SUM-RATE MAXIMIZATION FOR ACTIVE CHANNELS

By

Javad Mirzaee

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The undersigned hereby certify that they have read and recommend to the Faculty of Engineering and Applied Science for acceptance a thesis entitled "**Sum-rate Maximization for Active Channels**" by **Javad Mirzaee** in partial fulfillment of the requirements for the degree of **Master of Applied Science**.

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Supervisor: ________________________
Shahram ShahbazPanahi

Readers: ________________________
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Author: **Javad Mirzaee**

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Signature of Author

To my lovely parents.

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Abstract

In conventional wireless channel models, there is no control on the gains of different subchannels. In such channels, the transmitted signal undergoes attenuation and phase shift and is subject to multi-path propagation effects. We herein refer to such channels as passive channels. In this dissertation, we study the problem of joint power allocation and channel design for a parallel channel which conveys information from a source to a destination through multiple orthogonal subchannels. In such a link, the power over each subchannel can be adjusted not only at the source but also at each subchannel. We refer to this link as an active parallel channel. For such a channel, we study the problem of sum-rate maximization under the assumption that the source power as well as the energy of the active channel are constrained. This problem is investigated for equal and unequal noise power at different subchannels.

For equal noise power over different subchannels, although the sum-rate maximization problem is not convex, we propose a closed-form solution to this maximization problem. An interesting aspect of this solution is that it requires only a subset of the subchannels to be active and the remaining subchannels should be switched off. This is in contrast with passive parallel channels with equal subchannel signal-to-noise-ratios (SNRs), where water-filling solution to the sum-rate maximization under a total source power constraint leads to an equal power allocation among *all* subchannels. Furthermore, we prove that the number of active channels depends on the product of the source and channel powers. We also prove that if the total power available to the source and to the channel is limited, then in order to maximize the sum-rate via optimal power allocation to the source and to the active channel, half

of the total available power should be allocated to the source and the remaining half should be allocated to the active channel.

We extend our analysis to the case where the noise powers are unequal over different subchannels. we show that the sum-rate maximization problem is not convex. Nevertheless, with the aid of Karush-Kuhn-Tucker (KKT) conditions, we propose a computationally efficient algorithm for optimal source and channel power allocation. To this end, first, we obtain the feasible number of active subchannels. Then, we show that the optimal solution can be obtained by comparing a *finite* number of points in the feasible set and by choosing the best point which yields the best sum-rate performance. The worst-case computational complexity of this solution is linear in terms of number of subchannels.

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Oshawa, Ontario Javad Mirzaee
April 10, 2013.

Chapter 1

Introduction

1.1 Overview

In the last decade, the demand for fast and reliable wireless communications has increased drastically. Pioneered by the advances in technologies, such as very large scale integrated (VLSI) circuits and signal processing techniques, new wireless services have been emerged. These services are mostly dedicated to meet the requirements of high-quality video/audio streaming or even fast internet services. To this end, the systems should be able to support high data rates as well as reliable transmission in a resource-limited environment. These limitations can be either related to the system itself or imposed by the environment. For example, power and spectrum limitations are related to the system itself, while the fading effects are the limitations that are imposed by the surrounding environment.

In order to support reliable communication against the imposed limitations, it is essential to efficiently use the available resources such as power, time, frequency spectrum, and space. As an example, to combat the fading effects, it is well-known that diversity techniques can be utilized. Diversity can be implemented either in 1) time domain, through the repetition of the signal in different time slots, 2) frequency domain, through the repetition of the signal in different frequency bands, 3) space, transmission or reception of a signal through multi-antenna systems at the both transmitter and receiver side or 4) code, by repeating the signal using different codes. In all the aforementioned cases, different copies of a signal experience different fading states.

In conventional wireless channels, there is no control on the gain of different subchannels. In such channels, the transmitted signal undergoes attenuation and phase shift and is subject to multi-path propagation effects. We herein refer to such channels as passive channels. Shifting the focus from passive channels, in this dissertation, we study an energy-limited parallel channel, where the energy of each subchannel can be adjusted at a certain level. Such adjustable channels, herein referred to as active channels, differ from conventional passive links in the sense that not only the transmit power over different subchannels can be adjusted, but also the energy of each subchannel can be optimally controlled to optimize a certain performance criterion. We aim to maximize the sum-rate of such channels under the both transmit power and the channel energy constraint. The optimal solution to sum-rate maximization problem is obtained for equal and unequal noise powers over different subchannels. In the next subsection, we review the concept of a parallel channel and its properties.

1.2 Wireless Parallel Channel

Parallel channel refers to a link where the source and destination are able to communicate through different subchannels corrupted by independent noise. Most of the communication channels such as inter-symbol interference (ISI) channels, fading channels and multiple input multiple output (MIMO) channels can be categorized as parallel channels [1–7]. Below, we provide different applications of parallel channels:

Multi-tone transmission:

Multi-tone transmission deals with signaling over a number of different frequency bands, where each frequency band corresponds to one parallel subchannel [1]. Frequency division multiplexing (FDM) and orthogonal frequency division multiplexing (OFDM) are the examples of such parallel channels. The frequency bands may be non-overlapping or, as in OFDM, could be designed to be orthogonal. Practically, in digital subscriber line (DSL) and inter-symbol interference (ISI) channels, this technique is the prime solution.

Time varying fading channels:

Consider a frequency-flat fading channel where the gain of the channel varies over time. The channel can be interpreted as a parallel independent subchannels where each subchannel refers to one fading state [2, 6].

Multi-antenna communication:

Multi-antenna communication has been evolved to improve the performance of the communication systems. In such systems, the use of different antennas creates independent data tranmission paths which improve the performance of the systems in terms of bit error rate (BER) and capacity. For the case where the transmitter and receiver are equipped with multiple antennas, Telatar in [8] showed that singular value decomposition (SVD) of the channel matrix yields a set of parallel subchannels between the source and destination, and the gain of each of the parallel subchannels corresponds to the singular values of the MIMO channel. It is shown that the number of the independent parallel subchannels is equal to the rank of the channel response matrix [3, 4].

Dispersive channels:

In a time-dispersive channel or in a parallel channel with correlated noise , orthonormal transformation of the channel at the receiver or transmitter turns the channel into a set of parallel subchannels with uncorrelated noise [5].

In the above classification, different interpretations of parallel channels are presented. In each category, we are dealing with a set of parallel subchannels, where each subchannel is a time slot, a frequency band, a fading state or a singular value of the channel. It has been shown that water-filling power allocation scheme is the optimal solution to the sum-rate maximization of the parallel channels under a total power constraint [6, 7]. In the next section we review the water-filling power allocation scheme

1.3 Water-filling Power Allocation

Typically, under total transmit power constraint, water-filling power allocation scheme is the well-known solution to the sum-rate maximization problem of a point-to-point communication link which conveys the information between a source (transmitter) and a destination (receiver) through a set of parallel communication subchannels [9, 10]. In this subsection, we review the water-filling power allocation scheme. Let us consider a set of N parallel subchannels corrupted by independent noise. These parallel subchannels can be considered as transformations from a frequency selective, time varying, dispersive channel or the use of multiple antenna at both transmitter and receiver (see section 1.2). It is shown that, the sum-rate of this link is given by [11]:

$$\sum_{i=1}^{N} \log(1 + p_i\beta_i), \tag{1.3.1}$$

where p_i and β_i are, respectively, the assigned power and the signal-to-noise ratio (SNR) of the ith subchannel. Using the availability of channel state information (CSI) (the availability of β_i's), the problem is to find the optimal power allocation scheme which yields the maximum sum-rate when the power constraint $\sum_{i=1}^{N} p_i \leq P_{\mathrm{T}}$ is satisfied. Here, P_{T} is used to denote the total available power at the transmitter. Therefore, the optimization problem can be written as:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{N} \log(1 + p_i\beta_i) \\ \text{subject to} \quad & \sum_{i=1}^{N} p_i \leq P_T \\ & p_i \geq 0 \quad \text{and} \quad 1 < i < N, \end{aligned} \tag{1.3.2}$$

where $\mathbf{p} \triangleq [p_1, p_2, \ldots, p_N]^T$. Note that, at the optimum solution, the first constraint in (1.3.2) is satisfied with equality, otherwise, we can scale up the values of p_i which further increase the objective function. The solution to this optimization problem can be found with the aid of Lagrangian method. The Lagrange function is given by

$$\mathcal{L}(\lambda, \boldsymbol{\mu}, \mathbf{p}) = -\sum_{i=1}^{N} \log(1 + p_i\beta_i) + \lambda(\sum_{i=1}^{N} p_i - P_{\mathrm{T}}) - \boldsymbol{\mu}^T\mathbf{p}, \tag{1.3.3}$$

where the scalar λ as well as the $N \times 1$ vector $\boldsymbol{\mu}$ represent the Lagrange multipliers. Let us define p_i^*, λ^* and μ^* as the optimal values to p_i, λ and μ, respectively. Using the Karush-Kuhn-Tucker (KKT) conditions, we obtain that the optimal solution to (1.3.2) is required to satisfy the following conditions:

- Primal feasibility:

$$\mathbf{1}^T\mathbf{p}^* = P_{\mathrm{T}} \tag{1.3.4}$$

$$-\mathbf{p}^* \preceq \mathbf{0}. \tag{1.3.5}$$

- Dual feasibility:

$$\boldsymbol{\mu}^* \succeq 0. \tag{1.3.6}$$

- Complementary slackness:

$$\lambda^*(\mathbf{1}^T\mathbf{p}^* - P_{\mathrm{T}}) = 0 \tag{1.3.7}$$

$$\boldsymbol{\mu}^* \odot \mathbf{p}^* = 0. \tag{1.3.8}$$

- Stationary condition:

$$\frac{\partial \mathcal{L}(\lambda, \boldsymbol{\mu}, \mathbf{p})}{\partial p_i} = -\frac{h_i}{(1 + p_i^* h_i)} + \lambda^* - \mu_i^* = 0. \tag{1.3.9}$$

Now, assuming $p_i^* > 0$, then from (1.3.8), $\mu_i^* = 0$, and therefore, from (1.3.9), we have:

$$\frac{h_i}{(1 + p_i^* h_i)} = \lambda^*. \tag{1.3.10}$$

or equivalently

$$p_i^* = \frac{1}{\lambda^*} - \beta_i^{-1} \quad \text{for} \quad \frac{1}{\lambda^*} > \beta_i^{-1}. \tag{1.3.11}$$

If we assume that $p_i^* = 0$, from (1.3.8), we obtain that $\mu_i^* \neq 0$, and therefore, from (1.3.9), we conclude that $\beta_i + \mu_i^* = \lambda^*$, which means that $\frac{1}{\lambda^*} < \beta_i^{-1}$. Therefore,

$$p_i^* = \begin{cases} \frac{1}{\lambda^*} - \beta_i^{-1} & \text{if} \quad \frac{1}{\lambda^*} > \beta_i^{-1} \\ \\ 0 & \text{if} \quad \frac{1}{\lambda^*} < \beta_i^{-1}. \end{cases} \tag{1.3.12}$$

Defining $(x)^+ \triangleq \max\{0, x\}$, the optimal solution to (1.3.2) is given by

$$p_i^* = (\frac{1}{\lambda^*} - \beta_i^{-1})^+, \quad \text{for} \quad 1 < i < N. \tag{1.3.13}$$

The parameter λ is chosen to satisfy the equality of constraint in (1.3.2), This parameter is called water level. Visually, as shown in Figure. 1.1 , this power allocation technique is interpreted as pouring water over a surface given by the inverse gain of subchannels, hence, it is called water-pouring or water-filling [9, 12–14].

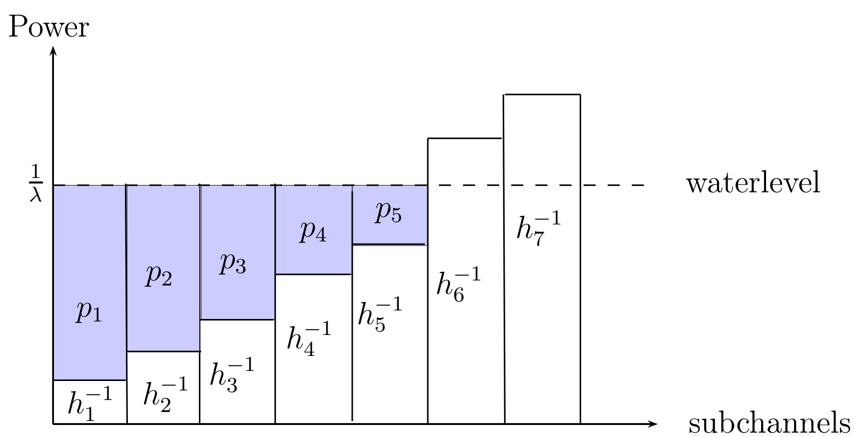

Figure 1.1: water-filling power allocation, $p_i^* = (\frac{1}{\lambda^*} - \beta_i^{-1})^+$, where $\sum_{i=1}^{N} p_i^* \leq P_T$.

As shown in Figure 1.1, the shaded regions refers to the allocated power to the corresponding subchannel. This power is a function of the inverse of the channel gain. Indeed, the more transmit power is allocated to the subchannels with better qualities.

1.4 Motivation

As mentioned earlier (see subsection 1.3), in water-filling scheme, channels with better qualities receive relatively more power. In a passive parallel channel, there is no control on the gains of different subchannels. However, one can easily observe that the maximum achievable sum-rate of a passive channel depends not only on the source transmit power but also on the quality of individual subchannels (i.e., on the power of the parallel channel). This obvious observation has motivated us to study the problem of sum-rate maximization for a parallel channel where not only can the source transmit power be adjusted but also the channel itself can be properly *designed* or optimally *adjusted* to achieve a higher sum-are compared to traditional passive channels. In order to design a channel, one can think of injecting power into different subchannels somewhere between the course and the destination, as in the relay networks. Alternatively, one may have some control over some parameters which determine the channel characteristics. Examples of such channels include single- and multi-user MIMO systems, where the antenna spacing can be adjusted to control the underlying MIMO channel(s) [15, 16]. Such adjustable parallel channels, herein referred to as active channels, differ from conventional passive links in the sense that their characteristics (such as the power of each individual subchannels) can be adjusted using a certain optimality criterion (such as sum-rate) under a constraint on the total energy of the channel.

1.5 Objective and Methodology

We study the problem of joint transmit power allocation and channel design for an active link which conveys information from a source to a destination through multiple orthogonal subchannels. In such a link, the power can be injected into the channel not only at the source but also at each subchannel. Assuming that the source power as well as the power injected by the active channel are constrained, we aim to jointly optimize the power of each subchannel and the transmit power allocated to each subchannel by the source such that the sum-rate is maximized for the both equal and unequal noise power over different subchannels. Compared to power allocation for a passive parallel channel, the optimization problem we consider has one additional constraint which limits the power of the active channel.

In the case of equal noise power for different subchannels, we show that the sum-rate maximization problem, under the both channel energy and transmit power, is not convex. Nevertheless, we show that KKT conditions can be used to develop a semi-closed form solution to this problem. Our results show that the maximum sum-rate is achieved by activating a certain number of subchannels, while the rest of the subchannels should be switched off. We show that the number of activated subchannels is unique and only depends on the product of transmit power and the channel energy. This number of activated subchannels is optimally found using steepest descent algorithm. Moreover, at the optimum, the total channel power and transmit power should be equally distributed among the activated subchannels.

For the case with unequal noise power over differen subchannels, we first formulate the sum-rate maximization problem of the parallel active channel, under the both channel energy and transmit power constraint. To solve the sum-rate maximization problem under the two aforementioned source and channel power constraints, we use KKT conditions to obtain a computationally efficient algorithm for source and channel power allocation. We first show that how KKT conditions can be used to determine how many subchannels can be active for the source power constraint to be feasible. Indeed, we develop a computationally efficient method to determine the feasible numbers of active channels. Then, for any feasible number of active channels, we obtain the optimal source power allocation. In fact, we show that for any feasible number of active channels, there are only zero, one, or two solutions for the optimal source power allocation. As such the optimal solution can be obtained by comparing a finite number of feasible points and choosing the best point which yields the best sum-rate performance.

