Task design to foster the competence in social decision making on mathematics education

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Diseño de tareas para promover la competencia en la toma de decisiones sociales en educación matemática

Resumen

Conforme la sociedad avanza, es cada vez más importante tratar matemáticamente una diversidad de aspectos, tener en cuenta riesgos e incertidumbres, clarificar los fundamentos de nuestra participación en la emisión de juicios y la toma de decisiones, junto con llevar a cabo deliberaciones críticas. Esta competencia se resume como "toma decisiones". Presentamos el diseño de una tarea sobre cómo salvar más vidas con desfibriladores externos automáticos, cuyo objetivo es promover dicha competencia en clases de matemáticas. Desde la perspectiva metodológica, usamos el Estudio de Clases para anticipar minuciosamente los procesos de los estudiantes al encontrar y resolver problemas. Revisamos el diseño de acuerdo con distintos puntos de vista, entre ellos: "¿En qué ocasiones y cómo deberían los estudiantes estudiar en el aula?" o "¿Qué tipo de materiales deberían ser presentados y cuándo?"

Palabras clave. Toma de decisiones sociales; Estudio de clases; tarea sobre desfibriladores externos automáticos; diagramas de Voronoi.

Task design to foster the competence in social decision making on mathematics education

Abstract

As society continues to progress, it is becoming increasingly more important to mathematically deal with issues, take risks and uncertainty into account, clarify the grounds for participating in judgements or decision making, and make critical deliberations. This competence is defined as "decision making". We present the design of a task about saving more lives with automatic external defibrillators (AEDs) in order to foster this competency in mathematics lessons. In methodological terms, we use Lesson Study to carefully anticipate the process of students' finding and solving problems. We review the design from different viewpoints, such as "What times and how should all the students in the class study?" or "When and what kind of materials should be presented?"

Keywords. Social decision making; Lesson Study; task on automatic external defibrillators; Voronoi diagrams.

Desenho de tarefas que promovam competências na tomada de decisões sociais em educação matemática

Resumo

À medida que a sociedade avança, torna-se cada vez mais importante tratar matematicamente uma diversidade de aspetos, ter em conta riscos e incertezas, clarificar os fundamentos da nossa participação

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na emissão de juízos e na tomada de decisões, juntamente com levar a cabo deliberações críticas. Esta competência resume-se como "tomada de decisão". Apresentamos o esquema de uma atividade sobre como salvar mais vidas com desfibrilhadores externos automáticos, cujo objetivo é promover aquela competência nas aulas de matemática. Desde uma perspetiva metodológica, usamos o Estudo de Classe para antecipar cuidadosamente os processos dos alunos ao encontrar e resolver problemas. Revimos o esquema de acordo com distintos pontos de vista, entre eles: "Em que ocasiões e como deveriam os alunos estudar nas aulas?" ou "Que tipo de materiais deveriam ser apresentados e quando?"

Palavras chave. Tomada de decisões sociais; Estudo de Classe; tarefa sobre desfibrilhadores externos automáticos; diagramas de Voronoi.

Conception de tâches pour favoriser la compétence de prise de décisions sociales en didactique des mathématiques

Résumé

Au fur et à mesure que la société progresse, il devient de plus en plus important de traiter mathématiquement un certain nombre de questions, de prendre en considération les risques et l'incertitude, de clarifier les motifs de notre participation dans l'émission de jugements ou la prise de décisions, et de réaliser des débats cruciaux. Cette compétence sera nommée « prise de décision ». Nous présentons la conception d'une tâche visant à sauver plus de vies avec des défibrillateurs externes automatiques, et dont l'objectif est de favoriser cette compétence dans le cours de mathématiques. Sur le plan méthodologique, nous utilisons la démarche des Lesson Studies pour anticiper soigneusement les processus de rencontre et de résolution de problèmes par les élèves. Nous examinons cette conception selon différents points de vue, tels que « À quels moments et comment les élèves doivent-ils étudier en classe » ou « Quand et quel type de matériels doit-on présenter ? »

Mots-clés. Prise de décision sociale; Lesson Study; tâche sur les défibrillateurs externes automatiques; diagrammes de Voronoi.

