Micro-Differential Evolution: Diversity Enhancement and Comparative Study

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.
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Abstract

Evolutionary algorithms (EAs), such as the differential evolution (DE) algorithm, suffer from high computational time due to large population size and nature of evaluation, to mention two major reasons. The micro-EAs employ a very small population size, which can converge to a reasonable solution quicker; while they are vulnerable to premature convergence as well as high risk of stagnation. One approach to overcome the stagnation problem is increasing the diversity of the population. In this thesis, a micro-differential evolution algorithm with vectorized random mutation factor (MDEVM) is proposed, which utilizes the small size population benefit while preventing stagnation through diversification of the population. The following contributions are conducted related to the micro-DE (MDE) algorithms in this thesis: providing Monte-Carlo-based simulations for the proposed vectorized random mutation factor (VRMF) method; proposing mutation schemes for DE algorithm with populations sizes less than four; comprehensive comparative simulations and analysis on performance of the MDE algorithms over variant mutation schemes, population sizes, problem types (i.e. uni-modal, multi-modal, and composite), problem dimensionalities, mutation factor ranges, and population diversity analysis in stagnation and trapping in local optimum schemes. The comparative studies are conducted on the 28 benchmark functions provided at the IEEE congress on evolutionary computation 2013 (CEC-2013) and comprehensive analyses are provided. Experimental results demonstrate high performance and convergence speed of the proposed MDEVM algorithm over variant types of functions.

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To my parents and reminder of my grandparents. To hearts beating to create happiness and smile.

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List of Acronyms

ACO ant colony optimization

AIS artificial immune system

AMGA2 archive-based micro-genetic algorithm

ANN artificial neural network

BFOA bacterial foraging optimization algorithm

BFV best fitness value

CMABC cooperative micro-artificial bee colony

CMBFA chaotic micro-bacterial foraging algorithm

CMF constant mutation factor

CSA clonal selection algorithm

CPU central processing unit

DE differential evolution

EAs evolutionary algorithms

ED economic dispatch

EEv elitistic evolution

EVTR error-value-to-reach

GAs genetic algorithms

G-ELM genetic extreme learning machine

HC heterogeneous computing

HDE hybrid differential evolution

LS local search

MABC micro-artificial bee colony

MDE micro-differential evolution

MDESM micro-differential evolution with scalar random mutation factor

MDEVM micro-differential evolution algorithm with vectorized random mutation factor

MAIS micro-artificial immune system

MBFOAs micro-bacterial foraging optimization algorithms

MNNs modular neural networks

MOHGA multi-objective optimization for hierarchical genetic algorithm

MOO multi-objective optimization

MSE mean square error

NSGA-II non-dominated sorting genetic algorithm

ODE opposition-based differential evolution

PMGA parallel micro-genetic algorithm

PSO particle swarm optimization

SMES simple multi-membered evolution strategy

SR stochastic ranking

SRMF scalar random mutation factor

SVC static var compensator

VMF vectorized mutation factor

VRMF vectorized random mutation factor

VTR value-to-reach

Chapter 1

Introduction

1.1 Motivation

Accuracy enhancement as well as increasing the convergence speed toward finding the global solution(s) in optimization problems have motivated many researchers to develop more efficient evolutionary and swarm intelligence algorithms. Such algorithms work based on a set of individuals, where optimal size setting of this parameter is imperative for algorithm performance [1]. Large population size setting in such algorithms supports a higher diversity of the population, which recombination of its diverse members offers a higher opportunity to the optimizer to locate the global solution(s) [2]-[4]. Although this diversity enhancement technique offers a better exploration of problem landscape, yet admits more function evaluations and as a consequence, lower convergence rate to the possible solution [2].

The population-based algorithms with large populations sizes often grant more reasonable results than small population size ones. Due to such performance and complexity of problems, most of the research works on population-based algorithms are focused on developing approaches with a large populations size with congenital computational complexities [38]. However, due to limited capabilities of hardware components, particularly in large-scale applications, which is one of the main reasons for evolutionary algorithms (EAs) development, employing modern algorithms with large population size and complex structures may not always be the best approach [37], [39]. Therefore, development of algorithms with small population sizes but reasonable performance, comparing to standard population-based algorithms, is of interest.

1.2 Objectives

The term micro-algorithm, denoted by μ-algorithm, refers to population-based algorithms with a small population size [4]. The micro-algorithms have been used in diverse applications, exceptionally due to their lighter hardware requirements and opportunity to operate in embedded systems with a memory saving approach [1]. The differential evolution (DE) algorithm is one of the state-of-the-art global optimization algorithms, which is popular due to its simplicity and effectiveness. In this thesis, the following principal contributions regarding the micro-differential evolution (MDE) algorithm are conducted:

- A comprehensive survey on micro-EAs.
- Proposing an enhanced version of the MDE algorithm, i.e., micro-differential evolution algorithm with vectorized random mutation factor (MDEVM), where the idea is supported by Monte-Carlo simulations.
- Proposing new mutation schemes for the DE algorithm.
- Comparative study and analysis of MDE algorithms in terms of variant mutation schemes and populations sizes.
- Comparative analysis on performance of the MDE algorithms on uni-modal, multi-modal, and composite problems solving.
- Comparative study and analysis on variant problem dimensions and mutation schemes for the MDE algorithms.
- Comparative study and analysis on variant ranges for mutation factor for the MDE algorithm.
- Comparative study and analysis on role of population diversity of the MDE algorithms in stagnation and trapping in local optimum and performance of the proposed MDEVM method to tackle with these scenarios.
- Comparative study and analysis on variant stopping conditions for the MDE algorithm.

1.3 Outline of the Thesis

This thesis is organized as follows: In chapter 2, a survey on micro-population based methods is presented. Then, a review of the DE algorithm and its variant micro schemes are presented in chapter 3. In chapter 4, the proposed diversity enhancement in MDE using vectorized random mutation factor (VRMF) is studied in detail. A comprehensive performance study on different types of MDE algorithm for a variety of population sizes, problem dimensionalities is performed in chapter 5. Finally, the thesis is concluded in chapter 6, along with directions for future works.

Chapter 2

Related Works

Many works have been conducted to propose efficient micro-algorithms. The research works can be categorized in four main groups which are genetic algorithms (GAs), particle swarm optimization (PSO) algorithms, DE-based algorithms, and other population-based approaches.

2.1 Micro-Genetic Algorithms

One of the earlier research works in this direction was a GA with five chromosomes [5]. The strategy in this micro-GA is to copy the best found chromosomes in the current population to the next generation. This work was tested on low-dimensional problems, which resulted a faster convergence speed compared to the classical GA. The idea of population reinitialization for micro-GA was another early work in the field [31]. In this approach, the best individual of each converged population, after a predefined number of generations, is replaced with a randomly selected individual in the population of the next iteration. The parallel version of micro-GA, called parallel micro-genetic algorithm (PMGA), is reported in [32]; which solves the ramp rate constrained economic dispatch (ED) problems for generating units with non-monotonically and monotonically increasing incremental cost functions. The PMGA is implemented on a thirty-two-processor Beowulf cluster and the reported results demonstrate feasibility of this approach in online applications. The micro-algorithms also have been employed in multi-objective optimization (MOO). The improved version of non-dominated sorting genetic algorithm (NSGA-II) with a specific population initialization strategy is embedded into the standard micro-GA to solve the MOO problems [10]. A micro-GA with a population size of four and a reinitialization strategy is proposed in [28], which can produce a major part of the Pareto front at a very low computational cost. Three forms of elitism and a memory are used to generate the initial population [28]. An improved version of micro-GA, called archive-based micro-genetic algorithm (AMGA2), for constrained MOO is proposed in [33]. This algorithm is based on a steady-state GA that preserves an external archive of best and divert candidate solutions. This small population-based approach facilitates the decoupling of the working population, the external archive, and the number of required solutions as the outcome of the optimization procedure. A model of multi-objective optimization for hierarchical genetic algorithm (MOHGA) based on the micro-GA approach for modular neural networks (MNNs) optimization is proposed in [34]. This approach is used in iris recognition. The MOHGA divides the input data into granules and sub-modules and then decides to split the data for training and testing phases. It is reported that this technique can obtain good results based on using less data [34]. The micro-GA has also been used for local fine tuning in an adaptive local search intensity manner for training recurrent artificial neural network (ANN) [35]. It is reported that this approach is useful for systems identification tasks. In [21] a multi-objective micro-genetic extreme learning machine (G-ELM) is proposed, which provides the appropriate number of hidden nodes in the machine for solving the problem and minimizes the mean square error (MSE) of the training phase. The micro-GA is applied successfully for many applications such as designing wave-guide slot antenna with dielectric lenses [36], detection of flaws in composites [30], and scheduling of a real-world pipeline network [29], where better performances compared to the standard GA are reported.

2.2 Micro-Particle Swarm Optimization Algorithms

The PSO is one of the well-known swarm intelligence algorithms where its small population size versions have been developed [6], [51], [7]. The micro-PSO method is proposed for high dimensional problems based on the Coulomb's law [6]. First achievement of this approach is removal of the burden for determining the suitable size of needed space to enclose the blacklisted solutions and the amount of repulsion needed to repel the particles. The other achievement is the flexibility of controlling the repulsion on particles through the use of a parameter which controls the amount of repulsion experienced by the particles at a particular position. The conducted simulation results on five high-dimensional benchmark functions demonstrate superior performance of micro-PSO versus the standard PSO with a large populations size. A five-particle micro-PSO is used in [41] to deal with constrained optimization problems. This method preserves population diversity by using a reinitialization process and incorporates a mutation operator to improve the exploratory capabilities of the algorithm. The reported results present competitive performance versus the simple multi-membered evolution strategy (SMES) and stochastic ranking (SR) method [41]. The micro-PSO is employed for MOO in [42]. This approach, comparing to PSO approach, produces reasonably good Pareto front approximations of moderate dimensional problems with a small number of objective function evaluations (only 3000 calls per run). In another micro-PSO algorithm, a parallel master-slave model of cooperative micro-PSO is introduced [7], in which the original search space is decomposed into subspaces with smaller dimensions. Then, five individuals are considered in each subspace to identify suboptimal partial solution components. Its performance is assessed on a set of five widely used test problems with significant improvements in solution quality, compared to the standard PSO algorithm [7]. A cooperative PSO approach is proposed in [47], which uses a company of low-dimensional and low-cardinality sub-swarms to deal with complex high-dimensional problems. Promising results are reported using these methods, tested with five widely used test problems. A clonal selection algorithm (CSA), which belongs to the family of artificial immune system (AIS), in conjunction with a micro-PSO (CS^2P^2SO) is introduced in [46] as a hybrid scheme. In this hybridization, the strength of standard PSO algorithm is enhanced, where the micro-PSO helps to find the optimum solution with less memory requirement and the CSA increases the exploration capability while reducing the chance of convergence to a local minima. Simulations are conducted on only four benchmark functions, where competitive performance is reported. A mixed-integer-binary small-population PSO is proposed in [58] for solving a problem of optimal power flow. The constraint handling technique used in this algorithm is based on a strategy to generate and keep its four decision variables in feasible space through heuristic operators. In this way, the algorithm focuses its search procedure on the feasible solution space to obtain a better objective value. This technique improves the final solution quality as well as the convergence speed [58]. The micro-PSO has been developed for many applications such as motion estimation [40], power system stabilizers design [43], [45], optimal design of static var compensator (SVC) damping controllers [44], reactive power optimization [48], short-term hydrothermal scheduling [50], reconfiguration of shipboard power system [52], and transient stability constrained optimal power flow [49].

2.3 Micro-Differential Evolution Algorithms

The DE algorithm works based on the scaled difference between two individuals of a population set, where the scaling factor is called the mutation factor. Due to reliability and simplicity of the DE algorithm, it has been employed in many science and engineering areas, such as, solving large capacitor placement problem [17] and synthesis of spaced antenna arrays [18]. Many research works have been conducted to enhance the DE algorithm, such as opposition-based differential evolution (ODE) [14], enhanced differential evolution using center-based sampling [15], and opposition-based adaptive differential evolution [16]. Some approaches toward reducing computational cost of DE-based algorithms by reducing the population size have been proposed, [8]-[9], [11]-[13]. In order to increase the exploration ability of MDE algorithm and to prevent stagnation, an extra search move is incorporated into the MDE algorithm in [1] by perturbing it along the axes. A local search procedure is hybridized with the MDE algorithm in [3] to tackle with high dimensional problems. However, the reported performance results are comparable with some other methods. As an application of MDE, a hybrid differential evolution (HDE) with population size of five is used for finding a global solution [60]. A gradually reducing population size method is proposed in [8]. This method is examined on 13 benchmark functions, where the results have demonstrated a higher robustness as well as efficiency compared to the parent DE [8]. In another approach [9], small size cooperative sub-populations are employed to find sub-components of the original problem concurrently. During cooperation of sub-populations, sub-components are combined to construct the complete solution of the problem. Performance evaluation of this method has been done on high-dimensional instances of five sample test problems with encouraging results reported in [9]. As a MDE application, it is employed for evolving an indirect representation of the bin packing problem, where acceptable performance is reported [11]. The idea of self-adaptive population size is carried out to test absolute encoding and relative encoding methods for DE [12]. The reported simulation results on 20 benchmark problems denote that in terms of the average performance and stability, the self-adaptive population size using relative encoding outperforms the absolute encoding method and the standard DE algorithm [12]. The idea of micro-ODE is proposed and evaluated for an image thresholding case study [13]. Performance of the proposed method is compared with the Kittler algorithm and the MDE. The micro-ODE method has outperformed these algorithms on 16 challenging test images and has demonstrated faster convergence speed due to embedding the opposition-based population initialization scheme [13].

2.4 Other Micro-Population-based Algorithms

Several other types of micro-population-based algorithms have been proposed in the literature. A cooperative micro-artificial bee colony (CMABC) approach for large-scale optimization is presented in [53]. This approach has combined the divide-and-conquer property of cooperative algorithms and low computational cost of micro-artificial bee colony (MABC) method. In case of employing micro-bacterial foraging optimization algorithms (MBFOAs) for solving optimization problems, in [54] the best bacterium is kept unaltered, whereas the other population members are reinitialized. It is reported that this approach

Table 2.1: Summary of related works in micro-population-based algorithms.

Population-based Algorithm	Related Research Works
Genetic Algorithm (GA)	[5],[10],[21],[28]-[36]
Particle Swarm Optimization (PSO)	[6],[7],[40]-[52],[58]
Differential Evolution (DE)	[1],[3],[8],[9],[11]-[18],[60]
Artificial Bee Colony (ABC)	[53]
Bacterial Foraging Optimization (BFO)	[54]
Artificial Immune System (AIS)	[56]
Elitistic Evolution (EEv)	[57]

has outperformed the standard bacterial foraging optimization algorithm (BFOA) with a larger population size [54]. For the environmental economic dispatch case study, a chaotic micro-bacterial foraging algorithm (CMBFA) with a time-varying chemotactic step size is proposed in [55]. It is reported that the convergence characteristic, speed, and solution quality of this method are better than the classical BFOA for a 3-unit system and the standard IEEE 30-bus test system. A micro-artificial immune system (MAIS) with five individuals (antibodies) from which only 15 clones are obtained is proposed in [56]. In this approach, the diversity is preserved by considering two simple but fast mutation operators in a nominal convergence manner that work together in a reinitialization process [56]. An other type of EAs, called elitistic evolution (EEv), is proposed for optimizing high-dimensional problems in [57]. This method works without using complex mechanisms such as Hessian or covariance matrix. This approach utilizes adaptive and elitism behaviour, in which a single adaptive parameter controls the evolutionary operators to provide reasonable local and global search abilities [57]. An efficient scheduler for heterogeneous computing (HC) and grid environments, based on parallel micro-cross generational elitist selection, heterogeneous recombination, and cataclysmic mutation, called pμ-CHC, is proposed in [59]. This method combines a parallel sub-populations model with a focused evolutionary search using a micro population and a randomized local search (LS) method.

Performance comparisons of algorithms such as ant colony optimization (ACO) and GA have demonstrated good scheduling in reduced execution times [59].

A summary of works up to our knowledge in micro-population-based algorithms is presented in Table 2.1.

Chapter 3

Differential Evolution

Conventionally, an optimizer has no knowledge about landscape structure to minimize/maximize an objective function in solving a black-box problem. The DE algorithm, similar to other algorithms in its category, starts its search procedure with some uniform random initial vectors and tries to improve them in each generation toward an optimal solution. The population $\mathbf{P} = \{\mathbf{X}_1, ..., \mathbf{X}_{N_P}\}$ consists of N_P vectors in generation g, where $\mathbf{X}_i$ is a D-dimensional vector defined as $\mathbf{X}_i = (x_{i,1}, ..., x_{i,D})$.

3.1 Differential Evolution Algorithm

Generally, a simple DE algorithm consists of three major operations which are mutation, crossover, and selection.

Mutation: This step selects three vectors randomly from the population such that $i_1 \neq i_2 \neq i_3 \neq i$ where $i \in \{1, ..., N_P\}$ and $N_P \geq 4$, for each vector $\mathbf{X}_i$, the mutant vector scheme "DE/Rand/1" is calculated as

$$\mathbf{V}_i = \mathbf{X}_{i_1} + F(\mathbf{X}_{i_2} - \mathbf{X}_{i_3}), \tag{3.1}$$

where the factor $F \in (0, 2]$ is a real constant number, which controls the amplification of the added differential variation of $(\mathbf{X}_{i_2} - \mathbf{X}_{i_3})$. The exploration of DE increases by selecting higher values for F. So far, four main mutation schemes are introduced [61],[62], summarized as

- DE/Best/1:

$$\mathbf{V}_i = \mathbf{X}_{i_{best}} + F(\mathbf{X}_{i_1} - \mathbf{X}_{i_2}), \tag{3.2}$$

- DE/Target-to-Best/1 (DE/T2B/1):

$$\mathbf{V}_i = \mathbf{X}_i + F(\mathbf{X}_{i_{best}} - \mathbf{X}_i) + F(\mathbf{X}_{i_1} - \mathbf{X}_{i_2}), \tag{3.3}$$

- DE/Rand/2:

$$\mathbf{V}_i = \mathbf{X}_{i_1} + F(\mathbf{X}_{i_2} - \mathbf{X}_{i_3}) + F(\mathbf{X}_{i_4} - \mathbf{X}_{i_5}), \tag{3.4}$$

- DE/Best/2:

$$\mathbf{V}_i = \mathbf{X}_{i_{best}} + F(\mathbf{X}_{i_1} - \mathbf{X}_{i_2}) + F(\mathbf{X}_{i_3} - \mathbf{X}_{i_4}), \tag{3.5}$$

where $\mathbf{X}_{i_{best}}$ is corresponding vector of the best objective fitness value in the population.

Crossover: The crossover operation increases diversity of the population by shuffling the mutant and parent vector as follows:

$$U_{i,d} = \begin{cases} V_{i,d}, & rand_d(0,1) \leq C_r \textit{ or } d_{rand} = d \\ x_{i,d}, & otherwise \end{cases}, \tag{3.6}$$

where $d = 1, ..., D$, $C_r \in [0,1]$ is the crossover rate parameter, and $rand(a,b)$ generates a real random number in the interval $[a,b]$ with a uniform distribution. Therefore, the trial vector $\mathbf{U}_i$ $\forall i \in \{1, ..., N_P\}$ can be generated as

$$\mathbf{U}_i = (U_{i,1}, ..., U_{i,D}). \tag{3.7}$$

Selection: The $\mathbf{U}_i$ and $\mathbf{X}_i$ vectors are evaluated and compared with respect to their fitness values; the one with better fitness is selected for the next generation.

3.2 Micro-Differential Evolution Algorithm

In MDE, a small population size is utilized. Employing small population sizes decreases the number of function calls, but unfortunately due to lack of diversity, it further increases the risk of premature convergence as well as chance of stagnation. The stagnation problem differs from the premature convergence. In stagnation scenario, the population still remains divert but unconverged after some generations, which prevents the optimization algorithms from processing [2]. Accordingly, reducing the population size while raising the diversity of the population is a key point to achieve a faster convergence speed throughout the time maintaining a low risk of premature convergence or stagnation.

Chapter 4

Proposed Diversity Enhancement via Vectorized Random Mutation

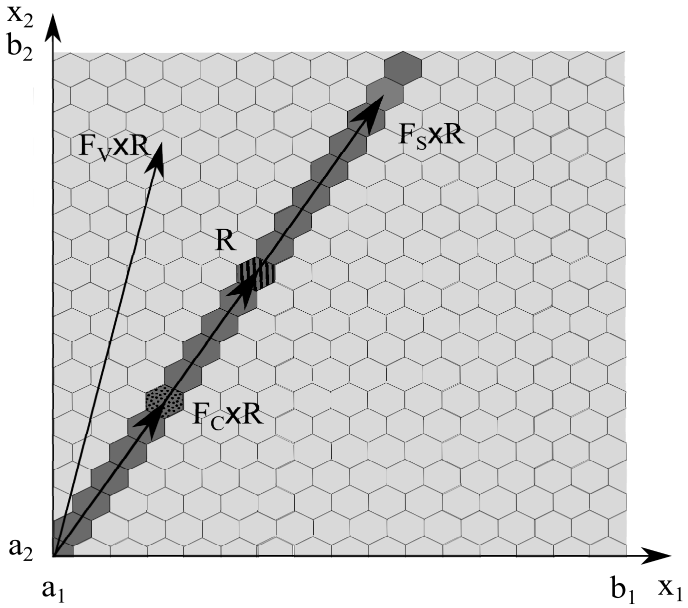
The transition from a scalar constant F to a scalar random F and to a vectorized random $\mathbf{F}$ in DE has an interesting inverse in PSO. In the proposed algorithm, the population size is considered very small compared to the DE algorithm. Reducing the population size results in a faster convergence rate but also a higher risk of stagnation. However, by increasing the population diversity it is possible to decrease the stagnation risk [2], [3]. In order to deliver diversity, the mutation factor F, as one of the most significant control parameters for the DE algorithm, can play a major role. Therefore, proper selection of F value is critical. The mutation factor F in the DE algorithm is a constant mutation factor (CMF), generally set to $F = 0.5$ [2], [14]. This factor can also be selected randomly from the interval $[0, 2]$ for each individual i in the population vector, $F_i = rand(0, 2)$ [3]. We call this algorithm the micro-differential evolution with scalar random mutation factor (MDESM), if the population size is very small. In the MDE algorithm, in order to increase the population diversity, we propose the idea of utilizing a random vector (not scalar) F for each individual in the population. This approach is called the MDEVM algorithm. Therefore, the mutation factor can be defined for each individual i as

$$\mathbf{F}_i = \{F_{i,1}, ..., F_{i,D}\},\ \forall i \in \{1, ..., N_P\}, \tag{4.1}$$

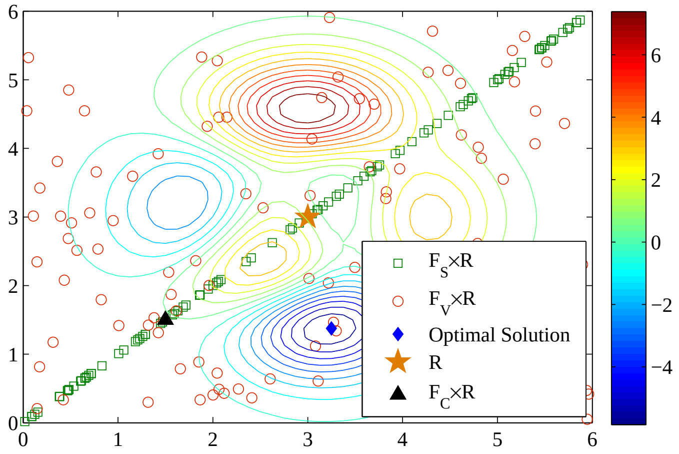
where $F_{i,j} = rand(0.1, 1.5)$, $\forall j \in \{1, ..., D\}$, [3].

4.1 Micro-Differential Evolution with Vectorized Random Mutation

The pseudocode of the proposed MDEVM approach is described in Algorithm 1. After generation the initial population, the mutation vector is computed by using the proposed mutation factor in Eq. (4.1). Then, the *crossover* and *mutation* procedures are conducted similar to the DE algorithm to generate the next population. The termination criterion is



(a) Mutation vector distribution of a random vector R, where each hexagon represents a point on the plane.



(b) Monte-Carlo simulation, plotted on a sample function.

Fig. 4.1: Diversity of mutation vector for a 2-D individual vector R on a 2-D map for constant (F_C), scalar random (F_S), and vectorized random (F_V) mutation factors.

met when the difference between best fitness value (BFV) and fitness value-to-reach (VTR) is less than fitness error-value-to-reach (EVTR), or the searching procedure exceeds the maximum number of function calls NFC_{Max}, i.e., $NFC \geq NFC_{Max}$. As mentioned, the only difference between DE and MDEVM is in the mutation amplification factor, F; which is a constant number in the DE and a uniform random *vector* in the proposed MDEVM algorithm.

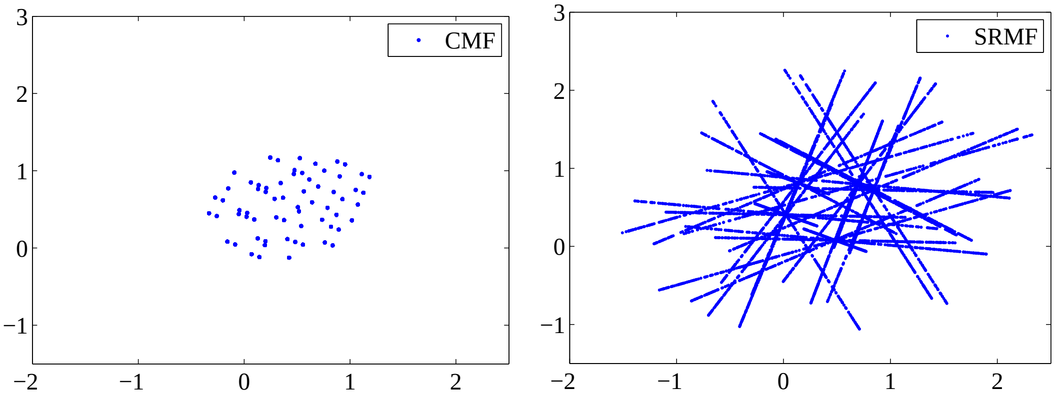
4.2 Monte-Carlo Simulations for Randomized Vectorized Mutation Factor

In order to visualize exploration abilities among CMF, scalar random mutation factor (SRMF), and VRMF, possible diversities of a 2-D individual sample vector $\mathbf{R}$ is presented in Figure 4.1(a). In order to have a better sense of variable space, it is constructed with hexagons, where each hexagon represents a point on the variable space. The landscape for variables x_1 and x_2 is limited to boundaries $[a_1, b_1]$ and $[a_2, b_2]$. Therefore, by having the sample vector $\mathbf{R}$, denoted with a dashed hexagon, effect of an arbitrary CMF on $\mathbf{R}$ is denoted by $F_C \times \mathbf{R}$, as a dotted hexagon. Therefore, diversity of the generated mutation vector $F_C \times \mathbf{R}$ is limited to one hexagon (i.e. the dotted hexagon) on the direction of vector $\mathbf{R}$. In the case of having an identical uniform random F for all variables of an individual, i.e. the SRMF scenario, the diversity of mutation vector $F_S \times \mathbf{R}$ is not just limited to one hexagon (i.e. the dotted hexagon), yet is along the vector $\mathbf{R}$, denoted by dark hexagons. Conversely, by randomizing F for each variable of each individual using a uniform random vector $\mathbf{F}$, i.e. $F_V \times \mathbf{R}$, the VRMF diversity covers the whole plane containing all the hexagons, which presents the highest exploration power.

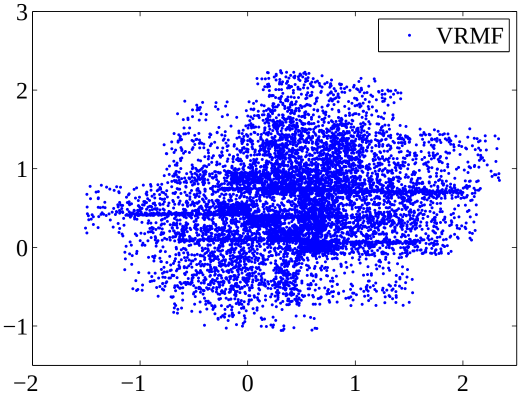
The diversities of CMF, SRMF, and VRMF are investigated by employing Monte-Carlo simulation on an arbitrary landscape in Figure 4.1(b). In this simulation for arbitrary vector $\mathbf{R} = [1, 1]$, 100 sample mutation vectors for each CMF, SRMF, and VRMF schemes with $F_C = 0.5$ and $F_S, F_V \in [0, 2]$ are generated, where the variables are limited as $x_1, x_2 \in [0, 3]$. The simulation illustrates that the VRMF scheme supports a higher diversity than the SRMF, where its diversity is limited to the points on a line. Strictly speaking, if all variables in the individual vector $\mathbf{R}$ are multiplied by a random scalar number, other points are generated on the same direction of the line which is indicated by vector $F_S \times \mathbf{R}$. In fact, the SRMF is generating points on the same direction as vector $\mathbf{R}$. If the relationship among the variables (variables' interaction) are linear, the mutation vector is doing fine (which is a very exceptional case, especially during solving real-world problems). However, when the VRMF scheme is utilized, the mutation vector has no restriction to explore any point on the search space with no linearity restriction, which was the case for SRMF. This discussion is valid for higher dimensions, where the line needs to be replaced with a plane or hyperplane.

By taking into account the crossover component of MDE algorithm, another Monte-Carlo simulation is conducted for CMF, SRMF, and VRMF schemes as presented in Figure 4.2. This simulations are conducted using the "DE/Rand/1" mutation scheme for a population size of $N_P = 5$ and $10,000$ times sample population generations from an identical uniform random population within a 2-D variables space, where each variable is uniform randomly selected as $x_i \in [0, 1]$. The crossover plays a decisive role in taking diversity into the populations, as presented for the CMF scheme in Figure 4.2(a). However as presented in Figure 4.2(b) and 4.2(c), the crossover also expands the diversity of SRMF and VRMF schemes dramatically such that almost the whole variable space is explored by the VRMF scheme.

