A novel approach for the identification of critical nodes and transmission lines for mitigating voltage instability in power networks

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Voltage collapse is a major issue combating the effectiveness and optimal operation of modern power systems in recent times. This has been a great threat to the security and the reliability of a modern power system and has been a growing concern to power system engineers, researchers and the utilities recently. A prompt identification of the sets of transmission lines whose outage could lead to a cascading failure and the sets of nodes where voltage collapse could erupt is therefore a vital issue for a reliable and secure power system operation. An alternative approach to solving these problems is therefore presented in this paper. The problem is viewed from the graph-theoretical perspective, considering the topological properties of the networks. Application of the fundamental circuit theory is employed and the Bus-to-Line matrix (BLM) is formulated. This matrix provides insights about the interconnections of the components within the network. This valuable information is captured and used for identifying the critical elements where a suitable location for reactive power support could be placed to avoid voltage collapse of the network. The effectiveness of the proposed approach is tested using a simple 10-bus power network. The results obtained are compared with that obtained from the existing approaches. The results obtained show a strong correlation and agreement and proved the efficiency of the proposed approach.

Keywords: voltage instability, reliability, security, cascading failure, Bus-to-Line matrix, critical nodes

Introduction

The operation of modern power systems, in which the integrity of the network, in terms of reliability and security will not be compromised, is a complex task. In the quest for higher security and reliability in the operation of modern power systems, modern power systems are now becoming highly interconnected, more complex and heavily loaded. Consequently, modern power systems are now becoming heavily stressed and vulnerable to voltage instability. The aftermath is the frequent occurrence of large-scale blackouts as a result of voltage collapse within the networks. This problem is usually experienced in topologically weak power networks, which are characterized by the shortage of reactive power within the networks (Sikiru, 2012). For voltage magnitude at every bus of typical power systems to be maintained and controllable at all times, there is a need for identifying a suitable location where reactive power devices could be located within the network (Sikiru, 2011; Dhadbanjan, 2009). Location of reactive po\|wer supports in maintaining an acceptable voltage limits within power system networks (Khan, 2013; Sikiru, 2011) is therefore an important problem to be solved in modern power systems.

The risk associated with this challenge is inevitable, which could be enormous and disastrous. This is why a power system should be well planned and reliably operated such that customers are not affected by most credible and critical contingencies that could result in total blackout within the network **(**Yamashita et al, 2008). Yamashita et al 2008) discussed extensively, the economic impacts of voltage collapse on both the developed and developing nations**.** The risk associated with this impact could be totally avoided or substantially minimized by quick identification of the insecure links and buses where voltage collapse could erupt within the networks (Alayande et al 2015; Taylor, 2011). In other words, prompt identification of critical elements within a modern power system is highly desirable to control and mitigate the frequent cascading failure and hence voltage collapse whose aftermath could be disastrous. Voltage collapse in power systems has been a serious concern to most power system utilities around the world in recent times. This challenge is usually accompanied by initial gradual and continuous decrease in the system voltage before it drops rapidly (Adebayo et al, 2018). Other contributing factors include stress power network which results from the network congestion or high real power loading of the network, the shortage of reactive power supports in the network, transformer tap changing characteristics etc. Several large-scale blackouts have been experienced in most countries around the world and this has raised a great concern among power system researchers in recent times (Adebayo et al, 2018; Yamashita et al, 2008).

One of the ways through which the solution can be provided is by reinforcing the network through transmission expansion planning. The main interest of the transmission owners and system operators is to ensure the system is voltage collapse-free but with the minimum investment cost. This approach however requires a huge amount of capital and this may limit its application in future power network. Moreover, transmission expansion planning requires line switching or additions, which greatly adds to the computational complexity of the problem. Consequently, obtaining solution to the problem, in a large modern power system, is therefore hampered. Hence, application of this method in solving the problem may be impractical and therefore not well acknowledged in modern power systems without additional computational tools.