1. Introduction

In Japan, task designers' outcomes are shown in mathematics textbooks, especially for primary school (schoolchildren from 6 to 12 years old) and junior high school (students from 12 to 15 years old). Textbooks are carefully designed to include not only good problems, but also key question; i.e., questions that would motivate students' thinking at a particular point in the lesson (see Figure 1). When designing a lesson, it is crucial for designers to anticipate students' thinking. Much attention is paid to every little detail during the lesson so that it enables the kind of envisaged thinking. In the background, a common agreement has been reached about mathematics teaching and learning based on problem solving, given the conviction that problem solving is powerful to foster students' understanding of mathematical contents (Hino, 2007). Japanese educators believe that the high quality of teaching and learning at both primary and junior high school levels is based on the textbooks in Japan. Thus textbook writing is the main work of designers in Japan.

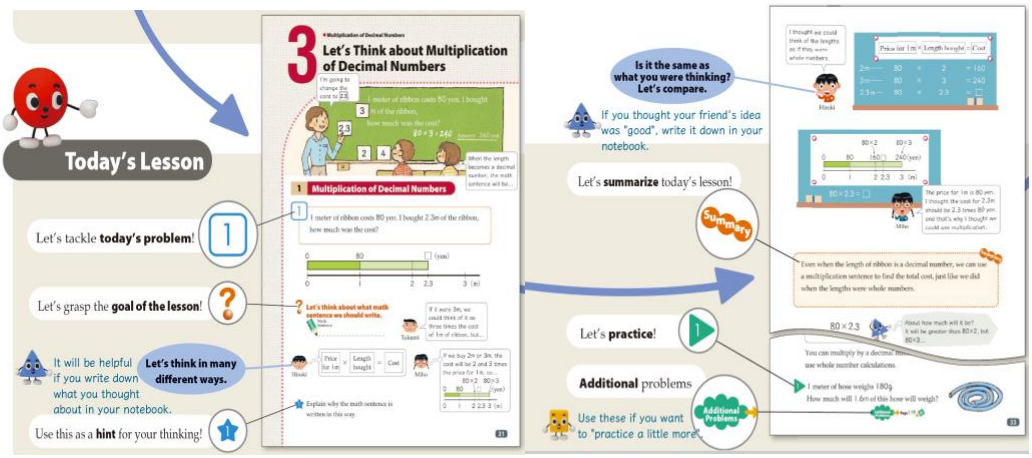

Figure 1. Excerpts from a Japanese textbook (Fujii et al., 2011, pp. 2-3)

2. Design framework and methodology

To understand design work in Japan, two main theoretical approaches should be considered. The first has to do with the way problem solving has been developed in Japan, which has given rise to a well-defined structure of a problem solving-based lesson: the so-called structured problem-solving approach. The second is related to the collective and thoughtful nature of the design process, and also to its connection with actual classrooms. Designing new tasks is normally done by the teams integrated by school teachers and university researchers following the lesson study cycle. We outline the main ideas of both approaches in the next sections.

2.1. Problem-solving lessons in Japan: The structured problem-solving approach

The Japanese structured-problem solving (SPS) methodology has been described extensively in research, and we refer to Stiegler and Hierbert's (1999) seminal work. Shimizu (1999) and Hino (2007) place the origin of this methodology in the 1960s, based on a traditional framework for planning and implementing lessons in Japan, and is clearly influenced by the NCTM's *Agenda for action* (National Council of Teachers of Mathematics, 1980). However in order to be able to really understand it, it is also important to consider, on the one hand, the growing interest in problem solving in curricular reforms in Japan from the 1950s onwards (Hino, 2007), the influence of Polya's work in Japan (Takahashi, 2008), and the collaboration of Japanese researchers with mathematics educators from the U.S.[1] and other countries (Hino, 2007); on the other hand, the impact that lesson study, as an extended professional practice, has had on polishing and disseminating it (Takahashi, 2006).

The SPS method is a didactical model that, albeit not exclusively, is closely connected to the development of a content-oriented epistemology, on the one hand, but which also values thinking processes and problem-solving skills, on the other hand. Thus the main focus lies in a profound understanding of mathematical concepts through engaging students in carefully designed situations (Takahashi, 2006). What the method

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[1] According to Hino (2007, p. 503), the "*first recommendation* [of the NCTM's *Agenda for action* (National Council of Teachers of Mathematics, 1980)] *"Problem solving should be the focus of school mathematics in the 1980s" was received by Japanese mathematics educators as a strong message*".

attempts is that "students create mathematical ideas and knowledge by themselves, by experiencing the process of problem solving" (Hino, 2007, p. 509).

