can be harvested from a variety of sources, is used to power up the relays for their transmission needs. Furthermore, we assume that the relays are connected to a single energy harvesting module that is equipped with a battery which stores the harvested energy. The energy harvesting module is responsible to distribute the harvested energy among the relays using power cables which connect all the relays to the harvested energy. This assumption means that the location of the relays are fixed and they all rely on the battery for their transmission need. The assumption that all the relays are connected to one single battery is essential and we will discuss the benefit of such an assumption in next section. The transmission framework consists of k equal-length orthogonal time intervals, called time frame, each with T seconds duration. Each time frame consists of two equal-length non-overlapping time slots. Assuming amplify-and-forward (AF) relaying protocol, in the first time slot the transmitter in the ith time frame sends its information symbols to the relays. The relays then broadcast the so-called amplified-and phase-adjusted versions of the signals they received in the previous time slot. We assume that at the beginning of ith time frame an energy packet E_{i-1}^r arrives and is stored in the battery. Note that each battery has a maximum capacity to store energy. We denote the capacity of the battery in our model as $B^{\max}$. Fig. 5.1 summarizes the proposed model.

Let us denote the transmitted symbol of the transmitter over the ith time frame, (when $(i-1)T \leq t \leq iT$), as $s_i(t)$ where the power of the transmitted symbol is assumed to be a fixed value, denoted as P_0. The $n_\mathrm{r} \times 1$ vector of the signals received
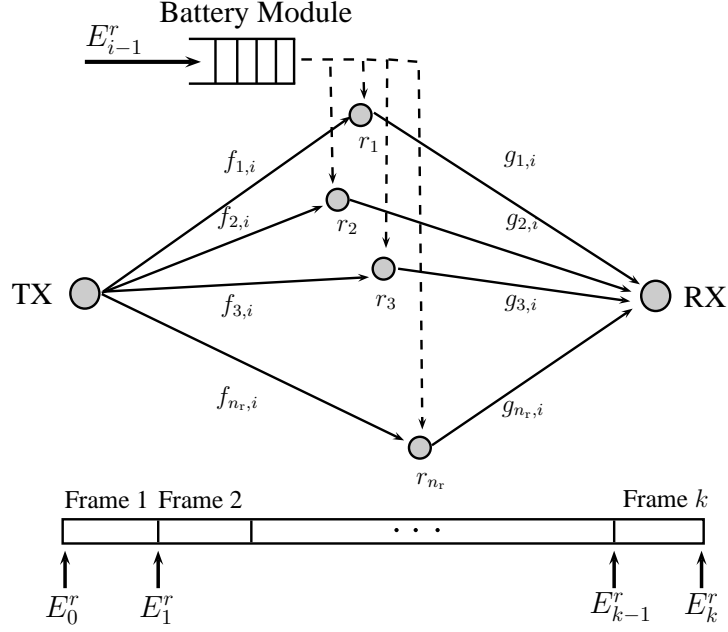
Figure 5.1: Energy harvesting in a two-hop relay assisted network.

at the relays over the ith time frame can be written as

$$\mathbf{x}_i(t) = \sqrt{P_0}\mathbf{f}_i s_i(t) + \mathbf{n}_i(t) \tag{5.1.1}$$

where $\mathbf{f}_i = [f_{1,i} \quad f_{2,i} \quad \cdots \quad f_{n_{\mathrm{r}},i}]^T$ is the $n_{\mathrm{r}} \times 1$ vector of the channel coefficients between the transmitter and the relays, $\mathbf{n}_i(t)$ is the $n_{\mathrm{r}} \times 1$ vector of the received noises at the relays over the ith time frame and $\mathbf{x}_i(t) = [x_{1,i}(t) \quad x_{2,i}(t) \quad \cdots \quad x_{n_{\mathrm{r}},i}(t)]^T$ is the $n_{\mathrm{r}} \times 1$ vector of the noisy signal received by the relays. The relays then process their received signals and broadcast modified versions of the signals that they received in the previous time slot to the receiver using AF relaying protocol. Defining $\tilde{x}_{j,i}(t) \triangleq \frac{x_{j,i}(t)}{\sqrt{P_0|f_{j,i}|^2+1}}$, for $j = 1, 2, ..., n_{\mathrm{r}}$, the $n_{\mathrm{r}} \times 1$ vector of the relay retransmitted signals can be written as

$$\mathbf{t}_i(t) = \mathbf{w}_i(t) \odot \tilde{\mathbf{x}}_i(t) \tag{5.1.2}$$

where $\mathbf{w}_i(t) = [w_{1,i}^*(t) \ w_{2,i}^*(t) \ \cdots \ w_{n_r,i}^*(t)]^T$ is the $n_r \times 1$ relay beamforming vector over the ith time frame and $\tilde{\mathbf{x}}_i(t) = [\tilde{x}_{1,i}(t) \ \tilde{x}_{2,i}(t) \ \cdots \ \tilde{x}_{n_r,i}(t)]^T$ is the $n_r \times 1$ vector of the normalized version of the received signals by the relays with unit power. Using (5.1.2), one can show that the sum of the instantaneous transmitted powers of all relays over the ith time frame can be written as

$$E\{\|\mathbf{t}_i(t)\|^2\} = \|\mathbf{w}_i(t)\|^2 = p_i(t) \tag{5.1.3}$$

where $p_i(t)$ is the total instantaneous power consumed by the relays over the ith time frame. Furthermore, using (5.1.2), one can write the received signal over the ith time frame at the receiver as

$$r_i(t) = \mathbf{t}_i^T(t)\mathbf{g}_i + n_i(t) = \sqrt{P_0}\mathbf{w}_i^H(t)\mathbf{G}_i\mathbf{f}_i s_i(t) + \mathbf{w}_i^H(t)\mathbf{G}_i\mathbf{n}_i(t) + \tilde{n}_i(t) \tag{5.1.4}$$

where $\mathbf{g}_i = [g_{1,i} \ g_{2,i} \ \cdots \ g_{n_r,i}]^T$ is the $n_r \times 1$ vector of the channel coefficients between the relays and the receiver, $\mathbf{G}_i$ is an $n_r \times n_r$ diagonal matrix whose jth diagonal element is $\mathbf{G}_i(j,j) = \frac{g_{j,i}}{\sqrt{P_0|f_{j,i}|^2+1}}$ and $\tilde{n}_i(t)$ is the received noise at the receiver over the ith time frame. We assume that all noises are complex Gaussian random processes with zero-mean and unit-variance, i.e., $E\{\mathbf{n}_i(t)\mathbf{n}_i^H(t)\} = \mathbf{I}$ and $E\{\tilde{n}_i(t)^2\} = 1$, where $E\{\cdot\}$ stands for the statistical expectation operator and $\mathbf{I}$ is an $n_r \times n_r$ identity matrix. One should note that in a discrete time signal model, the information theoretic rate is a function of the signal-to-noise-ratio (SNR), and thus is denoted as $R(\text{SNR})$ and the total number of transmitted bits can be written as $\delta t \times R(\text{SNR})$ where δt is the duration of the transmission. Moreover, if the bandwidth of the channel is large enough, the transmission scheme can accept small values for δt. Hence, one can write the instantaneous function for the number of the transmitted

bit as $\delta t \times R(\text{SNR}(t))$ [105].With a small abuse of notation, we define the instantaneous information theoretic rate in the ith time frame as

$$R_i(\mathbf{w}_i(t)) = \log_2\left(1 + \text{SNR}_i(\mathbf{w}_i(t))\right) \tag{5.1.5}$$

where we have obtained the instantaneous SNR over the ith time frame from (5.1.4) as $\text{SNR}_i(\mathbf{w}_i(t)) = \frac{P_0|\mathbf{w}_i^H(t)\mathbf{G}_i\mathbf{f}_i|^2}{|\mathbf{w}_i(t)^H\mathbf{G}_i|^2+1} = \frac{P_0\mathbf{w}_i^H(t)\left(\mathbf{G}_i\mathbf{f}_i\mathbf{f}_i^H\mathbf{G}_i^H\right)\mathbf{w}_i(t)}{\mathbf{w}_i^H(t)\left(\mathbf{G}_i\mathbf{G}_i^H\right)\mathbf{w}_i(t)+1} = \frac{\mathbf{w}_i^H(t)\left(P_0\mathbf{G}_i\mathbf{f}_i\mathbf{f}_i^H\mathbf{G}_i^H\right)\mathbf{w}_i(t)}{\mathbf{w}_i^H(t)\left(\mathbf{G}_i\mathbf{G}_i^H+\frac{1}{p_i(t)}\mathbf{I}\right)\mathbf{w}_i(t)}$. In the following sections, we use the proposed data model to design the parameters of interest, the relay beamfoming vectors $\{\mathbf{w}_i(t)\}_{i=1}^k$ and obtain their corresponding power consumptions $\{p_i(t)\}_{i=1}^k$, in our proposed energy harvesting system model, thereby allowing the relays to calculate the network parameters.

