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Theorizing in Design Research: Methodological reflections on developing and connecting theory elements for language-responsive mathematics classrooms

Susanne Prediger, TU Dortmund University (Germany)

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Teorizar en la Investigación de Diseño

Resumen

La Investigación de Diseño Didáctico centrada en contenido es una metodología de investigación con dos objetivos: 1) diseñar y mejorar los dispositivos de enseñanza y aprendizaje y 2) generar contribuciones teóricas para comprender procesos iniciados de enseñanza y aprendizaje de un cierto contenido. El artículo proporciona reflexiones metodológicas y ejemplos a fin de elaborar el significado de teorizar en el contexto de esta metodología. A partir de la distinción entre elementos teóricos categóricos, descriptivos, explicativos, normativos y predictivos con sus funciones y estructuras lógicas, los ejemplos muestran que teorizar en los estudios de Investigación de Diseño se puede concebir como un proceso de desarrollo sucesivo y conexión entre elementos teóricos, para preguntas-cómo (fundamentos de los dispositivos) y preguntas-qué (estructura del contenido de aprendizaje). Ilustro estas consideraciones para Investigaciones de Diseño Didáctico centradas en el contenido matemático y en el uso de la lengua, especialmente en relación con la comprensión conceptual de fracciones, de variables y de porcentajes en aulas con alumnos en proceso de aprendizaje de la lengua de instrucción.

Palabras clave. Investigación de Diseño Didáctico; teorizar; elementos teóricos; contenido matemático; uso de la lengua.

Theorizing in Design Research

Abstract

Topic-specific Didactical Design Research is a research methodology with two aims, 1) designing and improving teaching-learning arrangements and 2) generating theoretical contributions for understanding the initiated teaching-learning processes for a certain topic. The article provides methodological reflections and examples for elaborating the meaning of theorizing within this methodology. Starting from a distinction of categorial, descriptive, explanatory, normative and predictive theory elements with their functions and logical structures, the examples show that theorizing in Design Research studies can be conceived as a process of successively developing and connecting theory elements, for the how-questions (the rationales for the arrangements) and the what-questions (the structuring of the learning content). The considerations are illustrated for the case of topic-specific Didactical Design Research for language-responsive classrooms, particularly in relation to language learners' conceptual understanding of fractions, variables, and percentages.

Keywords. Didactical Design Research; theorizing; theory elements; mathematical content; language use.

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Teorizar em Investigação de Desenho

Resumo

A Investigação de Desenho Didático centrada no conteúdo é uma metodologia de investigação que combina dois objetivos: 1) construir e melhorar as práticas de ensino e aprendizagem e 2) criar contribuições teóricas para compreender os processos iniciados de ensino e aprendizagem de um certo conteúdo. O artigo proporciona reflexões metodológicas e exemplos com o intuito de elaborar o significado de teorizar no contexto desta metodologia. A partir da distinção entre elementos teóricos categoriais, descritivos, explicativos, normativos e preditivos com as suas funções e estruturas lógicas, os exemplos mostram que teorizar nos estudos de desenho de pesquisa pode ser concebido como um processo de desenvolvimento sucessivo e ligação entre elementos teóricos, para as perguntas-como (fundamentos das práticas) e as perguntas-o que (estrutura do conteúdo da aprendizagem). Ilustro estas considerações para investigações de desenho de pesquisa centradas no conteúdo matemático e no uso da língua, especialmente em relação com a compreensão conceptual das frações, de variáveis e de percentagens nas aulas de alunos em processo de aprendizagem da língua de instrução.

Palavras chave. Pesquisa de Desenho Didático; teorizar; elementos teóricos; conteúdo matemático; uso da língua.

Théorisation dans la Recherche sur la Conception

Résumé

La Recherche sur la Conception Didactique centrée sur un sujet spécifique est une méthodologie de recherche qui combine deux objectifs, 1) concevoir et améliorer les dispositifs d'enseignement-apprentissage et 2) produire des contributions théoriques pour comprendre ces processus d'enseignement-apprentissage liés à un sujet donné. L'article propose des réflexions méthodologiques et des exemples pour préciser ce que produire des théorisations peut signifier dans le cadre de cette méthodologie. Partant d'une distinction entre éléments théoriques catégoriels, descriptifs, explicatifs, normatifs et prédictifs avec leurs fonctions et leurs structures logiques, les exemples montrent que la théorisation dans la Recherche sur la Conception peut être conçue comme un processus où les éléments théoriques sont successivement développés et reliés, pour les questions-comment (la justification des dispositifs) et les questions-quoi (la structure du contenu didactique). Nous illustrons ces considérations dans le cas de la Recherche sur la Conception Didactique centrée sur un sujet spécifique et sur l'utilisation du langage, particulièrement en ce qui concerne la compréhension conceptuelle des fractions, variables et pourcentages avec des élèves en cours d'apprentissage de la langue.

Paroles clés. Recherche sur la Conception Didactique; théorisation; éléments théoriques; contenu mathématique; utilisation de la langue.

1. Introduction

Within the last 25 years, Design Research has been established as a research methodology that systematically combines two aims: (1) improving subject-matter classroom teaching by designing teaching-learning arrangements for a certain topic and (2) generating theoretical contributions by empirical research in order to understand the initiated teaching-learning processes for a certain topic (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Barab & Squire, 2004; Gravemeijer & Cobb, 2006). The resulting local theory serves as a rationale for the design and aims in the future for generalizations for further classroom contexts and topics.

These dual aims, *improving* designs and *theorizing*, are often pursued in Design Research projects (e.g., in many of the case studies in Plomp & Nieveen, 2013; van den Akker, Gravemeijer, McKenney & Nieveen, 2006). However, processes and possible outcomes of improving designs are much better reported on a methodological level than processes and possible outcomes of theorizing. This article is dedicated to methodologically elaborating what theorizing means in Design Research, in which the

processes are perhaps more complex than in other research approaches. This intention contains the questions of 1) how processes of empirically grounded theory generation can be conducted in Design Research, 2) how possible outcomes of theorizing in Design Research can be distinguished according to their function and structure, and 3) how the outcomes are connected in the steps of theorizing.

Section 2 of this article starts from general methodological considerations on theory elements with different functions and structures and explores types of theory elements that can be generated by topic-specific *Didactical* Design Research studies that focus on not only *how* to learn, but also *what* to learn. Section 3 illustrates the abstract methodological considerations on one exemplary field of Design Research, namely topic-specific Didactical Design Research on fostering language learners' conceptual understanding. This section lists typical needs for theory elements in this exemplary field and provides insights into processes of theorizing in which theory elements of different statuses are combined into a web of theory elements.

As a whole, this article suggests that theorizing can be conceived in Design Research studies as a process of successively elaborating a web of intertwined theory elements with different functions, namely, categorial, descriptive, normative, explanatory, and predictive functions, on *how*-questions and *what*-questions that are successively intertwined. This conceptualization and the typical steps of theorizing will be explained in the following sections.

2. Methodological foundations: Theory elements as outcomes of theorizing

2.1. Distinction of theory elements with different functions and structures

The role of theories for educational research studies is twofold: On the one hand, theories influence (but do not determine) the design decisions and the methods and perceptions in the empirical investigations of the teaching-learning processes that have been initiated (theories as a framework for research). On the other hand, empirical investigation aims at generating and eventually testing or refining theories (theoretical contributions as outcomes of research). The interplay between theories as frameworks and outcomes of research applies to all kinds of research in mathematics education (Mason & Waywood, 1996; Prediger, 2015). In Design Research in particular, it is fueled by the iterativity and interactivity between theory-generating and theory-guided experimenting. Whereas the role of theories as frameworks have often been discussed (e.g., Cobb et al., 2003; Mason & Waywood, 1996), the process of theorizing is worth further methodological reflections. For this purpose, this section provides distinctions for different kinds of theory elements that can be outcomes of research.