1.6 Outline of Dissertation

In this dissertation we focus on sum-rate maximization problem in an energy-limited active channels. Here, we aim to jointly optimize the power of each subchannel as well as the transmit power allocated to different subchannels such that the sum-rate is maximized. The remainder of this thesis is organized as follows:

In Chapter 2, we first review the recent research results on resource allocation schemes in different applications of parallel channels. Then, we proceed to the recent solutions to sum-rate maximization problem in active channels.

In Chapter 3, we study the sum-rate maximization problem for active channels. and, we assume an equal noise power over different subchannels. We show that the sum-rate maximization problem is not convex. In such a case, we apply the KKT conditions to derive the necessary optimality conditions. Then, we propose an efficient closed-form solution to sum-rate maximization problem. We then compare the performance of the active channel versus the passive channel in terms of the maximum sum-rate.

In Chapter 4, we extend the analysis in Chapter 3 to the case where unequal noise powers are considered over different subchannels. Similar to the case discussed in Chapter 3, we show that the sum-rate maximization problem is not convex. Nevertheless, with the aid of KKT conditions, we propose a computationally efficient algorithm for optimal source and channel power allocation. To this end, we first obtain the feasible number of active subchannels. We then show that the optimal solution can be obtained by comparing a finite number of points in the feasible set and by choosing the best point which yields the best sum-rate performance.

In Chapter 5, we present the concluding remarks as well as the potential future work in this area of research.

Chapter 2

Literature Review

As we introduced in Chapter 1, parallel channel is a model which fits to the various type of wireless communication technologies that are the basis for the future communication systems. Parallel channels have been widely used in modern communication systems. The application of such parallel channels can be found in MIMO systems, frequency hopping spread spectrum (FH-SS) scheme, time division multiplexing (TDM) systems, and OFDM-based communication schemes. In this chapter, we review the techniques which have been used to maximize the capacity of the parallel channels.

2.1 Power Allocation in Parallel Channels

Recently, the optimal resource allocation over parallel subchannels have been studied in literature. In this section, we review the current trends on the power allocations in parallel channel.

OFDM Systems

OFDM-based communications can also be considered as a set of parallel subchannels each of which corresponds to one frequency band. In the context of OFDM systems, under the availability of channel state information at the transmitter side (CSIT), water-filling power allocation scheme has been considered to design an efficient communication link [4, 17, 18]. This power allocation policy is mostly used to maximize the throughput of the system and the spectral efficiency [19–22] as well as to minimize the bit error rate (BER) [23, 24].

Under the situation where partial CSI is available, a modified water-filling is proposed for the solution of sum-rate maximization problem in a MIMO channel [25]. The authors in [26], aim to minimize the bit error rate in a parallel channel with partially available CSIT, while, under the same circumstances, the optimal power allocation has been developed in [27], to maximize the spectral efficiency. In [28], a correlated MIMO channel with partial CSIT is considered. The authors obtain an upper bound for the sum-rate through power allocation. Then, it is shown that the *statistical water-filling* (i.e., the water-filling power allocation over the mean of the channel gains instead of the instantaneous channel gains), leads to the maximum of the upper bound. The authors in [29], generalize the analysis in [28], by assuming Nakagami-m fading over each subchannel. Furthermore, unlike [28], they maximize the exact sum-rate of the parallel channel rather than its upper bound. They show that the statistical water-filling power allocation scheme proposed in [28], results in the maximum of sum-rate.

Multiuser orthogonal frequency division multiple accces (OFDMA) systems can also be considered as parallel channel, where each user corresponds to one subchannel. For such systems, the power and subcarrier allocation are well investigated in [30–34]. In downlink scenarios, it has been shown that the sum-rate is maximized when each subcarrier is allocated to only the user which reveals the best channel gain for that subcarrier, while the total power should be distributed among the subcarriers using a water-filling scheme [31, 32].

Time Dispersive Channels

A time dispersive, or equivalently, frequency selective channel can be considered as a multi-tap channel, where each tap corresponds to only one parallel subchannel. The resource allocation over this type of the parallel channels is investigated in [17, 18, 35–38].

For frequency selective single-input single-output (SISO) channels, the authors in [35], aim to find the optimal power allocation which yields the maximum sum-rate. They show that the water-filling scheme is the optimal solution to sum-rate maximization problem. The authors in [17, 18, 36–38], study a frequency selective channel between two transceivers with a colored noise. To maximize the sum-rate of such channel under a fixed total consumed power, the authors in [17], design a bank of finite impulse response (FIR) filters at the transmitter and receiver to decompose the frequency-selective channel into a set of parallel frequency-flat subchannels with uncorrelated noise over the subchannels. Then, they propose a power allocation algorithm to obtain the maximum sum-rate. They show that their algorithm yields the same result as water-filling power allocation scheme when the number of subchannels approaches infinity.

MIMO Channels

Recently, the use of MIMO channels has attracted a significant attention from the research community. Compared to SISO channels, the use of multi-antenna at both transmitter and receiver sides increases the diversity of the system and offers a considerable improvement in the capacity of the link through the creation of a set of independent parallel subchannels. [4, 8, 39].

One of the challenges in MIMO communication systems is to optimally allocate the available resources, such as power, to achieve the capacity of the channel [4, 9, 40–65] or to minimize the bit error rate and the mean squared error [17, 66–72]. In this regard, the capacity of the MIMO channels is well-studied in literature. In most cases, the transmitter and receiver are designed to transform the MIMO channel into a set of parallel subchannels. At the optimum, the capacity is the sum of water-filled singular (eigen-) subchannels that arise from the orthogonalization of the MIMO channel matrix [8, 39].

In the case of frequency selective MIMO channels, a multi-tone transmission is a well-known capacity achieving technique where each subcarrier experiences a flat fading MIMO channel [4, 39, 73]. It is shown that the capacity of a frequency selective MIMO channel is achievable when the transmitter and receiver is designed such that the channel matrix at each subcarrier is diagonalized. Then, the water-filling solution should be used to allocate the available power to each subchannel [4, 9, 40]. This type of power allocation requires that the channel state information (CSI) be available at both transmitter and receiver sides.

To minimize mean square errors, (MSEs), the authors in [39] consider a MIMO channel and aim to jointly design the precoder and decoder of this transmission system using a weighted minimum mean-squared error (MMSE) criterion subject to a total transmit power constraint. In [71, 74], the authors generalize the joint optimization of the pre-coder and decoder of MIMO channels 1) to achieve the maximum sum-rate, 2) to minimize the un-weighted MMSE, and 3) to satisfy a certain QoS over each subchannel, under a total power constraint. According to their criteria, at the optimum, the MIMO channel is decomposed into a set of parallel subchannels, where each subchannel corresponds to only one eigen mode. Then, to achieve their goals, the water-filling power allocation policy is used (see also, [17,68–70,72,75,76] and the references therein). Under the same structure, the authors in [66, 77], efficiently design the precoder and decoder to maximize the signal-to-interference-plus-noise ratio (SINR) and minimize the bit error rate (BER) of the MIMO channel, respectivelly. Under total power constraint, they show that the water-filling scheme can optimally achieve the capacity.

The aforementioned results, mostly, focus on a simple waterfiling technique which requires a single water level and a total transmit power constraint. Therefore, the optimal power allocation can be obtained by calculating the waterlevel which satisfies the power constraint with equality. To find the waterlevel, different approaches have been proposed which can be classified as *iterative* and *exact* algorithms. In the iterative algorithms, the value of water-filling can be obtained through an iterative procedure [70, 78–80], while the exact algorithms leads to the exact value for waterlevel within a finite number of iterations [17, 71, 81]. The result of iterative algorithms converges to the solution of the exact algorithms when the number of iterations tends to infinity. Unlike the above single-level water-filling solution, in some applications, a multi-level water-filling solution has been deducted. For example, in [82], the authors study the power minimization problem in a point-to-point MIMO communication scheme with a set of quality-of-service (QoS) constraints. They show that the solution is a multi-level water-filling scheme, where each water level satisfies one QoS constraint. Moreover, in [83], the joint transmitter-receiver beamformer design has been considered to minimize the maximum BERs of the MIMO channel under multiple quality of service constraints. The solution to this problem as well as the maximization of the harmonic mean of SINRs of subchannels lead to a multilevel water-filling solution. In terms of the implementation of the multiple water level water-filling solution, the authors in [84], propose a practical solution to general multiple water level water-filling problems.

2.2 Sum-rate Maximization for Active Channels

In the context of energy constrained active channels, the sum-rate maximization problem has been studied extensively in [15, 16, 85, 86]. Here, we provide a summary of different application of the active channels:

2.2.1 MIMO Active Channel

In MIMO communications, the motivations behind jointly optimal source power allocation and channel design subject to two constraints, one on the source transmit power and one the channel energy constraints are that, first, for the class of energy-constrained channels, an upper bound on the MIMO channel capacity can be found; and second, the characteristics of the channels with the best sum-rate can be obtained. The characteristics of the capacity achieving channel could then be used to guide the design of adaptive antenna arrays [15, 16, 85].

Single User MIMO Active Channel:

Among the literature, the investigation in [85] and [15] focus on single user MIMO active channels with equal noise power over different subchannels. In [85], the authors study the capacity of a point-to-point multi-antenna Gaussian channel with the freedom of perturbing the antennas location at the transmitter or at the receiver side. Indeed, the authors aim to maximize the sum-rate of the MIMO channel under both transmit power and MIMO channel energy constraints. It is shown that, for sufficiently large SNRs, the maximum sum-rate is achievable by creating a set of parallel subchannels where the power of all subchannels are equal. In [85], using eigen value decomposition, the MIMO channel is transformed into a set of parallel subchannel each of which corresponds to one eigen mode. The strength of each subchannel is defined by the square of the corresponding eigen values of the channel matrix. Furthermore, the antenna relocating possibility can modify a new eigen values of the channel matrix. Therefore, the problem of the sum-rate maximization corresponds to optimal positioning of the transmitter/reciver antennas to achieve the equal eigen values, thereby, satisfying the channel energy constraint.

In [15], the authors investigate the capacity of a point-to-point MIMO channel under the transmit power constraint as well as the channel norm constraint. They show that the maximum sum-rate is obtained when the channel has equal singular values for all of its non-zero eigen modes. Then, the total transmit power is equally distributed among a certain number of eigen modes. To obtain the optimal number of eigen modes, a global search should be conducted.

Multi User MIMO Active Channel :

The study in [16] considers a multi-user MIMO system with channel energy constraint and assume that the noise powers are equal over different subchannels. The authors assume a k-user network where the transmitter and the receiver are equipped with

N_t and N_r antennas, respectively. For such network, the authors in [15] derive an upper bound for the capacity when $N_t \geq kN_r$. The authors in [16] are looking for the best maximum sum-rate over all possible channel states which satisfy the channel norm constraint for $N_t \geq kN_r$. It is shown that, for large values of SNR, the bound is achieved when the user channels are mutually orthogonal to each other. This result is analogous to the result in point-to-point MIMO channel [15]. Furthermore, for each user, the channel energy and the transmit power are equally distributed among the non-zero MIMO eigen modes. A further optimization required to find the optimal number of eigne modes.

2.2.2 Relay-assisted Communication

Another application of the active channel is in asynchronous one- or two-way AF-based multi-relay channels, where the end-to-end channel impulse response can be adjusted by properly adjusting amplification weight of the relays [87,88] and/or their locations. The problem of sum-rate maximization for one- and two-way relay networks have been studied in numerous studies [89–93]. In all of these published results, the constraints that are often used are either individual or total relay power constraints or a total power constraints. The channel norm constraint that we herein study is different from the widely used total or individual relay power constraints. However, the norm of such channels can be written in terms of the individual relay powers, or inversely, given channel gain $\tilde{h}_i$'s, one can obtain the relay powers. Hence, one can use the optimal subchannel powers to design the relay channel, for example, by choosing the location of the relays with respect to the transmitter and the receiver. The authors of [94] study the problem of sum-rate maximization for a multi-antenna multi-carrier relay channel. The constraint considered in [94] is a total transmit power constraint which limits the sum of the relay and source powers. The solution provided in [94] relies on a high-SNR approximation but there is no guarantee that this solution results in high values of SNR, meaning that the sub-carrier powers are not guaranteed to result in high values of SNR in each subcarrier. Furthermore, the approximate solution provided in [94] has a water-filling structure.

2.3 Research Contribution

In this thesis, an active channel refers to a parallel channel whose subchannel gains can be adjusted within a bound on the norm of the channel. In this thesis, we study the sum-rate maximization for an active parallel channel subject to two constraints, one on the source total transmit power and one on the channel energy.

For the case where equal noise power is considered over differen subchannels, we prove that in order to achieve the maximum sum-rate, only a certain number of subchannels should be turned on and the rest of the subchannels should be switched off. This is in contrast with passive parallel channels with equal subchannel SNRs, where water-filling solution to the sum-rate maximization under a total source power constraint leads to an equal power allocation among *all* subchannels. The number of active subchannels is proven to depend on the product of the source and channel powers. Also, we show that when sum-rate is maximized, different active subchannels receive the same level of powers. We prove that if the total power available to the source and to the channel is limited, then in order to maximize the sum-rate via optimal power allocation to the source and to the active channel, half of the total available power should be allocated to the source and the remaining half should be allocated to the active channel.

For unequal noise power over different subchannels, we show that the sum-rate maximization problem is not convex. Neverthless, we use the Karush-Kuhn-Tucker (KKT) conditions and obtain a computationally efficient algorithm for optimal source and channel power allocation. We showed that not all subchannels but only a subset of them may receive transmit power from the source. Then, for any feasible number of active subchannels, we obtained the optimal source power allocation. In fact, we prove that for any feasible number of active subchannels, there are only zero, one, or two solutions for the optimal source power allocation. As such, the optimal solution can be obtained by comparing a finite number of points in the feasible set and by choosing the point which yields the best sum-rate performance. The worst-case computational complexity of our solution is linear in the number of subchannels. Our analysis and simulation results showed that active channels can offer significantly higher sum-rate compared to their passive counterpart which rely on water-filling scheme for source power allocation across subchannels.

Chapter 3

Sum-rate Maximization for Active Channel: Equal Noise Power Over Different Subchannels

3.1 System Model and Sum-rate Maximization

Consider an active channel which conveys information from a transmitter (source) to a receiver (destination) through N orthogonal parallel subchannels. The transmitter allocates power $\tilde{p}_i$ to the ith channel. The gain of the ith subchannel is represented by the complex number $\tilde{h}_i$. The received signal over the ith subchannel is modeled as $x_i = \tilde{p}_i \tilde{h}_i s_i + n_i$, where s_i and n_i are the transmitted signal and received noise of the ith subchannel, respectively. This data model fits very well, for example, to multi-carrier relay system, where the relay noise is negligible [95]. In this chapter, we restrict our analysis to the case where the noise powers are the same at different parallel subchannels. The case with unequal subchannel noise powers will be studied in the next chapter. We assume that the total source transmit power is limited to $P_{\rm s}$, that is $\sum_{i=1}^{N} \tilde{p}_i \leq P_{\rm s}$. Also, the total power of the channel is assumed to be limited to P_c. The problem of maximizing the sum-rate subject to two constraints, one on the source transmit power and one on the total power of the parallel channel, can be written as

$$\begin{aligned} \max_{\tilde{\mathbf{p}}\succeq\mathbf{0},\tilde{\mathbf{h}}\succeq\mathbf{0}} \quad & \sum_{i=1}^{N} \log_2(1+\tilde{p}_i|\tilde{h}_i|^2) \\ \text{subject to} \quad & \mathbf{1}^T\tilde{\mathbf{p}} \leq P_\text{s}, \\ & \mathbf{1}^T\tilde{\mathbf{h}} \leq P_\text{c}\,, \end{aligned} \tag{3.1.1}$$

where $\tilde{\mathbf{p}} \triangleq [\tilde{p}_1 \;\; \tilde{p}_2 \;\; \cdots \;\; \tilde{p}_N]^T$ and $\tilde{\mathbf{h}} \triangleq [|\tilde{h}_1|^2 \;\; |\tilde{h}_2|^2 \;\; \cdots \;\; |\tilde{h}_N|^2]^T$. One application of the optimization problem (3.1.1) is in asynchronous one- or two-way AF-based multi-relay channels, where the end-to-end channel impulse response can be adjusted by properly choosing the amplification weights of the relays [87–89] and/or their locations. Note that when applied to asynchronous relay channels, the channel norm constraint used in (3.1.1) is different from widely used total or individual relay power constraints. However, the norm of such channels can be written in terms of the individual relay powers, or inversely, given channel gains, $\tilde{h}_i$'s, one can obtain the relay powers. Hence, one can use the optimal $\tilde{h}_i$ to design the relay channel, for example, by choosing the location of the relays with respect to the transmitter and the source. Another application of (3.1.1) is that it can be used for optimal power allocation and channel design for single- or multi-user multiple-input multiple-output systems where the location of antennas are to be chosen carefully such that the sum-rate is maximized. One more application of the optimization problem (3.1.1) is joint power allocation for multi-career multi-antenna systems [94].