5.2   Problem Formulation

Using the data model developed in the previous section, we aim to design the re-lay beamfoming vectors $\{\mathbf{w}_i(t)\}_{i=1}^k$ and obtain their corresponding total relay power consumptions $\{p_i(t)\}_{i=1}^k$. Having the knowledge of the harvested anergies over all time frames apriori, we propose two different approaches to obtain the parameters of the network, where each of them has its own application. In order to obtain the parameters of interest, i.e., the beamforming vectors over all time frames and the corresponding power consumption of each relay, we aim to maximize the total transmitted bits (the overall throughput) in the network under several constraints on the total consumed power by the relays over each time frame. Let us consider the

following optimization problem:

$$\max_{\mathcal{W},\mathcal{P}} \sum_{i=1}^{k} \int_{(i-1)T+\frac{T}{2}}^{iT} R_i(\mathbf{w}_i(t))dt \tag{5.2.1a}$$

$$\text{s.t.} \quad \|\mathbf{w}_i(t)\|^2 = p_i(t), \ p_i(t) \geq 0 \ \ i = 1, 2, ..., k \tag{5.2.1b}$$

$$\sum_{i=1}^{l} \int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t)dt \leq \sum_{i=0}^{l-1} E_i^r, \quad \text{for } l = 1, 2, ..., k \tag{5.2.1c}$$

$$\sum_{i=0}^{l} E_i^r - \sum_{i=1}^{l} \int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t)dt \leq B^{\max}, \quad \text{for } l = 1, 2, ..., k-1, \tag{5.2.1d}$$

where we define $\mathcal{W} = \{\mathbf{w}_1(t), \mathbf{w}_2(t), \cdots, \mathbf{w}_k(t), \text{for all } t\}$, as the set of the beamforming vectors and $\mathcal{P} = \{p_1(t), p_2(t), \cdots, p_k(t), \text{for all } t\}$, as the set of total relay power, over different time frames. The objective function in (5.2.1a) is the overall throughput of the network over all time frames. Note that the factor $\frac{T}{2}$ in the boundary of the integral represent the fact that the relays transmit the received information symbols only over half of each of the time frames. The constraints in (5.2.1b) stand for the relation between the relays beamforming vectors and their corresponding total consumed power over all time frames. Due to the fact that the battery cannot store the energy which has yet to be harvested, a set of constraints is needed to be applied to our design problem. Hence, the so-called *energy causality constraints* in (5.2.1c) are used to ensure that the total energy consumed by all relays up to the lth time frame is less than the total harvested energy over the same sets of time frames. The constraints in (5.2.1d) are used to ensure that the remaining energy in the battery up to the arrival of the lth time frame is less than the battery maximum capacity, otherwise an overflow would occur in the battery. When overflow occurs in the battery, the excessive energy must be eliminated, e.g., grounding the extra amount of energy. These constraints ensure that the battery overflow will not occur. To simplify the

optimization problem (5.2.1), let us rewrite this problem in the following equivalent form:

$$\max_{\mathcal{P}} \sum_{i=1}^{k} \max_{\mathbf{w}_i(t)} \int_{(i-1)T+\frac{T}{2}}^{iT} R_i(\mathbf{w}_i(t)) dt$$

$$\text{s.t.} \qquad \|\mathbf{w}_i(t)\|^2 = p_i(t),$$

$$\sum_{i=1}^{l} \int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t) dt \leq \sum_{i=0}^{l-1} E_i^r, \quad \text{for } l = 1, 2, ..., k$$

$$\sum_{i=0}^{l} E_i^r - \sum_{i=1}^{l} \int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t) dt \leq B^{\max}, \quad \text{for } l = 1, 2, ..., k-1. \qquad (5.2.2)$$

We note that the inner maximization problem in (5.2.2) can be written as

$$\max_{\|\mathbf{w}_i(t)\|^2 = p_i(t)} \int_{(i-1)T+\frac{T}{2}}^{iT} R_i(\mathbf{w}_i(t)) dt = \int_{(i-1)T+\frac{T}{2}}^{iT} \max_{\|\mathbf{w}_i(t)\|^2 = p_i(t)} R_i(\mathbf{w}_i(t)) dt$$

$$= \int_{(i-1)T+\frac{T}{2}}^{iT} \log_2 \left(1 + \text{SNR}_i^{\max}(p_i(t)) \right) dt$$

$$= \int_{(i-1)T+\frac{T}{2}}^{iT} R_i^{\max}(p_i(t)) dt \qquad (5.2.3)$$

where $\text{SNR}_i^{\max}(p_i(t)) = \max_{\|\mathbf{w}_i(t)\|^2 = p_i(t)} \text{SNR}_i(\mathbf{w}_i(t))$ and with a small abuse of notation we have used $R_i^{\max}(p_i(t))$ instead of $R_i(\mathbf{w}_i^{\circ}(t))$, where $\mathbf{w}_i^{\circ}(t)$ is the optimal beamforming vector over the ith time frame at time t for a given value of $p_i(t)$. One can easily show that $\text{SNR}_i^{\max}(p_i(t))$ can be written as

$$\text{SNR}_i^{\max}(p_i(t)) = \max_{\|\mathbf{w}_i(t)\|^2 = p_i(t)} \text{SNR}_i(\mathbf{w}_i(t))$$

$$= \max_{\|\mathbf{w}_i(t)\|^2 = p_i(t)} \frac{\mathbf{w}_i^H(t) \left(P_0 \mathbf{G}_i \mathbf{f}_i \mathbf{f}_i^H \mathbf{G}_i^H \right) \mathbf{w}_i(t)}{\mathbf{w}_i^H(t) \left(\mathbf{G}_i \mathbf{G}_i^H + \frac{1}{p_i(t)} \mathbf{I} \right) \mathbf{w}_i(t)}$$

$$= P_0 p_i(t) \mathbf{h}_i^H \left(p_i(t) \mathbf{Q}_i + \mathbf{I} \right)^{-1} \mathbf{h}_i \qquad (5.2.4)$$

where we used the following definitions: $\mathbf{h}_i = \mathbf{G}_i\mathbf{f}_i$ and $\mathbf{Q}_i = \mathbf{G}_i\mathbf{G}_i^H$. We also note that the optimal beamforming vector in the ith time frame can be written as

$$\mathbf{w}_i^o(t) = \kappa\big(p_i(t)\mathbf{Q}_i + \mathbf{I}\big)^{-1}\mathbf{h}_i \tag{5.2.5}$$

where $\kappa = \dfrac{p_i(t)}{\mathbf{h}_i^H\big(p_i(t)\mathbf{Q}_i+\mathbf{I}\big)^{-2}\mathbf{h}_i}$. See [106] for more detail.

Lemma 1: The power distribution $p_i(t)$ over the ith time frame must be a constant value to deliver the maximum throughput.

Proof: See Appendix A.

For a given constant power allocation p_i over the ith time frame, the maximum achievable rate can be written as

$$R_i^{\max}(p_i) = \log_2\big(1 + \mathrm{SNR}_i^{\max}(p_i)\big) = \log_2\big(1 + P_0 p_i \mathbf{h}_i^H\big(p_i\mathbf{Q}_i + \mathbf{I}\big)^{-1}\mathbf{h}_i\big), \tag{5.2.6}$$

where with a small abuse of notation, we have used p_i instead of $p_i(t)$. We also note that the overall throughput over the ith time frame is given by $\frac{T}{2}R_i^{\max}(p_i)$. Using the result of Lemma 1 along with (5.2.4), the optimization problem (5.2.2) can be rewritten as

$$\max_{\substack{p_i \geq 0 \\ i=1,2,...,k}} \quad \frac{T}{2}\sum_{i=1}^{k} R_i^{\max}(p_i)$$

$$\text{s.t} \quad \frac{T}{2}\sum_{i=1}^{l} p_i \leq \sum_{i=0}^{l-1} E_i^r, \quad \text{for } l = 1, 2, ..., k$$

$$\sum_{i=0}^{l} E_i^r - \frac{T}{2}\sum_{i=1}^{l} p_i \leq B^{\max}, \quad \text{for } l = 1, 2, ..., k-1. \tag{5.2.7}$$

Lemma 2: For a fixed value of P_0, $\mathrm{SNR}_i^{\max}(p_i)$ is a concave function of p_i.