Niss (2007) defines theory as an "organized network of concepts (including ideas, notions, distinctions, terms, etc.) and claims about some extensive domain . . . consisting of objects, processes, situations, and phenomena. . . . In a theory, the concepts are linked in a connected hierarchy . . . [and] the claims are either basic hypotheses . . . or statements obtained from the fundamental claims" (Niss, 2007, p. 1308). He emphasizes the idea that several elements of a theory are connected in a network to form a theory. In line with the general philosophy of science, he distinguishes two logical structures of these elements: concepts and claims.

In general philosophy of science, concepts are called *categories* or *constructs* and claims are called *propositions*. The propositions are further distinguished by pointing to different *functions* of theories: A theory is a language entity in propositional or

categorial form that orders the phenomena of a domain and describes the relevant features of its objects and their relations to each other; explains by general laws and allows predictions for the occurrence of phenomena" (Thiel, 1996, p. 262). Similarly, McKenney and Reeves (2012) list the following functions of theories: "describe, explain, predict, or even prescribe how to change or affect certain phenomena" (p. 32).

Some research studies aim at one single function, for example, describing students' typical misconceptions by an interview study for generating a *descriptive* theory or validating one single hypothesis by a randomized controlled trial to achieve empirical evidence for *predictive* theory elements. Usually, however, theories serve more than one function, as they are composed by several theory elements with different functions. Rather than speaking about the function of one theory, Prediger (2015) therefore introduced plurality by referring to the functions of specific theory *elements*.

In particular, Design Research studies usually aim at complex local instruction theories, which do not address only one function but combine theory elements with different functions (Prediger, 2015). Beck and Krapp (2006, p. 39) applied the classical distinctions of functions from general philosophy of science to the academic discipline of psychology, with the functions of theories or theory elements corresponding to different logical structures. These functions and structures enable both experienced and novice researchers to distinguish theory elements and then methodologically reflect on different requirements for their empirical foundation. The functions and structures listed in Table 1 will be explained in the following and illustrated by examples in the next section.

Table 1. Five theory elements and their functions and structures (adapted from Beck & Krapp, 2006, p. 39 and Prediger, 2015, p. 652)

Theory elements	**Function**	**Structure**
Categorial	Constructs providing a language and thinking tool for perceiving and distinguishing	Conceptual structure, i.e., constructs, categories, relations
Descriptive	Describing a certain phenomenon qualitatively or quantitatively, focused by specific categories	Propositions stating existence, categorial hierarchies, or frequencies
Normative	Specifying and justifying aims and rationales (e.g., learning goals or process qualities)	Propositions with an aim-reason structure
Explanatory	Explaining, giving causes, or identifying backgrounds	Propositions with cause-effect or phenomenon-background structure
Predictive	Purposeful acting or predicting effects	Propositions in "in order to" or in "if-then" structure

- *Categorial theory elements* have the function of providing a language and thinking tool for perceiving and distinguishing phenomena. Their logical structure is conceptual, which means that descriptive elements usually consist of categories, constructs, and relations. Many researchers in mathematics education research have emphasized the relevance of categories or constructs for a theory's descriptive and explanatory power (e.g., Niss, 2007, p. 1308). diSessa and Cobb (2004) emphasize: "[Theoretical categories] enable us to discriminate between relations that are

necessary and those that are contingent. They delineate classes of phenomena that are worthy of inquiry and specify how to look and what to see in order to understand them. This last characteristic—epigrammatically, 'teaching us how to see'." (p. 79). Both authors have additionally emphasized that it can be the invention of an important category that brings a phenomenon into a new quality of being, for example, the construct didactical contract, the role of new categories are called ontological innovations. Empirical research that generates new categories must make sure to unfold an added value for articulating phenomena. The methodologies for generating categories in processes of data-led successive refinement have been carefully reflected, for example, in grounded theory (Strauss & Corbin, 1990).

The categorial type is decisive for all further theory elements, as they provide the language to describe, set aims, and explain or predict in propositional theory elements:

- *Descriptive theory elements* serve to describe a certain phenomenon qualitatively or quantitatively. They answer typical questions such as: What characterizes this area? Which phenomena and relations can occur? In which frequencies? Descriptive theory elements consist of propositions of different logical structures, for example, propositional structures describing features ("M has characteristics C" or "M can be C_1, C_2, or C_3"), categorial hierarchies ("Every *x* is also *y*"), or frequencies of occurrences ("20% of students have the characteristics C_1 and 30% have C_2"). Empirical research that generates new descriptive findings must make sure that the phenomena and eventually frequencies are adequately described with validity and reliability, depending on the adequacy of the categories (Strauss & Corbin, 1990).
- *Explanatory theory elements* serve to explain, give causes, or identify backgrounds of described phenomena, thus they answer to questions such as: Why does this phenomenon occur? What might be the background? Their logical structure consists of propositions with cause-effect or phenomenon-background structure ("Phenomenon *x* occurs because of *y*" or "phenomenon *x* can be traced back to phenomenon *y*"). Empirical research that generates new explanatory theory elements requires categorial and descriptive findings and empirical evidence that the phenomena are related to the claimed background. In qualitative research, this is shown by analyses in which the interplay between phenomenon and background is unpacked (e.g., by contrasting cases in interpretative methodologies; e.g., Yin, 1994; Strauss & Corbin, 1990). Although the categorial and descriptive components are required for explanatory elements, they are sometimes generated at the same time in qualitative research. In quantitative research, explanatory findings require methods that can identify explaining factors statistically, e.g., in statistical path models or regression models for testing hypotheses on potential connections, whereas purely correlative findings do not have explanatory power (Creswell, 2003).
- *Normative theory elements* serve to specify and justify aims and rationales, for example, by questions such as: Which aims shall be reached (e.g., by an instruction)? In which context are they justified? Normative theory elements can refer, for example, to content learning goals but also to process qualities (e.g., participation of all students) that should be reached in a teaching-learning arrangement. The logical structure of normative theory elements consists of propositions connecting the aims to reasons why the aim should be reached ("Students should acquire learning goal *x* because this is required for literacy aspect *y*" or "the learning process should reach process quality *z* because this has been shown to enhance *w*"). Making normative elements explicit is important because

they are crucial due to their role as explicit or implicit components of predictive theory elements. Whereas the aim itself in a normative theory element cannot be "proven" empirically, the justification of this aim can refer to explanatory theory elements and therefore have an empirical foundation (Prediger, 2015).

- *Predictive theory elements* serve to ground purposeful acting or predict effects of a design element or structural element (such as specific access to the structure of the content). They answer to questions such as: What can be done to reach a certain aim? What could happen if a decision is taken in a specific way? Their logical structures can be an "in-order-to" structure ("in order to reach aim *x* you are advised to do *y*") or an "if-then" structure ("if you do *y*, you could reasonably expect *y*," obviously not in a deterministic logical sense). Empirical research that generates new predictive findings is mostly interventionist: in qualitative research approaches by contrasting several cases (Yin, 1994) and in quantitative research approaches, for example, by the classical format of a randomized controlled trial.

This distinction of theory elements with their functions and logical structures helps to identify kinds of possible theoretical contributions and can hence guide the targeted theorizing process. Since different logical structures require different empirical warrants, it can also support the methodological reasoning of the researcher.

In order to guide theorizing in *Design Research* processes, the next section unpacks typical theory elements in Educational and Didactical Design Research. It then shows that the five kinds of theory elements rarely occur in isolated ways, but are consequently intertwined in Design Research.

2.2. Typical theory elements in Educational and Didactical Design Research

Design Research studies generate practical and theoretical contributions in two domains: i) The design of the teaching-learning arrangements (i.e., how-questions; focused on all kinds of Educational Design Research, see Plomp & Nieveen, 2013); and ii) The structure of the learning content (i.e., what-questions; additionally focused in the sub-area of Didactical Design Research, see Gravemeijer & Cobb, 2006; Prediger & Zwetzschler, 2013; Bakker, 2018).

In these distinctions, Educational Design Research is used as the overarching term for all kinds of Design Research in education with different focus domains (e.g., van den Akker et al., 2006), and Didactical Design Research refers to those Educational Design Research approaches that additionally focus on the learning content (which resonates with the European Didactic tradition; see Blum, Artigue, Mariotti, Sträßer & van den Heuvel-Panhuizen 2019). Didactical Design research is often conducted in subject-matter education research, whereas Educational Design Research without didactical focus on learning contents can be conducted in general educational sciences.