Note that in (3.1.1), for any fixed $\tilde{\mathbf{h}}$, the maximization over $\tilde{\mathbf{p}}$ is the traditional sum-rate maximization problem under total power constraint. This maximization over $\tilde{\mathbf{p}}$

is convex and leads to the well-known water-filling power allocation scheme at the transmitter for fixed channel. Similarly, for any fixed $\tilde{\mathbf{p}}$, the maximization over $\tilde{\mathbf{h}}$ is also convex and leads to the well-known water-filling power allocation scheme across subchannels. However, as shown in the appendix B, the objective function of the optimization problem (3.1.1) is not concave. Nevertheless, in what follows, we show how this problem can be solved efficiently. It can be readily seen that at the optimal solution, the two constraints in (3.1.1) will be satisfied with equality. Otherwise, if, for example, the entries of the optimal $\tilde{\mathbf{p}}$ are such that $\mathbf{1}^T\tilde{\mathbf{p}} < P_\mathrm{s}$, one can scale up all entries of such optimal $\tilde{\mathbf{p}}$ such that $\mathbf{1}^T\tilde{\mathbf{p}} = P_\mathrm{s}$ holds true and this new $\tilde{\mathbf{p}}$ further increases the objective function, thereby contradicting the optimality.

Note also that if the ith entry of $\tilde{\mathbf{p}}$ is zero, the corresponding entry in $\tilde{\mathbf{h}}$ will be zero and vice versa. Let n represent the number of non-zero entries of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{h}}$. Then, without loss of optimality, we can express the optimization problem (3.1.1) as

$$\begin{aligned}
\max_{n}\ \max_{\mathbf{p},\mathbf{h}} \quad & \sum_{i=1}^{n} \log_2(1 + p_i h_i) \\
\text{subject to} \quad & \mathbf{1}^T\mathbf{p} = P_\mathrm{s}, \\
& \mathbf{1}^T\mathbf{h} = P_\mathrm{c} \\
& \mathbf{p} \succ \mathbf{0} \\
& \mathbf{h} \succ \mathbf{0}, \quad n \in \{1, 2, \ldots, N\}\,, \qquad (3.1.2)
\end{aligned}$$

where $\mathbf{p}$ and $\mathbf{h}$ are $n \times 1$ vectors which capture the non-zero entries of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{h}}$, respectively, and h_i is the ith entry of $\mathbf{h}$. For any fixed n, the Lagrangian function corresponding to the inner maximization in (3.1.2) can be expressed as

$$\begin{aligned}
\mathcal{L}(\mathbf{p}, \mathbf{h}) = \; & - \sum_{i=1}^{n} \log_2(1 + p_i h_i) + \lambda_1(\mathbf{1}^T\mathbf{p} - P_\mathrm{s}) \\
& + \lambda_2(\mathbf{1}^T\mathbf{h} - P_\mathrm{c}) - \boldsymbol{\mu}_1^T\mathbf{p} - \boldsymbol{\mu}_2^T\mathbf{h}\,, \qquad (3.1.3)
\end{aligned}$$

where p_i and h_i are the ith entry of $\mathbf{p}$ and $\mathbf{h}$, respectively, and the scalars λ_1 and λ_2 as well as the $n \times 1$ vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ represent the Lagrange multipliers. Applying the KKT conditions, the optimal solution is required to satisfy the following conditions[1]:

- Primal feasibility:

$$\mathbf{1}^T\mathbf{p} = P_{\mathrm{s}} \tag{3.1.4}$$

$$\mathbf{1}^T\mathbf{h} = P_{\mathrm{c}} \tag{3.1.5}$$

$$-\mathbf{p} \prec \mathbf{0} \tag{3.1.6}$$

$$-\mathbf{h} \prec \mathbf{0}. \tag{3.1.7}$$

- Dual feasibility:

$$\boldsymbol{\mu}_1 \succeq 0, \quad \boldsymbol{\mu}_2 \succeq 0. \tag{3.1.8}$$

- Complementary slackness:

$$\lambda_1(\mathbf{1}^T\mathbf{p} - P_{\mathrm{s}}) = 0 \tag{3.1.9}$$

$$\lambda_2(\mathbf{1}^T\mathbf{h} - P_{\mathrm{c}}) = 0 \tag{3.1.10}$$

$$\boldsymbol{\mu}_1 \odot \mathbf{p} = 0 \tag{3.1.11}$$

$$\boldsymbol{\mu}_2 \odot \mathbf{h} = 0. \tag{3.1.12}$$

- Stationary condition:

$$\frac{\partial \mathcal{L}(\mathbf{p}, \mathbf{h})}{\partial p_i} = \frac{1}{\ln 2}\frac{-h_i}{(1 + p_i h_i)} + \lambda_1 - \mu_{1,i} = 0 \tag{3.1.13}$$

$$\frac{\partial \mathcal{L}(\mathbf{p}, \mathbf{h})}{\partial h_i} = \frac{1}{\ln 2}\frac{-p_i}{(1 + p_i h_i)} + \lambda_2 - \mu_{2,i} = 0. \tag{3.1.14}$$

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[1]Note that the constraints in maximization problem (3.1.2) satisfy linear constraint qualifications, and therefore, KKT conditions are necessary for the optimal solution [96].

In (3.1.13) and (3.1.14), $\mu_{1,i}$ and $\mu_{2,i}$ are the ith entries of $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, respectively. It follows from the primal feasibility condition and complementary slackness that $\mu_{1,i} = \mu_{2,i} = 0$ holds true for $i = 1, 2, ..., n$. Hence, (3.1.13) and (3.1.14) can be rewritten, respectively, as

$$\frac{1}{\ln 2} \frac{-h_i}{(1 + p_i h_i)} + \lambda_1 = 0, \quad \frac{1}{\ln 2} \frac{-p_i}{(1 + p_i h_i)} + \lambda_2 = 0. \tag{3.1.15}$$

Using (3.1.15), we can write[2] $\frac{h_i}{p_i} = \frac{\lambda_1}{\lambda_2}$, for $i = 1, 2, \ldots N$. Moreover, using the second equation in (3.1.15) for $i \neq j$, we obtain that

$$0 = \frac{1}{\ln 2} \frac{-p_i}{(1 + p_i h_i)} + \lambda_2 = \frac{1}{\ln 2} \frac{-p_j}{(1 + p_j h_j)} + \lambda_2 \,, \tag{3.1.16}$$

or

$$\frac{p_i}{(1 + p_i h_i)} = \frac{p_j}{(1 + p_j h_j)} \,. \tag{3.1.17}$$

Defining $\kappa \triangleq \frac{\lambda_1}{\lambda_2} = \frac{h_i}{p_i}$, we can rewrite (3.1.17) as $\dfrac{p_i}{1 + \kappa {p_i}^2} = \dfrac{p_j}{1 + \kappa {p_j}^2}$ or, equivalently, as

$$(p_i - p_j)(1 - \kappa p_i p_j) = 0 \,. \tag{3.1.18}$$

It follows from (3.1.18) that for any subchannel index j, either $p_j = p_1$ or $p_j = 1/\kappa p_1$ must hold true. Let n_2 be the number of those subchannels for which $p_j = 1/\kappa p_1$ holds true. In this case, using the fact that $h_j = \kappa p_j$, the sum-rate can be written as $(n - n_2) \log_2(1 + \kappa p_1^2) + n_2 \log_2(1 + 1/(\kappa p_1^2)) = n \log_2(1 + \kappa p_1^2) - n_2 \log_2(\kappa p_1^2)$. It now becomes obvious that in order to maximize the sum-rate, n_2 has to be 0, and hence, $p_j = p_1$ holds true, for every j. Using a similar approach, we can prove that $h_i = h_j$ holds true. As a result, using (3.1.19) along with the facts that $\mathbf{1}^T \mathbf{p} = P_{\mathrm{s}}$

[2]Note that $p_i \neq 0$ and $h_i \neq 0$.

and $\mathbf{1}^T\mathbf{h} = P_{\rm c}$, we conclude that for any given n, the optimal values of p_i and h_i are given by

$$p_i = \frac{1}{n}P_{\rm s}, \quad h_i = \frac{1}{n}P_{\rm c}. \tag{3.1.19}$$

To obtain the optimal number of active subchannels, n, using (3.1.19), we can write the optimization problem (3.1.2) as

$$\max_{n} \quad n\log_2(1+\frac{P_{\rm s}P_{\rm c}}{n^2}). \tag{3.1.20}$$

The following lemma helps us to find the optimal value of n in an efficient manner.

Lemma 1: *The function* $s(x) \triangleq x\log_2(1+\frac{P_{\rm s}P_{\rm c}}{x^2})$ *has a unique real-valued maximizer for* $x > 0$.

Proof: We know that $s(0) = 0$ and $s(+\infty) = 0$. As such, $s(x)$ has at least one maximum for $x > 0$. To prove that this maximum is unique, we show that for $x > 0$, the function $s(x)$ has a unique inflection point, where the second derivative of the function $s(x)$ vanishes. The second derivative of $s(x)$ with respect to x can be obtained as

$$\frac{\partial^2 s(x)}{\partial x^2} = -\frac{1}{\ln 2}\frac{\frac{2P_{\rm s}P_{\rm c}}{x^2}}{\left(1+\frac{P_{\rm s}P_{\rm c}}{x^2}\right)^2}\left(\frac{P_{\rm s}P_{\rm c}}{x^3}-\frac{1}{x}\right). \tag{3.1.21}$$

Equating (3.1.21) to zero, we obtain the only non-negative solution to this equation as $x = \sqrt{P_{\rm s}P_{\rm c}}$. It is obvious that $\frac{\partial^2 s(x)}{\partial x^2} > 0$, for $x \in (0, +\sqrt{P_{\rm s}P_{\rm c}}]$, and therefore, the function $s(x)$ is a concave function of x for $x \in (0, +\sqrt{P_{\rm s}P_{\rm c}}]$. Also, as $\frac{\partial^2 s(x)}{\partial x^2} < 0$ for $x > +\sqrt{P_{\rm s}P_{\rm c}}$, the function $s(x)$ is convex for $x \in [+\sqrt{P_{\rm s}P_{\rm c}}, +\infty)$. Hence, $s(x)$ has only one inflection point at $x = \sqrt{P_{\rm s}P_{\rm c}}$. Otherwise, if there were any other inflection point in the interval $(+\sqrt{P_{\rm s}P_{\rm c}}, +\infty)$, the second derivative should become negative somewhere in this interval, and this is obviously not happening. As such, the maximizer of $s(x)$ is unique and it resides in the interval $(0, +\sqrt{P_{\mathrm{s}}P_{\mathrm{c}}}]$, where $s(x)$ is concave. ■

It follows from Lemma 1 that in order to find the real-valued maximizer of $s(x)$, the following steepest ascent algorithm can be used: $x^{(k)} = x^{(k-1)} + \xi \frac{\partial s(x)}{\partial x}\Big|_{x=x^{(k-1)}}$, where $x^{(k)}$ is the value of x at the kth iteration and ξ is the parameter that controls the stability and convergence of the algorithm. Once the steepest ascent algorithm has converged to the global maximizer of $s(x)$, we can obtain the optimal value of n using the following procedure: If the global maximizer of $s(x)$ is larger than or equal to N, then the optimal value of n is equal to N. In this case, all subchannels will be turned on. If the global maximizer of $s(x)$ is smaller than N, then the optimal value of n is either the largest integer number which is smaller than or equal to the global maximizer of $s(x)$ or the smallest integer number which is larger than or equal to the global maximizer of $s(x)$. As such, the optimal number of active channels can be found in an efficient manner. The following lemma reveals another interesting aspect of active parallel channels.

Lemma 2: *If the total available power is limited, i.e., if $P_{\mathrm{s}} + P_c \leq P_T$ for a given P_T, then in order to maximize the sum-rate via optimal power allocation to the source and to the active channel, half of the total available power should be allocated to the transmitter and the remaining half should be allocated to the active channel.*

Proof : To prove this, let us assume $P_{\mathrm{s}} + P_c \leq P_T$, where P_T is the maximum total available power. Then, using, (3.1.20), the sum-rate maximization under a total power constraint can be written as

$$\max_{n,P_{\rm s}P_{\rm c}} \quad n\log_2(1+\frac{P_{\rm s}P_{\rm c}}{n^2}), \quad \text{subject to} \;\; P_{\rm s}+P_{\rm c}\leq P_T.$$

For any fixed n, the maximization over $P_{\rm s}$ and $P_{\rm c}$ leads to maximizing the product $P_{\rm s}P_{\rm c}$ subject to $P_{\rm s}+P_{\rm c}\leq P_T$. The solution to this maximization problem is well-known to be $P_{\rm s}=P_{\rm c}=0.5P_T$. ■

Interestingly, the optimal values of $P_{\rm s}$ and $P_{\rm c}$ are independent of the optimal number of active subchannels. As such, in this case the optimal number of active channels, n, can be obtained by using the very same steepest ascent based method, which we outlined above, for $P_{\rm s}=P_{\rm c}=0.5P_T$.

Note that the authors of [94] study the problem of sum-rate maximization for a multi-antenna multi-carrier relay channel. The constraint considered in [94] is a total transmit power constraint which limits the sum of the relay and source power. In our work, we consider two constraints, one on the source transmit power, and one on the channel norm. Moreover, the solution provided in [94] is different from ours. Indeed the method of [94] relies on a high-SNR approximation but there is no guarantee that this solution results in high values of SNR, meaning that the sub-carrier powers are not guaranteed to result in high values of SNR in each subcarrier. Furthermore, the approximate solution provided in [94], while not applicable to the problem we are considering, has a water-filling structure, whereas our methods does not fit into such a water-filling structure.

3.2 Simulation Results

Fig. 3.1 shows the maximum sum-rate, that can be achieved by an active channel with $N = 64$ subchannels, versus the total available power P_T. In this figure, we also compare this maximum sum-rate with the maximum sum-rate achieved by two passive parallel channels, one with equal subchannel SNRs and one with unequal subchannel SNRs. In the passive channel with equal subchannel SNRs, all subchannel SNRs are equal to 0 dB, while in the case of unequal subchannel SNRs, the subchannel SNRs are different in each simulation. Indeed, in the latter case, the flat fading subchannel coefficients are drawn from i.i.d. complex Gaussian distribution with zero mean and unit variance. Hence, in the case of unequal subchannel SNRs, in each simulation the subchannel SNRs are different, however, when averaged over different channel realizations, all subchannel SNRs are equal to 0 dB. For the cases of passive channels, the total available power is allocated to the transmitter, while in the case of active channel, the total transmit power is divided between the transmitter and the channel. Indeed, in this figure, for the active channel, three different scenarios are considered: $P_{\mathrm{s}} = P_{\mathrm{c}}$, $P_{\mathrm{s}} = 3P_{\mathrm{c}}$ and $P_{\mathrm{s}} = P_{\mathrm{c}}/3$, where $P_{\mathrm{s}} + P_{\mathrm{c}} = P_T$. Fig. 3.1 shows that at high values of total transmit power, the active channel outperforms the passive channel cases. However, in low values of total transmit power, the passive channels offer a higher sum-rate as compared to the active channel. The reason is that the passive channels considered here correspond to a feasible scenario in an equivalent active channel where the total available power is $P_T + N$. Note that any passive channel is a special case (or a feasible point) in an active channel problem where the channel energy is bounded to be less than the actual channel energy of the passive channel. Hence, for low values of total transmit power, the power injected by the active channel into the signal paths are lower than those amounts of power injected by the two passive channels into the signal paths. It can also be seen in Fig. 3.1 that the active channel yields the best sum-rate when the total transmit power is divided equally between the transmitter and the channel, i.e., when $P_\mathrm{s} = P_\mathrm{c} = 0.5P_T$. Interestingly, for fixed $P_\mathrm{s} + P_\mathrm{c}$, the active channel offers the same sum-rate for both cases of $P_\mathrm{s} = 3P_\mathrm{c}$ and $P_\mathrm{s} = P_\mathrm{c}/3$. This is well-justified, as (3.1.20) shows that the sum-rate is a function of the product $P_\mathrm{s}P_\mathrm{c}$.

