Joint Channel Pairing and Power Allocation Optimization in Two-Way Multichannel Relaying

by

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Abstract

We consider two-way amplify-and-forward relaying in a multichannel system with two end nodes and a single relay, using a two-slot multi-access broadcast (MABC) as well as time-division broadcast (TDBC) relaying strategies. We investigate the problem of joint subchannel pairing and power allocation to maximize the achievable sum-rate in the network, under an individual power budget at each node. To solve this challenging joint optimization problem, an iterative approach is proposed to decompose the problem into pairing optimization and joint power allocation optimization, and solve them iteratively.

For given power allocation, we first consider the problem of subchannel pairing at the relay to maximize the achievable sum rate in TDBC-based network. Unlike in the one-way relaying case, our result shows that there exists no explicit SNR-based subchannel pairing strategy that is optimal for sum-rate maximization for two-way relaying.

Nonetheless, for TDBC-based two way relaying, we formulate the pairing optimization as an axial 3-D assignment problem which is NP-hard, and propose an iterative optimization method to solve it with complexity $\mathcal{O}(N^3)$. Based on SNR over each subchannel, we also propose sorting-based algorithms for scenarios with and without direct link, with a low complexity of $\mathcal{O}(N \log N)$.

For the joint power allocation at the relay and the two end nodes, we propose another iterative optimization procedure to optimize the power at the two end nodes and at the relay iteratively. By using different forms of optimization parameters, the sum-rate maximization problem turns out to be convex and the optimal solutions can be obtained for each subproblem.

The simulation first demonstrates the proposed sorting-based pairing algorithm offers the performance very close to the iterative optimization method. Then, shows the gain of joint optimization approach over other pairing-only or power-allocation-only optimization

approaches.

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Chapter 1

Introduction

1.1 Overview

As our society moves toward information centricity, the need to have information accessible at any time and anywhere takes on a new level. Wireless technology has become an indispensable part of our life. Remote controllers, vehicle smart keys, cellular system and WiFi access are all examples of wireless communication systems.

Traditional wireless communications are based on point-to-point communication, i.e., only two nodes are involved in the communication network. These two nodes are: the Base Station (BS) and Mobile Station (MS) in a cellular environment, access point and laptop in wireless Local Area Networks (LANs), or two MSs in peer-to-peer communications. One of the most severe impairments to wireless communications is channel fading. Fading results in a significant loss in the transmitted power compared to the noise power. Hence, when the signal experiences a deep fading, the receiver can not decode it. So far, substantial research has been done and many techniques have been established to reduce the influence of fading. A widely used technique to combat the effects of channel fading is diversity. Typical examples include spatial, time and frequency diversity. The diversity shows how to improve the reliability of the transmission by using the available resources and sending multiple copies of the initial information to the destination.

Recently, cooperative communication has attracted great attention due to the ability to improve the spectral efficiency, extend the coverage area, and mitigate channel impairments [1, 2]. To fully utilize these advantages in cooperative communication systems, efficient wireless resource allocation is significant. Specifically, the problem formulation may differ remarkably in terms of optimization objectives, relay strategies, transmit power constraints, and system frameworks [3].

1.2 Relay Network

Relay network is a critical branch of cooperative wireless communication schemes, where both terminal nodes (or the sources and the destinations) are exchanging their signal with the help of one or multiple intermediate nodes. Because this bidirectional communication can improve bandwidth efficiency, it has recently receive substantially attention [4]. In such circumstance, the transceiver may not transmit signal direct with receiver because of the poor quality of a direct transmission link or long distance.

The main idea of relay network is firstly proposed by the Van Der Meulen (in 1971) [5], who studied the upper and lower bounds of the channel capacity, and proved relay technique can improve spectral efficiency and channel performance. The wireless relay networks are generally categorized as: relay models, resource allocations, diversity combination approach, performance metrics, coding strategies and particular relay modes [6]. For the relay models, it can be classified into: one-way relaying, two-way relaying, multiple access relaying and multi-node model. Regarding to the resource allocation term, it can be categorized as: orthogonal channel, duplex system and coding level.

Applying relaying techniques in wireless networking can potentially improve the entire network performance, such as capacity and transmission range [7]. Relay network deals with the situation that one or more multiple intermediate nodes consciously help transceivers to get the information from the other transceivers. The introduction of relay nodes creates more degrees of freedom in the system design, which can help to improve the performance, but also complicates the design process.

1.3 Multichannel Communication

With the strong demand for multimedia services and broadband wireless applications with higher data rate and wider bandwidth with fast and seamless connectivity everywhere and any time. Multichannel communication technique has initiated to improve the wireless system performance. Digital bandpass modulation techniques can be broadly classified in two categories. The first is single-carrier modulation, where data is transmitted by using a single radio frequency carrier. The other is multicarrier modulation, where data is transmitted by simultaneously modulating multiple frequency carriers. The basic idea of multi-carrier modulation is to divide the transmitted bit stream into different sub-streams and send these over many sub-channels. The concept of multichannel transmission was first explicitly proposed by Chang [8] in 1966. The sub-channels are orthogonal under ideal propagation conditions. The number of sub-streams is chosen to ensure that each sub-channel has a bandwidth less than the channel coherence bandwidth, so the sub-channels experience relatively flat fading. Therefore, the intersymbol interference (ISI) on each subchannel is small. Examples of such multichannel system include an Orthogonal Frequency Division Multiplexing (OFDM) system, where multiple subchannels (or subcarriers) are used for transmission. Combining relay network with OFDM-based transmission is a powerful technique to increase date rates over broadband wireless network. In order to exploit the potential abilities of OFDM-based relay networks, it is important to design efficient resource allocation strategies such as: deciding which relay node to cooperate with, which set of subchannels to operate on, and with how much power to transmit the signals [9].

In OFDM [10–12], the entire channel is divided into many narrow-band subchannels, which are transmitted in parallel to maintain high-data-rate transmission and, at the same time, to increase the system duration to combat ISI [13]. OFDM is attractive because it admits relatively easy solutions to some difficult challenges that are encountered when using single-carrier modulation schemes on wireless channels. It has been the underlying system for the current 4G and future wireless system such as LTE [14] and LTE-advanced [15]. What is more, the emerging next-generation wireless systems adopt a multichannel relaying architecture for broadband access and coverage improvement.

1.4 One-Way Relaying

In classical one-way relaying network, there are three nodes: one source node, one destination node, and one relay node. The transmission of signal completes in two time slots. In the first time slot, the source node send the data to the relay node, while in second time slot, the relay transfers processed signals to the destination node. Many transmitting schemes have proposed in the literature based on different relaying techniques, such as the amplify-and-forward (AF) [16, 17], decode–and-forward (DF) [16, 18, 19], selective relaying (SR) [16], compress-and-forward(CF) [17], coded cooperation (CC) [20] etc. AF relays retransmit the signal without decoding while DF relays decode the received signal, encode the signal again, and transmit. The AF technique is limited to amplify and adjust the phase of the received signal before retransmitting it to the destination because there is no need to detect the transmitted signals at the relays. While for DF technique, it is usually used when noise at the relay is high and amplifying the signals will amplify the noise as well [21]. However, the drawback side of DF is power consuming and increasing the design complexity of the relays [22]. When the source and relay node power is limited, the relay node and the end nodes know channel state information (CSI), the power allocation are the key to improve the entire system performance [23].

For one-way relaying, under given power allocation, channel pairing design and optimization have been investigated for various network setups [24–27]. Joint optimization of system resources, such as channel pairing, power allocation and channel assignment, has been investigated in [26, 28], where efficient numerical algorithms were devised to solve the complex joint optimization problems. In [29], distributed relay beamforming under individual relay power budget is researched.

1.5 Two-Way Relaying

In traditional half-duplex dual-hop AF relay networks, the source and destination nodes require four time slots to finish both the incoming and outgoing transmissions, which makes the spectral efficiency lower. The full-duplex mode, although better than the half-duplex, it is difficult to eliminate the self-interference at relay node. In order to compensate the drawbacks, Shannon in [30] firstly proposed the concept of two-way relaying communication, [4] further indicated that the spectral efficiency of two-way relaying is remarkable higher than the one-way relaying. Comparing with one-way relaying, two-way relaying offers substantial advantage in achievable sum-rate due to its bi-directional concurrent data transmission. The main idea of two-way relaying is to let relay re-transmit a processed version of the signal it receives from both terminal nodes, and each node can recover the transmitted data from the original node after cancelling the self-interference generated by its own transmission. Since the process is similar to network coding, but is done at symbol level, it is also called two-way relay with analog network coding [31, 32]. Two-way channels without relay were first proposed by [30]. It was later introduced in [5] from information theoretic point of view. For a two-way relaying system, beyond conventional relay network problems, there exists channel pairing problem, where the relay can select outgoing channels for data forwarding.

There are many literature related to this area. Under total power constrain, [33] present an optimal joint relay selection and power allocation scheme to achieve the maximization of SNR in two-way relaying network. The authors show that this problem has a close-form solution and requires only a single integer parameter to be broadcasted to all relays. While in [34], the system model is two single-antenna transceivers and n single-antenna relays. It aims at optimally obtaining the beamforming coefficients as well as the transceivers' powers. It proposed two approaches to achieve their goal, one is minimizing the total power subject to two constrains on the transceiver's received SNR, another is an SNR balancing technique. In [35], energy-efficient relay selection and power allocation scheme is studied for two-way relay channel based on analog network coding, with the object of minimizing power consumption at required end-to-end rates. Four new half-duplex protocols and four existing half-duplex protocols are compared in [36], where a comprehensive treatment of 8 possible half-duplex bi-directional relaying protocals are discussed. A tone permutation at the relay and power allocation for relay and end nodes are studied in [37], where a dual decomposition technique is proposed for power allocation and a greedy strategy is employed for tone permutation.

1.5.1 MABC Two-Way Relaying

For the two-phase multi-access broadcast (MABC) relaying strategy, the choices of incoming/outgoing channels between the relay and the two end nodes are tied to each other. There are two time slots in MABC scheme, at first time slot, terminal 1 and terminal 2 transmit data to relay, while at second time slot, relay transmits signal back to terminal 2 and terminal 1. The strategy showed in Fig. 1.1. An example of the system model and two way relaying pairing is given in Fig. 1.2, 1.3. Since two sources know their own transmitted messages, they can subtract the self-interference before decoding.

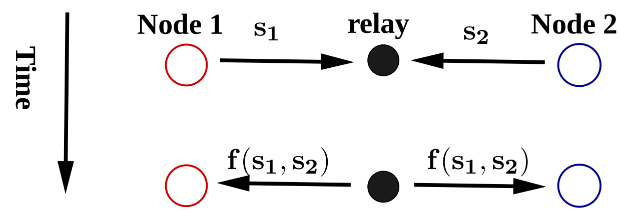

Figure 1.1: MABC-based two way relaying scheme

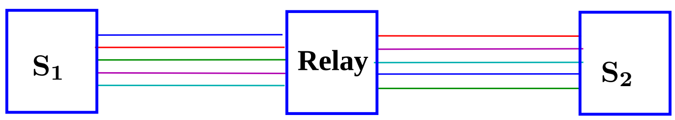

Figure 1.2: MABC-based two way relaying

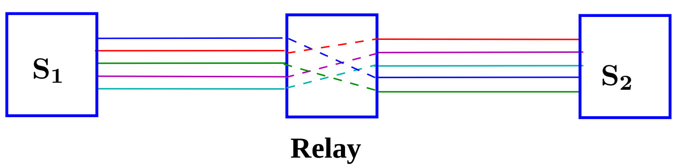

Figure 1.3: MABC-based two way relaying pairing

Furthermore, simultaneous transmission at the end nodes and at the relay complicates the received SNR structure, thereby making power allocation for two-way relaying a more difficult task. Due to these factors, joint channel pairing and power allocation problem is especially challenging.

There is few existing work addressing joint channel pairing and power allocation design in a two-way relay network. Under given power allocation, the pairing problem for MABC two-way relaying is considered in [38] and [39], where a numerical optimization algorithm and low-complexity pairing strategies were proposed, respectively. Pairing algorithms were also proposed in [40] for time-division broadcast (TDBC) two-way relaying.

Under the total power constraint in the network, the optimal power allocation in an MABC two-way OFDM system was obtained in [7], and joint power allocation and subcarrier assignment for a multi-relay system was investigated in [41]. In [42], considering a two-way DF MABC relaying network, where the author propose a relay-selection technique to improve the diversity gain. Joint channel pairing and power allocation problem for individual power constraints remains an open problem.

1.5.2 TDBC Two-Way Relaying

For time division broadcast(TDBC) two-way relaying scheme, we have three time slots to complete the entire transmission. As you can see in Fig. 1.4, at first time slot, Node 1 transmits signal to relay. At second time slot, Node 2 transmits data to relay. At third time slot, relay combines the received signal and forwards it to Node 1 and Node 2.

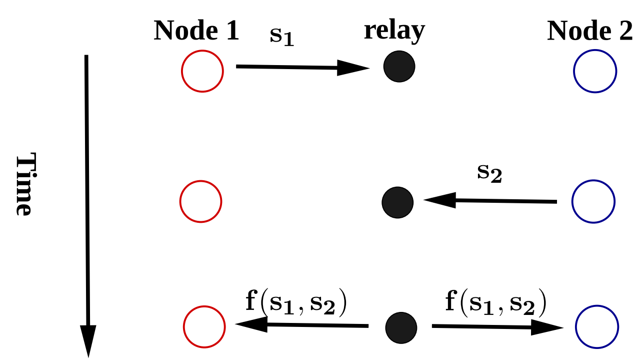

Figure 1.4: TDBC-based two way relaying scheme

Since the transmission from each terminal node to the relay is performed in different slots, channel pairing is no longer just between incoming and outgoing channels, but is among the two incoming channels and the outgoing channel. For broadband systems with large number of subchannels, designing efficient pairing strategies is thus important. An example of the system model is given in Fig. 1.5.

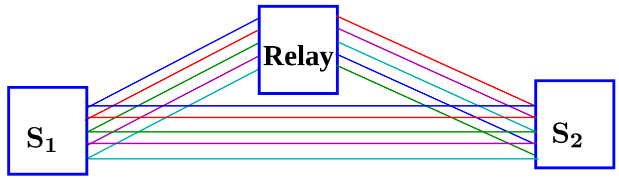

Figure 1.5: TDBC-based two way relaying

There have been many recent works on channel pairing design and optimization in two-way relaying in various network setups, either under given power allocation [24–27], or jointly with other resources, such as power and/or channel assignment in a multi-user case [26, 28, 43]. While in [44], two major AF-based protocals, that is, analog network coding and TDBC, in bidirectional relay networks with relay selection are studied. In [45], buffer in relay station is discussed under TDBC scheme.

In [46], a multiuser two-way relay system with TDBC protocol where multiple nodes compete to exchange information with another multiple nodes through the help of a single half-duplex AF relay is considered. A tight closed-form lower bound for the system outage probability over Nakagami-m fading channels is discussed with integer fading parameter. An asymptotic expression for the outage probability in the high SNR regine is also acquired. A multiuser two-way relaying network with TDBC scenario, where one multi-antenna BS and one out of M single-antenna MSs exchanging signal with the help of one single-antenna AF relay is considered in [47]. The authors first present an optimal joint user-antenna selection strategy, which minimizes the network outage probability. Then, by fixing the power allocation parameter at the relay, a low-complexity suboptimal algorithm is proposed.

In [48], a diversity-multiplexing tradeoff of the four-phase DF protocol is established in the half-duplex, non-separated two-way relay channel. The multiple access channel phase of hybrid broadcast protocol is not necessary to achieve optimal performance as compared with TDBC protocol.An energy-efficient power allocation strategy for MABC and TDBC two-way systems with multi-relay, under a specific transmission data rate of terminal nodes, aiming to minimize the system energy consumption is proposed in [49].

Adaptive Relay-Assisted/ Direct Transmission (ARDT) proposed in [50] is a simple and efficient protocol which adaptively applies the direct link between two terminal nodes with only channel state information at one side, and it validly enhances the spectrum efficiency of TDBC scenario. Joint power allocation and relay selection for multi-relay network was combined with ARDT protocol in [51], where each relay optimize its own forwarding power to maximize the minimum end to end SNR towards two terminal nodes, and the optimal relay is then selected to assist.

1.6 Motivation

Many existing research work in two-way relaying network field are focused on channel pairing design under given power allocation or channel assignment in a multi-user case. However, due to the complexity of joint channel pairing and power allocation problem for individual power constraints, joint optimization problem remains an open problem.

For MABC-based two-way relaying system, We aim to maximize the achievable sum rate in the network by jointly optimizing pairing strategy and power allocation at each node, under an individual power budget at each node. To achieve this goal, we proposed an iterative approach to decomposed the problem into pairing optimization and joint power allocation optimization, and solve them iteratively.

For TDBC-based two-way relaying system, the problem of channel pairing is more complicated due to the concurrent transmission than that of one-way relaying. Specifically, the choice of incoming/outgoing channels between the relay and two terminal nodes are tied to each other, even though channel strength on each side can be drastically different. In addition, the received signals at the relay from both side create additional noise amplification to the forwarded signal to be considered. For a given power allocation, in one way relay, the optimal pairing is a simple strategy based on sorted-SNR at the first and second hops. This strategy is attractive due to its optimality and low-complexity with $\mathcal{O}(N \log N)$ for N subchannels. In light of these results, for TDBC-based two-way relaying, one natural question to ask is whether a similar explicit SNR-based pairing scheme would still be optimal. After we proof that there exist no explicit SNR-based subchannel pairing strategy that is optimal for sum rate maximization, we propose two suboptimal pairing strategies, then use similar iterative approach as MABC-based two-way relaying to solve the joint pairing and power allocation problem in TDBC-based two-way relaying network.

1.7 Thesis Contribution

In this thesis, we consider joint optimization of channel pairing and power allocation design in a multichannel MABC-based as well as TDBC-based two-way relying system. We aim to maximize the achievable sum rate in the network by jointly optimizing pairing strategy and power allocation at each node, under an individual power budget at each node.

Joint Pairing and Power Allocation Optimization – MABC-Based Two-Way Relaying We propose an iterative approach to solve the challenging joint optimization problem. Specifically, the problem is decomposed into pairing and joint power allocation problems and solved iteratively. For the joint power allocation at the relay and two end nodes, we propose another iterative optimization procedure to optimize the power at the two end nodes and at the relay iteratively. By transforming the SNR expression with respect to different form of optimization parameters, each power optimization problem turns out to be a convex problem and the optimal solutions can be obtained. The simulation performance demonstrates the gain of joint optimization approach over other pairing-only or power-allocation-only optimization approaches.

Joint Pairing and Power Allocation Optimization – TDBC-Based Two-Way Relaying

First, we show that, unlike one-way relaying, there exist no explicit SNR-based subchannel pairing strategy that is optimal for sum rate maximization in TDBC-based two-way relaying, regardless whether direct link exists or not.