<u>Theory elements in the design of teaching-learning arrangements (how-questions)</u>

Educational Design Research studies have been roughly characterized as follows (e.g., Plomp & Nieveen, 2013): Design researchers start from formulating a problem (*descriptive* elements) that is not yet in line with the intended aims (*normative* elements); hence, they set out to develop or refine design principles (*predictive* heuristics or theory elements connecting specific options for design and acting towards the intended aims) for an orientation towards how to reach their goals. During the design experiment cycles, they develop not only a practical solution for the initial problem that has particular design elements (such as a task or support means), but also

descriptive and explanatory findings that further elaborate the underlying design principles. After the initial design experiment cycles, the if-then structure of the refined design principle can be conceived as a hypothesis to be tested, e.g., by controlled trials. In this characterization, design principles are the core theory element with a predictive function and an if-then structure. As early as 1999, van den Akker decomposed the logical structure of design principles as follows (see Figure 1):

> "If you want to design <intervention X> for the <purpose/function Y> in <context Z>, then you are best advised to give <that intervention> the <characteristics A, B, and C> [substantive emphasis], and to do that via <procedures K, L, and M> [procedural emphasis], because of <arguments P, Q, and R>." (van den Akker, 1999, p. 9)

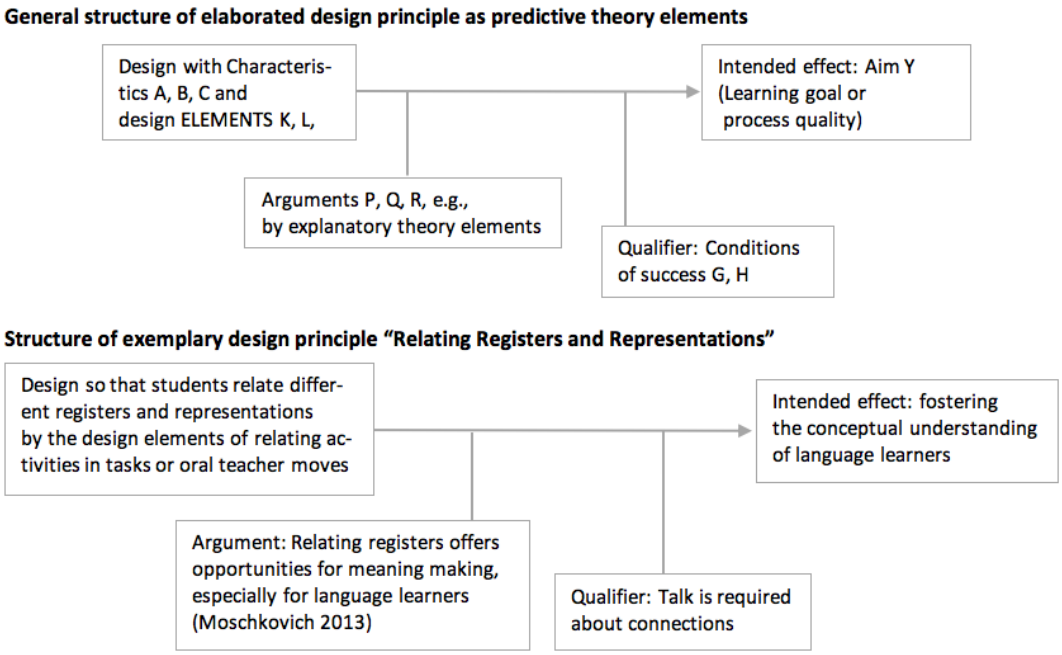

Figure 1. Logical structure of a design principle as predictive theory elements (adapted from van den Akker, 1999, visualized in Toulmin's 1969 scheme)

The example in Figure 1 may illuminate the logical structure for the exemplary design principle of relating registers and representations (Prediger & Wessel, 2013). It also illustrates the interpretation of qualifiers (conditions of success) added to van den Akker's (1999) characterization in the following reformulation of the principle:

> If you want to design language-responsive mathematics teaching-learning arrangements for fostering the conceptual understanding of language learners, then you are best advised to systematically *relate different registers and representations*. This can be realized by *relating activities* (translating, matching, operative variations…), initiated by tasks or oral teacher moves, because relating representations has been shown to enhance conceptual understanding in general, and relating registers must be added for taking into account the learning needs of language learners. However, talking about how the representations and registers are connected is a crucial condition of success since not all students discover the relevant structures by themselves.

The example shows how the articulation of the complex in-order-to structure of the predictive theory element requires various other theory elements: Purpose Y is the normative aim (in this case, fostering conceptual understanding) for Context Z (in this case, language learners). The normative aim itself can be justified by a normative theory element (in this case, conceptual understanding is the learning goal that requires

the most attention for language learners because it is not acquired automatically; see Prediger et al., 2018; Adler, 2001). In the articulation of the design principle, the normative aim is not justified but taken for granted. The articulation of the characteristics A, B, and C (in this case, relating registers and representations) requires the distinctions of typical categorial elements by which the area of study can be captured (in this case, semiotic representations and language registers of everyday language, school academic language, and technical language). The characteristics and procedures can often be concretized by design elements (in this case, a list of relating activities and a list of teacher moves). The argument within the design principle is required for the claim of the connection between the characteristics and the intended effect; this is a typical example of a predictive element for the design of teaching-learning arrangements. Key to van den Akker's (1999) characterization of design principles is the idea that the claim of an if-then structure needs to be justified by explanatory theory elements ("because of arguments P, Q, R"). In the concrete case, the explanatory theory elements justifying the claim refer to specific language learning needs of language learners (Prediger & Wessel, 2013).

The design principle in its complete structure provides the theoretical background for the functioning of the design: The design elements are practical tools for its realization. In this way, design principles and elements reflect the interplay of practical and theoretical aims. The conjectured if-then or in-order-to structure can be put to an empirical test, which is often beyond the scope of the first Design Research project.

Due to the complexity in which design principles relate different theory elements, McKenney and Reeves (2012) emphasize: "'design principles' is probably the most prevalent term used to characterize the kind of prescriptive theoretical understanding developed through educational design research . . . [as they] integrate descriptive, explanatory and predictive understanding to guide the design of interventions" (p. 35).

Thus, in Educational Design Research, the design principles provide answers to how-questions: how to design the teaching-learning arrangement so that the teaching-learning processes reach an intended aim (a learning goal or process quality). In Didactical Design Research, however, these typical answers to how-questions are complemented by treating what-questions that concern the content structures.

<u>Theory elements concerning the content structures (what-questions)</u>

In Didactical Design Research, the how-questions are complemented (and refined) by the what-questions (Gravemeijer & Cobb, 2006; Prediger & Zwetzschler, 2013), which should be guided by theory elements. Whereas some Educational Design Research treats the what-questions as answered and only as a practical base for treating how-questions, Didactical Design researchers emphasize that the major theorizing outcome can be an empirically grounded hypothetical *learning trajectory* (Bakker, 2018; Doorman, this issue) or local topic-specific instruction theory (Gravemeijer & Cobb, 2006). Obviously, the learning trajectory is rarely a unidimensional linear trajectory. It can also be a complete learning landscape (Confrey, 2006).

The theory elements for what-questions can be generated and refined by empirical research. The research often starts with a vague idea of the learning content, which becomes increasingly concise while analyzing students' learning pathways (van den Heuvel-Panhuizen, 2005). In these cases, categorial, descriptive, and explanatory theory elements are generated by systematically contrasting the intended perspectives on subject-matter aspects to the students' individual perspectives and the content is

often put in another structure, with other priorities, starting points, and connections. This has been often shown in the research approach of Didactical Reconstruction (Duit, Gropengießer & Kattmann, 2005).