Another solution to the problem is the placement of reactive power supports such as reactors or FACTs devices at various locations within the system where their influence would be most effective. Traditionally, generating stations are usually located at a far distance from the load centres due to health and developmental reasons. However, since it is not an easy task to transport reactive power over a long distance, there is a need to find an alternative way of compensating the transmission lines adequately in order to increase the efficiency of the transmission network (Taylor, 2011). In order to reduce the overall investment cost of installing VAR sources, it is important to install them in such locations, where the least VAR amount is needed to ensure system voltage security against all severe contingencies. In addition, optimal design and operation of power systems is usually faced with the determination of suitable locations of reactive power devices within the networks. The basic AC power-flow equations on which most of the power system solutions depend are mathematically complex and nonlinear in nature. This makes convergence to the real solution practically impossible in most large-sized and radial power networks. This, however, compounded the challenge of identifying a suitable location for the reactive power resources such as SVC, Capacitor banks, etc. Although, this approach is found helpful in resolving the issue, to some extent, its main bottleneck is how to quickly identify suitable locations where the effect of such reactive power supports could be most effective. Conventionally, most authors, in the open literature, have formulated the problem as a nonlinear optimization problem, which are subjected to certain system constraints (Ginarsa et al., 2013). The main challenges in this case are indeed non-convexity of the models and computational difficulties in terms of time and computer memory space. This has posed a lot of challenging tasks and concern for power system operators and planners. A quick and an efficient methodology, for identifying such suitable locations where the reactive power supports could be placed in order to prevent voltage collapse, is therefore, of utmost importance. Moreover, effective identification of locations where reactive power support could be placed requires that all the influential links and nodes that could lead to violations of network integrity in terms of reliability and security be quickly identified and monitored (Ginarsa et al., 2013; Ziari et al.,2010).

This challenge has attracted the attention of several researchers in recent times. For instance, Mustafa et al. (2005) considered an improved methodology, for the allocation of individual generator usage, in a deregulated power system. This approach is, however, time consuming. Moreso, it is solely dependent on the solved power-flow, whose solution may not be guaranteed and could only be obtained (if exists) through iterative means. This problem is more pronounced in a large ill-conditioned power system. More recently, Mafizul Islam1et al. (2013) presented and approach which focused on the influence and management of reactive power on the power system stability and security in a restructured or deregulated power system. An approach for solving reactive power cost allocation problem, based on power tracing principle is proposed by Mala De et al. (2012). This technique solves the problem of bidirectional reactive power-flow within power networks. The authors also considered reactive power supplied by line charging capacitance as a separate reactive source with their associated reactive load and loss allocated to it. Total system reactive demand and loss were first allocated to all the sources within the system such as generators, synchronous condensers, capacitors and line charging capacitance. A quadratic reactive cost function is adopted for the generators while all other sources are assumed to have a constant cost per unit MVAR supplied. Khalid et al. (2008) proposed a novel method for identifying the reactive power transfer between generators and load using modified Kirchhoff's laws. The bus admittance matrix is partitioned to decompose the current of the load buses as a function of generator current and voltage based on the solved power-flow solution and the network parameters. The decomposed demand reactive power is obtained from the decomposed currents. This work also creates an appropriate artificial neural network (ANN) for practical 25-bus equivalent power system of south Malaysia for testing the effectiveness of the ANN output compared to that of the modified nodal equation method. The results show that ANN output provides more promising results in terms of accuracy. However, the application of this approach is limited and may not be suitable for large power network due to convergence issue and large computational time associated with it.

A graph theory-based approach in conjunction with the power-flow analysis of the network is proposed by Wu et al. (2000) to solve an allocation problem in a deregulated economy. This approach presented promising results when applied to determine the allocation of individual network participants (generators and loads) to the flows on the transmission lines as well as the active power transfer between individual network participants in a deregulated economy. Although the approach has been shown to be effective, its capability in the identification of influential network elements without running the repetitive and time-consuming power-flow analysis, which is iterative-based approach, has not been investigated in a holistic view.

The contributions offered by this approach to the active stream of research are two folds. Firstly, the computational complexity involved in the traditional approaches is avoided in this approach. An attempt is, therefore, made to explore the benefit derived from the inherent structure of power networks and the interconnections that exist between the network components, without the need for solving the time-consuming power-flow equations, in identifying the influential nodes and lines within a power system network. Secondly, this approach is suitable for practical applications due to its simplicity in identifying the influential nodes and lines within the networks. This will help the system operators in making faster decisions for voltage collapse prevention before it actually occurs, thereby protecting the integrity of the networks, most especially, during critical outages.

The remainder of the paper is organized as follows. Section 2 presents the application of graph theory to a power network and how the advantages inherent in the graph-theoretical approach could be explored in solving power system problems. The description of a case study used to test the effectiveness of the proposed approach is presented in section 3. Section 4 presents the results, the discussions as well as the comparison of the results obtained with the results obtained using the existing methods while the study is concluded in section 5.