The SPS method relies on the so-called *open-closed tasks* (Souma, 1987). These are tasks that "apart from stimulating conjecture and application of guessing, should lead to multiple methods of solution etc., be constructed so as to later stimulate a discourse on theory that should stay somewhat focused on the well-defined subject that the teacher aims to cover" (Asami-Johansson, 2011, p. 2553). In the same vein, Fujii (2015, p. 276) stresses that: "the task should be understandable by the students with minimal teacher intervention; it should be solvable by at least some students (but not too quickly), and it should lend itself to multiple strategies". Around these tasks, an SPS lesson is structured in different phases (Fujii, 2015, p. 276):

1. Presenting the problem for the day (5-10 min)
2. Problem solving by the students (10-20 min)
3. Comparing and discussing (*neriage*) (10-20 min)
4. Summing up by the teacher (*matome*) (5 min)

2.2. Collective and thoughtful design: The Lesson Study cycle

The design of a Japanese Mathematics textbook is affected by the outcomes of some *Lesson Studies* processes as most designers' careers are teachers and/or discussants (external experts, the so-called *koshi*) in *Lesson Study* cycles.

Lesson Study is not merely about designing or improving a single lesson (Lewis, Perry & Hurd, 2004), but *Lesson Study* incorporates two important features of task design. It firstly involves a thoughtful and careful anticipation of students' thinking processes. Secondly, it includes a detailed description of how the lesson would be implemented (the so-called lesson plan). Thirdly, the task is evaluated during the post-lesson discussion, based on concrete evidence about students' thinking collected during the lesson (Fujii, 2015).

Fujii (2016) defines *Lesson Study* as a collective endeavour that is structured as follows (Figure 2):

1. *Goal setting.* Consider long-term goals for student learning and development. Identify the gaps between these long-term goals and current reality. Formulate the research theme.
2. *Lesson planning.* Collaboratively plan a "research lesson" designed to address the goals. Prepare a "lesson proposal", a document that describes the research theme, content goals, the connections between the current content and related content from former and later grades, the rationale for the chosen approach, a detailed plan for the research lesson, anticipated students' thinking, data collection, and more.
3. *Research lesson.* One team member teaches the research lesson while the other members of the planning team, staff members from across the school and, usually, a *knowledgeable other* observe and collect data.
4. *Post-lesson discussion.* During a formal lesson colloquium, observers share the data from the lesson to enlighten student learning, disciplinary content, lesson and unit design, and broader teaching and learning issues.
5. *Reflection.* Document the cycle to consolidate and carry forward learnings, as

well as new questions for the next Lesson Study cycle. Write a report or bulletin that includes the original research lesson proposal, student data from the research lesson, and reflections on what was learned.

Figure 2. Lesson Study cycle (Fujii, 2016, p. 412)

Lesson Study is a collaborative learning system. The important point to mention here is that Lesson Study is normally used by designers to develop well-crafted lessons to ensure students' understanding of mathematical contents.

In the next sections, we provide an example of the design of a lesson carried out as part of the project "Next-Generation Education Responding to the Society of 2030 to Children, New Educational and Evaluation Models", that the Tokyo Gakugei University is undertaking in collaboration with the OECD, the Japanese Ministry of Education, Culture, Sports, Science and Technology, and the University of Tokyo. This paper forms part of that research.

3. Designing a lesson to foster competence in social decision making

3.1. Background of the design process

The purpose of education is to liberate people. Having said that, as the necessary conditions for a person to become liberated vary depending on the time or society in which (s)he lives, it is natural to think that education cannot but change accordingly.

Globalisation has brought diversity to values, whose renewal is being accelerated by the evolution of information networks. Social issues have become increasingly more diverse and complex, and now interact with one another in a close and complicated manner. In response to this social trend, the OECD has developed "The Future of Education and Skills 2030 Project" since 2015 (OECD, 2018).

The aim of the project is to help countries find answers to two far-reaching questions: What knowledge, skills, attitudes and values will today's students need to thrive in and shape their world? How can instructional systems develop these knowledge, skills, attitudes, and values effectively?

It mentions the need for broader education goals. For instance, it needs to equip students with the skills they need to become active, responsible and engaged citizens. It requires a broad set of knowledge, skills, attitudes and values in action. In response to these demands, what can we do in mathematics education?

Swan (2014) distinguished among the educational goal of the lesson, the products that students might produce to give evidence for achieving that goal, the genres of mathematical tasks that will guide the task design, and the classroom activities that they intend to result from these tasks.