The intended learning trajectory is the holistic composite unit in which several theory elements are combined (Doormann, this issue). To understand and lead the theorizing processes in a reflected way, it is worth unpacking the intended learning trajectory into its theory elements, as Table 2 will show. Formulating the intended learning trajectory (predictive theory elements) and capturing students' individual learning pathways (descriptive and explanatory theory elements) requires detailed categorial elements in order to articulate both on a micro level. In the beginning, the categories are often humble and are then successively refined in the iterative cycles.

2.3. Process model for topic-specific Didactical Design Research

All Design Research is an iterative combination of research-based design and design-based research. Research-based design works with existing (perhaps still humble) theories, and design-based research aims at generating, testing and refining theories. This interplay is key for theory elements guiding how- and what-questions.

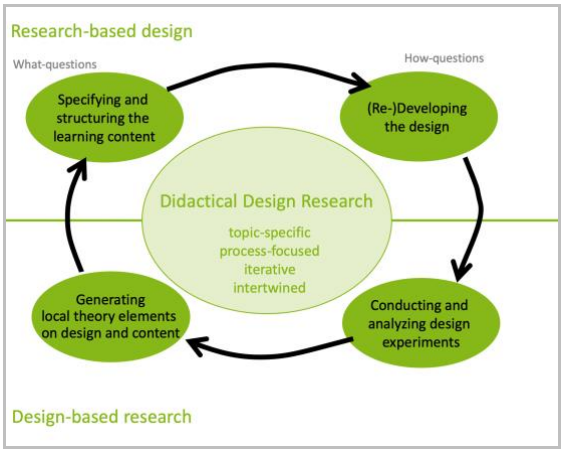

Figure 2. Process model with working areas for topic-specific Didactical Design Research (Prediger & Zwetzschler, 2013; Hußmann et al., 2013)

In the process model of topic-specific Didactical Design Research in Figure 2 (Hußmann et al., 2013; Prediger & Zwetzschler, 2013), specifying and structuring the learning content was thus articulated as its own working area. The model includes the three classical working areas of developing and redeveloping the design of the teaching-learning arrangement, conducting and analyzing design experiments and generating local theory elements, and includes an additional fourth working area that focusses the what-questions. This includes the methodological demand that the empirical analysis also informs the refined structuring of the learning content and that theorizing also refers to theory elements on the learning content listed in Table 2.

Table 2 shows the typical how- and what-questions and the different functions of theory elements. Although the research processes are never linear, the propositional theory elements usually start from (perhaps still humble versions of) normative theory elements and humble predictive heuristics and then elaborate them by iteratively refining and connecting categorial, descriptive and explanatory elements. The refined predictive elements are considered to be the major outcome, as they condense the other elements. Hußmann and Prediger (2016) have suggested a further analysis of categories that support distinguishing and relating aspects of the learning content.

Table 2. Typical theory elements in Didactical Design Research and their functions

	How-questions for theory elements on the design of teaching-learning arrangement	**What-questions for theory elements or structuring the content**
Categorial theory elements	Categories for design principles, process qualities, characteristics of design elements	Categories for distinguishing and relating aspects of the learning content
Normative theory elements	Which process quality should be reached in order to achieve a later learning goal and why?	What should students learn (and why)? (unpacked learning content goals)
Humble predictive heuristics	Which design principles should be applied for which aim?	In which (still vague) learning trajectory can the learning content be structured?
Descriptive theory elements	Which situational effects can the design principles and design elements unfold in the teaching-learning pathways? And how does that relate to the intended effects?	What learning pathways do students usually take along the intended learning trajectory? And how does that relate to the intended learning trajectory?
Explanatory theory elements	Which background do the (non-) effects of design principles and design elements have? Under which conditions of success do they have the intended effects?	What can explain the students' typical perspectives, learning pathways and obstacles? (e.g., which aspects are crucial for learning the next one?) What can explain the differences between the intended learning trajectory and the individual learning pathways?
Refined predictive theory elements	Elaborated design principles: Which design characteristics and elements can be applied for which intended aim and which explanatory element justifies the expectation of these effects and which conditions of success must be considered?	What relations between aspects of the learning contents must be considered? In which refined learning trajectory (or learning landscape) can the relevant aspects of the learning content be structured in order to increase access for all students?

3. Theorizing in the field of language-responsive mathematics classrooms: Exemplary insights into processes of developing the interplay of theory elements

To illustrate the abstract considerations in action, the article refers to the exemplary research area of fostering language learners' development of mathematical concepts. As the insightful overviews of Radford and Barwell (2016), Barwell et al. (2016), and Planas (2018) have shown, research on language and mathematics education has been established for 40 years, and several theoretical frameworks exist that were borrowed from linguistics and elaborated for mathematics education research. However, Design Research is only a recently emerging research methodology in this research area.

3.1. Needs for theory elements for fostering language learners' development of mathematical concepts: The how- and the what-questions

In Germany, interest in language was fueled by the results of large-scale assessments showing an achievement gap for multilingual students (OECD, 2007).

The MuM research group in Dortmund ("Mathematiklernen unter Bedingungen der Mehrsprachigkeit", or "Mathematics learning under conditions of language diversity") was founded in 2009 in order to improve mathematics classroom practices so that the achievement gap can be reduced. Whereas these early descriptive findings provided limited support for where to start, the following explanatory findings helped to sharpen the focus of the Design Research:

- Explanatory findings on the relevance of the language proficiency factor: Large-scale assessments showed that the achievement gap between monolinguals and multilinguals (OECD, 2007) can be traced back to a language gap. It is not multilingualism but rather language proficiency in the academic language of instruction that influences most mathematics achievement and mathematical language growth (Prediger et al., 2018; replicating findings of Cummins, 1986). So the achievement gap can be statistically explained by a language gap to a large extent. As a consequence, the MuM group identified the target group of multilingual and monolingual language learners for the succeeding Design Research.
- Explanatory findings on the challenging forms of mathematical knowledge: The language gap between students with high and low academic language proficiency is much smaller for procedural skills than for conceptual understanding, even if the items assessing them do not contain any reading obstacles (Prediger et al., 2018). As a consequence, the MuM group decided to focus on conceptual understanding as the learning goal in view (Other goals were also treated, such as reading and proving).
- Explanatory findings on existing classroom practices: In classroom observation studies with socially underprivileged students, conceptual talk turned out to occur much less often than in classrooms attended by socially privileged students. Thus, the achievement gap can be traced back to a learning opportunity gap when classroom practices are limited to procedural discourse (Setati, 2005; DIME, 2007). As a consequence, the MuM group decided that for enhancing language learners' conceptual understanding, the relevant process quality is engaging students in conceptual discourses.

All these considerations led to the two major research questions formulated as:

RQ1- *How can multilingual and monolingual language learners' conceptual understanding be fostered in spite of their limited academic language proficiency in the language of instruction?*

RQ2- *What language demands are relevant for students' learning pathways towards conceptual understanding?*

These research questions aim at generating predictive theory elements that can inform the design of teaching-learning arrangements and concrete classroom practices. To articulate them, categorial, normative, descriptive, and explanatory theory elements are also required. The research that can generate these theory elements is based on functional linguistic background theories, conceptualizing language as a process and as a means for understanding (Solano-Flores, 2010). Since individual learning processes are always embedded in classroom discourses, they have to be combined with interactionist background theories accounting for learning as increasing participation in classroom discourse practices (Quasthoff, Heller & Morek, 2017), such as the combination of theories suggested in Erath, Prediger, Quasthoff and Heller (2018).

Additionally, the didactics of second language education provided the general design principle of macro-scaffolding, according to which language learning

trajectories should be sequenced from students' everyday language via more structured language (i.e., academic language) towards the technical language of mathematics (Gibbons, 2002; similarly described in other terms by Adler, 2001).

