Image Noise Removal in Nakagami Fading Channels via Bayesian Estimator

Xu Huang
School of Information SCiences and Engineering
University of Canberra, ACT 2601
Canberra, Australia
Xu.Huang@canberra.edu.au

A.C. Madoc
Center for Actuarial Studies, Department of Economics,
University of Melbourne, VIC 3051, Australia
a_c_m81@hotmail.com

Dharmendra Sharma
School of Information SCiences and Engineering
University of Canberra, ACT 2601
Canberra, Australia
Dharmendra.Sharma@canberra.edu.au

***Abstract*—A maximum likelihood for Bayesian estimator based on α-stable was discussed in our previous papers. It is in terms of closer to a realistic situation, and unlike previous methods used for Bayesian estimator, for the case discussed here it is not necessary to know the variance of the noise. The Bayesian estimator here is based on in a Nakagami fading channel. Our previous research results has been extended to that Bayesian estimator that we investigated is still working well for the image noise removal in Nakagami fading channels. As an example, an improved Bayesian estimator (soft and hard threshold methods), is illustrated in our discussion.**

Keywords—digital image, image noise removal, Bayesian estimator, wavelet, wireless communications, Nakagami m- fading channels.

I. Introduction

The Nakagami *m*-distribution has founded many applications in technical sciences. It has been shown by extensive empirical measurement that this distribution is an appropriate model for wireless links [1-3]. A wide variety of fading effect can be modeled as Nakagami fading with different m parameters, including Rayleigh and one-side Gaussian fading as special cases when m equals to 1 and 0.5, respectively. Nakagami distribution is also suitable for modeling the output statistics of diversity combining system that are employed extensively to mitigate multipath faded effect. It is obvious that generation of correlated Nakagami fading channels is therefore an essential issue for a laboratory test of wireless systems or subsystems to operate in such a fading environment. Some papers have shown it is possible to have flexible algorithm with the ability to generate correlated Nakagami fading branches with arbitrary fading parameters and correlations [2,3].

Wireless telephones are not only convenient but are also providing flexibility and versatility. According to the nature of a particular application, wireless communications can be used in home-based and industrial systems or in commercial and military environments. One of major proposal of this paper is that what will happen when a image is communicated by a Nakagami-m distribution fading channels and whether the previous methods works for image noises removal via Bayesian estimator [4-7].

It is well known that noise degrades the performance of any image compression algorithm. In many cases, images degraded even before they are encoded. It is obvious that linear filtering techniques used in many image-processing applications, are attractive due to their mathematical simplicity, and efficiency in the presence of additive Gaussian noise. However, they also blur sharp edges, make some distortions of lines and fine image details, less effectively remove tailed noise, and poorly treat the presence of signal-dependent noise. For example, emission and transmission tomography images are usually contaminated by quantum noise, which is Poisson noise. Unlike additive Gaussian noise, Poisson noise is signal-dependent, and separating signal from noise is difficult. Several groups have discussed that wavelet sub band coefficients have highly non-Gaussian statistics and the general class of α-stable distributions has also been shown to accurately model heavy-tailed noise [8-10].

Wavelet transform is a powerful tool for recovering signals from noise and has been of considerably interest [11-13]. In fact, wavelet theory combines many existing concepts into a global framework and hence becomes a powerful tool for several domains of application.

Donoho gives some minimum thresholds for several threshold schemes, titled "universal thresholds" [14]. These explicitly depend on the standard deviation of noise, where the standard deviation is assumed to be known. In practice, the standard deviation can be readily estimated using the methods discussed in [15], [16]. For some applications the optimal threshold can be computed. An approach different from "universal thresholds" is presented by Nason [17], in which cross-validation is used. Two approaches to cross validation are used, namely ordinary cross validation (OCV) and generalized cross validation (GCV): each is used to minimize the least-squares error between the original (which is the unknown value) function and its estimate based on the noisy observation.

Modeling the statistics of raw images is a challenging task because of the high dimensionality of the signal and the complexity of statistical structures that are prevalent. Numerous papers discuss modeling the statistics of raw images, including Bayesian processing assuming proper modeling of the prior probability density function of the signal, but they dealt with the Gaussian noise, or with the symmetric stochastic distributions.

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In this paper it is carefully discussed that a wavelet-based maximum likelihood for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images from images contaminated by Nakagami fading channels that simulated by the method [2, 3].

As an example, a color image and its image contaminated by Poisson noise will be shown using the discussed method.

II. ALPHA-STABLE DISTRIBUTIONS AND LOG LIKELIHOOD

It is well known that the symmetric alpha-stable distribution ($S\alpha S$) distribution is defined by its characteristic function:

$$\phi(\omega) = \exp(j\delta\omega - \gamma|\omega|^{\alpha}), \tag{1}$$

The parameters α, γ, and δ describe completely a $S\alpha S$ distribution. The characteristic exponent α controls the heaviness of the tails of the stable density. α can take values in (0,2]; while α = 1 and 2 are the Cauchy and Gaussian cases respectively. There is not closed-form expression known for the general $S\alpha S$ probability density function (PDF). Thus, it is useful when using the principle of maximum likelihood estimation. The dispersion parameter γ (γ >0) refers to the spread of the PDF. The location parameter δ is analogous to the mean of the PDF, which, for our following discussion, will be the same assumption as that in [8].