He supposed *factual knowledge and procedural fluency*, *conceptual understanding*, *strategic competence* and *critical competence* as the goals and a framework for the task design. In our study, we add "decision making" to them as *a broad set of knowledge, skills, attitudes and values in action* (OECD, 2018).

The term "decision making" as used herein is employed to refer to the competence activated by individuals in problematic situations in which social decisions need to be taken (Nishimura, 2016, p. 21):

> *Mathematically formulate problems that require decision making in the real world, conduct mathematical approaches, follow the steps to achieve mathematical results, create multiple options, and clarify grounds to build a consensus for a certain decision.*

In this type of decision-making process, whether manifest or potential, various values of the concerned parties are expressed and affect the whole process from the manner of grasping the problems in question to the necessary mathematical selection, usage, interpretation, and consensus building required for problem solving, as well as the content and quality of this decision making.

The lesson we present here was developed mainly by Ms. Chiharu Honda, who works at the Tokyo Gakugei University International School (TGUISS), and by Mr. Keiichi Nishimura, a professor at the Tokyo Gakugei University. Mathematics teachers within TGUISS developed, and are developing, the mathematics textbook for the students through Lesson Study. Consequently, they have some research lessons every year. Mr. Nishimura is also developing tasks with some of them and sometimes gives advice after the research lesson as *Koshi* (an external expert) who is invited by university instructors and supervisors from the Board of Education.

Ms. Honda and Mr. Nishimura reported about the lesson to another colleague, who gave some comments and pieces of advice.

3.2. Outline of the classroom activity

During this classroom activity, the students discuss a social issue, "*Let's save more lives with AEDs.*"

Firstly, the students consider whether an AED (Automated External Defibrillator) is adequately installed in the area where they live. Ideally, AEDs should be placed in each household, but this is impractical. Therefore, students must determine the appropriate intervals within which AEDs should be placed by making trade-offs between ideals and reality.

Secondly, they discuss where to place additional AEDs in groups. They create mathematical models based on their assumptions and the data about the population distribution in the city, and make their decisions in groups. These are the options presented during the class. Afterwards, they compare each group's ideas critically and build a consensus.

They may also be expected to propose the idea that it should be made clear which AED to get whenever one is needed. Here the expectation is that the students will find out the following three facts: (1) that the intersecting points of the two circles of the same radius with two AEDs at the centres are at the same distance from the AEDs; (2) that every point on the line segment between the two intersecting points is at the same distance from the two AEDs; (3) that every point on the extended line of the above-mentioned segment (the perpendicular bisector of the two circles' centres) is at the same distance from the AEDs (Figure 3). This task is also designed for those students who have not studied perpendicular bisectors to review this activity and to learn how to construct drawings. As in Figure 4, the drawing of areas zoned with the nearest AED is shown as a *Voronoi diagram*.

Figure 3. Circular model of the area covered by AEDs

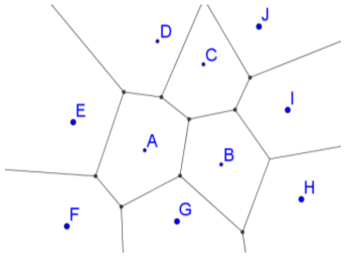


Figure 4. The Voronoi diagram of AEDs

3.3. Goal and plan of the lesson

The lesson was designed to fulfil the following two major goals:

i) Decision making

Students should be able to:

- Read a variety of materials and set up problems related to AEDs
- Explore a satisfactory answer by trading off clarified conditions and values
  ・Make assumptions, set conditions, and determine the AEDs intervals
- Make decisions based on mathematical evidence
  ・Use data on residents' age composition and the like, and consider where to install additional AEDs
  ・Consider the characteristics of perpendicular bisectors and the like, express the proximity of each AED from a certain area in a drawing, and evaluate or change the locations of AEDs
- Discuss critically and from various angles, based on mutually understood values and standpoints.

ii) Attitudes and values to:

- Foster the students' attitudes to formulate and make decisions on social issues as mathematical problems
- Foster the students' thinking that they can improve society through their efforts.

Table 1 outlines the three lessons that were designed around the problem of AEDs in an area of the city.

Table 1. Plan of the lesson

Lesson	Activity
1	Read materials on AEDs and think about the relevant gaps between ideals and reality. Imagine the time or running speed to get an AED and determine installation intervals.

2	Think about how to check whether AEDs are placed within the intervals that the students have decided upon. Read data on residents' age composition and the like, and think about where to add and install AEDs.
3	Think about how to examine which AED is closest to a certain location.