A few low-complexity suboptimal pairing strategies are then proposed. We first formulate the pairing optimization as an axial 3-D assignment problem (3-DAP) which is NP-hard, and propose an iterative optimization method to solve it with complexity $\mathcal{O}(N^3)$. Based on SNR over each subchannel, we also propose sorting-based algorithms for both with and without direct link scenarios. The algorithms have complexity of only $\mathcal{O}(N \log N)$, which is the same as that of the one-way relaying case. The complexity reduction is substantial especially for broadband multichannel systems of 10-20MHz bandwidth with $N \geq 1024$. The simulation results also show the proposed algorithm offers the performance very close to the iterative optimization method.

We propose a similar iterative approach as that in the MABC-based two-way relaying to solve the challenging joint optimization problem. Specifically, the problem is decomposed into pairing and joint power allocation problems and solved iteratively. For the joint power allocation at the relay and two end nodes, compare to MABC-based two-way relaying, we propose one more step to optimize the fraction α at relay. By transforming the SNR expression with respect to different form of optimization parameters, each power optimization problem turns out to be a convex problem and the optimal solutions can be obtained. The simulation performance demonstrates the gain of joint optimization approach with different pairing strategy over other pairing-only or power-allocation-only optimization approaches with or without direct link.

1.8 Thesis Organization

In Chapter 2, joint pairing and power allocation optimization of MABC-based two way relaying will be proposed. In Chapter 3, joint pairing and power allocation optimization of TDBC-based two way relaying will be proposed. Final conclusions and necessary mathematical derivations will be given in Chapter 4 and 5, respectively.

Chapter 2

Joint Pairing and Power Allocation Optimization in Multichannel MABC-Based Two-Way Relaying

2.1 System Model

We consider an MABC-based two-way relay network including two end nodes (Nodes 1 and 2) and one relay node, all equipped with single antenna. All nodes exchange information in a multichannel system with N subchannels, where each subchannel experiences frequency flat fading. We assume that transmitting and receiving signals to and from the relay are over the same set of N subchannels, and that the relay channels to and from each end node are reciprocal.

Under the MABC relay protocol, in the first phase, both end nodes transmit their signals to the relay at the same time. The received signal at the relay over the nth subchannel is given by

$$y_{rn} = \sqrt{P_{1n}} h_{1n} s_{1n} + \sqrt{P_{2n}} h_{2n} s_{2n} + v_{rn} \tag{2.1}$$

where s_{1n} and s_{2n} are signals transmitted by Nodes 1 and 2 with unit-power over the nth subchannel, respectively, P_{1n} and P_{2n} are the transmit power at Nodes 1 and 2 over the nth subchannel, respectively, h_{1n} and h_{2n} are the channel coefficients over the nth subchannel from the relay to Nodes 1 and 2, respectively, and v_{rn} is additive white Gaussian noise (AWGN) with variance σ^2 at the relay receiver over the nth subchannel.

In the second phase, the relay normalizes the power of the received signal over the nth subchannel and retransmits it over the mth subchannel with power P_{rm}. The received signals at Nodes 1 and 2 over the mth subchannel are given by

$$y_{1,mn} = h_{1m} w_{mn} y_{rn} + v_{1m},$$
$$y_{2,mn} = h_{2m} w_{mn} y_{rn} + v_{2m}$$

where v_{1m} and v_{2m} are AWGN with variance σ^2 over the mth subchannel at the receivers of nodes 1 and 2, respectively, and w_{mn} is the relay power coefficient given by

$$w_{mn} = \sqrt{\frac{P_{rm}}{P_{1n}|h_{1n}|^2 + P_{2n}|h_{2n}|^2 + \sigma^2}}. \tag{2.2}$$

We assume that the channel pairing scheme is known at Nodes 1 and 2. Thus, each end node can cancel the self-interference in its respective received signal, before performing detection. The residual signals after self-cancellation at Nodes 1 and 2 are given by

$$\tilde{y}_{1,mn} = \sqrt{P_{2n}} w_{mn} h_{1m} h_{2n} s_{2n} + w_{mn} h_{1m} v_{rn} + v_{1m}$$
$$\tilde{y}_{2,mn} = \sqrt{P_{1n}} w_{mn} h_{2m} h_{1n} s_{1n} + w_{mn} h_{2m} v_{rn} + v_{2m}.$$

The post-self-cancellation received SNR on the mth subchannel at Nodes 1 and 2, for the signal transmitted over the nth subchannel from each respective source, is respectively given by

$$\mathrm{SNR}_{1,mn} = \frac{P_{2n}|w_{mn}|^2|h_{1m}|^2|h_{2n}|^2}{\sigma^2(1+|w_{mn}|^2|h_{1m}|^2)} \tag{2.3}$$
$$\mathrm{SNR}_{2,mn} = \frac{P_{1n}|w_{mn}|^2|h_{1n}|^2|h_{2m}|^2}{\sigma^2(1+|w_{mn}|^2|h_{2m}|^2)}. \tag{2.4}$$

The pairing of the incoming subchannels to the relay and the outgoing subchannels to the end nodes can be described using a permutation function $p(\cdot)$, where $m = p(n)$, for $n = 1, \cdots, N$. Different permutation functions provide different pairing schemes. As a special case, the traditional two-way relaying with direct pairing, *i.e.*, the same subchannel is used for incoming signal and outgoing signal at the relay, can be expressed as $n = p(n)$. An example of the relay system with $N = 2$ is given in Fig. 2.1.

The sum-rate for Nodes 1 and 2 achieved over the paired nth and mth subchannels, under a given pairing function $m = p(n)$, can express as

$$R_{mn} = \log\left(1+\mathrm{SNR}_{1,mn}\right) + \log\left(1+\mathrm{SNR}_{2,mn}\right). \tag{2.5}$$

The overall sum-rate achieved in the multichannel system is given by

$$R = \sum_{n=1, m=p(n)}^{N} R_{mn}. \tag{2.6}$$

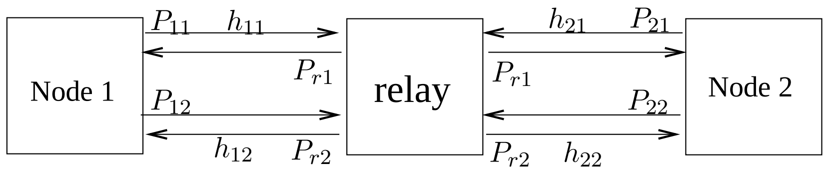

Figure 2.1: An MABC two-way relay system with $N = 2$.

2.2 Joint Two-Way Pairing and Power Optimization

Substituting the expression of w_{mn} in (2.2) into the SNR expressions in (2.3) and (2.4), we can re-write $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ in terms of P_{1n}, P_{2n}, and P_{rm} as

$$\mathrm{SNR}_{1,mn} = \frac{P_{2n}P_{rm}|h_{1m}|^2|h_{2n}|^2/\sigma^2}{\sigma^2 + P_{rm}|h_{1m}|^2 + P_{1n}|h_{1n}|^2 + P_{2n}|h_{2n}|^2} \tag{2.7}$$

$$\mathrm{SNR}_{2,mn} = \frac{P_{1n}P_{rm}|h_{1n}|^2|h_{2m}|^2/\sigma^2}{\sigma^2 + P_{rm}|h_{2m}|^2 + P_{1n}|h_{1n}|^2 + P_{2n}|h_{2n}|^2}. \tag{2.8}$$

Thus, the sum-rate R_{mn} in (2.5) is a function of $\{P_{1n}, P_{2n}, P_{rm}\}$. Let $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_r$ denote the $N \times 1$ vectors containing the power allocated on each subchannel at Nodes 1 and 2, and at the relay, respectively, with $[\mathbf{p}_1]_n = P_{1n}$, $[\mathbf{p}_2]_n = P_{2n}$, and $[\mathbf{p}_r]_m = P_{rm}$. Let P_{tot} denote the power budget at each node[1]. Our goal is to maximize the sum rate in (2.6) by jointly optimizing the subchannel pairing strategy $p(\cdot)$ and the power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$ under the individual power constraint P_{tot} at each node. We formulate this joint optimization problem as follows

[1]We assume the same power budget at each node for simplicity.

$$
\begin{aligned}
\textbf{(P0)}: \max_{\boldsymbol{\Phi},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_r} \quad & \sum_{n=1}^{N}\sum_{m=1}^{N} \phi_{mn} R_{mn} \\
\text{s.t.} \quad & \sum_{n=1}^{N} \phi_{mn} = 1, \sum_{m=1}^{N} \phi_{mn} = 1, \phi_{mn} \in \{0,1\}, \forall m, n, \\
& \sum_{n=1}^{N} P_{jn} \leq P_{\text{tot}}, \text{for } j = 1,2,\ \sum_{m=1}^{N} P_{rm} \leq P_{\text{tot}}, \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0
\end{aligned}
$$

where ϕ_{mn} is a binary variable indicating the pairing outcome of subchannels n and m, and $\boldsymbol{\Phi}$ is an $N \times N$ matrix with $[\boldsymbol{\Phi}]_{mn} = \phi_{mn}$. Note that $\boldsymbol{\Phi}$ and $p(\cdot)$ are one-to-one correspondent and can be used interchangeably for a pairing strategy.

The joint optimization problem **P0** is a mixed-integer programming which is difficult to solve. We propose an iterative method in which we separate **P0** into two sub-problems: 1) An optimal paring problem under given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$; and 2) an optimal power allocation problem under given pairing strategy $\boldsymbol{\Phi}$. In the following, we first address the two optimization problems separately, and then present the iterative approach for the joint optimization.

2.2.1 Subchannel Pairing Optimization

We first consider the subchannel pairing optimization problem, when power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$ is given. The sum-rate in (2.5) can be re-written as

$$R_{mn} = \log\left(1 + \text{SNR}_{mn}^{\text{eff}}\right) \tag{2.9}$$

where $\mathrm{SNR}_{mn}^{\mathrm{eff}}$ is the effective received SNR combining both end nodes and relay with given a pairing function $p(\cdot)$, defined by

$$\mathrm{SNR}_{mn}^{\mathrm{eff}} \triangleq \mathrm{SNR}_{1,mn} + \mathrm{SNR}_{2,mn} + \mathrm{SNR}_{1,mn}\,\mathrm{SNR}_{2,mn}\,. \tag{2.10}$$

Combining the paired incoming subchannel n from the two end nodes and the outgoing subchannel m from the relay, $\mathrm{SNR}_{mn}^{\mathrm{eff}}$ can be viewed as the effective received SNR over this path. It is a function of subchannels paired, as well as power allocated to the paired subchannels at each end node and the relay $\{P_{1n}, P_{2n}, P_{rm}\}$.

For given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$, the joint optimization problem **P0** reduces to the subchannel pairing problem which is reformulated into the following optimization problem

$$\begin{aligned} \textbf{(P1)}: \quad & \max_{\boldsymbol{\Phi}} \sum_{n=1}^{N}\sum_{m=1}^{N} \phi_{mn} \log(1+\mathrm{SNR}_{mn}^{\mathrm{eff}}) \\ & \text{s.t.} \ \sum_{n=1}^{N}\phi_{mn} = 1, \sum_{m=1}^{N}\phi_{mn} = 1, \\ & \phi_{mn} \in \{0,1\},\ \forall m,n. \end{aligned}$$

The above optimization problem is known as the two-dimensional assignment problem. It was discussed in [39], where both an optimal solution and low-complexity suboptimal solutions are given. The authors first proof that unlike one-way relaying, there exist no explicit SNR-based subchannel pairing strategy that is optimal for sum-rate maximization in MABC two-way relaying case. Then, they proposed a low-complexity SNR-based suboptimal pairing scheme, *i.e.*, , SNR^{eff}-Greedy algorithm, which have much lower complexity($\mathcal{O}(N^2 \log N)$) as compared to optimal solution($\mathcal{O}(N^3)$) by using the Hungarian Algorithm [52].

2.2.2 Joint Power Allocation Optimization

With a fixed pairing strategy $p(\cdot)$ (or $\mathbf{\Phi}$), the optimization problem **P0** reduces to the joint optimization of power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$ at Nodes 1 and 2 and at the relay, given by

$$\begin{aligned}
\textbf{(P2)}: \max_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_r} \quad & \sum_{\substack{n=1 \\ m=p(n)}}^{N} \log(1+\mathrm{SNR}_{1,mn}) + \log(1+\mathrm{SNR}_{2,mn}) \\
\text{s.t.} \quad & \sum_{n=1}^{N} P_{jn} \le P_{\text{tot}},\ j=1,2, \quad \sum_{m=1}^{N} P_{rm} \le P_{\text{tot}}, \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0.
\end{aligned}$$

From (2.7) and (2.8), we observe that $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are not jointly convex with respect to (w.r.t.) $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$. Thus, the optimization problem **P2** is non-convex and thus is difficult to solve. Instead, we separate this joint power optimization problem into two sub-problems, and solve them iteratively. Specifically, we separate power allocation at the relay $\mathbf{p}_r$, and those at end nodes $\{\mathbf{p}_1, \mathbf{p}_2\}$, for sum-rate maximization.

2.2.2.1 Joint Optimization of $\{\mathbf{p}_1, \mathbf{p}_2\}$ Given $\mathbf{p}_r$

The objective in **P2** can be rewritten as

$$\begin{aligned}
\max_{\mathbf{p}_r} \max_{\mathbf{p}_1,\mathbf{p}_2} \quad & \sum_{\substack{n=1 \\ m=p(n)}}^{N} \log(1+\mathrm{SNR}_{1,mn}) + \log(1+\mathrm{SNR}_{2,mn}) \qquad (2.11) \\
\text{s.t.} \quad & \sum_{n=1}^{N} P_{jn} \le P_{\text{tot}},\ j=1,2, \quad \sum_{m=1}^{N} P_{rm} \le P_{\text{tot}}, \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0.
\end{aligned}$$

Since $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are not jointly convex w.r.t. $\{\mathbf{p}_1, \mathbf{p}_2\}$, given $\mathbf{p}_r$, the inner maximization over $\{\mathbf{p}_1, \mathbf{p}_2\}$ is non-convex and thus might not have a computational efficient solution. However, from (2.3) and (2.4), we see that $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ can be expressed in terms of of $\{P_{2n}, w_{mn}\}$ and $\{P_{1n}, w_{mn}\}$, respectively. If we fix the relay power coefficients $\{w_{mn}\}$ instead of $\mathbf{p}_r$ at the relay, the inner maximization above turns out to be convex.

Let $\mathbf{w}$ be the relay power coefficient vector with $[\mathbf{w}]_n = w_{mn}$, where $m = p(n)$. From w_{mn} in (2.2), the joint power optimization problem **P2** can be rewritten as

$$\begin{aligned}
\textbf{(P2')}: \quad & \max_{\mathbf{w}} \max_{\mathbf{p}_1, \mathbf{p}_2} \sum_{\substack{n=1 \\ m=p(n)}}^{N} \log(1 + \mathrm{SNR}_{1,mn}) + \log(1 + \mathrm{SNR}_{2,mn}) \\
\text{s.t.} \quad & \sum_{n=1}^{N} |w_{mn}|^2 (P_{1n}|h_{1n}|^2 + P_{2n}|h_{2n}|^2 + \sigma^2) \le P_{\text{tot}}, \\
& \sum_{n=1}^{N} P_{jn} \le P_{\text{tot}}, \; j = 1, 2, \quad \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0.
\end{aligned}$$

where $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are expressed in (2.3) and (2.4), respectively, as functions of $\{P_{1n}, P_{2n}, w_{mn}\}$. For given $\mathbf{w}$, the inner maximization of **P2'** is given by

$$\begin{aligned}
\textbf{(P2'a)}: \quad & \max_{\mathbf{p}_1, \mathbf{p}_2} \sum_{\substack{n=1 \\ m=p(n)}}^{N} \log(1 + \mathrm{SNR}_{1,mn}) + \log(1 + \mathrm{SNR}_{2,mn}) \\
\text{s.t.} \quad & \sum_{n=1}^{N} |w_{mn}|^2 (P_{1n}|h_{1n}|^2 + P_{2n}|h_{2n}|^2 + \sigma^2) \le P_{\text{tot}}, \\
& \sum_{n=1}^{N} P_{jn} \le P_{\text{tot}}, \; j = 1, 2, \quad \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0.
\end{aligned}$$

From (2.3) and (2.4), $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are both linear with respect to P_{1n} and P_{2n} respectively, thus the objective in **P2'a** is jointly convex with respect to $\{\mathbf{p}_1, \mathbf{p}_2\}$. Therefore, the optimization problem **P2'a** is convex and can be solved by standard convex optimization tools.

2.2.2.2 Optimization of $\mathbf{p}_r$ Given $\{\mathbf{p}_1, \mathbf{p}_2\}$

With given pair of $\{\mathbf{p}_1, \mathbf{p}_2\}$, the optimization problem **P2** becomes

$$\textbf{(P2'b)}: \quad \max_{\mathbf{p}_r} \sum_{\substack{n=1 \\ m=p(n)}}^{N} \log(1+\mathrm{SNR}_{1,mn}) + \log(1+\mathrm{SNR}_{2,mn})$$
$$\text{s.t.} \sum_{\mathrm{m=1}}^{\mathrm{N}} \mathrm{P_{rm}} \leq \mathrm{P_{tot}}, \quad \mathbf{p}_\mathrm{r} \succcurlyeq 0.$$

Given $\{\mathbf{p}_1, \mathbf{p}_2\}$, we see from (2.7) and (2.8) that $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are both concave functions of P_{rm}. As a result, the objective in **P2'b** is concave with respect to $\mathbf{p}_r$, and the optimization problem **P2'b** is convex.

However, the expression of $\mathrm{SNR}_{1,mn}$ in (2.7) w.r.t. $\mathbf{p}_r$ has a complicated fractional form that cannot be easily implemented by standard convex optimization tools for a solution. Therefore, we obtain the solution for **P2'b** using Karush-Kuhn-Tucker (KKT) conditions [53].

Denote $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]$ as the vector of Lagrange multipliers corresponding to the non-negative relay power constrains on each subchannel. Denote ν as the Lagrange multiplier corresponding to the relay power budget constraint. Since $\mathrm{SNR}_{1,mn}$ and $\mathrm{SNR}_{2,mn}$ are now only functions of P_{rm}, to explicit show this dependency, we denote the sum-rate objective in **P2'b** as $\sum_{m=1}^{N} R_m(P_{rm})$, where

$$R_m(P_{rm}) \triangleq \log(1+\mathrm{SNR}_{1,mn}) + \log(1+\mathrm{SNR}_{2,mn}).$$

It is easy to see that at optimality, the relay power constraint is attained at the equality, *i.e.*,

$\sum_{m=1}^{N} P_{rm} = P_{\text{tot}}$. Thus, using the KKT conditions, we have

$$\mathbf{p}_r \succcurlyeq 0,\ \boldsymbol{\lambda} \succcurlyeq 0,\ \sum_{m=1}^{N} P_{rm} = P_{\text{tot}},\ \lambda_m P_{rm} = 0,$$
$$R'_m(P_{rm}) - \lambda_m + \nu = 0, \quad m = 1, \ldots, N \tag{2.12}$$

where $R'_m(P_{rm})$ denote the derivative of $R_m(P_{rm})$ w.r.t. P_{rm}, and the value of ν should ensure $\sum_{m=1}^{N} P^o_{rm} = P_{\text{tot}}$. From (3.27), if the optimal $P^o_{rm} > 0$, we have $\lambda_m = 0$. Thus, P^o_{rm}, for $m = 1, \cdots, N$, should satisfy

$$P^o_{rm} > 0 \ \text{ and } R'_m(P^o_{rm}) + \nu = 0, \ \text{ or } \quad P^o_{rm} = 0. \tag{2.13}$$

The above solution for P^o_{rm} can be viewed as a variation of classical waterfilling solution, the detail of solution is provided in Appendix A.