By keeping the stated Monte-Carlo simulation settings, the diversity analysis on CMF, SRMF, and VRMF schemes is extended for variable space dimensions $D \in \{1, ..., 1000\}$ and populations sizes $N_P \in \{5, 50\}$ as shown in Figure 4.3. In these simulations, the average of centroid distance and pairwise distance measures are considered. The average

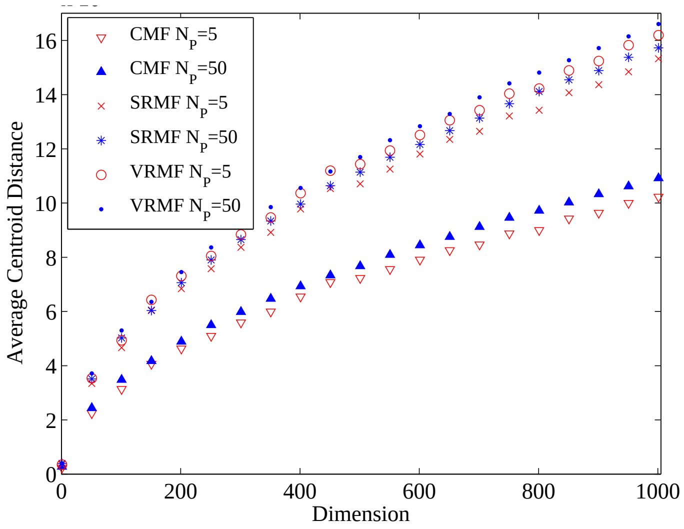


(a) Constant Mutation Factor (b) Scalar Random Mutation Factor

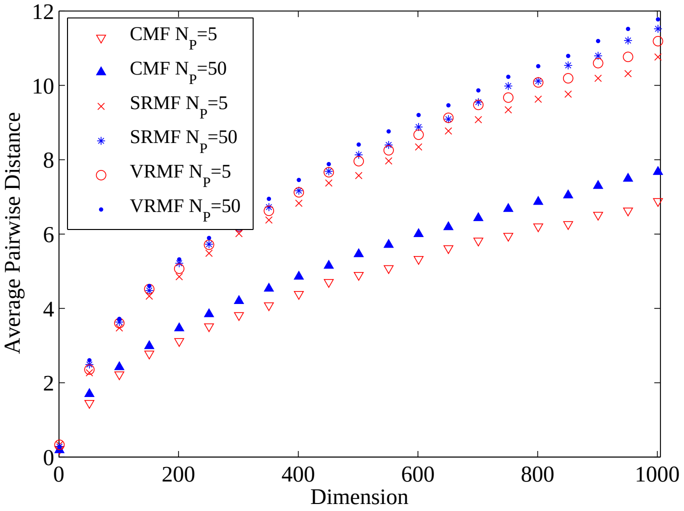


(c) Vectorized Random Mutation Factor

Fig. 4.2: Monte-Carlo simulation of population diversity for $D = 2$ and $N_P = 5$ after 10,000 random generation by considering the crossover operator.



(a) Centroid Distance



(b) Pairwise Distance

Fig. 4.3: Average centroid and pairwise distances for the Monte-Carlo simulation of population diversity for dimensions 1 to 1,000 and $N_P \in \{5, 50\}$ after 10,000 random generaion by considering the mutation and crossover operators.

of centroid distance demonstrates distance of each population's individual with the centroid of the population. This measure shows diversity of the population. The pairwise distance measure presents the average of distances between individual pairs in a population. This measure demonstrated the diversity of population as well as how far can individuals are spreaded on the landscape.

The average of distances between sample population individuals and the centroid of samples for each dimension is computed as

$$C_D = \frac{1}{N_P} \sum_{i=1}^{N_P} \sqrt{\sum_{d=1}^{D} (x_{i,d} - x_d^c)^2}, \tag{4.2}$$

where the centroid of the population is $\mathbf{X}^c = (x_1^c, ..., x_D^c)$, computed as

$$x_d^c = \frac{1}{N_P} \sum_{i=1}^{N_P} x_{i,d}, \ \forall d \in \{1, ..., D\}. \tag{4.3}$$

As Figure 4.3(a) demonstrates, the CMF has the least diversity for both $N_P = 5$ and $N_P = 50$ compared to SRMF and VRMF schemes. This is while the VRMF scheme has the highest diversity and as the dimensionality of problem increases, its diversity is improved more comparing to the CMF and SRMF schemes. It is obvious that the $N_P = 50$ has a higher diversity than the $N_P = 5$ in all schemes, but this diversity improvement is much less than the diversity that the VRMF scheme can deliver into the population with a much smaller population size, i.e. $N_P = 5$. The comparison among CMF with $N_P = 5$ and CMF with $N_P = 50$ and VRMF with $N_P = 5$ clearly indicates the performance of VRMF scheme with small population size is higher in term of diversity enhancement.

In order to study the diversity based on the average pairwise distance, it is computed as

$$P_D = \frac{1}{N_P(N_P-1)} \sum_{i=1}^{N_P} \sum_{\substack{j=1 \\ i \neq j}}^{N_P} \sqrt{\sum_{d=1}^{D} (x_{i,d} - x_{j,d})^2}. \tag{4.4}$$

The average pairwise distances for different dimensions and populations sizes $N_P = 5$ and $N_P = 50$ are illustrated in Figure 4.3(b). The performance results for this diversity measurement criterion also clearly demonstrates strength of the VRMF with small populations size.

Algorithm 1 Micro-Differential Evolution with Vectorized Mutation (MDEVM)

```
Procedure MDEVM
$g = 0$
    //Initial Population Generation
for $i = 1 \rightarrow N_P$ do
    for $d = 1 \rightarrow D$ do
        $\mathbf{X}_{i,d} = x_d^{min} + rand(0,1) \times (x_d^{max} - x_d^{min})$
    end for
    $\mathbf{P}_i^g = \mathbf{X}_i$
end for
    //End of Initial Population Generation
while $(|BFV - VTR| > EVTR \,\&\, NFC < NFC_{Max})$ do
    for $i = 1 \rightarrow N_P$ do
    //Mutation
        Select three random population vectors from $\mathbf{P}^g$ where $(i_1 \neq i_2 \neq i_3 \neq i)$
        for $d = 1 \rightarrow D$ do
            $F = rand(0.1, 1.5)$
            $\mathbf{V}_{i,d} = \mathbf{X}_{i_1,d} + F(\mathbf{X}_{i_2,d} - \mathbf{X}_{i_3,d})$
        end for
    //End of Mutation
    //Crossover
        for $d = 1 \rightarrow D$ do
            if $rand(0,1) < C_r$ or $d_{rand} = d$ then
                $U_{i,d} = V_{i,d}$
            else
                $U_{i,d} = x_{i,d}$
            end if
        end for
    //End of Crossover
    //Selection
        if $f(\mathbf{U}_i) \leq f(\mathbf{X}_i)$ then
            $\mathbf{X}'_i = \mathbf{U}_i$
        else
            $\mathbf{X}'_i = \mathbf{X}_i$
        end if
    //End of Selection
    end for
    $\mathbf{X}_i = \mathbf{X}'_i, \forall i \in \{1, ..., N_P\}$
    $g = g + 1$
    $\mathbf{P}^g = \{\mathbf{X}_1, ..., \mathbf{X}_{N_P}\}$
end while
```

Chapter 5

Simulation Results

In this section, performance of the proposed MDEVM algorithm is compared with the MDE and MDESM algorithms. To do so, the parameter setting and employed benchmark functions (i.e. CEC-2013 testbed [19]) are described in the next subsection. Then, the comprehensive experimental series to analyze the MDE, MDESM, and MDEVM schemes' performance are presented in details.

Table 5.1: Parameter setting for all conducted experiments

Parameter	Description	Value
Cr	Crossover Probability Constant	0.9
NFC_{Max}	Maximum Number of Function Calls	$1e3 \times D$
$EVTR$	Objective Function Error Value to Reach	1e-8
N_{Run}	Number of Runs	30

5.1 Benchmark Functions and Parameters Setting

All the experiments have been conducted on the CEC-2013 testbed [19]. It is comprised of 28 benchmark functions and an improved version of CEC-2005 [20] counterpart with additional test functions and modified formula for the composite functions, oscillations, and symmetric-breaking transforms. This testbed is divided into three categories which are uni-modal functions ($f_1 - f_5$), multi-modal functions ($f_6 - f_{20}$), and composite functions ($f_{21} - f_{28}$) [19]. Parameters setting for all the experiments are presented in Table 5.1 adapted from the literature [3], [14], [19], unless a change is mentioned. The reported values are averaged for $N_{Run} = 30$ independent runs per function per algorithm to minimize the effect of the stochastic nature of the algorithms on the reported results.

The mutation schemes presented by Eq. 3.1 to Eq. 3.5 are the five main schemes [61], [62]. However, the "DE/Rand/1", "DE/Best/1", "DE/T2B/1", and "DE/Best/2" schemes are standard DE algorithms where the standard population size is $N_P \geq 4$ and the scheme “DE/Rand/2” works for $N_P \geq 5$. However, we have used these mutation schemes for small population sizes, i.e. $N_P < 4$, and have employed the “DE/Rand/1” mutation vector scheme for MDE algorithm with $N_P = 2$ as illustrated in Table 5.2.

5.2 Experimental Series 1: Mutation Schemes and Population Size Analysis

Performance of the MDE, MDESM, and MDEVM schemes are evaluated for mutation schemes provided in Table 5.2 and population sizes $N_P \in \{2, 3, 4, 5, 6, 50\}$ for dimension $D = 50$ regarding the best error (Best) and standard deviation of best errors (Std) criteria as reported in Tables A.1 to A.15. The Wilcoxon test results are reported in terms of pair-wise comparisons, where the symbols “+”, “=”, and “-” indicate a statistically better, equivalent, and worse performance, respectively, compared with the algorithm in the column label [63]. Number of Wilcoxon pair-wise comparisons for the results are presented in Table 5.3.

In $N_P = 2$ case, the proposed MDEVM “DE/T2B/1” has the highest performance among other schemes with success in 25 and 17 benchmark functions compared to the MDE and MDESM approaches, respectively. Totally, the MDEVM method has acceptable performance among all mutation schemes and methods. By considering the population size of $N_P = 3$, the proposed MDEVM method using the “DE/Best/1” mutation schemes has the best performance among all with acceptable performance over other mutation schemes comparing with MDE and MDESM methods. For population size $N_P = 4$, which is the standard population size for MDE algorithms, four mutation schemes are analysed, where same as before, the MDEVM method on “DE/Best/1” , “DE/T2B/1”, and “DE/Rand/1” schemes have the highest performance. However, it has poor performance for the “DE/Best/2” mutation scheme. The same order is obvious for the $N_P = 5$

Table 5.2: Mutation vector (MV) schemes for population sizes $N_P \in \{2, 3, 4\}$ and $N_P \geq 5$.

N_P	MV	$\mathbf{V}_i$
2	DE/Rand/1	$\mathbf{X}_1 + F(\mathbf{X}_2)$
	DE/Best/1	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/T2B/1	$\mathbf{X}_i + F(\mathbf{X}_{best} - \mathbf{X}_i) + F(\mathbf{X}_1 - \mathbf{X}_2)$
3	DE/Rand/1	$\mathbf{X}_1 + F(\mathbf{X}_2 - \mathbf{X}_3)$
	DE/Best/1	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/T2B/1	$\mathbf{X}_i + F(\mathbf{X}_{best} - \mathbf{X}_i) + F(\mathbf{X}_1 - \mathbf{X}_2)$
4	DE/Rand/1	$\mathbf{X}_1 + F(\mathbf{X}_2 - \mathbf{X}_3)$
	DE/Best/1	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/T2B/1	$\mathbf{X}_i + F(\mathbf{X}_{best} - \mathbf{X}_i) + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/Best/2	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2) + F(\mathbf{X}_3 - \mathbf{X}_4)$
$5\leq$	DE/Rand/1	$\mathbf{X}_1 + F(\mathbf{X}_2 - \mathbf{X}_3)$
	DE/Best/1	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/T2B/1	$\mathbf{X}_i + F(\mathbf{X}_{best} - \mathbf{X}_i) + F(\mathbf{X}_1 - \mathbf{X}_2)$
	DE/Best/2	$\mathbf{X}_{best} + F(\mathbf{X}_1 - \mathbf{X}_2) + F(\mathbf{X}_3 - \mathbf{X}_4)$
	DE/Rand/2	$\mathbf{X}_1 + F(\mathbf{X}_2 - \mathbf{X}_3) + F(\mathbf{X}_4 - \mathbf{X}_5)$

and $N_P = 6$ scenarios, where all five mutation schemes are analysed. By increasing the population diversity toward using the size $N_P = 50$, the MDEVM method has very poor performance for all mutation schemes, except the "DE/Best/1" scheme.

The difference between DE and MDE algorithms is in population size which delivers diversity into the population. Combining the VRMF technique with the DE algorithm results in extra diversity which results a poor performance of the algorithm. Using the standalone DE-algorithm may result in better performance, but by the cost of more number of function calls. Therefore, utilizing the MDE algorithm with small population sizes can deliver both higher diversity and performance into the algorithm. In overall, the "DE/Best/1", "DE/Rand/1", and "DE/T2B/1" schemes have the best performance among the mutation schemes for MDEVM. Since the "DE/Best/2" and "DE/Rand/2" schemes have more exploration capability due to incorporating more population individuals and therefore, resulting in extra diversity in the population which prevents the algorithm from

Table 5.3: Number of Wilcoxon rank-sum test comparisons (reference: MDEVM) for MDEVM against MDE and MDESM schemes on CEC 2013 benchmark functions and population sizes $N_P \in \{2, 3, 4, 5, 6, 50\}$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2", and "DE/Rand/2".

N_P	MV	MDEVM/MDE			MDEVM/MDESM		
		+	=	-	+	=	-
2	DE/Rand/1	0	23	5	2	19	7
	DE/Best/1	14	11	3	15	9	4
	DE/T2B/1	25	3	0	17	9	2
3	DE/Rand/1	11	10	7	7	11	10
	DE/Best/1	24	4	0	20	5	3
	DE/T2B/1	17	9	2	12	10	6
4	DE/Rand/1	20	4	4	12	5	11
	DE/Best/1	21	7	0	19	5	4
	DE/T2B/1	20	4	4	17	1	10
	DE/Best/2	2	3	23	0	4	24
5	DE/Rand/1	19	2	7	12	8	8
	DE/Best/1	24	2	2	16	7	5
	DE/T2B/1	19	2	7	15	4	9
	DE/Best/2	12	6	10	6	7	15
	DE/Rand/2	0	1	27	1	1	26
6	DE/Rand/1	13	5	10	13	7	8
	DE/Best/1	21	3	4	18	6	4
	DE/T2B/1	19	5	4	15	2	11
	DE/Best/2	11	5	12	7	4	17
	DE/Rand/2	1	1	26	1	1	26
50	DE/Rand/1	1	2	25	1	3	24
	DE/Best/1	10	5	13	2	6	20
	DE/T2B/1	0	3	25	0	4	24
	DE/Best/2	0	2	26	0	3	25
	DE/Rand/2	0	2	26	1	2	25

fast convergence toward the possible optimal solution. In Figure 5.1, a summary of better performance counting of all schemes is presented, where $N_P = 5$ has the highest number of success for all mutation schemes on average.

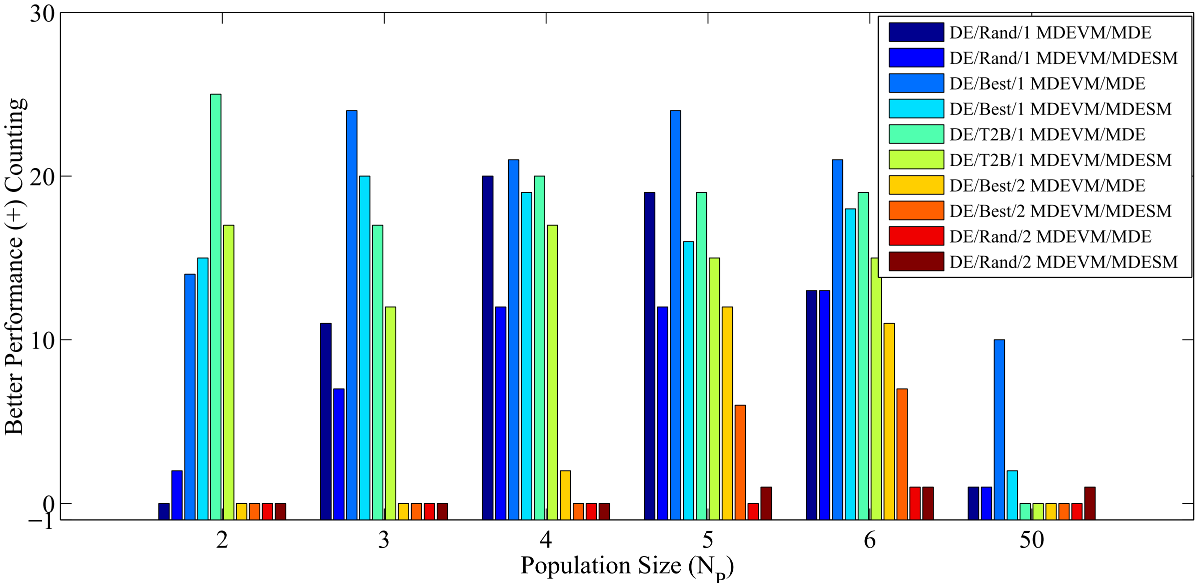
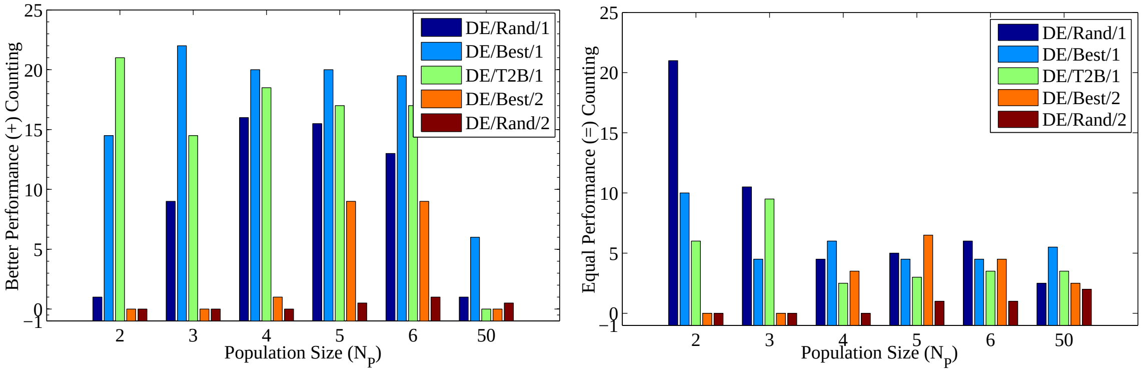


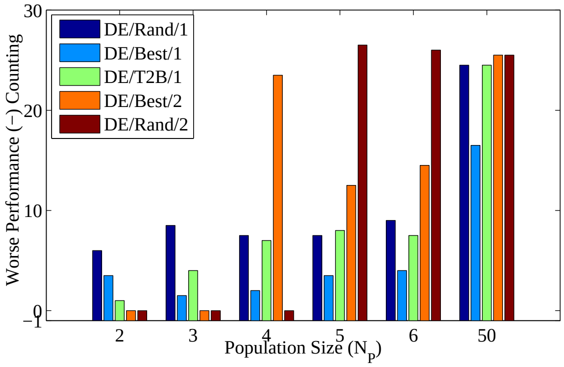
Fig. 5.1: The better (+) performance counting for the MDEVM/MDE and MDEVM/MDESM comparisons for different mutation schemes and populations sizes.

In order to have a deeper look, average of better, equal, and worse performance counting for the MDEVM/MDE and MDEVM/MDESM comparisons are presented in Figure 5.2. Regarding the average of better and equivalent performances results as shown in Figure 5.2(a) and Figure 5.2(b), it is clear that the "DE/Best/1" scheme has the most number of successes. In terms of worse performance comparison as in Figure 5.2(c), it is interesting that as the population size increases, the number of worse performance counts, particularly for the "DE/T2B/1", "DE/Best/2", and "DE/Rand/2" mutation schemes, increase dramatically.

To have an analyse from different benchmark functions types viewpoint, the approach with the best error value for each benchmark function is illustrated as in Figure 5.3. The dash line separates the uni-modal, multi-modal, and composite, benchmark functions types. For the uni-modal and multi-modal functions, the VRMF method with the "DE/T2B/1" mutation scheme and $N_P = 6$ has the best performance. For the composite functions,



(a) Better (+) performance (b) Equal (=) performance



(c) Worse (-) performance

Fig. 5.2: Average of MDEVM/MDE and MDEVM/MDESM performances for different mutation schemes and populations sizes.

the SRMF method with the "DE/T2B/1" mutation scheme and $N_P = 6$ has the best performance.

It is obvious that the more population size results in better performance with more number of function calls. In overall, the "DE/Best/1" mutation scheme with population size of $N_P = 5$ is recommended as the well-performance scheme among the all. Further analysis are conducted on this scheme in depth, including the popular scheme "DE/Rand/1".

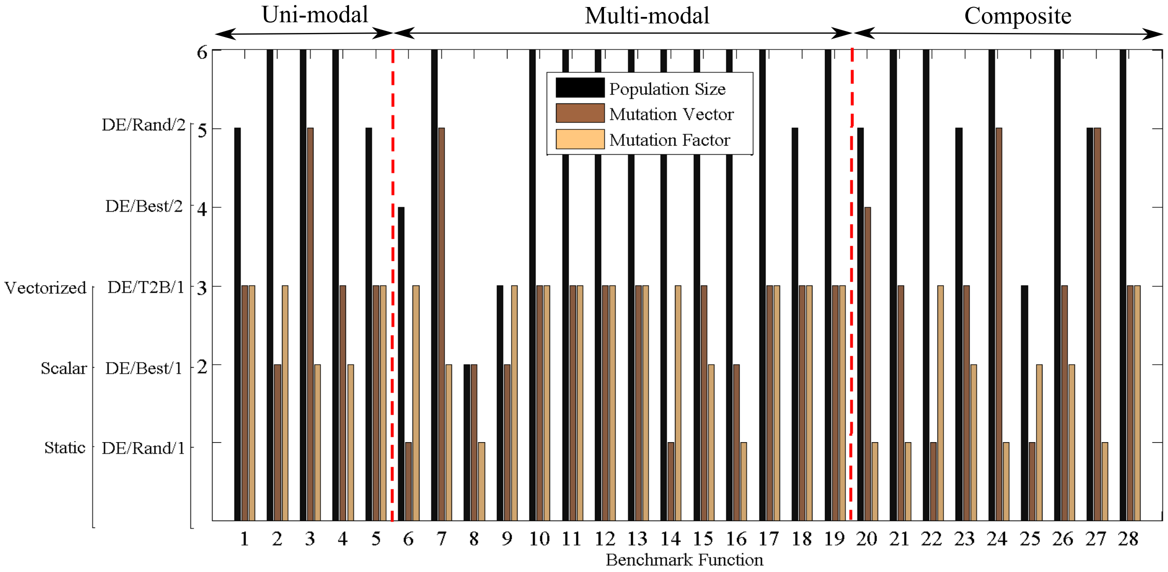
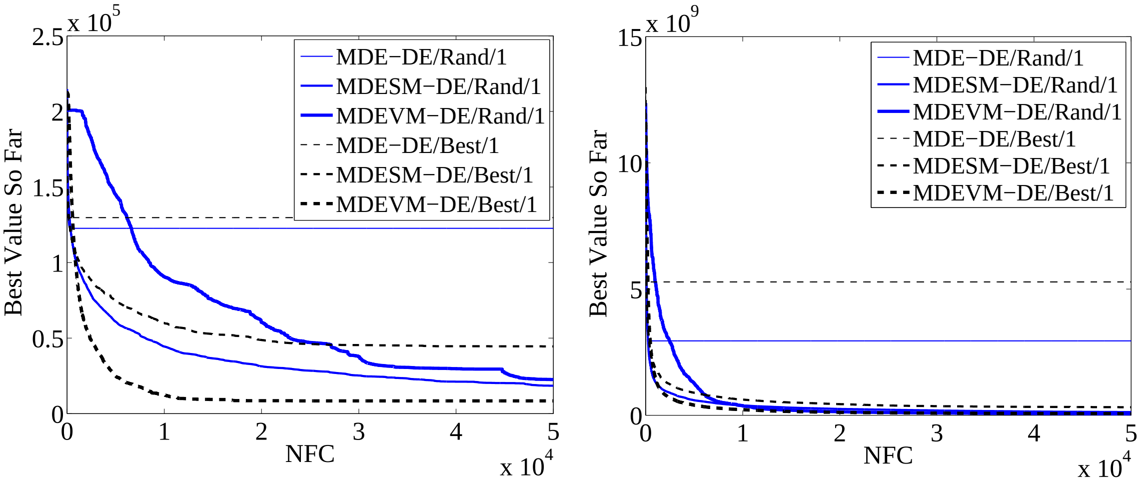


Fig. 5.3: Components of highest performance algorithm with respect to the best error for each benchmark function, and function families uni-modal ($f_1 - f_5$), multi-modal ($f_6 - f_{19}$), and composite ($f_{20} - f_{28}$).

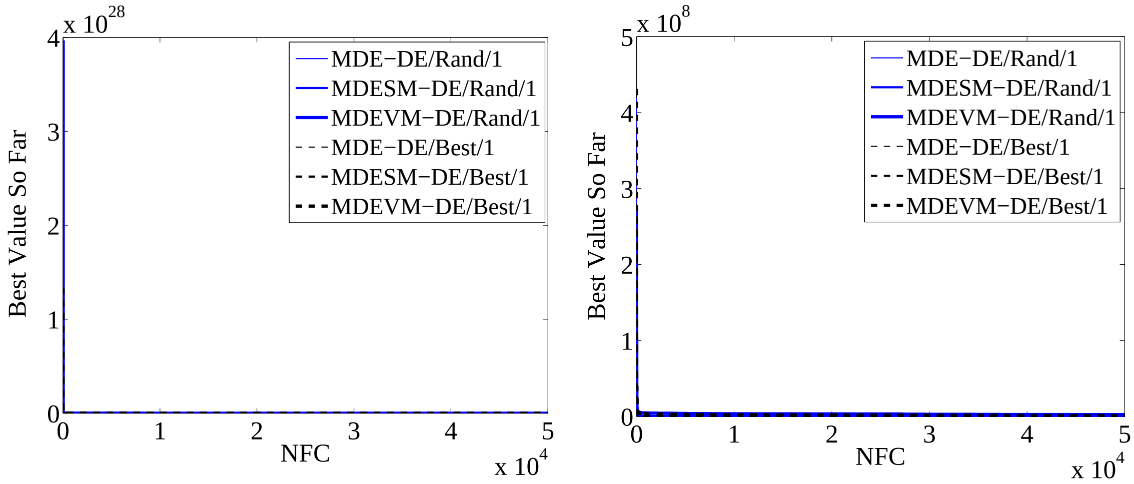
In Figures 5.4 to 5.10, performances of the MDE, MDESM, and MDEVM methods for the "DE/Rand/1" and "DE/Best/1" mutation schemes and different number of function calls are presented. As an example for the f_1 in Figure 5.4(a), the MDEVM method with the "DE/Best/1" has converged faster and the MDEVM method with the "DE/Rand/1" is going to converge with a sharp slope. By assigning a higher number of possible function calls, this method can outperform the MDEVM method with the "DE/Best/1" converged best error. The algorithms for different number of function calls are discussed further in the current section. The same situation is obvious for f_{14}, f_{20} and f_{22}.

5.3 Experimental Series 2: Dimensionality Effects

In this subsection, performance of the proposed MDEVM method is compared with the MDE and MDESM methods for population size $N_P = 5$ and dimension $D \in \{10, 30, 50, 100\}$

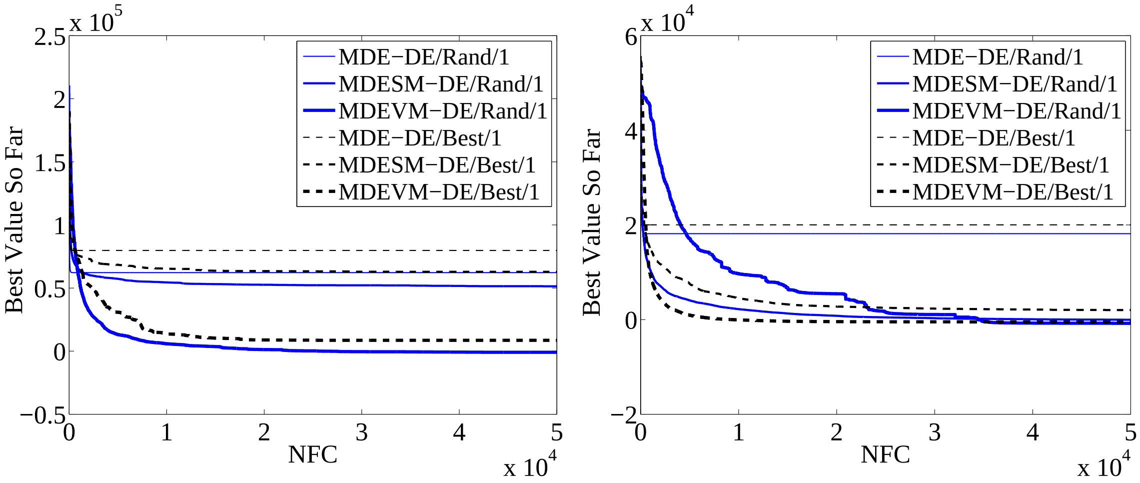


(a) f_1 (b) f_2

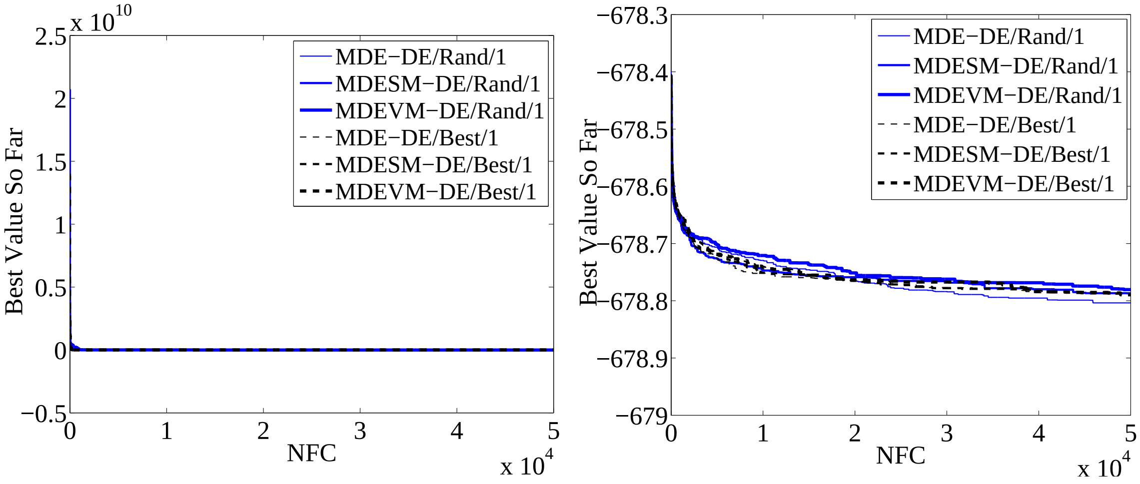


(c) f_3 (d) f_4

Fig. 5.4: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_1 to f_4.