4. Research lesson

The lessons were video-recorded and observed by some teachers. In particular, Lesson 3 was observed by almost 100 external participants in addition to the developing team, because it was offered as an open research lesson.

In this paper, Lesson 2 is described in detail, while Lessons 1 and 3 are only outlined.

The participants in the research lesson class were 28 first-year students at TGUISS (7th grade, Junior High School, aged 12-13) who had not learned the characteristics and construction of perpendicular bisectors.

4.1. Lesson 1 (Outline): "Let's think about the conditions for AEDs installation"

To grasp the situation and enter problematic issues, a collection of graphs was presented to the students. Each graph offered some interesting information about the social issue they had to explore. A first graph informed about the relation between the start time of first aid and the possibility of survival (Figure 5). A second one included information about the survival and social rehabilitation rates due to AEDs. A third one included information about the number of installed AEDs, the number of times AEDs where used, and witnessed cardiopulmonary arrests nationwide. A fourth one was about the places where cardiopulmonary arrests were witnessed. A fifth one contained information about the age-specific numbers of attended sufferers of such a condition.

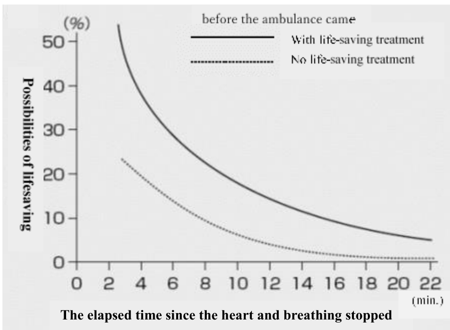


Figure 5. The relationship between the first aid start time and the lifesaving possibility

After examining the graphs, the students were asked to think about the situation and point out some ideas. Some examples of the proposed ideas are the following:

- The sooner the sufferer is treated after the arrest, the more likely he or she is to survive.
- The number of installed AEDs is increasing, but the number of times AEDs were used is not.
- Many sufferers are the elderly, and 75% of arrests occurred at the sufferer's home. There may be no AEDs at or near the places where they are needed.

In response, the students conducted a trade-off between the possible increase in saved lives made by AEDs and the installation cost before determining that the appropriate distance between installed AEDs should be "300 metres." This decision was based on a set of conditions that they considered important: "within 3 minutes" (getting access to an AED should take no longer than 3 min), "running at the speed of 150 metres per minute" (average speed of a person when trying to reach an AED) and "1 minute to remove an AED" (average time needed to take the equipment from its location).

4.2. Lesson 2: "Are there enough AEDs installed in the areas where we live?"

The locations of installed AEDs are confirmed on the map publicised by Nerima Ward, where students live. The aim of the lesson is to analyse whether AEDs are placed with intervals of 300 metres, as students determined in Lesson 1. The three expected responses from the students are indicated below:

A. Use rulers and compasses to check whether each interval between AEDs is within 300 metres.

B. Draw circles of the radius of 300 metres with AEDs as the centre for the check.

C. Draw circles of the radius of 150 metres with AEDs as the centre for the check.

The third strategy (C) is easy to comprehend in visual terms as the parts where the circles do not overlap indicate the areas where there is no AED within the 300-metre range. Therefore, many students are expected to use this method.

After this approach of finding the areas where insufficient AEDs are installed, other factors to consider than installation intervals are deliberated. Looking back on Lesson 1, some of the students are expected to think, "AEDs should be placed in areas where there are many elderly people." Each group of students then proposes three candidate sites where AEDs should be additionally installed together with the reasons, using the map that describes each area in different colours according to the numbers of residents aged 65 years or older. For example, the students may prioritise and choose areas outside any of the circles where there are many residents aged 65 years or older.

In addition, some students may be expected to point out the fact that it is not enough to only add three sites as there are areas with no AED included in any of the circles. This awareness of a problem is taken in account to be utilised in Lesson 3.