2.2.2.3 Iterative Procedure for Joint Power Optimization

We now solve the joint power optimization problem **P2** by iteratively solving the optimization subproblems **P2'a** and **P2'b** to update $\mathbf{p}_r$ and $\{\mathbf{p}_1, \mathbf{p}_2\}$, respectively.

Specifically, let $\{\mathbf{p}_1^l, \mathbf{p}_2^l, \mathbf{p}_r^l\}$ denote the power allocation solutions obtained after the lth iteration, and let $\mathbf{w}^l$ be the corresponding relay power coefficient obtained from $\{\mathbf{p}_1^l, \mathbf{p}_2^l, \mathbf{p}_r^l\}$. At the $(l+1)$th iteration:

1. Given $\mathbf{w}^l$, we solve the joint optimization problem **P2'a** to obtain $\{\mathbf{p}_1^{l+1}, \mathbf{p}_2^{l+1}\}$;
2. Given $\{\mathbf{p}_1^{l+1}, \mathbf{p}_2^{l+1}\}$, we solve the optimization problem **P2'b** to obtain $\mathbf{p}_r^{l+1}$, and obtain $\mathbf{w}^{l+1}$.

Repeat steps 1-2 until the sum-rate objective in **P2** converges, and we obtain a local maximum solution for **P2**.

2.2.3 Joint Pairing and Power Allocation Iterative Optimization

Finally, to solve the original joint subchannel pairing and power optimization problem **P0**, we iteratively solve the optimization problems **P1** and **P2**. The kth iteration contains two steps:

1. Given power allocation $\{\mathbf{p}_1^k, \mathbf{p}_2^k, \mathbf{p}_r^k\}$, we solve subchannel pairing problem **P1** and obtain pairing permutation function $p^{k+1}(\cdot)$;

2. Given $p^{k+1}(\cdot)$, the joint power allocation optimization problem **P2** is solved using iterative approach described in Section 3.2.2.4.

The above procedure is repeated until the value of the sum-rate objective converges.

Note that the convergence of this iterative approach is guaranteed, since the value of the sum-rate objective in each step of the iterative procedure is non-decreasing. However, the original joint optimization problem may have multiple local maxima, and the global convergence is not guaranteed. Thus, for a better result, typically we need a few initialization trials and select the one with the highest objective value. Our iterative joint optimization approach is summarized in Algorithm 1.

Algorithm 1: Iterative Optimization Approach to Solve **P0**

Initialize: Set $\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0, p^0(\cdot), \epsilon$; Set $k = 0$;

Compute R^0 in (2.6) with $\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0, p^0(\cdot)$. Set $\Delta R^0 > \epsilon$.

while $\Delta R^k > \epsilon$ **do**

// Given $p^k(\cdot)$, solve power optimization in **P2**

Set $l = 0$, $\tilde{\mathbf{p}}_1^0 = \mathbf{p}_1^k$, $\tilde{\mathbf{p}}_2^0 = \mathbf{p}_2^k$, $\tilde{\mathbf{p}}_r^0 = \mathbf{p}_r^k$;

Let $\tilde{R}^l$ be the objective value in **P2** at the lth iteration.

Compute $\tilde{R}^0$ with $\{\tilde{\mathbf{p}}_1^0, \tilde{\mathbf{p}}_2^0, \tilde{\mathbf{p}}_r^0\}$. Set $\Delta\tilde{R}^0 > \epsilon$.

while $\Delta\tilde{R}^l > \epsilon$ **do**

Compute $\mathbf{w}^l$ in (2.2) based on $\{\tilde{\mathbf{p}}_1^l, \tilde{\mathbf{p}}_2^l, \tilde{\mathbf{p}}_r^l, p^k(\cdot)\}$;

Given $\mathbf{w}^l$, solve **P2'a** to obtain $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}\}$;

Given $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}\}$, solve **P2'b** to obtain $\tilde{\mathbf{p}}_r^{l+1}$;

Compute $\tilde{R}^{l+1}$ using $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}, \tilde{\mathbf{p}}_r^{l+1}, p^k(\cdot)\}$;

Set $\Delta\tilde{R}^{l+1} = \tilde{R}^{l+1} - \tilde{R}^l$;

Set $l \leftarrow l + 1$;

end

Output: $\mathbf{p}_1^{k+1} = \tilde{\mathbf{p}}_1^l$, $\mathbf{p}_2^{k+1} = \tilde{\mathbf{p}}_2^l$, $\mathbf{p}_r^{k+1} = \tilde{\mathbf{p}}_r^l$;

Given $\{\mathbf{p}_1^{k+1}, \mathbf{p}_2^{k+1}, \mathbf{p}_r^{k+1}\}$, solve the pairing optimization problem **P1** to obtain $p^{k+1}(\cdot)$;

Compute R^{k+1} using $\{\mathbf{p}_1^{k+1}, \mathbf{p}_2^{k+1}, \mathbf{p}_r^{k+1}, p^{k+1}(\cdot)\}$;

Set $\Delta R^{k+1} = R^{k+1} - R^k$;

Set $k \leftarrow k + 1$;

end

Output: $\mathbf{p}_1^k$, $\mathbf{p}_2^k$, $\mathbf{p}_r^k$ and $p^k(\cdot)$;

2.3 Simulation Results

We evaluate the performance of the proposed algorithm through simulations using an OFDM system with N subchannels. The channel gain over each subchannel is complex Gaussian with zero mean and variance σ_h^2, where σ_h^2 follows a pathloss model $\sigma_h^2 = K_o d^{-\kappa}$ with pathloss exponent $\kappa = 3$ and $K_o = 1$. The receiver noise is assumed AWGN with variance $\sigma^2 = 1$. Let d_{12}, d_{1r}, and d_{r2} denote the distance between two end nodes, that between Node 1 and the relay, and that between Node 2 and the relay, respectively. The total power at each node is set to $P_{\text{tot}} = N$. We define $\overline{\text{SNR}} \triangleq P_{\text{tot}} d_{12}^{-\kappa}/\sigma^2$ as the average SNR from Node 1 to Node 2 over the direct path. We set $\overline{\text{SNR}} = 2$ dB in our simulations.

Convergence Behavior We first study the convergence behavior of the iterative power allocation optimization method in Section 3.2.2.4 and iterative joint pairing and power allocation algorithm in Algorithm 1. We set the relay to be at the middle point between the two end nodes, *i.e.*, $d_{1r} = d_{r2}$.

Fig. 2.2 plots the average sum-rate per subchannel versus the number of iterations by solving the joint power allocation optimization problem **P2** using the iterative approach in Section 3.2.2.4. The pairing scheme $p(\cdot)$ is randomly generated at the beginning but is fixed during the experiment. A few randomly generated initializations for $\{\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0\}$ are used to study the convergence behavior and performance to the local maxima. Each curve corresponds to a different initialization. Similarly, Figs. 2.3 and 2.4 plot the average sum-rate per subchannel versus the number of iterations under Algorithm 1 for solving the joint optimization problem **P0** for $N = 8$ and $N = 64$, respectively. Each curve corresponds to a different initialization for $\{\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0, p^0(\cdot)\}$, which are randomly generated. The same set of channel realizations are used for Figs. 2.2-2.4. From Figs. 2.2-2.4, we see that, for the iterative optimization for both **P2** and **P0**, the sum-rate converges in just a few iterations. In addition, we see that several local maximum points may exist and different initialization may converge to different local maxima, although the difference is not large. This shows that a few initializations are required to improve the performance.

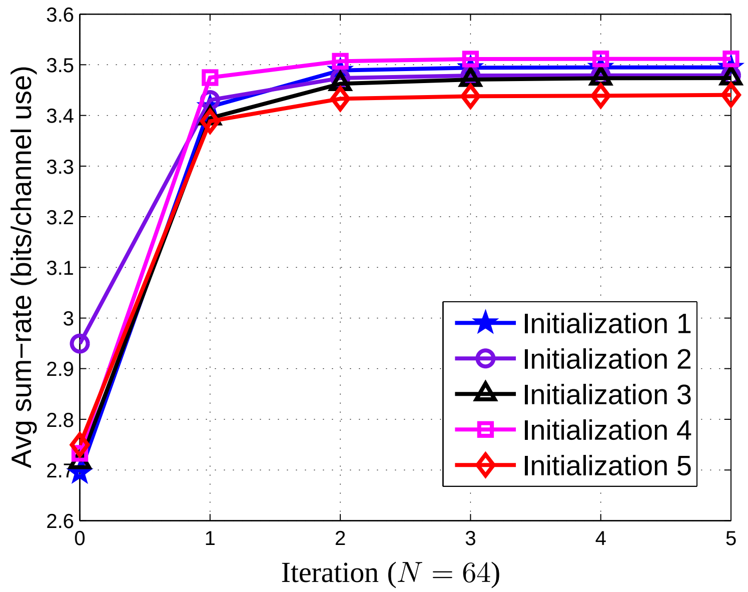

Figure 2.2: Average sum-rate vs. iterations for solving joint power allocation problem **P2**.

Performance We investigate the performance among different strategies as N increases. Specifically, four schemes are compared: 1) Equal power allocation at all nodes and direct pairing; 2) Pairing only: pairing is optimized using **P1** by Hungarian Algorithm, while equal power allocation is assumed; 3) Power allocation only: only **P2** is solved by the proposed iterative procedure, while a random pairing is given; 4) Joint pairing and power allocation: our proposed Algorithm 1 to solve **P0**. We assume the relay is at the middle point between the two end nodes, *i.e.*, $d_{1r} = d_{r2}$.

Fig. 2.5 plots the average sum-rate per subchannel vs. N. It can be seen that the performance of joint pairing and power allocation optimization in Algorithm 1 is the best. The additional performance gain of joint optimization over the pairing-only and the power-allocation-only schemes is clearly seen. The spectral efficiency under the pairing-only

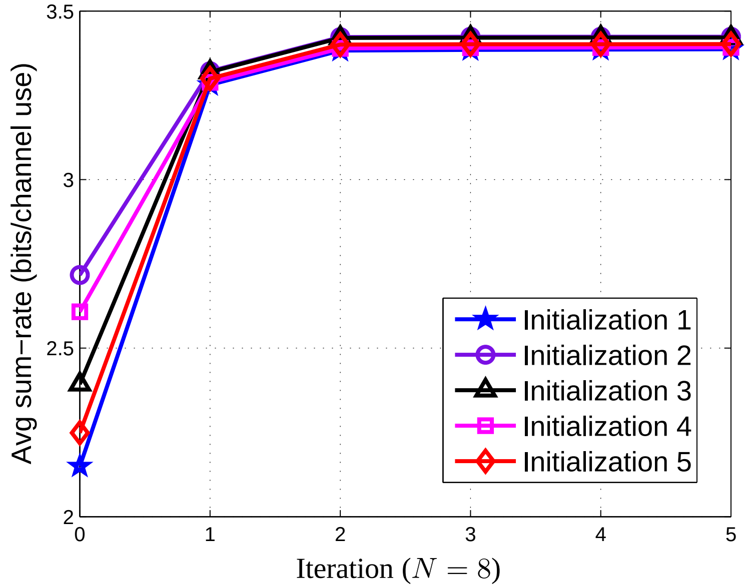

Figure 2.3: Average sum-rate vs. iterations for solving joint paring and power allocation problem **P0**, when number of subchannel N is 8.

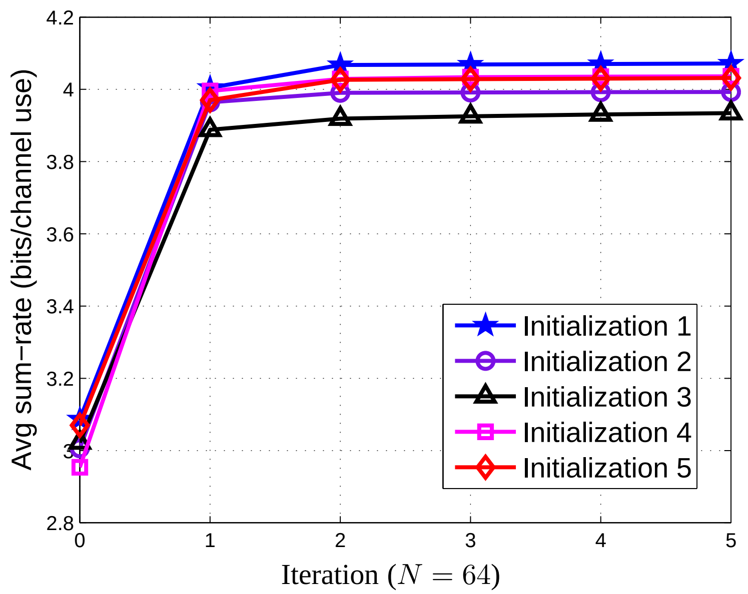

Figure 2.4: Average sum-rate vs. iterations for solving joint paring and power allocation problem **P0**, when number of subchannel N is 64

scheme and joint optimization (Algorithm 1) increases with N, due to the pairing gain increasing with N as discussed earlier. For the other two schemes, the spectral efficiency remains almost flat as N increases, as they do not exploit the pairing benefit.

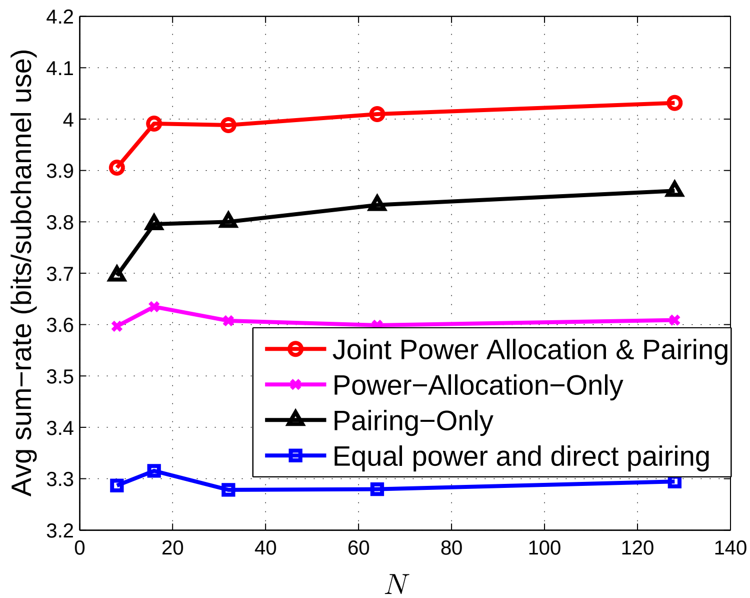

Figure 2.5: Average Sum-rate per subchannel vs. N $(\mathrm{SNR}_{sd} = 2\mathrm{dB})$

Finally, we show the performance of average sum-rate versus the relay position between the two end notes in Fig. 2.6. We set $N = 32$. Performances under the four schemes are again compared. Again, the sum-rate increases as the relay moves towards to the middle point between Nodes 1 and 2. The best performance is when the relay is at the middle point to benefit. Comparing different schemes, we see that when the relay is at the middle point, the gain due to pairing alone exceeds the gain due to power allocation alone, indicating the significance of subchannel pairing. The additional performance gain of joint pairing and power allocation optimization over the pairing-only and the power-allocation-only schemes is clearly seen at any relay location.

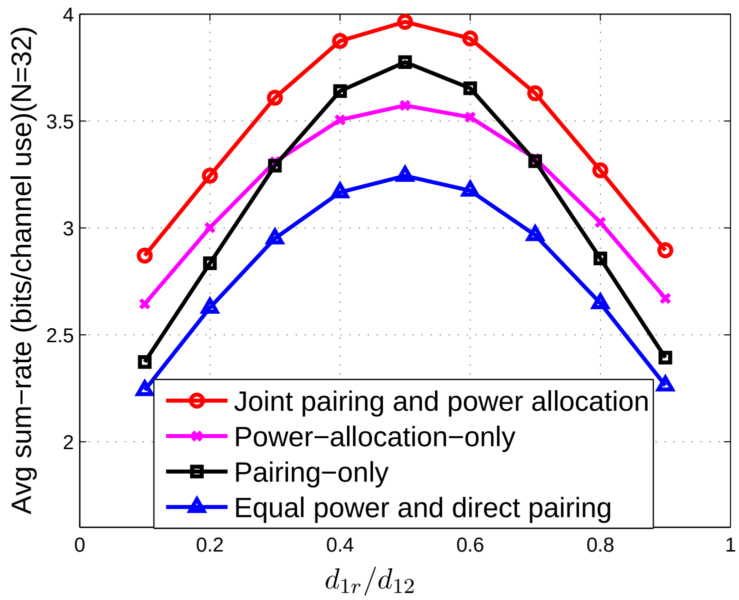

Figure 2.6: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 32$).

2.4 Summary

In this chapter, the joint subchannel paring and power allocation optimal problem for a multichannel MABC-based two-way relay network is considered. The objective is to maximize the sum-rate of both end nodes. The joint optimization is a difficult problem, especially for two-way relaying. We proposed an iterative algorithm which solves the pairing and power allocation problem iteratively. For the joint power allocation among the two end nodes and the relay, an iterative optimization procedure was proposed to solve the power allocation at the relay and at two end nodes iteratively. By transforming the SNR expression w.r.t. different form of optimization parameters, each power optimization problem turns out to be a convex problem and can be solved to obtain the optimal solution.

Finally, for the original joint subchannel pairing and power allocation problem, we proposed an additional iterative algorithm to solve the pairing and power allocation subproblems iteratively. The simulation performance demonstrates the gain of joint optimization approach over other pairing-only or power-allocation-only optimization approaches.