Proof: See Appendix B.

Since $\mathrm{SNR}_i^{\max}(p_i)$ is a concave function of p_i and $\log_2(\cdot)$ is a concave and increasing function of its argument, $R_i^{\max}(p_i)$ is also a concave function of p_i. Moreover, the two

sets of constraints in (5.2.7) are linear in terms of the total power consumed in each time frame. Therefore, the optimization problem (5.2.7) is a convex optimization problem. There are several efficient algorithms, e.g., interior point algorithm, can be used to obtain the optimal solution to the design problem (5.2.7). In the following sections, we introduce two different scenarios where we aim to obtain the optimal power consumption for all relays over all time frames at the beginning of the first time frame.

5.3   Offline case

We note that the optimal solution to the optimization problem (5.2.7) can be obtained if the values of the energies arrived at the beginning of all time frames as well as the CSI for all links over all time frames are known apriori. We refer to the solution for this case as *offline solution*, meaning that, at any time instance, the CSI and the knowledge of the values of the energies which are to arrive over all future time frames are available apriori. In the sequel, we provide some properties of the optimal solution to the optimization problem (5.2.7) by using Karush-Kuhn-Tucker (KKT) conditions. Let us define the Lagrangian corresponding to the optimization problem (5.2.7) as

$$\mathcal{L}(\mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu}) = -\frac{T}{2} \sum_{i=1}^{k} R_i^{\max}(p_i) + \sum_{l=1}^{k} \lambda_l \left(\frac{T}{2} \sum_{i=1}^{l} p_i - \sum_{i=0}^{l-1} E_i^r \right)$$
$$+ \sum_{l=1}^{k-1} \eta_l \left(\sum_{i=0}^{l} E_i^r - \frac{T}{2} \sum_{i=1}^{l} p_i - B^{\max} \right) - \sum_{l=1}^{k} \mu_l p_l \qquad (5.3.1)$$

where we have used the following notations: $\mathbf{p} \triangleq [p_1 \quad p_2 \quad \cdots \quad p_k]$, $\boldsymbol{\lambda} \triangleq [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_k]$, $\boldsymbol{\eta} \triangleq [\eta_1 \quad \eta_2 \quad \cdots \quad \eta_k]$ and, $\boldsymbol{\mu} \triangleq [\mu_1 \quad \mu_2 \quad \cdots \quad \mu_k]$. We note

that $\boldsymbol{\lambda}$, $\boldsymbol{\eta}$ and $\boldsymbol{\mu}$ are the vectors of dual variables. The KKT conditions for the Lagrangian function in (5.3.1) are given by

Primal and Dual Feasibility:

$$\mathbf{p} \succcurlyeq 0, \ \boldsymbol{\lambda} \succcurlyeq 0, \ \boldsymbol{\eta} \succcurlyeq 0, \ \boldsymbol{\mu} \succcurlyeq 0$$

$$\frac{T}{2} \sum_{i=1}^{l} p_i \leq \sum_{i=0}^{l-1} E_i^r, \quad \text{for } l = 1, 2, \cdots, k$$

$$\sum_{i=0}^{l} E_i^r - \frac{T}{2} \sum_{i=1}^{l} p_i \leq B^{\max}, \quad \text{for } l = 1, 2, \cdots, k-1 \qquad (5.3.2)$$

Complementary Slackness:

$$\lambda_l \left(\frac{T}{2} \sum_{i=1}^{l} p_i - \sum_{i=0}^{l-1} E_i^r \right) = 0, \quad \text{for } l = 1, 2, \cdots, k \qquad (5.3.3a)$$

$$\eta_l \left(\sum_{i=0}^{l} E_i^r - \frac{T}{2} \sum_{i=1}^{l} p_i - B^{\max} \right) = 0, \quad \text{for } l = 1, 2, \cdots, k \qquad (5.3.3b)$$

$$\mu_l p_l = 0, \quad \text{for } l = 1, 2, \cdots, k \qquad (5.3.3c)$$

Stationarity:

$$\frac{\partial \mathcal{L}(\mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu})}{\partial p_i} = -\frac{T}{2} \frac{dR_i^{\max}(p_i)}{dp_i} + \frac{T}{2} \sum_{l=i}^{k} \lambda_l$$

$$-\frac{T}{2} \sum_{l=i}^{k-1} \eta_l - \mu_i = 0, \quad \text{for } i = 1, 2, \cdots, k-1. \qquad (5.3.4)$$

$$\frac{\partial \mathcal{L}(\mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu})}{\partial p_k} = -\frac{T}{2} \frac{dR_k^{\max}(p_k)}{dp_k} + \frac{T}{2} \lambda_k - \mu_k = 0. \qquad (5.3.5)$$

Let us define $\nu_i(p_i) \triangleq \frac{T}{2} \frac{dR_i^{\max}(p_i)}{dp_i} = \frac{T}{2} \frac{\frac{dSNR_i^{\max}(p_i)}{dp_i}}{1+SNR_i^{\max}(p_i)} = \frac{T}{2} \frac{P_0 \mathbf{h}_i^H (p_i \mathbf{Q}_i + \mathbf{I})^{-2} \mathbf{h}_i}{1 + P_0 p_i \mathbf{h}_i^H (p_i \mathbf{Q}_i + \mathbf{I})^{-1} \mathbf{h}_i}$. We note that $R_i^{\max}(p_i)$ is a concave and increasing function of its argument (see Appendix B), which means $\nu_i(p_i) > 0$ and $\frac{d\nu_i(p_i)}{dp_i} < 0$. In other words, $\nu_i(p_i)$ is a strictly decreasing

function of p_i, therefore $\nu_i^{-1}(\cdot)$ exists. One can see that, for $i = 1, 2, \cdots, k-1$, if at the optimum, the optimal power, denoted as p_i^{o}, is equal to zero, then $\mu_i^{\text{o}} \geq 0$, and if $p_i^{\text{o}} > 0$ then, $\mu_i^{\text{o}} = 0$. Using (5.3.3c) and (5.3.4), for $i = 1, 2, \cdots, k-1$, one can write

$$\text{if } \mu_i^{\text{o}} > 0, \text{ then } p_i^{\text{o}} = 0 \Rightarrow -\nu_i(0) + \frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} = \mu_i^{\text{o}} \tag{5.3.6}$$

$$\text{if } p_i^{\text{o}} > 0, \text{ then } \mu_i^{\text{o}} = 0 \Rightarrow -\nu_i(p_i^{\text{o}}) + \frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} = 0 \ . \tag{5.3.7}$$

We note that (5.3.6) and (5.3.7) are mutually exclusive. This means that only one of the two constraints in (5.3.6) and (5.3.7) can happen at the same time. To show this, noting that $\nu_i(0) = \frac{T}{2}P_0\|\mathbf{h}_i\|^2$, one can realize from (5.3.6) that $\frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} \geq \nu_i(0) = \frac{T}{2}P_0\|\mathbf{h}_i\|^2$ must hold true if $\mu_i^{\text{o}} > 0$. Moreover, using the fact that $\nu_i(p_i)$ is a positive and decreasing function of p_i, when $p_i > 0$, then one can write $0 \leq \nu_i(p_i^{\text{o}}) < \nu_i(0) = \frac{T}{2}P_0\|\mathbf{h}_i\|^2$. This fact leads us to rewrite (5.3.7) as $0 \leq \frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} = \nu_i(p_i^{\text{o}}) \leq \nu_i(0) = \frac{T}{2}P_0\|\mathbf{h}_i\|^2$, and hence obtain the optimal power consumption p_i^{o} as $p_i^{\text{o}} = \nu_i^{-1}\left(\frac{T}{2}\left(\sum_{l=i}^{k}\lambda_l^{\text{o}} - \sum_{l=i}^{k-1}\eta_l^{\text{o}}\right)\right)$. These discussions prove that (5.3.6) and (5.3.7) are mutually exclusive and they can be rewritten as

$$\text{if } \frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} \geq \frac{T}{2}P_0\|\mathbf{h}_i\|^2 \iff p_i^{\text{o}} = 0 \tag{5.3.8}$$

$$\text{if } \frac{T}{2}\sum_{l=i}^{k}\lambda_l^{\text{o}} - \frac{T}{2}\sum_{l=i}^{k-1}\eta_l^{\text{o}} < \frac{T}{2}P_0\|\mathbf{h}_i\|^2 \iff p_i^{\text{o}} = \nu_i^{-1}\left(\frac{T}{2}\left(\sum_{l=i}^{k}\lambda_l^{\text{o}} - \sum_{l=i}^{k-1}\eta_l^{\text{o}}\right)\right) \tag{5.3.9}$$