In the actual lesson (implemented with the 12-13-year old students at TGUISS), the students used the distributed maps and thought about how to check whether AEDs were installed at 300-metre intervals. They worked individually for 3.5 minutes and in small groups for 2 minutes Those who adopted strategy (A) made presentations before those who adopted strategy (B). Many students supported idea (B). The whole class discussion on this issue was as follows:

S1 *"Every 300 metres", given in writing in the last class, meant that you can go, get an AED and return within 3 minutes if you run 150 metres a minute. At 150 metres, it's 2 minutes there and back, so the place you can get to in that time, within 2 minutes, is an area within 150 metres of an AED. Therefore, if the size of the circles increases, then the circle or the AED can cover the AED which is located here, I think.*

Figure 6. A student explaining her approach to the task

T	*Well, do you have any questions? Are you okay with that? From the last class, you well remember the assumption "walk 150 metres per minute to get an AED". With a radius of 150 metres, you said that you can find the area covered by one AED. Let me draw it. Well, this circle shows the areas covered by this AED.*
T	*Oh, here. Yes, then, what did S2 say now, immediately after having seen S1's presentation?*
S2	*For example, if these circles overlap, the area within the circles is covered, but if they are separated, those who are in the uncovered areas cannot use AEDs: they cannot get back within 300 m.*
T	*We can see this. Is that clear? Okay then, almost all the AEDs are placed within 300 m of the school, but are there any problems? The distances are reasonable, but where they are placed are… S3.*
S3	*The left side. There aren't any AEDs in the residential area.*
T	*That's right. As a whole, the fact that there are no AEDs in the residential area is a problem. Are there any problems where the AEDs are? S4.*
S4	*Too much overlapping.*
T	*Oh, they overlap too much.*
S4	*Three or four circles overlap on the upper right side of the map, so the AEDs should be a little further apart.*
T	*Okay, you mean they should be separated a bit more. Too many circles overlap in a certain area. S5, do you have the same idea? Okay, yours is different.*
S5	*All the AEDs are placed in schools or big buildings. As we learned in the previous lesson, many people need AEDs at home. We cannot utilise them immediately if they are in big buildings.*

Student S1 reviewed the condition of an AED being installed within intervals of 300 metres and explained that she had drawn circles with a radius of 150 metres with AEDs at the centre, which indicated the ranges covered by each AED. In the previous lesson, the sites where AEDs have become necessary were considered as the starting point to calculate the round-trip distance within 2 minutes. However, Student 1 considered an AED as a new starting point here.

Students S2, S4 and S5 examined this drawing critically and arrived at more concrete problematic points related to the locations of AEDs.

Knowing the problem that the students pointed out, the teacher displayed a map from jSTAT MAP (https://jstatmap.e-stat.go.jp/gis/nstac) showing the population of each area by age (Figure 7).

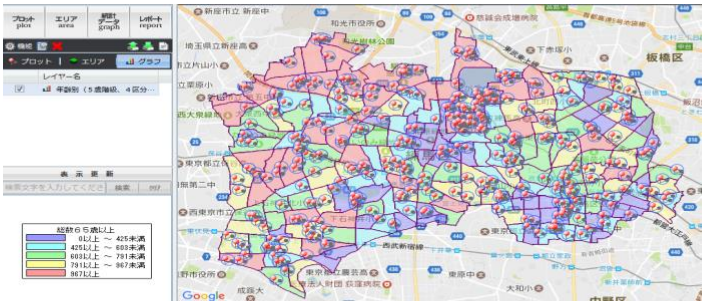


Figure 7. Small area statistics of census (age-specific numbers of households) and installation sites of AEDs

The presentation of the results is described below:

S *Well, around here. The reason is that although many people, especially many old people, live there, only a few AEDs are placed. So we think this place is the most important one.*

T *Thank you. That means around here. Do you have the same idea? Oh, every group is of the same opinion. We want to propose to Nerima Ward that AEDs are required around there. The reason is that both the entire population and the population of those who are aged 65 years and more are large, but there are no AEDs. So then, has any group decided the second place?*

S *This area has fewer AEDs, so we want to add some here. And that area's entire population and the population of those aged 65 years and over are also large. So we want to place more AEDs there.*

T *OK, next, here and there. Does any group have the same idea? Oh, yes, group 3. What have you discussed? You have discussed another thing. S6, okay, stand up. Can you explain?*

S6 *Our concern is this area. Other groups say that the red part, which has many old people, is important, but other areas cannot be considered less important because there are fewer old people in them. That does not mean that no-one lives in those areas. So there being no AEDs there seems to be a problem. Some AEDs should be placed in such areas.*

Every group prioritised the areas in which both the entire population and that of seniors were large, but no AEDs were available. When considering the second priority, one of the groups regarded it as a problem because some areas with fewer elderly people had no AEDs, but there were some residents. These ideas seem to be the result of students seeking society's betterment, and critically considering the current situation based on the acquired data.

