BOCR Framework for Decision Analysis ⋆

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Abstract: This paper considers establishing a framework for modellig decision analysis problems where the analyst must cope with uncertainty, multiple objectives, multiple attributes and multiple actors. These probems rise when considering large scale and complex decision problems encountered in real world applications in domains such as risk assessment and managment, infrastrucutres planning, complex process monitoring, supply chain planning, etc. To tackle this modelling chalenges, we propose to use BOCR (benefit, opportunity, cost, and risk) paradigm to identify attributes that must characterize an alternative with regard to a given objective. Then Bayesian network and/or AHP (analytic hierrarchy process) analysis can be used to assess the values of these later attributes. Finally an aggregation method based on satisficing game is developed that permit to evaluate each alternative by two measures: selectability degree constructed using "positive" attributes (benefit and opportunity) and the rejectability degree built on "negative" attributes (cost and risk).

Keywords: Decision Analysis, Multi-Objectives, Multi-Attributes, Multi-Actors, BOCR, Bayesian Network, AHP, Satisficing Game.

1. INTRODUCTION

Any real world decision analysis (selecting, sorting, ranking, benchmarking, ..., alternatives) problem is characterized by at least one of the following features.

- Multiple attributes or criteria: alternatives to be ranked, sorted or chosen are characterized by many attributes.
- Multiple objectives: decisions are made when seeking to satisfy many objectives.
- Multiple actors (stakeholders): for a number of practical decision making problems, the (possible antagonist) opinions regarding the importance of attributes as well as objectives of many actors have to be taken into account.
- Uncertainty: the realization of objectives or the attributes defining alternatives as well as actors opinions may be subjected to uncertainty.

The majority of contributions related to these problems encountered in the literature concern the case of either multiple attributes or multiple objectives rarely taking into account uncertainty and multiplicity of actors. The approaches developed for this purpose can roughly be divided into two schools: aggregation methods and outranking methods. The aggregation methods consist in aggregating objectives into a single objective or transforming some objectives or criteria into constraints using different procedures leading to well known methods such as weighting methods, constraint methods and goal programming methods, see for instance (18; 8; 9) and value oriented methods such as that of AHP, see (17). The advantage of these methods is that efficient and broad algorithms developed for single objective optimization problems (see (7; 10; 11) and references therein) can be used to solve the resulting problems. The second category of approaches is constituted by that of outranking methods where a partial order of alternatives is derived by an interactive procedure between analyst and decision maker(s) (see (4; 5; 16; 23)).

The main drawbacks of the methods evoked previously that we want to overcome in this paper are that these methods rarely consider uncertainty and multiplicity of actors and most important these methods consider that alternatives are characterize by the same attributes or criteria whereas in practice this assumption can be defeated. In this paper we consider that the important thing to consider is the adequacy between attributes characterizing an alternatives and the pursued objectives or goals. To aid structuring the process to elicitate attributes for a given objective and a considered alternative, we propose to use the so called BOCR (benefit, opportunity, cost, and risk) analysis. The remainder of the paper is organized in the following: the second section presents the BOCR analysis framework developed in this paper and the third section presents an aggregation procedure based on satisficing game approach to derive a decision analysis procedure using two aggregated measures, namely the selectability and the rejectability measures. Finally a conclusion is presented in the fourth section. Because of limited space, an application will be considered only during the presentation.

⋆ BOCR stands for Benefit, Opportunity, Cost, and Risk

2. BOCR FRAMEWORK FOR DECISION ANALYSIS

Decision analysis process begins by defining decision *objectives* or *goals*. From these objectives, potential *alternatives* and *attributes* that characterize them are identified and/or elicitated. The triplet (attributes, Alternatives, Objectives) is referred to in this paper as decision analysis space; this space is formed by three axis, namely attributes axis known as $a-axis$, alternatives axis, referred to as $A-axis$ and finally objectives axis or $O-axis$. Any pair of these axis will define a plane and a point in this plane may mean many things:

- the $a-A$ *plane* constituted by $a-axis$ and $A-axis$ will represent a measurement plane so that each point of this plane will be characterized by the value of given attribute a for an alternative A;
- the plane formed by $a-axis$ and $O-axis$ that we refer to as $a-O$ *plane* represent some qualitative relationships; does a given attribute a works towards or against the achievement of a considered or pursued objective O;
- finally the $A-O$ *plane* defined by $A-axis$ and $O-axis$ represent the plane where one will determine how well a given alternative does satisfy a considered objective.