However, the general linguistics and language education frameworks were not sufficient for generating a mathematics-specific teaching-learning theory that can inform the design of teaching-learning arrangements for specific mathematical concepts. Hence, theoretical needs occurred to refine ideas and to develop categorial elements that allow more topic-specific research for different mathematical topics. In the discussion document for ICMI-Study 21, *Mathematics Education and Language Diversity* (ICMI, 2009), topic-specific theoretical needs were articulated as follows:

> "But there has been little systematic focus on whether and how the demands of multilingualism and mathematics change with different domains of mathematics. In the mathematics classroom, we are not teaching and learning undefined and vague objects and processes, but mathematical objects and processes, with their own differing and specific natures and structures." (ICMI, 2009, p. 302)

Hence, Bailey (2007) and Prediger and Hein (2017) called for systematic research for specifying language demands. Especially for the topic-specific questions, the existing general frameworks had to be refined and enriched by categorial, descriptive, and explanatory theory elements that allow the topic-specific learning content to be specified and structured. The following section shows how the processes of theorizing involved topic-specific and topic-independent theory elements.

3.2. Insights into steps of theorizing in a series of Design Research projects

Based on these considerations, Design Research projects have been completed in the MuM group for the mathematical topics of *fractions* (Prediger & Wessel, 2013; Wessel, 2015), *percentages* (Pöhler & Prediger, 2015), *variables* (Prediger & Krägeloh, 2015), *functions* (Prediger & Zindel, 2017) and *proving* (Prediger & Hein, 2017). Currently, Design Research projects are being conducted for qualitative calculus (Şahin-Gür & Prediger, 2018), probability, proportionality, and arithmetical expressions and laws. All of these projects combine several design experiment cycles, some of them in laboratory settings (i.e., with 2-4 students and the researchers as design experiment leaders) and most of them later in whole-class settings (i.e., with the teachers as design experiment leaders). These experiments are videotaped and analyzed with respect to successively refined research questions aiming at articulating, testing, and refining conjectures for explanatory and predictive theory elements.

In the following, the work on theorizing in this series of Design Research projects is sketched in order to illustrate typical theorizing processes, which are later summarized in Table 3. The projects started from the *normative theory element*:

> Students should develop conceptual understanding for mathematical concepts (e.g., understanding equivalence of fractions) because this is required for applying mathematical knowledge (according to Hiebert & Carpenter, 1992).

This normative theory element contains the key *categorial element* of conceptual understanding (Hiebert & Carpenter, 1992) and gains its relevance for the specific target group of language learners by the cited *explanatory theory elements*:

> Language gaps particularly occur for conceptual understanding for which underprivileged students seem to have less learning opportunities in classroom interaction (see Section 3.1).

The existing qualitative research (e.g., Setati, 2005; DIME, 2007) suggests the following conjecture for a *predictive theory element*:

> If language learner students are engaged in the discourse practice of explaining meanings, then this can enhance their development of conceptual understanding.

In the beginning, the conjecture could not be tested in regular classrooms because the condition (the "if" in the if-then structure) was hardly observable. So, the Design Research studies in laboratory settings set out to create teaching-learning arrangements in which the condition is satisfied before the conjectured effects could be investigated (interventionist nature of Design Research; see Cobb et al., 2003). Their design followed three design principles (DP) that were rather vague and humble in the beginning (and therefore not in the final logical structure) and successively refined with respect to the target group of language learners and the mathematical concepts in view (similarly articulated by Moschkovich, 2013):

> *DP1. Relating semiotic representations (symbolic, graphical, and concrete manipulative representations) and different language registers (everyday language, school academic language, and technical language).*
>
> *DP2. Engaging students in rich discourse practices, especially in explaining meanings.*
>
> *DP3. Macro-scaffolding: Sequence language-learning opportunities from students' everyday language resources via more structured academic language towards technical language.*

DP1, relating representations and registers, traces back to the 1960s, when the general design principle of using multiple representations in order to enhance students' conceptual understanding was articulated (Bruner, 1966; Lesh, 1979). With respect to the target group of language learners and the background theory that emphasizes language as a resource for meaning making, this design principle was refined into the principle of relating representations and registers (Prediger & Wessel, 2013; Prediger, Clarkson & Bose, 2016). The term "relating" was prioritized over "switching between" due to explanatory findings that conceptual understanding is endangered when students see no connections between different registers and representations (Cramer et al., 1997; Prediger & Wessel, 2013). The design principle was elaborated from prior design principles by unifying traditions and integrating empirical research results. However, it can only be applied successfully for a specific mathematical topic with a more detailed specification of which representations and language means are most relevant for this topic (e.g., the fraction bar or the graphical scaffolds for proving).

DP2, engaging students in rich discourse practices, could be derived from existing research (DIME, 2007; Setati, 2005; Adler, 2001). However, the categorial element discourse practices had to be tightened regarding its theoretical background in order to specify what the design principle means and why it is crucial. This included a theoretical explanation of how individual discourse competence and interactively established discourse practice are intertwined in individual and social perspectives and in the knowledge constitution of mathematical conceptual knowledge, for which the discourse practice of explaining meanings is highly relevant (Erath et al., 2018).

The main idea of DP3, the design principle of macro-scaffolding, had already been articulated in different contexts (e.g., Gibbons, 2002, for second language education; Adler, 2001, for second language learners in mathematics). However, the categories of sequencing and registers were too general to inform the topic-specific design of

language learning opportunities, and the relation to the conceptual learning trajectories was not yet part of the theories. The necessary empirically grounded refinement of the design principles first required their realization in teaching-learning arrangements for specific topics. So that task design could follow well-established learning trajectories, we started the projects with two topics for which the specification of aspects on a conceptual learning trajectory has been well provided by the mathematics education literature: fractions (Cramer et al., 1997; Lesh, 1979) and algebraic expressions (Mason et al., 1985). The empirical analyses of the initiated teaching-learning processes (Prediger & Wessel, 2013; Wessel, 2015, on fractions and Prediger & Krägeloh, 2015, on algebraic expressions) revealed similar findings: Engaging students in rich discourse practices involved two different discourse practices: 1) reporting calculation strategies and procedures and 2) explaining meanings. Whereas the former was empirically identified as easier and more familiar to most language learners, the latter was very difficult and less familiar. The crucial explanatory finding with respect to the role of language was that language learners' everyday language resources were often not sufficient for expressing their informal ideas in the academic meaning-making practices. Hence, a mediating language between the students' everyday language resources and the target language had to be specified and characterized (Prediger & Krägeloh, 2015).

By analyzing from a functional perspective the language means used by the most successful students whose learning pathways corresponded best with the intended conceptual learning trajectories, we could specify that these students utilized a specific vocabulary: For instance, to make sense of the equivalence of fractions, students require the fraction bar or a rectangle as a graphical representation (see Figure 3), but also vocabulary such as "part of the whole," "structuring the whole into finer grained pieces." In a process of grounded theory, these elements were identified and later termed the *meaning-related vocabulary* (Wessel, 2015; Pöhler & Prediger, 2015).

This invention of a new categorial element turned out to be highly relevant for identifying language means that can support discourse practices, as it is substantially different from the formal vocabulary (such as "numerator," "denominator," "to multiply by"), which is most often focused on by teachers (Prediger, 2019).

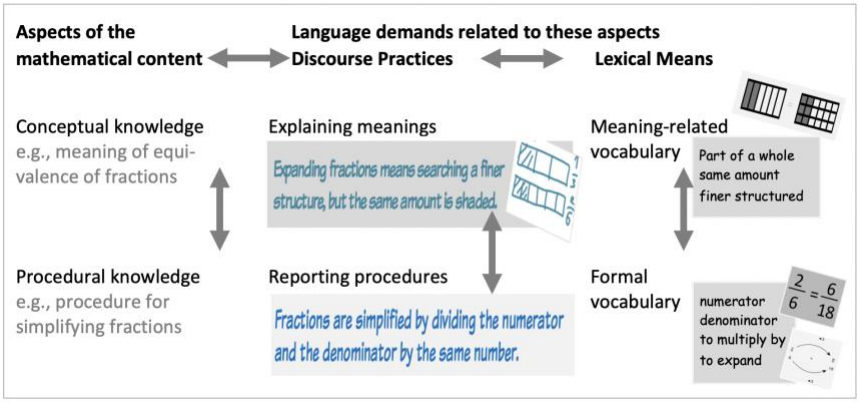

Figure 3. Categories for specifying the learning content on the conceptual and language sides

These categories and their relation (Figure 3) resonate with the underlying functional perspective on language (for identifying the language demands most relevant for specific content goals, see Solano-Flores, 2010) and support to account for the interplay of the discourse level of language (in which discourse practices are the

crucial category) as well as the vocabulary level (which are not considered in a self-contained way, but as language means for articulating distinct discourse practices).