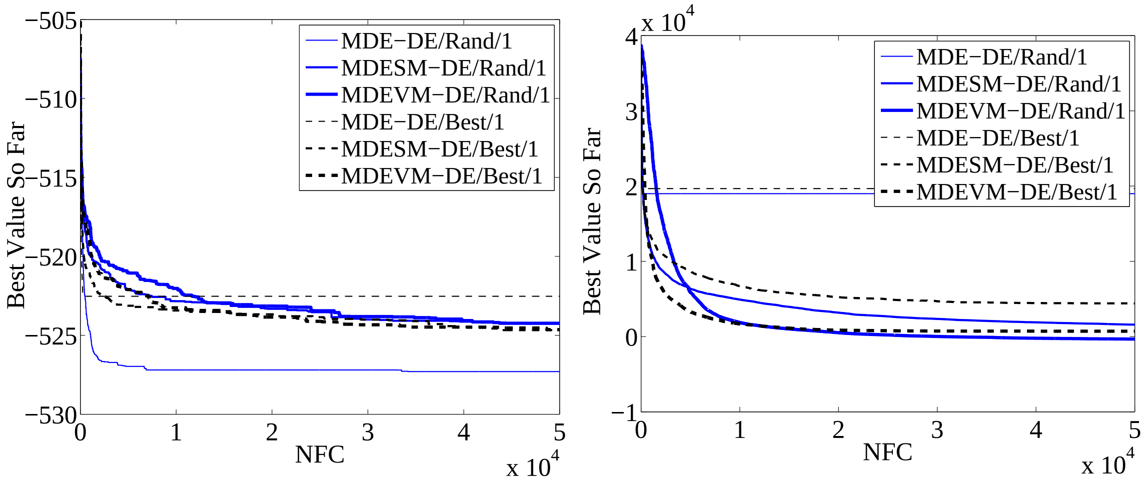


(a) f_5 (b) f_6



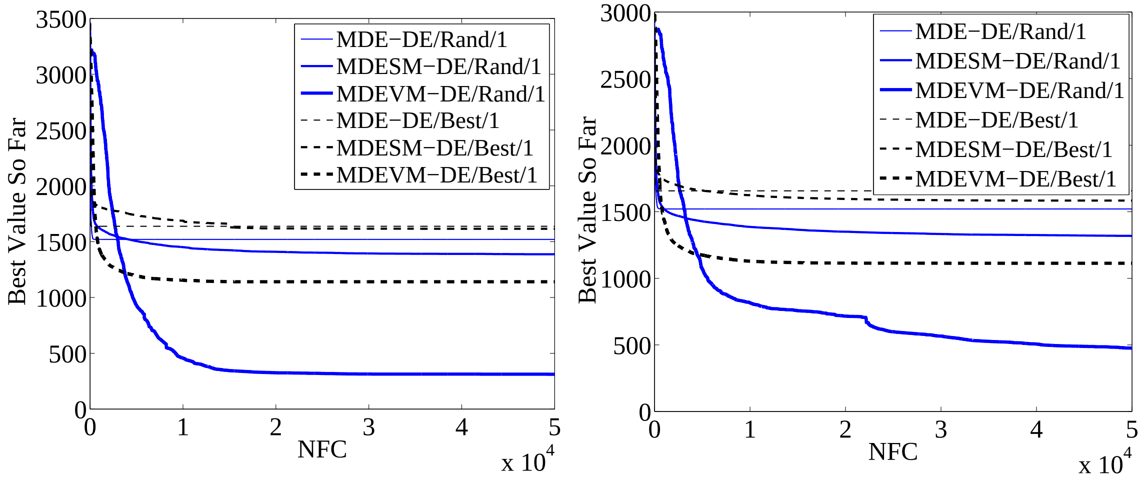
(c) f_7 (d) f_8

Fig. 5.5: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_5 to f_8.



(a) f_9

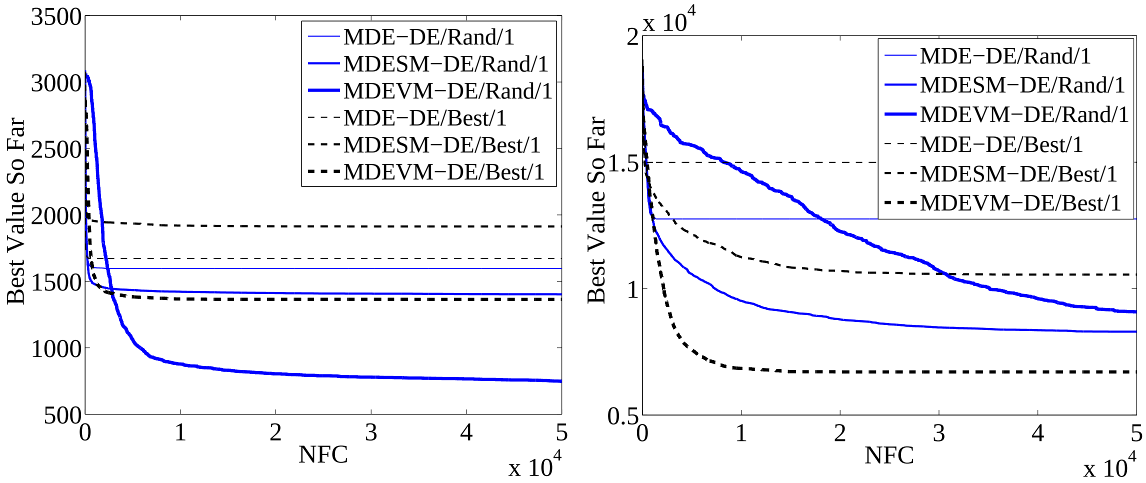
(b) f_{10}



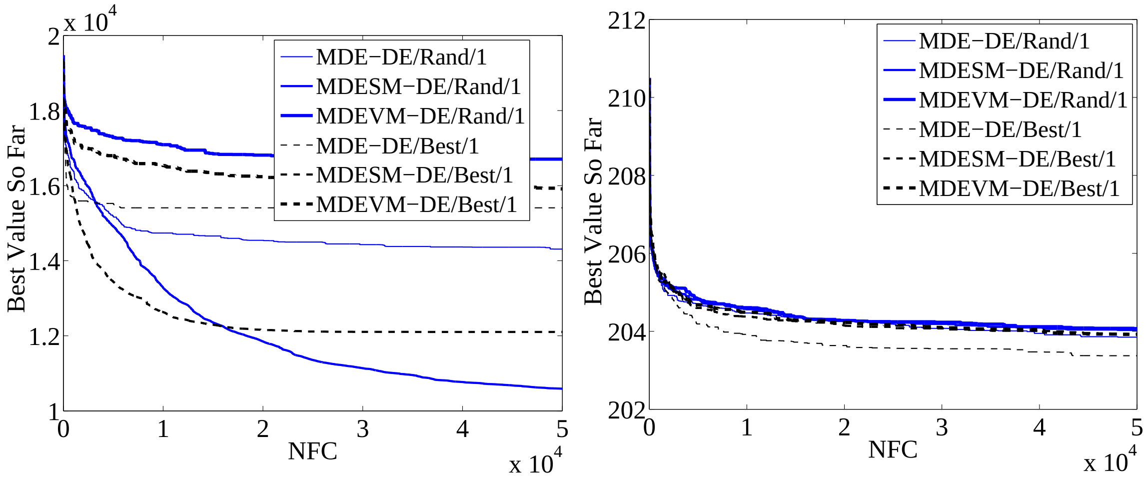
(c) f_{11}

(d) f_{12}

Fig. 5.6: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_9 to f_{12}.

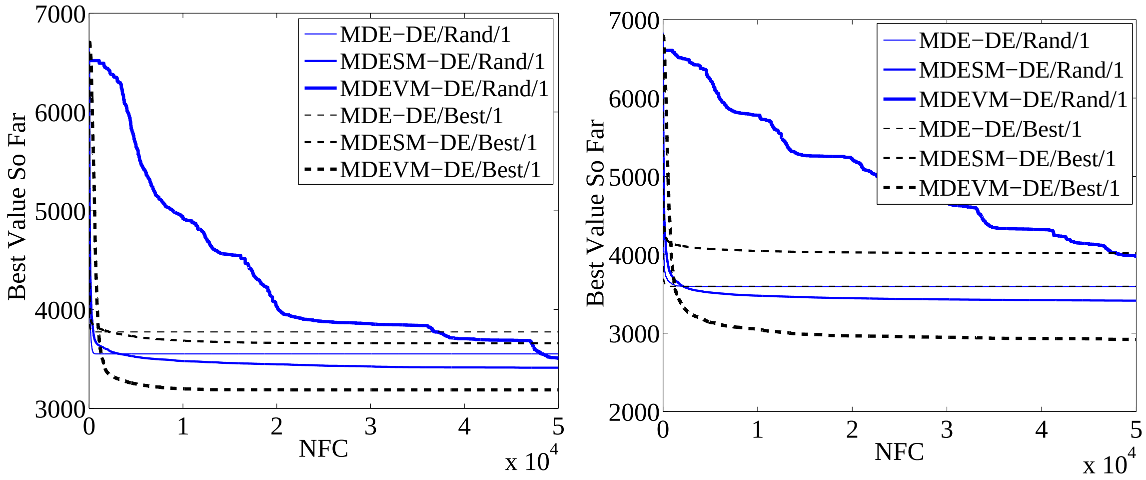


(a) f_{13} (b) f_{14}

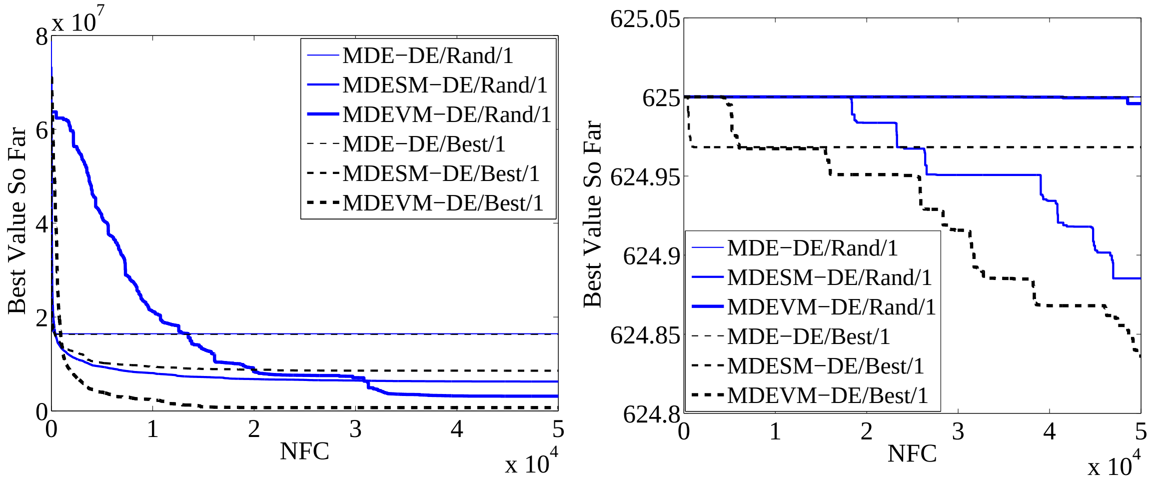


(c) f_{15} (d) f_{16}

Fig. 5.7: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_{13} to f_{16}.

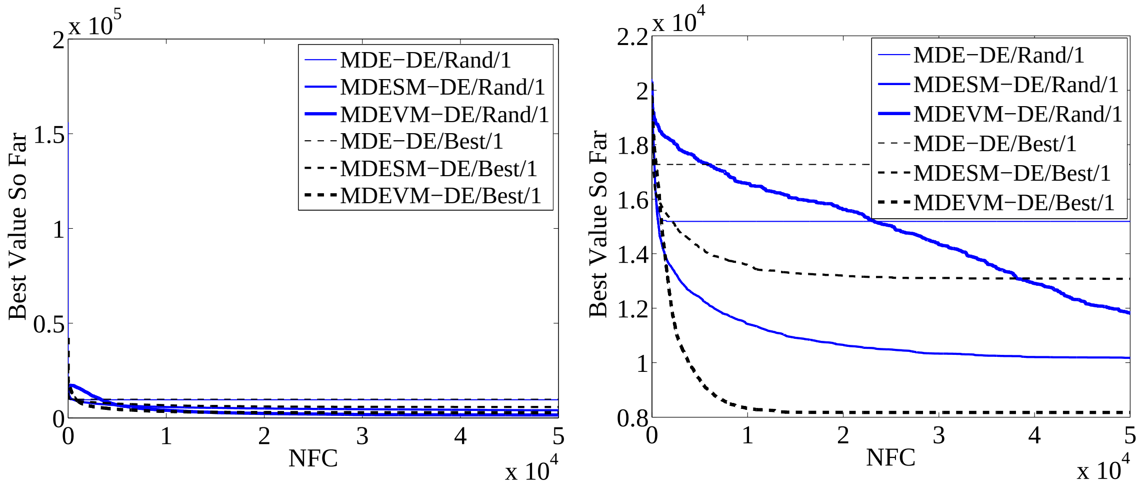


(a) f_{17} (b) f_{18}

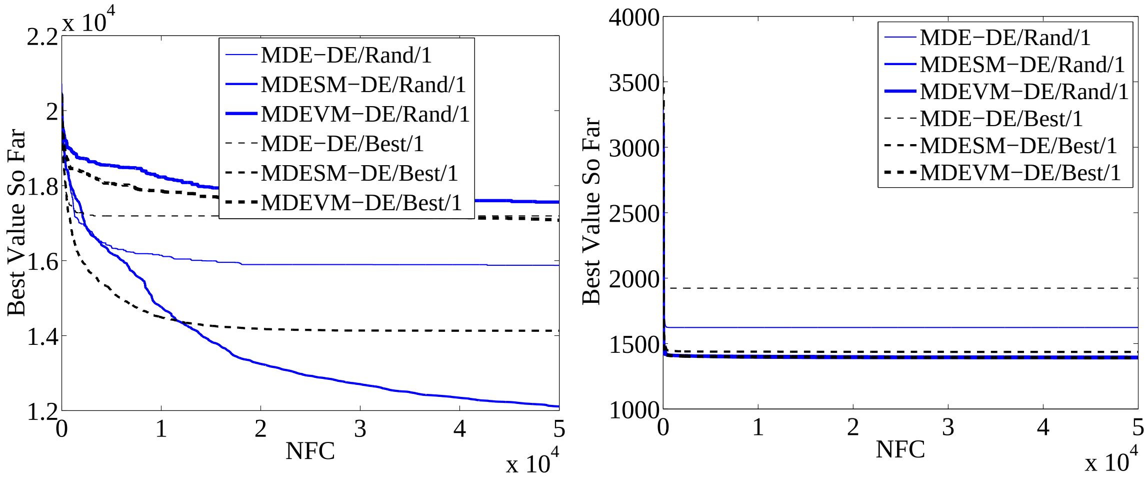


(c) f_{19} (d) f_{20}

Fig. 5.8: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_{17} to f_{20}.

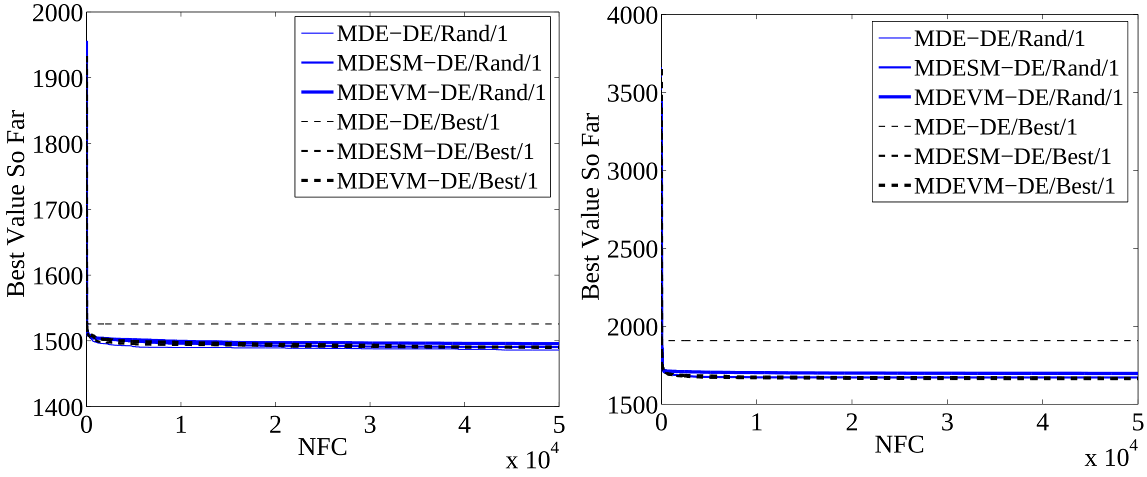


(a) f_{21} (b) f_{22}

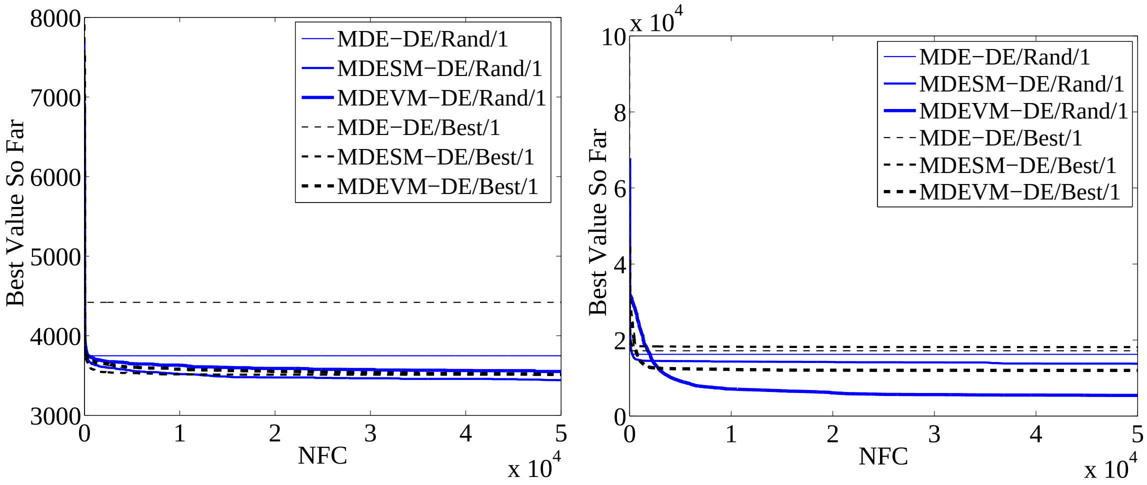


(c) f_{23} (d) f_{24}

Fig. 5.9: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark functions f_{21} to f_{24}.



(a) f_{25} (b) f_{26}



(c) f_{27} (d) f_{28}

Fig. 5.10: Performance comparison among the MDE, MDESM, and MDEVM schemes for the DE/Rand/1 and DE/Best/1 mutation schemes, dimension $D = 50$, population size $N_P = 5$, and benchmark function f_{25} to f_{28}.

Table 5.4: Number of Wilcoxon rank-sum test comparisons (reference: MDEVM) for MDEVM against MDE and MDESM schemes on CEC 2013 benchmark functions and population size $N_P = 5$ for dimension $D \in \{10, 30, 50, 100\}$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

D	MV	MDEVM/MDE			MDEVM/MDESM		
		+	=	-	+	=	-
10	DE/Rand/1	23	3	2	12	13	3
	DE/Best/1	26	0	2	24	3	1
30	DE/Rand/1	22	4	2	17	5	6
	DE/Best/1	21	5	2	20	6	2
50	DE/Rand/1	19	2	7	12	8	8
	DE/Best/1	24	2	2	16	7	5
100	DE/Rand/1	18	3	7	16	5	7
	DE/Best/1	20	5	3	15	9	4

with mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1" regarding the best error (Best) and standard deviation of best errors (Std) criteria, Table A.7 to Table A.9, and Tables A.16 to A.18. By considering the MDEVM method as the reference algorithm, summary of the Wilcoxon test results are reported in terms of pair-wise comparisons in Table 5.4. The results clearly demonstrate that the proposed MDEVM method has outperformed the MDE and MDESM methods for different dimensions. The MDESM method shows a better performance than the MDE method, which is due to the SRMF diversity enhancement technique used in this scheme. Both "DE/Rand/1" and "DE/Best/1" mutation schemes have competitive performances over all dimensions and MDE schemes. In overall, it is clear that the randomizing the mutation factor for each variable space delivers diversity into the populations and increases the performance of search toward finding the optimal solution.

Table 5.5: Summary of performance results for the MDESM approach with $\mathbf{F} \in [0, 2]$, versus the MDESM with $F \in [0.1, 1.5]$ and MDEVM methods with $\mathbf{F} \in [0.1, 1.5]$ denoted with index F, for $N_P = 5$ and $D = 30$.

MV	MDESM/MDESM$_F$			MDESM/MDEVM$_F$		
	+	=	-	+	=	-
DE/Rand/1	14	12	2	6	7	15
DE/Best/1	14	14	0	7	9	12

5.4 Experimental Series 3: Mutation Factor's Range Analysis

The most common mutation factor in the literature is $F = 0.5$, selected from the recommended range $F \in [0, 2]$, [62]. Recently, different values for F and its range has been proposed, such as $F = 0.7$ in [1] and $F \in [0.1, 1.5]$ in [3]. Therefore, some experiments are conducted in this subsection to analyse effect of mutation factor range, on the performance of the MDESM and MDEVM approaches. By considering $N_P = 5$ for dimension $D = 30$ and mutation vector schemes "DE/Rand/1" and "DE/Best/1", the best error, standard deviation, and Wilcoxon rank-sum test results by considering the MDESM and MDEVM algorithms as references are presented in Tables A.19 and A.20. The summary of results are presented in Tables 5.5 and Table 5.6. The mutation factor ranges are considered as $F \in [0, 2]$ and $\mathbf{F} \in [0, 2]$ for MDESM and MDEVM approaches. The approaches with the range $F \in [0.1, 1.5]$ and $\mathbf{F} \in [0.1, 1.5]$ are denoted by index F, which are MDESM$_F$ and MDEVM$_F$. The W_S and W_V demonstrate performance of the MDESM and MDEVM methods versus the MDESM$_F$ and MDEVM$_F$ methods, respectively.

As demonstrated in Table 5.5, the MDESM method has almost better performance than the MDESM$_F$ method. However, the MDEVM$_F$ method has outperformed the MDESM method due to the delivered diversity by the VRMF approach into the MDEVM$_F$ method.

Table 5.6: Summary of performance results for the MDEVM approach with $\mathbf{F} \in [0, 2]$, versus the MDESM with $F \in [0.1, 1.5]$ and MDEVM methods with $\mathbf{F} \in [0.1, 1.5]$ denoted with index F, for $N_P = 5$ and $D = 30$.

MV	MDEVM/MDESM$_F$			MDEVM/MDEVM$_F$		
	+	=	-	+	=	-
DE/Rand/1	19	5	4	3	25	0
DE/Best/1	19	6	3	19	3	6

In Table 5.6, the MDEVM is compared with the MDESM$_F$ and MDEVM$_F$ methods. The results demonstrate that selecting $\mathbf{F}$ in the interval $[0, 2]$ has a better performance than the limited interval $[0.1, 1.5]$. The comparison between the MDEVM and MDEVM$_F$ also shows almost equal performance. Overall, better performance of the MDEVM method is obvious, since the MDEVM method has diversity from both VRMF and wider mutation factor range $[0, 2]$.

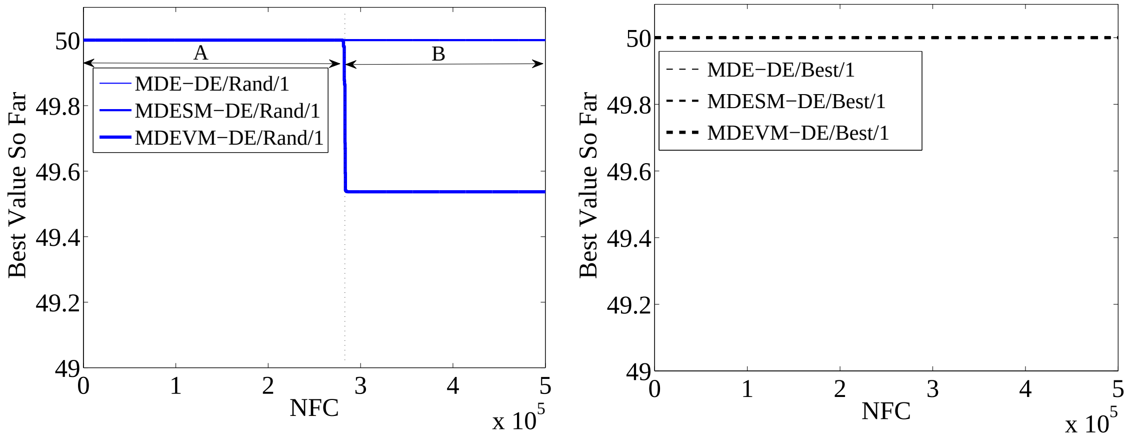
5.5 Experimental Series 4: Population's Diversity Analysis

The vectorized mutation factor (VMF) method can empower the MDE algorithm to escape trapping in local optima and decrease the stagnation risk. In order to analyze the effect of randomization of mutation factor on the population diversity by considering the centroid distance measure and performance of the MDE algorithm, the best-value-so-far and population diversity plots of the MDE, MDESM, and MDEVM methods are presented for composite functions f_{20} to f_{22} in Figures 5.11 to 5.13. The simulations are conducted for dimension $D = 100$, population size $N_P = 5$, and "DE/Rand/1" and "DE/Best/1" mutation schemes. Conductive to have a better sense of analysis, the maximum number of function calls is considered $NFC_{Max} = 5,000D$.

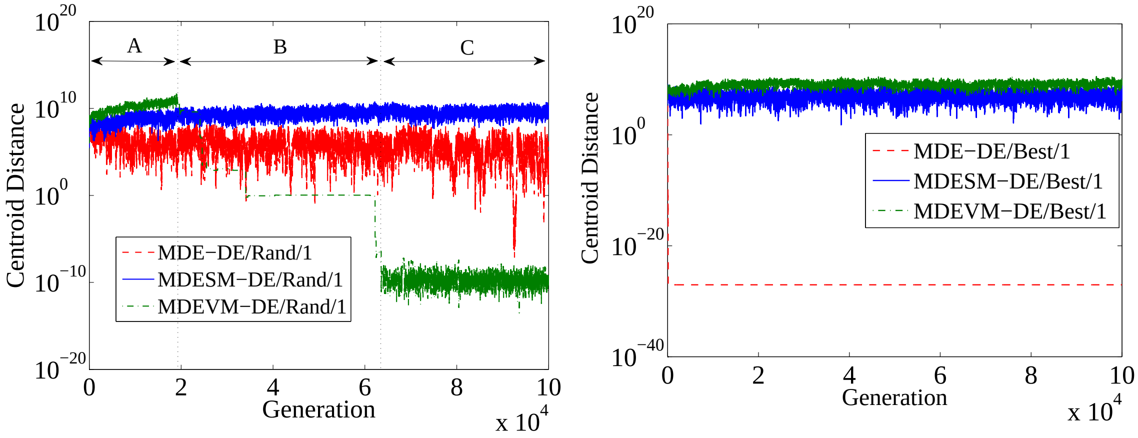
The MDEVM method for the mutation scheme "DE/Rand/1" has the best performance for the function f_{20} as shown in Figure 5.11(a), denoted by "B". The population diversities in Figure 5.11(c) and for the "DE/Best/1" mutation scheme in Figure 5.11(d), clearly show that while the MDE and MDESM methods for both mutation schemes are stagnated, due to almost static large value of centroid distance value, the MDEVM method for the "DE/Rand/1" has escaped from the stagnation denoted by "A" while trying to converge in generations denoted by "B". When the centroid distance value is large, and the performance of algorithm in finding the solution is almost static with respect to the best-value-so-far measure, the population is considered stagnated. For situation of trapping in a local minimum, the population is not divert and the centroid distance value is small, while having a poor best-value-so-far performance.

For the f_{21} case, the MDEVM method using the "DE/Rand/1" and "DE/Best/1" schemes has the best performance, as shown in Figure 5.12(a) and Figure 5.12(b). The MDE algorithm is trapped in local minimum for both mutation schemes, while the MDESM method has better capability to escape from both stagnation and local optimum trapping, denoted by "C" in Figure 5.12(c) and Figure 5.12(d). The MDEVM has the best best-value-so-far for both mutation schemes. For the "DE/Rand/1" mutation scheme, the population's centroid distance shows a similar convergence trend to the MDESM method, but has achieved a much better best-value-so-far at he beginning generations and then trapped in the local minimum, as denoted in part "C" of Figure 5.12(a). The same performance is obvious for the "DE/Best/1" mutation scheme as shown in Figure 5.12(b), where in part "A" it is converged to a solution. The corresponding centroid distance behaviour is well-illustrated in Figure 5.12(d). In part "A", which is the exploration phase, the population's diversity is decreased and it is converged, as shown in part "B". In "D", it has trapped but recovered fast to the same level as part "B".

The power of VMF is well illustrated for the benchmark function f_{22} as shown in Figure 5.13(a) and Figure 5.13(b). In Figure 5.13(a), it is clear that the VMF technique

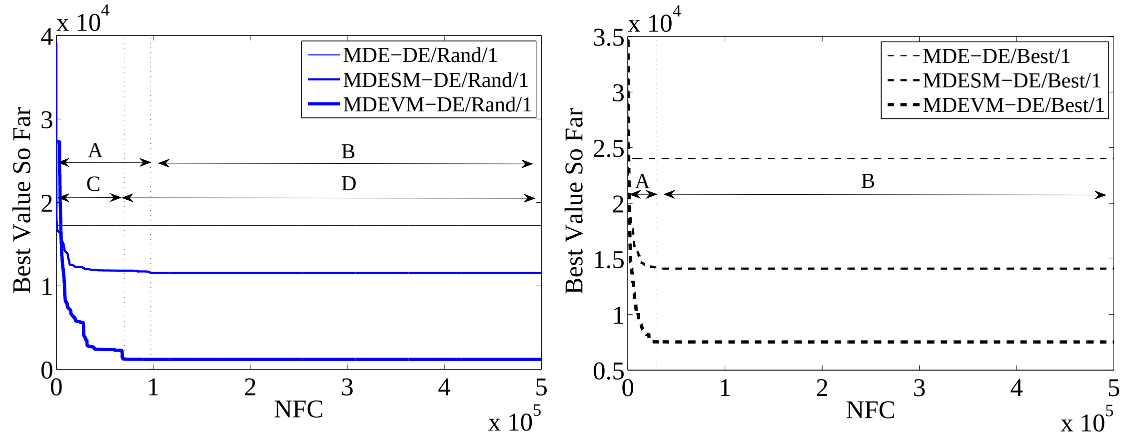


(a) DE/Rand/1 (b) DE/Best/1



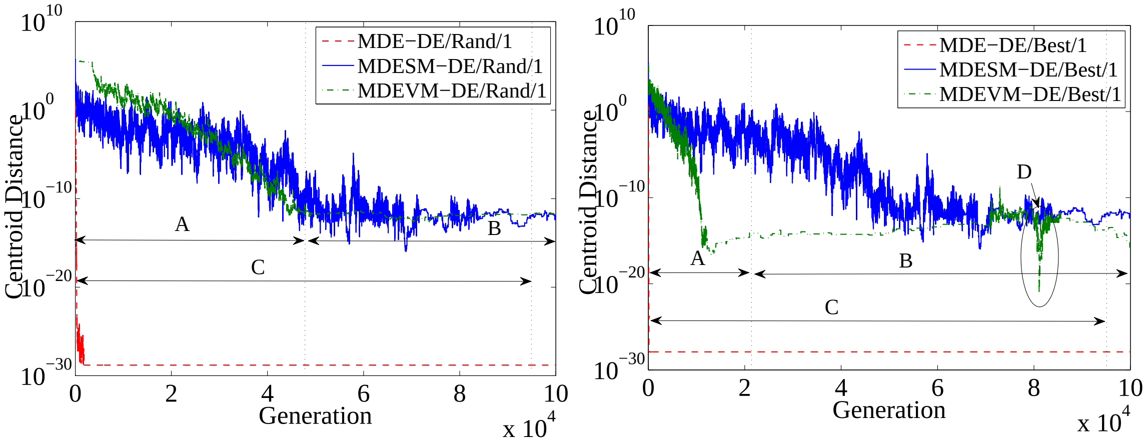
(c) DE/Rand/1 (d) DE/Best/1

Fig. 5.11: Performance comparison and population centroid distance analysis among the MDE, MDESM, and MDEVM schemes for the maximum number of function calls $NFC_{Max} = 5,000D$, dimension $D = 100$, population size $N_P = 5$, and DE/Rand/1 and DE/Best/1 mutation schemes for f_{20}.