4.3. Lesson 3 (Outline): "Let's clearly show which AED to go get"

For the areas where no additional AED will be installed, it is necessary to clarify at least which AED is closest to each one. Thus, the magnified drawing of some portion of the map prepared in Lesson 2 (Figure 8) was presented before the students were questioned, "*How will we be able to clearly show which AED to go and get in all the areas, and whether or not one is installed within a round-trip distance of 300 metres?*"

Figure 8. Part of the map

Almost all of the students started by focusing on the overlapping portion of the two circles, and most of these students moved on to connect the intersecting points of the circles with one line.

This idea was shared by all the students in the class before the meaning of the line segment was questioned. The students answered that the sites on the straight line connecting the two intersection points were at the same distance from the two AEDs, and that the line segment could act as a "borderline". As for the areas outside the circles, some students learned to find the points which were at the same distance from Points B and C (Figure 9).

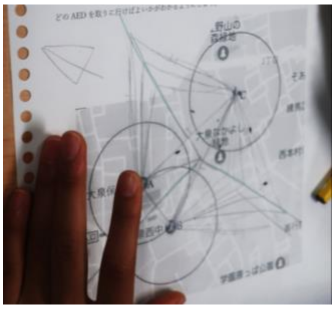


Figure 9. Student's worksheet: *diagram of AEDs*

4.4. Post-lesson discussion of the open research lesson

After giving the lesson at TGUISS, we did not have enough time to discuss the design of the task during the post-lesson discussion. The reason was that the lesson was offered on an open research lesson day held at the school (it had 12 research lessons on seven subjects, with over 300 participants) and some sessions on some projects had been planned after our discussion. Nevertheless, some teachers who observed the research lesson (Figure 10) reported on the students' problem-solving process that they observed. For instance:

- Some students found out the borderline through detecting the aggregate of the points at the same distance from the two AEDs. However, they may not have recognised the domain.
- Although the teacher drew the pattern of slanted lines within the circle, the students might not have noticed its meaning (Figure 11).
- It was difficult for the students to change the radius. They did not draw circles other than those with the 150-metre radius.

Figure 10. Lesson observation

Figure 11. Writing on the blackboard

5. Reflection

Considering the results of the students' activities, we can state that students almost fulfilled the goal set out in Section 3.3. We based our statement on the following considerations:

- They read various materials and found problematic points related to AEDs
- They made assumptions, conducted trade-offs between lives that can be saved by AEDs and installation costs, set conditions, and determined the intervals of AEDs within which they were to be installed
- They examined and selected routes, arrived at "circle models," and deliberated the situation regarding AED installation
- They considered the population distribution of the elderly and the intervals of installed AEDs by determining the sites of additional installations
- They learned to express their own value and discuss in groups or as a whole class by determining within which intervals to install AEDs or by examining where to place additional AEDs. That is, they learned to hold critical and multi-faceted discussions based on the stances and values on which they mutually agreed.

Furthermore, as regards social issues, we report about students' change in attitude towards engaging in mathematical thinking and making judgements. The following comments that the students' made support our affirmation:

- *I realised that, if you think mathematically, even about trivial things such as the location of AEDs, you can contribute to society by helping save someone's life. I will use mathematics more to improve society.*
- *Through the mathematical activities, I was able to learn about many things and connect them to influential issues in society. I've come to think about what I can do and how I can become a person who will improve society. I think I have grown.*
- *I was able to engage in the activity of using actual social data in checking*

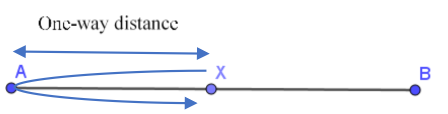
whether the necessary numbers of AEDs were installed, hoping to change people's lives.

In addition, through the activity of expressing the proximity of AEDs to a certain site in a drawing during Lesson 3, the students found out the characteristics of the perpendicular bisectors, which they had not learned. Due to time constraints, a review of these activities and a study of how to construct perpendicular bisectors were omitted in the lessons. However, it can be stated that this task design has demonstrated that the students can find how to construct perpendicular bisectors on their own.

6. Modification of design

When considering the differences observed between the expected and actual responses of the students in class, and as a result of the post-lesson discussion, three adjusting points were found in relation to the task design, as mentioned below.