One of Wessel's (2015) major results was the explanatory conjecture that meaning-related vocabulary might be a key to language learners' access to the discourse practice of explaining meanings and might therefore a crucial step in their learning pathways towards conceptual understanding. After these categories, prescriptive conjectures (design principles), and explanatory conjectures were established in the Design Research project on fractions (Wessel, 2015), they were systematically applied and elaborated to structure the learning content in the next project on percentages.

Starting from predictive design heuristics developed in Realistic Mathematics Education (the level principle for structuring an intended conceptual learning trajectory; see Gravemeijer, 1999; van den Heuvel-Panhuizen, 2003), the macro-scaffolding principle (here DP3) could be refined by combing the conceptual learning trajectory with a language learning trajectory in which the general sequence from everyday language via more structured academic language towards technical language could now be realized topic specifically as a sequence of discourse practices and corresponding vocabularies (see Figure 4).

	Conceptual learning trajectory towards a mathematical concept (e.g. percentages)	**Language learning trajectory for different discourse practices**
Level I	Constructing first meanings for the problems and representations (here for percentages in the download bar)	Intuitive use of *students' everyday resources* for discussing first ideas (e.g. "how much was downloaded", "how much is left")
Level II	Developing informal strategies in the context and graphical representation (here for determining rates and amounts)	Establish common *meaning-related vocabulary* for explaining meanings (e.g. part of the whole, old price, new price, discount)
Level III	Developing more formal procedures, independent from context & representations (e.g. proportional reasoning)	Introduce *formal vocabulary* in the technical register for reporting procedures (e.g. base, amount, rate)
Level IV	Applying the mathematical concepts and procedures to more complex problems (also in non-familiar contexts)	Introduce extended reading vocabulary for comprehending texts on non-familiar contexts (e.g. VAT, net, fat content)

Figure 4. Embedding the categorial distinctions from Figure 3 into the structuring of a dual learning trajectory according to the macro-scaffolding principle

Whereas Figure 3 mainly contained explanatory elements, Figure 4 combines them into *predictive theory elements*, claiming that the combined conceptual and language learning trajectory can provide a better access for language learners. Hence, this project started to investigate the conjecture for a *predictive theory element:*

Conjecture: If the learning content is structured along the intended dual learning trajectory with its specific emphasis on the discourse practice of explaining meanings and the corresponding meaning-related vocabulary, then language learners' access to conceptual understanding can be fostered.

As Cobb, Jackson and Dunlap (2016) have pointed out, elaborating conjectures on learning trajectories involves

- "Demonstrating that the students would not have developed particular forms of mathematical reasoning but for their participation in the design study.
- Documenting how each successive form of reasoning emerged as a reorganization of prior forms of reasoning.

- Identifying the specific aspects of the classroom learning environment that were necessary rather than contingent in supporting the emergence of these successive forms of reasoning." (p. 490).

Hence, the Design Research project on percentages investigated the predictively conjectured learning trajectories and design principles by

- designing a teaching-learning arrangement in line with this dual learning trajectory and the three design principles DP1, DP2, and DP3 and
- qualitatively investigating the students' meaning-making processes by means of semiotic chains (Presmeg, 1998), as they capture the individual concept and language learning pathways.

Earlier design experiment cycles showed that students could not follow the conceptual learning trajectory without the discursive and vocabulary support to explain meanings. With the developed refined categories, the generic questions in Table 2, "What learning pathways do students usually take along the intended learning trajectory? And how does that relate to the intended learning trajectory?" could be refined topic specifically: The learning trajectory consisted of a conceptual trajectory towards percentages. The students' individual learning pathways were captured by a qualitative analysis with semiotic chains as the analytic tool to trace students' meaning-making processes. For each mathematical concept (base, rate, and amount), this tool identified the language means used by the student Beren for her individual meaning constructions (see Figure 5). The interpretative results showed that Beren's (and also other students') individual learning pathways were quite aligned with the intended language learning trajectory, starting from students' everyday language resources (colored in white in Figure 5), the commonly established meaning-related vocabulary (in light grey), and the formal vocabulary (in dark grey). These findings were interpreted as a first qualitative evidence for the conjecture.

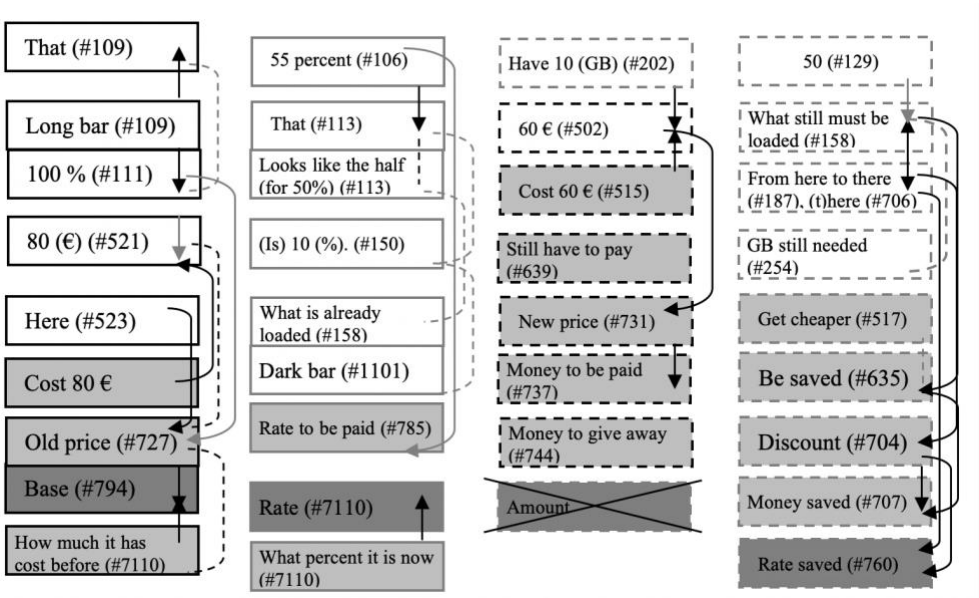

Figure 5. Semiotic chains capturing individual language means for meaning making (Pöhler & Prediger, 2015, p. 1717; arrows in expressions used for explaining later expressions)

Quantitative evidence for the functioning of the design (with its principles and structuring) cannot be provided by the research format of Design Research. That is why the succeeding study was conducted as a classical quasi-randomized control trial with 594 students. The intervention group was taught using the teaching-learning

arrangement that followed the intended dual learning trajectory towards percentages, while the control group used the traditional textbook curriculum. The ANOVA with repeated measurements from pre-test to post-test showed that the intervention group had significantly higher learning gains than the control group ($F(1, 678)=18$, $p<0.001$), even if the effect was small ($\eta^2=0.036$; Prediger & Neugebauer, 2019).

3.3. Looking back

Although this quantitative evidence for the predictive theory element could only be provided by a quasi-randomized controlled field trial, the Design Research in the earlier studies was necessary: 1) to find the most relevant focus of the Design Research, which requires a relevant explanatory focus and well-defined goals; 2) to coin the categorial elements for capturing the most important aspects of the learning content (conceptual understanding, discourse practices of reporting procedures and explaining meanings, and formal vocabulary and meaning-related vocabulary); 3) to reveal the explanatory findings, which connect the different aspects of the learning content, and to refine the design principles; 4) to develop a teaching-learning arrangement in which all descriptive and explanatory findings enter into the relevant predictive conjectures; and 5) to generate explanatory theory elements that justify the predictive conjecture (because the controlled trial confirms effectiveness but does not explain it).

This exemplary insight into a series of Design Research projects provides an example for typical theorizing processes for the successive intertwinement of theory elements that in the beginning are unconnected and vague. Obviously, the research processes are never as linear as they are reported: They always contain deviations, attempts at categorizing that are later refuted due to missing explanatory power, and so on. However, typical steps of theorizing can be seen in Table 3.