Graph -Theoretical Approach in power systems

In recent times, graph theory is now becoming an interesting approach to solving most engineering problems. Its applications have been widely deployed and well researched in most fields of engineering and sciences such as transportation engineering problem (Alayande et al., 2017). In this paper, the role of the graph theory, in identifying influential elements, within power system networks to prevent voltage collapse within the network, is investigated. In this section, a power network is modelled as a graph in order to actually capture the benefits inherent in the interconnections of the network for identifying critical elements whose outage could lead to voltage collapse within power system.

Suppose a complex network such as power systems can be represented as a directed graph $G=(V,E)$ with sets of vertices V and a set of edges or loops E. According to the fundamental concepts in graph theory, two vertices i and j are said to be connected, in an undirected graph G, if they are neighbours; a path exists between the two vertices u and v. A connected graph G is one whose all pairs of distinct vertices are connected. A graph G is said to be undirected if none of the edges has orientations. In a graph G, a branch is a link that connects any two vertices u and v and it is referred to as an edge. A set of edges whose removal could lead to separation of a connected graph G into two different components is termed a cutset. Suppose a complex power system is reduced to a simple graph with no multiple loops or edges, the degree of any node k is the number of neighbours associated with bus k. In other words, the degree of a given node within a network graph indicates the number of nodes which are adjacent to that node.

In a complex network such as power system, which could be described as a graph, whose vertices are connected by edges, the following simplifying assumptions hold: the edges are ordered pairs with at most one link joining any two buses and no self-loop exists and all the edges of the graph are not weighted or similar. Considering these assumptions, the elements of the bus-to-line incidence matrix or adjacency matrix $A(G)$ for an undirected graph $G(V,E)$ has its $ij-th$ element equals to 1 if a connection exists between two vertices i and j and 0 if no connection exists. Mathematically,

$$A(G)_{i,j}\begin{cases} 1, & if\ i\ is\ connected\ to\ j \\ 0, & if\ i\ is\ not\ connected\ to\ j \end{cases} \tag{1}$$

For a directed graph,

$$A(G)_{i,j}=\begin{cases} 1, & if\ line\ j\ starts\ at\ bus\ i \\ -1, & if\ line\ j\ ends\ at\ bus\ i \\ 0, & if\ j\ is\ not\ connected\ to\ bus\ i \end{cases} \tag{2}$$

Hence, the degree of a node can be defined in terms of the adjacency matrix, as the sum of the elements of the corresponding row (or column). The importance of the degree distribution is found in representing the global connectivity of a given network. The node degree can simply be expressed as

$$D(G)_{i,i} = \sum_{j=1}^{n} \left| a_{ij} \right| \tag{3}$$

where n is the total number of buses and A is a symmetric matrix.

The Laplacian Matrix, L of any given graph G is defined as

$$L(G)_{i,j} = D(G)_{i,j} - W(G)_{i,j} \tag{4}$$

where W is the weight matrix associated with the vertices of the graph.

Based on the fundamental theorem and the preliminary results obtained by Wu et al. (2000), the following characteristics of BLM are identified:

(1) The summation of all nonzero elements in each column is zero based on Kirchhoff's current law at each node within the network.

(2) The total number of positive 1s in both BOLIM is equal to the number of negative 1s in BILIM.

Algorithm Development for the Formulation of Bus-to-Line Incidence Matrix

This section presents the three matrices developed for solving the identified problem based on graph theory. These matrices are termed Bus-to-Line Incidence Matrix (BLM), Bus-Inflow Line Incidence Matrix (BILIM) and Bus-Outflow Line Incidence Matrix (BOLIM). The BLM matrix is easily formulated from the graphical representation of the network as shown in Figure 1. The BILIM matrix contains the elements of the sinks while the elements of BOLIM correspond to the sources. Based on the upstream and downstream power tracing, pure sources and pure sinks are identified. The rows with zero element values in BOLIM correspond to the pure sink buses while the rows having their elements all zeros in BILIM correspond to the pure source buses. Having identified the pure source buses within the network, corresponding columns (lines) in the BILIM matrix are deleted and the reduced or equivalent network for the network in terms of the two matrices BOLIM and BILIM is easily generated. The process is continued until there is only one pure source or pure sink within the network. During the process, if loop-flow occurs, the bus where it exists supersedes other identified buses where reactive power support needs to be placed.