(a) DE/Rand/1

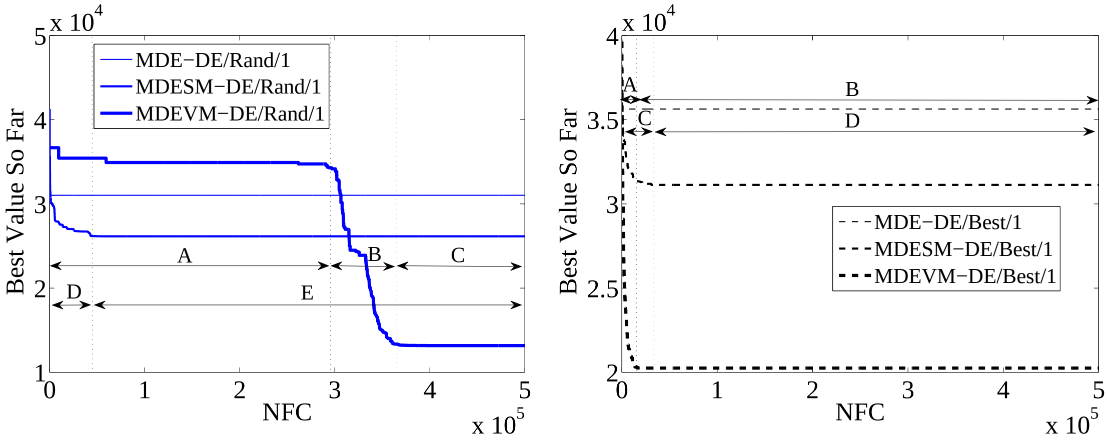
(b) DE/Best/1



(c) DE/Rand/1

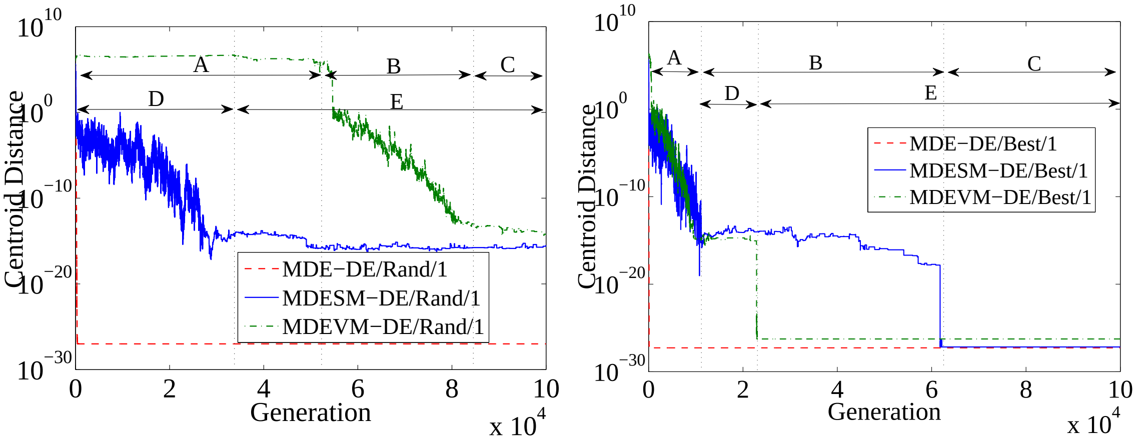
(d) DE/Best/1

Fig. 5.12: Performance comparison and population centroid distance analysis among the MDE, MDESM, and MDEVM schemes for the maximum number of function calls $NFC_{Max} = 5,000D$, dimension $D = 100$, population size $N_P = 5$, and DE/Rand/1 and DE/Best/1 mutation schemes for f_{21}.



(a) DE/Rand/1

(b) DE/Best/1



(c) DE/Rand/1

(d) DE/Best/1

Fig. 5.13: Performance comparison and population centroid distance analysis among the MDE, MDESM, and MDEVM schemes for the maximum number of function calls $NFC_{Max} = 5,000D$, dimension $D = 100$, population size $N_P = 5$, and DE/Rand/1 and DE/Best/1 mutation schemes for f_{22}.

has escaped the DE-algorithm from stagnation (denoted by "A") approximately at $NFC = 3e{+}5$ and with a sudden movement, as denoted by "B", it has reached a better performance than the other methods in "C". This is clearly shown in Figure 5.13(c), that the MDEVM algorithm is rescued from stagnation ("A") and gradually converging as shown in "B" and "C". This is while the MDE algorithm is completely trapped in a local minimum, since its best-value-so-far remains constant for all $NFCs$ and the population diversity is extremely low for all generations, i.e. almost $1e-28$ in Figure 5.13(c). The MDESM has tried to converge (part "D" of Figure 5.13(c)) to the solution as presented in part "E" of Figure 5.13(a). However, its exploration is stopped as shown in parts "E" and "E" of the Figure 5.13(a) and Figure 5.13(c), respectively, and no further improvements are achieved. For the "DE/Best/1" mutation scheme, the MDE is trapped in a local minimum similar to the "DE/Best/1" mutation scheme, as shown in Figure 5.13(b) and Figure 5.13(d). The MDESM has achieved better performance by converging its population toward a solution as denoted by "C" and "A" in Figure 5.13(b) and Figure 5.13(d), respectively. In further generations, although it has spent some time in generations denoted by "B" in Figure 5.13(d) to find a better solution, but it has been trapped finally in a local minimum as illustrated in part "C" of the Figure 5.13(d). The MDEVM has experienced the similar trend as the MDESM (parts "A", "D", and "E" for centroid distance in Figure 5.13(d)), but with better performance from part "A" toward part "B" of Figure 5.13(b).

The centroid diversity measure along the best-so-far-value analysis clearly have demonstrated performance of the MDE, MDESM, and MDEVM algorithms in stagnation and local optimum trapping scenarios. The results clearly indicate a successful performance of the VRM approach in delivering diversity into the population. Particularly that after some generation where the algorithm is trapped in local optimum or stagnated, it is rescued and moved toward better solutions, while the other algorithms could not survive.

5.6 Experimental Series 5: Number of Function Calls Analysis

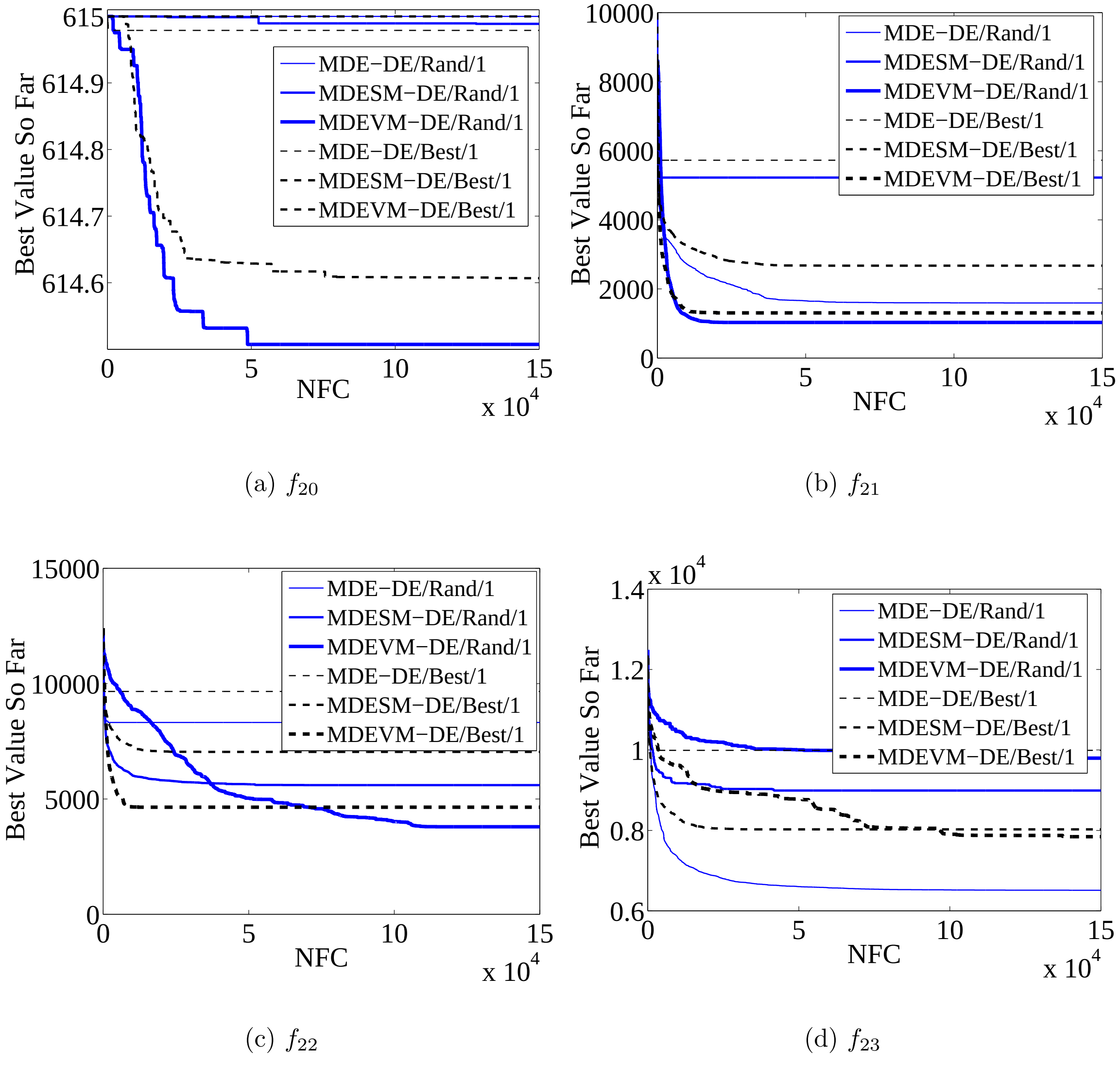
In order to analyse performance of the MDE, MDESM, and proposed MDEVM algorithms in term of solution accuracy by assigning a higher number of function calls than the previous experiments, i. e. $NFC_{Max} = 5,000D$, the algorithms are compared to solve the set of benchmark functions for $D = 30$ and $N_P = 5$. The best error, standard deviation, and Wilcoxon rank-sum results by considering the MDEVM as the reference algorithm are presented in Table A.21. Summary of the performance results are presented in Table 5.7. The comparison of number of "better", "equal", and "worse" performance results for $NFC_{Max} = 5,000D$ and $NFC_{Max} = 1,000D$ presented in Table 5.4 are shown using the arrows. The upward arrows demonstrate for how many more function the $NFC_{Max} = 5,000D$ setting has achieved better results. The results demonstrate that by allowing more number of function calls, the proposed MDEVM algorithm can obtain better performance than other algorithms. For the "DE/Best/1" mutation scheme as an example, the MDEVM algorithm in comparison with the MDE algorithm with $NFC_{Max} = 5,000D$ can achieve "better" results for more four benchmark functions than the $NFC_{Max} = 1,000D$ setting, i. e. increasing from 21 to 25 number of "+" results as stated in Table 5.7. The MDEVM also has one less failure versus the MDE for this mutation scheme. In case of MDEVM versus the MDESM, the MDEVM has achieved two more "better" records, from 20 to 22, and two less failure from 2 to zero. For the "DE/Rand/1" mutation scheme, the MDEVM has one more success than both the MDE and MDESM algorithms.

Best value so far of the MDE, MDESM, and MDEVM methods for the "DE/Rand/1" and "DE/Best/1" mutation schemes and the composite functions f_{20} to f_{27} with $D = 30$ and $NFC_{Max} = 5,000D$ is presented in Figures 5.14 and 5.15. The performance results show that by providing enough number of function calls to the algorithm, the MDEVM method due to its VMF technique is capable of performing better than the other algorithm,

Table 5.7: Summary of Wilcoxon rank-sum test comparisons (reference: MDEVM) for MDEVM against MDE and MDESM schemes on CEC 2013 benchmark functions and population size $N_P = 5$ for dimension $D = 30$ with $NFC_{Max} = 5,000D$ and mutation vector (MV) schemes “DE/Rand/1” and “DE/Best/1”. The number next to the upward arrows present additional number of success comparing with the $NFC_{Max} = 1,000D$ and the downward arrow represents the vice-versa.

MV	MDEVM/MDE			MDEVM/MDESM		
	+	=	-	+	=	-
DE/Rand/1	23 ↑1	3 ↓1	2 -0	18 ↑1	4 ↓1	6 -0
DE/Best/1	25 ↑4	2 ↓3	1 ↓1	22 ↑2	6 -0	0 ↓2

while they are stagnated or trapped. Particularly this is obvious in Figure 5.14(c) for f_{22} and in Figure 5.14(d) for f_{23} for mutation schemes “DE/Rand/1” and “DE/Best/1”, respectively.



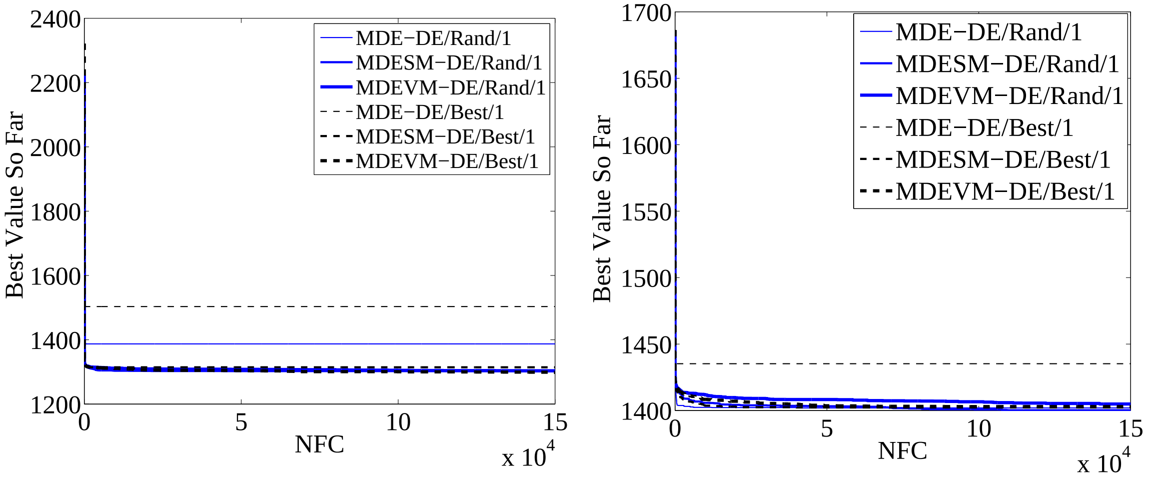
(a) f_{20}

(b) f_{21}

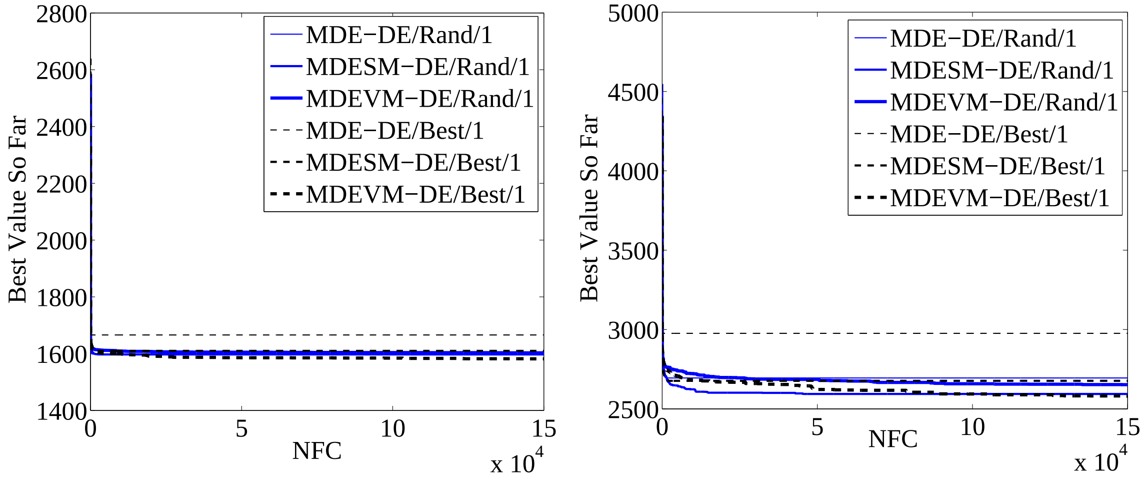
(c) f_{22}

(d) f_{23}

Fig. 5.14: Performance comparison among the MDE, MDESM, and MDEVM schemes for maximum number of function calls $NFC_{Max} = 5,000D$, dimension $D = 30$, population size $N_P = 5$, mutation schemes DE/Rand/1 and DE/Best/1, and benchmark functions $f_{20} - f_{23}$.



(a) f_{24} (b) f_{25}



(c) f_{26} (d) f_{27}

Fig. 5.15: Performance comparison among the MDE, MDESM, and MDEVM schemes for maximum number of function calls $NFC_{Max} = 5,000D$, dimension $D = 30$, population size $N_P = 5$, mutation schemes DE/Rand/1 and DE/Best/1, and benchmark functions $f_{24} - f_{27}$.

Chapter 6

Conclusion and Future Work

6.1 Contributions

In this thesis, we have proposed an enhanced version of the MDE algorithm based on the important capability of the mutation factor to provide diversity in the population, i.e. the micro-differential evolution using vectorized random mutation factor (MDEVM) algorithm. In this approach, in contrast to the standard micro DE, the mutation factor F is selected randomly for each variable of each individual in the population. In this case, the population can provide much higher diversity during the search process. In order to analyse the performance of the proposed MDEVM algorithm, we have conducted experiments for different schemes of the mutation factor in the MDE algorithm, which are constant mutation factor (standard DE-algorithm), scalar random mutation factor (randomized mutation factor for each individual), and the proposed vectorized random mutation factor (MDEVM algorithm). The simulation results clearly demonstrate performance superiority of the MDEVM algorithm.

6.2 Conclusion

In evolutionary algorithms (EAs), population size is critical in term of providing diversity into searching procedure. Particularly in the differential evolution (DE) algorithm, where correct selection of the mutation factor is likewise a crucial parameter in delivering diversity into the population. Normally large population sizes provide higher diversity with higher computational cost, which can provide less chance of stagnation and premature convergence due to its high exploration capability. Additionally, DE can generate a limited number of mutant vectors by using a constant mutation factor. The DE algorithm with small population size, micro-DE (MDE) algorithm, convergence to a solution is faster than standard DE algorithm. Yet, the chance of stagnation and premature convergence increases too. To avoid such situations, diversity should be increased while keeping the convergence speed of algorithm high. The crossover technique is one of the method to inject diversify into the population, where in conjunction with a better mutation scheme it can provide a higher diversity and possible faster finding of solution.

6.3 Future Directions

Since the population size of MDEVM is small, a basic parallel processing method can be proposed to evaluate each individual on one central processing unit (CPU), in case of a population size four for quad-core CPUs, using a shared memory. In order to design fast but reliable optimization algorithms to tackle with real-time applications, mostly in embedded systems, micro-algorithms can be one of the promising approaches. The proposed algorithm can be utilized in different real-world applications to facilitate fast but accurate computation such as in localization in wireless sensor networks, resource allocation in communication systems, and vehicle navigation.

Appendices

Appendix A

Comprehensive Tables of Results

Table A.1: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{14}$ and population size $N_P = 2$ for dimension $D = 50$ and mutation vector (MV) schemes “DE/Rand/1”, “DE/Best/1”, and “DE/T2B/1”.

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	1.73E+05	3.29E+04	1.96E+05	2.63E+04	=	1.72E+05	3.25E+04	=
	DE/Best/1	1.15E+05	2.07E+04	1.25E+05	2.66E+04	=	1.41E+05	3.30E+04	+
	DE/T2B/1	7.42E+04	2.42E+04	1.22E+05	3.99E+04	+	8.01E+04	3.28E+04	+
2	DE/Rand/1	6.18E+09	5.50E+09	6.99E+09	7.86E+09	=	4.20E+09	6.11E+09	=
	DE/Best/1	1.31E+09	2.65E+09	4.67E+09	4.53E+09	+	2.82E+09	6.06E+09	+
	DE/T2B/1	7.74E+08	9.61E+08	5.33E+09	6.05E+09	+	1.16E+09	2.73E+09	+
3	DE/Rand/1	6.91E+19	6.11E+34	3.47E+20	1.52E+29	=	4.29E+16	4.18E+29	=
	DE/Best/1	2.71E+13	2.04E+21	2.37E+16	2.43E+30	+	2.34E+14	1.62E+25	+
	DE/T2B/1	3.88E+12	1.29E+19	3.68E+18	1.51E+28	+	1.27E+14	3.89E+23	+
4	DE/Rand/1	4.70E+05	1.94E+07	2.56E+05	9.00E+06	=	5.33E+05	3.10E+09	=
	DE/Best/1	2.25E+05	1.77E+06	1.18E+05	7.48E+08	-	1.60E+05	7.10E+05	-
	DE/T2B/1	1.86E+05	3.10E+06	2.19E+05	9.50E+08	+	1.51E+05	4.82E+06	-
5	DE/Rand/1	8.04E+04	9.12E+04	1.30E+05	8.09E+04	=	1.13E+05	1.43E+05	=
	DE/Best/1	6.24E+04	4.84E+04	5.37E+04	1.24E+05	-	6.16E+04	1.14E+05	-
	DE/T2B/1	1.50E+04	5.49E+04	6.73E+04	1.22E+05	+	3.25E+04	1.47E+05	+
6	DE/Rand/1	4.02E+04	2.06E+04	3.01E+04	2.54E+04	=	2.54E+04	1.67E+04	=
	DE/Best/1	1.08E+04	1.25E+04	1.44E+04	1.44E+04	+	1.97E+04	1.88E+04	+
	DE/T2B/1	5.82E+03	7.67E+03	2.05E+04	1.85E+04	+	9.53E+03	1.42E+04	+
7	DE/Rand/1	2.54E+07	8.75E+12	3.86E+06	9.67E+11	-	2.25E+05	1.77E+14	=
	DE/Best/1	1.14E+04	3.94E+09	5.39E+05	6.24E+10	+	2.38E+06	2.65E+09	+
	DE/T2B/1	2.77E+02	3.45E+07	1.41E+06	3.66E+11	+	5.12E+04	1.77E+10	+
8	DE/Rand/1	2.12E+01	6.80E-02	2.11E+01	4.90E-02	-	2.12E+01	5.68E-02	=
	DE/Best/1	2.11E+01	4.09E-02	2.10E+01	1.03E-01	=	2.11E+01	4.53E-02	=
	DE/T2B/1	2.11E+01	3.72E-02	2.13E+01	8.86E-02	+	2.10E+01	4.73E-02	=
9	DE/Rand/1	7.09E+01	1.70E+00	7.29E+01	1.58E+00	=	7.25E+01	1.32E+00	+
	DE/Best/1	6.43E+01	3.48E+00	7.84E+01	4.04E+00	+	7.14E+01	4.65E+00	+
	DE/T2B/1	6.91E+01	2.13E+00	8.27E+01	3.64E+00	+	7.41E+01	4.78E+00	+
10	DE/Rand/1	2.51E+04	8.93E+03	2.56E+04	8.75E+03	=	2.62E+04	8.67E+03	=
	DE/Best/1	1.61E+04	7.65E+03	1.70E+04	9.72E+03	+	1.93E+04	8.87E+03	+
	DE/T2B/1	5.31E+03	4.76E+03	2.07E+04	9.08E+03	+	1.34E+04	6.41E+03	+
11	DE/Rand/1	2.67E+03	1.15E+03	2.80E+03	8.35E+02	=	2.25E+03	1.15E+03	=
	DE/Best/1	1.74E+03	6.97E+02	2.04E+03	8.52E+02	=	1.74E+03	7.63E+02	=
	DE/T2B/1	1.52E+03	6.18E+02	2.01E+03	8.86E+02	+	1.81E+03	5.07E+02	=
12	DE/Rand/1	2.51E+03	9.08E+02	2.41E+03	8.92E+02	=	2.32E+03	7.19E+02	=
	DE/Best/1	1.86E+03	4.10E+02	1.89E+03	6.22E+02	=	1.77E+03	5.73E+02	-
	DE/T2B/1	1.53E+03	4.85E+02	2.21E+03	8.47E+02	+	1.83E+03	3.41E+02	=
13	DE/Rand/1	2.16E+03	6.54E+02	1.92E+03	6.78E+02	=	2.21E+03	7.69E+02	=
	DE/Best/1	1.58E+03	6.24E+02	1.77E+03	7.76E+02	=	1.48E+03	5.91E+02	=
	DE/T2B/1	1.38E+03	5.43E+02	1.99E+03	6.27E+02	+	1.82E+03	5.42E+02	+
14	DE/Rand/1	1.70E+04	7.63E+02	1.56E+04	9.41E+02	=	1.50E+04	7.38E+02	-
	DE/Best/1	1.13E+04	1.24E+03	1.34E+04	1.05E+03	+	1.50E+04	9.61E+02	+
	DE/T2B/1	9.63E+03	1.08E+03	1.64E+04	1.07E+03	+	1.26E+04	1.24E+03	+

Table A.2: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{15} - f_{28}$ and population size $N_P = 2$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", and "DE/T2B/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
15	DE/Rand/1	1.72E+04	8.13E+02	1.57E+04	1.04E+03	-	1.51E+04	8.30E+02	-
	DE/Best/1	1.31E+04	8.46E+02	1.44E+04	1.11E+03	+	1.37E+04	1.07E+03	+
	DE/T2B/1	1.09E+04	2.02E+03	1.72E+04	7.72E+02	+	1.31E+04	9.46E+02	=
16	DE/Rand/1	3.23E+00	3.77E-01	3.30E+00	3.04E-01	=	3.46E+00	3.07E-01	+
	DE/Best/1	2.85E+00	4.27E-01	2.69E+00	1.27E+00	-	3.21E+00	5.08E-01	=
	DE/T2B/1	3.15E+00	3.13E-01	5.79E+00	1.47E+00	+	2.28E+00	6.60E-01	-
17	DE/Rand/1	5.04E+03	8.59E+02	5.82E+03	6.46E+02	=	5.71E+03	7.02E+02	=
	DE/Best/1	3.24E+03	7.76E+02	2.97E+03	8.91E+02	=	3.34E+03	7.73E+02	=
	DE/T2B/1	3.40E+03	8.16E+02	3.53E+03	1.11E+03	+	3.28E+03	8.80E+02	=
18	DE/Rand/1	5.07E+03	8.38E+02	5.01E+03	8.01E+02	=	4.42E+03	8.85E+02	=
	DE/Best/1	3.49E+03	8.68E+02	3.51E+03	6.43E+02	=	2.74E+03	1.06E+03	=
	DE/T2B/1	3.48E+03	7.40E+02	3.19E+03	1.19E+03	=	3.03E+03	1.04E+03	=
19	DE/Rand/1	1.52E+07	7.78E+07	9.17E+06	4.28E+07	=	1.93E+07	4.38E+07	=
	DE/Best/1	2.19E+06	3.87E+07	9.31E+06	3.92E+07	=	3.61E+06	4.73E+07	=
	DE/T2B/1	3.85E+06	1.78E+07	9.09E+06	4.11E+07	+	4.09E+06	3.23E+07	+
20	DE/Rand/1	2.50E+01	0.00E+00	2.50E+01	4.15E-14	=	2.50E+01	1.36E-05	=
	DE/Best/1	2.50E+01	0.00E+00	2.50E+01	0.00E+00	=	2.50E+01	0.00E+00	=
	DE/T2B/1	2.45E+01	8.81E-02	2.50E+01	0.00E+00	=	2.50E+01	4.46E-10	=
21	DE/Rand/1	1.46E+04	1.02E+05	1.24E+04	3.22E+06	=	1.07E+04	3.56E+07	=
	DE/Best/1	8.80E+03	2.27E+04	7.34E+03	4.20E+06	=	7.75E+03	2.51E+03	-
	DE/T2B/1	6.31E+03	6.24E+03	8.67E+03	4.29E+05	+	6.92E+03	2.70E+03	=
22	DE/Rand/1	1.75E+04	8.45E+02	1.81E+04	5.18E+02	=	1.62E+04	9.05E+02	-
	DE/Best/1	1.16E+04	1.34E+03	1.72E+04	7.25E+02	+	1.56E+04	1.04E+03	+
	DE/T2B/1	1.01E+04	1.27E+03	1.79E+04	7.78E+02	+	1.42E+04	1.19E+03	+
23	DE/Rand/1	1.72E+04	7.28E+02	1.62E+04	7.02E+02	-	1.54E+04	9.21E+02	-
	DE/Best/1	1.38E+04	9.50E+02	1.58E+04	7.69E+02	+	1.46E+04	9.95E+02	+
	DE/T2B/1	1.10E+04	1.93E+03	1.76E+04	6.55E+02	+	1.35E+04	1.19E+03	+
24	DE/Rand/1	4.20E+02	1.03E+03	4.20E+02	1.38E+03	=	3.97E+02	1.41E+03	-
	DE/Best/1	3.71E+02	2.95E+01	7.75E+02	8.30E+02	+	3.84E+02	1.04E+03	+
	DE/T2B/1	3.58E+02	1.08E+01	5.29E+02	7.53E+02	+	3.86E+02	9.88E+02	+
25	DE/Rand/1	3.97E+02	9.20E+00	3.93E+02	6.08E+00	-	3.88E+02	6.37E+00	-
	DE/Best/1	3.65E+02	7.87E+00	4.24E+02	8.55E+01	+	3.70E+02	1.56E+01	+
	DE/T2B/1	3.67E+02	5.33E+00	4.18E+02	1.36E+02	+	3.71E+02	1.40E+01	+
26	DE/Rand/1	5.19E+02	1.60E+03	5.08E+02	7.41E+03	=	4.80E+02	5.69E+03	=
	DE/Best/1	4.66E+02	7.35E+01	5.32E+02	1.40E+03	+	4.86E+02	1.34E+03	+
	DE/T2B/1	4.64E+02	9.36E+00	5.33E+02	2.10E+03	+	4.79E+02	1.03E+03	+
27	DE/Rand/1	2.47E+03	9.95E+01	2.37E+03	1.71E+03	=	2.23E+03	2.24E+03	-
	DE/Best/1	1.94E+03	1.27E+02	2.77E+03	2.73E+03	+	2.17E+03	2.86E+03	+
	DE/T2B/1	1.85E+03	9.38E+01	2.63E+03	3.42E+03	+	1.98E+03	4.54E+02	+
28	DE/Rand/1	1.54E+04	9.32E+04	1.78E+04	5.25E+04	=	1.45E+04	1.10E+05	=
	DE/Best/1	1.01E+04	2.58E+04	1.28E+04	1.14E+05	=	1.40E+04	4.30E+04	=
	DE/T2B/1	1.08E+04	1.62E+04	1.28E+04	7.25E+04	=	1.18E+04	4.25E+04	=