In Fig. 3.2, we have shown the sum-rate versus number of subchannels N, for different scenarios as explained above. As can be seen from this figure, when $P_T = 30$ dBW, the passive channel outperforms the active channel for small values of N. In this case, as the number of the subchannels, N is increased, the performance of the active channel saturates as the number of active channels reaches a certain value which depends only on the product $P_\mathrm{s}P_\mathrm{c}$. This value does not change when the number of subchannels, N is increased. At the same time, the performance of the passive channels improves consistently, as the number of subchannels increases. As P_T is increased from 30 to 35 dBW, the active channels outperform the passive counterparts in a wider range of N. This is due to the fact that for $P_T = 35$ dBW, number of active channels saturates at a higher value as compared to the case where $P_T = 30$ dBW.

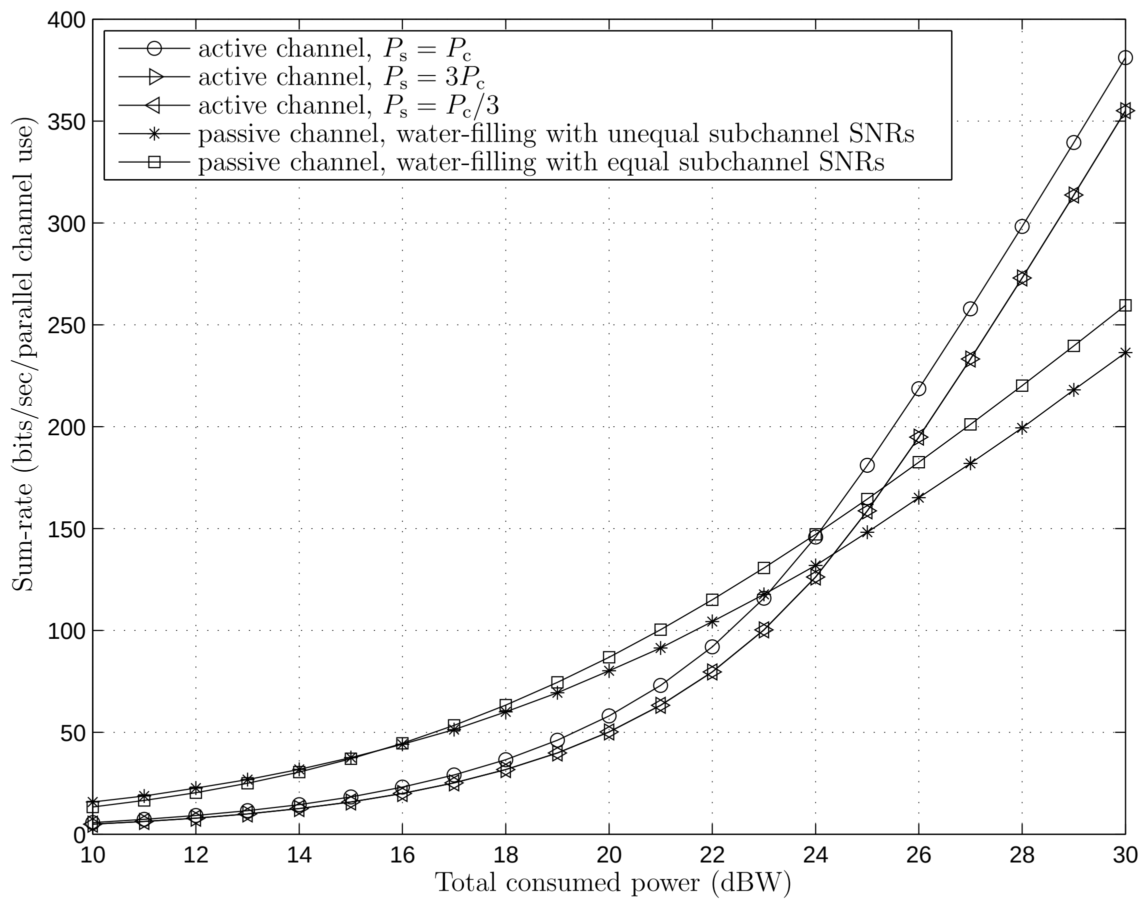

Figure 3.1: Maximum sum-rate versus the total consumed power for both active and passive channels, $N = 64$.

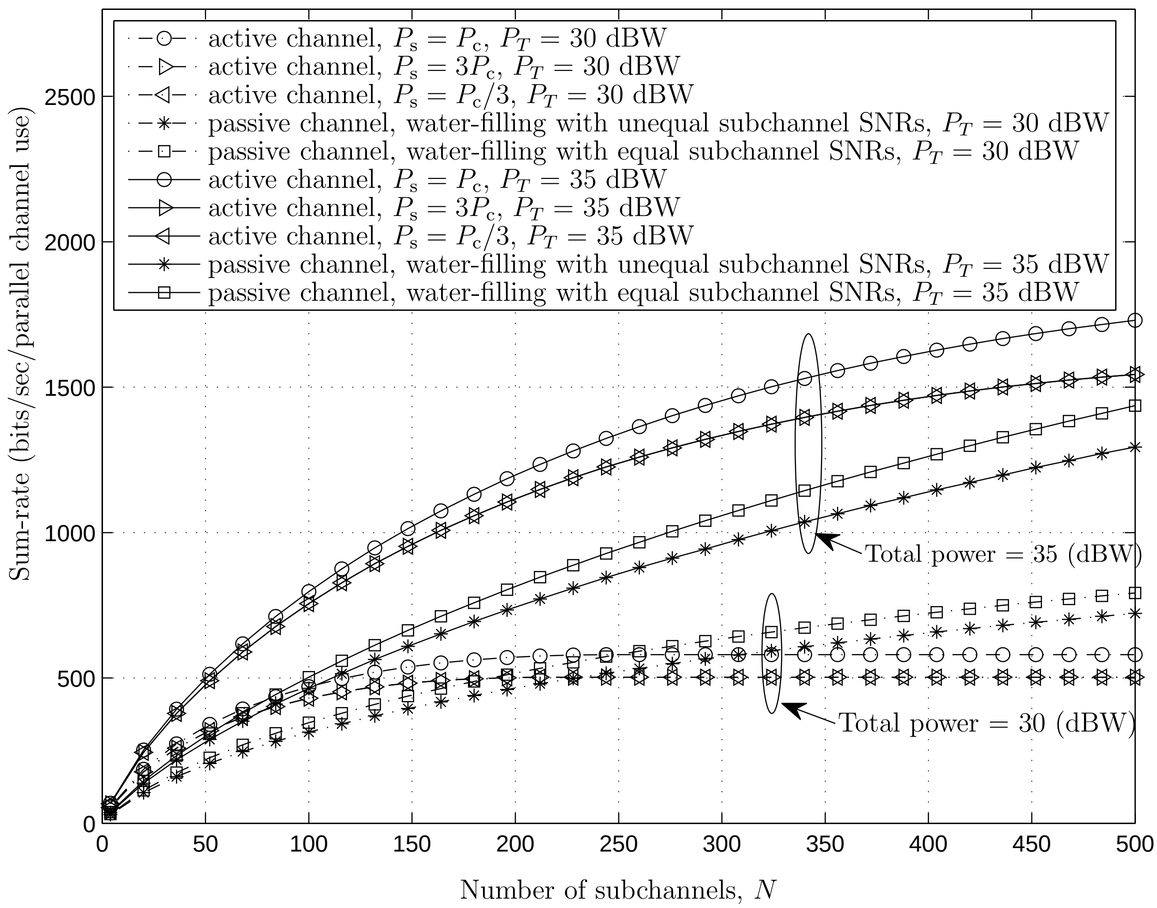

Figure 3.2: Maximum sum-rate versus number of subchannels, for active and passive channels. and for different values of total consumed power.

Chapter 4

Sum-rate Maximization for Active Channel: Unequal Noise Power Over Different Subchannels

4.1 System Model and Sum-Rate Maximization

We consider an active channel which conveys information from a transmitter (source) to a receiver (destination) through a set of N parallel orthogonal subchannels . The communication channel is assumed to be active in the sense that the energy injected into each subchannel can be adjusted to a certain level based on an optimality criterion such as sum-rate. We assume that $\tilde{p}_i$ is the transmit power allocated to the ith subchannel whose channel gain is represented by $\tilde{h}_i$. In Chapter 3, we studied the case where noise powers over different subchannels are all equal. In this chapter, we assume unequal noise powers over different subchannels, and use $\tilde{\alpha}_i$ to denote the inverse of the noise power over the ith subchannel. Without loss of generality, we assume that

$$\tilde{\alpha}_N \geq \tilde{\alpha}_{N-1} \geq ... \geq \tilde{\alpha}_1. \tag{4.1.1}$$

We further assume that the total transmit power is limited to P_{s}, that is, $\sum_{i=1}^{N} \tilde{p}_i \leq P_{\mathrm{s}}$. Moreover, the norm of the active channel is constrained to be smaller than or equal to P_{c}. We herein aim to maximize the sum-rate under two constraints, one on the total transmit power of the source, and one on the total energy of the parallel channel. Mathematically, we solve the following problem:

$$\begin{aligned} \max_{\tilde{\mathbf{p}}\succeq\mathbf{0},\tilde{\mathbf{h}}\succeq\mathbf{0}} \quad & \sum_{i=1}^{N} \log_2(1+\tilde{\alpha}_i\tilde{p}_i|\tilde{h}_i|^2) \\ \text{subject to} \quad & \mathbf{1}^T\tilde{\mathbf{p}} \leq P_{\mathrm{s}} \\ & \mathbf{1}^T\tilde{\mathbf{h}} \leq P_{\mathrm{c}} \end{aligned} \tag{4.1.2}$$

where $\tilde{\mathbf{p}} \triangleq [\tilde{p}_1 \;\; \tilde{p}_2 \;\; \cdots \;\; \tilde{p}_N]^T$ and $\tilde{\mathbf{h}} \triangleq [|\tilde{h}_1|^2 \;\; |\tilde{h}_2|^2 \;\; \cdots \;\; |\tilde{h}_N|^2]^T$. Compared to power allocation in a passive parallel channel, the optimization problem (4.1.2) has one additional constraint which limits the energy of the active channel. Such a constraint can be used for optimal power allocation and channel design in single- or multi-user multiple-input multiple-output systems with unequal subchannel noise powers [15,16,85]. Another application of the optimization problem (4.1.2) is in asynchronous one- or two-way AF-based multi-relay channels, where the end-to-end channel impulse response can be adjusted by properly adjusting amplification weight of the relays [87, 88, 97] and/or their locations. Note that when applied to asynchronous relay channels, the channel norm constraint used in (4.1.2) is different from widely used total or individual relay power constraints. However, the norm of such channels can be written in terms of the individual relay powers, or inversely, given channel gains, $\tilde{h}_i$'s, one can obtain the relay powers. Hence, one can use the optimal $\tilde{h}_i$ to design the relay channel, for example, by choosing the location of the relays with respect to the transmitter and the source and/or by using the right amount of the relay transmit power.

In Appendix A, it has been shown that the optimization problem (4.1.2) is not convex. Nevertheless, we herein show how the optimization problem (4.1.2) can be solved efficiently. To this end, note that at the optimum, the two constraints in (4.1.2) are satisfied with equality. Otherwise, if at the optimum, $\mathbf{1}^T\tilde{\mathbf{p}} < P_{\mathrm{s}}$ and/or $\mathbf{1}^T\tilde{\mathbf{h}} < P_{\mathrm{c}}$, we can scale up the elements of the optimal value of $\tilde{\mathbf{p}}$ and/or those of the optimal $\tilde{\mathbf{h}}$ such that $\mathbf{1}^T\tilde{\mathbf{p}} = P_{\mathrm{s}}$ and/or $\mathbf{1}^T\tilde{\mathbf{h}} = P_{\mathrm{c}}$, whereas the new $\tilde{\mathbf{p}}$ and/or the new $\tilde{\mathbf{h}}$ further increases the objective function, thereby contradicting the optimality. Hence, the optimization problem (4.1.2) can be rewritten as

$$\begin{aligned} \max_{\tilde{\mathbf{p}}\succeq\mathbf{0},\tilde{\mathbf{h}}\succeq\mathbf{0}} \quad & \sum_{i=1}^{N} \log_2(1+\tilde{\alpha}_i\tilde{p}_i\tilde{h}_i) \\ \text{subject to} \quad & \mathbf{1}^T\tilde{\mathbf{p}} = P_{\mathrm{s}} \\ & \mathbf{1}^T\tilde{\mathbf{h}} = P_{\mathrm{c}} \end{aligned} \tag{4.1.3}$$

where $\tilde{h}_i$ is the ith entry of $\tilde{\mathbf{h}}$. Note that if the ith entry of the optimal value of $\tilde{\mathbf{p}}$ is zero, the corresponding entry in the optimal $\tilde{\mathbf{h}}$ will be zero and vice versa, otherwise power will be wasted. Without loss of generality, let n represent the number of non-zero entries of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{h}}$. Non-zero entries of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{h}}$ correspond to the n largest entries of $\tilde{\boldsymbol{\alpha}}$. We define $\boldsymbol{\alpha}$ to capture the n largest entries of $\tilde{\boldsymbol{\alpha}}$, i.e, $\boldsymbol{\alpha} = [\tilde{\alpha}_{N-n+1} \quad \tilde{\alpha}_{N-n+2} \quad \cdots \quad \tilde{\alpha}_N]^T$. Note that in light of (4.1.1), $\alpha_i \geq \alpha_1$, that is the elements of the vector $\boldsymbol{\alpha}$ are ordered in non-descending order with α_1 being the smallest entry of $\boldsymbol{\alpha}$. Then, we can express the optimization problem (4.1.2) as

$$\begin{aligned}
\max_{n} \quad \max_{\mathbf{p},\mathbf{h}} \quad & \sum_{i=1}^{n} \log_2(1+\alpha_i p_i h_i) \\
\text{subject to} \quad & \mathbf{1}^T\mathbf{p} = P_\mathrm{s} \\
& \mathbf{1}^T\mathbf{h} = P_\mathrm{c} \\
& \mathbf{p} \succ \mathbf{0} \\
& \mathbf{h} \succ \mathbf{0}
\end{aligned} \tag{4.1.4}$$

where we use $\mathbf{p}$ and $\mathbf{h}$ to denote the non-zero entries of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{h}}$, respectively, and p_i and h_i are the corresponding ith entries of $\mathbf{p}$ and $\mathbf{h}$.