The procedural steps involved in the algorithm development for the proposed approach are as follows:

(1) Model the network as a weighted directed graph.
(2) Perform arbitrary numbering on the network vertices.
(3) Perform arbitrary numbering of the network edges.
(4) Indicate the directions of power flows on the network edges. For example, power going out of the bus could be assumed to be positive while that coming into the bus would be assumed to be negative.
(5) Compute the elements of BLM and form the BLM matrix.
(6) Formulate the matrices BILIM and BOLIM from BLM matrix.
(7) Identify the sources and the corresponding sinks within the network.
(8) Perform a matrix reduction on matrix BILIM and its corresponding matrix BOLIM.
(9) Continue the last two steps until the matrices BILIM and BOLIM are no longer reducible.
(10) Identify the critical elements such as weak nodes weak and weak transmission lines within the network.
(11) End.

Numerical Illustration

A numerical example is presented in this section to illustrate the effectiveness of the approach proposed in this paper. For the sake of clear illustration, we consider a simple Southern Indian 10-bus power grid whose single-line diagram is presented in Figure 1. The transmission line data is adapted from (Alayande, 2015). This system comprises of 12 transmission lines, 10 buses out of which 3 are generators that are geographically located within the network and the remaining 7 buses are load buses.

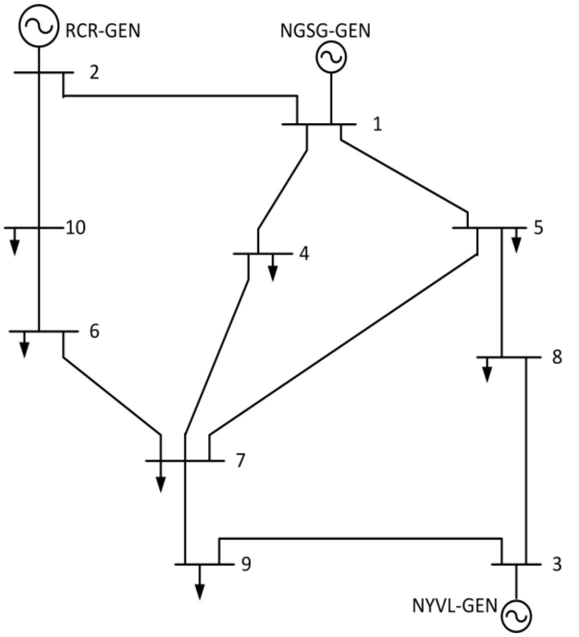

Figure 1: A simple 10-bus network (Alayande et al. 2015)

Results and discussions

The benefit derived from the analogy of atomic structures is first explored to determine the structural levels of each node within the network in this section. Table 1 presents the structural levels of each node in a 10-bus network of Figure 1 and their corresponding node degrees for the network under consideration. The structural level in the context of this paper refers to the combination of network participants or elements whose structural interconnections form a circular path or shell-like structure. This provides necessary information about the topological distance of each load bus with reference to the generator buses within the network, which could easily be captured without the need to carry out the repetitive and time-consuming power flow analysis. The generators are assigned zero (0) being the reference nodes and are presented in Table 1. The results obtained for the degree of each network load is also presented in Table 1.

Table 1: Structural level identification and Bus-to-Line Incidence Matrix (BLM)

Bus no.	Bus type	Structural level	Node degree	Transmission line											
				a	b	c	d	e	f	g	h	i	j	k	m
1	Generator	0	3	1	1	1	0	0	0	0	0	0	0	0	0
2	Generator	0	2	0	0	-1	1	0	0	0	0	0	0	0	0
3	Generator	0	2	0	0	0	0	0	0	0	1	1	0	0	0
4	Load	1	2	-1	0	0	0	0	1	0	0	0	0	0	0
5	Load	1	3	0	-1	0	0	1	0	1	0	0	0	0	0
6	Load	2	2	0	0	0	0	0	0	0	0	0	1	-1	0
7	Load	2	3	0	0	0	0	0	-1	-1	0	0	-1	0	1
8	Load	1	2	0	0	0	0	-1	0	0	-1	0	0	0	0
9	Load	1	2	0	0	0	0	0	0	0	0	-1	0	0	-1
10	Load	1	2	0	0	0	-1	0	0	0	0	0	0	1	0

Table 2: Bus-Inflow Line Incidence Matrix (BILIM)

Bus no.	Transmission line											
	a	b	c	d	e	f	g	h	i	j	k	m
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	1	1	0	0	1	0	0
8	0	0	0	0	1	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	1
10	0	0	0	1	0	0	0	0	0	0	0	0