Table A.3: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{14}$ and population size $N_P = 3$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", and "DE/T2B/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	7.77E+01	1.12E+05	1.09E+05	2.37E+04	=	6.51E+04	2.54E+04	=
	DE/Best/1	2.84E+04	2.26E+04	1.12E+05	2.55E+04	+	1.07E+05	2.03E+04	+
	DE/T2B/1	7.36E+01	9.20E+04	8.99E+04	2.46E+04	=	2.48E+04	1.53E+04	+
2	DE/Rand/1	1.38E+09	1.79E+09	1.68E+09	3.28E+09	+	6.11E+08	1.36E+09	-
	DE/Best/1	1.35E+08	4.21E+08	3.21E+09	4.19E+09	+	1.31E+09	2.48E+09	+
	DE/T2B/1	2.79E+07	2.83E+09	2.30E+09	4.23E+09	+	1.13E+08	2.74E+08	=
3	DE/Rand/1	5.51E+11	7.28E+19	8.90E+13	5.32E+21	+	1.11E+12	6.58E+19	=
	DE/Best/1	2.77E+11	4.87E+15	2.48E+14	5.78E+26	+	1.84E+11	1.33E+21	-
	DE/T2B/1	1.42E+11	2.50E+22	1.08E+14	1.21E+26	=	1.01E+11	3.70E+13	-
4	DE/Rand/1	3.52E+05	3.07E+06	1.40E+05	1.60E+05	-	1.39E+05	1.14E+06	-
	DE/Best/1	1.44E+05	6.47E+05	1.49E+05	5.20E+07	+	1.84E+05	7.23E+05	=
	DE/T2B/1	2.20E+05	1.22E+06	1.37E+05	4.15E+05	-	8.66E+04	1.77E+05	-
5	DE/Rand/1	2.06E+02	1.44E+04	4.16E+04	5.81E+04	+	2.09E+04	6.72E+04	+
	DE/Best/1	6.68E+03	1.03E+05	3.08E+04	7.36E+04	+	4.33E+04	4.77E+04	+
	DE/T2B/1	5.21E+01	3.19E+04	1.82E+04	7.10E+04	+	1.08E+04	4.91E+04	+
6	DE/Rand/1	1.92E+02	1.96E+04	1.65E+04	9.51E+03	=	5.41E+03	1.04E+04	=
	DE/Best/1	3.15E+03	4.55E+03	1.68E+04	9.78E+03	+	1.18E+04	1.14E+04	+
	DE/T2B/1	2.70E+02	1.82E+04	1.34E+04	9.15E+03	+	9.39E+02	1.90E+03	+
7	DE/Rand/1	6.14E+02	1.54E+06	5.68E+04	8.04E+07	+	4.57E+03	5.72E+06	+
	DE/Best/1	5.59E+02	4.01E+06	9.91E+03	3.78E+11	+	3.42E+03	8.96E+07	+
	DE/T2B/1	3.11E+02	4.52E+06	1.33E+04	4.36E+08	+	1.80E+03	2.43E+06	=
8	DE/Rand/1	2.11E+01	3.73E-02	2.11E+01	3.72E-02	=	2.11E+01	5.19E-02	=
	DE/Best/1	2.12E+01	3.16E-02	2.11E+01	3.47E-02	=	2.11E+01	3.65E-02	=
	DE/T2B/1	2.11E+01	3.26E-02	2.11E+01	4.06E-02	=	2.11E+01	2.57E-02	=
9	DE/Rand/1	7.15E+01	1.57E+00	6.29E+01	4.22E+00	=	6.29E+01	3.84E+00	-
	DE/Best/1	5.48E+01	4.56E+00	7.50E+01	4.00E+00	+	7.01E+01	5.18E+00	+
	DE/T2B/1	7.16E+01	1.58E+00	7.07E+01	4.00E+00	-	6.38E+01	3.21E+00	-
10	DE/Rand/1	6.83E+02	1.29E+04	1.44E+04	9.16E+03	+	6.75E+03	5.56E+03	=
	DE/Best/1	2.45E+03	2.65E+03	1.72E+04	7.22E+03	+	1.12E+04	6.00E+03	+
	DE/T2B/1	5.81E+02	1.68E+04	1.55E+04	7.07E+03	=	2.36E+03	2.39E+03	=
11	DE/Rand/1	4.13E+02	9.43E+02	1.63E+03	5.46E+02	+	1.34E+03	6.16E+02	+
	DE/Best/1	1.31E+03	4.45E+02	1.38E+03	6.51E+02	+	1.73E+03	7.40E+02	+
	DE/T2B/1	8.49E+02	1.10E+03	1.46E+03	7.62E+02	=	1.19E+03	4.52E+02	+
12	DE/Rand/1	7.72E+02	1.06E+03	1.60E+03	3.10E+02	=	1.36E+03	5.29E+02	=
	DE/Best/1	1.22E+03	3.69E+02	1.79E+03	4.32E+02	+	1.60E+03	3.79E+02	+
	DE/T2B/1	1.22E+03	5.76E+02	1.63E+03	4.17E+02	+	1.22E+03	3.33E+02	+
13	DE/Rand/1	7.31E+02	1.08E+03	1.51E+03	4.41E+02	=	1.04E+03	3.97E+02	=
	DE/Best/1	1.16E+03	3.24E+02	1.45E+03	5.28E+02	+	1.51E+03	4.63E+02	+
	DE/T2B/1	1.28E+03	8.41E+02	1.52E+03	4.76E+02	+	1.45E+03	4.58E+02	+
14	DE/Rand/1	1.44E+04	1.00E+03	1.20E+04	1.12E+03	-	1.03E+04	1.00E+03	-
	DE/Best/1	7.84E+03	1.17E+03	1.44E+04	7.92E+02	+	1.35E+04	1.08E+03	+
	DE/T2B/1	5.52E+03	3.20E+03	1.38E+04	1.06E+03	+	9.06E+03	9.87E+02	=

Table A.4: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{15} - f_{28}$ and population size $N_P = 3$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", and "DE/T2B/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
15	DE/Rand/1	1.42E+04	7.28E+02	1.15E+04	1.18E+03	-	1.09E+04	1.04E+03	-
	DE/Best/1	9.41E+03	1.77E+03	1.40E+04	8.29E+02	+	1.32E+04	9.43E+02	=
	DE/T2B/1	8.11E+03	1.85E+03	1.32E+04	9.54E+02	=	9.69E+03	1.22E+03	+
16	DE/Rand/1	3.27E+00	2.89E-01	2.26E+00	5.05E-01	=	2.63E+00	4.09E-01	=
	DE/Best/1	3.35E+00	2.90E-01	2.65E+00	8.00E-01	=	2.39E+00	5.95E-01	-
	DE/T2B/1	2.66E+00	4.13E-01	2.27E+00	8.33E-01	=	2.92E+00	4.78E-01	+
17	DE/Rand/1	1.11E+03	1.57E+03	2.50E+03	7.68E+02	+	3.09E+03	8.78E+02	+
	DE/Best/1	2.12E+03	8.45E+02	3.07E+03	8.00E+02	+	3.52E+03	8.73E+02	+
	DE/T2B/1	1.30E+03	1.70E+03	2.37E+03	5.50E+02	+	2.56E+03	6.83E+02	+
18	DE/Rand/1	1.43E+03	1.39E+03	2.71E+03	6.78E+02	+	2.93E+03	8.00E+02	+
	DE/Best/1	2.69E+03	5.92E+02	3.22E+03	5.31E+02	=	3.20E+03	8.03E+02	+
	DE/T2B/1	1.85E+03	1.68E+03	2.51E+03	6.04E+02	+	2.21E+03	6.58E+02	+
19	DE/Rand/1	6.62E+03	4.58E+07	7.09E+06	2.12E+07	=	3.80E+06	1.73E+07	=
	DE/Best/1	8.68E+05	1.26E+07	5.68E+06	3.18E+07	+	1.61E+06	2.41E+07	+
	DE/T2B/1	7.05E+03	3.76E+07	6.54E+06	3.26E+07	+	4.24E+05	9.12E+06	+
20	DE/Rand/1	2.50E+01	5.05E-06	2.50E+01	5.33E-04	=	2.50E+01	0.00E+00	+
	DE/Best/1	2.50E+01	0.00E+00	2.50E+01	0.00E+00	=	2.47E+01	5.87E-02	=
	DE/T2B/1	2.50E+01	0.00E+00	2.50E+01	1.24E-10	=	2.45E+01	8.95E-02	=
21	DE/Rand/1	4.32E+02	5.19E+04	6.93E+03	2.30E+03	=	6.11E+03	3.22E+03	=
	DE/Best/1	4.49E+03	2.31E+03	6.35E+03	1.29E+04	+	7.47E+03	2.10E+03	+
	DE/T2B/1	4.06E+02	7.94E+03	6.84E+03	2.10E+03	+	3.83E+03	8.75E+02	+
22	DE/Rand/1	1.36E+04	1.08E+03	1.29E+04	1.27E+03	-	1.00E+04	1.27E+03	-
	DE/Best/1	8.11E+03	1.18E+03	1.55E+04	7.87E+02	+	1.35E+04	1.04E+03	+
	DE/T2B/1	5.20E+03	3.60E+03	1.52E+04	1.12E+03	+	9.74E+03	1.26E+03	=
23	DE/Rand/1	1.65E+04	6.57E+02	1.24E+04	1.09E+03	-	1.12E+04	1.06E+03	-
	DE/Best/1	1.00E+04	1.56E+03	1.58E+04	6.47E+02	+	1.48E+04	8.25E+02	+
	DE/T2B/1	1.36E+04	7.63E+02	1.42E+04	9.19E+02	+	1.05E+04	1.67E+03	-
24	DE/Rand/1	3.84E+02	6.07E+00	4.99E+02	5.83E+02	+	3.67E+02	2.01E+02	=
	DE/Best/1	3.65E+02	9.21E+00	4.50E+02	7.08E+02	+	3.95E+02	7.96E+02	+
	DE/T2B/1	3.81E+02	4.74E+00	4.14E+02	5.37E+02	+	3.59E+02	6.80E+01	=
25	DE/Rand/1	3.77E+02	6.43E+00	3.59E+02	3.91E+01	-	3.50E+02	9.81E+00	-
	DE/Best/1	3.53E+02	7.19E+00	4.02E+02	1.08E+02	+	3.77E+02	1.24E+01	=
	DE/T2B/1	3.80E+02	3.99E+00	3.91E+02	4.21E+01	+	3.54E+02	7.85E+00	-
26	DE/Rand/1	4.87E+02	5.80E+00	4.81E+02	6.72E+02	-	2.71E+02	2.94E+02	-
	DE/Best/1	4.49E+02	1.19E+01	5.00E+02	1.33E+03	+	3.09E+02	8.92E+02	-
	DE/T2B/1	2.05E+02	5.33E+01	4.89E+02	9.28E+02	+	2.16E+02	9.66E+01	=
27	DE/Rand/1	2.18E+03	5.03E+01	2.32E+03	1.19E+03	+	1.95E+03	2.79E+02	-
	DE/Best/1	1.90E+03	8.61E+01	2.51E+03	1.35E+03	+	2.05E+03	4.55E+02	+
	DE/T2B/1	2.09E+03	5.75E+01	2.22E+03	1.70E+03	+	1.94E+03	1.07E+02	-
28	DE/Rand/1	7.60E+02	1.34E+04	1.19E+04	6.39E+03	+	9.80E+03	1.41E+04	+
	DE/Best/1	9.60E+03	1.11E+04	1.21E+04	1.52E+04	+	1.11E+04	2.00E+04	+
	DE/T2B/1	5.31E+03	2.31E+04	1.20E+04	8.53E+03	=	1.10E+04	1.10E+04	=

Table A.5: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{14}$ and population size $N_P = 4$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", and "DE/Best/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	3.77E-04	8.97E+04	9.69E+04	2.28E+04	+	2.18E+04	1.65E+04	+
	DE/Best/1	5.53E+03	1.57E+04	8.56E+04	2.11E+04	+	4.14E+04	2.56E+04	+
	DE/T2B/1	3.05E-08	9.86E+02	9.78E+04	1.96E+04	+	5.50E+02	9.17E+03	+
	DE/Best/2	1.65E+05	2.65E+04	9.33E+04	2.37E+04	-	2.85E+03	1.14E+04	-
2	DE/Rand/1	5.55E+08	6.48E+08	1.89E+09	2.49E+09	+	1.53E+08	2.15E+08	-
	DE/Best/1	6.82E+07	9.45E+07	1.35E+09	2.59E+09	+	2.67E+08	4.25E+08	+
	DE/T2B/1	8.31E+06	8.39E+08	1.72E+09	2.10E+09	+	2.60E+07	4.49E+07	+
	DE/Best/2	5.11E+09	2.50E+09	1.25E+09	1.96E+09	-	2.87E+07	5.90E+07	-
3	DE/Rand/1	6.03E+10	4.32E+11	1.43E+12	2.68E+21	+	1.05E+11	1.98E+13	=
	DE/Best/1	1.40E+11	2.50E+11	4.52E+14	1.76E+20	+	4.06E+11	6.54E+17	+
	DE/T2B/1	1.28E+10	8.01E+10	9.99E+13	1.69E+21	+	3.21E+10	5.67E+10	+
	DE/Best/2	3.39E+12	1.50E+23	1.21E+12	1.04E+21	-	3.45E+10	1.29E+11	-
4	DE/Rand/1	3.88E+05	1.43E+06	1.23E+05	1.14E+05	-	7.74E+04	9.23E+05	-
	DE/Best/1	1.49E+05	3.90E+05	1.59E+05	1.26E+05	+	1.16E+05	3.82E+05	-
	DE/T2B/1	1.95E+05	1.33E+06	1.19E+05	4.77E+04	-	2.09E+04	1.55E+05	-
	DE/Best/2	3.31E+05	1.32E+07	1.64E+05	3.23E+05	-	3.57E+04	1.25E+05	-
5	DE/Rand/1	7.72E-02	4.99E+01	2.97E+04	3.95E+04	+	1.57E+04	3.72E+04	+
	DE/Best/1	6.76E+03	4.18E+04	3.63E+04	8.19E+04	+	1.55E+04	4.42E+04	+
	DE/T2B/1	9.11E-04	2.55E+02	3.07E+04	4.49E+04	+	1.19E+03	1.78E+04	+
	DE/Best/2	1.23E+04	2.55E+04	1.87E+04	7.34E+04	+	5.09E+03	3.43E+04	=
6	DE/Rand/1	4.05E+01	5.34E+01	1.01E+04	8.24E+03	+	1.40E+03	2.27E+03	+
	DE/Best/1	5.90E+02	1.23E+03	1.32E+04	9.32E+03	+	3.06E+03	5.31E+03	+
	DE/T2B/1	4.24E+01	7.74E+01	1.09E+04	7.93E+03	+	3.04E+02	3.35E+02	+
	DE/Best/2	2.89E+04	1.52E+04	7.25E+03	6.53E+03	-	3.79E+02	8.89E+02	-
7	DE/Rand/1	2.83E+02	2.52E+04	4.06E+03	6.80E+06	+	5.31E+02	3.75E+06	+
	DE/Best/1	4.32E+02	1.30E+05	4.91E+03	2.41E+07	+	8.82E+03	5.80E+06	+
	DE/T2B/1	2.01E+02	1.64E+02	5.90E+03	1.83E+07	+	1.40E+03	2.58E+05	+
	DE/Best/2	2.87E+03	3.26E+07	1.32E+03	1.12E+06	-	2.74E+02	2.57E+05	-
8	DE/Rand/1	2.11E+01	4.64E-02	2.11E+01	4.49E-02	=	2.10E+01	4.60E-02	=
	DE/Best/1	2.11E+01	3.49E-02	2.11E+01	6.17E-02	=	2.11E+01	3.79E-02	-
	DE/T2B/1	2.11E+01	3.95E-02	2.11E+01	4.00E-02	=	2.11E+01	4.79E-02	=
	DE/Best/2	2.12E+01	4.05E-02	2.11E+01	3.61E-02	-	2.11E+01	3.28E-02	-
9	DE/Rand/1	7.06E+01	1.72E+00	6.27E+01	3.78E+00	-	6.22E+01	3.58E+00	=
	DE/Best/1	6.57E+01	2.25E+00	6.73E+01	3.78E+00	+	6.79E+01	3.60E+00	=
	DE/T2B/1	7.03E+01	2.10E+00	6.99E+01	2.86E+00	=	6.54E+01	3.48E+00	-
	DE/Best/2	7.34E+01	1.42E+00	6.49E+01	3.46E+00	-	7.09E+01	1.83E+00	=
10	DE/Rand/1	1.62E+01	1.50E+03	1.15E+04	5.72E+03	+	1.88E+03	1.48E+03	+
	DE/Best/1	2.21E+03	1.57E+03	1.32E+04	6.91E+03	+	4.82E+03	3.77E+03	+
	DE/T2B/1	7.05E+00	1.74E+02	1.14E+04	4.82E+03	+	5.11E+02	6.59E+02	+
	DE/Best/2	1.95E+04	6.39E+03	1.27E+04	5.48E+03	-	9.32E+02	1.29E+03	-
11	DE/Rand/1	2.97E+02	1.71E+02	1.59E+03	5.08E+02	+	9.78E+02	4.25E+02	+
	DE/Best/1	1.19E+03	5.56E+02	1.49E+03	4.80E+02	+	1.24E+03	5.81E+02	+
	DE/T2B/1	3.50E+02	2.74E+02	1.46E+03	5.40E+02	+	8.41E+02	4.16E+02	+
	DE/Best/2	1.16E+03	8.45E+02	1.24E+03	4.05E+02	+	1.03E+03	4.16E+02	-
12	DE/Rand/1	4.01E+02	4.16E+02	1.26E+03	4.70E+02	+	1.14E+03	4.22E+02	+
	DE/Best/1	1.15E+03	3.62E+02	1.50E+03	4.70E+02	+	1.35E+03	4.76E+02	+
	DE/T2B/1	4.31E+02	2.30E+02	1.17E+03	3.45E+02	+	9.57E+02	2.11E+02	+
	DE/Best/2	1.99E+03	4.57E+02	1.20E+03	3.88E+02	-	1.16E+03	3.02E+02	-
13	DE/Rand/1	5.95E+02	3.54E+02	1.35E+03	3.77E+02	+	1.27E+03	4.46E+02	+
	DE/Best/1	1.06E+03	4.94E+02	1.33E+03	4.01E+02	=	1.21E+03	4.24E+02	+
	DE/T2B/1	7.81E+02	2.13E+02	1.13E+03	3.51E+02	+	1.11E+03	2.65E+02	+
	DE/Best/2	1.74E+03	4.75E+02	1.45E+03	2.75E+02	-	1.31E+03	3.04E+02	-
14	DE/Rand/1	9.72E+03	1.75E+03	1.12E+04	1.08E+03	+	7.74E+03	8.61E+02	-
	DE/Best/1	6.59E+03	9.64E+02	1.39E+04	8.44E+02	+	1.03E+04	1.10E+03	+
	DE/T2B/1	4.31E+03	3.25E+03	1.32E+04	9.59E+02	+	5.93E+03	1.01E+03	+
	DE/Best/2	1.50E+04	9.45E+02	1.18E+04	1.53E+03	-	7.20E+03	8.47E+02	-

Table A.6: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{15} - f_{28}$ and population size $N_P = 4$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", and "DE/Best/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
15	DE/Rand/1	1.43E+04	7.88E+02	1.26E+04	8.42E+02	-	8.61E+03	1.74E+03	-
	DE/Best/1	9.45E+03	1.53E+03	1.38E+04	8.99E+02	=	1.18E+04	1.31E+03	+
	DE/T2B/1	1.39E+04	6.82E+02	1.17E+04	8.50E+02	-	7.96E+03	1.96E+03	-
	DE/Best/2	1.58E+04	7.69E+02	1.18E+04	1.14E+03	-	8.12E+03	2.32E+03	-
16	DE/Rand/1	3.45E+00	2.93E-01	3.05E+00	4.05E-01	=	2.98E+00	3.99E-01	=
	DE/Best/1	3.19E+00	3.68E-01	2.32E+00	7.50E-01	=	2.61E+00	5.98E-01	=
	DE/T2B/1	3.41E+00	3.00E-01	2.66E+00	4.66E-01	=	2.90E+00	3.93E-01	-
	DE/Best/2	3.23E+00	3.25E-01	2.90E+00	4.80E-01	=	3.14E+00	3.77E-01	=
17	DE/Rand/1	7.51E+02	2.48E+03	2.17E+03	5.33E+02	+	2.00E+03	7.26E+02	+
	DE/Best/1	2.12E+03	6.89E+02	2.62E+03	7.45E+02	=	1.83E+03	8.99E+02	-
	DE/T2B/1	9.04E+02	4.01E+02	2.01E+03	5.40E+02	+	1.75E+03	6.00E+02	+
	DE/Best/2	4.91E+03	7.49E+02	1.74E+03	8.27E+02	-	2.24E+03	7.16E+02	-
18	DE/Rand/1	8.70E+02	2.82E+03	2.18E+03	5.52E+02	=	2.30E+03	6.26E+02	=
	DE/Best/1	2.42E+03	7.39E+02	2.68E+03	6.15E+02	=	3.07E+03	7.87E+02	+
	DE/T2B/1	9.23E+02	3.70E+02	1.97E+03	4.65E+02	+	1.56E+03	6.11E+02	+
	DE/Best/2	5.16E+03	6.38E+02	1.88E+03	8.15E+02	-	1.71E+03	8.97E+02	-
19	DE/Rand/1	1.34E+02	1.14E+07	1.50E+06	9.91E+06	+	1.02E+06	1.15E+07	+
	DE/Best/1	5.59E+05	3.84E+06	1.28E+06	1.69E+07	+	1.05E+06	1.59E+07	+
	DE/T2B/1	9.07E+01	4.21E+04	1.06E+06	2.29E+07	+	8.10E+04	1.25E+06	+
	DE/Best/2	7.84E+06	2.65E+07	1.53E+06	1.46E+07	-	4.71E+04	3.07E+06	-
20	DE/Rand/1	2.45E+01	1.48E-01	2.50E+01	3.53E-05	+	2.45E+01	1.44E-01	-
	DE/Best/1	2.45E+01	1.87E-01	2.50E+01	6.44E-05	=	2.50E+01	0.00E+00	+
	DE/T2B/1	2.45E+01	1.35E-01	2.45E+01	1.30E-01	-	2.43E+01	1.31E-01	-
	DE/Best/2	2.50E+01	0.00E+00	2.50E+01	0.00E+00	=	2.45E+01	9.13E-02	=
21	DE/Rand/1	1.01E+02	4.83E+02	6.86E+03	2.46E+03	+	3.71E+03	1.04E+03	+
	DE/Best/1	2.84E+03	8.87E+02	6.95E+03	2.16E+03	+	5.14E+03	1.24E+03	+
	DE/T2B/1	2.00E+02	3.70E+02	6.05E+03	2.27E+03	+	1.36E+03	7.02E+02	+
	DE/Best/2	6.49E+03	9.15E+03	6.39E+03	1.90E+03	-	2.36E+03	7.93E+02	-
22	DE/Rand/1	9.41E+03	2.20E+03	1.28E+04	1.49E+03	+	8.54E+03	1.31E+03	-
	DE/Best/1	5.42E+03	1.36E+03	1.57E+04	6.69E+02	+	1.27E+04	1.10E+03	+
	DE/T2B/1	4.44E+03	3.16E+03	1.32E+04	1.18E+03	+	7.19E+03	1.60E+03	+
	DE/Best/2	1.59E+04	7.08E+02	1.26E+04	1.71E+03	-	7.68E+03	1.10E+03	-
23	DE/Rand/1	1.56E+04	5.43E+02	1.35E+04	1.05E+03	-	1.01E+04	1.75E+03	-
	DE/Best/1	8.95E+03	2.27E+03	1.51E+04	7.99E+02	+	1.18E+04	1.34E+03	=
	DE/T2B/1	1.50E+04	6.33E+02	1.45E+04	7.92E+02	=	9.31E+03	1.76E+03	-
	DE/Best/2	1.66E+04	6.29E+02	1.24E+04	1.21E+03	-	8.93E+03	2.19E+03	-
24	DE/Rand/1	3.90E+02	3.13E+00	4.10E+02	2.61E+02	+	3.67E+02	3.25E+02	-
	DE/Best/1	3.61E+02	9.04E+00	4.12E+02	5.18E+02	+	3.79E+02	4.76E+02	+
	DE/T2B/1	3.84E+02	5.07E+00	3.99E+02	4.63E+02	+	3.52E+02	2.34E+02	-
	DE/Best/2	3.91E+02	7.21E+00	3.76E+02	2.64E+02	-	3.60E+02	8.67E+00	-
25	DE/Rand/1	3.75E+02	5.21E+00	3.78E+02	2.54E+01	+	3.73E+02	6.50E+00	-
	DE/Best/1	3.79E+02	4.46E+00	3.90E+02	3.40E+01	+	3.73E+02	7.40E+00	=
	DE/T2B/1	3.81E+02	4.67E+00	3.71E+02	3.15E+01	-	3.64E+02	7.79E+00	-
	DE/Best/2	3.85E+02	6.15E+00	3.62E+02	1.09E+01	-	3.76E+02	4.90E+00	-
26	DE/Rand/1	4.85E+02	4.65E+00	3.86E+02	4.26E+02	=	2.11E+02	9.61E+01	-
	DE/Best/1	4.47E+02	1.52E+01	4.92E+02	8.59E+02	+	2.35E+02	5.04E+02	-
	DE/T2B/1	2.07E+02	5.24E+01	3.56E+02	5.49E+02	+	2.04E+02	9.22E+01	-
	DE/Best/2	4.86E+02	7.77E+00	3.82E+02	8.74E+02	-	2.09E+02	3.41E+02	-
27	DE/Rand/1	2.13E+03	4.61E+01	2.32E+03	5.41E+02	+	1.81E+03	1.06E+02	-
	DE/Best/1	2.14E+03	3.89E+01	2.45E+03	1.23E+03	+	1.94E+03	1.10E+02	=
	DE/T2B/1	2.09E+03	5.12E+01	2.15E+03	1.56E+03	+	1.98E+03	6.30E+01	-
	DE/Best/2	2.24E+03	7.01E+01	2.03E+03	3.57E+02	=	1.89E+03	9.61E+01	-
28	DE/Rand/1	4.03E+02	2.06E+03	1.17E+04	4.27E+03	+	1.03E+04	4.76E+03	+
	DE/Best/1	5.12E+03	5.98E+03	1.03E+04	3.94E+03	+	8.79E+03	4.56E+03	+
	DE/T2B/1	4.21E+02	2.79E+03	1.03E+04	2.70E+03	+	8.41E+03	7.71E+03	+
	DE/Best/2	1.34E+04	4.49E+04	9.44E+03	1.11E+04	-	9.46E+03	4.24E+03	-

Table A.7: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{10}$ and population size $N_P = 5$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2" and "DE/Rand/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	1.03E-02	6.74E+04	8.34E+04	1.90E+04	+	1.90E+03	1.01E+04	+
	DE/Best/1	2.91E+01	8.95E+03	7.79E+04	2.41E+04	+	1.06E+04	1.65E+04	+
	DE/T2B/1	8.93E-09	3.78E-03	7.55E+04	1.90E+04	+	2.47E+01	3.82E+03	+
	DE/Best/2	6.80E+01	1.02E+05	6.65E+04	1.87E+04	+	4.88E+01	6.45E+03	=
	DE/Rand/2	1.51E+05	2.70E+04	4.11E+04	1.48E+04	-	2.04E+01	2.99E+03	-
2	DE/Rand/1	2.15E+07	2.94E+07	7.54E+08	1.56E+09	+	3.87E+07	4.74E+07	+
	DE/Best/1	2.04E+07	3.07E+07	1.86E+09	2.57E+09	+	1.09E+08	1.70E+08	+
	DE/T2B/1	1.45E+07	1.64E+08	1.70E+09	2.70E+09	+	1.83E+07	2.83E+07	=
	DE/Best/2	1.31E+09	1.04E+09	7.10E+08	1.27E+09	-	1.59E+07	2.75E+07	-
	DE/Rand/2	4.80E+09	2.71E+09	2.54E+08	4.23E+08	-	9.56E+06	2.26E+07	-
3	DE/Rand/1	2.42E+10	2.77E+11	4.00E+12	3.05E+18	+	4.86E+10	1.07E+11	=
	DE/Best/1	2.45E+10	1.37E+11	2.43E+13	4.45E+20	+	7.63E+10	2.59E+11	+
	DE/T2B/1	9.57E+09	7.56E+10	8.55E+12	4.58E+20	+	2.72E+10	1.90E+10	=
	DE/Best/2	8.76E+11	4.22E+18	3.22E+12	3.59E+18	+	1.84E+10	5.36E+10	-
	DE/Rand/2	1.33E+14	2.30E+24	2.07E+11	1.01E+15	-	3.19E+10	3.88E+10	-
4	DE/Rand/1	2.81E+05	2.00E+06	1.31E+05	1.36E+05	-	5.63E+04	2.68E+05	-
	DE/Best/1	1.16E+05	1.94E+06	1.27E+05	1.59E+05	+	7.04E+04	3.08E+05	-
	DE/T2B/1	2.05E+05	8.02E+05	1.14E+05	5.40E+04	-	1.65E+04	6.79E+04	-
	DE/Best/2	1.04E+06	1.83E+07	1.16E+05	5.13E+05	-	2.07E+04	1.19E+05	-
	DE/Rand/2	2.32E+05	1.42E+07	9.55E+04	1.40E+05	-	6.10E+04	1.53E+05	-
5	DE/Rand/1	1.25E-01	1.32E+02	1.90E+04	3.21E+04	+	1.09E+04	3.77E+04	+
	DE/Best/1	2.77E+02	1.26E+04	3.48E+04	3.55E+04	+	1.59E+04	3.34E+04	+
	DE/T2B/1	1.41E-04	1.69E-01	1.55E+04	3.36E+04	+	1.73E+02	2.99E+03	+
	DE/Best/2	1.04E+02	1.06E+03	1.92E+04	3.65E+04	+	1.14E+03	3.24E+04	+
	DE/Rand/2	6.67E+04	2.91E+04	1.65E+04	1.96E+04	-	1.26E+02	9.06E+03	-
6	DE/Rand/1	4.34E+01	4.85E+01	9.58E+03	6.31E+03	+	4.09E+02	3.58E+02	+
	DE/Best/1	1.25E+02	2.79E+02	1.03E+04	7.21E+03	+	9.12E+02	1.57E+03	+
	DE/T2B/1	4.29E+01	3.30E+01	8.63E+03	5.92E+03	+	1.28E+02	1.45E+02	+
	DE/Best/2	7.36E+01	9.49E+03	7.85E+03	4.58E+03	+	1.21E+02	2.19E+02	=
	DE/Rand/2	2.58E+04	1.35E+04	2.53E+03	2.48E+03	-	1.07E+02	1.49E+02	-
7	DE/Rand/1	1.92E+02	2.04E+02	3.86E+02	2.25E+05	+	6.23E+02	2.14E+05	+
	DE/Best/1	2.77E+02	2.10E+04	2.53E+03	6.97E+06	+	3.02E+03	2.10E+07	+
	DE/T2B/1	1.85E+02	1.09E+02	1.09E+04	5.13E+07	+	2.59E+02	1.04E+05	+
	DE/Best/2	6.83E+02	1.08E+05	9.06E+02	8.13E+05	+	2.90E+02	2.07E+05	-
	DE/Rand/2	1.86E+04	2.99E+09	2.00E+02	4.89E+04	-	1.72E+02	5.48E+03	-
8	DE/Rand/1	2.11E+01	5.80E-02	2.11E+01	3.49E-02	+	2.11E+01	3.67E-02	=
	DE/Best/1	2.11E+01	3.59E-02	2.11E+01	3.16E-02	=	2.11E+01	3.21E-02	=
	DE/T2B/1	2.11E+01	4.17E-02	2.11E+01	3.77E-02	=	2.11E+01	3.33E-02	=
	DE/Best/2	2.11E+01	6.61E-02	2.11E+01	4.14E-02	+	2.11E+01	4.79E-02	=
	DE/Rand/2	2.12E+01	3.84E-02	2.11E+01	4.08E-02	-	2.11E+01	5.55E-02	-
9	DE/Rand/1	7.23E+01	1.38E+00	6.55E+01	3.90E+00	-	6.82E+01	2.36E+00	=
	DE/Best/1	6.52E+01	3.01E+00	7.11E+01	3.73E+00	+	6.97E+01	3.40E+00	=
	DE/T2B/1	7.30E+01	1.47E+00	6.49E+01	3.16E+00	-	6.62E+01	3.24E+00	-
	DE/Best/2	7.11E+01	1.69E+00	6.02E+01	3.87E+00	-	6.73E+01	2.30E+00	-
	DE/Rand/2	7.28E+01	1.41E+00	6.48E+01	2.49E+00	-	7.34E+01	1.24E+00	+
10	DE/Rand/1	1.09E+01	1.55E+02	1.34E+04	4.21E+03	+	8.51E+02	8.03E+02	+
	DE/Best/1	2.34E+02	7.58E+02	1.08E+04	5.94E+03	+	2.03E+03	2.09E+03	+
	DE/T2B/1	2.49E+00	6.27E+02	9.49E+03	4.06E+03	+	2.38E+02	3.80E+02	+
	DE/Best/2	6.65E+02	3.85E+03	8.08E+03	4.43E+03	+	2.63E+02	3.65E+02	-
	DE/Rand/2	2.65E+04	5.11E+03	4.09E+03	2.95E+03	-	1.95E+02	4.73E+02	-

Table A.8: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{11} - f_{20}$ and population size $N_P = 5$ for dimension $D = 50$ and mutation vector (MV) schemes “DE/Rand/1”, “DE/Best/1”, “DE/T2B/1”, “DE/Best/2” and “DE/Rand/2”.