Table 3. Typical theorizing steps in Didactical Design Research (examples for percentages)

Typical steps of theorizing	Examples
Identifying an interesting phenomenon and developing categories for describing and explaining it	• Language learners' obstacles in engaging in conceptual discourse practices
Refining categories in order to increase their explanatory power	• "Discourse practice"→ *refined* into "reporting procedures" versus "explaining meanings" • "School academic register in general"→ *refined* into "explaining meanings and meaning-related vocabulary" • "Meaning-related vocabulary"→ topic specifically *refined* into "part of the whole" and "old price vs. new price" for percentages
Connecting two descriptive elements to explanatory elements	• "Language learners do not participate in explaining meanings" and "the formal vocabulary does not support explaining meanings"→ *connected* into "Language learners cannot engage in explain meanings when they miss meaning-related vocabulary"
Transforming an explanatory theory element into a normative element	• "Missing opportunities for participating in conceptual discourse practices to explain the conceptual achievement gaps of language learners"→ *transformed* into the process

	quality "Wide participation in conceptual discourse practices is the process quality to aim at"
Transforming an explanatory theory element into a conjecture for a predictive theory element	• "Language learners cannot engage in explain meanings when they miss meaning-related vocabulary"→ *transformed* into "If we focus the language learning trajectory on establishing common meaning-related vocabulary, then students' conceptual learning pathways can be strengthened"
Refining a predictive theory element by adding a qualifier	• "If we focus the language learning trajectory on establishing common meaning-related vocabulary, then students' conceptual learning pathways can be strengthened"→ *refined* by adding the qualifier "Condition of success: Meaning-related vocabulary is systematically connected to formal vocabulary"
Refining a normative or predictive theory element on design (how) by integrating an explanatory theory element on the content (what)	• Design principle of relating representations and registers→ *refined* by topic-specific argument "Relating registers is crucial for strengthening students' meaning-making processes by the meaning-related vocabulary of 'part of the whole and its finer structures,' as this is crucial for understanding the relation of amount and base in percentages"
Connecting by integrating what- and how-aspects into topic-specific explanatory and predictive theory elements	• The percent bar can serve as a graphical scaffold to construct meanings in situations and for amount and base. Therefore, it is the central graphical representation in the design principle of relation of representations and should be strengthened by collecting joint meaning-related vocabulary. The design principle of macro-scaffolding is therefore sharpened by the levels of the language trajectory in Figure 4.
…	

4. Theorizing as successively elaborating networks of intertwined theory elements

Even if the preceding section could only provide rough sketches of the approaches and results from a series of topic-specific Didactical Design Research projects on language-responsive mathematics teaching-learning arrangements, the example might serve as an illustration of the complexities of theorizing.

Unlike sometimes implicitly assumed (e.g., by US Congress, 2001), providing a theoretical base for teaching-learning arrangements does not only involve the falsification or validation of predictive hypotheses. This is only the very last step. Instead, theorizing starts much earlier and contains several theorizing steps (Table 3), for example, setting and refining the goals (normative theory elements), describing the aspects of the learning content and typical phenomena of the teaching-learning processes, and explaining typical obstacles on students' learning pathways, typical patterns of the functioning of design elements and design principles, and conditions of success. These normative, descriptive, and explanatory theory elements always require suitable categories in which phenomena can be captured and goals can be articulated.

Usually, theorizing in Design Research is an iterative process, starting with vague design heuristics and humble categories (Cobb et al., 2003, p. 9). By several theorizing steps (Table 3), the categories can be iteratively refined and explanatory theory elements can be more and more connected into a web of categories and propositions

(in the status of conjectures or with empirical evidence for their validity). Specifically, good Didactical Design Research requires the succeeding intertwinement of what- and how-questions that cannot be separated in the end of some projects.

Although topic-specific Didactical Design Research can only reveal contributions to local theories (bound to the specific topics and the specific teaching-learning arrangements in which they were investigated), the generated theory elements can inform research beyond the topic-specific theory. The transfer of topic-specific constructs to new topics must itself be a part of a new Design Research project, but when research groups conduct series of Design Research projects for different mathematical topics, they can develop theory elements with an increasing range and specify in greater detail what is really topic-specific.

This shows why theorizing is always a long-term process that transcends the single project.

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References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht, Netherlands: Kluwer.

Bailey, A. L. (2007). *The language demands of school: Putting academic English to the test*. New Haven, CT: Yale.

Bakker, A. (2018). *Design Research in education*. London, England: Routledge.

Barab, S., & Squire, K. (Eds.) (2004). Design-Based Research: Clarifying the terms. *Journal of the Learning Sciences, 13*(1), Special Issue.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N. …Villavicencio, M. (Eds.) (2016). *Mathematics education and language diversity*. *The 21st ICMI Study*. Dordrecht, Netherlands: Springer.

Beck, K., & Krapp, A. (2006). Wissenschaftstheoretische Grundfragen der Pädagogischen Psychologie. In A. Krapp & B. Weidenmann (Eds.), *Pädagogische Psychologie* (4th ed., pp. 33-73). Weinheim, Germany: Beltz.

Blum, W., Artigue, M., Mariotti, M. A., Sträßer, R. & van den Heuvel-Panhuizen, M. (Eds.) (2019). *European traditions in Didactics of Mathematics*. Cham, Switzerland: Springer.

Bruner, J. (1966). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32(1)*, 9-13.

Cobb, P., Jackson, K., & Dunlap, C. (2016). Design Research: An analysis and critique. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 481-503). New York: Routledge.

Confrey, J. (2006). The evolution of design studies as methodology. In K. R. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 135-152). Cambridge, MA: Cambridge University Press.

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational Number Project: Fraction lessons for the middle grades - Level 1*. Dubuque, IA: Kendall/Hunt Publishing.

Creswell, J.W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Cummins, J. (1986). Language proficiency and academic achievement. In J. Cummins & M. Swain (Eds.), *Bilingualism in education: Aspects of theory, research and practice* (pp. 138-161). London, England: Longman.

DIME - Diversity in Mathematics Education Center for Learning and Teaching (2007). Culture, race, power in mathematics education. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 405-433). Charlotte, NC: Information Age.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences, 13*(1), 77-103.

Duit, R., Gropengießer, H., & Kattmann, U. (2005). Towards science education that is relevant for improving practice: The model of educational reconstruction. In H. E. Fischer (Ed.), *Developing standards in research on science rducation* (pp. 1-9). London, England: Taylor & Francis.

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: The case of explaining to learn and learning to explain. *Educational Studies in Mathematics, 99*(2), 161-179.

Gibbons, P. (2002). *Scaffolding language, scaffolding learning. Teaching second language learners in the mainstream classroom*. Portsmouth, England: Heinemann.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1*(2), 155-177.

Gravemeijer, K., & Cobb, P. (2006). Design Research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research: The design, development and evaluation of programs, processes and products* (pp. 17-51). London, England: Routledge.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Hußmann, S., & Prediger, S. (2016). Specifying and structuring mathematical topics – a four-level approach for combining formal, semantic, concrete, and empirical levels exemplified for exponential growth. *Journal für Mathematik-Didaktik, 37*(Suppl. 1), 33-67.

Hußmann, S., Thiele, J., Hinz, R., Prediger, S., & Ralle, B. (2013). Gegenstandsorientierte Unterrichtsdesigns entwickeln und erforschen - Fachdidaktische Entwicklungsforschung im Dortmunder Modell. In M. Komorek & S. Prediger (Eds.), *Der lange Weg zum Unterrichtsdesign: Zur Begründung und Umsetzung genuin fachdidaktischer Forschungs- und Entwicklungsprogramme* (pp. 19-36). Münster, Germany: Waxmann.

ICMI (2009/2016). ICMI Study 21 Discussion Document: Mathematics Education and Language Diversity. Re-published in Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N. … Villavicencio, M. (Eds.). (2016). *Mathematics education and language diversity: The 21st ICMI Study* (pp. 297-308). Dordrecht, Netherlands: Springer.