One can merge (5.3.8) and (5.3.9) to write the optimal power consumption over the ith time frame, $i = 1, \cdots, k-1$, as

$$p_i^{\text{o}} = \max\left\{0, \nu_i^{-1}\left(\frac{T}{2}\left(\sum_{l=i}^{k}\lambda_l^{\text{o}} - \sum_{l=i}^{k-1}\eta_l^{\text{o}}\right)\right)\right\} . \tag{5.3.10}$$

Similarly, using (5.3.5), one can write the optimal power consumption over the kth time frame as

$$p_k^o = \max \left\{ 0, \nu_k^{-1} \left(\frac{T}{2} \lambda_k^o \right) . \right\} \qquad (5.3.11)$$

The optimal total power consumption over all time frames, obtained in (5.3.10) and (5.3.11), must satisfy the primal feasibility constraints in (5.3.2). The optimal dual variables in (5.3.10) and (5.3.11) can be interpreted as the solution of the so-called directional-water-filling algorithm that is introduced in [105] as a variation of the conventional water-filling algorithm. Algorithm 4 summarizes the steps to obtain the optimal solution.

Algorithm 4 : Directional water-filling algorithm to solve (5.3.10) and (5.3.11)

Step 1. Set the iteration index as $u = 1$.

Step 2. Given the values of E_{i-1}^r, for $i = u, u+1, \cdots, k$, find the first index d where $E_{d-1}^r > E_{l-1}^r$, for $l = u, u+1, \cdots, d$, then calculate $\nu_l(\tilde{p}_l)$ as the water levels, where $\tilde{p}_l = \frac{2E_{l-1}^r}{T}$.

Step 3. If water levels are equalized, i.e., $\nu_m(\tilde{p}_m) = \nu_n(\tilde{p}_n)$ for all $m, n \in \{u, u+1, \cdots, d\}$, set $p_m^o = \tilde{p}_m$ and go to step 4 otherwise, change the level of powers, using bisection method to, p_m^o such that $\nu_m(p_m^o)$ is equalized, and that the conditions $\frac{T}{2} \sum_{m=1}^d p_m^o = \sum_{m=1}^d E_{m-1}^r$ and $\frac{T}{2} p_m < B^{\max}$ are not violated, then go to step 4.

Step 4. If $u > k$ stop the algorithm otherwise, set $u = d + 1$ and go to step 2.

5.4 Semi-offline case

In the previous section, we obtained the optimal values of the total power consumption over each time frame if the values of the energies arrived at the beginning of all time frames as well as the CSI for all links over all time frames are known apriori. However, if the values of the energies E_i^r, $i = 0, 1, \cdots, k - 1$, are known[1] apriori but the channel coefficients are known only at the beginning of the upcoming time frame, one can not solve the optimization problem (5.2.7). However, if the statistics of the channels are available, one can consider a different scenario where the decision for transmission over each time frame is made based on the average throughput criterion. In other words, one can maximize the average transmitted bits over all time frames under several constraints on the power vector $\mathbf{p}$. In this scenario, we assume that $f_{j,i}$ and $g_{j,i}$, $j = 1, 2, \cdots, n_{\mathrm{r}}$ and $i = 1, 2, \cdots, k$, are drawn from a complex Gaussian distribution with zero-mean and unit-variance. Let us consider a modified version of the optimization problem (5.2.7), where we have replaced the instantaneous throughput with the average throughput, that is we consider the following optimization problem

$$
\max_{\mathbf{p}} \quad \mathrm{E}\left\{\frac{T}{2}\sum_{i=1}^{k}\left\{R_i^{\mathrm{max}}(p_i)\right\}\right\}
$$

$$
\mathrm{s.}t \quad \frac{T}{2}\sum_{i=1}^{l}p_i \leq \sum_{i=0}^{l-1}E_i^r, \quad \text{for } l = 1, 2, ..., k
$$

$$
\sum_{i=0}^{l}E_i^r - \frac{T}{2}\sum_{i=1}^{l}p_i \leq B^{\mathrm{max}}, \quad \text{for } l = 1, 2, ..., k - 1 \tag{5.4.1}
$$

where $\mathrm{E}\{\,\cdot\,\}$ is the expectation operator is taken over all channel realizations. In appendix C, we obtain a semi-closed-form expression for $\mathrm{E}\left\{R_i^{\mathrm{max}}(p_i)\right\}$, and thereby

---

[1]For most of the energy harvesting technologies, the values of energies that are to arrive for a specific duration of time can be calculated precisely.

showing that this expression does not depend on the index of the time frame. This means that for a given value of s, $\mathrm{E}\left\{R_i^{\max}(s)\right\} = \mathrm{E}\left\{R_j^{\max}(s)\right\} = \psi(s)$, where $j \neq i$. Hence, one can rewrite the optimization problem (5.4.1) as

$$\max_{\mathbf{p}} \quad \frac{T}{2}\sum_{i=1}^{k}\psi(p_i)$$

$$\text{s.t.} \quad \frac{T}{2}\sum_{i=1}^{l}p_i \leq \sum_{i=0}^{l-1}E_i^r, \quad \text{for } l = 1, 2, ..., k$$

$$\sum_{i=0}^{l}E_i^r - \frac{T}{2}\sum_{i=1}^{l}p_i \leq B^{\max}, \quad \text{for } l = 1, 2, ..., k-1. \tag{5.4.2}$$

We note that the expectation and the summation operators in the objective function of (5.4.1) are linear operators, hence they preserve concavity. This means that the objective function in (5.4.2) is a concave function of $\mathbf{p}$. As the objective function in (5.4.2) does not depend on the CSI over the ith time frame, one can show that the optimal power policy for the optimization problem (5.4.2) can be obtained based on only considering the constraints. Indeed, if in an optimization problem, the objective function is a summation of the same concave function, evaluated over different optimization variables, and when the constraints of the optimization problem are affine functions of the optimization variables, then one can obtain the optimal power policy regardless of the shape of the objective function [105, 107]. In order to obtain the optimal value for the total consumed power over each time frame, let us first plot the so-called energy tunnel graph. The energy tunnel is the area restricted between the cumulative energy $\sum_{i=0}^{l-1}E_i^r$ as the upper-bound of the graph and $\max(0, \sum_{i=0}^{l}E_i^r - B^{\max})$ as the lower bound of the graph, for different time frame index l. Figs. 5.2 and 5.3 show two different examples of this tunnel for two different energy arrival profiles. The cumulative energy spent by the optimal power

allocation, forms a continuous curve and must stay within this tunnel to conform to energy feasibility constraints. A power allocation that crosses the upper bound $\sum_{i=0}^{l-1} E_i^r$, spends more energy than the available energy, and the one that crosses the lower bound $\max(0, \sum_{i=0}^{l} E_i^r - B^{\mathrm{max}})$, causes a battery overflow. Therefore, the set of feasible energy consumptions lies within this tunnel. The optimal energy consumption policy corresponds to a plot that connects the origin of the graph to the end point of the upper-bound of the graph while having the minimum length. Such an optimal energy consumption policy is indeed a piece-wise linear curve which resides in the energy tunnel, has the minimum total length and connects the origin of the graph to end point of the upper bound of the consumed cumulative energy. Then this optimal path allows us to calculate the optimal constant power consumption over each time frame. The authors in [107] proposed a low complexity algorithm to calculate the optimal power consumption corresponding to the optimization problem (5.4.2), by first finding the optimal energy consumption path as described earlier. Then the optimal power consumption can be calculated by obtaining the slope of the optimal energy consumption path over each time interval. We further use the results of the Appendix C to evaluate the average throughput of the users based on the optimal power policy that is described earlier.