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
11	DE/Rand/1	2.84E+02	2.88E+02	1.40E+03	3.77E+02	+	1.03E+03	4.02E+02	+
	DE/Best/1	9.72E+02	3.63E+02	1.22E+03	4.44E+02	+	9.15E+02	5.27E+02	-
	DE/T2B/1	3.00E+02	1.74E+02	1.17E+03	3.47E+02	+	4.94E+02	2.92E+02	+
	DE/Best/2	3.77E+02	1.14E+02	1.14E+03	4.41E+02	+	7.88E+02	3.67E+02	+
	DE/Rand/2	2.57E+03	5.78E+02	6.12E+02	2.71E+02	-	7.34E+02	2.83E+02	-
12	DE/Rand/1	2.75E+02	2.27E+02	1.32E+03	3.82E+02	+	9.93E+02	3.41E+02	+
	DE/Best/1	1.01E+03	3.24E+02	1.21E+03	4.17E+02	+	1.27E+03	3.85E+02	+
	DE/T2B/1	3.25E+02	1.57E+02	1.10E+03	3.96E+02	+	7.80E+02	2.38E+02	+
	DE/Best/2	6.02E+02	9.58E+02	1.26E+03	2.54E+02	+	8.98E+02	3.67E+02	+
	DE/Rand/2	2.30E+03	4.37E+02	9.98E+02	2.06E+02	-	7.22E+02	2.89E+02	-
13	DE/Rand/1	5.78E+02	1.88E+02	9.64E+02	2.97E+02	+	1.21E+03	2.42E+02	+
	DE/Best/1	9.30E+02	2.94E+02	1.28E+03	3.45E+02	+	1.38E+03	3.95E+02	+
	DE/T2B/1	5.20E+02	1.41E+02	1.32E+03	2.13E+02	+	8.49E+02	2.62E+02	+
	DE/Best/2	6.14E+02	6.73E+02	1.14E+03	2.83E+02	+	9.43E+02	2.68E+02	+
	DE/Rand/2	2.21E+03	4.85E+02	8.61E+02	2.14E+02	-	6.82E+02	3.13E+02	-
14	DE/Rand/1	4.20E+03	4.05E+03	1.08E+04	9.62E+02	+	6.50E+03	1.14E+03	=
	DE/Best/1	5.11E+03	9.32E+02	1.31E+04	8.07E+02	+	8.96E+03	1.01E+03	+
	DE/T2B/1	5.77E+03	3.36E+03	1.18E+04	9.44E+02	+	5.48E+03	9.65E+02	-
	DE/Best/2	1.52E+04	5.73E+02	1.05E+04	1.49E+03	-	5.88E+03	9.23E+02	-
	DE/Rand/2	1.62E+04	6.85E+02	6.76E+03	2.52E+03	-	4.73E+03	1.09E+03	-
15	DE/Rand/1	1.54E+04	4.76E+02	1.24E+04	8.31E+02	-	7.41E+03	1.19E+03	-
	DE/Best/1	1.47E+04	4.58E+02	1.40E+04	7.25E+02	-	1.01E+04	1.31E+03	-
	DE/T2B/1	1.50E+04	5.45E+02	1.27E+04	8.62E+02	-	8.21E+03	2.09E+03	-
	DE/Best/2	1.55E+04	6.94E+02	1.10E+04	1.38E+03	-	7.40E+03	1.82E+03	-
	DE/Rand/2	1.62E+04	8.12E+02	1.37E+04	5.60E+02	-	9.17E+03	1.36E+03	-
16	DE/Rand/1	3.56E+00	2.41E-01	3.01E+00	3.97E-01	-	3.28E+00	3.36E-01	=
	DE/Best/1	2.99E+00	3.46E-01	1.94E+00	7.73E-01	-	2.89E+00	3.91E-01	=
	DE/T2B/1	3.34E+00	3.24E-01	2.51E+00	4.27E-01	=	3.24E+00	3.67E-01	=
	DE/Best/2	3.18E+00	3.16E-01	3.43E+00	2.98E-01	=	3.54E+00	3.19E-01	=
	DE/Rand/2	3.33E+00	3.28E-01	2.67E+00	4.17E-01	=	2.88E+00	4.41E-01	=
17	DE/Rand/1	4.68E+02	2.59E+03	2.00E+03	6.52E+02	=	1.56E+03	6.70E+02	=
	DE/Best/1	1.71E+03	5.92E+02	2.10E+03	6.76E+02	+	2.21E+03	6.89E+02	+
	DE/T2B/1	4.83E+02	2.63E+02	1.95E+03	4.32E+02	+	1.12E+03	4.79E+02	+
	DE/Best/2	6.81E+02	2.74E+03	2.27E+03	5.32E+02	=	2.07E+03	4.66E+02	=
	DE/Rand/2	5.00E+03	6.52E+02	1.47E+03	3.75E+02	-	1.39E+03	6.21E+02	-
18	DE/Rand/1	1.03E+03	2.55E+03	2.41E+03	4.68E+02	=	2.15E+03	6.00E+02	=
	DE/Best/1	1.71E+03	5.50E+02	2.24E+03	5.22E+02	+	1.57E+03	7.99E+02	-
	DE/T2B/1	5.86E+02	2.86E+02	1.98E+03	4.49E+02	+	1.49E+03	4.49E+02	+
	DE/Best/2	6.89E+02	2.67E+03	2.25E+03	4.71E+02	=	1.30E+03	5.46E+02	=
	DE/Rand/2	5.41E+03	5.75E+02	1.36E+03	9.43E+02	-	1.15E+03	5.46E+02	-
19	DE/Rand/1	1.25E+02	1.19E+07	1.94E+06	1.49E+07	+	9.26E+04	5.81E+06	+
	DE/Best/1	1.56E+03	2.04E+06	2.30E+06	1.31E+07	+	7.44E+05	1.24E+07	+
	DE/T2B/1	2.27E+01	1.92E+02	1.19E+06	8.39E+06	+	1.48E+03	4.35E+05	+
	DE/Best/2	4.40E+03	4.27E+07	1.06E+06	8.34E+06	=	6.28E+03	1.48E+06	+
	DE/Rand/2	1.68E+07	3.12E+07	5.23E+05	6.01E+06	-	2.04E+03	9.86E+06	-
20	DE/Rand/1	2.49E+01	2.31E-02	2.50E+01	0.00E+00	+	2.45E+01	2.12E-01	-
	DE/Best/1	2.45E+01	2.36E-01	2.50E+01	0.00E+00	+	2.45E+01	1.21E-01	+
	DE/T2B/1	2.45E+01	1.38E-01	2.45E+01	2.02E-01	-	2.50E+01	8.33E-03	+
	DE/Best/2	2.50E+01	0.00E+00	2.40E+01	1.82E-01	=	2.45E+01	1.84E-01	-
	DE/Rand/2	2.50E+01	0.00E+00	2.45E+01	1.98E-01	-	2.45E+01	1.98E-01	-

Table A.9: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{21} - f_{28}$ and population size $N_P = 5$ for dimension $D = 50$ and mutation vector (MV) schemes “DE/Rand/1”, “DE/Best/1”, “DE/T2B/1”, “DE/Best/2” and “DE/Rand/2”.

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
21	DE/Rand/1	2.02E+02	4.82E+02	6.54E+03	1.39E+03	+	2.02E+03	7.16E+02	+
	DE/Best/1	5.55E+02	7.76E+02	6.52E+03	1.97E+03	+	3.13E+03	9.86E+02	+
	DE/T2B/1	2.00E+02	3.43E+02	5.60E+03	1.28E+03	+	3.41E+02	6.35E+02	+
	DE/Best/2	3.11E+02	3.36E+03	5.48E+03	1.22E+03	+	9.19E+02	8.31E+02	=
	DE/Rand/2	9.82E+03	9.31E+03	4.91E+03	9.31E+02	-	4.57E+02	6.12E+02	-
22	DE/Rand/1	4.82E+03	4.44E+03	1.23E+04	1.12E+03	+	7.32E+03	1.16E+03	=
	DE/Best/1	4.76E+03	1.06E+03	1.45E+04	8.94E+02	+	1.05E+04	1.03E+03	+
	DE/T2B/1	5.54E+03	3.05E+03	1.14E+04	1.27E+03	+	6.10E+03	1.27E+03	+
	DE/Best/2	1.52E+04	8.29E+02	1.11E+04	1.21E+03	-	6.51E+03	1.63E+03	-
	DE/Rand/2	1.67E+04	5.75E+02	8.22E+03	2.51E+03	-	5.39E+03	7.40E+02	-
23	DE/Rand/1	1.52E+04	6.18E+02	1.27E+04	8.51E+02	-	8.37E+03	1.84E+03	-
	DE/Best/1	1.48E+04	5.27E+02	1.48E+04	7.96E+02	=	9.77E+03	1.88E+03	-
	DE/T2B/1	1.55E+04	5.46E+02	1.36E+04	8.42E+02	-	6.89E+03	2.38E+03	-
	DE/Best/2	1.63E+04	5.72E+02	1.21E+04	1.26E+03	-	8.49E+03	1.63E+03	-
	DE/Rand/2	1.61E+04	6.82E+02	1.41E+04	5.25E+02	-	7.81E+03	1.90E+03	-
24	DE/Rand/1	3.86E+02	4.36E+00	3.90E+02	2.24E+02	+	3.65E+02	1.01E+01	-
	DE/Best/1	3.64E+02	6.23E+00	5.03E+02	4.32E+02	+	3.73E+02	2.12E+02	=
	DE/T2B/1	3.82E+02	4.55E+00	4.28E+02	4.48E+02	+	3.55E+02	7.75E+00	-
	DE/Best/2	3.89E+02	5.03E+00	3.72E+02	7.43E+01	=	3.59E+02	7.83E+00	-
	DE/Rand/2	4.00E+02	5.48E+00	3.52E+02	1.96E+02	-	3.69E+02	6.29E+00	-
25	DE/Rand/1	3.85E+02	3.88E+00	3.63E+02	1.23E+01	-	3.73E+02	4.82E+00	-
	DE/Best/1	3.83E+02	3.51E+00	3.92E+02	3.02E+01	+	3.72E+02	7.53E+00	=
	DE/T2B/1	3.86E+02	4.19E+00	3.67E+02	2.24E+01	-	3.78E+02	4.29E+00	-
	DE/Best/2	3.87E+02	5.07E+00	3.56E+02	9.17E+00	-	3.80E+02	4.20E+00	-
	DE/Rand/2	3.96E+02	6.05E+00	3.62E+02	5.63E+00	-	3.82E+02	3.71E+00	-
26	DE/Rand/1	4.90E+02	4.28E+00	4.59E+02	5.63E+01	-	2.10E+02	5.02E+01	-
	DE/Best/1	2.04E+02	7.28E+01	4.87E+02	7.23E+02	+	2.09E+02	9.15E+01	=
	DE/T2B/1	4.80E+02	5.02E+00	4.34E+02	3.01E+02	-	2.04E+02	8.32E+01	-
	DE/Best/2	4.93E+02	4.62E+00	3.49E+02	2.00E+02	-	2.02E+02	1.05E+02	-
	DE/Rand/2	5.01E+02	7.36E+00	2.80E+02	3.93E+01	-	4.51E+02	1.02E+01	-
27	DE/Rand/1	2.04E+03	5.54E+01	2.16E+03	2.26E+02	+	1.98E+03	8.64E+01	-
	DE/Best/1	2.11E+03	3.33E+01	2.33E+03	1.06E+03	+	2.02E+03	8.13E+01	=
	DE/T2B/1	2.17E+03	4.28E+01	2.36E+03	8.92E+02	+	1.94E+03	7.65E+01	-
	DE/Best/2	2.19E+03	4.41E+01	1.96E+03	2.23E+02	-	1.84E+03	9.22E+01	-
	DE/Rand/2	2.32E+03	6.31E+01	1.68E+03	1.27E+02	-	2.07E+03	3.37E+01	-
28	DE/Rand/1	4.35E+02	3.05E+03	1.00E+04	9.63E+03	+	7.42E+03	2.26E+03	+
	DE/Best/1	2.86E+03	3.61E+03	1.05E+04	3.36E+03	+	1.05E+04	1.04E+04	+
	DE/T2B/1	4.00E+02	2.27E+03	1.02E+04	1.64E+03	+	7.98E+03	2.10E+03	+
	DE/Best/2	4.79E+02	2.74E+03	9.51E+03	4.52E+03	+	3.86E+03	2.80E+03	+
	DE/Rand/2	1.58E+04	1.31E+04	7.72E+03	2.44E+03	-	4.14E+03	2.97E+03	-

Table A.10: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{10}$ and population size $N_P = 6$ for dimension $D = 50$ and mutation vector (MV) schemes “DE/Rand/1”, “DE/Best/1”, “DE/T2B/1”, “DE/Best/2” and “DE/Rand/2”.

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	1.66E-02	4.95E+04	7.86E+04	1.89E+04	+	4.19E+02	5.28E+03	+
	DE/Best/1	1.02E-01	2.72E+03	7.32E+04	2.37E+04	+	1.33E+03	1.24E+04	+
	DE/T2B/1	1.27E-08	1.17E-04	7.41E+04	1.53E+04	+	1.69E+00	1.04E+03	+
	DE/Best/2	4.86E+01	1.05E+05	6.53E+04	1.50E+04	=	5.45E-01	1.58E+03	-
	DE/Rand/2	1.97E+04	4.13E+04	1.92E+04	1.18E+04	-	2.82E+00	1.90E+03	-
2	DE/Rand/1	2.04E+07	6.06E+07	7.89E+08	1.05E+09	+	1.95E+07	3.85E+07	=
	DE/Best/1	6.07E+06	2.47E+07	6.28E+08	2.19E+09	+	4.68E+07	5.59E+07	+
	DE/T2B/1	1.54E+07	2.10E+08	8.87E+08	1.69E+09	+	8.83E+06	1.58E+07	-
	DE/Best/2	1.60E+08	1.63E+09	6.85E+08	4.64E+08	=	1.62E+07	2.41E+07	-
	DE/Rand/2	3.09E+09	1.59E+09	1.05E+08	1.21E+08	-	1.28E+07	1.67E+07	-
3	DE/Rand/1	3.65E+10	4.45E+11	4.74E+12	1.55E+17	+	3.39E+10	3.23E+10	-
	DE/Best/1	8.45E+09	5.02E+10	1.03E+12	2.67E+19	+	4.62E+10	7.39E+10	+
	DE/T2B/1	1.37E+10	1.24E+11	4.23E+13	9.56E+18	+	9.47E+09	2.59E+10	-
	DE/Best/2	2.28E+11	7.54E+18	1.06E+12	5.87E+17	+	2.01E+10	4.06E+10	-
	DE/Rand/2	1.49E+13	1.03E+21	7.88E+10	5.76E+13	-	8.29E+09	2.93E+10	-
4	DE/Rand/1	2.88E+05	2.65E+06	1.01E+05	9.59E+04	-	4.01E+04	1.29E+05	-
	DE/Best/1	1.07E+05	6.60E+05	1.00E+05	1.37E+05	-	3.03E+04	4.25E+04	-
	DE/T2B/1	2.45E+05	1.30E+06	1.27E+05	7.70E+04	-	1.29E+04	5.82E+04	-
	DE/Best/2	6.64E+05	6.75E+06	8.27E+04	9.56E+04	-	2.13E+04	6.16E+05	-
	DE/Rand/2	5.54E+05	8.15E+06	9.07E+04	1.11E+05	-	3.47E+04	3.27E+05	-
5	DE/Rand/1	6.30E-01	4.94E+01	2.41E+04	3.16E+04	+	6.35E+02	2.69E+04	+
	DE/Best/1	5.80E-01	2.87E+03	2.12E+04	3.58E+04	+	3.17E+03	3.02E+04	+
	DE/T2B/1	6.44E-04	5.29E-02	1.75E+04	1.89E+04	+	4.94E+01	1.20E+03	+
	DE/Best/2	1.66E+02	1.73E+03	1.19E+04	1.72E+04	+	6.54E+01	7.54E+03	-
	DE/Rand/2	1.04E+04	7.34E+03	1.08E+04	2.82E+04	+	6.24E+01	1.45E+04	-
6	DE/Rand/1	4.12E+01	6.27E+01	7.49E+03	4.06E+03	+	2.31E+02	2.48E+02	+
	DE/Best/1	4.94E+01	1.87E+02	5.82E+03	7.93E+03	+	3.91E+02	5.86E+02	+
	DE/T2B/1	4.30E+01	4.54E+01	5.04E+03	5.01E+03	+	9.05E+01	5.98E+01	+
	DE/Best/2	1.40E+02	5.18E+03	4.20E+03	4.02E+03	+	7.09E+01	1.05E+02	=
	DE/Rand/2	8.77E+03	1.81E+04	7.47E+02	8.40E+02	-	1.27E+02	9.84E+01	-
7	DE/Rand/1	2.37E+02	2.81E+02	7.72E+02	2.80E+06	+	2.53E+02	1.16E+04	+
	DE/Best/1	2.00E+02	5.23E+03	2.54E+03	3.10E+06	+	5.24E+02	2.06E+06	+
	DE/T2B/1	2.08E+02	2.01E+02	1.44E+03	4.92E+05	+	1.39E+02	1.00E+04	-
	DE/Best/2	5.40E+02	6.83E+02	2.57E+02	3.09E+05	-	2.59E+02	1.76E+04	-
	DE/Rand/2	1.82E+03	3.70E+06	1.73E+02	2.54E+04	-	1.39E+02	4.20E+04	-
8	DE/Rand/1	2.11E+01	4.03E-02	2.11E+01	3.63E-02	-	2.11E+01	4.76E-02	=
	DE/Best/1	2.11E+01	4.82E-02	2.10E+01	5.77E-02	=	2.12E+01	2.92E-02	=
	DE/T2B/1	2.11E+01	4.14E-02	2.11E+01	3.66E-02	=	2.11E+01	4.80E-02	=
	DE/Best/2	2.11E+01	4.34E-02	2.11E+01	4.89E-02	-	2.11E+01	4.08E-02	-
	DE/Rand/2	2.12E+01	3.96E-02	2.11E+01	3.63E-02	-	2.12E+01	3.11E-02	-
9	DE/Rand/1	7.21E+01	1.48E+00	6.19E+01	3.57E+00	-	6.85E+01	1.89E+00	=
	DE/Best/1	7.23E+01	1.36E+00	6.95E+01	3.68E+00	=	6.86E+01	3.11E+00	=
	DE/T2B/1	7.23E+01	1.60E+00	6.48E+01	3.28E+00	-	6.44E+01	3.34E+00	-
	DE/Best/2	7.27E+01	1.55E+00	6.28E+01	3.63E+00	-	6.71E+01	2.97E+00	=
	DE/Rand/2	7.13E+01	1.86E+00	6.38E+01	3.10E+00	-	7.23E+01	1.25E+00	+
10	DE/Rand/1	2.85E+01	1.14E+02	9.79E+03	4.01E+03	+	2.83E+02	5.81E+02	+
	DE/Best/1	1.88E+01	2.80E+02	1.33E+04	3.54E+03	+	7.38E+02	8.01E+02	+
	DE/T2B/1	2.29E+00	1.44E+02	8.24E+03	5.27E+03	+	1.15E+02	1.69E+02	+
	DE/Best/2	3.03E+02	1.28E+03	5.43E+03	3.11E+03	+	1.57E+02	2.33E+02	-
	DE/Rand/2	1.27E+04	6.64E+03	1.31E+03	1.57E+03	-	1.07E+02	2.50E+02	-

Table A.11: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{11} - f_{20}$ and population size $N_P = 6$ for dimension $D = 50$ and mutation vector (MV) schemes “DE/Rand/1”, “DE/Best/1”, “DE/T2B/1”, “DE/Best/2” and “DE/Rand/2”.

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
11	DE/Rand/1	2.61E+02	1.44E+02	8.21E+02	4.78E+02	+	7.90E+02	4.09E+02	+
	DE/Best/1	6.49E+02	4.22E+02	1.25E+03	3.54E+02	+	1.09E+03	4.32E+02	+
	DE/T2B/1	2.45E+02	1.01E+02	1.07E+03	2.36E+02	+	6.70E+02	3.46E+02	+
	DE/Best/2	2.87E+02	5.32E+02	1.09E+03	3.57E+02	+	8.30E+02	2.97E+02	+
	DE/Rand/2	9.25E+02	8.23E+02	6.69E+02	3.04E+02	-	4.53E+02	3.55E+02	-
12	DE/Rand/1	4.68E+02	1.56E+02	1.05E+03	3.31E+02	+	8.97E+02	2.50E+02	+
	DE/Best/1	6.32E+02	2.52E+02	1.15E+03	3.01E+02	+	8.94E+02	3.30E+02	+
	DE/T2B/1	1.97E+02	1.84E+02	1.08E+03	2.52E+02	+	5.09E+02	1.94E+02	+
	DE/Best/2	5.85E+02	1.49E+02	1.01E+03	2.72E+02	+	7.24E+02	2.93E+02	+
	DE/Rand/2	1.21E+03	7.13E+02	7.33E+02	2.71E+02	-	6.37E+02	2.90E+02	-
13	DE/Rand/1	6.31E+02	1.37E+02	1.24E+03	2.44E+02	+	9.19E+02	3.15E+02	+
	DE/Best/1	7.96E+02	3.14E+02	1.18E+03	2.45E+02	+	1.33E+03	3.46E+02	+
	DE/T2B/1	4.99E+02	7.95E+01	1.08E+03	2.63E+02	+	8.93E+02	2.42E+02	+
	DE/Best/2	5.79E+02	1.10E+02	1.10E+03	2.73E+02	+	9.47E+02	2.04E+02	+
	DE/Rand/2	1.34E+03	7.21E+02	8.31E+02	2.51E+02	-	7.47E+02	2.75E+02	-
14	DE/Rand/1	4.17E+03	4.17E+03	9.46E+03	1.20E+03	=	5.93E+03	7.20E+02	+
	DE/Best/1	4.48E+03	8.73E+02	1.20E+04	1.05E+03	+	7.81E+03	1.04E+03	+
	DE/T2B/1	4.47E+03	3.25E+03	1.07E+04	9.78E+02	=	5.48E+03	6.54E+02	+
	DE/Best/2	1.54E+04	5.54E+02	1.02E+04	7.10E+02	-	4.97E+03	9.10E+02	-
	DE/Rand/2	1.55E+04	5.73E+02	4.80E+03	2.68E+03	-	4.34E+03	9.20E+02	-
15	DE/Rand/1	1.57E+04	4.15E+02	1.20E+04	1.01E+03	-	7.78E+03	1.23E+03	-
	DE/Best/1	1.44E+04	5.54E+02	1.25E+04	7.12E+02	-	8.61E+03	1.07E+03	-
	DE/T2B/1	1.50E+04	5.16E+02	1.17E+04	7.98E+02	-	6.62E+03	2.53E+03	-
	DE/Best/2	1.60E+04	5.51E+02	1.03E+04	1.56E+03	-	6.95E+03	2.10E+03	-
	DE/Rand/2	1.56E+04	8.01E+02	1.37E+04	4.20E+02	-	8.56E+03	2.30E+03	-
16	DE/Rand/1	3.04E+00	4.07E-01	2.98E+00	4.52E-01	=	3.43E+00	2.61E-01	=
	DE/Best/1	3.43E+00	2.97E-01	1.76E+00	7.42E-01	-	3.20E+00	3.78E-01	=
	DE/T2B/1	2.88E+00	3.71E-01	2.93E+00	3.61E-01	=	3.06E+00	4.04E-01	=
	DE/Best/2	2.90E+00	3.82E-01	3.46E+00	2.62E-01	=	2.84E+00	4.55E-01	=
	DE/Rand/2	2.89E+00	4.12E-01	3.01E+00	3.97E-01	=	3.47E+00	3.32E-01	=
17	DE/Rand/1	3.55E+02	2.65E+03	2.00E+03	6.18E+02	=	1.44E+03	6.10E+02	=
	DE/Best/1	9.76E+02	6.27E+02	2.38E+03	4.17E+02	+	1.95E+03	7.04E+02	+
	DE/T2B/1	2.72E+02	1.66E+02	1.55E+03	4.86E+02	+	1.19E+03	3.65E+02	+
	DE/Best/2	6.33E+02	2.49E+03	1.82E+03	5.26E+02	+	1.67E+03	5.97E+02	+
	DE/Rand/2	4.32E+03	5.79E+02	1.02E+03	4.14E+02	-	1.18E+03	5.45E+02	-
18	DE/Rand/1	6.32E+02	2.78E+03	1.77E+03	5.68E+02	=	1.28E+03	5.62E+02	=
	DE/Best/1	1.10E+03	6.07E+02	1.90E+03	6.73E+02	+	1.94E+03	6.76E+02	+
	DE/T2B/1	5.94E+02	1.36E+02	1.90E+03	3.81E+02	+	8.54E+02	4.14E+02	+
	DE/Best/2	7.51E+02	2.53E+03	2.07E+03	3.69E+02	+	1.56E+03	5.07E+02	+
	DE/Rand/2	4.89E+03	6.69E+02	1.22E+03	3.63E+02	-	1.07E+03	5.51E+02	-
19	DE/Rand/1	3.63E+01	1.06E+07	1.52E+06	8.07E+06	+	8.05E+04	4.27E+06	+
	DE/Best/1	3.47E+02	1.71E+05	2.08E+06	1.06E+07	+	3.05E+04	3.64E+06	+
	DE/T2B/1	1.87E+01	1.26E+02	3.51E+05	3.82E+06	+	8.00E+02	4.70E+04	+
	DE/Best/2	6.11E+02	2.59E+07	1.43E+06	6.08E+06	=	2.01E+03	1.63E+05	+
	DE/Rand/2	1.32E+07	3.64E+07	2.71E+05	2.00E+06	-	8.51E+02	3.94E+06	-
20	DE/Rand/1	2.50E+01	2.32E-04	2.45E+01	1.27E-01	-	2.45E+01	2.41E-01	=
	DE/Best/1	2.45E+01	2.29E-01	2.45E+01	1.24E-01	+	2.45E+01	1.21E-01	+
	DE/T2B/1	2.45E+01	8.93E-02	2.50E+01	0.00E+00	+	2.45E+01	1.47E-01	-
	DE/Best/2	2.50E+01	2.91E-13	2.50E+01	0.00E+00	=	2.45E+01	1.88E-01	=
	DE/Rand/2	2.50E+01	1.27E-12	2.45E+01	2.43E-01	-	2.45E+01	2.44E-01	-

Table A.12: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{21} - f_{28}$ and population size $N_P = 6$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2" and "DE/Rand/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
21	DE/Rand/1	2.06E+02	2.90E+02	6.39E+03	1.23E+03	+	8.86E+02	6.92E+02	+
	DE/Best/1	2.53E+02	9.07E+02	6.32E+03	1.14E+03	+	1.81E+03	8.36E+02	+
	DE/T2B/1	1.00E+02	3.84E+02	5.34E+03	8.80E+02	+	3.21E+02	3.61E+02	+
	DE/Best/2	5.30E+02	2.78E+03	5.64E+03	1.50E+03	+	1.71E+02	5.46E+02	-
	DE/Rand/2	7.48E+03	2.69E+03	3.75E+03	6.78E+02	-	3.43E+02	5.58E+02	-
22	DE/Rand/1	4.40E+03	3.73E+03	1.05E+04	1.23E+03	=	6.26E+03	1.12E+03	+
	DE/Best/1	5.49E+03	8.20E+02	1.34E+04	1.05E+03	+	8.46E+03	1.34E+03	+
	DE/T2B/1	5.00E+03	3.41E+03	1.21E+04	9.35E+02	+	6.23E+03	1.34E+03	+
	DE/Best/2	1.49E+04	7.52E+02	1.05E+04	1.26E+03	-	6.54E+03	1.20E+03	-
	DE/Rand/2	1.56E+04	6.63E+02	7.13E+03	2.59E+03	-	5.56E+03	1.02E+03	-
23	DE/Rand/1	1.59E+04	5.69E+02	1.18E+04	1.07E+03	-	8.21E+03	1.82E+03	-
	DE/Best/1	1.50E+04	5.57E+02	1.47E+04	6.62E+02	=	1.01E+04	1.34E+03	-
	DE/T2B/1	1.53E+04	5.96E+02	1.29E+04	9.46E+02	-	7.77E+03	2.42E+03	-
	DE/Best/2	1.64E+04	5.38E+02	1.24E+04	1.17E+03	-	7.26E+03	1.92E+03	-
	DE/Rand/2	1.64E+04	4.78E+02	1.40E+04	6.03E+02	-	6.96E+03	2.73E+03	-
24	DE/Rand/1	3.86E+02	5.05E+00	3.75E+02	2.02E+02	-	3.59E+02	8.21E+00	-
	DE/Best/1	3.77E+02	5.14E+00	4.13E+02	3.59E+02	+	3.55E+02	2.72E+02	=
	DE/T2B/1	3.88E+02	4.02E+00	4.08E+02	4.10E+02	+	3.57E+02	7.31E+00	-
	DE/Best/2	3.89E+02	5.44E+00	3.65E+02	2.55E+01	-	3.55E+02	1.11E+01	-
	DE/Rand/2	3.94E+02	5.91E+00	3.40E+02	1.37E+01	-	3.72E+02	8.11E+00	-
25	DE/Rand/1	3.86E+02	4.46E+00	3.61E+02	9.38E+00	-	3.78E+02	5.10E+00	-
	DE/Best/1	3.78E+02	4.85E+00	3.80E+02	2.14E+01	+	3.64E+02	8.67E+00	-
	DE/T2B/1	3.75E+02	5.15E+00	3.63E+02	1.73E+01	=	3.77E+02	5.29E+00	+
	DE/Best/2	3.86E+02	6.25E+00	3.56E+02	1.07E+01	-	3.78E+02	4.41E+00	-
	DE/Rand/2	3.85E+02	6.17E+00	3.76E+02	4.81E+00	-	3.81E+02	4.28E+00	-
26	DE/Rand/1	4.82E+02	5.98E+00	3.11E+02	3.20E+01	-	4.49E+02	1.20E+01	-
	DE/Best/1	4.55E+02	8.51E+00	4.39E+02	4.84E+02	-	2.11E+02	4.01E+02	=
	DE/T2B/1	4.89E+02	3.81E+00	3.36E+02	5.52E+02	=	2.02E+02	9.36E+01	-
	DE/Best/2	4.90E+02	5.74E+00	3.41E+02	5.05E+02	-	2.03E+02	6.93E+01	-
	DE/Rand/2	4.98E+02	4.69E+00	2.19E+02	6.68E+01	-	4.49E+02	1.24E+01	-
27	DE/Rand/1	2.22E+03	3.53E+01	1.95E+03	1.92E+02	-	1.92E+03	9.86E+01	-
	DE/Best/1	2.14E+03	4.35E+01	2.27E+03	3.75E+02	+	1.98E+03	8.96E+01	=
	DE/T2B/1	2.16E+03	3.79E+01	2.21E+03	6.23E+02	+	2.03E+03	4.83E+01	-
	DE/Best/2	2.16E+03	6.43E+01	1.91E+03	1.48E+02	-	1.90E+03	9.84E+01	-
	DE/Rand/2	2.25E+03	5.23E+01	1.95E+03	8.18E+01	-	2.02E+03	7.50E+01	-
28	DE/Rand/1	4.03E+02	2.17E+03	8.99E+03	1.56E+03	+	4.55E+03	2.70E+03	+
	DE/Best/1	6.47E+02	2.64E+03	9.93E+03	2.53E+03	+	1.02E+04	2.55E+03	+
	DE/T2B/1	4.00E+02	2.05E+03	8.61E+03	2.13E+03	+	8.03E+02	2.97E+03	+
	DE/Best/2	5.53E+02	2.15E+03	9.12E+03	3.17E+03	+	3.57E+03	2.88E+03	+
	DE/Rand/2	8.06E+03	8.01E+03	6.98E+03	2.65E+03	-	7.70E+02	2.27E+03	-