Chapter 3

Joint Pairing and Power Allocation Optimization in Multichannel TDBC-Based Two-Way Relaying

3.1 System Model

We consider a wireless relay network where one relay node help two terminal nodes (Nodes 1 and 2) to exchange information in a multichannel system with N subchannels. We assume that the relay channels to and from each terminal node are reciprocal and that each terminal node has perfect knowledge of the channel state information. Assuming transmission over each subchannel experiences flat fading, we denote h_{1i} and h_{2i} as the channel coefficient over the ith subchannel from the relay to Nodes 1 and 2, respectively, and the channel coefficient of the direct link between the two terminal nodes over the ith subchannel as h_{0i}. We consider a three-phase TDBC-based two-way transmission strategy and assume the channel coefficient remains unchanged within the duration of three-phase transmission. In phases one and two, Nodes 1 and 2 transmit their data to the relay. The received signal at the relay over the ith subchannel in Phase j, denoted as r_{ji}, is given by

$$r_{ji} = \sqrt{P_{ji}} h_{ji} s_{ji} + v_{ji}, \quad j = 1, 2;\ i = 1, \cdots, N \tag{3.1}$$

where s_{ji} and P_{ji} are the information symbol and transmitted power of Node j ($j = 1, 2$), respectively while v_{ji} is the additive white Gaussian noise (AWGN) with variance σ^2 at the relay on the ith subchannel in Phase j. The signals received at Nodes 1 and 2 from the direct link over the ith subchannel during Phases 2 and 1 are given by

$$y^{\mathrm{d}}_{\tilde{j}i} = \sqrt{P_{ji}} h_{0i} s_{ji} + n^{\mathrm{d}}_{\tilde{j}i}, \quad j = 1, 2;\ i = 1, \cdots, N \tag{3.2}$$

where $\tilde{j} = 1$ (or 2) for $j = 2$ (or 1); $n^{\mathrm{d}}_{\tilde{j}n}$ is the AWGN with variance σ^2 at Node $\tilde{j}$, for $\tilde{j} = 1, 2$. In the third phase, the relay performs both pairing and power amplification for forwarding. Specifically, the relay combines the received signals from Nodes 1 and 2 on the kth and mth subchannels, respectively, and retransmits the combined signal over the nth subchannel with power P_{rn}. The received signals at Nodes j over the nth subchannel are given by

$$y_{jn} = h_{jn}(w_{1nk} r_{1k} + w_{2nm} r_{2m}) + n_{jn}, \quad j = 1, 2 \tag{3.3}$$

where n_{jn} is the receiver noise over the nth subchannel at Node j in the third phase, respectively with variance σ^2; the coefficient w_{1nk} and w_{2nm} are given by

$$w_{1nk} = \sqrt{\frac{\alpha_n P_{rn}}{P_{1k}|h_{1k}|^2 + \sigma^2}}, \ w_{2nm} = \sqrt{\frac{(1-\alpha_n) P_{rn}}{P_{2m}|h_{2m}|^2 + \sigma^2}} \tag{3.4}$$

where P_{rn} is the relay transmit power over the nth subchannel, and $\alpha_n \in [0, 1]$ is the fraction applied to forward signal from relay over the nth subchannel.

Assume Nodes 1 and 2 know the pairing scheme used at the relay, and thus, they can cancel the self-interference received, before performing detection. The residual signal after self-cancellation at Node 1 is given by

$$\begin{aligned}\tilde{y}_{1n} =& \sqrt{P_{2m}} w_{2nm} h_{1n} h_{2m} s_{2m} + h_{1n}(w_{1nk} v_{1k} + w_{2nm} v_{2m}) \\ &+ n_{1n}. \end{aligned} \tag{3.5}$$

Using (3.2) and (3.5), the received signal from the direct link over the mth subchannel and that from the relay over the nth subchannel will be combined for the detection of transmitted symbol s_{2m} from Node 2. It is known that the maximum ratio combining (MRC) is optimal in the sense that it results the highest received SNR output given by

$$\mathrm{SNR}_{1nm} = \frac{P_{2m}|w_{2nm}|^2|h_{1n}|^2|h_{2m}|^2}{\sigma^2(1 + (|w_{1nk}|^2 + |w_{2nm}|^2)|h_{1n}|^2)} + \frac{P_{2m}|h_{0m}|^2}{\sigma^2} \tag{3.6}$$

$$\mathrm{SNR}_{2nk} = \frac{P_{1k}|w_{1nk}|^2|h_{2n}|^2|h_{1k}|^2}{\sigma^2(1 + (|w_{1nk}|^2 + |w_{2nm}|^2)|h_{2n}|^2)} + \frac{P_{1k}|h_{0k}|^2}{\sigma^2} \tag{3.7}$$

where the first terms are the post-cancellation received transceiver's SNRs over the relay after self-cancellation, and the second terms are the SNR from the direct link. Similarly we can write the received SNR at Node 2, denoted as SNR_{2nk}, for transmitted symbol s_{1k} from Node 1. An example of the system model is given in Fig 3.1.

Note that for this three-phase two-way relaying scheme, the channel pairing involves three subchannels: the two incoming subchannels to the relay from Nodes 1 and 2, and the common outgoing subchannel to the Nodes 1 and 2. The pairing can be described using two permutation functions $k = p(n), m = q(n)$, for $n = 1, \cdots, N$. In other words, $p(\cdot)$ and $q(\cdot)$ provide specific pairing strategies of subchannels to and from the relay for Nodes 1 and 2, respectively.

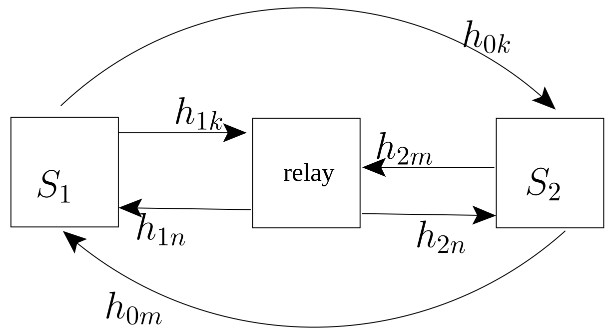

Figure 3.1: System model for TDBC-based two way relaying

The system sum-rate of Nodes 1 and 2 under a given pairing function $k = p(n)$ and $m = q(n)$ can express as

$$R_{nmk} = \log\left(1 + \mathrm{SNR}_{1nm}\right) + \log\left(1 + \mathrm{SNR}_{2nk}\right). \tag{3.8}$$

The overall sum-rate achieved in multichannel system is given by

$$R = \sum_{\substack{n=1 \\ k=p(n), m=q(n)}}^{N} R_{nmk} \tag{3.9}$$

3.2 Joint Two-Way Pairing and Power Optimization

Using (3.4), we can rewrite SNR_{1nm} and SNR_{2nk} in (3.6) and (3.7) in terms of P_{1k}, P_{2m}, P_{rn} and α_n as

$$\begin{aligned}\mathrm{SNR}_{1nm} =& \frac{P_{2m}(1-\alpha_n)P_{rn}|h_{1n}|^2|h_{2m}|^2(P_{1k}|h_{1k}|^2+\sigma^2)/\sigma^2}{(P_{2m}|h_{2m}|^2+\sigma^2)(P_{1k}|h_{1k}|^2+\alpha_n P_{rn}|h_{1n}|^2+\sigma^2)+(1+\alpha_n)P_{rn}|h_{1n}|^2(P_{1k}|h_{1k}|^2+\sigma^2)} \\ &+ \frac{P_{2m}|h_{0m}|^2}{\sigma^2}\end{aligned} \tag{3.10}$$

$$\text{SNR}_{2nk} = \frac{P_{1k}\alpha_n P_{rn}|h_{2n}|^2|h_{1k}|^2(P_{2m}|h_{2m}|^2+\sigma^2)/\sigma^2}{(P_{1k}|h_{1k}|^2+\sigma^2)(P_{2m}|h_{2m}|^2+(1-\alpha_n)P_{rn}|h_{2n}|^2+\sigma^2)+\alpha_n P_{rn}|h_{2n}|^2(P_{2m}|h_{2m}|^2+\sigma^2)} + \frac{P_{2m}|h_{0m}|^2}{\sigma^2}. \quad (3.11)$$

The sum-rate R_{nmk} in (3.8) is a function of power allocation at each end node and the relay, *i.e.*, $\{P_{1k}, P_{2m}, P_{rn}, \alpha_n\}$. Let $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_r$ and $\boldsymbol{\alpha}$ be the $N \times 1$ vectors containing the power allocation and fraction on each subchannel at Nodes 1and 2, and at the relay, respectively, with $[\mathbf{p}_1]_k = P_{1k}$, $[\mathbf{p}_2]_m = P_{2m}$, $[\mathbf{p}_r]_n = P_{rn}$ and $[\boldsymbol{\alpha}]_n = \alpha_n$. Let P_{tot} be the power budget at each node[1]. Our goal is to maximize the sum-rate in (3.9) by jointly optimizing the subchannel pairing strategy $p(\cdot)$ and $q(\cdot)$ and the power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}\}$ under the individual power constraint P_{tot}. The joint optimization problem is formulated as the following:

$$\begin{aligned}
\textbf{(P0)}: \max_{\boldsymbol{\Phi}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}} & \sum_{m=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N} \phi_{nmk} R_{nmk} \\
\text{s.t.} \quad & \sum_{m=1}^{N}\sum_{k=1}^{N} \phi_{nmk} = 1, \forall n; \ \sum_{n=1}^{N}\sum_{k=1}^{N} \phi_{nmk} = 1, \forall m \\
& \sum_{m=1}^{N}\sum_{n=1}^{N} \phi_{nmk} = 1, \forall k; \ \phi_{nmk} \in \{0,1\}, \forall m, k, n. \\
& \sum_{k=1}^{N} P_{1k} \le P_{\text{tot}}, \sum_{m=1}^{N} P_{2m} \le P_{\text{tot}}, \ \sum_{n=1}^{N} P_{rn} \le P_{\text{tot}}. \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0, \alpha_n \in [0,1], \forall n
\end{aligned}$$

where ϕ_{nmk} is a binary variable indicating the pairing outcome of subchannels n, m and k, and $\boldsymbol{\Phi}$ is a three-dimensional matrix with $[\boldsymbol{\Phi}]_{nmk} = \phi_{nmk}$. The joint optimization problem **P0** is a mixed-integer programming problem which is difficult to solve. We propose an

[1]We assume the same power budget at each node for simplicity.

iterative method in which we separate **P0** into two sub-problems: an optimal paring problem under given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\ \boldsymbol{\alpha}\}$, and an optimal power allocation problem under given pairing strategy $\boldsymbol{\Phi}$.

3.2.1 Subchannel Pairing Optimization

We first consider the subchannel pairing optimization problem, when power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}\}$ is given. The sum-rate in (3.8) can be re-written as

$$R_{nmk} = \log\left(1 + \mathrm{SNR}^{\mathrm{eff}}_{nmk}\right) \tag{3.12}$$

where $\mathrm{SNR}^{\mathrm{eff}}_{nmk}$ is the effective received SNR combining both end nodes and relay with given a pairing function $p(\cdot)$ and $q(\cdot)$, defined by

$$\mathrm{SNR}^{\mathrm{eff}}_{nmk} \triangleq \mathrm{SNR}_{1nm} + \mathrm{SNR}_{2nk} + \mathrm{SNR}_{1nm}\,\mathrm{SNR}_{2nk}\,. \tag{3.13}$$

Combining the paired incoming subchannel k and m from two end nodes and outgoing subchannel n from the relay, $\mathrm{SNR}^{\mathrm{eff}}_{nmk}$ can be considered as the received effective SNR over this path. It is a function of subchannels paired, as well as power allocated to the paired subchannels at each end node and the relay.

The sum-rate expression in (3.12) is now in the same format as that in a traditional one-way relaying system. For a one-way relaying system, it has been shown that for given power allocations at the relay, the optimal pairing strategy for end-to-end sum-rate maximization is an *SNR-sorting* based pairing strategy [25, 27], where the subchannel at the 1st hop with the kth highest SNR is paired with the subchannel at the 2nd hop with the kth highest SNR. This optimal pairing strategy uses explicit SNR ordering and thus is efficient in computation with $\mathcal{O}(N \log N)$ complexity (determined by the sorting complexity). The efficiency of this strategy comes from the fact that it can separately and independently sort the incoming and outgoing channels according to a certain SNR-based metric. Due to the computational benefit, for two-way relaying, we are interested in investigating whether such a similar SNR-based efficient pairing strategy also exists. In the following, we show that, unfortunately, such SNR-based pairing strategy is not optimal for two-way relaying.

3.2.1.1 Sub-optimality of SNR-Based Pairing

Re-grouping the variables in (3.10) and (3.11), we can re-write the received SNR expressions as

$$\mathrm{SNR}_{1nm} = \frac{\gamma_{2m}\theta_{1n}}{1 + \theta_{1n} + \gamma_{2m} + \theta_{1n}\frac{\alpha_n}{1-\alpha_n}\frac{\gamma_{2m}+1}{\gamma_{1k}+1}} + \eta_{1m} \tag{3.14}$$

$$\mathrm{SNR}_{2nk} = \frac{\gamma_{1k}\theta_{2n}}{1 + \theta_{2n} + \gamma_{1k} + \theta_{2n}\frac{1-\alpha_n}{\alpha_n}\frac{\gamma_{1k}+1}{\gamma_{2m}+1}} + \eta_{2k} \tag{3.15}$$

where γ_{1k} and γ_{2m} are the received SNRs at the relay from Nodes 1 and 2 over the kth and the mth subchannels, respectively; θ_{jn} is the received SNR at Nodes j from the relay over the nth subchannel; and η_{1m} and η_{2k} are the received SNRs at Nodes 1 and 2 from the direct link over the kth and mth subchannels, respectively. They are expressed as:

$$\gamma_{1k} = \frac{P_{1k}|h_{1k}|^2}{\sigma^2}, \qquad \gamma_{2m} = \frac{P_{2m}|h_{2m}|^2}{\sigma^2} \tag{3.16}$$

$$\theta_{1n} = \frac{(1-\alpha_n)P_{rn}|h_{1n}|^2}{\sigma^2}, \qquad \theta_{2n} = \frac{\alpha_n P_{rn}|h_{2n}|^2}{\sigma^2} \tag{3.17}$$

$$\eta_{1m} = \frac{P_{2m}|h_{0m}|^2}{\sigma^2}, \qquad \eta_{2k} = \frac{P_{1k}|h_{0k}|^2}{\sigma^2} \tag{3.18}$$

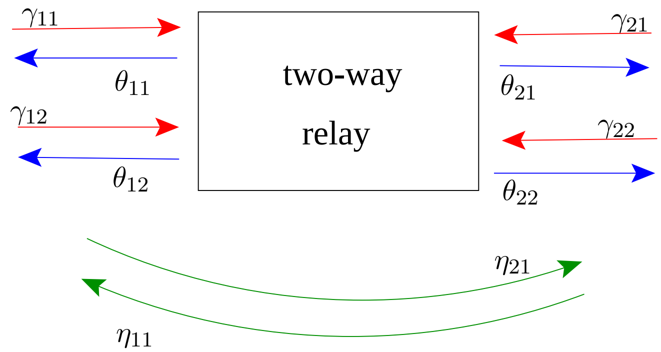

Figure 3.2: SNR-based two way relaying pairing

Thus, the received SNRs at Nodes 1 and 2 are functions of SNR over each of the six subchannels used for transmissions, *i.e.*, the kth and mth subchannels to the relay and over the direct link, and the nth outgoing subchannels to Nodes 1 and 2. An example of the SNR-based pairing is given in Fig 3.2.

Based on (3.14) and (3.15), $\mathrm{SNR}_{nmk}^{\mathrm{eff}}$ is a function of $(\gamma_{1k}, \gamma_{2m}, \theta_{1n}, \theta_{2n}, \eta_{1m}, \eta_{2k})$ expressed as

$$\textit{Direct}: \ \mathrm{SNR}_{nmk}^{\mathrm{eff}} = \Phi(\gamma_{1k}, \gamma_{2m}, \theta_{1n}, \theta_{2n}, \eta_{1m}, \eta_{2k}) \tag{3.19}$$

$$\textit{No direct}: \ \mathrm{SNR}_{nmk}^{\mathrm{eff}} = \tilde{\Phi}(\gamma_{1k}, \gamma_{2m}, \theta_{1n}, \theta_{2n}). \tag{3.20}$$

Define $f, g, h : R^2 \rightarrow R$, where $f(\gamma_{1k}, \eta_{2k})$ is a function of SNRs on the kth subchannel from Node 1 to relay and Node 2 over the direct link, $g(\gamma_{2m}, \eta_{1m})$ is a function of SNRs on the mth subchannel from Node 2 to relay and Node 1 over the direct link, and $h(\theta_{1n}, \theta_{2n})$ is a function of SNRs on outgoing n subchannel from relay to Nodes 1 and 2 ($\tilde{h}(\theta_{1n}, \theta_{2n})$ for the no direct link case). For a SNR-based pairing strategy, the pairing will be based on the value of f, g and h over each subchannel. We intend to find out whether $\mathrm{SNR}_{nmk}^{\mathrm{eff}}$ can be expressed as

$$\textit{Direct}: \quad \Phi(\gamma_{1k}, \gamma_{2m}, \theta_{1n}, \theta_{2n}, \eta_{1m}, \eta_{2k}) = \Psi(f(\gamma_{1k}, \eta_{2k}), g(\gamma_{2m}, \eta_{1m}), h(\theta_{1n}, \theta_{2n})) \tag{3.21}$$

$$\textit{No direct}: \quad \tilde{\Phi}(\gamma_{1k}, \gamma_{2m}, \theta_{1n}, \theta_{2n}) = \tilde{\Psi}(\gamma_{1k}, \gamma_{2m}, \tilde{h}(\theta_{1n}, \theta_{2n})) \tag{3.22}$$

where $\Psi(f, g, h)$ is only a function of f, g and h, and similarly for $\tilde{\Psi}(\gamma_{1k}, \gamma_2 m, \tilde{h})$. The following result shows that such functions cannot be found for the optimal pairing.

Lemma 1. *1) There do not exist functions $f(x_1, y_1)$, $g(x_2, y_2)$, $h(x_3, y_3)$, and $\Psi(x, y, z)$ satisfying* (3.21)*; 2) There do not exist functions $\tilde{h}(x_1, y_1)$ and $\tilde{\Psi}(x, y, z)$ satisfying* (3.22).

$Proof$: See Appendix B.

Following Lemma 1, we have the following conclusion.

Proposition 1. *For the TDBC-based two-way relaying multichannel system with or without direct link, with given power allocations at Nodes 1 and 2 and the relay, there exist no explicit SNR-based subchannel pairing strategy optimal for sum-rate maximization.*

Recall that, when there is no direct link, the explicit SNR-based pairing strategy is optimal for a one-way relaying system [24–26]. Proposition 1 indicates that the TDBC-based two-way multichannel relaying system lacks of an efficient (explicit) optimal pairing strategy, regardless of availability of direct link. Using exhaustive search among a total of $(N!)^2$ possible pairing combinations results in the computational complexity of $\mathcal{O}\left(2^N\right)$, which is impractical when N is large. Thus, our focus is to design low-complexity suboptimal pairing strategies with good performance, which is particularly important for broadband systems with large N.