If a variable $\hat{\theta}$ is unbiased it follows that

$$E(\hat{\theta} - \theta) = 0 \tag{2}$$

which can be expressed as:

$$\int_{-\infty}^{\infty}\cdots\int(\hat{\theta} - \theta) f_{\vec{\mathbf{x}};\theta}(\vec{\mathbf{x}};\theta) d\vec{\mathbf{x}} = 0 \tag{3}$$

where $\vec{x}(\xi) = [x_1(\xi), x_2(\xi), \ldots, x_N(\xi)]^T$ and $f_{\vec{x};\theta}(\vec{x};\theta)$ is the joint density of $\vec{x}(\xi)$, which depends on a fixed but unknown parameter. Following [15-17] we have

$$\mathrm{var}(\hat{\theta}) \geq -\frac{1}{E\{\partial^2 \ln f_{\vec{\mathbf{x}};\theta}(\vec{\mathbf{x}};\theta)/\partial\theta^2\}} \tag{4}$$

The function $\ln f_{\vec{x};\theta}(\vec{x};\theta)$ is well known as the "*log likelihood"* function of θ (LLF). Its maximum likelihood estimate can be obtained from the equation:

$$\frac{\partial \ln f_{\vec{x};\theta}(\vec{x};\theta)}{\partial\theta} = 0 \tag{5}$$

The first order of differential log likelihood function with respect to θ is called the maximum likelihood (ML) estimate. If the efficient estimate does not exist, then the ML estimate will not achieve the lower bound and hence it is difficult to ascertain how closely the variance of any estimate will approach the bound.

It is noted that the value of about 1.5 is strongly recommended if there is no information about α due to the 2nd order simulations of the LLF for an alpha-stable [7].

III. NAKAGAMI FADING CHANNEL AND WAVE-BASED BAYESIAN ESTIMATOR

If we take the probability density of θ as $p(\theta)$; and the posterior density function as $f(\theta | x_1, \ldots, x_n)$, then the updated probability density function of θ is as follows:

$$f(\theta | x_1, \ldots, x_n) = \frac{f(\theta, x_1, \ldots, x_n)}{f(x_1 \ldots, x_n)}$$

$$= \frac{p(\theta) f(x_1, \ldots, x_n | \theta)}{\int f(x_1, \ldots, x_n | \theta) p(\theta) d\theta} \tag{6}$$

If we estimate the parameters of the prior distributions of the signal s and noise q components of the wavelet coefficients c, we may use the parameters to form the prior PDFs of $P_s(s)$ and $P_q(q)$, hence the input/output relationship can be established by the Bayesian estimator, namely, let input/output of the Bayesian estimator = *BE*, we have:

$$BE = \frac{\int P_q(q) P_s(s) s ds}{\int P_q(q) P_s(s) ds} \tag{7}$$

$P_s(s)$ is the prior PDF of the signal component of the wavelet coefficients of the ultrasound image and $P_q(q)$ is the PDF of the wavelet coefficients corresponding to the noise.

In order to be able to construct the Bayesian processor in (7), we must estimate the parameters of the prior distributions of the signal (s) and noise (q) components of the wavelet coefficients (d). Then, we use the parameters to obtain the two prior PDFs $P_q(q)$ and $P_s(s)$ and the nonlinear input-output relationship *BE*.

Consider the moments of the Nakagami distribution NK(m,Ω). Following [2], we have the probability density function (PDF) of NK(m,Ω) as below:

$$E[z^r] = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right) \int_0^{\infty} z^{2m+r-1} \exp\left(-\frac{m}{\Omega} z^2\right) dz \tag{8}$$

and we also have

$$E[z^r] = \frac{\Gamma(m + \frac{r}{2})}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{r/2} \tag{9}$$

The Gamma function Γ(m) is defined by

$$\Gamma(m) = \int_0^{\infty} x^{m-1} e^{-x} dx \tag{10}$$

The following methodology is that following the way that introduced in [2] to obtain a Nakagami source, then build our BE by equation (7). We, in the next section, take an example to illustrate the processing for noise removal.

Figure 1 shows our BE in terms of various m values of Nakagami fading channels. We follow the equation (7) and pick four different m values, namely m = 0.6, 1.0, 2.5 and 15. The corresponding Nakagami PDFs are shown in Figure 2.