4.2 KKT Conditions

In what follows, we use the KKT conditions to obtain the necessary condition that the solution to (4.1.4) must satisfy. To do so, for any n, the Lagrangian function corresponding to the inner maximization over $\mathbf{p}$ and $\mathbf{h}$ can be expressed as

$$\begin{aligned}
\mathcal{L}(\mathbf{p},\mathbf{h}) = & - \sum_{i=1}^{n} \log_2(1+\alpha_i p_i h_i) + \lambda_1(\mathbf{1}^T\mathbf{p} - P_\mathrm{s}) \\
& + \lambda_2(\mathbf{1}^T\mathbf{h} - P_\mathrm{c}) - \boldsymbol{\mu}_1^T\mathbf{p} - \boldsymbol{\mu}_2^T\mathbf{h}
\end{aligned} \tag{4.2.1}$$

where the scalars λ_1 and λ_2 as well as the $n \times 1$ vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ represent the Lagrange multipliers. Based on the KKT conditions, the optimal solution must satisfy the following conditions:

- Primal feasibility:

$$\mathbf{1}^T\mathbf{p} = P_{\rm s} \tag{4.2.2}$$

$$\mathbf{1}^T\mathbf{h} = P_{\rm c} \tag{4.2.3}$$

$$-\mathbf{p} \prec \mathbf{0} \tag{4.2.4}$$

$$-\mathbf{h} \prec \mathbf{0}. \tag{4.2.5}$$

- Dual feasibility:

$$\boldsymbol{\mu}_1 \succeq 0, \quad \boldsymbol{\mu}_2 \succeq 0, \tag{4.2.6}$$

- Complementary slackness:

$$\lambda_1(\mathbf{1}^T\mathbf{p} - P_{\rm s}) = 0 \tag{4.2.7}$$

$$\lambda_2(\mathbf{1}^T\mathbf{h} - P_{\rm c}) = 0 \tag{4.2.8}$$

$$\boldsymbol{\mu}_1 \odot \mathbf{p} = 0 \tag{4.2.9}$$

$$\boldsymbol{\mu}_2 \odot \mathbf{h} = 0 \tag{4.2.10}$$

- Stationary condition:

$$\frac{\partial \mathcal{L}(\mathbf{p},\mathbf{h})}{\partial p_i} = \frac{1}{\ln 2}\frac{-\alpha_i h_i}{(1+\alpha_i p_i h_i)} + \lambda_1 - \mu_{1,i} = 0 \tag{4.2.11}$$

$$\frac{\partial \mathcal{L}(\mathbf{p},\mathbf{h})}{\partial h_i} = \frac{1}{\ln 2}\frac{-\alpha_i p_i}{(1+\alpha_i p_i h_i)} + \lambda_2 - \mu_{2,i} = 0, \tag{4.2.12}$$

  where $\mu_{1,i}$ and $\mu_{2,i}$ are the ith entries of $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, respectively.

It follows from (4.2.4) and (4.2.9) as well as from (4.2.5) and (4.2.10) that $\mu_{1,i} = \mu_{2,i} = 0$ holds true for $i = 1, 2, ..., n$. Hence, the conditions in (4.2.11) and (4.2.12) can be rewritten, respectively, as

$$\frac{1}{\ln 2}\frac{-\alpha_i h_i}{(1+\alpha_i p_i h_i)} + \lambda_1 = 0 \tag{4.2.13}$$

$$\frac{1}{\ln 2}\frac{-\alpha_i p_i}{(1+\alpha_i p_i h_i)} + \lambda_2 = 0\,. \tag{4.2.14}$$

It follows from (4.2.13) and (4.2.14) that

$$\frac{h_i}{p_i} = \frac{\lambda_1}{\lambda_2}, \quad \text{for } i = 1, 2, \cdots, n\,. \tag{4.2.15}$$

Defining $\kappa \triangleq \frac{\lambda_1}{\lambda_2}$ and using (4.2.15) along with the first and the second constraints in (4.1.4), we can obtain κ as

$$\kappa = \frac{P_\mathrm{c}}{P_\mathrm{s}}\,. \tag{4.2.16}$$

Using (4.2.15) and (4.2.16), the maximization problem (4.1.4) can be rewritten as

$$\begin{aligned} \max_{n} \;\; \max_{\mathbf{p}} \quad & \sum_{i=1}^{n} \log_2(1+\kappa\alpha_i p_i^2) \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p} = P_\mathrm{s} \\ & \mathbf{p} \succ \mathbf{0}\,. \end{aligned} \tag{4.2.17}$$

Solving the optimization problem (4.2.17) means that we are looking for the optimal number of the activated subchannels and their corresponding allocated powers such that the sum-rate is maximized subject to a constraint on the source transmit power. Note that if $\alpha_i > \alpha_j$, then at the optimum, $p_i > p_j$. Otherwise, we could swap the optimal p_i and the optimal p_j, thereby increasing the cost function in (4.2.17), without violating the constraint. This implies that as the elements of $\boldsymbol{\alpha}$ are sorted in non-descending order, the elements of the optimal vector $\mathbf{p}$ are also sorted in non-descending order.

In the sequel, we simplify the optimization problem (4.2.17) showing that it can be equivalently written as optimally finding the number of active subchannels, n, and the power of the weakest subchannel among the nth strongest subchannel. To this end, note that or any $i \neq j$, we obtain from (4.2.14) that

$$0 = \frac{1}{\ln 2}\frac{-\alpha_i p_i}{(1+\alpha_i p_i h_i)} + \lambda_2 = \frac{1}{\ln 2}\frac{-\alpha_j p_j}{(1+\alpha_j p_j h_j)} + \lambda_2 \tag{4.2.18}$$

or

$$\frac{\alpha_i p_i}{(1+\alpha_i p_i h_i)} = \frac{\alpha_j p_j}{(1+\alpha_j p_j h_j)}\,. \tag{4.2.19}$$

As $h_i = \kappa p_i$, we can rewrite (4.2.19) as

$$\frac{\alpha_i p_i}{1+\alpha_i \kappa {p_i}^2} = \frac{\alpha_1 p_1}{1+\alpha_1 \kappa {p_1}^2}\,, \quad for \quad i = 1, ..., n. \tag{4.2.20}$$

It follows from (4.2.20) that, for given p_1, the optimal p_i must satisfy the following quadratic equation:

$$(\kappa\alpha_i\alpha_1 p_1)p_i^2 - (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)p_i + \alpha_1 p_1 = 0, \quad for \quad i = 1, ..., n. \tag{4.2.21}$$

Solving (4.2.21) yields the following two possible solutions for p_i in terms of p_1:

$$p_i^+(p_1) = \frac{(\alpha_i + \kappa\alpha_i\alpha_1 p_1^2) + \sqrt{\Delta_i}}{2(\kappa\alpha_i\alpha_1 p_1)} \tag{4.2.22}$$

$$p_i^-(p_1) = \frac{(\alpha_i + \kappa\alpha_i\alpha_1 p_1^2) - \sqrt{\Delta_i}}{2(\kappa\alpha_i\alpha_1 p_1)} \tag{4.2.23}$$

where $\Delta_i \triangleq (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)^2 - 4\kappa\alpha_i\alpha_1^2 p_1^2$. Note that

$$\begin{aligned}
\Delta_i &= (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)^2 - 4\kappa\alpha_i\alpha_1^2 p_1^2 \\
&\geq (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)^2 - 4\kappa{\alpha_i}^2\alpha_1 p_1^2 \\
&= (\alpha_i - \kappa\alpha_i\alpha_1 p_1^2)^2 \\
&\geq 0.
\end{aligned} \tag{4.2.24}$$

where, in the first inequality, we have used the fact that $\alpha_i \geq \alpha_1$, for $i > 1$. It follows from (4.2.24) that both values of $p_i^+(p_1)$ and $p_i^-(p_1)$ are real. Nevertheless, using the fact that the elements of the optimal vector $\mathbf{p}$ are ordered in a non-descending order, we now show that the solution $p_i^-(p_1)$, given in (4.2.23), is not acceptable. To do so, at the optimal p_1, it is required that

$$p_i^-(p_1) \geq p_j^-(p_1), \qquad \text{for} \quad \alpha_i > \alpha_j. \tag{4.2.25}$$

Using (4.2.23) in (4.2.25), we can write

$$\frac{1+\kappa\alpha_1 p_1^2}{2\kappa\alpha_1 p_1} - \sqrt{\frac{(1+\kappa\alpha_1 p_1^2)^2}{4\kappa^2\alpha_1^2 p_1^2} - \frac{1}{\kappa\alpha_i}} \geq \frac{1+\kappa\alpha_1 p_1^2}{2\kappa\alpha_1 p_1} - \sqrt{\frac{(1+\kappa\alpha_1 p_1^2)^2}{4\kappa^2\alpha_1^2 p_1^2} - \frac{1}{\kappa\alpha_j}} \tag{4.2.26}$$

which leads us to $\alpha_i \leq \alpha_j$. This contradicts with the earlier assumption that $\alpha_i > \alpha_j$. As such, there cannot be more than one subchannel whose power is given by $p_i^-(p_1)$. Now assume that the ith channel power is given by $p_i^-(p_1)$ and the rest are given by $p_i^+(p_1)$. Then, assuming that

$$p_i^-(p_1) \geq p_j^+(p_1), \qquad \text{for} \quad \alpha_i > \alpha_j. \tag{4.2.27}$$

we arrive at $-\alpha_i^{-1} > \alpha_j^{-1}$ which contradicts with the fact that α_i is positive. Hence, we conclude that no subchannel power can be given by $p_i^-(p_1)$ and $p_i^+(p_1)$ is the only acceptable solution to (4.2.21). For the sake of simplicity, we hereafter drop the superscript $^+$ from $p_i^+(p_1)$. Then, given p_1, the optimal value of p_i is given by

$$p_i(p_1) = \frac{1+\kappa\alpha_1 p_1^2}{2\kappa\alpha_1 p_1} + \sqrt{\frac{(1+\kappa\alpha_1 p_1^2)^2}{4\kappa^2\alpha_1^2 p_1^2} - \frac{1}{\kappa\alpha_i}} > 0\,. \tag{4.2.28}$$

Using (4.2.28), the optimization problem in (4.2.17) can be equivalently written as

$$
\begin{aligned}
\max_{p_1, n>2} \quad & \sum_{i=1}^{n} \log_2(1+\kappa\alpha_i p_i^2(p_1)) \\
\text{subject to} \quad & p_1 + \sum_{i=2}^{n} p_i(p_1) = P_{\mathrm{s}} \\
& 0 < p_1 < P_{\mathrm{s}}
\end{aligned}
\tag{4.2.29}
$$

where we have used the assumption that $p_i(p_1) > 0$, for $i = 2, 3, \ldots, n$. Note that in (4.2.29), we have excluded the case $n = 1$, as for $n = 1$, the solution is simply to assign all the power to the strongest subchannel and deactivate all the other subchannels.

To solve (4.2.29), we propose to use a search procedure over n, where we obtain the optimal value of p_1 for every value of n. The pair of n and the corresponding optimal value of p_1 is then used to calculate the cost function. The pair which leads to the highest value of the cost function is introduced as the solution to (4.2.29). Note that not every value of n is feasible. In order for a particular value of n to be feasible, the corresponding feasible set must not be empty. In other words for a ceratin value of n to be feasible, the first constraint in (4.2.29) must have a solution in terms of p_1 in the interval $(0, P_{\mathrm{s}})$. In the next section, we show that for any value of n, this constraint has only zero, one, or two solutions for p_1 in the interval $(0, P_{\mathrm{s}})$. Hence, the solution to (4.2.29) belongs to a set of finite number of pairs (n, p_1) which are the solutions to the second constraint in (4.2.29) such that $0 < p_1 < P_{\mathrm{s}}$. This property simplifies the search procedure as we need to examine only a countable number of pairs (n, p_1) to see which pair results in the highest value of the sum-rate.

4.3 Feasibility And Solution

In this section, we present an efficient algorithm to find the feasible values of $n > 2$. Let us rewrite the first constraint in (4.2.29) as

$$f^n(p_1) = P_s. \quad (4.3.1)$$

where $f^n(p_1) \triangleq \mathbf{1}^T\mathbf{p} = p_1 + \sum_{i=2}^{n} p_i(p_1)$. To solve (4.2.29), we need to find the values of p_1 and n which satisfy (4.3.1) and which at the same time result in the largest value for the objective function. However, the equality constraint in (4.3.1) may not be feasible for every n. In order for (4.3.1) to be feasible for a certain n, the following inequality must hold true:

$$\min_{p_1} f^n(p_1) \leq P_s\,, \qquad \text{for} \qquad p_1 \in (0, P_s). \quad (4.3.2)$$

Indeed, if for any n, the minimum value of $f^n(p_1)$, when $p_1 \in (0, P_s)$, is greater than P_s, there is no solution for p_1 satisfying the equality in (4.3.1), and therefore, that particular value of n is not feasible. Hence, in order to reject the infeasible values of n, we can find, for a certain n, the minimum value of $f^n(p_1)$ for $p_1 \in (0, P_s)$ and compare that minimum value with P_s. If this minimum value is greater than P_s, then that value of n is rejected, otherwise, that specific value of n remains in the feasible set. Let $f_n^{\min}$ represent the minimum value of $f^n(p_1)$, when $p_1 \in (0, P_s)$.

We now find the minimum value of $f^n(p_1)$, when $p_1 \in (0, P_s)$. To do so, note that $f_n^{\min}$ is the same as the global minimum of $f^n(p_1)$, if the global minimizer of $f^n(p_1)$ is in the interval $(0, P_s)$. Let us study the properties of the global minimizer of $f^n(p_1)$ (which may not be in the interval $(0, P_s)$). We will later use these properties to obtain the minimizer of $f^n(p_1)$ in the interval $(0, P_s)$. The following lemma presents these properties.

Lemma 3: : *Let* $p_{1,n}^{\min}$ *denote the global minimizer of* $f^n(p_1)$ *for* $n = 2, 3, ..., N$. *Then, the following statements are true for any* n*:*

a) $p_{1,n}^{\min}$ *is unique.*

b) The inequality $p_{1,n}^{\min} < \dfrac{1}{\sqrt{\kappa\alpha_1}}$ *holds true.*

c) $f^n(p_1)$ *is monotonically decreasing for* $p_1 \in (0, p_{1,n}^{\min})$ *and it is monotonically increasing for* $p_1 \in (p_{1,n}^{\min}, P_\mathrm{s})$.

Proof: See Appendix C.

It follows from part (a) of Lemma 3 that the equation

$$\frac{\partial f^n(p_1)}{\partial p_1} = 0 \tag{4.3.3}$$

has a unique solution for $p_1 > 0$. This solution, referred to as $p_{1,n}^{\min}$, can be easily obtained using a bisection algorithm, as explained in the sequel. Let $p_l = 0$ and $p_u = \frac{1}{\sqrt{\kappa\alpha_1}}$ be, respectively, the lower and upper bounds of the solution to (4.3.3). If for any given value of p_1 in the interval $[p_l, p_u]$ (say $p_1 = (p_l + p_u)/2$), we have that $\dfrac{\partial f^n(p_1)}{\partial p_1} > 0$, then the solution to (4.3.3) is smaller than that value of p_1. Hence, we can choose that value of p_1 to be a new value of p_u. If, for the chosen p_1, we have $\dfrac{\partial f^n(p_1)}{\partial p_1} < 0$, then the solution to (4.3.3) is larger than that value of p_1. Hence we can choose that value of p_1 to be a new value of p_l. This process can be repeated until the change in the value of p_1 is small enough. The so-obtained value of p_1 is then introduced as $p_{1,n}^{\min}$. If $p_{1,n}^{\min} \in (0, P_\mathrm{s}]$ (see Figs. 4.1(a), 4.1(c), 4.1(d), or 4.1(f)), then we have $f_n^{\min} = f_n(p_{1,n}^{\min})$. If $p_{1,n}^{\min} > P_\mathrm{s}$ (see Fig. 4.1(b) or 4.1(e)), as the function $f^n(p_1)$ is monotonically decreasing for $p_1 < p_{1,n}^{\min}$ (see part (c) of Lemma 3), then this function attains its lowest value in the interval $(0, P_\mathrm{s}]$ at $p_1 = P_\mathrm{s}$. In this case, $f_n^{\min} = f_n(P_\mathrm{s})$. We have summarized this process as Algorithm 1.