3.2.1.2 Low-Complexity Suboptimal Pairing Strategy

For given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}\}$, the optimization problem **(P0)** reduces to the subchannel pairing problem and can be formulated as

$$\begin{aligned}
\textbf{(P1)}: \quad & \max_{\phi_{nmk}} \sum_{m=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N} \phi_{nmk} \log(1+\mathrm{SNR}_{nmk}^{\text{eff}}) \\
& s.t. \sum_{m=1}^{N}\sum_{k=1}^{N} \phi_{nmk} = 1, \forall n; \ \sum_{n=1}^{N}\sum_{k=1}^{N} \phi_{nmk} = 1, \forall m \\
& \sum_{m=1}^{N}\sum_{n=1}^{N} \phi_{nmk} = 1, \forall k; \ \phi_{nmk} \in \{0,1\}, \forall m,k,n.
\end{aligned}$$

The above optimization problem is essentially an axial 3-DAP [54]. In contrast to the 2-D assignment problem which can be solved efficiently by algorithms such as the Hungarian Algorithm [52], the 3-DAP is an NP-hard problem. This problem has been studied extensively in literature, and various heuristic methods have been proposed to solve it.

Iterative Optimization Here, we use an iterative approach to solve the 3-DAP by reducing it into a 2-D assignment problem and solving it iteratively, an idea which was first proposed in [55].

Let C_{mk}, C_{nm}, and C_{nk} be the 2-D permutation matrices over the pair (m,k), (n,m), and (n,k), respectively, under given permutations $p(\cdot)$ and $q(\cdot)$. Let C_{nmk} be the 3-D permutation matrix over the tuple (n,m,k). Determining the optimal 3-D permutation matrix under a fixed 2-D permutation matrix is essentially a 2-D assignment problem. Since there are three 2-D permutation matrices, the lth iteration involves three steps as follow. An example of the procedure (3.23) is given in Fig 3.3, where each column stands for kth, mth and nth subchannel respectively.

$$\text{Fix}\, C_{mk}^{(l)} \Rightarrow \text{find optimal}\, C_{nm}^{(l+1)} \tag{3.23}$$

$$\text{Fix}\, C_{nm}^{(l+1)} \Rightarrow \text{find optimal}\, C_{nk}^{(l+1)} \tag{3.24}$$

$$\text{Fix}\, C_{nk}^{(l+1)} \Rightarrow \text{find optimal}\, C_{mk}^{(l+1)}. \tag{3.25}$$

The optimal solution in each optimization above can be obtained by the 2-D assignment problem through the Hungarian algorithm. Note that the above procedures in each iteration will effectively determine the two permutations $p(\cdot)$ and $q(\cdot)$. The iteration repeats the above procedures (3.23)-(3.25) to heuristically optimizes the sum-rate in (3.9) until no more improvement can be achieved. An example of the procedure (3.23) is given in Fig 3.3, where each column stands for kth, mth and nth subchannel respectively. The iterative optimization approach is summarized in Algorithm 2.

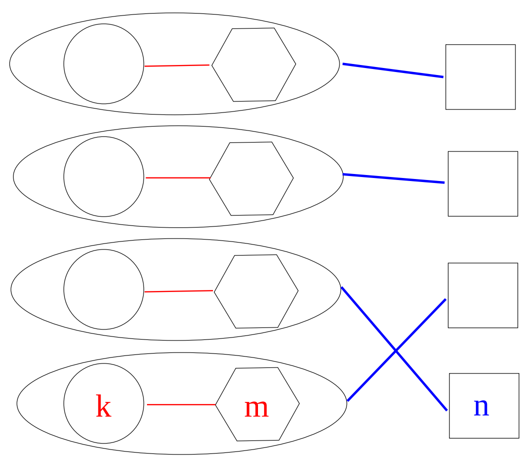

Figure 3.3: Iterative Optimization Pairing

Algorithm 2: Iterative Optimization Algorithm

1): Initialize P_{mk}, threshold ϵ

2): Set $l \leftarrow 0$, $R^{(0)} \leftarrow 0$ for $R^{(0)}$ in (3.9)

while $R^{(l+1)} - R^{(l)} > \epsilon$ **do**

Obtain $C_{nm}^{(l+1)}$, $C_{nk}^{(l+1)}$, $C_{mk}^{(l+1)}$ using (3.23), (3.24), and (3.25), respectively, using the Hungarian algorithm.

Update $R^{(l+1)}$ in (3.9) with $C_{nm}^{(l+1)}$, $C_{nk}^{(l+1)}$.

$l \leftarrow l + 1$.

end while

Complexity: The complexity of Hungarian algorithm is $\mathcal{O}\left(N^3\right)$, thus the complexity of each iteration is also $\mathcal{O}\left(N^3\right)$. From simulations, we show that the performance converges in a couple of iterations.

Sorting-Based Algorithm It is known that for one-way relaying, without the direct link, the optimal pairing strategy is an explicit SNR-based pairing [24–26]. We utilize this result to propose a similar sorting strategy for TDBC-based two-way relaying.

Without Direct Link Let $\{\gamma_{1(k)}\}$ denote the sorted version of $\{\gamma_{1k}\}$, *i.e.*, $\gamma_{1(k)} \geq \gamma_{1(k+1)}$. Similarly, we denote $\{\gamma_{2(m)}\}$. For the nth outgoing subchannel θ_{1n} and θ_{2n}, assuming θ_{1n} and θ_{2n} are ranked the n_1th among $\{\theta_{1n}\}$, and the n_2th among $\{\theta_{2n}\}$, respectively. Our proposed algorithm is to pair $\gamma_{1(n_2)}$ with θ_{2n}, and $\gamma_{2(n_1)}$ with θ_{1n}. In this case, in each one-way direction, the SNR-sorting based pairing is used, which is optimal if there is no interference from the other direction. The summary of this procedure is given in Algorithm 3. An example of the sorting-based pairing strategy is given in Fig 3.4.

With Direct Link When we consider the direct link, a sorting-based strategy has been proposed in [27] for one-way relaying which is optimal under a fixed-gain power amplification. Adopting that sorting strategy into the two-way relaying case, we can adjust our ranking metric by including the direct link, *i.e.*, $\frac{\gamma_{1k}}{1+\eta_{2k}}$ and $\frac{\gamma_{2m}}{1+\eta_{1m}}$. Algorithm 4 provides a

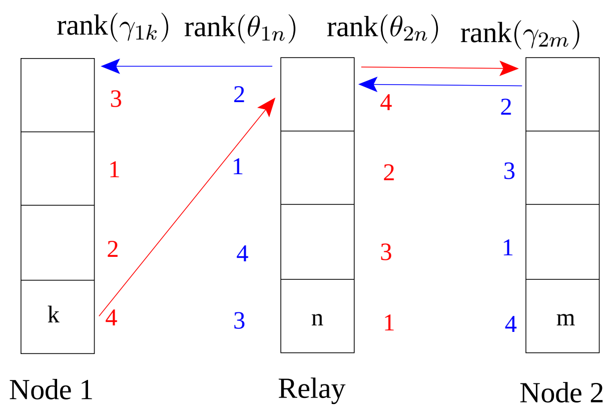

Figure 3.4: Sorting-based pairing

summary of the procedure in this case.

Complexity: The complexity of sorting a queue of length N is $\mathcal{O}(N \log N)$. Thus, the total complexity of the sorting-based strategy is $\mathcal{O}(N \log N)$. Comparing to $\mathcal{O}\left(N^3\right)$ of Algorithm 2, we see that the complexity of Algorithms 3 and 4 is extremely low. It maintains the same pairing complexity as in the one-way case. As we will see from simulation studies that the performance of such sorting-based strategy is close to that of the optimal pairing, demonstrating the effectiveness and the excellent performance of the proposed suboptimal algorithm.

Special case – direct pairing It is known that for one-way relaying, the optimal pairing strategy is an explicit SNR-based pairing [25, 27], where the nth strongest subchannels, measured by SNR, over the first hop and second hop are paired. If we directly apply this one-way optimal pairing strategy to two-way relaying, it is easy to see that it is simply equivalent to the direct pairing case, *i.e.*, $n = m = k$. As we will see from simulation studies that the performance of such direct pairing strategy for two-way relaying is inferior to that of the iterative optimization algorithm and sorting-based algorithm we proposed above.

Algorithm 3: Sorting-Based Algorithm (without direct link)

1): Sorting γ_{1k}, γ_{2m}, θ_{1n} and θ_{2n} in descending order to obtain rank(γ_{1k}), rank(γ_{2m}), rank(θ_{1n}) and rank(θ_{2n}).

2): Perform pairing

for $n = 1 \to N$ **do**

For rank$(\gamma_{1k^*}) =$ rank(θ_{2n}), pair (n, k^*);

For rank$(\gamma_{2m^*}) =$ rank(θ_{1n}), pair (n, m^*).

end for

3): Obtain the pairing result $\{(n, m^*, k^*) : n = 1, \ldots, N\}$

Algorithm 4: Sorting-Based Algorithm (include direct link)

1): Sorting $(\gamma_{1k}/(1+\eta_{2k}))$, $(\gamma_{2m}/(1+\eta_{1m}))$, θ_{1n} and θ_{2n} in descending order to obtain rank$(\gamma_{1k}/(1+\eta_{2k}))$, rank$(\gamma_{2m}/(1+\eta_{1m}))$, rank$(\theta_{1n})$ and rank(θ_{2n}).

2): Perform pairing

for $n = 1 \to N$ **do**

For rank$\left(\frac{\gamma_{1k^*}}{1+\eta_{2k^*}}\right) =$ rank(θ_{2n}), pair (n, k^*).

For rank$\left(\frac{\gamma_{2m^*}}{1+\eta_{1m^*}}\right) =$ rank(θ_{1n}), pair (n, m^*).

end for

3): Obtain the pairing result $\{(n, m^*, k^*) : n = 1, \ldots, N\}$

3.2.2 Joint Power Allocation Optimization

With given pairing strategy $p(\cdot)$ and $q(\cdot)$, the optimization problem **P0** reduces to the joint optimization of power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}\}$ at each end nodes and at the relay, given by

$$
\begin{aligned}
\textbf{(P2)}:\ \max_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_r,\boldsymbol{\alpha}} & \sum_{\substack{n=1\\ k=p(n),\, m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right)+\log\left(1+\mathrm{SNR}_{2nk}\right) \\
\text{s.t.} \quad & \sum_{k=1}^{N} P_{1k} \leq P_{\text{tot}}, \sum_{m=1}^{N} P_{2m} \leq P_{\text{tot}}, \sum_{n=1}^{N} P_{rn} \leq P_{\text{tot}}. \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0, \alpha_n \in [0,1], \forall n.
\end{aligned}
$$

From (3.10) and (3.10), we observe that SNR_{1nm} and SNR_{2nk} are not jointly convex with respect to (w.r.t.) $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r, \boldsymbol{\alpha}\}$. Thus the optimization problem **P2** is non-convex and difficult to solve. We propose to separate this joint power optimization problem into three sub-optimization problems, and solve them iteratively. Specifically, given $\mathbf{p}_r$ and $\boldsymbol{\alpha}$, we first optimize $\{\mathbf{p}_1, \mathbf{p}_2\}$ to maximize the objective in **P2**. Then, using the obtained $\{\mathbf{p}_1, \mathbf{p}_2\}$ with given $\mathbf{p}_r$, we optimize $\boldsymbol{\alpha}$. Finally, with obtained $\{\mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\alpha}\}$, we optimize $\mathbf{p}_r$.

3.2.2.1 Joint Optimization of $\mathbf{p}_1$ and $\mathbf{p}_2$ Given $\boldsymbol{\alpha}$ and $\mathbf{p}_r$

The objective in **P2** can be rewritten as

$$
\begin{aligned}
\max_{\mathbf{p}_r,\boldsymbol{\alpha}} & \Big\{\max_{\mathbf{p}_1,\mathbf{p}_2,} \sum_{\substack{n=1\\ k=p(n),\, m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right)+\log\left(1+\mathrm{SNR}_{2nk}\right)\Big\} \\
\text{s.t.} \quad & \sum_{k=1}^{N} P_{1k} \leq P_{\text{tot}}, \sum_{m=1}^{N} P_{2m} \leq P_{\text{tot}}, \sum_{n=1}^{N} P_{rn} \leq P_{\text{tot}}. \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \mathbf{p}_r \succcurlyeq 0, \alpha_n \in [0,1], \forall n.
\end{aligned} \tag{3.26}
$$

However, from (3.10) and (3.11), we see that, given $\mathbf{p}_r$ and $\boldsymbol{\alpha}$, the inner maximization problem with respect to $\{\mathbf{p}_1, \mathbf{p}_2\}$ is not convex and thus might not have a computationally efficient solution. From (3.6) and (3.7), we see that SNR_{1nm} and SNR_{2nk} can be expressed in terms of of $\{P_{2m}, w_{1nk}, w_{2nm}\}$ and $\{P_{1k}, w_{1nk}, w_{2nm}\}$, respectively. If we fix the relay power coefficients w_{1nk} and w_{2nm} instead of $\mathbf{p}_r$ and $\boldsymbol{\alpha}$ at the relay, the inner maximization above turns out to be convex.

Let $\mathbf{w}_1$ and $\mathbf{w}_2$ be the power coefficient vector with $[\mathbf{w}_1]_k = w_{1nk}$ and $[\mathbf{w}_2]_m = w_{2nm}$, where k is related to n based on the pairing result $k = p(n)$ and m is related to n based on the pairing result $m = q(n)$. Using (3.4), the joint power optimization problem **P2** can then be rewritten as

$$
\begin{aligned}
\textbf{(P2')}: \quad & \max_{\mathbf{w}_1,\mathbf{w}_2} \max_{\mathbf{p}_1,\mathbf{p}_2} \sum_{\substack{n=1 \\ k=p(n),\, m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right) + \log\left(1+\mathrm{SNR}_{2nk}\right) \\
\text{s.t.} \quad & \sum_{n=1}^{N} [|w_{1nk}|^2 (P_{1k}|h_{1k}|^2 + \sigma^2) + \\
& |w_{2nm}|^2 (P_{2m}|h_{2m}|^2 + \sigma^2)] \le P_{tot}, \\
& \sum_{k=1}^{N} P_{1k} \le P_{\text{tot}}, \sum_{m=1}^{N} P_{2m} \le P_{\text{tot}}. \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \alpha_n \in [0,1], \forall n.
\end{aligned}
$$

where SNR_{1nm} and SNR_{2nk} are expressed in (3.6) and (3.7), respectively, as functions of $\{P_{1n}, P_{2n}, w_{1nk}, w_{2nm}\}$. For given $\mathbf{w}$, the inner maximization of **P2'** is given by

$$
\begin{aligned}
\textbf{(P2'a)}: \quad & \max_{\mathbf{p}_1,\mathbf{p}_2} \sum_{\substack{n=1 \\ k=p(n),\, m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right) + \log\left(1+\mathrm{SNR}_{2nk}\right) \\
\text{s.t.} \quad & \sum_{n=1}^{N} [|w_{1nk}|^2 (P_{1k}|h_{1k}|^2 + \sigma^2) + \\
& |w_{2nm}|^2 (P_{2m}|h_{2m}|^2 + \sigma^2)] \le P_{tot}, \\
& \sum_{k=1}^{N} P_{1k} \le P_{\text{tot}}, \sum_{m=1}^{N} P_{2m} \le P_{\text{tot}}. \\
& \mathbf{p}_1 \succcurlyeq 0, \mathbf{p}_2 \succcurlyeq 0, \alpha_n \in [0,1], \forall n.
\end{aligned}
$$

From (3.6) and (3.7), SNR_{1nm} and SNR_{2nk} are both linear with respect to P_{1k} and P_{2m}, thus the objective in **P2'a** is jointly convex with respect to $\{\mathbf{p}_1, \mathbf{p}_2\}$. Therefore, the optimization problem **P2'a** is convex and can be solved by standard convex optimization tools.

3.2.2.2 Optimization of $\boldsymbol{\alpha}$ Given $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_r$

With given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_r\}$, the optimization problem **P2** becomes

$$\textbf{(P2'b)}: \max_{\boldsymbol{\alpha}} \sum_{\substack{n=1 \\ k=p(n), m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right) + \log\left(1+\mathrm{SNR}_{2nk}\right)$$
$$\text{s.t.} \quad \alpha_{\mathrm{n}} \in [0,1], \forall \mathrm{n}.$$

Let $R(\alpha_n) = \log(1+\mathrm{SNR}_{nmk}^{\mathrm{eff}})$, and $R'(\alpha_n)$ be the derivative of $R(\alpha_n)$. Since $R'(\alpha_n) = 0$ leads a quadratic equation as shown in the Appendix C, the roots α_{n1} and α_{n2} of $R'(\alpha_n) = 0$ is the maximum or the minimum points of $R(\alpha_n)$. If α_{n1} and α_{n2} are within the interval $[0,1]$, we then compare $R(\alpha_{n1})$ and $R(\alpha_{n2})$ with $R(0)$ and $R(1)$, the maximum one is the optimal solution for $R(\alpha_n)$. The detail of solution is in Appendix C.

3.2.2.3 Optimization of $\mathbf{p}_r$ Given $\mathbf{p}_1$, $\mathbf{p}_2$ and $\boldsymbol{\alpha}$

With given power allocation $\{\mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\alpha}\}$, the optimization problem **P2** becomes

$$\textbf{(P2'c)}: \max_{\mathbf{p}_r} \sum_{\substack{n=1 \\ k=p(n), m=q(n)}}^{N} \log\left(1+\mathrm{SNR}_{1nm}\right) + \log\left(1+\mathrm{SNR}_{2nk}\right)$$
$$\text{s.t.} \quad \sum_{\mathrm{m}=1}^{\mathrm{N}} \mathrm{P}_{\mathrm{rn}} \leq \mathrm{P}_{\mathrm{tot}}, \quad \mathbf{p}_{\mathrm{r}} \succcurlyeq 0.$$

Given $\{\mathbf{p}_1, \mathbf{p}_2\}$, we see from (3.10) and (3.11) that SNR_{1nm} and SNR_{2nk} are both concave functions of P_{rn}. As a result, the objective in **P2'c** is concave with respect to $\mathbf{p}_r$. Thus, the optimization problem **P2'c** is convex. However, the objective in **P2'c** with respect to $\mathbf{p}_r$ has a complicated fractional form that cannot be easily implemented using standard convex optimization tools. Therefore, we obtain the solution for **P2'c** using Karush-Kuhn-Tucker (KKT) conditions [53].