Table 3: Bus-Outflow Line Incidence Matrix (BOLIM)

Bus no	Transmission line											
	a	b	c	d	e	f	g	h	I	J	k	m
1	1	1	1	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	1	1	0	0	0
4	0	0	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	1	0	0
7	0	0	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0

The structural levels of other buses are determined based on their topological interconnections to the nearest generator bus in the network. All the generator buses on structural level 0 are connected on a circular path, which is the innermost circle of Figure 1. The next circular path is the circle that connects all the buses on the structural level 1 while the outermost circle connects all the buses that are on the structural level 2 as presented in Table 1. Also presented in Table 1 are the results obtained for the formulation of BLM matrix for the network from which the BILIM in Table 2 and BOLIM in Table 3 are extracted. The elements of the BLM matrix are obtained by finding the absolute values of all the negative elements contained in BLIM while positive elements of the BLIM matrix are used to form the Bus-Outflow Line Incidence Matrix (BOLIM).

Considering the developed algorithm, pure source buses within the network are identified as buses 1 and 3 while the pure sink buses are identified as buses 8 and 9. The identified pure source buses are therefore deleted in the BILIM, which correspond to the rows 1 and 3. The columns that correspond to pure source buses in BOLIM matrix are those columns whose elements of BOLIM are 1s on rows 1 and 3. Therefore, columns a, b, c, h and i in BOLIM are also deleted. The results

Table 4: Bus-Inflow Line Incidence Matrix (BILIM)

Bus no	Transmission line						
	d	e	f	g	j	k	m
2	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0
7	0	0	1	1	1	0	0
8	0	1	0	0	0	0	0
9	0	0	0	0	0	0	1
10	1	0	0	0	0	0	0

Table 5: Bus-Outflow Line Incidence Matrix (BOLIM)

Bus no	Transmission line						
	d	e	F	g	j	k	m
2	1	0	0	0	0	0	0
4	0	0	1	0	0	0	0
5	0	1	0	1	0	0	0
6	0	0	0	0	1	0	0
7	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0

Based on the results presented in Table 5, pure source buses within the network are identified as 2, 4 and 5 while the pure sink buses are buses 8 and 9. Therefore, the identified pure source buses and their correspondence columns are deleted. These correspond to the rows 2, 4 and 5 in Table 4 and columns d, e and f in Table 5. The results obtained for the reduced BILIM and BOLIM matrices are presented in Tables 6 and 7 respectively.

Table 6: Bus-Inflow Line Incidence Matrix (BILIM)

Bus no	Transmission line		
	j	k	m
6	0	1	0
7	0	0	0
8	0	0	0
9	0	0	1
10	1	0	0

Table 7: Bus-Outflow Line Incidence Matrix (BOLIM)

Bus no	Transmission line		
	j	k	m
6	1	0	0
7	0	0	1
8	0	0	0
9	0	0	0
10	0	1	0

In a similar manner, pure source buses within the reduced network are buses 7 and 8 while pure sink bus is bus 8. Therefore, from Table 6, the pure source buses are deleted which correspond to the rows 7 and 8 and their corresponding columns in Table 7 are also deleted which correspond to line m. The result obtained for the reduced BILIM and BOLIM matrices, after the deletion, are presented in Table 8 and 9 respectively. It can be seen from Tables 8 and 9 that bus 9 corresponds to a pure source bus and pure sink. Therefore, row 9 in Tables 9 and 10 is deleted. The results obtained for the reduced equivalent BILIM and BOLIM matrices are presented in Tables 10 and 11.

Table 8: Bus-Inflow Line Incidence Matrix (BILIM)

Bus no	Transmission line	
	j	k
6	0	1
9	0	0
10	1	0

Table 9: Bus-Outflow Line Incidence Matrix (BOLIM)

Bus no	Transmission line	
	j	k
6	1	0
9	0	0
10	0	1

It can be seen that no pure source or pure sink bus exists in Tables 10 and 11. This shows the end of line and bus deletion within the network. Furthermore, the transmission lines not deleted as can be seen in Tables 10 and 11 are lines j and k while the buses that are not deleted until the end of the process are buses 6 and 10. By employing the electrical characteristics of the network, the most influential node and weakest transmission lines could be easily identified. Hence, the most suitable and appropriate location where the reactive support could be placed can easily be identified. Line j corresponds to the transmission line joining buses 6 and 7 with an electrical distance of 248.7593 while line k represents the transmission line joining buses 6 and 10 with an electrical distance of 14.6716.