5.5   Simulation Results

We consider a network consisting of a transmitter-receiver pair and $n_{\mathrm{r}} = 10$ relay nodes. The relay nodes are connected to an energy harvesting module. We assume that the energy harvesting module has a battery with maximum capacity of 10 energy units and the energy arrival rate is λ per unit of time. We further assume that the
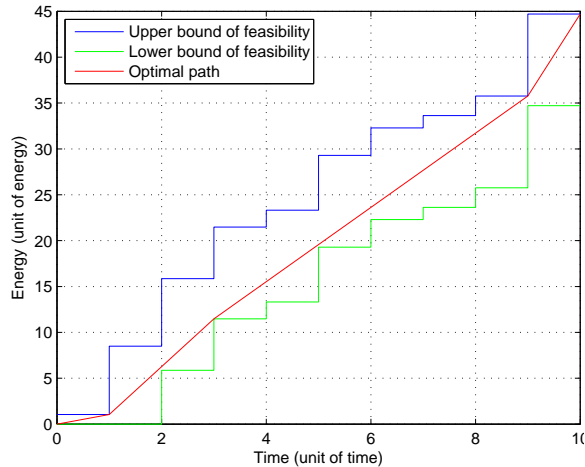
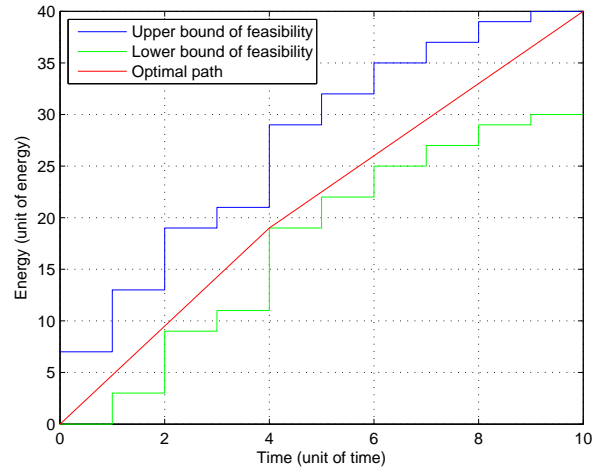Figure 5.2: Optimal path for power consumption for two different energy arrival profiles, with $B^{\max} = 10$.

Figure 5.3: Optimal path for power consumption for two different energy arrival profiles, with $B^{\max} = 10$.

relays are randomly distributed between the transmitter-receiver pair. Indeed, we assume that the channel coefficient between the transmitter (the receiver) and each relay, over each time interval, is drawn from a complex Gaussian random distribution with zero mean and unit variance.

In Fig. 5.4, we show the average value of the throughput of our scheme versus the energy arrival rate λ for the offline and semi-offline cases. One can observe that, for both offline and semi-offline cases, the average throughput of the network increases as λ increases. Indeed, the larger the value of λ is, the higher the average amount of the harvested energy will be. Hence, the average throughput of the network increases as the amount of the harvested energy increases. We also observe from Fig. 5.4 that the rate of increase of the value of the average throughput of the network increases

as λ increases. As can be seen from this figure, the offline case outperform the semi-offline case, due to the fact that the strong assumption of the availability of full CSI is assumed in the former case. Although assuming the availability of full CSI, over all time frames, is not practical but the offline case has been considered as a benchmark for comparison in the literature [105]. We also observe from Fig. 5.4 that the gap between the value of the average throughput of the offline case and semi-offline case remains constant as λ increases. However, for any fixed value of λ, this gap is higher for larger values of k.

Fig. 5.5 depicts the average value of the throughput of the network versus the number of time frames for the offline and the semi-offline cases. We observe that the average throughput of the network increases monotonically as the number of the time frames increases. Indeed, the value of the objective function in each of (5.2.7) and (5.4.2) is a summation of per-time-frame throughput of the network. Hence, for any fixed value of λ, as the number of time frames increases, the throughput of the network increases monotonically. We also observe that that the gap between the value of the average throughput of the offline case and semi-offline case increases as k increases. One can quantify the gap between the average throughput of the offline case and semi-offline case as the sum of per-time-frame gap. Hence, as k increases, the gap between the value of the average throughput of the offline case and semi-offline case increases.

5.6 Conclusions

In this chapter, In this paper, we considered a unidirectional collaborative relay network consisting of n_{r} relay nodes and a transmitter-receiver pair. Considering no
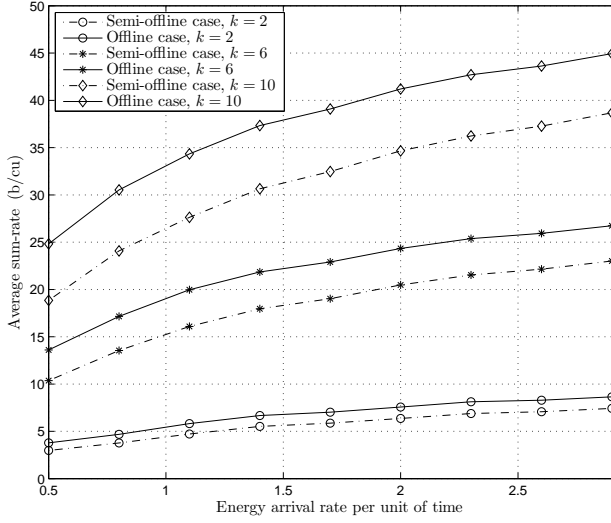
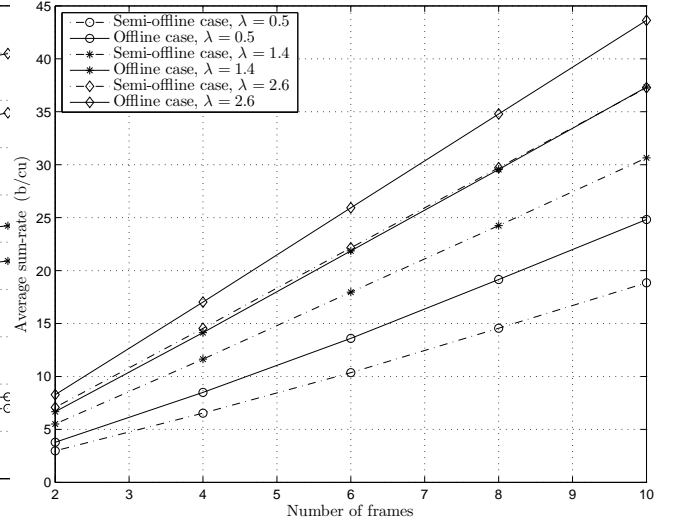Figure 5.4: Average throughput versus energy arrival rate (λ) for the offline and the semi-offline cases.

Figure 5.5: Average throughput versus the number of frames (k) for offline and the semi-offline cases.

direct link between the source and the destination, the relays assist the two end nodes to establish a unidirectional communication. The relays are connected to a central energy harvesting module with a battery that has a capacity of B^{max}. In each time frame, a specific amount of the harvested energy will be allocated to each relay. Assuming an AF relaying protocol, the relays collectively materialize a network beamformer to establish a link between the transmitter and the receiver. For such a relay network, we studied two different scenarios.

In the first scenario, we considered the case where the global CSI is available (i.e., the offline case) and we formulated an optimization problem to maximize the throughput of the network subject to two affine sets of constraints on the total power consumed by the relays over each time frame. The first set of constraints are energy causality constraints which ensure that the energy which have been harvested up to

any given time frame may be consumed. The second set of constraints are to prevent overflow of the battery at any given time frame by optimally using the available energy. We proved that such an optimization problem is convex hence it is amenable to a computationally efficient solution. We observe that for both offline- and semi-offline cases, the average throughput of the network increases as λ increases. We also observe that the gap between the value of the average throughput of the offline case and semi-offline case remains constant as λ increases. However, for any fixed value of λ, this gap is higher for larger values of k.