Denote $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]$ as the vector of Lagrange multipliers corresponding to the non-negative power constrains on each subchannel. Denote ν as the Lagrange multiplier corresponding to the power budget constraint. Since SNR_{1nm} and SNR_{2nk} are now only functions of P_{rn}, to explicit show this dependency, we denote the sum-rate objective in **P2'c** as $\sum_{n=1}^{N} R_n(P_{rn})$, where

$$R_n(P_{rn}) \triangleq \log(1 + \mathrm{SNR}_{1nm}) + \log(1 + \mathrm{SNR}_{2nk}).$$

It is easy to see that at the optimum, the relay power constraint is attained at the equality, *i.e.*, $\sum_{n=1}^{N} P_{rn} = P_{\text{tot}}$. Thus, using the KKT conditions, we have

$$\begin{aligned} \mathbf{p}_r \succcurlyeq\ 0,\ \boldsymbol{\lambda} \succcurlyeq 0,\ \sum_{n=1}^{N} P_{rn} = P_{\text{tot}},\ \lambda_n P_{rn} = 0, \\ R_n'(P_{rn}) - \lambda_n + \nu = 0, \quad n = 1, \ldots, N \end{aligned} \tag{3.27}$$

where $R_n'(P_{rn})$ denote the derivative of $R_n(P_{rn})$ w.r.t. P_{rn}, and the value of ν should ensure $\sum_{n=1}^{N} P_{rn}^o = P_{\text{tot}}$. From (3.27), if the optimal $P_{rn}^o > 0$, we have $\lambda_n = 0$. Thus, P_{rn}^o, for $n = 1, \cdots, N$, should satisfy

$$\begin{aligned} P_{rn}^o > 0 \ \text{ and } R_n'(P_{rn}^o) + \nu = 0 \\ \text{or } \ P_{rn}^o = 0, \quad n = 1, \cdots, N. \end{aligned} \tag{3.28}$$

The above solution for P_{rn}^{o} can be viewed as a variation of classical waterfilling solution, the detail of solution is provided in Appendix. A.

3.2.2.4 Iterative Procedure for Joint Power Optimization

We solve the joint power optimization problem **P2** by iteratively solving the optimization subproblems **P2'a**, **P2'b**, and **P2'c** to update $\{\mathbf{p}_1, \mathbf{p}_2\}$, $\boldsymbol{\alpha}$, and $\mathbf{p}_r$, respectively. Specifically, let $\{\mathbf{p}_1^l, \mathbf{p}_2^l, \mathbf{p}_r^l, \boldsymbol{\alpha}^l\}$ be the power allocation solutions obtained after the lth iteration, and let $\mathbf{w}_1^l$ and $\mathbf{w}_2^l$ be the corresponding relay power coefficient obtained from $\{\mathbf{p}_1^l, \mathbf{p}_2^l, \mathbf{p}_r^l, \boldsymbol{\alpha}^l\}$. At the $(l+1)$th iteration:

1. Given $\mathbf{w}_1^l$ and $\mathbf{w}_2^l$, we solve the joint optimization problem **P2'a** to obtain $\{\mathbf{p}_1^{l+1}, \mathbf{p}_2^{l+1}\}$;

2. Given $\{\mathbf{p}_1^{l+1}, \mathbf{p}_2^{l+1}\}$ and $\mathbf{p}_r^l$, we solve the optimization problem **P2'b** to obtain $\boldsymbol{\alpha}^{l+1}$;

3. Given $\{\mathbf{p}_1^{l+1}, \mathbf{p}_2^{l+1}, \boldsymbol{\alpha}^{l+1}\}$, we solve the optimization problem **P2'c** to obtain $\mathbf{p}_r^{l+1}$.

Repeat steps 1-3 until the sum-rate objective in **P2** converges, and we obtain a local maximum solution for **P2**.

3.2.3 Joint Pairing and Power Allocation Iterative Optimization

Finally, to solve the original joint optimization problem **P0**, we iteratively solve the optimization problems **P1** and **P2**. The sth iteration contains two steps:

1. Given power allocation $\{\mathbf{p}_1^s, \mathbf{p}_2^s, \mathbf{p}_r^s, \boldsymbol{\alpha}^s\}$, we solve subchannel pairing problem **P1** and obtain pairing permutation $p^{s+1}(\cdot)$ and $q^{s+1}(\cdot)$;

2. Given pairing permutation function $p^{s+1}(\cdot)$ and $q^{s+1}(\cdot)$, the power allocation problem **P2** is solved using iterative approach mentioned in Section 3.2.2.4.

The above procedure is repeated until the sum-rate converges.

Note that the convergence of this iterative approach is guaranteed as the value of the sum-rate objective in each step of the iterative procedure is non-decreasing. However, the original joint optimization problem may have multiple local maxima. Thus, typically we need a few initialization trials and select the one with the best performance. The iterative joint optimization approach is summarized in Algorithm 5.

Algorithm 5: Iterative Optimization to Solve **P0**

Initialize: *Set* $\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0$, $\boldsymbol{\alpha}^0$, $p^0(\cdot)$, $q^0(\cdot)$, ϵ;

Set $s = 0$; Compute R^0 in (3.8) with $\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0$, $\boldsymbol{\alpha}^0$, $p^0(\cdot)$, $q^0(\cdot)$. Set $\Delta R^0 > \epsilon$.

while $\Delta R^s > \epsilon$ **do**

// Given $p^s(\cdot)$, $q^s(\cdot)$ solve power optimization in **P2**

Set $l = 0$, $\tilde{\mathbf{p}}_1^0 = \mathbf{p}_1^s$, $\tilde{\mathbf{p}}_2^0 = \mathbf{p}_2^s$, $\tilde{\mathbf{p}}_r^0 = \mathbf{p}_r^s$, $\tilde{\boldsymbol{\alpha}}^0 = \boldsymbol{\alpha}^s$;

Let $\tilde{R}^l$ be the objective value in **P2** at the lth iteration.

Compute $\tilde{R}^0$ with $\{\tilde{\mathbf{p}}_1^0, \tilde{\mathbf{p}}_2^0, \tilde{\mathbf{p}}_r^0\}, \tilde{\boldsymbol{\alpha}}^0$. $\Delta\tilde{R}^0 > \epsilon$.

while $\Delta\tilde{R}^l > \epsilon$ **do**

Obtain $\mathbf{w}_1^l$, $\mathbf{w}_2^l$ using (3.4) based on $\tilde{\mathbf{p}}_1^l$, $\tilde{\mathbf{p}}_2^l$ $\tilde{\mathbf{p}}_r^l$ and $\tilde{\boldsymbol{\alpha}}^l$;

Given $\mathbf{w}_1^l$, $\mathbf{w}_2^l$ solve **P2'a** to obtain $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}\}$;

Given $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}\}$, and $\tilde{\mathbf{p}}_r^l$ solve **P2'b** to obtain $\tilde{\mathbf{p}}_r^{l+1}$;

Given $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}\}$, and $\tilde{\boldsymbol{\alpha}}^{l+1}$ solve **P2'c** to obtain $\tilde{\mathbf{p}}_r^{l+1}$;

Compute $\tilde{R}^{l+1}$ based on $\{\tilde{\mathbf{p}}_1^{l+1}, \tilde{\mathbf{p}}_2^{l+1}, \tilde{\mathbf{p}}_r^{l+1}, \boldsymbol{\alpha}^{l+1}, p^s(\cdot), q^s(\cdot)\}$.

Set $l \leftarrow l + 1$;

end

Output: $\mathbf{p}_1^{s+1} = \tilde{\mathbf{p}}_1^l$, $\mathbf{p}_2^{s+1} = \tilde{\mathbf{p}}_2^l$, $\mathbf{p}_r^{s+1} = \tilde{\mathbf{p}}_r^l$, $\boldsymbol{\alpha}^{s+1} = \tilde{\boldsymbol{\alpha}}^l$;

Given $\{\mathbf{p}_1^{s+1}, \mathbf{p}_2^{s+1}, \mathbf{p}_r^{s+1}, \boldsymbol{\alpha}^{s+1}\}$, solve pairing optimization problem **P1** to obtain $p^{s+1}(\cdot)$ and $q^{s+1}(\cdot)$;

Compute R^{s+1} based on $\{\mathbf{p}_1^{s+1}, \mathbf{p}_2^{s+1}, \mathbf{p}_r^{s+1}, \boldsymbol{\alpha}^{s+1}, p^{s+1}(\cdot), q^{t+1}(\cdot)\}$;

Set $\Delta R^{s+1} = R^{s+1} - R^s$;

Set $s \leftarrow s + 1$;

end

Output: $\mathbf{p}_1^s$, $\mathbf{p}_2^s$, $\mathbf{p}_r^s$, $\boldsymbol{\alpha}^s$, $p^s(\cdot)$ and $q^s(\cdot)$;

3.3 Simulation Results

We evaluate the performance of the proposed pairing strategy and iterative joint optimization of pairing and power allocation through simulations using an OFDM system with N subchannels. The channel gain over each subchannel is complex Gaussian with distribution $\mathcal{CN}(0, \sigma_h^2)$, where σ_h^2 follows the pathloss model $\sigma_h^2 = K_o d^{-\kappa}$ with pathloss exponent $\kappa = 3$ and $K_o = 1$. Let d_{12}, d_{1r} and d_{r2} denote the distance between Nodes 1 and 2, Node 1 to the relay, and Node 2 to the relay, respectively. We define the average SNR from Node 1 to Node 2 over the direct path as $\mathrm{SNR}_{12} \triangleq \frac{P_T}{N} d_{12}^{-k} / \sigma^2$.

3.3.1 Performance Comparison under Pairing Schemes

We first fix the power allocation and compare the performance under different pairing strategies. We chose α_n in (3.4) such that $w_{1nk} = w_{2mk}$ [2]. Let d_{12}, d_{1r} and d_{r2} denote the distance between Nodes 1 and 2, Node 1 to the relay, and Node 2 to the relay, respectively. We define the average SNR from Node 1 to Node 2 over the direct path as $\mathrm{SNR}_{12} \triangleq \frac{P_T}{N} d_{12}^{-k} / \sigma^2$. Besides the proposed Algorithms 2-4, we also consider the following schemes for benchmark comparison 1) A random pairing scheme, in which $p(\cdot)$ and $q(\cdot)$ is a random permutation; 2) Direct pairing, *i.e.*, $k = m = n$.

We first show the convergence behavior under the Iterative Optimization over iteration in Fig. 3.5. We set $N = 128$ and assume no direct link. For initialization, we use the pairing result under either the random pairing or the sorting-based algorithm. We see that the performance under the two initialization methods is different only in first a couple of iterations. This indicates that the convergence is not sensitive to the initial pairing used. Furthermore, the convergence is fast with typically only 3-4 iterations needed.

[2] In our simulation study, we observe that the sum-rate performance under this specific power allocation is higher than other power allocations. Therefore, we choose this particular setting. Joint optimization of pairing and power allocation will show in following section.

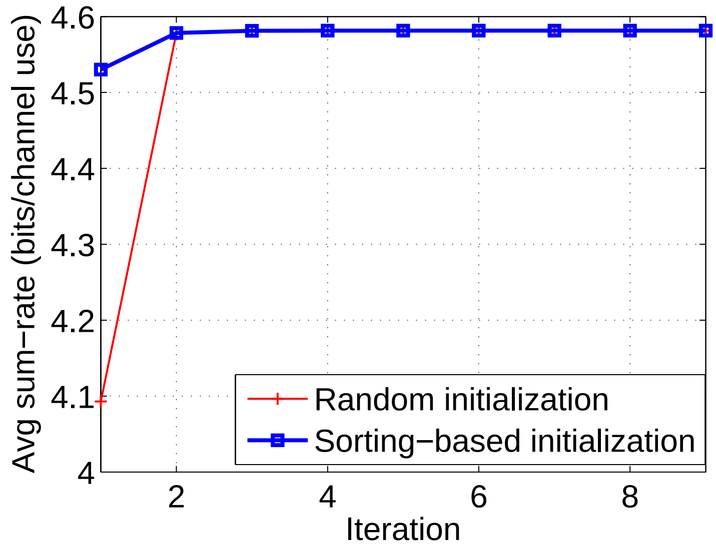

Figure 3.5: Convergence behavior under the Iterative Optimization scheme over iteration

Then, we compare the sum-rate performance among different pairing schemes as N increases. We assume $d_{1r} = d_{r2}$, and set $\mathrm{SNR}_{12} = 2dB$. Fig. 3.6 and Fig. 3.7 show the average sum-rate per subchannel vs. N, with or without direct link, respectively. From Fig. 3.6, we observe that the sum-rate per subchannel under each pairing algorithm increases as N increases, indicating higher pairing gain is achieved as N increases. We also see that the performance of the proposed sorting-based algorithm is very close to the interactive optimization algorithm, while the latter has significantly higher complexity than the former. As expect, the random pairing and direct pairing provides the lowest performance as they do not actively seeking pairing to improve the sum-rate. Fig. 3.7 shows similar trend when the direct link is considered. We also observe that using Algorithm 4 that includes direct link SNR in the metric for pairing improves upon the performance of Algorithm 3 that does not involves the direct link for pairing.

Next, we show the performance vs. different relay locations between two terminal nodes under the proposed pairing algorithms. We set $N = 128$. Shown in Fig. 3.8 (without

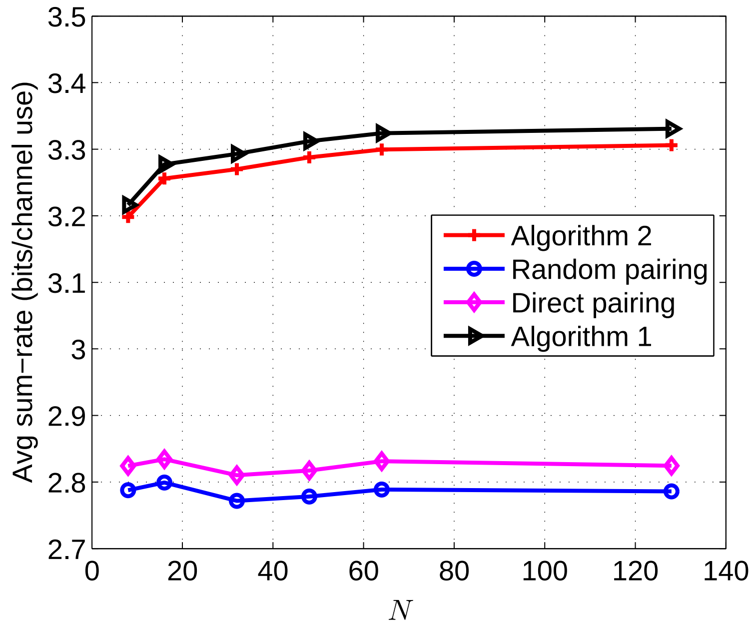

Figure 3.6: Sum rate per subchannel vs. N (No direct link; $\mathrm{SNR}_{12} = 2dB$)

direct link) and Fig. 3.9 (with direct link), the sum-rate is maximized as the relay moves to the middle point between Nodes 1 and 2, and the pairing gain is the highest as the relay is moved to the middle between Nodes 1 and 2.

3.3.2 Performance under Joint Pairing and Power Allocation Strategies

We now study the performance under the joint pairing and power allocation.

Convergence Behavior We first study the convergence behavior of the iterative power allocation optimization method in Section 3.2.2.4 and iterative joint pairing and power allocation algorithm in Algorithm 5. We assume the relay is at the middle point between the two end nodes, *i.e.*, $d_{1r} = d_{r2}$. We set subchannel number as $N = 64$ for power allocation optimization problem **P2**, $N = 32$ for iterative optimization problem **P0**.

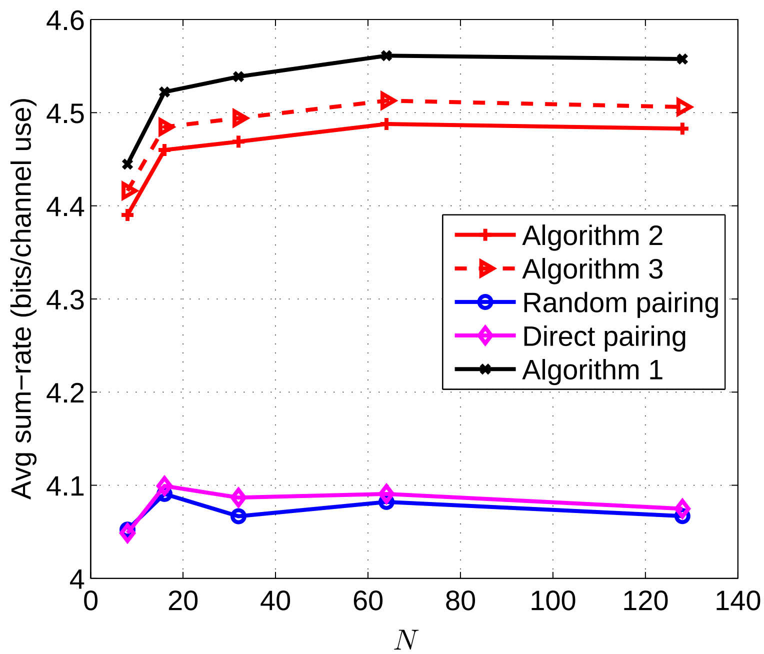

Figure 3.7: Sum rate per subchannel vs. N (With direct link; $\mathrm{SNR}_{12} = 2dB$)

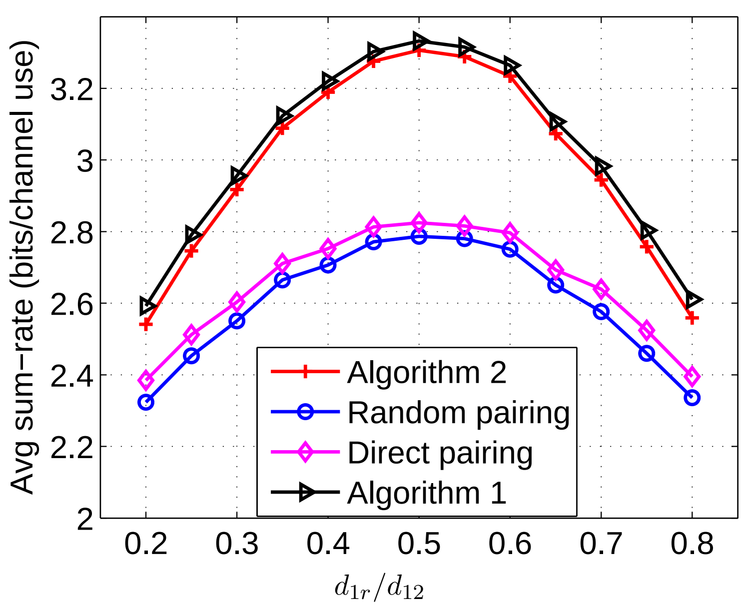

Figure 3.8: Sum rate per subchannel vs. d_{1r}/d_{r2} ($N = 128$; No direct link)