Table 10: Bus-Inflow Line Incidence Matrix (BILIM)

Bus	Transmission line	
	j	k
6	0	1
10	1	0

Table 11: Bus-Outflow Line Incidence Matrix (BOLIM)

Bus	Transmission line	
	j	k
6	1	0
10	0	1

Therefore, bus 6 is identified as the most influential bus within the network, where the location of a reactor will have an influence on the operation of the system. Also, line j, which connects buses 6 and 7 is the most critical transmission line within the network under study. This information about transmission lines within the network could be of utmost importance during the outage of a critical transmission line.

Contingency Analysis

In this section, we carry out a contingency analysis based on N-1 criterion. This is necessary in order to further confirm and affirm the results obtained from the proposed approach. The influence of a critical outage of the identified transmission line, which connects buses 6 and 7, based on the N-1 criterion, on the integrity of the network is investigated. Power-flow analysis is carried out to further test the efficiency of the method proposed in this paper. Table 12 presents the results obtained from the power-flow solution as well as the eigenvalue and eigenvector analyses. From Table 12, for the base case, it is seen that the voltage magnitudes at buses 6 and 7 are out of the prescribed limits of 0.95-1.05 pu and the active power loss of 18.573 MW is obtained. When the most influential transmission line (the line connecting buses 6 and 7) is outaged, the results show that buses 4, 7 and 9 are outside the prescribed limits and the active power loss within the network increases to 19.960 MW. However, by injecting reactive power at the identified weak bus 6, the results show a considerable improvement in the voltage profile and the active power loss is significantly reduced to 17.35 MW.

Based on the eigenvalue and eigenvector analyses (Sikiru et al., 2012), the smallest eigenvalues for both normal and outage operations are found to be associated with bus 6 with a value of 0.0000 as presented in the last two columns of Table 13. This further confirms that bus 6 is the weakest bus in the network. Also, elements of the eigenvector associated with this smallest eigenvalue, for all the buses, are found to be equal to the value of 0.380 as depicted in Figure 3. This indicates that the network is topologically weak and needs to be reinforced by injection of reactive power at the weakest bus 6.

Table 12: Results obtained for a critical outage on line 6-7 in a simple 10-bus network

Bus No	Bus type	Voltage Magnitude (pu)			Eigenvalue	
		Base Case	During Outage	With VAR Injection at bus 6	After Outage	Before Outage
1	Generator	1.0000	1.0000	1.0000	-	-
2	Generator	1.0500	1.0500	1.0500	-	-
3	Generator	1.0300	1.0300	1.0300	-	-
4	Load	1.0481	1.0672	1.0443	100.1787	540.5414
5	Load	1.0367	1.0466	1.0281	63.1080	81.6696
6	Load	1.0544	0.0008	1.0248	0.0000	0.00000
7	Load	1.0547	1.0705	1.0289	6.2914	58.2337
8	Load	1.0171	1.0337	1.0130	6.2914	41.0213
9	Load	1.0353	1.0548	1.0264	6.2914	31.6160
10	Load	1.031	1.0478	1.0494	36.1679	23.1600
Active Power Loss		18.573 MW	19.960 MW	17.35 MW		