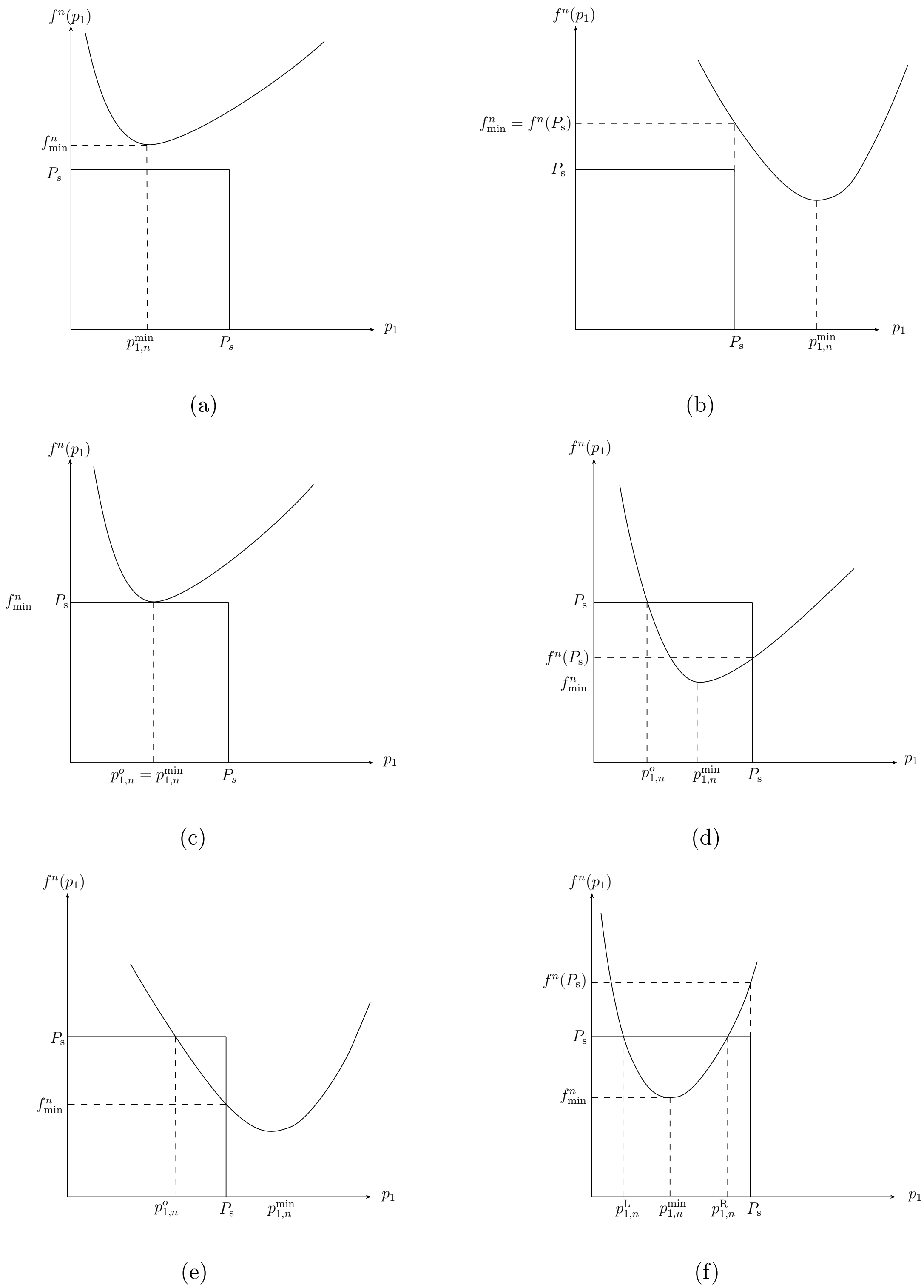

Figure 4.1: Geometric representation of the location of $f^n_{\min}$ with respect to P_s.

Algorithm 1 Bisection algorithm to find $f^n_{\min} = \min_{0<p_1<P_s} f^n(p_1)$

Step 1. Set ϵ as the desired stopping criterion.

Step 2. Initialize $k = 0$, $p_l = 0$ and $p_u = \frac{1}{\sqrt{\kappa\alpha_1}}$.

Step 3. Choose $p_t^{(k)} = (p_l + p_u)/2$.

Step 4. Calculate $\dfrac{\partial f^n(p_1)}{\partial p_1}$ at $p_1 = p_t^{(k)}$ using

$$\frac{\partial f^n(p_1)}{\partial p_1} = 1 + \sum_{i=2}^{n} \frac{\left(\alpha_1\, \kappa\, \mathrm{p_1}^2 - 1\right)\left(\sqrt{\Delta_i} + b_i\right)}{2\, \alpha_1\, \kappa\, \mathrm{p_1}^2 \sqrt{\Delta_i}},$$

where $\Delta_i = b_i^2 - 4\kappa\alpha_i\alpha_1^2 p_1^2$ and $b_i = (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)$.

Step 5. If $\left.\dfrac{\partial f^n(p_1)}{\partial p_1}\right|_{p_1=p_t^{(k)}} > 0$, set $p_u = p_t^{(k)}$. If $\left.\dfrac{\partial f^n(p_1)}{\partial p_1}\right|_{p_1=p_t^(k)} < 0$ set $p_l = p_t^{(k)}$

Step 6. If $|p_t^{(k+1)} - p_t^{(k)}| > \epsilon$, set $k = k + 1$ and go to Step 3.

Step 7. Set $p_{1,n}^{\min} = p_t^{(k)}$.

Step 8. If $p_{1,n}^{\min} > P_s$, set $f_n^{min} = f_n(P_s)$ otherwise set $f_n^{\min} = f_n(p_{1,n}^{\min})$, where

$$f^n(p_1) = p_1 + \sum_{i=2}^{n} \left(\frac{1 + \kappa\alpha_1 p_1^2}{2\kappa\alpha_1 p_1} + \sqrt{\frac{(1 + \kappa\alpha_1 p_1^2)^2}{4\kappa^2\alpha_1^2 p_1^2} - \frac{1}{\kappa\alpha_i}} \right).$$

As mentioned earlier, the so-obtained value of $f_n^{\min}$ can be used to determine whether the corresponding value of n is feasible or not. To do so, let us consider the following three possible cases:

1. If for a certain value of n, $f^n_{\text{min}} > P_{\text{s}}$ holds true, then there is no solution for p_1 which can satisfy $f^n(p_1) = P_{\text{s}}$, or, equivalently, to meet the KKT conditions. Therefore, the chosen n is not feasible. Figs.4.1(a) and 4.1(b) correspond to this situation, where the function $f^n(p_1)$ does not intersect with the horizontal line with the height P_{s} in the interval $(0, P_{\text{s}}]$.

2. If for a certain value of n, $f^n_{\text{min}} = P_{\text{s}}$ holds true, then there is only one value for p_1 which satisfies (4.3.1) or the KKT conditions. This solution is, then the optimal value of p_1 for that n, is given by $p^o_{1,n} = p^{\min}_{1,n}$. This situation is shown in Fig. 4.1(c). Note that in practice, the probability of this case is zero, given the random nature α_i's.

3. If for a certain value of n, $f^n_{\text{min}} < P_{\text{s}}$ holds true, then using part (c) of Lemma 3, there is at least one solution to (4.3.1) in the interval of $(0, P_{\text{s}}]$. Based on the value of $f^n(P_{\text{s}})$ and the location of $p^{\min}_{1,n}$ with respect to P_{s}, the following scenarios are possible:

   subcase 3.1) If $f^n(P_{\text{s}}) < P_{\text{s}}$ and $p^{\min}_{1,n} < P_{\text{s}}$ (see Fig. 4.1(d)), then (4.3.1) has a unique solution in the interval of $(0, p^{\min}_{1,n}]$ and there is no solution when $p_1 \in [p^{\min}_{1,n}, P_{\text{s}}]$. The uniqueness of this solution stems from the fact that $f_n(p_1)$ is monotonically decreasing in the interval of $(0, p^{\min}_{1,n})$, and hence, it can intersect with the horizontal line with height P_{s} only once in the interval $(0, P_{\text{s}})$. Also, the reason that (4.3.1) has no solution in the interval $[p^{\min}_{1,n}, P_{\text{s}})$ is that the function

$f_n(p_1)$ is monotonically increasing in this interval and that $f^n(P_\mathrm{s}) < P_\mathrm{s}$.

subcase 3.2) If $f^n(P_\mathrm{s}) < P_\mathrm{s}$ and $p_{1,n}^{\min} > P_\mathrm{s}$ (see Fig. 4.1(e)), then (4.3.1) has only one solution in the interval $(0, P_\mathrm{s})$.

subcase 3.3) If $f^n(P_\mathrm{s}) > P_\mathrm{s}$ and $p_{1,n}^{\min} < P_\mathrm{s}$ (see Fig. 4.1(f)), then (4.3.1) has two solutions in the interval $(0, P_\mathrm{s})$. One of these solutions (denoted as $p_{1,n}^{\mathrm{L}}$) is in the interval $(0, p_{1,n}^{\min})$ and the second solution (denoted as $p_{1,n}^{\mathrm{R}}$) is in the interval $(p_{1,n}^{\min}, P_\mathrm{s})$. Note that for a certain value of n, one of two values of $p_{1,n}^{\mathrm{L}}$ and $p_{1,n}^{\mathrm{R}}$ should be chosen as the optimal value of p_1 for that value of n. This can be done by calculating the corresponding value of the objective function for both values and choosing the one which leads to the largest value of this objective function. Note that if the case $p_{1,n}^{\mathrm{R}} = P_\mathrm{s}$ is not feasible and in such a case, $p_{1,n}^{\mathrm{L}}$ is the solution for the chosen n.

In any case, the solution(s) to (4.3.1) (if exists) can be obtained using a simple bisection method.

Note that if the constraint (4.3.1) is infeasible for a certain value of n, it will be infeasible for $m > n$. The reason is that

$$f_m(p_1) = f_n(p_1) + \sum_{i=n+1}^{m} p_i(p_1) > f_n(p_1) \tag{4.3.4}$$

It follows from (4.3.4) that if n is not feasible, i.e. if $f_n(p_1) > P_\mathrm{s}$ in the interval $(0, P_\mathrm{s}]$, then $f_m(p_1) > f_n(p_1) > P_\mathrm{s}$ in this interval. In other words, m is also infeasible. This reduces the computational complexity as we do not need to check all values of n. Indeed, we can start from $n = 2$ and check the feasibility of all values of $n \geq 2$. As soon as we find an infeasible value for n, we stop the search. Our proposed solution is summarized as Algorithm 2.

Algorithm 2 Finding the optimal solution to power control for active channels

Step 1. Define

$$p_i(z) = \left(\frac{1+\kappa\alpha_1 z^2}{2\kappa\alpha_1 z} + \sqrt{\frac{(1+\kappa\alpha_1 z^2)^2}{4\kappa^2\alpha_1^2 z^2} - \frac{1}{\kappa\alpha_i}} \right) \quad (4.3.5)$$

Step 2. For $n = 1$, choose $p^o_{1,n} = P_\mathrm{s}$ and calculate the corresponding sum-rate as $R(n) = \log_2(1 + \alpha_1 P_\mathrm{s} P_\mathrm{c})$.

Step 3. Choose $n = n + 1$.

Step 4. If $n > N$, go to Step 14

Step 5. Use Algorithm 1 to obtain $f^n_{\min}$ and $p^{\min}_{1,n}$.

Step 6. If $f^n_{\min} > P_\mathrm{s}$, then go to Step 14.

Step 7. If $f^n_{\min} = P_\mathrm{s}$, then $p^o_{1,n} = p^{\min}_{1,n}$. Use (4.3.5) to obtain $p_i(p^o_{1,n})$, and then calculate the sum-rate as $R(n) = \sum_{i=1}^n \log_2(1 + \kappa\alpha_i p_i^2(p^o_{1,n}))$, where $\kappa = \frac{P_\mathrm{c}}{P_\mathrm{s}}$. Then, go to Step 14.

Step 8. If $f^n(P_\mathrm{s}) < P_\mathrm{s}$ and $p^{\min}_{1,n} < P_\mathrm{s}$, use a bisection method to find the solution $p^o_{1,n}$ to $f^n(p_1) = P_\mathrm{s}$ in the interval $(0, p^{\min}_{1,n})$. Then, go to Step 12.

Step 9. If $f^n(P_\mathrm{s}) < P_\mathrm{s}$ and $p^{\min}_{1,n} > P_\mathrm{s}$, use a bisection method to find the solution $p^o_{1,n}$ to $f^n(p_1) = P_\mathrm{s}$ in the interval $(0, P_\mathrm{s})$. Then, go to Step 12.

Step 10. If $f^n(P_\mathrm{s}) > P_\mathrm{s}$ and $p^{\min}_{1,n} < P_\mathrm{s}$, use bisection methods to obtain $p^\mathrm{L}_{1,n}$ and $p^\mathrm{R}_{1,n}$ in the intervals $(0, p^{\min}_{1,n})$ and $(p^{\min}_{1,n}, P_\mathrm{s})$, respectively.

Step 11. If $\sum_{i=1}^n \log_2(1 + \kappa\alpha_i p_i^2(p^\mathrm{L}_{1,n})) \geq \sum_{i=1}^n \log_2(1 + \kappa\alpha_i p_i^2(p^\mathrm{R}_{1,n}))$, then $p^o_{1,n} = p^\mathrm{L}_{1,n}$, otherwise $p^o_{1,n} = p^\mathrm{R}_{1,n}$.

Step 12. Use (4.3.5) to obtain $p_i(p^o_{1,n})$ and then calculate the sum-rate as $R(n) = \sum_{i=1}^n \log_2(1 + \kappa\alpha_i p_i^2(p^o_{1,n}))$, where $\kappa = \frac{P_\mathrm{c}}{P_\mathrm{s}}$.

Step 13. Go to Step 3

Step 14. Find the maximum value of the sum-rate and the corresponding value of n as $R_{max} = \max_n R(n)$ and $n^o = \arg\max_n R(n)$, respectively.

Step 15. Find the optimum value of p_1 as $p^o_1 = p^o_{1,n^o}$.

Step 16. Use (4.3.5) to calculate the optimal value of power of the ith subchannels as $p^0_i = p_i(p^o_1)$, for $i = 1, 2, \ldots, n^o$. For $n^o < i < N$, choose $p^o_i = 0$

Step 17. Calculate the optimal value of subchannel gains as $h^o_i = \kappa p^o_i$.

4.4 Simulation Results

In this section, we compare the performance of an active channel with that of a passive channel with the same number of subchannels. For the active channel, the total consumed power is defined as $P_{\mathrm{T}} = P_{\mathrm{s}} + P_{\mathrm{c}}$, and the following three scenarios are considered: 1) $P_{\mathrm{s}} = P_{\mathrm{c}} = P_{\mathrm{T}}/2$, 2) $P_{\mathrm{s}} = 3P_{\mathrm{c}} = 3P_{\mathrm{T}}/4$ and 3) $P_{\mathrm{s}} = P_{\mathrm{c}}/3 = P_{\mathrm{T}}/4$. For the active channel, the noise powers of different subchannels (i.e., α_i's) are modeled as i.i.d exponentially distributed random variables with rate 0.5. For the passive channel, we assume that we have no control over the subchannels, and hence, the total available power is consumed at the source. Also, each subchannel of the passive channel is assumed to have a gain which is modeled as complex Gaussian random variable with variance 1. To maximize the sum-rate of the passive channel, we use water-filling power allocation scheme.

In Fig. 4.2, we plot the maximum sum-rate of the active channel as well as the maximum sum-rate of the passive channel versus the total consumed power, for $N = 16$. This figure shows that, at large values of P_{T}, the active channel significantly outperforms its passive counterpart in terms of the maximum sum-rate. This is due to the fact that when maximizing the sum-rate of the active channel, we have more degrees of freedom in our optimization problem as compared to the case when we maximize the sum-rate of the passive channel. However, for small values of the total consumed power, water-filling solution to the sum-rate maximization of the passive channel performs slightly better than the proposed solution to sum-rate maximization for the active channel. The reason is that the passive channel considered here corresponds to a feasible point in an active channel problem where the total available power is, in average, $P_{\mathrm{T}} + N$. Indeed, for the passive channel considered here, the norm of the channel is not zero but it is equal to N, in average. For small values of the total transmit power, the power assigned to the active channel is smaller than the power of the passive channel. This relatively low channel power of the active scheme wastes the advantages of the additional degrees of freedom offered by this scheme, resulting in a lower sum-rate as compared to the passive channel for small values of the total transmit power. As the total available power is increased, the active channel receives increasingly more power, thereby gaining sum-rate advantages over the passive channel.