As the second scenario, we considered the case where only the statistics of the channels are available. For this scenario, we resorted to maximizing the average throughput of the network under the same constraints of the previous design problem. We proved that the objective function is concave, and thus the solution to such an optimization problem does not depend on the shape of the objective function. Using the concept of energy-tunnel, we used an computationally efficient algorithm to optimally calculate the total power consumed by the relays over each time frame and obtain the corresponding optimal beamforming vector. We observe that the average throughput of the network increases monotonically as the number of the time frames increases.

5.7 Appendix

5.7.1 Proof of constant transmit power over each time interval

Let us consider the optimal power profile as $p_i(t)$ and define a constant power policy $p_i^* = \dfrac{\int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t)dt}{\frac{T}{2}} = \dfrac{2E_i^{av}}{T}$ over the ith time frame, $t \in [(i{-}1)T{+}\frac{T}{2} \ \ iT)$, where E_i^{av} is the total available energy to be consumed over the ith time frame. We now prove that the constant power policy p_i^* results in an average data rate which is higher than that for the optimal power profile $p_i(t)$, i.e., $R_i^{\max}(p_i^*) \geq \frac{2}{T}\int_{(i-1)T+\frac{T}{2}}^{iT} R_i^{\max}(p_i(t))dt$, that contradicts the optimality. To show this, let us first consider the following equality

$$p_i^* = \frac{2\int_{(i-1)T+\frac{T}{2}}^{iT} p_i(t)dt}{T} = \frac{2\lim\limits_{m\longrightarrow\infty}\sum_{j=1}^{m} p_i\left((i-1)T + \frac{T}{2} + j\,\Delta t\right)\Delta t}{T} \qquad (5.7.1)$$

where $\Delta t = \frac{T}{2m}$. Since $R_i^{\max}(\cdot)$ is a concave function of its argument (see Appendix 2), hence one can show that

$$
\begin{aligned}
R_i^{\max}(p_i^*) = R_i^{\max}\left(\lim_{m\longrightarrow\infty}\sum_{j=1}^{m} p_i\big((i-1)T + \frac{T}{2} + j\,\Delta t\big)\frac{2\Delta t}{T}\right) &\geq \\
= \lim_{m\longrightarrow\infty}\sum_{j=1}^{m} R_i^{\max}\left(p_i\big((i-1)T + \frac{T}{2} + j\,\Delta t\big)\right)\frac{2\Delta t}{T} & \\
= \frac{2}{T}\int_{(i-1)T+\frac{T}{2}}^{iT} R_i^{\max}\Big(p_i(t)\Big)dt. & \qquad (5.7.2)
\end{aligned}
$$

This shows that the optimal power policy $p_i(t)$ delivers lower average rate than the constant power policy p_i^*, thereby contradicting optimality.

5.7.2 Proof of strictly concavity of $\mathrm{SNR}_i^{\max}(p_i)$

To prove the concavity of $\mathrm{SNR}_i^{\max}(p_i)$, we need to prove that the second order derivative of this function with respect to p_i is always negative. To do so, let us first calculate the first derivative of $\mathrm{SNR}_i^{\max}(p_i)$ with respect to p_i as

$$
\begin{aligned}
\frac{d\,\mathrm{SNR}_i^{\max}(p_i)}{dp_i} &= P_0\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-1}\mathbf{h}_i + P_0 p_i\mathbf{h}_i^H\frac{d}{dp_i}\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-1}\mathbf{h}_i \\
&= P_0\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-1}\mathbf{h}_i - P_0 p_i\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-2}\mathbf{Q}_i\mathbf{h}_i \\
&= P_0\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-2}\mathbf{h}_i. \quad\quad\quad (5.7.3)
\end{aligned}
$$

In the derivation of (5.7.3), we have used the fact that if $\mathbf{A}(\alpha)$ is a diagonal matrix, then $\dfrac{d\mathbf{A}^{-1}(\alpha)}{d\alpha} = -\mathbf{A}^{-1}(\alpha)\dfrac{d\mathbf{A}(\alpha)}{d\alpha}\mathbf{A}^{-1}(\alpha)$. One can see form (5.7.3) that $\dfrac{d\,\mathrm{SNR}_i^{\max}(p_i)}{dp_i}$ is always positive which means that $\mathrm{SNR}_i^{\max}(p_i)$ is an increasing function of p_i. We derive the second derivative of $\mathrm{SNR}_i^{\max}(p_i)$ over p_i as

$$
\frac{d^2\,\mathrm{SNR}_i^{\max}(p_i)}{dp_i^2} = \frac{d}{dp_i}\left(P_0\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-2}\mathbf{h}_i\right) = -2P_0\mathbf{h}_i^H\left(p_i\mathbf{Q}_i + \mathbf{I}\right)^{-3}\mathbf{Q}_i\mathbf{h}_i \quad (5.7.4)
$$

which is always negative. This completes the proof that $\mathrm{SNR}_i^{\max}(p_i)$ is a strictly concave function of p_i. We also note that $\log_2(\cdot)$ is a concave and increasing function of its argument, hence $R_i^{\max}(p_i) = \log_2\left(1 + \mathrm{SNR}_i^{\max}(p_i)\right)$ is also a concave function of p_i.

5.7.3 Semi-closed form expression for the average rate in the third scenario

To calculate the average rate of the user over a time frame, let us first recall that

$$\text{SNR}_i^{\max}(p_i) = P_0 p_i \mathbf{h}_i^H \left(p_i \mathbf{Q}_i + \mathbf{I} \right)^{-1} \mathbf{h}_i = \sum_{j=1}^{n_\text{r}} \frac{P_0 p_i |f_{j,i} \mathbf{G}_i(j,j)|^2}{p_i \mathbf{Q}_i(j,j) + 1}$$

$$= \sum_{j=1}^{n_\text{r}} \frac{P_0 p_i |f_{j,i} \mathbf{G}_i(j,j)|^2}{p_i |\mathbf{G}_i(j,j)|^2 + 1} = \sum_{j=1}^{n_\text{r}} \frac{P_0 p_i |f_{j,i}|^2 |g_{j,i}|^2}{p_i |g_{j,i}|^2 + P_0 |f_{j,i}|^2 + 1}. \tag{5.7.5}$$

Note that since the distribution of $f_{j,i}$ as well as that of $g_{j,i}$ are considered to be complex Gaussian random variables with zero-means and unit-variances, the distributions of $\alpha_{j,i} = |f_{j,i}|^2$ and $\beta_{j,i} = |g_{j,i}|^2$ are exponential with parameter 1, i.e., $\alpha_{j,i} \sim e^{-\alpha_{j,i}}$ and $\beta_{j,i} \sim e^{-\beta_{j,i}}$. To find the average rate, let us first find the cumulative distribution function (c.d.f) of $Z_{j,i}(p_i) = \frac{P_0 p_i |f_{j,i}|^2 |g_{j,i}|^2}{p_i |g_{j,i}|^2 + P_0 |f_{j,i}|^2 + 1}$, i.e., $F_{Z_{j,i}(p_i)}(z_j)$, as

$$F_{Z_{j,i}(p_i)}(z_j) = Pr \left\{ Z_{j,i}(p_i) \leq z_j \right\} = Pr \left\{ \frac{P_0 p_i \alpha_{j,i} \beta_{j,i}}{p_i \beta_{j,i} + P_0 \alpha_{j,i} + 1} \leq z_j \right\}$$

$$= Pr \left\{ P_0 \alpha_{j,i} (p_i \beta_{j,i} - z_j) \leq z_j (p_i \beta_{j,i} + 1) \right\}. \tag{5.7.6}$$

where $Pr\{\cdot\}$ is the probability of an event. Using the definition of the total probability for disjoint events, one can rewrite the equation (5.7.6) as

$$F_{Z_{j,i}(p_i)}(z_j) = Pr \left\{ P_0 \alpha_{j,i} (p_i \beta_{j,i} - z_j) \leq z_j (p_i \beta_{j,i} + 1), \ p_i \beta_{j,i} - z_j \geq 0 \right\}$$

$$+ Pr \left\{ P_0 \alpha_{j,i} (p_i \beta_{j,i} - z_j) \leq z_j (p_i \beta_{j,i} + 1), \ p_i \beta_{j,i} - z_j < 0 \right\}$$