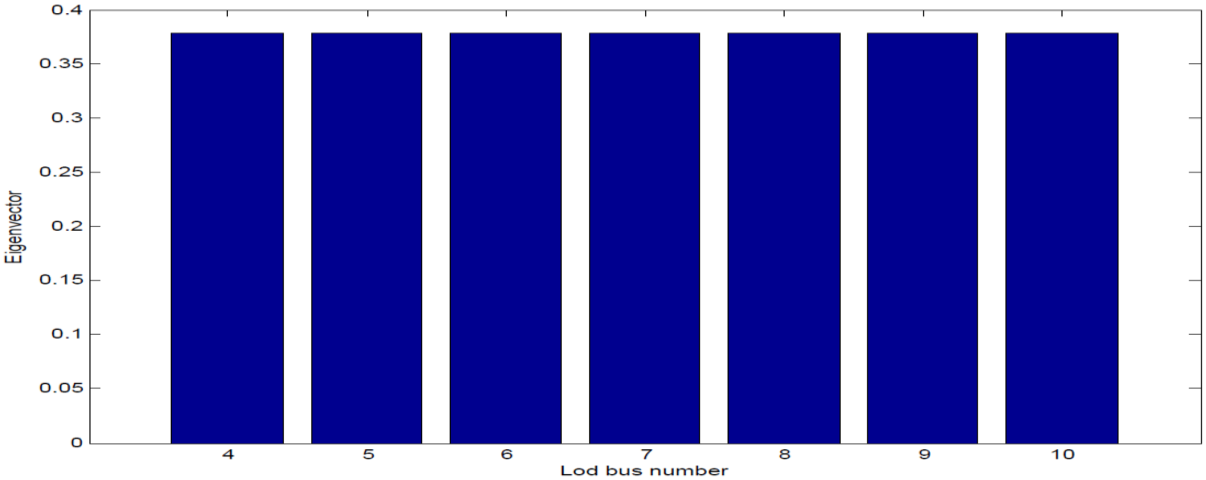

Figure 3. Elements of the eigenvector associated with the weakest bus in a 10-bus network

Comparison of approaches

Table 13 presents the comparison of the results obtained from the existing approaches with that obtained using the proposed approach.

The results obtained in this paper are in strong agreement with the results obtained using inherent structural characteristics as presented by Alayande et al. (2015). The results obtained using the proposed approach also compare well with the existing approach of eigenvalue decomposition presented by Sikiru, 2011 and T-index approach proposed by Dhadbanjan, 2009. All the three existing methods are in agreement with the most suitable location identified by the proposed method where the influence of reactive power support within the network will be greatly appreciated.

The main benefit of the proposed approach over the existing approaches is that this approach can easily identify both influential nodes and transmission lines without the need to perform power-flow analysis. The issues associated with the slack bus identification are therefore avoided. Moreso, a considerable amount of time is therefore saved as there is no need for performing the time consuming power flow analysis before the solution could be arrived at as opposed to the solution approach presented by Wu et al. (2000). Furthermore, it does not require partitioning analysis, which is highly required for the solution to be obtained in the approach presented by Alayande et al. (2015).

Table 13: Comparison of approaches for weakest load bus identification

Load bus no	Ranking			
	Proposed approach	Coupling strength index-based approach (Alayande et al., 2015)	Eigenvalue-based approach (Sikiru, 2011)	T-Index-Based approach (Dhadbanjan, 2009)
4	7th	7th	7th	5th
5	6th	5th	6th	3rd
6	**1st**	**1st**	**1st**	**1st**
7	4th	2nd	5th	2nd
8	5th	6th	4th	7th
9	3rd	4th	3rd	4th
10	2nd	3rd	2nd	6th

We also compare the results obtained based on our approach with that obtained through the use of coupling strength index-based approach (Alayande, Jimoh and Yusuff, 2015) for the identification of weakest transmission line. The results of the comparison are presented in table 14. From the results presented in Table 14, it can be seen that both methods identified the line connecting buses 6 and 7 as the weakest transmission line. The main benefit of the proposed approach over the existing approaches is that this approach can easily identify both influential nodes and transmission lines without the need to perform power-flow analysis. A considerable amount of time is therefore saved as there is no need for performing the time consuming power flow analysis before the solution could be arrived at as opposed to the solution approach presented by Wu et al. (2000). Furthermore, it does not require partitioning analysis, which is highly required for the solution to be obtained in the approach presented by Alayande et al. (2015).

Table 14: Comparison of approaches for critical transmission line identification within Load-Load attraction region

Bus No – Bus No	Ranking	
	Proposed approach	Coupling strength index-based approach (Alayande et al., 2015)
4 – 7		0.0276
5 – 8		0.0425
6 – 7		**6.1881**
6 – 10		0.0250
7 – 9		0.0765
5-7		0.0436

Conclusion

In this paper, identification of a suitable location of a reactor where the operation of power systems could be improved based on graph theory has been considered. The network matrices termed BLIM, BILIM and BOLIM are developed, from graph theory, for identifying the influential nodes and links within power grids. A Southern Indian 10-bus power grid is used as the case study. The results obtained compared well with that obtained by the existing methods as documented in the literature. The comparison shows a strong agreement with the existing results obtained by the past researchers using different methods. This confirms the accuracy of the proposed approach. Based on the results obtained, the proposed approach could be helpful as an alternative algorithm for locating a suitable nodes, where a reactive power support could be placed for efficient operation of power systems. Moreso, by this method, prompt identification of weak transmission lines and critical buses whose outage could have adverse effect on the stability of the network is made easy. It could also be useful to the system operators in identifying an influential transmission line for power rerouting during an outage of a critical transmission line within the network. The information, if provided could safeguard the network from collapsing.