As can be seen from Fig. 4.2, the active channel yields the same maximum sum-rate for $P_{\mathrm{s}} = 3P_{\mathrm{c}}$ and $P_{\mathrm{s}} = P_{\mathrm{c}}/3$. This is consistent with the fact that in the optimization problem (4.1.4), the objective function does not change if we swap h_i and p_i. As can be seen from this figure, the maximum sum-rate of the active channel is achieved when half of the total available power is allocated to the source, while the remaining half is assigned to the channel. This observation is explained below. Consider the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p},\mathbf{h}} \quad & \sum_{i=1}^{n} \log_2(1 + \alpha_i p_i h_i) \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p} + \mathbf{1}^T\mathbf{h} = P_{\mathrm{T}} \\ & \mathbf{p} \succcurlyeq \mathbf{0}, \quad \mathbf{h} \succcurlyeq \mathbf{0}\,. \end{aligned}$$

This optimization problem can be equivalently written as

$$\begin{aligned} \max_{\boldsymbol{\beta}} \ \max_{\mathbf{p},\mathbf{h}} \quad & \sum_{i=1}^{n} \log_2(1 + \alpha_i p_i h_i) \\ \text{subject to} \quad & p_i + h_i = \beta_i \\ & \mathbf{1}^T\boldsymbol{\beta} = P_{\mathrm{T}} \\ & \mathbf{p} \succcurlyeq \mathbf{0}, \quad \mathbf{h} \succcurlyeq \mathbf{0}, \quad \boldsymbol{\beta} \succcurlyeq \mathbf{0}\,. \end{aligned}$$

or as

$$
\begin{aligned}
\max_{\boldsymbol{\beta}} \quad & \sum_{i=1}^{n} \max_{p_i,h_i} \; \log_2(1+\alpha_i p_i h_i) \\
\text{subject to} \quad & p_i + h_i = \beta_i \\
& \mathbf{1}^T \boldsymbol{\beta} = P_{\mathrm{T}} \\
& \mathbf{p} \succcurlyeq \mathbf{0}, \quad \mathbf{h} \succcurlyeq \mathbf{0}, \quad \boldsymbol{\beta} \succcurlyeq \mathbf{0}\,.
\end{aligned}
$$

where $\boldsymbol{\beta} \triangleq [\beta_1, \beta_2, \ldots, \beta_n]^T$. It is obvious that the inner maximization is solved when $p_i = h_i = \beta_i/2$. As such, at the optimum, half of the total available power has to be assigned to the source and the remaining half has to be assigned to the channel.

In Fig. 4.3, we plot the average number of activated subchannels in the passive channel as well as the average number of activated subchannels in the active channel for the same aforementioned three power allocation scenarios. Although, for small values of P_{T}, the active channel performs slightly worse than the passive channel (see Fig. 1), the number of activated subchannels of the active channel, in average, is much smaller than that number for the passive channel. For example, at $P_{\mathrm{T}} = 12$ (dBW), the active channel uses, in average, 4 out of 16 subchannels, while the passive channel utilizes 13 subchannels. However, the maximum sum-rate of the passive channel is only about 5 (bits/sec/parallel channel use) higher than that of the active channel. At moderate values of P_{T}, for example when $P_{\mathrm{T}} = 22$ dBW, compared to the passive channel, the active channel yields higher sum-rate with using, in average, less number of subchannels. For large values of P_{T}, both active and passive channels utilize the same number of subchannels, however, the active channel achieves a significantly higher sum-rate. These features of the active channel is well explained by the fact that the active channel offers more degrees of freedom. Indeed, in the active channel both the source transmit power allocation strategy and the channel are designed to achieve a higher sum-rate, while in the passive channel, we have no control over the channel and can only adjust the source power allocation scheme.

In Fig. 4.4, we have shown the maximum sum-rate for both active and passive channels, versus number of the total available subchannels N, for $P_{\mathrm{T}} = 30$ (dBW) and $P_{\mathrm{T}} = 35$ (dBW). For small number of available subchannels, the active channel results in a higher sum-rate compared to its passive counterpart. As the number of available subchannels increases, the maximum sum-rate of the active channel is saturated, while the maximum sum-rate in the passive channel is increased consistently. The reason for this saturation behavior of the active channel is that beyond a certain value of N, the problem becomes infeasible, and no matter how many subchannels are available, the corresponding power allocation scheme does not result in a higher sum-rate. When P_{T} increases from 30 (dBW) to 35 (dBW), the active channel performs better than the passive channel in a wider range of N. To explain why the performance of the passive passive channel improves as N is increased, one should note that the power of the passive channel increases with N. However for an active channel, the proposed power allocation scheme does not activate all subchannels, but uses only a subset of them.

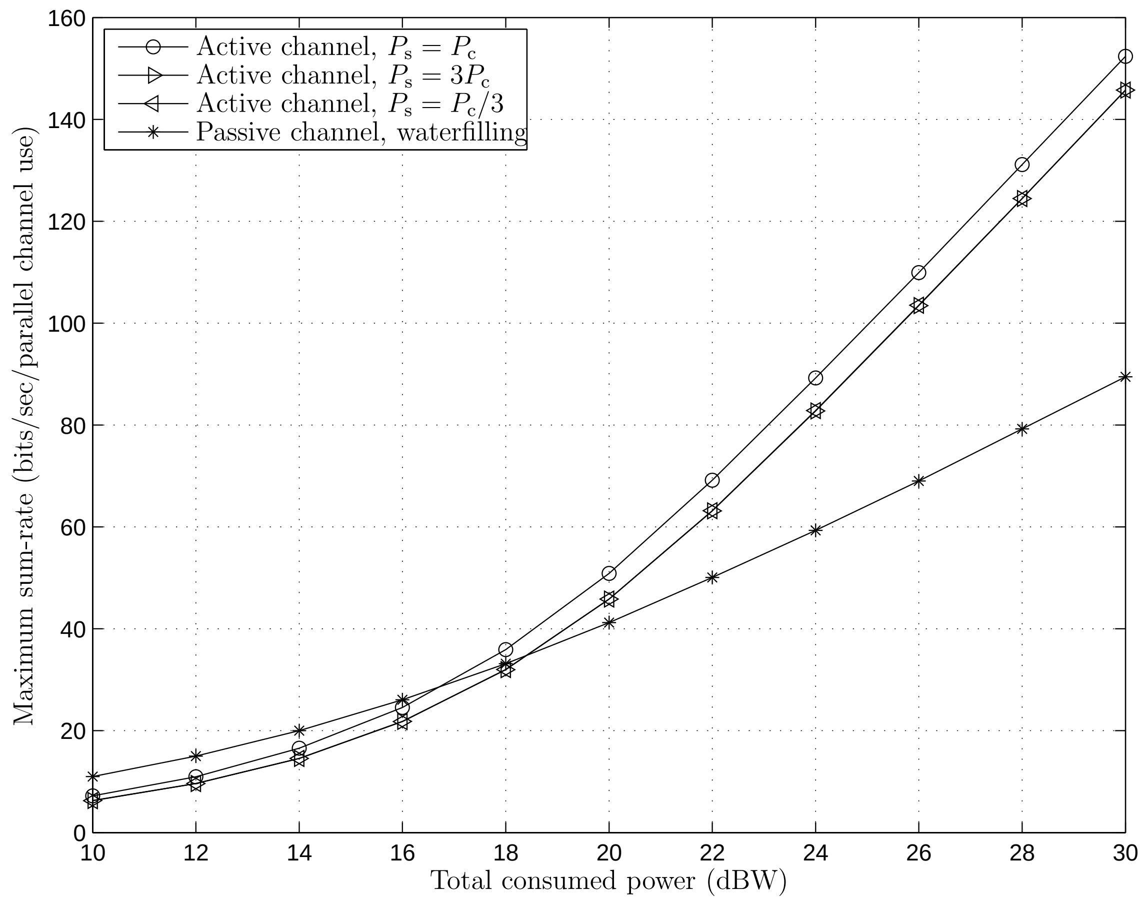

Figure 4.2: Maximum sum-rate versus the total consumed power for both active and passive channels with $N = 16$ subchannels.

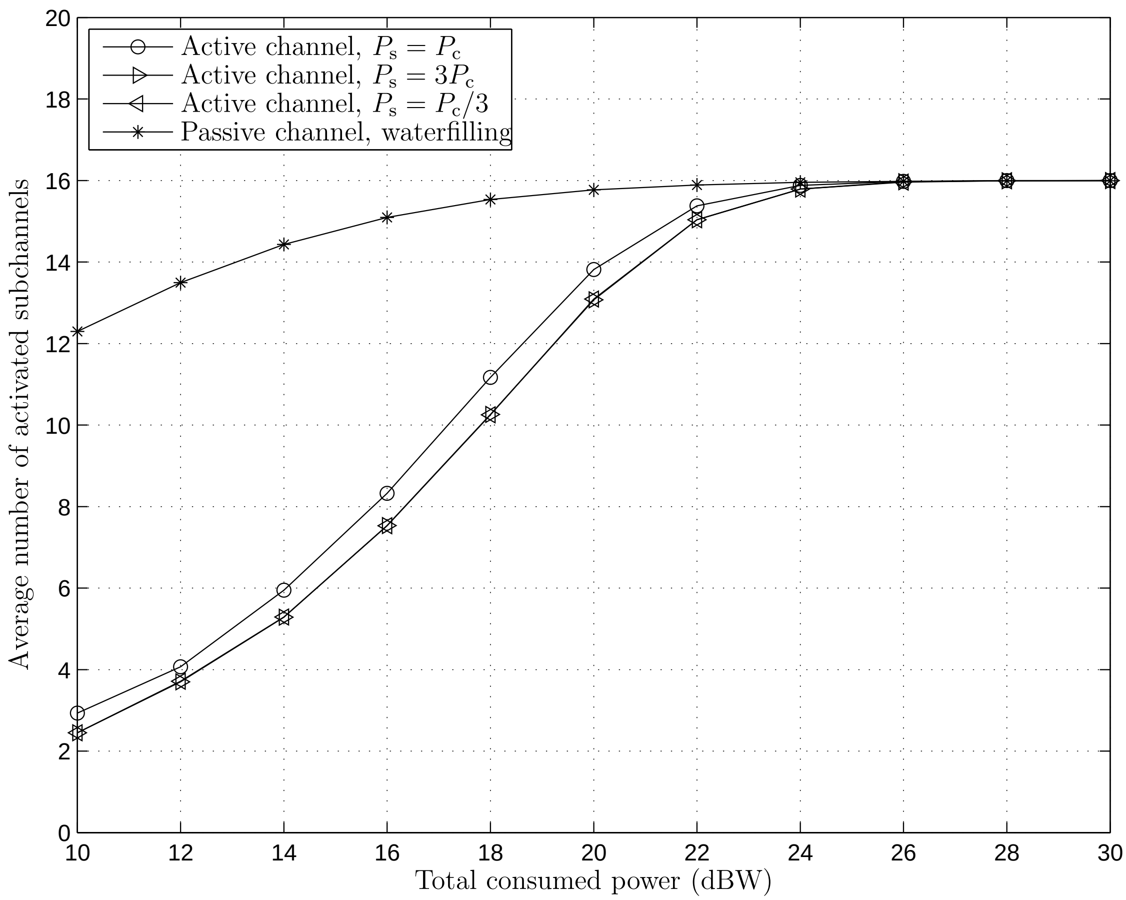

Figure 4.3: Average number of activated subchannels versus the total consumed power for both active and passive channels with $N = 16$ subchannels.

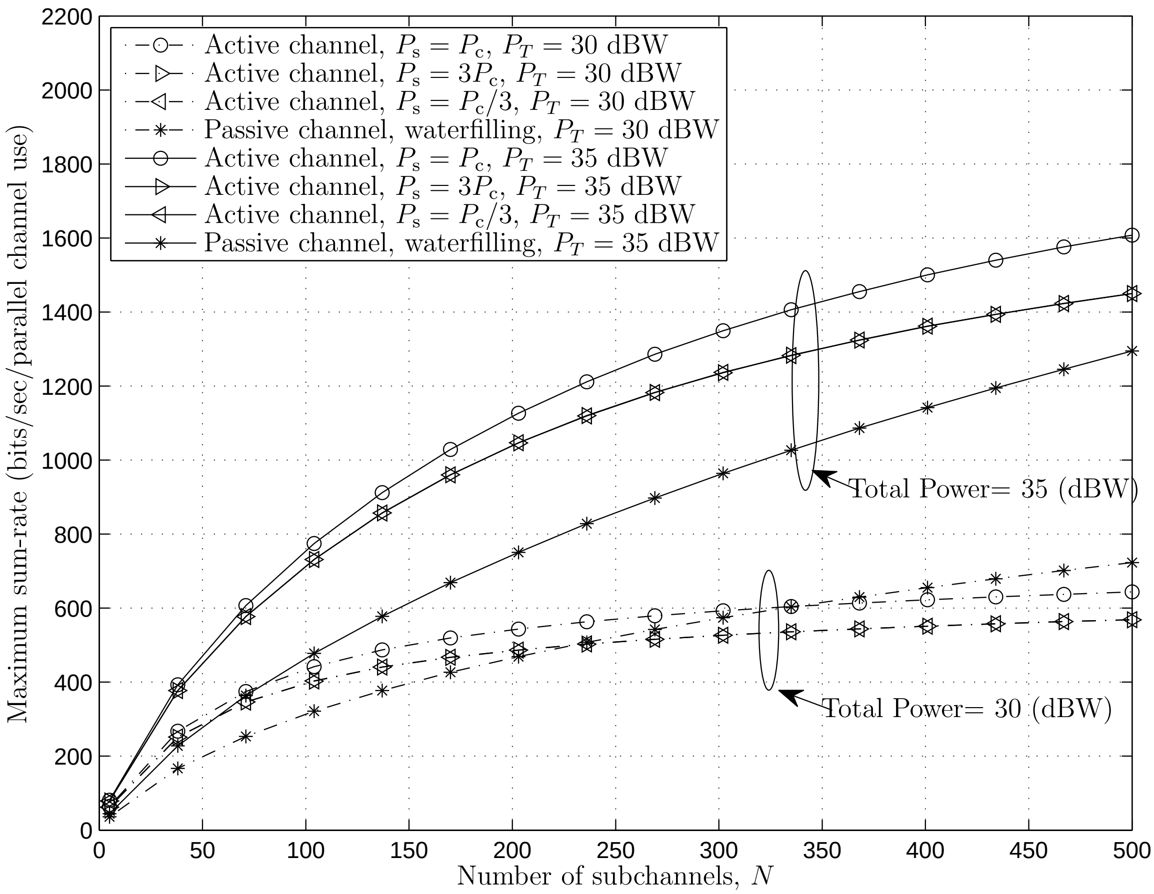

Figure 4.4: Maximum sum-rate versus number of subchannels, for active and passive channels, and for different values of total consumed power

Chapter 5

Conclusions And Future work

The maximum achievable sum-rate of a passive channel depends not only on the source transmit power but also on the quality of individual subchannels (i.e., on the power (or strength) of the parallel channel). This obvious observation has motivated us to study the problem of sum-rate maximization for a parallel channel where not only can the source transmit power be adjusted but also the channel itself can be properly designed or optimally adjusted to achieve a higher sum-rate compared to traditional passive channels. This channel where its energy can be controlled at a certain level refers to active channel. Throughout this dissertation, we studied the joint optimization of the channel energy and transmit power over a set of active parallel subchannels. The sum-rate maximization of such channels is investigated under two constraints, one on the energy of channel and one on the transmit power. This problem is investigated in two cases: equal and unequal noise power over different subchannels.

For equal subchannel noise powers, we proved that in order to achieve the maximum sum-rate, only a certain number of subchannels should be turned on and the rest of the subchannels should be switched off. This is in contrast with passive parallel channels with equal subchannel SNRs, where water-filling solution to the sum-rate maximization under a total source power constraint leads to an equal power allocation among *all* subchannels. The number of active subchannels is proven to depend on the product of the source and channel powers. We have also shown that when sum-rate is maximized, different active subchannels receive the same level of powers. We have also proven that if the total power available to the source and to the channel is limited, then in order to maximize the sum-rate via optimal power allocation to the source and to the active channel, half of the total available power should be allocated to the source and the remaining half should be allocated to the active channel.

The sum-rate maximization problem is further investigated for unequal subchannel noise powers. To solve this problem under source and channel power constraints, we used KKT conditions to obtain a computationally efficient algorithm for source and channel power allocation. We showed that how KKT conditions can be used to determine how many subchannels can be active for the source power constraint to be feasible. Indeed, we developed a computationally efficient method to determine the feasible numbers of active subchannels. Then, for any feasible number of active subchannels, we obtained the optimal source power allocation. In fact, we showed that for any feasible number of active channels, there are only zero, one, or two solutions for the optimal source power allocation. As such the optimal solution can be obtained by comparing a finite number of feasible points and choosing the best point which yields the best sum-rate performance. We showed that activating the whole subchannels does not necessarily lead to the maximum sum-rate. Moreover, it is proven that at the optimum, half of the total power should be assigned to the subchannels and the remaining half should be allocated to each subchannel at the source.