Lesh, R. (1979). Mathematical learning disabilities. In R. Lesh, D. Mierkiewicz, & M. Kantowski (Eds.), *Applied mathematical problem solving* (pp. 111-180). Columbus, OH: Ericismeac.

Mason, J., & Waywood, A. (1996). The role of theory in mathematics education and research. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 1055-1089). Dordrecht, Netherlands: Kluwer.

Mason, J., Graham, A., Pimm, D., & Gowar, N. (1985). *Routes to/Roots of Algebra*. Milton Keynes, England: University Press.

McKenney, S., & Reeves, T. (2012). *Conducting educational design research*. London, England: Routledge.

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English Language Learners. *Journal of Urban Mathematics Education, 6*(1), 45-57.

Niss, M. (2007). Reflections on the state of and trends in research in mathematics teaching and learning. From here to utopia. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1293-1312). Reston, VA: NCTM.

OECD (2007). *PISA 2006. Science competencies for tomorrow's world* (Vol. 2). Paris: OECD.

Planas, N. (2018). Language as resource: A key notion for understanding the complexity of mathematics learning. *Educational Studies in Mathematics, 98*(3), 215-229.

Plomp, T., & Nieveen, N. (2013). *Educational Design Research: Illustrative cases*. Enschede: SLO, Netherlands Institute for Curriculum Development.

Pöhler, B., & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories. *Eurasia Journal of Mathematics, Science, & Technology Education, 11*(6), 1697-1722.

Prediger, S. (2015). Theorien und Theoriebildung in didaktischer Forschung und Entwicklung. In R. Bruder, L. Hefendehl-Hebeker, B. Schmidt-Thieme & H.-G. Weigand (Eds.), *Handbuch der Mathematikdidaktik* (pp. 643-662). Heidelberg, Germany: Springer.

Prediger, S. (2019, online first). Promoting and investigating teachers' pathways towards expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal*. doi: 10.1007/s13394-019-00258-1

Prediger, S., & Hein, K. (2017). Learning to meet language demands in multi-step mathematical argumentations: Design Research on a subject-specific genre. *European Journal of Applied Linguistics, 5*(2), 309-335.

Prediger, S., & Krägeloh, N. (2015). The epistemic role of languages while constructing meaning for the variable as generalizers. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms* (pp. 89-108). Rotterdam, Netherlands: Sense Publishers.

Prediger, S., & Neugebauer, P. (2019, in preparation). *Can monolingual and multilingual students equally profit from a language-responsive instructional approach for percentages? Differential effectiveness in a field trial study*. Manuscript in preparation.

Prediger, S., & Wessel, L. (2013). Fostering German language learners' constructions of meanings for fractions – Design and effects of a language- and mathematics-integrated intervention. *Mathematics Education Research Journal, 25*(3), 435-456.

Prediger, S., & Zindel, C. (2017). School academic language demands for understanding functional relationships: A design research project on the role of language in reading and learning. *Eurasia Journal of Mathematics, Science and Technology Education, 13*(7b), 4157-4188.

Prediger, S., & Zwetzschler, L. (2013). Topic-specific Design Research with a focus on learning processes: The case of understanding algebraic equivalence in grade 8. In T. Plomp & N. Nieveen (Eds.), *Educational Design Research: Illustrative cases* (pp. 407-424). Enschede: SLO, Netherlands Institute for Curriculum Development.

Prediger, S., Clarkson, P., & Bose, A. (2016). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas … M. Villavicencio (Eds.), *Mathematics education and language diversity. The 21st ICMI Study* (pp. 193-215). Dordrecht, Netherlands: Springer.

Prediger, S., Wilhelm, N., Büchter, A., Gürsoy, E., & Benholz, C. (2018). Language proficiency and mathematics achievement – Empirical study of language-induced obstacles in a high stakes test, the central exam ZP10. *Journal für Mathematik-Didaktik, 39*(Supp. 1), 1-26.

Presmeg, N. C. (1998). A semiotic analysis of students' own cultural mathematics. Research forum report. In A. Olivier & K Newstead (Eds.), *Proceedings of the 22nd Conference of PME* (Vol. 1, pp. 136-151). Stellenbosch, SA: University.

Quasthoff, U., Heller, V., & Morek, M. (2017). On the sequential organization and genre-orientation of discourse units in interaction. *Discourse Studies, 19*(1), 84-110.

Radford, L., & Barwell, R. (2016). Language in mathematics education research. In A. Gutiérrez, G. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education. The journey continues* (pp. 275-313). Rotterdam, Netherlands: Sense Publishers.

Şahin-Gür, D., & Prediger, S. (2018). "Growth goes down, but of what?" A case study on language demands in qualitative calculus. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 99-106). Umeå, Sweden: PME.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447-466.

Solano-Flores, G. (2010). Function and form in research on language and mathematics Education. In J. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 113-149). Charlotte, NC: IAP.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research. Grounded theory procedures and techniques*. Newbury Park, CA: Sage.

Thiel, C. (1996). Theorie. In J. Mittelstraß (Ed.), *Enzyklopädie Philosophie und Wissenschaftstheorie* (4th ed., pp. 260-270). Stuttgart, Germany: Springer.

Toulmin, S. E. (1969). *The uses of argument*. Cambridge, MA: Cambridge University Press.

US Congress (2001). *No child left behind act*. Washington D.C.

van den Akker, J. (1999). Principles and methods of development research. In J. v. Akker, R. M. Branch, K. Gustafson, N. Nieveen, & T. Plomp (Eds.), *Design approaches and tools in education and training* (pp. 1-14). Boston, MA: Kluwer.

van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Educational Design Research: The design, development and evaluation*. London, England: Routledge.

van den Heuvel-Panhuizen, M. (2003). The didactical use of models in Realistic Mathematics Education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35.

van den Heuvel-Panhuizen, M. (2005). Can scientific research answer the 'what' question of mathematics education? *Cambridge Journal of Education, 35*(1), 35-53.

Wessel, L. (2015). *Fach- und sprachintegrierte Förderung durch Darstellungsvernetzung und Scaffolding. Ein Entwicklungsforschungsprojekt zum Anteilbegriff*. Heidelberg, Germany: Springer Spektrum.

Yin, R. K. (1994). *Case study research* (2nd ed.). Thousand Oaks, CA: Sage.

Author's contact details

Susanne Prediger, Technische Universität Dortmund (Germany)
prediger@math.uni-dortmund.de

Theorizing in Design Research

Methodological reflections on developing and connecting theory elements for language-responsive mathematics classrooms

Susanne Prediger, TU Dortmund

Extended abstract

Topic-specific Didactical Design Research is a research methodology with two aims, 1) designing and improving teaching-learning arrangements and 2) generating theoretical contributions for understanding the initiated teaching-learning processes for a certain topic. The article provides methodological considerations of theory elements and then illustrates the abstract considerations on one exemplary field of Design Research for elaborating the meaning of theorizing within this methodology. Starting from a distinction of categorial, descriptive, explanatory, normative and predictive theory elements with their different functions and logical structures, it is shown that these kinds of theory elements rarely occur in isolated ways, but are consequently intertwined in Design Research. A number of examples show that theorizing in Design Research studies can be conceived as a process of successively developing and connecting theory elements, for the how-questions (the educational rationales for the teaching-learning arrangements) and the what-questions (the didactical structuring of the learning content). In this respect, all Design Research is an iterative combination of research-based design and design-based research. Research-based design works with existing theories and design-based research aims at generating, testing and refining theories, in an interplay that creates the possibilities for theory elements to guide the how- and the what-questions. The final part of the article discusses the case of topic-specific Didactical Design Research for language-responsive classrooms, particularly in relation to the design of teaching-learning arrangements and concrete classroom practices for fostering language learners' conceptual understanding of fractions, variables, and percentages. The research questions underlying the case are as follows: i) How can multilingual and monolingual language learners' conceptual understanding be fostered in spite of their limited academic language proficiency in the language of instruction? ii) What language demands are relevant for students' learning pathways towards conceptual understanding?