References

Adebayo, I. G., Jimoh, A. A., Yusuff, A. A., and Sun, Y., Alternative method for the identification of critical nodes leading to voltage instability in a power system, African Journal of Science, Technology, Innovation and Development, pp 323-333, 2018. Doi:10.1080/20421338.2018.1461967

Alayande, A. S., Jimoh, A. A., and Yusuff, A. A., Identification of Critical Buses and Weak Transmission Lines Using Inherent Structural Characteristics Theory, *IEEE Power and Energy Engineering Conference (APPEEC), 2015 IEEE PES Asia-Pacific*, pp.1-5, 15-18 Nov. 2015, Brisbane, Australia, IEEExplore DOI: 10.1109/APPEEC.2015.7380974

Alayande, A. S., Solving Power System Problems Based on Network Structural Characteristics, D.Tech. Thesis, Department of Electrical Engineerig, *Tshwane University of Technology*, Pretoria, South Africa, 2017.

Dhadbanjan, T., and Chintamani, V., Evaluation of Suitable Locations for Generation Expansion in Restructure Power Systems: A Novel Concept of T-Index, *International Journal of Emerging Electric Power Systems*, vol. 10, issue 1, article 4, 2009.

Ginarsa, I. M., Soeprijanto, A., and Purnomo, M. H., Controlling Chaos and Voltage Collapse Using An ANFIS-Based Composite Controller-Static Var Compensator in Power Systems, *International Journal of Electrical Power & Energy Systems*, Vol. 463, pp.79-88, 2013.

Khalid, S. N., Mustafa, M.W., Shareef, H., Khairuddin, A. Kalam, A. and Maungthan Oo, A., A Novel Reactive Power Transfer Allocation Method with the Application of Artificial Neural Network, *2008 IEEE Australasian University Power Engineering Conference (AUPEC),* Sydney, NSW, Australia, 2008.

Khan, B., and Agnihotri, G., Optimal Transmission Network Usage and Loss Allocation Using Matrices Methodology and Cooperative Game Theory, *International Journal of Electrical, Robotics, Electronics and Communications Engineering*, World Academy of Science, Engineering and Technology, Vol:7 No2, pp142 -149, 2013.

Mafizul Islam1, Sk, Alam2, M., and Mohammad Y. Sk, A New Approach of Reactive Loss Allocation in

Deregulated Power System, *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering*, Vol. 2, Issue 7, pp. 2948-2954, July 2013

Mala De and Goswami, S. K., Reactive Power Cost Allocation by Power Tracing Based Method, *Journal of Energy Conversion and Management*, No. 64, pp. 43–51, 2012.

Mustafa, M. W., Shareef, H., and Ahmad, M. R., An Improved Usage Allocation Method for Deregulated Transmission Systems, *IEEE 2005 International Power Engineering Conference*, Singapore, 2005.

Sikiru, T. H., Jimoh, A. A., and Agee, J. T., Optimal Location of Network Devices using a Novel Inherent Network Topology Based Technique, *IEEE Africon 2011* - The Falls Resort and Conference Centre, Livingstone, Zambia, Pp.1-4, 13 - 15 September 2011.

Sikiru, T.H., Jimoh A.A., Hamam Y., Agee J.T., & Ceschi R., Classification of networks based on inherent structural characteristics. In 6th IEEE Transmission and distribution Latin America Conference. Montevideo, Uruguay, 2012.

Taylor, J. A., Conic Optimization of Electric Power Systems, in partial fulfilment of the requirements for the degree of Doctor of Philosophy, *Massachusetts Institute of Technology*, June 2011.

Wu, F.F., Ni, Y., and Wei, P., Power Transfer Allocation for Open Access Using Graph Theory—Fundamentals and Applications in Systems Without Loopflow, *IEEE Transactions On Power Systems*, vol. 15, no. 3, pp. 923-929, 2000.

Yamashita, K., Joo, S., Li, J., Zhang, P., and Liu, C., Analysis, control, and economic impact assessment of major blackout events, European transactions on electrical power, vol 18, pp.854–871, 2008.

Ziari, I., Ledwich, G., Ghosh, A., Cornforth, D., and Wishart, M., Optimal Allocation and Sizing of Capacitors to Minimize the Transmission Line Loss and to Improve the Voltage Profile, *Journal of Computers and Mathematics with Applications*, vol. 60, pp. 1003-1013, 2010.