The first point has to do with the activity of checking whether AEDs are installed at 300-metre intervals during Lesson 2. Although it had been predicted that the number of students who would draw and check circles with a 150-metre radius with AEDs as their centres, and the number of students who would set the radius as 300 metres, would be nearly the same, in fact almost no student adopted the 150-metre radius. The possible reasons for this could lie in the activity of determining the intervals during Lesson 1. When examining the installation intervals, the students were considering sites where AEDs had become necessary as the start point for calculating distance. To develop the circle model with a 150-metre radius, however, one needs to shift the starting point to the AED. When teaching students to make this shift, it is necessary to make them aware of the starting point when thinking about installation intervals by using drawings which serve that purpose, as exemplified by Figure 12.



A, B: Location of AED
X : Needed position of AED

Figure 12. Diagram to consider the installation interval of AED

The second point has to do with the activity of producing a drawing to indicate which AED to go and get, taught during Lesson 3. It was more difficult than expected for the students to think up an idea of drawing other circles than those with a 150-metre radius. As a starter, the task of thinking only about the inner side of a circle can be given to the students. While searching for the significance of the overlapping or non-overlapping portions of two circles, the students detected the aggregate of points at the same distance from the two AEDs and found out the borderline. Afterwards, they thought about the outer sides of the circles in the same way. With this approach, more students are likely to reach the idea of enlarging circles' radii.

The third point has to do with modifying "the circle model." During Lesson 3, the magnified map helped some students arrive at and suggest that the focus should be placed not on distance, but on routes. For example, it is possible for students to collect data on direct distances and routes between AEDs and certain sites, examine their relations, and, as a result, modify the circle model's radius.

7. Conclusion

This paper presents a case study of designing a problem-solving task following the Lesson Study methodology. This task aimed to foster the social decision-making competence in mathematics. Such tasks or lessons seek a different mathematics education from the conventional one in qualitative terms. In conventional mathematics education, the issues related to human emotions, such as value, are regarded as irrelevant, as if to say that only neutral objects independent of the actual world were to be discussed. We believe that the task and lesson on the AEDs mentioned herein prove the need for task design to foster the ability to clearly express values which forms a basis for the mathematical selection, usage and interpretation required for comprehension and solving problems in decision making.

Importantly, the decision-making competence does not exist independently of factual knowledge and procedural fluency, conceptual understanding, strategic competence and critical competence, but interacts with them all. When solving social problems, not only conventional mathematics, but also new mathematical methods or tools, are sometimes created. The design discussed herein can be stated as being valuable insofar that it includes the activity of finding how to construct perpendicular bisectors that the students have not yet learnt, but falls in line with the context.

In such a task design, it is important to carefully foresee the process of the students' finding and solving problems, and appropriately plan out what and how to discuss as a whole class, or when and what kind of materials should be presented *through* Lesson Study.

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Task design to foster the competence in social decision making on mathematics education

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Extended abstract

The purpose of education is to liberate people. As the necessary conditions for a person to become liberated vary depending on the time and society in which (s)he lives, mathematics education cannot but change accordingly. In this study, we added “decision making” to the goal and a framework for the task design as *a broad set of knowledge, skills, attitudes and values in action* (OECD, 2018). In this type of decision-making process, whether manifest or potential, various values of the concerned parties are expressed and affect the whole process from the manner of grasping the problems in question to the mathematical selection, usage, interpretation, and consensus building required for problem solving, as well as the content and quality of decision making. We developed the task (classroom activity) of saving more lives with AEDs (Automated External Defibrillators) in order to fostering this competence. As a task design methodology, we adopted *Lesson Study*, which incorporates the need to anticipate the process of students’ finding and solving problems carefully, and reviews the design after it is implemented. First, students must determine appropriate intervals within which AEDs should be placed by making trade-offs between ideals and reality in the task. Second, they discuss where to place additional AEDs in groups. They create mathematical models based on their assumptions and data on the population distribution in the town, and make their decisions in groups. It is possible that students find they are able to improve their society using mathematics. Further, they are expected to propose the idea that it should be made clear which AED to get whenever one is needed. That is, they develop the *Voronoi diagram* concept. In problem-solving situations, students not only have to use the mathematics that they have learnt, but also frequently have to use mathematical methods or tools that are new to them. It can be stated that the design discussed herein is valuable insofar that it includes the activity of finding how to construct perpendicular bisectors that the students have not learnt yet in line with the context. In this kind of task design, it is important to foresee the process of students finding and solving problems, and appropriately planning what and how to discuss as a whole class, or when and what kind of materials should be presented.