$$= Pr \left\{ \alpha_{j,i} \leq \frac{z_j (p_i \beta_{j,i} + 1)}{P_0 (p_i \beta_{j,i} - z_j)}, \ \beta_{j,i} \geq \frac{z_j}{p_i} \right\}$$

$$+ Pr \left\{ \alpha_{j,i} \geq \frac{z_j (p_i \beta_{j,i} + 1)}{P_0 (p_i \beta_{j,i} - z_j)}, \ \beta_{j,i} < \frac{z_j}{p_i} \right\}$$

$$= Pr \left\{ \alpha_{j,i} \leq \frac{z_j (p_i \beta_{j,i} + 1)}{P_0 (p_i \beta_{j,i} - z_j)}, \ \beta_{j,i} \geq \frac{z_j}{p_i} \right\} + Pr \left\{ \alpha_{j,i} \geq 0, \ \beta_{j,i} < \frac{z_j}{p_i} \right\},$$

$$\tag{5.7.7}$$

where we have used the fact that when $\beta_{j,i} < \frac{z_j}{p_i}$, then $\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)} < 0$ and since $\alpha_{j,i}$ is always non-negative, then the constraint $\alpha_{j,i} \geq \frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}$ for this case is simplified to $\alpha_{j,i} \geq 0$. To further simplify (5.7.7), one can write

$$Pr\left\{\alpha_{j,i} \leq \frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}, \beta_{j,i} \geq \frac{z_j}{p_i}\right\} = \int_{\frac{z_j}{p_i}}^{\infty} \left(\int_0^{\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}} e^{-\alpha_{j,i}} \times e^{-\beta_{j,i}} d\alpha_{j,i}\right) d\beta_{j,i}$$

$$= \int_{\frac{z_j}{p_i}}^{\infty} \left(1 - e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\beta_{j,i}} d\beta_{j,i}. \quad (5.7.8)$$

$$Pr\left\{\alpha_{j,i} \geq 0, \beta_{j,i} < \frac{z_j}{p_i}\right\} = Pr\left\{\alpha_{j,i} \geq 0\right\} \times Pr\left\{0 \leq \beta_{j,i} \leq \frac{z_j}{p_i}\right\}$$

$$= 1 \times \int_0^{\frac{z_j}{p_i}} e^{-\beta_{j,i}} d\beta_{j,i} = 1 - e^{-\frac{z_j}{p_i}}. \quad (5.7.9)$$

From (5.7.8)-(5.7.9), one can show that

$$F_{Z_{j,i}(p_i)}(z_j) = \left(\int_{\frac{z_j}{p_i}}^{\infty} \left(1 - e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\beta_{j,i}} d\beta_{j,i}\right) + \left(1 - e^{-\frac{z_j}{p_i}}\right) \quad (5.7.10)$$

We now derive the probability density function (p.d.f) of $Z_{j,i}(p_i)$, denoted by $\mathfrak{F}_{Z_{j,i}(p_i)}(z_j)$, by finding the derivative of $F_{Z_{j,i}(p_i)}(z_j)$ as

$$\mathfrak{F}_{Z_{j,i}(p_i)}(z_j) = \frac{dF_{Z_{j,i}(p_i)}(z_j)}{dz} = \frac{d}{dz_j}\left(\int_{\frac{z}{p_i}}^{\infty} \left(1 - e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\beta_{j,i}} d\beta_{j,i}\right) + \frac{1}{p_i} e^{-\frac{z_j}{p_i}},$$

$$(5.7.11)$$

where

$$\frac{d}{dz_j}\left(\int_{\frac{z_j}{p_i}}^{\infty} \left(1 - e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\beta_{j,i}} d\beta_{j,i}\right)$$

$$= -\frac{1}{p_i}\left(1 - \lim_{\beta_{j,i} \to (\frac{z_j}{p_i})^+} e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\frac{z_j}{p_i}}$$

$$+ \int_{\frac{z_j}{p_i}}^{\infty} \left(\frac{p_i\beta_{j,i}(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)^2} e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}}\right) e^{-\beta_{j,i}} d\beta_{j,i}. \quad (5.7.12)$$

One can show that $\lim_{\beta_{j,i} \to (\frac{z_j}{p_i})^+} e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}} \approx 0$, hence we can rewrite (5.7.11) as

$$
\mathfrak{F}_{Z_{j,i}(p_i)}(z_j) = \left(-\frac{1}{p_i}e^{-\frac{z_j}{p_i}} + \int_{\frac{z_j}{p_i}}^{\infty} \left(\frac{p_i\beta_{j,i}(p_i\beta_{j,i}+1)}{P_0\left(p_i\beta_{j,i}-z_j\right)^2}e^{-\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}} \right)e^{-\beta_{j,i}}d\beta_{j,i} \right)
$$

$$
+ \frac{1}{p_i}e^{-\frac{z_j}{p_i}}
$$

$$
= \int_{\frac{z_j}{p_i}}^{\infty} \left\{ \frac{p_i\beta_{j,i}(p_i\beta_{j,i}+1)}{P_0\left(p_i\beta_{j,i}-z_j\right)^2} \right\} \times e^{-\left(\frac{z_j(p_i\beta_{j,i}+1)}{P_0(p_i\beta_{j,i}-z_j)}+\beta_{j,i} \right)}d\beta_{j,i}. \tag{5.7.13}
$$

As mentioned earlier, $\psi(s) = \mathrm{E}\left\{ R_i^{\max}(s) \right\}$ does not depend on the index of time frame, hence one calculate the average value of the rate as

$$
\psi(p_i) = \mathrm{E}\left\{ R_i^{\max}(p_i) \right\}
$$

$$
= \mathrm{E}\left\{ \log_2\left(1 + \mathrm{SNR}_i^{\max}(p_i)\right) \right\}
$$

$$
= \mathrm{E}\left\{ \log_2\left(1 + \sum_{j=1}^{n_r} Z_{j,i}(p_i)\right) \right\}
$$

$$
= \int_{z_1=0}^{\infty} \cdots \int_{z_{n_r}=0}^{\infty} \log_2\left(1 + \sum_{j=1}^{n_r} z_j\right) \times
$$

$$
\mathfrak{F}_{Z_{1i}(p_i)}(z_1) \times \cdots \times \mathfrak{F}_{Z_{n_r i}(p_i)}(z_{n_r})dz_1 \cdots dz_{n_r}. \tag{5.7.14}
$$

One can also note that the p.d.f of the sum of several independent random variables is the convolution of the p.d.f of all random variables, i.e., $\mathfrak{F}(\sum_{j=1}^{n_r} Z_{j,i}(p_i)) = \mathfrak{F}_{Z_{1i}(p_i)}(z_1) \circledast \mathfrak{F}_{Z_{2i}(p_i)}(z_2) \circledast \cdots \circledast \mathfrak{F}_{Z_{n_r i}(p_i)}(z_{n_r}) = \mathfrak{F}_{\tilde{Z}_i(p_i)}(\tilde{z}_i)$, where $\circledast$ stands for the convolution operator. Hence, if we denote $\tilde{Z}_i(p_i) = \sum_{j=1}^{n_r} Z_{j,i}(p_i)$, then we can easily

show that the equation (5.7.14) can be rewritten as

$$
\begin{aligned}
\psi_i(p_i) &= \int_{\tilde{z}=0}^{\infty} \log_2\left(1 + \tilde{z}_i\right) \mathfrak{F}_{\tilde{z}_i}(\tilde{z}_i) d\tilde{z}_i \\
&= \int_{\tilde{z}=0}^{\infty} \log_2\left(1 + \tilde{z}_i\right) \times \\
&\qquad \mathcal{F}^{-1}\left\{\mathcal{F}\left\{\mathfrak{F}_{Z_{1i}(p_i)}(\tilde{z}_i)\right\} \times \cdots \times \mathcal{F}\left\{\mathfrak{F}_{Z_{n_r i}(p_i)}(\tilde{z}_i)\right\}\right\} d\tilde{z}_i. \qquad (5.7.15)
\end{aligned}
$$

where $\mathcal{F}(\cdot)$ and $\mathcal{F}^{-1}(\cdot)$ stand for Fourier transform and inverse Fourier transform, respectively. Note that the p.d.f of each of $Z_{j,i}(s)$ does not depend on the index of time frame and on the index of relays, as all channel coefficients are assumed to be independent and identically distributed. Hence, with a small abuse of notation, we use $Z(p_i)$ and $\tilde{Z}(p_i)$ instead of $Z_{j,i}(p_i)$ and $\tilde{Z}_i(p_i)$, respectively, one can rewrite (5.7.15) as

$$
\psi(p_i) = \int_{\tilde{z}=0}^{\infty} \log_2\left(1 + \tilde{z}\right) \mathcal{F}^{-1}\left\{\left(\mathcal{F}\left\{\mathfrak{F}_{Z(p_i)}(\tilde{z})\right\}\right)^{n_r}\right\} d\tilde{z}. \qquad (5.7.16)
$$