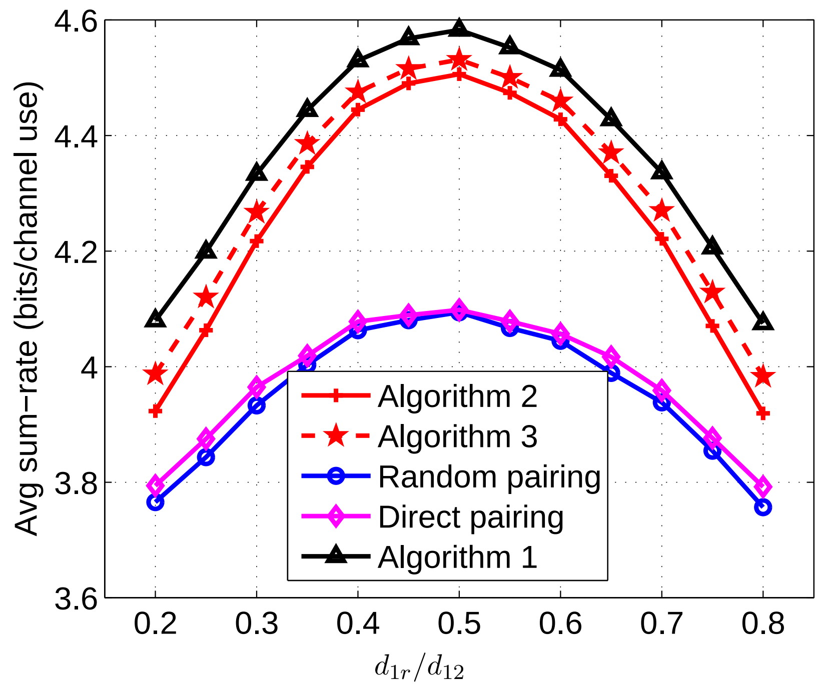

Figure 3.9: Sum rate per subchannel vs. d_{1r}/d_{r2} ($N = 128$; With direct link)

Fig. 3.10 plots the average sum-rate per subchannel versus the number of iterations by solving the joint power allocation optimization problem **P2** using the iterative approach in Section 3.2.2.4. The pairing scheme $p(\cdot)$ and $q(\cdot)$ is randomly generated at the beginning but is fixed during the experiment. A few randomly generated initializations for $\{\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0, \boldsymbol{\alpha}^0\}$ are used to test the convergence behavior and the local maximum . Each curve corresponds to a different initialization. Similarly, Fig. 3.11 and Fig. 3.12 plots the average sum-rate per subchannel versus the number of iterations under Algorithm 5 for solving the joint optimization problem **P0**. Fig. 3.11 uses Algorithm 2 as its pairing strategy, while Fig. 3.12 uses Algorithm 3 as its pairing strategy. Each curve corresponds to a different initialization for $\{\mathbf{p}_1^0, \mathbf{p}_2^0, \mathbf{p}_r^0, \boldsymbol{\alpha}^0, p^0(\cdot), q^0(\cdot)\}$ which are again randomly generated. The same set of channel realizations are used for Figs. 3.10-3.12 and all of them are without direct link. From Figs. 3.10-3.12, we see that, for the iterative optimization for both

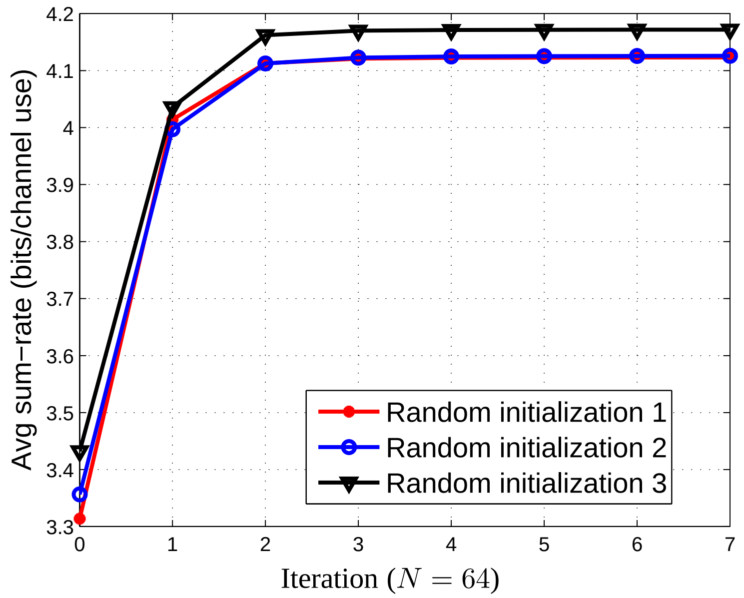

Figure 3.10: Average sum-rate vs. iterations for solving joint power allocation problem **P2** (without direct link).

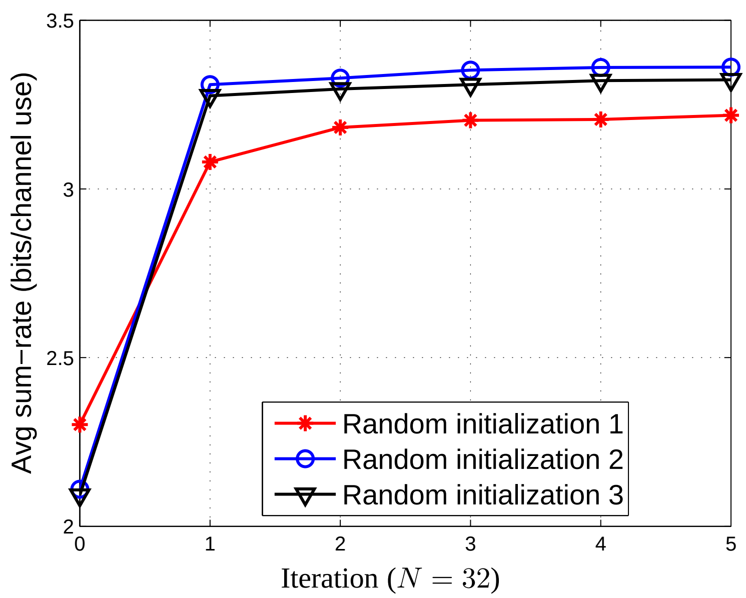

Figure 3.11: Average sum-rate vs. iterations for solving joint pairing and power allocation problem **P0**, (iterative pairing algorithm and without direct link)

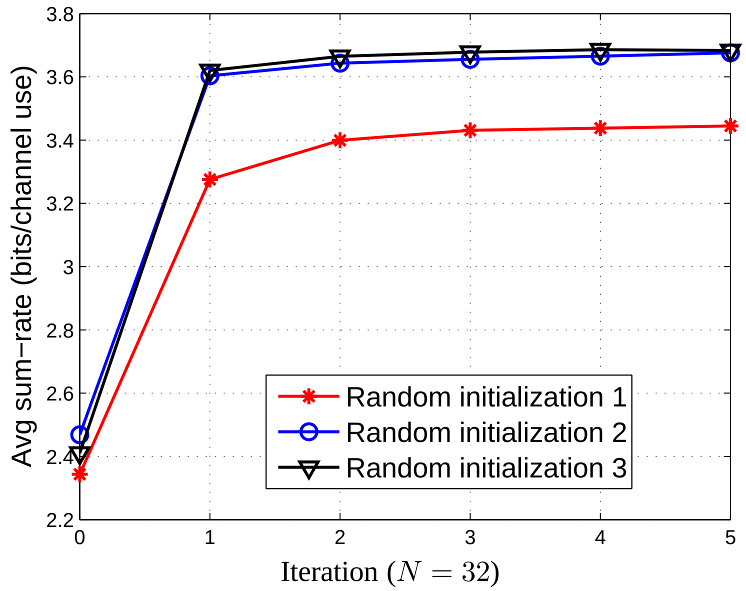

Figure 3.12: Average sum-rate vs. iterations for solving joint pairing and power allocation problem **P0**, (sorting-based paring strategy and without direct link)

P2 and **P0**, the sum-rate converges in just a few iterations. In addition, we see that several local maximum points may exist and different initializations may converge to different local maxima, although the difference is not large. This shows that a few initializations are required to improve the performance.

Performance We study the performance of average sum-rate versus the relay position between the two end notes with or without direct link, respectively. We set subcannel as $N = 8$ for without direct link and $N = 32$ for with direct link. Performances under four schemes are compared: 1) Equal power allocation at all nodes and no pairing (*i.e.*, the same incoming and outgoing subchannels); 2) Pairing only: pairing is optimized using **P1** with two pairing strategies: Iterative Optimization Algorithm and Sorting-Based Algorithm, while equal power allocation is assumed; 3) Power allocation only: only **P2** is solved by proposed power allocation iterative procedure, while a random pairing is given;

4) Power allocation and pairing: our proposed Algorithm 5 to solve **P0**.

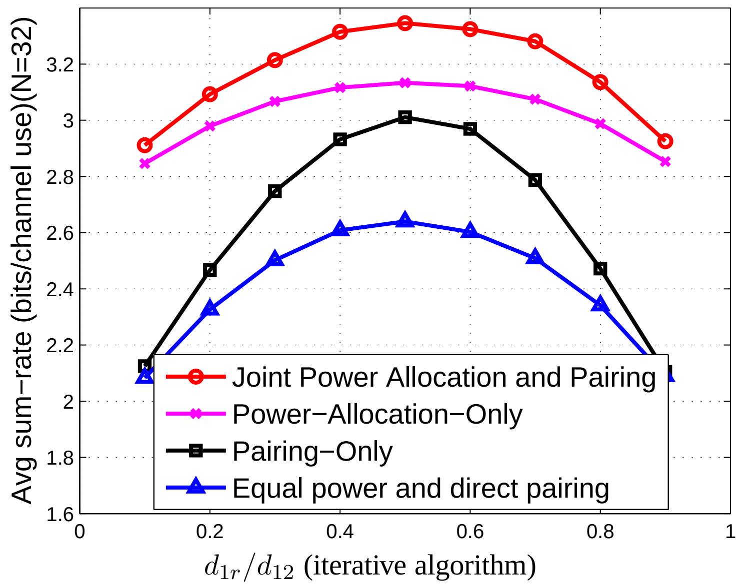

Figure 3.13: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 8$; Iterative Pairing Algorithm; no direct link)

Shown in Figs. 3.13, 3.14(without direct link) and Figs. 3.15, 3.16(with direct link), the additional performance gain of joint optimization over the pairing-only and the power-allocation-only schemes is clearly seen. The sum-rate increases as the relay moves towards the middle point between Nodes 1 and 2. Note that, for two-way relaying, it is desirable to have the relay at the middle point to benefit both end nodes for.

Then we compare the performance of our proposed joint pairing and power allocation Algorithm 5 under two different suboptimal pairing schemes: Iterative Optimization 2 and Sorting-Based Algorithm 3, 4 with or without direct link. We set subcannel as $N = 32$.

Shown in Fig. 3.17(without direct link) and Fig. 3.18(with direct link), Sorting-Based Algorithm 3, have better joint pairing and power allocation optimization performance than Iterative Optimization 2 without direct link, while Iterative Optimization 2 have better joint optimization performance than Sorting-Based Algorithm 4 with direct link.

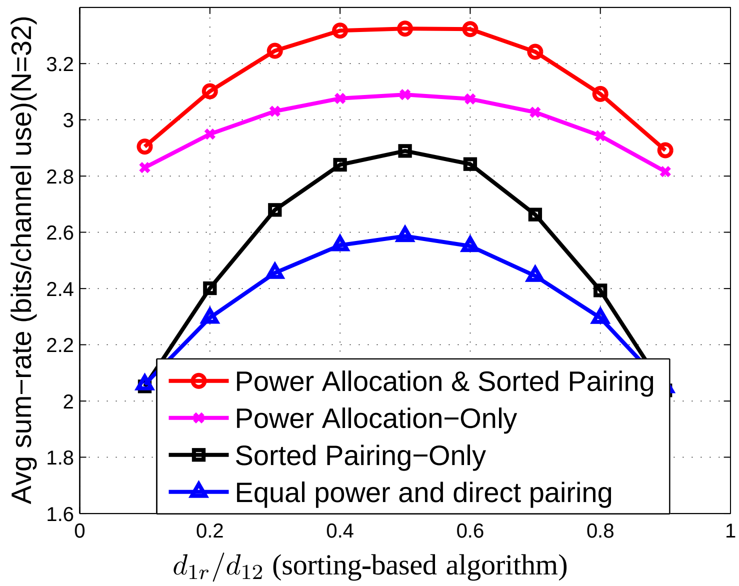

Figure 3.14: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 8$; Sorting-Based Pairing Strategy; no direct link)

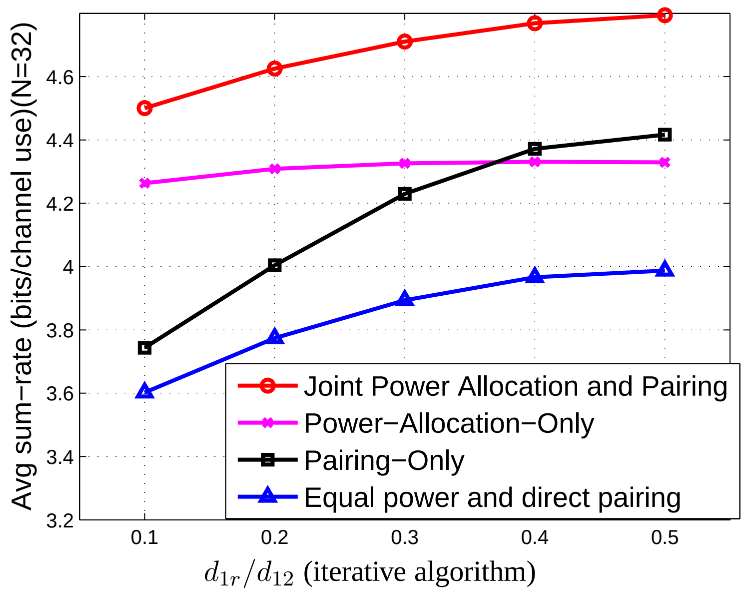

Figure 3.15: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 32$; Iterative Pairing Algorithm; with direct link)

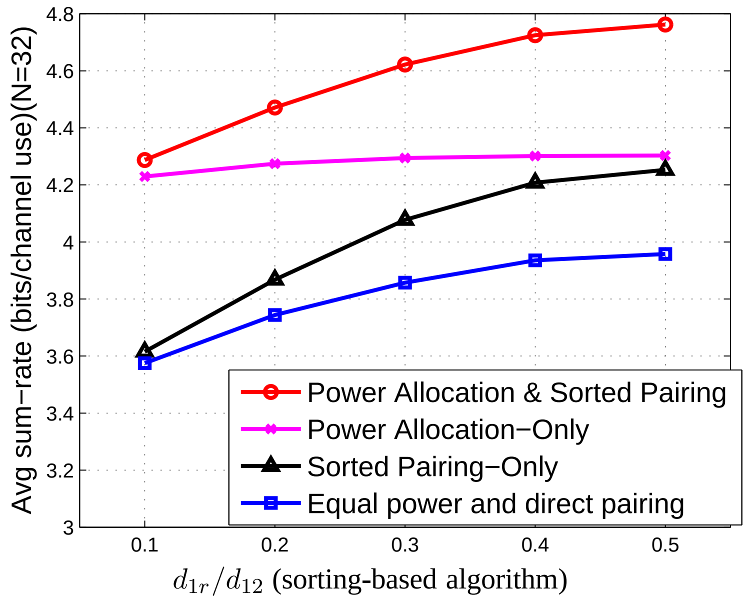

Figure 3.16: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 32$; Sorting-Based Pairing Strategy; with direct link)

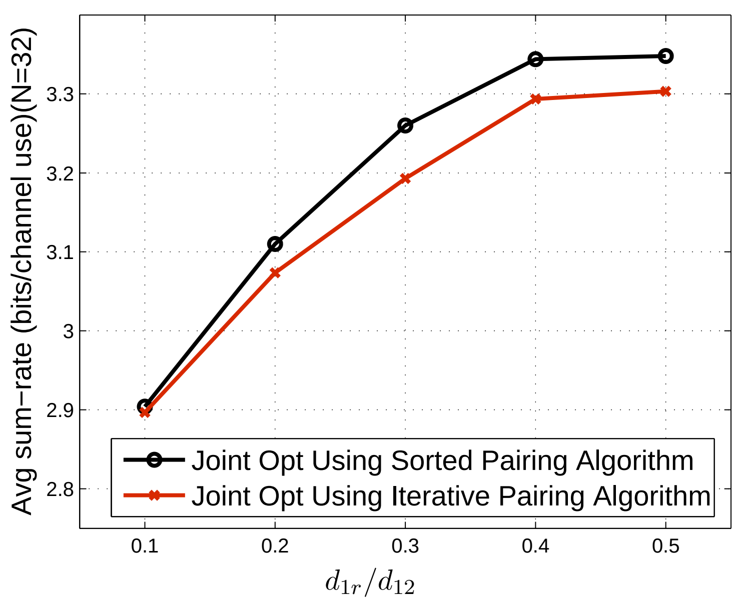

Figure 3.17: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 32$; no direct link)

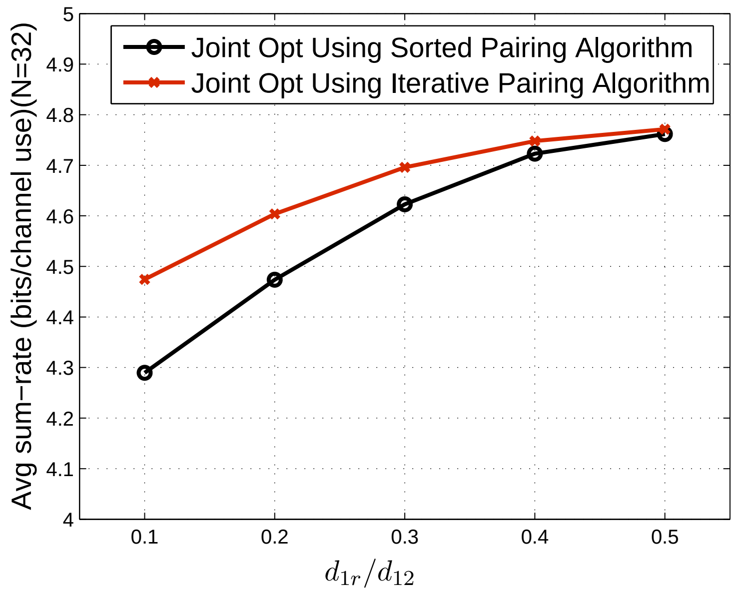

Figure 3.18: Average sum-rate per subchannel vs. d_{1r}/d_{12} ($N = 32$; with direct link)

3.4 Summary

In this chapter, the joint paring and power allocation optimal problem for TDBC-based two-way AF relaying among two terminal nodes and one relay is considered. The objective is to maximize the sum-rate of both end nodes. We have first studied the subchannel pairing problem, which the subchannel pairing schemes are designed to maximize the achievable data rate under certain power allocation in the network. Unlike in the one-way case, we showed that there exists no efficient SNR-based pairing strategy that is sum-rate optimal for two-way pairing. Formulating the pairing optimization as an axial 3-D assignment problem, we proposed an iterative optimization method to solve it with complexity $\mathcal{O}(N^3)$. Based on SNR over each subchannel, we also proposed sorting-based algorithms for scenarios with and without direct link, with a low complexity of $\mathcal{O}(N \log N)$. The complexity reduction is substantial especially for broadband systems, and the simulation performance is shown to be very close to the iterative optimization method.