This description is summarized on the following Figure 1

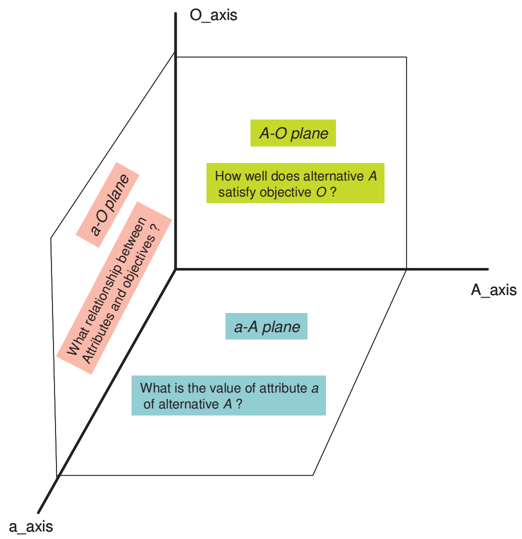

Fig. 1. Decision analysis space

The decision analysis space depicted on Figure 1 is a valuable source of information that may aid structuring decision analysis. One of main drawbacks of existing decision analysis methods encountered in literature is that all alternatives are characterized by same attributes. In this paper we consider the point of view that strength the relationship between attributes defining an alternative and the pursued objectives. By so doing, attributes may differ from one alternative to another and depend on the objectives and alternatives that is on a point in the $A-O$ *plane*. Attributes elicitation process is then a mapping from this $A-O$ *plane* onto $a-axis$. The process of elicitating attributes given an alternative and an objective is not an easy task in general and needs a methodology to structure it. To facilitate this process we introduce supporting and rejecting attributes concepts for any objective o that are given by the following definition.

Definition 1. An objective o is said to be supported (respect. rejected) by an attribute a if and only if its variation is positively (respect. negatively) correlated with the variations of that attribute. Otherwise the objective and the attribute are said to be mutually neutral.

Now to make precise these notions we put them in the framework of BOCR analysis similar to (17) where benefit and opportunity constitute the supportability attributes whereas cost and risk are considered as rejectability attributes for a given objective. Elicitation process is then carried by searching for benefit attributes (B_attributes), cost attributes (C_attributes), opportunity attributes (O_attributes) and risk attributes (R_attributes), as defined by the following definition, for a given pair of objective and alternative.

Definition 2. Benefit attributes (B_attributes) and cost attributes (C_attributes) are attributes that are not subjected to *uncertainty* and that work towards (respectively against) the achievement of a given objective. On the contrary, opportunity attributes (O_attributes) and risk attributes (R_attributes) are *uncertain* attributes that may enhance (respectively contrariate) the achivement of the considered objective or goal.

Given an objective o and an alternative A, an AHP type analysis can be considered to decompose each type of attributes going from general statements to more measurable attributes as shown by Figure 2.

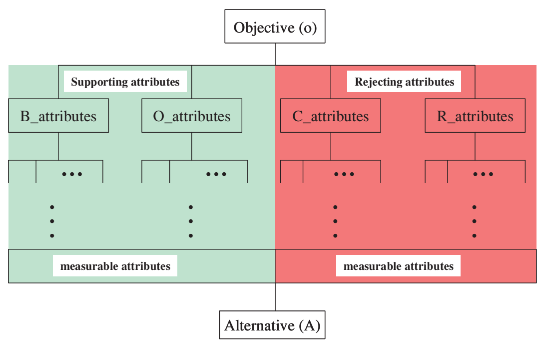

Fig. 2. BOCR analysis to elicitate attributes for an alternative A with regard to an objective o

We will denote by $\mathcal{A}$ the universe of alternatives, by $\mathcal{B}$ the set of measurable benefit attributes, by $\mathcal{O}$ the set of measurable opportunity attributes, by $\mathcal{C}$ the set of measurable cost attributes and finally by $\mathcal{R}$ the set of measurable risk attributes. We will give in the following paragraphs, a procedure to elicitate and assessed the value of these attributes for each pair constituted by an objective o and an alternative A.