Note that $\psi(p_i)$ does not have a closed-form expression, however, the relays can create a look-up table for different values of p_i and P_0, and evaluate $\psi(p_i)$ numerically. We also note that one can obtain $\psi(p_i)$ in (5.7.16) using numerical monte-carlo simulations thereby approximating $\psi(p_i)$ as

$$
\begin{aligned}
\psi(p_i) &= \mathrm{E}\left\{R_i^{\max}(p_i)\right\} = \mathrm{E}\left\{\log_2\left(1 + \mathrm{SNR}_i^{\max}(p_i)\right)\right\} \\
&\simeq \lim_{m\to\infty} \frac{1}{m} \times \sum_{k=1}^{m} \log_2\left(1 + \{\mathrm{SNR}_i^{\max}(p_i)\}_k\right) \qquad (5.7.17)
\end{aligned}
$$

where $\{\mathrm{SNR}_i^{\max}(p_i)\}_k$ stands for the kth realization of the $\mathrm{SNR}_i^{\max}(p_i)$ over the ith time frame.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In Chapter 3, we studied optimal resource sharing between two pairs of transceivers which exploit a network of n_{r} relays. We considered a communication framework in which the primary pair leases out a portion of its spectral and temporal resources to the secondary pair in exchange for using the relays to guarantee a minimum data rate for the primary transceivers. We proposed three approaches with different pros and cons. In each approach, we formulated an optimization problem in order to optimally calculate the corresponding design parameters.

As the first approach, we maximize the secondary transceivers rates while guaranteeing a minimum data rate for the primary transceivers and limiting the total powers consumed in the primary and the secondary network to be less than predefined thresholds. We showed that for the primary and the secondary transceivers, the design problem can be simplified into two SNR balancing problems, each with its own semi-closed-form solution.

In the second approach, we replaced the two separate constraints on the total power consumed in the primary and secondary networks used in the first approach,

with a constraint on the total power consumed in the whole time frame. We proved that the optimization problem in this approach can be simplified to a simple line search problem with low complexity. Furthermore, we showed that the second approach is superior to the first approach as in the latter approach one can optimally allocate the available power between the primary and the secondary transceivers.

The third approach combines the two aforementioned methods to materialize spectrum leasing and sharing for the case when the primary network is active with a certain probability.

In Chapter 4, we investigated the temporal, spectral and relay resource sharing problem between two pairs of transceivers which exploit a network of one or multiple relays in a multi-carrier scenario. We proposed two spectrum leasing and resource sharing approaches, each of which has its own application. In each approach, we formulated an optimization problem in order to calculate the corresponding design parameters.

As the first approach, we considered a multi-relay scenario and maximized the average sum-rate of the secondary transceivers while guaranteeing an average sum-rate for the primary transceivers and under two spectral power masks to limit the total power consumed in each network over each subchannel. We showed that for the primary (secondary) transceivers, the design problem can be turned into a linear programming problem.

In the second approach, we considered maximization of the secondary network sum-rate subject to per network total power constraints while guaranteeing a minimum sum-rate for the primary transceivers. In this approach, we considered two

different scenarios namely a single-relay case and a multi-relay case. For the single relay case, we used a high SNR approximation and developed an iterative convex search algorithm which applies the biconcavity of the approximated objective function in terms of the design parameters, namely the vector of the total powers allocated to each network over different subchannels and the vector of rate of the primary network over all subchannels. In the case of a multi-relay scenario, we expanded the design parameters to five vectors and showed that the problem is concave in any parameter vector, given the other four vectors are fixed. Doing so, we proposed an alternate convex search algorithm to introduce a solution to the underlying problem.

In Chapter 5, we considered a unidirectional collaborative relay network consisting of n_{r} relay nodes and a transmitter-receiver pair. Considering no direct link between the source and the destination, the relays assist the two end nodes to establish a unidirectional communication. The relays are connected to a central energy harvesting module with a battery that has a capacity of B^{max}. In each time frame, a specific amount of the harvested energy will be allocated to each relay. Assuming an AF relaying protocol, the relays collectively materialize a network beamformer to establish a link between the transmitter and the receiver. For such a relay network, we studied two different scenarios.

In the first scenario, we considered the case where the global CSI is available (i.e., the offline case) and we formulated an optimization problem to maximize the throughput of the network subject to two affine sets of constraints on the total power consumed by the relays over each time frame. The first set of constraints are energy causality constraints which ensure that the energy which have been harvested up to any given time frame may be consumed. The second set of constraints are to prevent

overflow of the battery at any given time frame by optimally using the available energy. We proved that such an optimization problem is convex hence it is amenable to a computationally efficient solution. We observe that for both offline- and semi-offline cases, the average throughput of the network increases as λ increases. We also observe that the gap between the value of the average throughput of the offline case and semi-offline case remains constant as λ increases. However, for any fixed value of λ, this gap is higher for larger values of k.

As the second scenario, we considered the case where only the statistics of the channels are available. For this scenario, we resorted to maximizing the average throughput of the network under the same constraints of the previous design problem. We proved that the objective function is concave, and thus the solution to such an optimization problem does not depend on the shape of the objective function. Using the concept of energy-tunnel, we used an computationally efficient algorithm to optimally calculate the total power consumed by the relays over each time frame and obtain the corresponding optimal beamforming vector. We observe that the average throughput of the network increases monotonically as the number of the time frames increases.

6.2   Future Work

Energy harvesting in wireless relay networks is a promising technology to enable a long-lasting communication between wireless nodes through providing sustainable energy supply. The nodes in such networks are capable of harvesting energy from different sources in the environment, e.g., motion and vibration, light and infra-red radiation, RF radio waves and etc. In some specific wireless devices, e.g., randomly

distributed relay nodes or wireless sensors, where each node is supplied by a battery, it is highly restrictive or even impossible to replace/recharge the batteries when they are deployed. Hence, energy harvesting has been introduced to alleviate the bottleneck of the power supplies. Due to the renewable nature of the aforementioned energy sources, energy harvesting can provide supply of power with a theoretically unlimited lifetime. The challenge, however, is that due to the random nature of the profile of the harvested energy, one needs to provide an optimal policy to use the harvested energy in order to sustain a specific design requirement. Several possible research areas for future work can be summarized as they follows

1. Assuming a bidirectional communication, one can extend our study of the optimal resource sharing and spectrum leasing problem between two pairs of transceivers to the case where there exist multiple pairs of transceivers for both single- and multi-carrier scenarios. In such a design approach, one can maximize the smaller of the sum-rate of each pair of transceivers under per-network power constraints or average total power constraint. One can benefit from the results of this thesis to elaborate on the solution to the problem of resource sharing and spectrum leasing in the multi-pair scenario.

2. Moreover, in the unidirectional communication, one can investigate the optimal resource sharing problem between the transmitter-receiver pair and the relays for the case where both the relays, the transmitter and the receiver are equipped with energy harvesting modules. In this case, one can maximize the overall throughput of the network under the constraints on the causality of energy arrivals and the battery capacity and obtain the optimal policy to allocate the harvested energy between the relay nodes as well as the users.

3. Furthermore, one can consider the problem of maximizing the lifetime of the battery such that the overall throughput of the network is above a predefined threshold. This means that the throughput demand of the network must be satisfied while the lifetime of the battery is maximized. A fair comparison between the throughput maximization and the battery lifetime maximization schemes is needed to be well studied.

4. In addition, one can extend our studies on bidirectional networks to the case where the relay nodes, the users in the two pairs of transceivers, or both, are equipped with energy harvesting modules and solve the corresponding optimization problems under a new set of constraints.

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