Table A.13: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{10}$ and population size $N_P = 50$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2" and "DE/Rand/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	1.33E+05	1.77E+04	1.75E-05	1.20E-03	-	3.86E-01	4.27E+00	-
	DE/Best/1	1.84E+03	2.91E+03	2.62E+04	7.08E+03	+	2.70E-05	1.18E-01	-
	DE/T2B/1	9.53E+04	1.77E+04	1.88E+04	5.00E+03	-	1.24E-06	5.26E-04	-
	DE/Best/2	1.33E+05	1.43E+04	8.81E-09	6.91E-09	-	8.04E-06	6.54E-02	-
	DE/Rand/2	1.20E+05	1.65E+04	3.02E+03	1.48E+03	-	5.92E+00	5.54E+01	-
2	DE/Rand/1	1.86E+09	1.58E+09	1.66E+07	1.78E+07	-	8.56E+06	1.33E+07	-
	DE/Best/1	1.89E+08	8.55E+08	7.99E+07	1.28E+08	-	1.79E+06	2.73E+06	-
	DE/T2B/1	2.67E+09	9.49E+08	4.72E+07	5.98E+07	-	1.06E+06	2.54E+06	-
	DE/Best/2	3.45E+09	1.37E+09	7.55E+06	6.00E+06	-	1.58E+06	3.20E+06	-
	DE/Rand/2	3.09E+09	1.67E+09	1.21E+09	3.77E+08	-	1.38E+07	1.83E+07	-
3	DE/Rand/1	3.77E+13	1.34E+14	1.04E+09	4.26E+09	-	1.74E+09	1.40E+10	-
	DE/Best/1	3.06E+11	2.51E+12	3.53E+10	2.98E+11	-	1.53E+09	3.91E+09	-
	DE/T2B/1	1.55E+13	3.55E+18	3.65E+10	3.41E+10	-	2.83E+08	1.34E+09	-
	DE/Best/2	3.05E+13	1.20E+20	7.64E+08	2.25E+10	-	9.23E+08	1.17E+10	-
	DE/Rand/2	5.82E+13	4.49E+18	2.00E+12	5.12E+12	-	1.26E+10	2.91E+10	-
4	DE/Rand/1	1.88E+05	7.27E+05	1.14E+05	2.99E+04	-	1.60E+05	1.03E+05	-
	DE/Best/1	1.91E+05	4.41E+05	1.26E+04	1.04E+04	-	2.17E+04	1.54E+04	-
	DE/T2B/1	2.31E+05	6.64E+05	1.79E+04	6.99E+03	-	4.98E+04	2.25E+04	-
	DE/Best/2	2.64E+05	1.23E+06	6.71E+04	3.52E+04	-	5.91E+04	6.63E+04	-
	DE/Rand/2	2.06E+05	2.43E+06	2.02E+05	1.75E+05	-	1.85E+05	1.60E+05	-
5	DE/Rand/1	4.13E+04	1.22E+04	1.47E-02	4.45E+00	-	5.67E+00	3.32E+01	-
	DE/Best/1	1.20E+03	1.22E+03	3.00E+03	3.22E+03	+	8.70E-03	9.13E+00	-
	DE/T2B/1	2.56E+04	1.02E+04	2.28E+03	1.59E+03	-	3.47E-03	1.42E-02	-
	DE/Best/2	5.63E+04	2.19E+04	3.33E-05	5.48E-04	-	1.26E-03	8.20E-02	-
	DE/Rand/2	4.79E+04	2.56E+04	1.88E+03	7.21E+02	-	3.19E+01	3.77E+01	-
6	DE/Rand/1	1.98E+04	4.60E+03	4.54E+01	1.35E+01	-	4.30E+01	4.36E+01	-
	DE/Best/1	1.63E+02	2.59E+02	1.01E+03	6.37E+02	+	4.41E+01	3.82E+01	-
	DE/T2B/1	1.55E+04	5.28E+03	6.21E+02	4.00E+02	-	3.21E+01	3.24E+01	-
	DE/Best/2	2.20E+04	6.11E+03	3.88E+01	2.64E+01	-	3.97E+01	3.61E+01	-
	DE/Rand/2	1.60E+04	5.38E+03	4.22E+02	1.93E+02	-	5.31E+01	3.57E+01	-
7	DE/Rand/1	2.97E+03	3.79E+03	7.60E+01	4.35E+01	-	8.28E+01	3.28E+01	-
	DE/Best/1	3.80E+02	3.96E+02	1.15E+02	2.16E+02	-	1.03E+02	5.52E+01	-
	DE/T2B/1	1.35E+03	6.05E+05	9.02E+01	2.38E+01	-	8.59E+01	3.06E+01	-
	DE/Best/2	3.47E+04	9.19E+06	1.10E+02	7.16E+01	-	9.55E+01	2.49E+01	-
	DE/Rand/2	7.38E+03	2.13E+06	1.12E+03	5.43E+02	-	8.40E+01	5.45E+01	-
8	DE/Rand/1	2.11E+01	3.90E-02	2.11E+01	4.84E-02	-	2.11E+01	4.62E-02	=
	DE/Best/1	2.11E+01	4.01E-02	2.11E+01	3.96E-02	=	2.11E+01	4.12E-02	=
	DE/T2B/1	2.11E+01	4.45E-02	2.11E+01	5.28E-02	=	2.10E+01	5.61E-02	=
	DE/Best/2	2.12E+01	3.40E-02	2.11E+01	2.80E-02	-	2.11E+01	4.37E-02	-
	DE/Rand/2	2.12E+01	3.74E-02	2.11E+01	3.49E-02	-	2.11E+01	3.39E-02	-
9	DE/Rand/1	7.21E+01	1.40E+00	7.30E+01	1.40E+00	=	7.02E+01	1.55E+00	=
	DE/Best/1	7.24E+01	1.45E+00	4.72E+01	3.32E+00	-	5.76E+01	5.93E+00	=
	DE/T2B/1	7.12E+01	1.77E+00	3.72E+01	3.81E+00	-	6.53E+01	2.28E+00	=
	DE/Best/2	7.36E+01	1.24E+00	7.01E+01	1.91E+00	-	7.34E+01	1.26E+00	=
	DE/Rand/2	7.16E+01	1.68E+00	7.10E+01	2.05E+00	=	7.24E+01	1.39E+00	=
10	DE/Rand/1	1.86E+04	3.71E+03	2.68E+00	6.21E+00	-	3.58E+01	3.88E+01	-
	DE/Best/1	9.53E+02	8.71E+02	2.29E+03	1.11E+03	+	1.78E+00	1.68E+01	-
	DE/T2B/1	1.81E+04	2.15E+03	1.53E+03	8.43E+02	-	1.29E+00	1.57E+00	-
	DE/Best/2	1.95E+04	3.25E+03	2.15E-01	8.86E-01	-	1.96E+00	1.36E+01	-
	DE/Rand/2	1.87E+04	3.63E+03	3.00E+03	6.55E+02	-	9.83E+01	6.62E+01	-

Table A.14: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{11} - f_{20}$ and population size $N_P = 50$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2" and "DE/Rand/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
11	DE/Rand/1	2.09E+03	2.79E+02	2.64E+02	2.95E+01	-	9.57E+01	4.12E+01	-
	DE/Best/1	3.20E+02	8.62E+01	4.97E+02	1.53E+02	+	4.71E+02	1.02E+02	+
	DE/T2B/1	1.75E+03	2.66E+02	2.96E+02	5.40E+01	-	1.50E+02	6.95E+01	-
	DE/Best/2	2.16E+03	2.67E+02	1.95E+02	1.15E+02	-	3.17E+02	1.04E+02	-
	DE/Rand/2	2.04E+03	3.64E+02	4.83E+02	3.28E+01	-	1.05E+02	5.02E+01	-
12	DE/Rand/1	2.01E+03	2.42E+02	3.73E+02	2.12E+01	-	1.35E+02	9.77E+01	-
	DE/Best/1	6.02E+02	5.93E+01	5.14E+02	1.04E+02	=	3.62E+02	1.31E+02	=
	DE/T2B/1	1.90E+03	1.89E+02	2.87E+02	7.44E+01	-	2.18E+02	1.16E+02	-
	DE/Best/2	1.98E+03	2.73E+02	4.61E+02	4.74E+01	-	2.75E+02	1.32E+02	-
	DE/Rand/2	1.94E+03	2.14E+02	5.16E+02	4.87E+01	-	2.24E+02	6.73E+01	-
13	DE/Rand/1	1.86E+03	2.66E+02	3.74E+02	2.25E+01	-	3.92E+02	3.64E+01	-
	DE/Best/1	5.37E+02	8.85E+01	6.09E+02	8.80E+01	+	5.51E+02	1.61E+02	+
	DE/T2B/1	1.67E+03	2.23E+02	4.18E+02	1.02E+02	-	4.34E+02	5.59E+01	-
	DE/Best/2	2.06E+03	2.05E+02	4.63E+02	6.23E+01	-	4.36E+02	7.00E+01	-
	DE/Rand/2	2.08E+03	2.32E+02	5.56E+02	5.88E+01	-	4.17E+02	3.19E+01	-
14	DE/Rand/1	1.54E+04	4.26E+02	1.26E+04	6.50E+02	-	6.50E+03	1.25E+03	-
	DE/Best/1	1.13E+04	1.29E+03	7.28E+03	6.27E+02	-	4.83E+03	8.03E+02	-
	DE/T2B/1	1.48E+04	5.65E+02	1.27E+04	5.37E+02	-	1.33E+04	4.32E+02	-
	DE/Best/2	1.61E+04	4.11E+02	1.38E+04	4.49E+02	-	4.92E+03	6.04E+02	-
	DE/Rand/2	1.62E+04	4.71E+02	1.43E+04	4.57E+02	-	1.18E+04	6.09E+02	-
15	DE/Rand/1	1.55E+04	4.82E+02	1.38E+04	4.83E+02	-	1.45E+04	3.66E+02	-
	DE/Best/1	1.59E+04	3.74E+02	7.96E+03	2.75E+03	-	6.01E+03	2.26E+03	-
	DE/T2B/1	1.59E+04	4.69E+02	1.34E+04	5.04E+02	-	1.24E+04	6.42E+02	-
	DE/Best/2	1.62E+04	4.66E+02	1.42E+04	4.93E+02	-	7.39E+03	2.63E+03	-
	DE/Rand/2	1.60E+04	5.38E+02	1.52E+04	4.47E+02	-	1.44E+04	4.93E+02	-
16	DE/Rand/1	2.37E+00	4.83E-01	3.55E+00	2.76E-01	=	3.15E+00	2.97E-01	=
	DE/Best/1	3.14E+00	3.28E-01	3.47E+00	2.68E-01	=	3.19E+00	3.88E-01	=
	DE/T2B/1	3.39E+00	3.31E-01	3.05E+00	3.60E-01	=	3.35E+00	3.01E-01	=
	DE/Best/2	3.21E+00	3.02E-01	3.26E+00	4.03E-01	=	3.07E+00	4.25E-01	=
	DE/Rand/2	3.27E+00	3.51E-01	3.19E+00	3.32E-01	-	3.59E+00	2.58E-01	+
17	DE/Rand/1	4.43E+03	4.42E+02	3.99E+02	1.79E+01	-	2.13E+02	5.21E+01	-
	DE/Best/1	5.72E+02	1.36E+02	8.19E+02	2.51E+02	+	5.13E+02	2.20E+02	-
	DE/T2B/1	3.90E+03	4.05E+02	4.00E+02	1.49E+02	-	4.68E+02	7.45E+01	-
	DE/Best/2	4.82E+03	3.76E+02	4.91E+02	7.31E+01	-	2.94E+02	1.56E+02	-
	DE/Rand/2	4.01E+03	4.42E+02	6.12E+02	7.60E+01	-	3.08E+02	3.96E+01	-
18	DE/Rand/1	4.19E+03	4.21E+02	4.30E+02	1.47E+01	-	4.44E+02	4.22E+01	-
	DE/Best/1	7.14E+02	9.77E+01	8.48E+02	2.19E+02	+	4.04E+02	1.72E+02	-
	DE/T2B/1	3.62E+03	3.80E+02	7.06E+02	1.08E+02	-	5.30E+02	5.17E+01	-
	DE/Best/2	4.65E+03	3.40E+02	5.51E+02	5.63E+01	-	2.95E+02	1.07E+02	-
	DE/Rand/2	4.29E+03	3.92E+02	6.71E+02	6.57E+01	-	4.51E+02	8.10E+01	-
19	DE/Rand/1	1.06E+07	1.05E+07	3.06E+01	4.39E+00	-	3.88E+01	8.57E+01	-
	DE/Best/1	6.55E+03	3.94E+05	2.85E+04	1.52E+05	=	5.56E+01	1.37E+02	-
	DE/T2B/1	6.43E+06	8.64E+06	7.13E+03	4.13E+04	-	4.10E+01	7.70E+00	-
	DE/Best/2	1.23E+07	9.49E+06	4.34E+01	1.76E+01	-	4.10E+01	1.19E+02	-
	DE/Rand/2	8.00E+06	8.93E+06	9.22E+03	5.16E+04	-	5.63E+01	3.89E+02	-
20	DE/Rand/1	2.50E+01	3.49E-10	2.50E+01	9.57E-09	-	2.50E+01	1.64E-05	-
	DE/Best/1	2.50E+01	8.73E-13	2.30E+01	4.20E-01	=	2.45E+01	1.23E-01	=
	DE/T2B/1	2.50E+01	1.71E-07	2.15E+01	1.12E+00	=	2.50E+01	1.09E-08	=
	DE/Best/2	2.50E+01	1.96E-07	2.34E+01	4.61E-01	=	2.50E+01	1.88E-04	=
	DE/Rand/2	2.50E+01	2.24E-09	2.50E+01	8.63E-11	=	2.50E+01	2.47E-03	=

Table A.15: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_{21} - f_{28}$ and population size $N_P = 50$ for dimension $D = 50$ and mutation vector (MV) schemes "DE/Rand/1", "DE/Best/1", "DE/T2B/1", "DE/Best/2" and "DE/Rand/2".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
21	DE/Rand/1	1.02E+04	8.48E+02	2.00E+02	4.38E+02	-	2.16E+02	3.82E+02	-
	DE/Best/1	1.11E+03	8.51E+02	3.68E+03	3.20E+02	+	2.01E+02	3.77E+02	-
	DE/T2B/1	9.66E+03	1.15E+03	3.09E+03	2.93E+02	-	2.00E+02	3.05E+02	-
	DE/Best/2	1.05E+04	9.02E+02	2.00E+02	4.09E+02	-	2.00E+02	3.65E+02	-
	DE/Rand/2	9.86E+03	9.85E+02	1.55E+03	6.59E+02	-	2.56E+02	4.16E+02	-
22	DE/Rand/1	1.51E+04	4.63E+02	1.27E+04	6.76E+02	-	4.82E+03	1.75E+03	-
	DE/Best/1	8.78E+03	1.73E+03	7.46E+03	9.58E+02	-	4.97E+03	1.03E+03	-
	DE/T2B/1	1.63E+04	4.69E+02	1.31E+04	5.78E+02	-	1.33E+04	5.16E+02	-
	DE/Best/2	1.66E+04	4.02E+02	1.37E+04	6.21E+02	-	5.31E+03	7.70E+02	-
	DE/Rand/2	1.65E+04	4.45E+02	1.47E+04	4.47E+02	-	1.28E+04	6.41E+02	-
23	DE/Rand/1	1.58E+04	4.23E+02	1.46E+04	2.96E+02	-	1.52E+04	3.19E+02	-
	DE/Best/1	1.62E+04	4.15E+02	8.55E+03	2.61E+03	-	6.73E+03	1.58E+03	-
	DE/T2B/1	1.64E+04	3.87E+02	1.38E+04	6.29E+02	-	1.43E+04	4.69E+02	-
	DE/Best/2	1.65E+04	4.88E+02	1.47E+04	5.45E+02	-	9.85E+03	1.63E+03	-
	DE/Rand/2	1.68E+04	3.83E+02	1.59E+04	3.63E+02	-	1.51E+04	4.11E+02	-
24	DE/Rand/1	3.95E+02	4.46E+00	3.83E+02	3.87E+00	-	3.80E+02	4.89E+00	-
	DE/Best/1	3.86E+02	4.11E+00	3.40E+02	1.19E+01	-	3.55E+02	1.49E+01	-
	DE/T2B/1	3.91E+02	4.91E+00	3.20E+02	1.25E+01	-	3.74E+02	5.38E+00	-
	DE/Best/2	3.86E+02	7.44E+00	3.81E+02	4.68E+00	-	3.78E+02	5.87E+00	-
	DE/Rand/2	3.97E+02	5.27E+00	3.85E+02	4.39E+00	-	3.86E+02	4.30E+00	-
25	DE/Rand/1	3.74E+02	6.59E+00	3.76E+02	4.63E+00	+	3.84E+02	4.05E+00	+
	DE/Best/1	3.75E+02	5.04E+00	3.22E+02	1.37E+01	-	3.73E+02	5.18E+00	-
	DE/T2B/1	3.90E+02	4.21E+00	3.13E+02	8.71E+00	-	3.80E+02	4.60E+00	-
	DE/Best/2	3.95E+02	4.68E+00	3.83E+02	3.29E+00	-	3.79E+02	4.76E+00	-
	DE/Rand/2	3.91E+02	5.13E+00	3.83E+02	3.76E+00	-	3.83E+02	4.40E+00	-
26	DE/Rand/1	4.94E+02	6.16E+00	4.86E+02	3.33E+00	-	4.82E+02	4.48E+00	-
	DE/Best/1	4.90E+02	4.46E+00	2.23E+02	5.56E+01	-	4.28E+02	1.30E+01	-
	DE/T2B/1	4.96E+02	4.43E+00	2.12E+02	5.29E+01	-	4.63E+02	5.93E+00	-
	DE/Best/2	4.94E+02	7.09E+00	2.01E+02	5.33E+01	-	4.25E+02	1.99E+01	-
	DE/Rand/2	4.97E+02	7.32E+00	4.89E+02	3.41E+00	-	4.89E+02	2.99E+00	-
27	DE/Rand/1	2.16E+03	5.74E+01	2.03E+03	5.83E+01	-	2.15E+03	4.37E+01	-
	DE/Best/1	2.20E+03	4.30E+01	1.53E+03	1.07E+02	-	1.57E+03	1.84E+02	-
	DE/T2B/1	2.12E+03	6.48E+01	1.39E+03	1.09E+02	-	2.04E+03	5.68E+01	-
	DE/Best/2	2.29E+03	4.99E+01	2.12E+03	3.84E+01	-	2.12E+03	4.22E+01	-
	DE/Rand/2	2.31E+03	5.44E+01	2.20E+03	3.33E+01	-	2.15E+03	3.16E+01	-
28	DE/Rand/1	1.33E+04	1.90E+03	4.00E+02	1.25E+03	-	4.09E+02	1.15E+03	-
	DE/Best/1	1.15E+03	1.58E+03	3.78E+03	9.44E+02	+	4.04E+02	2.62E+03	=
	DE/T2B/1	1.27E+04	1.94E+03	3.58E+03	6.74E+02	-	4.00E+02	1.32E+03	-
	DE/Best/2	1.18E+04	2.12E+03	4.00E+02	1.49E+03	-	4.00E+02	2.23E+03	-
	DE/Rand/2	1.21E+04	1.88E+03	1.43E+03	1.35E+03	-	4.26E+02	1.16E+03	-

Table A.16: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{28}$ and population size $N_P = 5$ for dimension $D = 10$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	4.78E-01	1.83E+03	2.02E+03	8.44E+03	+	4.95E-02	3.05E+03	=
	DE/Best/1	5.01E-09	2.90E-07	2.86E+03	6.03E+03	+	2.28E-03	2.45E+03	+
2	DE/Rand/1	5.76E+05	6.48E+06	2.50E+06	6.23E+07	+	2.21E+05	3.79E+06	=
	DE/Best/1	1.13E+05	1.43E+06	7.76E+06	1.37E+08	+	1.73E+06	1.16E+07	+
3	DE/Rand/1	8.70E+07	6.56E+09	6.12E+09	3.18E+13	+	4.49E+07	3.34E+10	=
	DE/Best/1	9.93E+06	5.34E+09	9.41E+09	3.75E+17	+	3.30E+08	1.84E+16	+
4	DE/Rand/1	2.09E+04	1.29E+05	1.59E+04	4.32E+04	-	4.34E+03	5.68E+04	-
	DE/Best/1	9.06E+03	1.75E+05	1.18E+04	4.15E+04	+	1.00E+04	4.15E+04	+
5	DE/Rand/1	1.34E+00	7.28E+03	5.14E+02	1.00E+04	+	1.58E+01	2.17E+04	=
	DE/Best/1	8.35E-09	3.48E-01	5.45E+02	2.37E+04	+	5.93E+01	1.58E+04	+
6	DE/Rand/1	9.11E-04	5.89E+01	2.68E+02	6.73E+02	+	4.33E+00	8.13E+01	=
	DE/Best/1	5.14E-02	3.37E+01	2.20E+02	1.40E+03	+	2.41E+00	2.73E+02	+
7	DE/Rand/1	1.70E+01	5.30E+01	5.44E+01	6.34E+05	+	6.21E+01	1.63E+02	+
	DE/Best/1	7.92E+01	1.34E+02	6.91E+01	1.67E+06	-	1.10E+02	1.51E+04	+
8	DE/Rand/1	2.02E+01	1.20E-01	2.03E+01	2.02E-01	+	2.03E+01	8.27E-02	=
	DE/Best/1	2.02E+01	1.21E-01	2.04E+01	1.98E-01	+	2.03E+01	9.40E-02	=
9	DE/Rand/1	3.55E+00	1.81E+00	5.66E+00	1.74E+00	=	7.20E+00	1.03E+00	=
	DE/Best/1	5.15E+00	1.87E+00	8.35E+00	1.61E+00	+	8.39E+00	1.45E+00	+
10	DE/Rand/1	1.15E+01	1.01E+02	1.35E+02	5.93E+02	+	1.03E+01	1.56E+02	=
	DE/Best/1	2.14E+00	3.14E+01	2.14E+02	1.10E+03	+	2.15E+01	1.36E+02	+
11	DE/Rand/1	1.19E+01	4.51E+01	5.55E+01	8.78E+01	+	3.78E+01	8.18E+01	+
	DE/Best/1	2.19E+01	6.22E+01	7.23E+01	1.04E+02	+	4.55E+01	1.04E+02	+
12	DE/Rand/1	1.18E+01	5.57E+01	7.48E+01	8.44E+01	+	3.92E+01	5.32E+01	+
	DE/Best/1	1.39E+01	7.02E+01	6.62E+01	1.07E+02	+	5.40E+01	8.90E+01	+
13	DE/Rand/1	1.71E+01	4.58E+01	8.66E+01	7.03E+01	+	6.09E+01	6.01E+01	+
	DE/Best/1	5.92E+01	6.68E+01	8.76E+01	1.14E+02	+	7.90E+01	9.00E+01	+
14	DE/Rand/1	1.05E+02	3.08E+02	7.73E+02	5.11E+02	+	3.58E+02	3.67E+02	+
	DE/Best/1	1.43E+02	3.72E+02	1.19E+03	3.78E+02	+	6.75E+02	3.09E+02	+
15	DE/Rand/1	7.99E+02	3.85E+02	8.24E+02	4.77E+02	+	5.20E+02	3.68E+02	-
	DE/Best/1	5.01E+02	5.72E+02	1.62E+03	3.85E+02	+	8.12E+02	3.84E+02	+
16	DE/Rand/1	6.64E-01	3.85E-01	1.50E+00	6.36E-01	+	3.26E-01	5.08E-01	=
	DE/Best/1	1.07E+00	2.75E-01	8.47E-01	1.52E+00	-	5.90E-01	4.26E-01	-
17	DE/Rand/1	2.13E+01	5.78E+01	4.83E+01	1.20E+02	+	4.21E+01	1.00E+02	+
	DE/Best/1	2.78E+01	6.53E+01	7.32E+01	1.38E+02	+	4.93E+01	1.31E+02	+
18	DE/Rand/1	4.61E+01	9.11E+01	1.14E+02	8.28E+01	+	4.81E+01	7.64E+01	+
	DE/Best/1	3.93E+01	9.19E+01	1.69E+02	1.07E+02	+	9.87E+01	1.18E+02	+
19	DE/Rand/1	1.10E+00	1.02E+04	8.07E+02	5.02E+05	+	1.92E+01	1.16E+04	+
	DE/Best/1	6.10E-01	1.98E+02	7.88E+02	5.00E+05	+	6.36E+01	3.13E+05	+
20	DE/Rand/1	4.05E+00	2.62E-01	3.80E+00	3.92E-01	=	4.06E+00	2.13E-01	+
	DE/Best/1	3.51E+00	3.52E-01	3.80E+00	2.53E-01	+	3.70E+00	3.14E-01	+
21	DE/Rand/1	3.04E+02	4.74E+01	5.31E+02	2.69E+02	+	2.22E+02	1.38E+02	=
	DE/Best/1	1.00E+02	6.27E+01	5.42E+02	2.67E+02	+	1.13E+02	9.08E+01	+
22	DE/Rand/1	2.23E+02	4.99E+02	9.11E+02	4.42E+02	+	6.71E+02	3.59E+02	+
	DE/Best/1	4.20E+02	2.70E+02	1.57E+03	4.03E+02	+	7.38E+02	4.02E+02	+
23	DE/Rand/1	8.23E+02	4.63E+02	9.22E+02	5.33E+02	+	9.04E+02	4.41E+02	+
	DE/Best/1	7.65E+02	5.20E+02	2.06E+03	3.39E+02	+	1.22E+03	4.87E+02	=
24	DE/Rand/1	2.10E+02	4.78E+00	2.19E+02	6.04E+00	+	2.17E+02	4.45E+00	=
	DE/Best/1	1.58E+02	1.28E+01	2.21E+02	4.32E+01	+	2.21E+02	4.18E+00	+
25	DE/Rand/1	2.20E+02	2.22E+00	2.18E+02	4.38E+00	-	2.17E+02	4.02E+00	=
	DE/Best/1	2.17E+02	2.43E+00	2.23E+02	4.17E+00	+	2.20E+02	4.76E+00	=
26	DE/Rand/1	2.00E+02	5.76E+01	1.73E+02	6.43E+01	=	1.78E+02	5.05E+01	-
	DE/Best/1	1.54E+02	7.71E+01	2.04E+02	1.04E+02	+	2.00E+02	4.49E+01	+
27	DE/Rand/1	4.29E+02	4.48E+01	5.41E+02	2.30E+02	+	4.09E+02	2.15E+02	=
	DE/Best/1	4.00E+02	4.48E+01	5.75E+02	2.31E+02	+	4.91E+02	8.40E+01	+
28	DE/Rand/1	3.10E+02	2.28E+02	7.02E+02	4.30E+02	+	4.12E+02	3.03E+02	+
	DE/Best/1	1.00E+02	4.63E+02	9.18E+02	3.68E+02	+	7.97E+02	4.04E+02	+