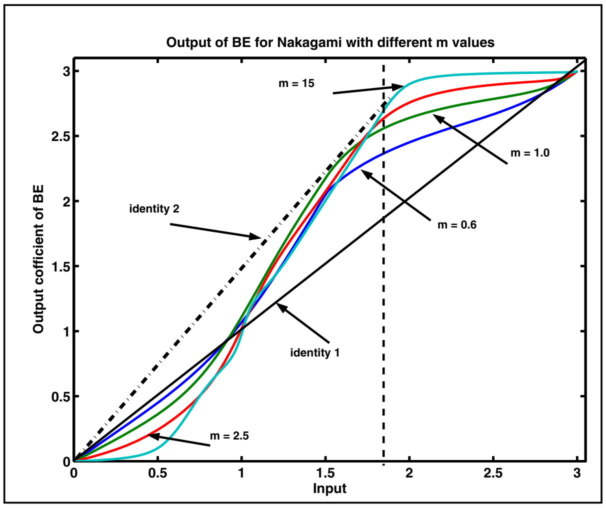

Figure 1: The output of BE for different Nakagami m values, namely m = 0.6, 1.0, 2.5 and 15.

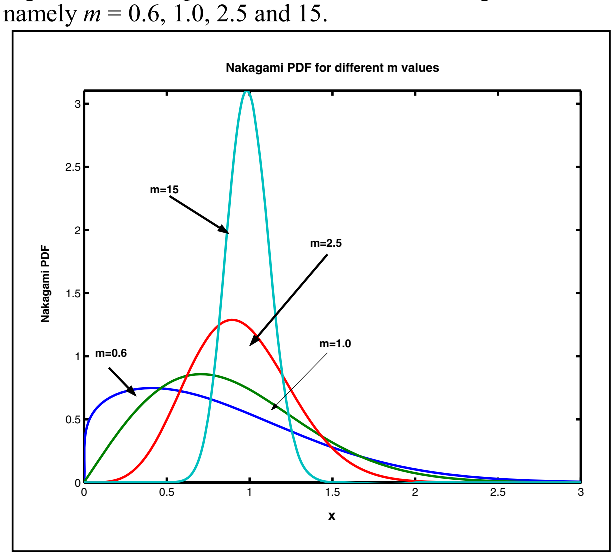

Figure 2: The PDFs of Nakagami fading channels for different m values shown in Figure 1.

Figure 2 clearly shows that the m values are associated with the PDFs of Nakagami fading channels. From Figure 1 we can see that the different m values make the output of BE are different. It is well know that the smaller value is the more fading effects would be for the Nakagami fading channels. If we take the identity 2 in Figure 1, we may observe that the curve makes the output of the BE "soft threshold" like and the cure corresponding to m = 0.6 closer to the "hard threshold". In fact the curves of the output of the BE based on Nakagami with different m values for the fading channels are different, in terms of shapes, from that in our previous results [4-7], which is inspected. But the tendencies of those curves of the BE for the different m values are similar to those previous results in comparison of that in [4-7]. The following example will support this belief.

IV. AN EXAMPLE

We first to have a Nakagami fading channels resources built by the methodology introduced in [2] (or [3]). Hence, it is it is expected to generate an n-by-1 correlated Nakagami vector z with fading parameter m and covariance matrix $\boldsymbol{R}_z$.

When we make measurements, we have no information about the noise value of the image we obtained. The only information one may have is from experience in judgment of the noise level, which becomes the outline of the denoising strategy.

(a) (b)

Figure 3: An image of the "Arms" of a University: (a) original (b) the contaminated (a) by Nakagami fading channel.

The Nakagami fading channel contaminate the image shown in Figure 1 as shown in Figure 2 (a), where the m deliberately chose 2.5 but just we knew it not the BE. As we previously described that in our papers [4-7] we used the so-called blind "noise removal" to denoise the contaminated image by Nakagami fading channel via the designed BE as shown in Figure 1. The result of the blind noise removal is shown in Figure 4.

(a) (b)

Figure 4: (a): the output of BE by the "blind noise removal" for the Figure 2; (b) the result of for the "matched BE" for the Figure 2

If we know some information about the natures of Nakagami fading channel, which is possible even we did not show this in this paper. Note that we can estimate the m value we are facing for the particular situation via the methods discussed in [18-20]. Then we can apply a almost matched BE to the contaminated image by the Nakagami fading channel. For this example, we use the matched BE for the m =

2.5 and the result is shown in Figure 4 (b), which is better than Figure 4 (a) shown by Table 1.

TABLE 1: THE RESULTS FOR THE NOISE REMOVAL OF THE CONTAMINATED BY THE NAKAGAMI FADING CHANNEL VIA "BLIND NOISE REMOVAL" AND "MATCHED NOISE REMOVAL".

METHOD	1	2	3	4	5
S/MSE	14.21	14.11	13.51	14.87	15.68

Note: Comparison of denoising results with BE in signal to mean square error (S/MSE) in dB. Here 1= soft thresholding; 2 = Hard thresholding; 3 = Homomorphic Wiener; 4 = BE (blind), 5 = BE (matched).

In order to make a comparison, a noise removal by traditional hard thresholding is shown in Figure 5.

Figure 5: The noise removal of the Figure 2 by hard thresholding.

V. CONCLUSION

The technique using the wavelet-based Bayesian estimator has been extended to Nakagami fading channels with different m parameters. The statistician's Bayesian estimator theory is used not only to simplify the selection of parameters but also in some situations to provide more precise images than other methods. It is noted that if we use the so-called matched BE with an estimation of m parameter, the result would be most encouraging as shown in this paper.

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