For the joint power allocation among the two end nodes and the relay, a further iterative optimization procedure was proposed to solve the power allocation at the relay and at two end nodes iteratively. By transforming the SNR expression with respect to different form of optimization parameters, each power optimization problem turns out to be a convex problem and can be solved to obtain the optimal solution.

Finally, for the original joint subchannel pairing and power allocation problem, we proposed an additional iterative algorithm which solves the pairing and power allocation subproblem iteratively. The simulation performance demonstrates the gain of joint optimization approach over other pairing-only or power-allocation-only optimization approaches.

Chapter 4

Conclusions

In this thesis, the joint subchannel paring and power allocation optimal problem for a multichannel MABC-based and TDBC two-way relay network is considered. Based on the theoretical analysis and numerical results, we can make conclusions as follow.

First, in MABC-based two way relaying, We proposed an iterative algorithm which solves the pairing and power allocation problem iteratively. For the joint power allocation among the two end nodes and the relay, a further iterative optimization procedure was proposed to solve the power allocation at the relay and at two end nodes iteratively. By transforming the SNR expression with respect to different form of optimization parameters, each power optimization problem turns out to be a convex problem and can be solved to obtain the optimal solution. The simulation performance demonstrates the gain of joint optimization approach over other pairing-only or power-allocation-only optimization approaches.

Then, in TDBC-based two way relaying, We studied the subchannel pairing problem, which the subchannel pairing schemes are designed to maximize the achievable data rate under certain power allocation in the network. Unlike in the one-way case, we showed that there exists no efficient SNR-based pairing strategy that is sum-rate optimal for two-way pairing. Formulating the pairing optimization as an axial 3-D assignment problem, we proposed an iterative optimization method to solve it with complexity $\mathcal{O}(N^3)$. Based on SNR over each subchannel, we also proposed sorting-based algorithms for scenarios with and without direct link, with a low complexity of $\mathcal{O}(N \log N)$. The complexity reduction is substantial especially for broadband systems, and the simulation performance is shown to be very close to the iterative optimization method.

Finally, in TDBC-based two-way relaying, the joint optimization is, on the other hand, a difficult problem. We proposed a similar iterative algorithm as MABC-based two way relaying, which solves the pairing and power allocation problem iteratively. For the joint power allocation among the two end nodes and the relay, a further iterative optimization procedure was proposed to solve the power allocation at the relay and at two end nodes iteratively. Unlike MABC-based two-way relaying, there is another parameter the fraction α need to consider. By transforming the SNR expression with respect to different form of optimization parameters, each power optimization problem turns out to be a convex problem and can be solved to obtain the optimal solution. The simulation performance demonstrates the gain of joint optimization approach over other pairing-only or power-allocation-only optimization approaches.

Appendix A

Derivation of the Optimal Solution P_{rm}^{o} in (2.13)

Let $a_m = \phi_n \frac{|h_{1m}|^2}{\sigma^2}, b_m = 1 + \phi_n + \gamma_n, c_m = \frac{|h_{1m}|^2}{\sigma^2}, d_m = \gamma_n \frac{|h_{2m}|^2}{\sigma^2}, e_m = \frac{|h_{2m}|^2}{\sigma^2}$. Then, $R_m(P_{rm})$ can be written as:

$$R_m = \log(1 + \frac{a_m P_{rm}}{b_m + c_m P_{rm}} + \frac{d_m P_{rm}}{b_m + e_m P_{rm}} + \frac{a_m d_m P_{rm}^2}{(b_m + c_m P_{rm})(b_m + e_m P_{rm})}) \tag{A.1}$$

By using KKT conditions, we have equation (3.27) and (2.13).

When $P_{rm}^{o} > 0$, the equation in (2.13), *i.e.*,

$$R'_m(P_{rm}^{o}) + \nu = 0$$

can be expressed as

$$A_m P_{rm}^4 + B_m P_{rm}^3 + C_m P_{rm}^2 + D_m P_{rm} + E_m = 0 \tag{A.2}$$

where

$$
\begin{aligned}
A_m &= \nu(a_m e_m^2 c_m + a_m d_m e_m c_m + c_m^2 e_m^2 + c_m^2 d_m e_m) \\
B_m &= \nu(2b_m c_m e_m a_m + 2b_m c_m^2 e_m + 2b_m c_m e_m d_m + 2b_m c_m e_m^2 \\
&+ b_m a_m e_m^2 + a_m d_m e_m b_m + a_m d_m c_m b_m + c_m^2 d_m b_m) \\
C_m &= \nu b_m^2(4c_m e_m + 2a_m e_m + a_m c_m + c_m^2 \\
&+ e_m d_m + 2c_m d_m + e^2 + a_m d_m) \\
&- (a_m b_m e_m^2 + b_m e_m a_m d_m + b_m c_m^2 d_m + b_m c_m a_m d_m) \\
D_m &= \nu b_m^3(a_m + 2c_m + d_m + 2e_m) \\
&- b_m(2b_m a_m e_m + 2b_m c_m d_m + 2b_m a_m d_m) \\
E_m &= \nu b_m^4 - b_m^3(a_m + d_m)
\end{aligned}
$$

Lagrange operator ν should ensure relay power constraint *i.e.*,

$$\sum_{m=1}^{N} P_{rm}^o = P_{\text{tot}}.$$

We can solve this problem by using bisection method: for a given ν, we can obtain a set of P_{rm}^o by solving quartic function (D.1), then, we adjust the value of ν based on the comparison of $\sum_{m=1}^{N} P_{rm}^o$ and relay power constraint P_{tot}. Finally, we can find the proper ν^o which satisfy relay power constraint and substitute ν^o in quartic function (D.1) to obtain the optimal relay power P_{rm}^o.

Appendix B

Proof of Lemma 1

We proof by contradiction. Assume there exist an optimal pairing strategy based on $f(\gamma_{1k}, \eta_{2k})$, $g(\gamma_{2m}, \eta_{1m})$ and $h(\theta_{1n}, \theta_{2n})$, and (3.21) holds. Then, the partial derivatives of $\Phi(\cdot)$ and $\Psi(\cdot)$ with respect to(w.r.t) the same variables should also be equal. The ratio of the partial derivatives of $\Psi(f(\gamma_{1k}, \eta_{2k}), g(\gamma_{2m}, \eta_{1m}), h(\theta_{1n}, \theta_{2n}))$ w.r.t. γ_{1k} and η_{2k} is given by

$$\frac{\partial \Psi}{\partial \gamma_{1k}} / \frac{\partial \Psi}{\partial \eta_{2k}} = \frac{\partial \Psi}{\partial f} \frac{\partial f}{\partial \gamma_{1k}} / \frac{\partial \Psi}{\partial f} \frac{\partial f}{\partial \eta_{2k}} = \frac{\partial f}{\partial \gamma_{1k}} / \frac{\partial f}{\partial \eta_{2k}}. \tag{B.1}$$

It follows that the following equality must hold

$$\frac{\partial \Phi}{\partial \gamma_{1k}} / \frac{\partial \Phi}{\partial \eta_{2k}} = \frac{\partial f}{\partial \gamma_{1k}} / \frac{\partial f}{\partial \eta_{2k}} \tag{B.2}$$

$$\frac{\partial \Phi}{\partial \phi_{m}} / \frac{\partial \Phi}{\partial \eta_{1,m}} = \frac{\partial g}{\partial \phi_{m}} / \frac{\partial g}{\partial \eta_{1,m}} \tag{B.3}$$

$$\frac{\partial \Phi}{\partial \lambda_{n}} / \frac{\partial \Phi}{\partial \theta_{n}} = \frac{\partial h}{\partial \lambda_{n}} / \frac{\partial h}{\partial \theta_{n}}. \tag{B.4}$$

From (3.14) and (3.15), $\Phi(\gamma_k, \mu_{2,k}, \phi_m, \eta_{1,m}, \lambda_n, \theta_n)$ in (3.19) is given by

$$\Phi(\gamma_k, \mu_{2,k}, \phi_m, \eta_{1,m}, \lambda_n, \theta_n) =$$
$$(1 + \frac{\phi_m \lambda_n}{2 + \gamma_k + \phi_m + 2\lambda_n} + \eta_{1,m})(1 + \frac{\gamma_k \theta_n}{2 + \phi_m + \gamma_k + 2\theta_n} + \mu_{2,k}) - 1.$$

Set X and Y as

$$\begin{aligned} X &= \phi_m \lambda_n (\mu_{2,k} + 1)(2 + \phi_m + 2\theta_n) + \gamma_k^2 \theta_n (\eta_{1,m} + 1) \\ &\quad + \gamma_k [\phi_m \lambda_n \theta_n + \theta_n (\eta_{1,m} + 1)(2 + \phi_m + 2\lambda_n) + \phi_m \lambda_n (\mu_{2,k} + 1)] \\ Y &= (2 + \gamma_k + \phi_m + 2\lambda_n)(2 + \phi_m + \gamma_k + 2\theta_n). \end{aligned}$$

Let X' is the derivative of X with respect to γ_{1k}, Y' is the derivative of Y with respect to γ_{1k} we have

$$\frac{\partial \Phi}{\partial \gamma_k} = \frac{X'Y - XY'}{(2 + \gamma_k + \phi_m + 2\lambda_n)^2 (2 + \phi_m + \gamma_k + 2\theta_n)^2}. \tag{B.5}$$

The expression of $\frac{\partial \Phi}{\partial \eta_{2k}}$ can be written as

$$\frac{\partial \Phi}{\partial \eta_{2k}} = 1 + \eta_{1,m} + \frac{\phi_m \lambda_n (2 + \phi_m + 2\theta_n + \gamma_k)}{(2 + \gamma_k + \phi_m + 2\lambda_n)(2 + \phi_m + \gamma_k + 2\theta_n)}. \tag{B.6}$$

From (B.5) and (B.6) we can see that left hand side of (B.2) depends on ϕ_m, $\eta_{1,m}$, λ_n, and θ_n, while the right hand side of (B.2) is not. Thus, the condition (B.2) does not hold. Similarly, we can show that (B.3) and (B.4) cannot hold either. This leads to contradiction of our earlier assumption.

Appendix C

Relay Power Fraction α Optimization in TDBC-Based Two-Way Relaying

Let $\hat{a_n} = \gamma_{2m}\theta'_{1n}$, $\hat{b_n} = 1 + \theta'_{1n} + \gamma_{2m}$, $\hat{c_n} = \theta'_{1n}\frac{\gamma_{2m}-\gamma_{1k}}{\gamma_{1k}+1}$, $\hat{d_n} = \gamma_{1k}\theta'_{2n}$, $\hat{e_n} = 1 + \theta'_{2n}\frac{\gamma_{1k}+1}{\gamma_{2m}+1} + \gamma_{1k}$, $\hat{f_n} = \theta'_{2n}\frac{\gamma_{2m}-\gamma 1k}{\gamma_{2m}+1}$, $\hat{g_n} = \eta_{1m}$, $\hat{h_n} = \eta_{2k}$.

Where $\theta'_{1n} = \frac{P_{rn}|h_{1n}|^2}{\sigma^2}$, $\theta'_{2n} = \frac{P_{rn}|h_{2n}|^2}{\sigma^2}$. $R(\alpha_n)$ can be written as

$$\begin{aligned} R(\alpha_n) = & \log(1 + \frac{\hat{a_n}(1-\alpha_n)}{\hat{b_n} + \hat{c_n}\alpha_n} + \hat{g_n}) \\ & + \log(1 + \frac{\hat{d_n}\alpha_n}{\hat{e_n} + \hat{f_n}\alpha_n} + \hat{h_n}) \end{aligned} \tag{C.1}$$

The derivation of $R(\alpha_n)$ is:

$$\begin{aligned} R'(\alpha_n) = & \frac{\hat{d_n}\hat{e_n}}{(\hat{e_n} + \hat{f_n}\alpha_n)(\hat{e_n} + \hat{e_n}\hat{h_n} + \hat{d_n}\alpha_n + \hat{f_n}\alpha_n + \hat{h_n}\hat{f_n}\alpha_n)} \\ & - \frac{\hat{a_n}(\hat{b_n} + \hat{c_n})}{(\hat{b_n} + \hat{c_n}\alpha_n)(\hat{a_n} + \hat{b_n} + \hat{b_n}\hat{g_n} - \hat{a_n}\alpha_n + \hat{c_n}\alpha_n + \hat{c_n}\hat{g_n}\alpha_n)} \end{aligned} \tag{C.2}$$

Let $R'(\alpha_n) = 0$, we obtain quadratic equation:

$$\hat{A}_n \alpha_n^2 + \hat{B}_n \alpha_n + \hat{C}_n = 0 \tag{C.3}$$

Where

$$\begin{aligned}
\hat{A}_n =& \hat{d}_n \hat{e}_n \hat{c}_n{}^2 - \hat{a}_n \hat{c}_n \hat{f}_n{}^2 - \hat{a}_n \hat{b}_n \hat{f}_n{}^2 + \hat{c}_n{}^2 \hat{d}_n \hat{e}_n \hat{g}_n - \hat{a}_n \hat{b}_n \hat{h}_n \hat{f}_n{}^2 \\
& - \hat{a}_n \hat{c}_n \hat{h}_n \hat{f}_n{}^2 - \hat{a}_n \hat{d}_n \hat{c}_n \hat{f}_n - \hat{a}_n \hat{d}_n \hat{e}_n \hat{c}_n - \hat{a}_n \hat{b}_n \hat{d}_n \hat{f}_n \\
\hat{B}_n =& 2\hat{b}_n \hat{c}_n \hat{d}_n \hat{e}_n - 2\hat{a}_n \hat{b}_n \hat{d}_n \hat{e}_n - 2\hat{a}_n \hat{b}_n \hat{e}_n \hat{f}_n - 2\hat{a}_n \hat{c}_n \hat{e}_n \hat{f}_n \\
& + 2\hat{b}_n \hat{c}_n \hat{d}_n \hat{e}_n \hat{g}_n - 2\hat{a}_n \hat{b}_n \hat{e}_n \hat{h}_n \hat{f}_n - 2\hat{a}_n \hat{c}_n \hat{e}_n \hat{h}_n \hat{f}_n \\
\hat{C}_n =& \hat{b}_n{}^2 \hat{d}_n \hat{e}_n - \hat{a}_n \hat{c}_n \hat{e}_n{}^2 - \hat{a}_n \hat{b}_n \hat{e}_n{}^2 - \hat{a}_n \hat{b}_n \hat{e}_n{}^2 \hat{h}_n \\
& - \hat{a}_n \hat{c}_n \hat{e}_n{}^2 \hat{h}_n + b_n^2 \hat{d}_n \hat{e}_n \hat{g}_n + \hat{a}_n \hat{b}_n \hat{d}_n \hat{e}_n
\end{aligned}$$

The roots α_{n1} and α_{n2} can be expressed as:

$$\alpha_{n1} = \frac{-\hat{B}_n + \sqrt{\hat{B}_n{}^2 - 4\hat{A}_n \hat{C}_n}}{2\hat{A}_n}$$
$$\alpha_{n2} = \frac{-\hat{B}_n - \sqrt{\hat{B}_n{}^2 - 4\hat{A}_n \hat{C}_n}}{2\hat{A}_n}$$

Appendix D

Derivation of the Optimal Solution P_{rn}^{O} in (3.28)

Let $a_n = \frac{\alpha_n}{1-\alpha_n}\frac{\gamma_{2m}+1}{\gamma_{1k}+1}$, $b_n = \frac{(1-\alpha_n)|h_{1n}|^2}{\sigma^2}$, $c_n = \gamma_{2m}, d_n = \gamma_{1k}$, $e_n = \frac{\alpha_n|h_{2n}|^2}{\sigma^2}$, $g_n = \eta_{1m}$, $h_n = \eta_{2k}$, $R_n(P_{rn})$ can be written as

$$\begin{aligned} R_n(P_{rn}) = \log(1 + &\frac{c_n b_n P_{rn}}{1 + c_n + b_n(1+a_n)P_{rn}} \\ &+ \frac{d_n e_n P_{rn}}{1 + d_n + e_n(1+\frac{1}{a_n})P_{rn}} \\ &+ \frac{c_n b_n d_n e_n P_{rn}^2}{(1 + c_n + b_n(1+a_n)P_{rn})(1 + d_n + e_n(1+\frac{1}{a_n})P_{rn})}) \end{aligned}$$

By using KKT conditions, we have equation (3.27) and (3.28).

Let $A_n = c_n b_n$, $B_n = 1 + c_n$, $C_n = b_n(1 + a_n)$, $D_n = d_n e_n$, $E_n = 1 + d_n$, $F_n = e_n(1 + \frac{1}{a_n})$. When $P_{rn}^o > 0$ equation (3.28) *i.e.*,

$$R_n'(P_{rn}^o) + \nu = 0$$

can be expressed as

$$\tilde{A}_n P_{rn}^4 + \tilde{B}_n P_{rn}^3 + \tilde{C}_n P_{rn}^2 + \tilde{D}_n P_{rn} + \tilde{E}_n = 0 \tag{D.1}$$

where

$$\begin{aligned}
\tilde{A}_n &= (C_n^2 F_n^2 + A_n C_n F_n^2 + C_n^2 D_n F_n + A_n C_n D_n F_n)\nu \\
\tilde{B}_n &= (2B_n C_n F_n^2 + A_n B_n F_n^2 + 2B_n C_n D_n F_n)\nu \\
&\quad + (A_n B_n D_n F_n + 2C_n^2 E_n F_n)\nu \\
&\quad + (2A_n C_n E_n F_n + C_n^2 D_n E_n + A_n C_n D_n E_n)\nu \\
\tilde{C}_n &= (4B_n C_n E_n F_n + 2A_n B_n E_n F_n + 2B_n C_n D_n E_n)\nu \\
&\quad + (A_n B_n D_n E_n + B_n^2 F_n^2 + B_n^2 D_n F_n)\nu \\
&\quad + (C_n^2 E_n^2 + A_n C_n E_n^2)\nu \\
&\quad + A_n B_n F_n^2 + D_n E_n C_n^2 + A_n D_n B_n F_n + A_n D_n C_n E_n \\
\tilde{D}_n &= (2B_n^2 E_n F_n + 2B_n C_n E_n^2 + A_n B_n E_n^2 + B_n^2 D_n E_n)\nu \\
&\quad + 2A_n B_n E_n F_n + 2D_n E_n B_n C_n + 2A_n D_n B_n E_n \\
\tilde{E}_n &= B_n^2 E_n^2 \nu + A_n B_n E_n^2 + D_n E_n B_n^2
\end{aligned}$$

Lagrange operator ν should ensure relay power constraint *i.e.*,

$$\sum_{n=1}^{N} P_{rn}^o = P_{\text{tot}}.$$

We can solve this problem by using bisection method: for a given ν, we can obtain a set of P_{rn}^o by solving quartic function (D.1), then, we adjust the value of ν based on the comparison of $\sum_{n=1}^{N} P_{rn}^{o}$ and relay power constraint P_{tot}. Finally, we can find the proper ν^{o} which satisfy relay power constraint and substitute ν^{o} in quartic function (D.1) to obtain the optimal relay power P_{rn}^{o}.

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