Table A.17: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{28}$ and population size $N_P = 5$ for dimension $D = 30$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	4.51E+00	4.02E+03	4.33E+04	1.62E+04	+	6.64E+02	5.48E+03	+
	DE/Best/1	6.29E-04	2.13E+03	5.91E+04	1.88E+04	+	2.15E+03	1.04E+04	+
2	DE/Rand/1	1.29E+07	2.46E+07	2.69E+08	7.63E+08	+	2.17E+07	4.76E+07	+
	DE/Best/1	6.33E+06	2.84E+07	5.93E+08	1.07E+09	+	2.33E+07	1.22E+08	+
3	DE/Rand/1	2.06E+10	6.46E+10	1.42E+11	2.35E+21	+	1.37E+10	5.27E+10	=
	DE/Best/1	4.42E+09	1.42E+12	3.70E+12	3.61E+21	+	2.78E+10	1.51E+12	+
4	DE/Rand/1	6.94E+04	2.73E+05	7.42E+04	7.24E+04	=	2.45E+04	4.31E+04	-
	DE/Best/1	3.61E+04	3.92E+05	6.03E+04	8.00E+04	=	3.76E+04	1.72E+05	+
5	DE/Rand/1	2.81E+02	2.60E+04	1.31E+04	3.96E+04	+	1.07E+03	2.99E+04	=
	DE/Best/1	1.17E-01	1.09E+03	2.12E+04	5.30E+04	+	1.40E+03	4.66E+04	+
6	DE/Rand/1	5.91E+01	1.31E+02	6.56E+03	5.66E+03	+	7.45E+01	3.64E+02	+
	DE/Best/1	2.38E+01	3.94E+02	7.66E+03	6.57E+03	+	3.12E+02	1.76E+03	+
7	DE/Rand/1	2.00E+02	2.16E+03	1.20E+03	7.09E+07	+	2.30E+02	5.01E+05	+
	DE/Best/1	1.90E+02	1.27E+05	6.76E+04	9.31E+07	+	3.75E+02	1.40E+07	+
8	DE/Rand/1	2.09E+01	4.30E-02	2.09E+01	6.22E-02	=	2.08E+01	6.63E-02	-
	DE/Best/1	2.09E+01	4.46E-02	2.09E+01	7.99E-02	=	2.10E+01	4.24E-02	=
9	DE/Rand/1	3.21E+01	2.08E+00	3.30E+01	2.89E+00	+	3.49E+01	2.00E+00	=
	DE/Best/1	3.88E+01	1.21E+00	3.69E+01	2.73E+00	-	3.66E+01	2.36E+00	=
10	DE/Rand/1	6.80E+01	2.47E+02	5.02E+03	3.26E+03	+	2.05E+02	5.62E+02	+
	DE/Best/1	1.49E+01	1.47E+02	5.26E+03	4.42E+03	+	4.90E+02	8.66E+02	+
11	DE/Rand/1	1.98E+02	2.19E+02	5.79E+02	3.58E+02	+	2.95E+02	2.42E+02	+
	DE/Best/1	2.67E+02	2.85E+02	9.06E+02	3.12E+02	+	4.10E+02	4.37E+02	+
12	DE/Rand/1	1.97E+02	1.61E+02	4.23E+02	2.90E+02	+	3.68E+02	2.78E+02	+
	DE/Best/1	3.81E+02	2.29E+02	6.39E+02	3.59E+02	+	5.65E+02	2.57E+02	+
13	DE/Rand/1	2.87E+02	1.52E+02	6.27E+02	3.03E+02	+	5.65E+02	2.99E+02	+
	DE/Best/1	4.62E+02	1.73E+02	8.42E+02	3.20E+02	+	8.22E+02	2.90E+02	+
14	DE/Rand/1	1.73E+03	1.25E+03	5.62E+03	6.94E+02	+	3.03E+03	8.82E+02	+
	DE/Best/1	2.75E+03	6.82E+02	6.43E+03	7.14E+02	+	4.32E+03	6.56E+02	+
15	DE/Rand/1	7.56E+03	3.90E+02	5.57E+03	8.75E+02	-	3.76E+03	7.84E+02	-
	DE/Best/1	5.45E+03	8.65E+02	5.97E+03	8.23E+02	=	4.91E+03	7.68E+02	-
16	DE/Rand/1	2.36E+00	3.06E-01	1.73E+00	5.80E-01	=	2.59E+00	3.34E-01	=
	DE/Best/1	2.45E+00	2.91E-01	1.63E+00	6.92E-01	=	1.23E+00	6.39E-01	=
17	DE/Rand/1	2.23E+02	3.70E+02	6.99E+02	4.43E+02	+	7.18E+02	4.77E+02	+
	DE/Best/1	5.33E+02	3.94E+02	1.22E+03	3.55E+02	+	1.23E+03	4.90E+02	+
18	DE/Rand/1	4.56E+02	2.95E+02	1.22E+03	3.81E+02	+	9.77E+02	3.03E+02	+
	DE/Best/1	5.28E+02	4.51E+02	1.16E+03	4.49E+02	+	1.07E+03	5.27E+02	+
19	DE/Rand/1	4.36E+02	1.01E+06	4.93E+05	7.07E+06	+	3.37E+04	3.34E+06	+
	DE/Best/1	7.73E+01	4.89E+05	3.11E+05	1.16E+07	+	4.96E+04	4.91E+06	+
20	DE/Rand/1	1.45E+01	2.17E-01	1.50E+01	3.98E-07	+	1.45E+01	1.25E-01	-
	DE/Best/1	1.45E+01	1.71E-01	1.50E+01	0.00E+00	+	1.45E+01	8.95E-02	+
21	DE/Rand/1	2.19E+02	4.20E+02	3.22E+03	1.07E+03	+	4.56E+02	5.79E+02	+
	DE/Best/1	2.10E+02	3.38E+02	3.31E+03	1.20E+03	+	6.08E+02	7.53E+02	+
22	DE/Rand/1	1.88E+03	1.53E+03	6.00E+03	8.71E+02	+	3.38E+03	8.44E+02	+
	DE/Best/1	2.30E+03	6.90E+02	7.43E+03	6.57E+02	+	4.23E+03	7.75E+02	+
23	DE/Rand/1	7.86E+03	4.43E+02	6.82E+03	8.80E+02	-	4.59E+03	7.76E+02	-
	DE/Best/1	3.87E+03	1.64E+03	7.60E+03	6.71E+02	=	4.97E+03	9.57E+02	+
24	DE/Rand/1	2.70E+02	9.58E+00	3.02E+02	1.20E+02	+	2.79E+02	1.69E+01	+
	DE/Best/1	2.99E+02	2.85E+00	3.21E+02	2.07E+02	+	2.86E+02	1.93E+02	-
25	DE/Rand/1	2.97E+02	3.09E+00	2.83E+02	9.27E+00	=	2.95E+02	4.42E+00	=
	DE/Best/1	2.95E+02	4.34E+00	3.03E+02	1.80E+01	+	2.92E+02	5.40E+00	=
26	DE/Rand/1	2.01E+02	3.79E+01	2.33E+02	5.78E+01	+	2.04E+02	6.00E+01	+
	DE/Best/1	3.78E+02	9.85E+00	2.61E+02	1.24E+02	-	2.04E+02	7.92E+01	=
27	DE/Rand/1	1.25E+03	3.96E+01	1.28E+03	1.09E+02	+	1.18E+03	6.33E+01	-
	DE/Best/1	1.18E+03	4.34E+01	1.43E+03	6.97E+02	+	1.17E+03	9.37E+01	=
28	DE/Rand/1	2.64E+03	1.50E+03	4.43E+03	2.97E+03	+	4.26E+03	3.39E+03	+
	DE/Best/1	2.74E+03	2.54E+03	7.42E+03	9.88E+03	+	3.81E+03	4.38E+03	+

Table A.18: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{28}$ and population size $N_P = 5$ for dimension $D = 100$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	1.92E+04	1.99E+05	2.25E+05	3.74E+04	+	6.00E+04	2.16E+04	=
	DE/Best/1	2.16E+04	2.93E+04	1.87E+05	4.12E+04	+	1.12E+05	3.26E+04	+
2	DE/Rand/1	9.68E+07	1.02E+08	7.70E+09	3.69E+09	+	3.37E+08	2.22E+08	+
	DE/Best/1	1.75E+08	2.00E+08	1.18E+10	5.07E+09	+	8.77E+08	1.01E+09	+
3	DE/Rand/1	3.00E+11	9.77E+11	5.51E+19	2.58E+25	+	1.40E+12	5.23E+15	+
	DE/Best/1	3.46E+11	2.71E+12	3.08E+20	4.69E+28	+	4.27E+14	1.29E+20	+
4	DE/Rand/1	4.38E+05	9.53E+05	3.19E+05	4.06E+05	-	1.23E+05	1.11E+06	-
	DE/Best/1	2.69E+05	2.28E+06	3.09E+05	2.52E+05	+	1.94E+05	7.36E+05	-
5	DE/Rand/1	2.40E+04	4.64E+04	1.08E+05	5.63E+04	+	4.18E+04	8.46E+04	+
	DE/Best/1	9.10E+03	5.73E+04	1.12E+05	5.45E+04	+	7.49E+04	7.17E+04	+
6	DE/Rand/1	1.92E+03	5.56E+04	5.03E+04	2.26E+04	+	5.41E+03	4.00E+03	+
	DE/Best/1	2.93E+03	3.68E+03	6.42E+04	2.06E+04	+	1.73E+04	9.37E+03	+
7	DE/Rand/1	2.87E+04	1.34E+07	4.99E+06	2.05E+09	+	1.14E+06	9.27E+07	+
	DE/Best/1	4.01E+04	2.57E+08	2.64E+07	1.19E+10	+	6.69E+06	6.67E+09	+
8	DE/Rand/1	2.13E+01	2.87E-02	2.12E+01	5.11E-02	-	2.13E+01	2.38E-02	=
	DE/Best/1	2.13E+01	2.86E-02	2.12E+01	5.01E-02	-	2.12E+01	4.19E-02	=
9	DE/Rand/1	1.61E+02	1.84E+00	1.43E+02	7.41E+00	-	1.48E+02	4.58E+00	-
	DE/Best/1	1.39E+02	5.13E+00	1.51E+02	5.18E+00	+	1.50E+02	6.21E+00	=
10	DE/Rand/1	2.51E+03	1.56E+03	3.56E+04	9.79E+03	+	5.56E+03	2.76E+03	+
	DE/Best/1	5.26E+03	2.57E+03	3.98E+04	9.77E+03	+	1.19E+04	4.27E+03	+
11	DE/Rand/1	2.44E+03	8.08E+02	3.78E+03	9.22E+02	+	2.94E+03	1.00E+03	+
	DE/Best/1	2.49E+03	9.43E+02	4.29E+03	9.79E+02	+	3.68E+03	1.01E+03	+
12	DE/Rand/1	2.12E+03	5.59E+02	3.81E+03	1.15E+03	+	3.03E+03	6.86E+02	+
	DE/Best/1	3.56E+03	5.57E+02	4.07E+03	7.65E+02	+	3.44E+03	9.49E+02	-
13	DE/Rand/1	2.59E+03	5.68E+02	3.46E+03	8.68E+02	+	3.27E+03	8.76E+02	+
	DE/Best/1	2.93E+03	7.07E+02	4.06E+03	9.65E+02	+	4.10E+03	8.06E+02	+
14	DE/Rand/1	1.26E+04	1.43E+03	2.62E+04	1.54E+03	+	1.76E+04	1.67E+03	+
	DE/Best/1	1.48E+04	1.59E+03	3.07E+04	1.04E+03	+	2.25E+04	1.83E+03	+
15	DE/Rand/1	3.18E+04	7.00E+02	2.73E+04	1.46E+03	-	1.99E+04	3.47E+03	-
	DE/Best/1	3.23E+04	8.49E+02	2.93E+04	1.20E+03	-	2.46E+04	1.82E+03	-
16	DE/Rand/1	3.57E+00	3.12E-01	3.87E+00	2.40E-01	=	3.90E+00	2.61E-01	=
	DE/Best/1	3.79E+00	2.95E-01	2.87E+00	5.05E-01	-	3.30E+00	3.18E-01	=
17	DE/Rand/1	4.97E+03	2.99E+03	6.43E+03	9.35E+02	+	5.16E+03	1.42E+03	+
	DE/Best/1	5.52E+03	1.26E+03	6.11E+03	8.72E+02	=	6.81E+03	1.47E+03	=
18	DE/Rand/1	5.06E+03	3.64E+03	5.91E+03	1.33E+03	+	5.87E+03	1.23E+03	+
	DE/Best/1	5.93E+03	1.22E+03	5.91E+03	1.22E+03	=	6.20E+03	1.03E+03	=
19	DE/Rand/1	3.98E+05	9.63E+07	2.17E+07	2.32E+07	+	5.74E+06	2.03E+07	+
	DE/Best/1	1.15E+06	3.52E+07	1.64E+07	2.38E+07	+	2.61E+06	4.37E+07	+
20	DE/Rand/1	5.00E+01	0.00E+00	5.00E+01	0.00E+00	=	5.00E+01	0.00E+00	=
	DE/Best/1	5.00E+01	0.00E+00	5.00E+01	0.00E+00	=	5.00E+01	0.00E+00	=
21	DE/Rand/1	2.85E+03	2.62E+03	1.24E+04	3.06E+03	+	6.28E+03	1.66E+03	+
	DE/Best/1	2.97E+03	3.13E+03	1.27E+04	4.92E+03	+	9.86E+03	1.66E+03	+
22	DE/Rand/1	1.20E+04	1.92E+03	2.84E+04	1.45E+03	+	2.04E+04	1.68E+03	+
	DE/Best/1	1.48E+04	1.66E+03	3.24E+04	1.29E+03	+	2.41E+04	2.20E+03	+
23	DE/Rand/1	3.23E+04	6.97E+02	2.89E+04	1.25E+03	-	2.13E+04	3.52E+03	-
	DE/Best/1	2.52E+04	1.91E+03	3.24E+04	9.58E+02	=	2.58E+04	2.63E+03	+
24	DE/Rand/1	5.99E+02	5.83E+00	6.20E+02	1.67E+03	+	5.70E+02	9.21E+02	-
	DE/Best/1	5.98E+02	6.37E+00	9.28E+02	1.44E+03	+	5.85E+02	1.87E+03	-
25	DE/Rand/1	5.98E+02	5.98E+00	5.80E+02	2.31E+01	-	6.00E+02	5.75E+00	=
	DE/Best/1	5.99E+02	5.77E+00	6.18E+02	9.77E+01	+	5.88E+02	8.38E+00	=
26	DE/Rand/1	7.04E+02	6.14E+00	6.85E+02	1.44E+03	-	3.02E+02	7.42E+02	-
	DE/Best/1	6.42E+02	1.76E+01	7.24E+02	1.98E+03	+	6.93E+02	8.05E+02	+
27	DE/Rand/1	4.31E+03	6.93E+01	4.31E+03	1.98E+03	+	4.05E+03	1.03E+02	-
	DE/Best/1	4.29E+03	6.10E+01	5.00E+03	3.09E+03	+	4.21E+03	7.03E+02	=
28	DE/Rand/1	2.23E+04	1.09E+04	2.78E+04	2.64E+04	=	2.99E+04	3.98E+04	+
	DE/Best/1	2.71E+04	2.78E+04	2.66E+04	1.77E+04	=	2.98E+04	8.56E+04	=

Table A.19: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test (references=MDESM and MDEVM) for the MDESM with $F \in [0,2]$ and MDEVM with $\mathbf{F} \in [0,2]$ approaches denoted with indexes S and V respectively, versus the MDESM with $F \in [0.1,1.5]$ and MDEVM with $\mathbf{F} \in [0.1,1.5]$ methods denoted with index F on CEC2013 benchmark functions $f_1 - f_{20}$ and population size $N_P = 5$ for dimension $D = 30$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDESM		MDEVM		MDESM$_F$				MDEVM$_F$			
		Best	Std	Best	Std	Best	Std	W$_S$	W$_V$	Best	Std	W$_S$	W$_V$
1	DE/Rand/1	6.64E+02	5.48E+03	4.51E+00	4.02E+03	8.80E+03	1.06E+04	+	+	5.15E+00	5.43E+03	=	+
	DE/Best/1	2.15E+03	1.04E+04	6.29E-04	2.13E+03	2.71E+04	2.08E+04	+	+	4.02E+03	1.36E+04	+	+
2	DE/Rand/1	2.17E+07	4.76E+07	1.29E+07	2.46E+07	6.25E+07	1.65E+08	+	+	9.47E+06	1.80E+07	-	=
	DE/Best/1	2.33E+07	1.22E+08	6.33E+06	2.84E+07	2.38E+08	4.53E+08	+	+	4.94E+07	1.05E+08	+	+
3	DE/Rand/1	1.37E+10	5.27E+10	2.06E+10	6.46E+10	4.46E+10	6.89E+14	+	+	3.96E+09	1.20E+11	=	=
	DE/Best/1	2.78E+10	1.51E+12	4.42E+09	1.42E+12	4.64E+11	6.87E+20	+	+	9.69E+10	4.83E+12	+	+
4	DE/Rand/1	2.45E+04	4.31E+04	6.94E+04	2.73E+05	8.77E+04	1.14E+05	+	=	1.06E+05	3.40E+05	+	+
	DE/Best/1	3.76E+04	1.72E+05	3.61E+04	3.92E+05	5.10E+04	1.98E+05	+	=	6.42E+04	1.94E+05	+	=
5	DE/Rand/1	1.07E+03	2.99E+04	2.81E+02	2.60E+04	1.16E+04	4.26E+04	+	+	7.70E+01	2.23E+04	-	=
	DE/Best/1	1.40E+03	4.66E+04	1.17E-01	1.09E+03	1.17E+04	7.47E+04	=	+	4.28E+03	5.46E+04	=	+
6	DE/Rand/1	7.45E+01	3.64E+02	5.91E+01	1.31E+02	3.37E+02	1.21E+03	+	+	2.98E+01	1.73E+02	-	=
	DE/Best/1	3.12E+02	1.76E+03	2.38E+01	3.94E+02	2.35E+03	5.97E+03	+	+	4.03E+02	1.49E+03	+	+
7	DE/Rand/1	2.30E+02	5.01E+05	2.00E+02	2.16E+03	2.60E+02	3.22E+06	=	+	1.40E+02	1.37E+03	-	=
	DE/Best/1	3.75E+02	1.40E+07	1.90E+02	1.27E+05	5.09E+02	3.70E+07	=	+	2.51E+02	2.94E+06	-	+
8	DE/Rand/1	2.08E+01	6.63E-02	2.09E+01	4.30E-02	2.09E+01	5.04E-02	=	=	2.10E+01	4.95E-02	=	+
	DE/Best/1	2.10E+01	4.24E-02	2.09E+01	4.46E-02	2.09E+01	4.79E-02	=	=	2.10E+01	4.22E-02	=	=
9	DE/Rand/1	3.49E+01	2.00E+00	3.21E+01	2.08E+00	3.44E+01	2.39E+00	-	+	3.96E+01	1.09E+00	=	=
	DE/Best/1	3.66E+01	2.36E+00	3.88E+01	1.21E+00	3.68E+01	2.25E+00	=	-	3.44E+01	2.60E+00	-	-
10	DE/Rand/1	2.05E+02	5.62E+02	6.80E+01	2.47E+02	6.81E+02	1.46E+03	+	+	3.01E+01	1.10E+02	-	=
	DE/Best/1	4.90E+02	8.66E+02	1.49E+01	1.47E+02	2.23E+03	2.48E+03	+	+	9.02E+02	1.09E+03	+	+
11	DE/Rand/1	2.95E+02	2.42E+02	1.98E+02	2.19E+02	4.89E+02	3.92E+02	=	+	3.09E+02	3.12E+02	+	=
	DE/Best/1	4.10E+02	4.37E+02	2.67E+02	2.85E+02	5.93E+02	4.41E+02	+	+	4.41E+02	3.07E+02	+	+
12	DE/Rand/1	3.68E+02	2.78E+02	1.97E+02	1.61E+02	6.47E+02	2.72E+02	+	+	2.48E+02	2.13E+02	-	=
	DE/Best/1	5.65E+02	2.57E+02	3.81E+02	2.29E+02	7.67E+02	3.30E+02	+	+	5.50E+02	2.34E+02	-	+
13	DE/Rand/1	5.65E+02	2.99E+02	2.87E+02	1.52E+02	4.88E+02	2.57E+02	=	+	2.83E+02	1.48E+02	-	=
	DE/Best/1	8.22E+02	2.90E+02	4.62E+02	1.73E+02	5.75E+02	4.05E+02	=	+	7.15E+02	2.14E+02	-	+
14	DE/Rand/1	3.03E+03	8.82E+02	1.73E+03	1.25E+03	3.42E+03	7.29E+02	=	+	2.09E+03	1.77E+03	-	=
	DE/Best/1	4.32E+03	6.56E+02	2.75E+03	6.82E+02	5.56E+03	7.92E+02	+	+	3.09E+03	8.19E+02	-	+
15	DE/Rand/1	3.76E+03	7.84E+02	7.56E+03	3.90E+02	4.70E+03	1.09E+03	+	-	7.94E+03	3.62E+02	+	=
	DE/Best/1	4.91E+03	7.68E+02	5.45E+03	8.65E+02	6.43E+03	8.09E+02	+	+	3.97E+03	1.45E+03	=	-
16	DE/Rand/1	2.59E+00	3.34E-01	2.36E+00	3.06E-01	2.27E+00	3.91E-01	=	=	2.46E+00	3.43E-01	=	=
	DE/Best/1	1.23E+00	6.39E-01	2.45E+00	2.91E-01	1.71E+00	5.12E-01	=	=	2.03E+00	4.02E-01	=	=
17	DE/Rand/1	7.18E+02	4.77E+02	2.23E+02	3.70E+02	9.80E+02	3.92E+02	=	+	3.54E+02	2.65E+02	-	=
	DE/Best/1	1.23E+03	4.90E+02	5.33E+02	3.94E+02	9.82E+02	6.01E+02	=	+	1.10E+03	3.44E+02	-	+
18	DE/Rand/1	9.77E+02	3.03E+02	4.56E+02	2.95E+02	8.49E+02	3.45E+02	=	+	5.15E+02	2.15E+02	-	=
	DE/Best/1	1.07E+03	5.27E+02	5.28E+02	4.51E+02	1.34E+03	4.51E+02	=	+	1.05E+03	3.26E+02	-	+
19	DE/Rand/1	3.37E+04	3.34E+06	4.36E+02	1.01E+06	1.78E+05	3.75E+06	=	+	1.35E+03	1.65E+06	-	=
	DE/Best/1	4.96E+04	4.91E+06	7.73E+01	4.89E+05	1.99E+05	9.58E+06	+	+	1.04E+05	9.56E+06	=	+
20	DE/Rand/1	1.45E+01	1.25E-01	1.45E+01	2.17E-01	1.45E+01	2.06E-01	-	=	1.45E+01	1.99E-01	-	=
	DE/Best/1	1.45E+01	8.95E-02	1.45E+01	1.71E-01	1.45E+01	1.24E-01	=	+	1.45E+01	1.50E-01	=	+

Table A.20: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test (references=MDESM and MDEVM) for the MDESM with $F \in [0, 2]$ and MDEVM with $\mathbf{F} \in [0, 2]$ approaches denoted with indexes S and V respectively, versus the MDESM with $F \in [0.1, 1.5]$ and MDEVM with $\mathbf{F} \in [0.1, 1.5]$ methods denoted with index F on CEC2013 benchmark functions $f_{21} - f_{28}$ and population size $N_P = 5$ for dimension $D = 30$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDESM		MDEVM		MDESM$_F$				MDEVM$_F$			
		Best	Std	Best	Std	Best	Std	W$_S$	W$_V$	Best	Std	W$_S$	W$_V$
21	DE/Rand/1	4.56E+02	5.79E+02	2.19E+02	4.20E+02	1.55E+03	5.37E+02	+	+	3.02E+02	3.41E+02	-	=
	DE/Best/1	6.08E+02	7.53E+02	2.10E+02	3.38E+02	2.71E+03	7.59E+02	+	+	1.61E+03	5.19E+02	=	+
22	DE/Rand/1	3.38E+03	8.44E+02	1.88E+03	1.53E+03	4.22E+03	7.89E+02	+	+	1.83E+03	7.41E+02	-	=
	DE/Best/1	4.23E+03	7.75E+02	2.30E+03	6.90E+02	6.11E+03	8.36E+02	+	+	4.04E+03	5.49E+02	-	+
23	DE/Rand/1	4.59E+03	7.76E+02	7.86E+03	4.43E+02	5.15E+03	1.06E+03	+	-	7.89E+03	3.55E+02	+	=
	DE/Best/1	4.97E+03	9.57E+02	3.87E+03	1.64E+03	6.16E+03	9.21E+02	+	=	5.20E+03	1.28E+03	=	+
24	DE/Rand/1	2.79E+02	1.69E+01	2.70E+02	9.58E+00	2.83E+02	1.28E+01	+	=	2.87E+02	4.31E+00	+	=
	DE/Best/1	2.86E+02	1.93E+02	2.99E+02	2.85E+00	2.97E+02	2.60E+02	=	-	2.77E+02	9.62E+00	-	-
25	DE/Rand/1	2.95E+02	4.42E+00	2.97E+02	3.09E+00	2.97E+02	3.09E+00	=	-	3.00E+02	2.93E+00	=	=
	DE/Best/1	2.92E+02	5.40E+00	2.95E+02	4.34E+00	2.89E+02	6.11E+00	=	=	2.81E+02	7.03E+00	-	-
26	DE/Rand/1	2.04E+02	6.00E+01	2.01E+02	3.79E+01	2.06E+02	6.23E+01	+	+	3.55E+02	9.80E+00	=	=
	DE/Best/1	2.04E+02	7.92E+01	3.78E+02	9.85E+00	2.55E+02	1.34E+02	=	-	2.04E+02	5.65E+01	-	-
27	DE/Rand/1	1.18E+03	6.33E+01	1.25E+03	3.96E+01	1.20E+03	7.28E+01	=	-	1.28E+03	3.09E+01	+	=
	DE/Best/1	1.17E+03	9.37E+01	1.18E+03	4.34E+01	1.21E+03	8.20E+01	=	=	1.07E+03	8.36E+01	-	-
28	DE/Rand/1	4.26E+03	3.39E+03	2.64E+03	1.50E+03	5.81E+03	3.61E+03	=	+	2.13E+03	1.19E+03	-	=
	DE/Best/1	3.81E+03	4.38E+03	2.74E+03	2.54E+03	5.77E+03	9.04E+03	=	+	4.93E+03	2.93E+03	=	+

Table A.21: Best error (Best), standard deviation (Std), and Wilcoxon rank-sum (W) test(reference=MDEVM) for MDEVM against MDE and MDESM schemes on CEC2013 benchmark functions $f_1 - f_{28}$ and population size $N_P = 5$ for dimension $D = 30$ with $NFC_{Max} = 5,000D$ and mutation vector (MV) schemes "DE/Rand/1" and "DE/Best/1".

f	MV	MDEVM		MDE			MDESM		
		Best	Std	Best	Std	W	Best	Std	W
1	DE/Rand/1	8.30E-09	5.75E-02	4.93E+04	1.39E+04	+	1.08E+01	4.88E+03	+
	DE/Best/1	5.30E-07	2.91E+03	4.18E+04	1.88E+04	+	8.64E+02	1.06E+04	+
2	DE/Rand/1	1.05E+06	2.47E+06	7.66E+08	4.55E+08	+	2.08E+06	1.72E+07	+
	DE/Best/1	1.67E+06	5.83E+06	3.97E+08	1.36E+09	+	2.70E+07	6.34E+07	+
3	DE/Rand/1	1.43E+09	3.39E+10	1.06E+13	1.31E+21	+	1.52E+10	5.73E+10	+
	DE/Best/1	1.95E+09	4.19E+10	2.22E+14	1.25E+23	+	3.65E+10	2.19E+11	+
4	DE/Rand/1	1.98E+04	3.78E+04	7.65E+04	7.34E+04	+	5.28E+03	1.56E+04	-
	DE/Best/1	2.67E+03	7.18E+04	9.41E+04	8.52E+04	+	1.26E+04	8.06E+04	=
5	DE/Rand/1	9.57E-09	1.45E-01	1.74E+04	4.95E+04	+	2.97E+03	2.86E+04	+
	DE/Best/1	6.13E-01	8.22E+03	5.66E+03	4.59E+04	+	5.31E+03	3.28E+04	+
6	DE/Rand/1	1.51E+01	1.03E+01	5.84E+03	5.90E+03	+	5.00E+01	2.02E+02	+
	DE/Best/1	2.65E+01	7.51E+01	8.00E+03	1.36E+04	+	3.59E+02	8.52E+02	+
7	DE/Rand/1	1.01E+02	2.17E+02	4.06E+03	4.01E+06	+	2.02E+02	2.29E+05	+
	DE/Best/1	2.22E+02	4.05E+04	1.47E+04	2.79E+08	+	3.48E+02	1.21E+07	+
8	DE/Rand/1	2.09E+01	4.69E-02	2.09E+01	4.45E-02	+	2.09E+01	5.85E-02	=
	DE/Best/1	2.09E+01	4.30E-02	2.09E+01	8.67E-02	=	2.08E+01	5.69E-02	=
9	DE/Rand/1	3.17E+01	1.32E+00	3.34E+01	2.36E+00	+	3.44E+01	2.05E+00	=
	DE/Best/1	3.53E+01	1.73E+00	3.85E+01	2.47E+00	+	3.83E+01	2.54E+00	+
10	DE/Rand/1	4.68E-02	4.66E-01	7.07E+03	2.07E+03	+	2.63E+01	3.13E+02	+
	DE/Best/1	5.52E+00	6.15E+01	7.12E+03	3.17E+03	+	3.43E+02	7.12E+02	+
11	DE/Rand/1	1.23E+02	1.35E+02	6.25E+02	3.08E+02	+	4.44E+02	3.06E+02	+
	DE/Best/1	4.48E+02	3.46E+02	7.50E+02	4.18E+02	+	6.84E+02	3.49E+02	+
12	DE/Rand/1	7.96E+01	1.34E+02	5.93E+02	2.36E+02	+	4.16E+02	2.60E+02	+
	DE/Best/1	2.78E+02	2.43E+02	9.46E+02	1.68E+02	+	6.10E+02	3.90E+02	+
13	DE/Rand/1	2.55E+02	1.31E+02	5.89E+02	2.74E+02	+	4.46E+02	3.63E+02	+
	DE/Best/1	4.94E+02	2.66E+02	8.18E+02	3.11E+02	+	8.46E+02	3.09E+02	+
14	DE/Rand/1	1.64E+03	3.54E+02	5.53E+03	7.15E+02	+	3.23E+03	6.17E+02	+
	DE/Best/1	2.53E+03	5.05E+02	6.64E+03	8.87E+02	+	3.70E+03	7.74E+02	+
15	DE/Rand/1	7.84E+03	4.05E+02	5.91E+03	6.72E+02	-	3.91E+03	7.90E+02	-
	DE/Best/1	3.74E+03	1.94E+03	7.45E+03	5.49E+02	+	4.77E+03	9.94E+02	=
16	DE/Rand/1	1.80E+00	3.03E-01	1.83E+00	4.28E-01	=	1.98E+00	2.90E-01	=
	DE/Best/1	1.97E+00	3.43E-01	1.34E+00	7.51E-01	=	1.42E+00	5.64E-01	=
17	DE/Rand/1	1.56E+02	1.39E+02	8.48E+02	4.13E+02	+	1.04E+03	3.12E+02	+
	DE/Best/1	6.54E+02	4.70E+02	1.30E+03	4.73E+02	+	1.08E+03	4.38E+02	+
18	DE/Rand/1	2.65E+02	1.92E+02	1.15E+03	2.80E+02	+	5.84E+02	6.06E+02	+
	DE/Best/1	4.51E+02	4.04E+02	1.19E+03	3.36E+02	+	9.86E+02	3.82E+02	+
19	DE/Rand/1	1.26E+01	2.80E+02	1.51E+06	4.42E+06	+	2.29E+04	2.88E+06	+
	DE/Best/1	2.59E+02	9.25E+05	1.55E+05	9.12E+06	+	1.00E+05	4.59E+06	+
20	DE/Rand/1	1.48E+01	4.64E-02	1.50E+01	7.99E-06	+	1.45E+01	4.32E-03	-
	DE/Best/1	1.45E+01	2.02E-01	1.46E+01	9.43E-02	+	1.45E+01	1.85E-01	+
21	DE/Rand/1	2.00E+02	8.52E+01	3.78E+03	5.20E+02	+	3.56E+02	4.15E+02	+
	DE/Best/1	2.32E+02	4.24E+02	2.87E+03	1.36E+03	+	1.05E+03	6.52E+02	+
22	DE/Rand/1	1.47E+03	1.49E+03	5.99E+03	1.04E+03	+	3.36E+03	8.46E+02	+
	DE/Best/1	2.32E+03	7.27E+02	7.33E+03	7.47E+02	+	4.57E+03	1.01E+03	+
23	DE/Rand/1	8.30E+03	2.54E+02	6.84E+03	5.07E+02	-	4.38E+03	7.03E+02	-
	DE/Best/1	4.06E+03	1.73E+03	7.70E+03	7.27E+02	+	5.29E+03	1.05E+03	=
24	DE/Rand/1	2.98E+02	3.07E+00	3.01E+02	1.03E+02	+	2.93E+02	7.15E+00	=
	DE/Best/1	2.65E+02	9.92E+00	3.17E+02	2.20E+02	+	2.96E+02	1.26E+01	+
25	DE/Rand/1	3.00E+02	2.79E+00	2.93E+02	5.56E+00	=	2.86E+02	5.26E+00	-
	DE/Best/1	2.99E+02	2.67E+00	2.99E+02	3.24E+01	-	2.92E+02	6.46E+00	=
26	DE/Rand/1	3.54E+02	1.24E+01	2.96E+02	3.29E+01	=	3.82E+02	6.29E+00	+
	DE/Best/1	2.02E+02	4.39E+01	2.81E+02	1.41E+02	+	3.97E+02	7.69E+00	+
27	DE/Rand/1	1.09E+03	2.79E+01	1.26E+03	1.98E+02	+	1.05E+03	7.79E+01	-
	DE/Best/1	9.90E+02	9.80E+01	1.43E+03	3.53E+02	+	1.26E+03	9.15E+01	+
28	DE/Rand/1	4.30E+02	1.06E+03	6.59E+03	2.24E+03	+	2.92E+03	2.71E+03	+
	DE/Best/1	2.19E+03	1.90E+03	6.49E+03	3.23E+03	+	6.28E+03	3.27E+03	+

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