5.1 Future work

In this thesis, sum-rate maximization of active parallel channels is extensively discussed. This work can be further investigated in one- or two-way relay networks to:

- Derive the achievable rate region in OFDM-based two-way relay networks
- Achieve the maximum sum-rate of asynchronous relay networks
- Maximize the minimum rate in asynchronous relay networks

Appendix A

proof of non-convexity of (3.1.1)

To show that the optimization problem in (4.1.2) is not convex we show that the Hessian matrix of the objective function is not negative definite. To show this, the Hessian matrix can be written as

$$\mathrm{b}lkdg\left\{\begin{bmatrix}\frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i^2} & \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i\partial\tilde{h}_i}\\ \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i\partial\tilde{h}_i} & \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{h}_i^2}\end{bmatrix}\right\}_{i=1}^{N} \tag{A.0.1}$$

where $f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})$ denotes the cost function in (4.1.2). For $i = 1, 2, \cdots, n$, we can write

$$\frac{\partial f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i} = \frac{1}{\ln 2}\frac{\tilde{h}_i}{1+\tilde{p}_i\tilde{h}_i} \tag{A.0.2}$$

$$\frac{\partial f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{h}_i} = \frac{1}{\ln 2}\frac{\tilde{p}_i}{1+\tilde{p}_i\tilde{h}_i} \tag{A.0.3}$$

$$\tag{A.0.4}$$

and hence, we obtain

$$\frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i^2} = \frac{1}{\ln 2}\frac{-\tilde{h}_i^2}{(1+\tilde{p}_i\tilde{h}_i)^2} \tag{A.0.5}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{h}_i^2} = \frac{1}{\ln 2}\frac{-\tilde{p}_i^2}{(1+\tilde{p}_i\tilde{h}_i)^2} \tag{A.0.6}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial\tilde{p}_i\partial\tilde{h}_i} = \frac{1}{\ln 2}\frac{1}{(1+\tilde{p}_i\tilde{h}_i)^2} \tag{A.0.7}$$

we can write the determinant of ith block in (B.0.1) as

$$\left| \begin{bmatrix} \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial p_i^2} & \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial \tilde{p}_i \partial \tilde{h}_i} \\ \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial \tilde{p}_i \partial \tilde{h}_i} & \frac{\partial^2 f(\tilde{\mathbf{p}},\tilde{\mathbf{h}})}{\partial \tilde{h}_i^2} \end{bmatrix} \right| = \frac{1}{\ln 2} \frac{\tilde{p}_i^2 \tilde{h}_i^2 - 1}{(1 + \tilde{p}_i \tilde{h}_i)^2} \tag{A.0.8}$$

As can be seen from (B.0.7) that the ith block of the Hessian matrix is not always negative definite. As such, the Hessian matrix is not always negative definite. Hence, the cost function in (4.1.2) is not concave.

Appendix B

Proof of non-convexity of (4.1.2)

To show that the optimization problem in (4.1.4) is not convex we show that the Hessian matrix of the objective function is not negative definite. To show this, Hessian matrix can be written as

$$\mathrm{b}lkdg\left\{\begin{bmatrix} \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i^2} & \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i \partial h_i} \\ \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i \partial h_i} & \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial h_i^2} \end{bmatrix}\right\}_{i=1}^{N} \tag{B.0.1}$$

where $g(\mathbf{p}, \mathbf{h})$ denotes the cost function in (4.1.2). Noting that

$$\frac{\partial g(\mathbf{p},\mathbf{h})}{\partial p_i} = \frac{1}{\ln 2}\frac{\alpha_i h_i}{1+\alpha_i p_i h_i} \qquad i = 1, 2, \cdots, n, \tag{B.0.2}$$

$$\frac{\partial g(\mathbf{p},\mathbf{h})}{\partial h_i} = \frac{1}{\ln 2}\frac{\alpha_i p_i}{1+\alpha_i p_i h_i} \qquad i = 1, 2, \cdots, n, \tag{B.0.3}$$

we can have

$$\frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i^2} = \frac{1}{\ln 2}\frac{-\alpha_i h_i^2}{(1+\alpha_i p_i h_i)^2} \qquad i = 1, 2, \cdots, n, \tag{B.0.4}$$

$$\frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial h_i^2} = \frac{1}{\ln 2}\frac{-\alpha_i p_i^2}{(1+\alpha_i p_i h_i)^2} \qquad i = 1, 2, \cdots, n, \tag{B.0.5}$$

$$\frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i \partial h_i} = \frac{1}{\ln 2}\frac{\alpha_i}{(1+\alpha_i p_i h_i)^2} \qquad i = 1, 2, \cdots, n\,. \tag{B.0.6}$$

We can write the determinant of ith block in (B.0.1) as

$$\mathrm{d}et \begin{bmatrix} \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i^2} & \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i \partial h_i} \\ \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial p_i \partial h_i} & \frac{\partial^2 g(\mathbf{p},\mathbf{h})}{\partial h_i^2} \end{bmatrix} = \frac{1}{\ln 2} \frac{\alpha_i^2 p_i^2 h_i^2 - 1}{(1 + \alpha_i p_i h_i)^2} \tag{B.0.7}$$

As can be seen from (B.0.7), the ith block of the Hessian matrix is not always negative definite which means that the optimization problem (4.1.4) is non-convex.

Appendix C

Proof of Lemma

The proofs of parts (a) and (b) consist of three steps: Step 1:) we show that, for $n = 2, ..., N$, the function $\sum_{i=2}^{n} p_i(p_1)$, has a unique minimizer at $p_1 = \frac{1}{\sqrt{\kappa\alpha_1}}$, Step 2:) we prove that the function $\sum_{i=2}^{n} p_i(p_1)$ is convex for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, and Step 3) using the previous two steps, we prove the uniqueness of the minimizer of $p_1 + \sum_{i=2}^{n} p_i(p_1)$.

Step 1) The uniqueness of the minimizer of $\sum_{i=2}^{n} p_i(p_1)$: To prove that the function $\sum_{i=2}^{n} p_i(p_1)$ has a unique minimizer, we show that for any $i \in 2, \cdots, N$, the function $p_i(p_1)$ has the same unique minimizer. To show this, we differentiate $p_i(p_1)$, given as in (4.2.22), with respect p_1 as

$$\frac{\partial p_i(p_1)}{\partial p_1} = \frac{(\alpha_1\, \kappa\, \mathrm{p_1}^2 - 1)\,\left(\sqrt{\Delta_i} + b_i\right)}{2\,\alpha_1\,\kappa\,\mathrm{p_1}^2\,\sqrt{\Delta_i}} \tag{C.0.1}$$

where $\Delta_i = b_i^2 - 4\kappa\alpha_i\alpha_1^2 p_1^2$ and $b_i = (\alpha_i + \kappa\alpha_i\alpha_1 p_1^2)$. Equating the derivative in (C.0.1) to zero yields

$$p_1 = \frac{1}{\sqrt{\kappa\alpha_1}}\,. \tag{C.0.2}$$

It follows from (C.0.1) that for any i, $\frac{\partial p_i(p_1)}{\partial p_1} > 0$, when $p_1 \in (\frac{1}{\sqrt{\kappa\alpha_1}}, \infty)$ and $\frac{\partial p_i(p_1)}{\partial p_1} \leq 0$ for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, and hence, we can write

$$\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} \begin{cases} > 0\,, & \text{for } p_1 \in (\frac{1}{\sqrt{\kappa\alpha_1}}, \infty) \\ = 0\,, & \text{for } p_1 = \frac{1}{\sqrt{\kappa\alpha_1}} \\ < 0\,, & \text{for } p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}) \end{cases} . \tag{C.0.3}$$

The proof of Step 1 is now complete.

Step 2) Convexity of $\sum_{i=2}^{n} p_i(p_1)$, when $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$: To show that the function $\sum_{i=2}^{n} p_i(p_1)$ is convex for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, it is sufficient to show that for any i, the function $p_i(p_1)$ is convex in this interval, or equivalently, that the second derivative of $p_i(p_1)$ is positive for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$. The second derivative of $p_i(p_1)$ can be obtained as

$$\frac{\partial^2 p_i(p_1)}{\partial p_1^2} = \frac{(\Delta_i)^{\frac{3}{2}} + b^3 - 6\kappa\alpha_1^2\alpha_i^2 p_1^2 - 2\kappa^3\alpha_1^4\alpha_i^2 p_1^6}{\alpha_1\, \kappa\, \mathrm{p_1}^3\, (\Delta_i)^{\frac{3}{2}}} . \tag{C.0.4}$$

Let us rewrite (C.0.4) as

$$\frac{\partial^2 p_i(p_1)}{\partial p_1^2} = \frac{(\Delta_i)^{\frac{3}{2}} + (\alpha_i + \alpha_i(\kappa\alpha_1 p_1^2))^3 - 6\alpha_1\alpha_i^2(\kappa\alpha_1 p_1^2) - 2\alpha_1\alpha_i^2(\kappa\alpha_1 p_1^2)^3}{p_1(\kappa\alpha_1 p_1^2)\,(\Delta_i)^{\frac{3}{2}}} . \tag{C.0.5}$$

Note that for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, there exists $0 < \epsilon < 1$ such that $\alpha_1\, \kappa\, \mathrm{p_1}^2 = 1 - \epsilon$. Hence, we can write

$$\frac{\partial^2 p_i(p_1)}{\partial p_1^2} = \frac{(\Delta_i)^{\frac{3}{2}} + (\alpha_i + \alpha_i(1-\epsilon))^3 - 6\alpha_1\alpha_i^2(1-\epsilon) - 2\alpha_1\alpha_i^2(1-\epsilon)^3}{p_1(1-\epsilon)\,(\Delta_i)^{\frac{3}{2}}} . \tag{C.0.6}$$

The denominator in (C.0.6) is positive for any $0 < \epsilon < 1$. The numerator can be written as

$$\Delta_i^{\frac{3}{2}} + 8\alpha_i^3 - \alpha_i^3\epsilon^3 + 6\alpha_i^3\epsilon^2 - 12\alpha_i^3\epsilon - 6\alpha_1\alpha_i^2 + 6\alpha_1\alpha_i^2\epsilon - 2\alpha_1\alpha_i^2 + 2\alpha_1\alpha_i^2\epsilon^3 + 6\alpha_1\alpha_i^2\epsilon - 6\alpha_1\alpha_i^2\epsilon^2$$
$$= \Delta_i^{\frac{3}{2}} + \alpha_1\alpha_i^2\epsilon^3 + 8\alpha_i^2(\alpha_i - \alpha_1) + 6\alpha_i^2\epsilon^2(\alpha_i - \alpha_1) - 12\alpha_i^2\epsilon(\alpha_i - \alpha_1) - \alpha_i^2\epsilon^3(\alpha_i - \alpha_1)$$
$$= \Delta_i^{\frac{3}{2}} + \alpha_1\alpha_i^2\epsilon^3 + \alpha_i^2(\alpha_i - \alpha_1)(-\epsilon^3 + 6\epsilon^2 - 12\epsilon + 8)$$
$$= \Delta_i^{\frac{3}{2}} + \alpha_1\alpha_i^2\epsilon^3 + \alpha_i^2(\alpha_i - \alpha_1)(2 - \epsilon)^3 \,. \qquad \text{(C.0.7)}$$

Using the fact that $\alpha_i > \alpha_1$ and that $\Delta_i > 0$ (see (4.2.24)), it can be easily seen that (C.0.7) is positive for any $0 < \epsilon < 1$. Therefore, $\frac{\partial^2 p_i(p_1)}{\partial p_1^2} > 0$ for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, and consequently,

$$\sum_{i=2}^{n} \frac{\partial^2 p_i(p_1)}{\partial p_1^2} > 0 \qquad for \qquad p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]. \qquad \text{(C.0.8)}$$

Indeed, (C.0.8) states that $\sum_{i=2}^{n} p_i(p_1)$ is convex for $p_1 \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}]$. This completes the proof of Step 2.

Step 3) The uniqueness of the minimizer of $f^n(p_1) = p_1 + \sum_{i=2}^{n} p_i(p_1)$: In order to prove the uniqueness of the minimizer of $f^n(p_1)$, we need to show that for any n, the solution to the following equation:

$$\frac{\partial f^n(p_1)}{\partial p_1} = 1 + \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} = 0, \qquad \text{(C.0.9)}$$

or, equivalently, to this one:

$$\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} = -1, \qquad n = 2, 3, ..., N. \qquad \text{(C.0.10)}$$

is unique. From (C.0.3) of Step 1, we observe that the solution to (C.0.10) is located in the interval of $(0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, because out of this interval, $\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1}$ is positive, and hence, (C.0.10) (or (C.0.9)) cannot be satisfied. Using this observation along with the convexity of $\sum_{i=2}^{n} p_i(p_1)$ in the interval $(0, \frac{1}{\sqrt{\kappa\alpha_1}}]$, as shown in (C.0.8) of Step 2, we conclude that the solution to (C.0.10) is unique. Therefore, the minimizer of $f^n(p_1)$, denoted by $p_{1,n}^{\min}$ is unique and

$$p_{1,n}^{\min} \in (0, \frac{1}{\sqrt{\kappa\alpha_1}}] \,. \tag{C.0.11}$$

The proofs of both parts (a) and (b) are now complete.

To prove the first statement of part (c), we note that in the interval $(0, p_{1,n}^{\min})$ the function $\sum_{i=2}^{n} p_i(p_1)$ is convex (Step 3). The convexity of the function $\sum_{i=2}^{n} p_i(p_1)$ implies that its derivative $\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1}$ is monotonically increasing. Hence, for any $p_1 \in (0, p_{1,n}^{\min})$, we have

$$\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} < \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1}\Big|_{p_1 = p_{1,n}^{\min}} = -1. \tag{C.0.12}$$

where the equality follows from the fact that $p_{1,n}^{\min}$ is the global minimizer of $f^n(p_1)$, i.e., (C.0.10) holds true at $p_1 = p_{1,n}^{\min}$. Using (C.0.12), we can write

$$\frac{\partial f^n(p_1)}{\partial p_1} = 1 + \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} < 0, \quad for \quad p_1 \in (0, p_{1,n}^{\min}). \tag{C.0.13}$$

It follows from (C.0.13) that $f^n(p_1)$ is monotonically decreasing for $p_1 \in (0, p_{1,n}^{\min})$. This completes the proof of the first statement of part (c).

We now prove the second statement of part (c). Using (C.0.3), we can write[1]

$$\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} \leq 0 \quad for \quad p_1 \in [p_{1,n}^{\min}, \frac{1}{\sqrt{\kappa\alpha_1}}) \tag{C.0.14}$$

[1]Note that according to part (a), $p_{1,n}^{\min} < \frac{1}{\sqrt{\kappa\alpha_1}}$ holds true.

In light of (C.0.8), the function $\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1}$ is monotonically increasing when $p_1 \in (p_{1,n}^{\min}, \frac{1}{\sqrt{\kappa\alpha_1}})$. Hence, for $p_1 \in (p_{1,n}^{\min}, \frac{1}{\sqrt{\kappa\alpha_1}})$, we can write

$$\sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} > \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1}\Big|_{p_1 = p_{1,n}^{\min}} > -1 \tag{C.0.15}$$

Hence, for $p_1 \in (p_{1,n}^{\min}, \frac{1}{\sqrt{\kappa\alpha_1}})$, we can write

$$\frac{\partial f^n(p_1)}{\partial p_1} = 1 + \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} > 0\,. \tag{C.0.16}$$

Therefore, $f^n(p_1)$ is monotonically increasing in the interval $(p_{1,n}^{\min}, \frac{1}{\sqrt{\kappa\alpha_1}})$.

When $p_1 \in [\frac{1}{\sqrt{\kappa\alpha_1}}, P_{\mathrm{s}}]$, using (C.0.3), we can write

$$\frac{\partial f^n(p_1)}{\partial p_1} = 1 + \sum_{i=2}^{n} \frac{\partial p_i(p_1)}{\partial p_1} > 0\,. \tag{C.0.17}$$

Using (C.0.17), we conclude that $f^n(p_1)$ is monotonically increasing in the interval $[\frac{1}{\sqrt{\kappa\alpha_1}}, P_{\mathrm{s}}]$. This completes the proof of the second statement of part (c). ■

The proof is now complete.

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