2.1 Benefit and Cost attributes elicitation and assessment

According to previous definition, for a given objective o and an identified alternative A, one will determine the sets $\mathcal{B}(o, A)$ and $\mathcal{C}(o, A)$ of benefit attributes and costs attributes respectively by answering a question of the form "what feature of alternative A free of uncertainty that work toward the achievement of objective o (respectively against the objective o) ? Once these sets are identified the value of a each attribute can be measured directly if possible (quantifiable attribute) or by a pairwise comparison procedure using AHP procedure for instance. No matter the procedure used to assess the value of attributes, they will be normalized using AHP procedure to obtain $b_A^o(a)$ the normalized value of benefit attribute $a \in \mathcal{B}(o, A)$ and $c_A^o(a)$ the normalized value of cost attributes $a \in \mathcal{C}(o, A)$.

2.2 Risk and Opportunity elicitation and assessment

Contrary to benefit and cost attributes elicitation and assessment, the case for risk and opportunity is not straightforward as these indicators are subjected to uncertainty that need a precise definition and measurement.

Introduction and definition Risk and opportunity are related to uncertainty in the realization of an objective. Formerly speaking risk and opportunity are jointly associated with the likelihood (probability) of something (an event or a sequence of events, a feature, ...) happening and the impact (severity, gravity, consequence, ...) on the realization of the objective if it does actually happen. Formerly, risk and opportunity that we will consider in this paper is given by the following definition.

Definition 3. The risk $R(a, A)$ (respectively the opportunity $O(a, A)$) related to an attribute a of an alternative A is defined by two measures: the probability of occurrence $\Pr(a)$ of of this attribute and its severity $S(a, A)$ (respecively its benefit $B(a, A)$). The severity (repectively benefit) measures the negative impact (respectively the positive impact) on the realization of the objective; it is generally expressed by the amount of some losses (economic loss, lives loss, etc.) (respectively some gain) or by the probability of no satisfaction (respectively the probability of realization) of the objective.

For sake of simplicity we will represent the risk $R(a, A)$ (respect. the opportunity $O(a, A)$) of an attribute a of an alternative A by its criticity $C(a, A)$ (respect. its probable benefit $B(a, u)\Pr(a)$) as given by equations (1) - (2),

$$R(a, A) = C(a, A) = S(a, A)\Pr(a) \quad (1)$$

$$O(a, A) = B(a, A)\Pr(a)) \quad (2)$$

though this aggregation is not appropriate when dealing with risk management because actions to prevent risk that consist in reducing $\Pr(a)$ and actions to mitigate risk, reduction of the severity $S(a, A)$ may be very different or infeasible given two events with the same criticity ; this remark holds for opportunity with effect considered in reverse form for indicators $\Pr(a)$ and $B(a, A)$. But as we are just interested in hierarchising risk (respect. opportunity) this aggregation is assumed here. Now that risk and opportunity have been defined, we must derive a procedure to assess them.

Risk and opportunity assessment Assessment process is a purely analytic activity where the analyst is willing to characterize the risks (or the opportunity) faced by an organization or a system by following some procedures. In risk (respectively opportunity) assessment in the decision analysis framework considered here, the analyst will attempt to answer the following set of triple questions, given an objective o and an alternative A.

- What can go wrong ? (respectively what can go better ?). Identification of all events, scenarios or features of alternative u that may have a negative (respectively a positive) effect on the realization of the objective; these attributes will be regrouped in the sets $\mathcal{R}(o, A)$ and $\mathcal{O}(o, A)$ respectively.
- What is the likelihood that it would go wrong ? (respectively it will go better ?): the probability of occurrence of that events, scenarios or features.
- And, what are the consequences (negative in the case of risk and positive in the case of opportunity). Characterization of the impact on the objective if that events do occur.

Answers to these questions help analysts identify risk attributes set $\mathcal{R}(o, A)$ and opportunity attributes set $\mathcal{O}(o, a)$ for any pair of objective o and alternative A. The consequences depend on the attributes as well as on the *state* of the alternative, that is all things that make it more or less vulnerable (in the risk case) or that permit it to exploit the attribute (opportunity case). Identifying attributes and the state of the alternative and their interaction leading to severity or benefit for a given pair (a, A) is a complex process for different reasons: it may be difficult to obtain direct attributes and/or to assess their probability of occurrence; in the same way the state of the alternative may be a result of complex interaction of many variables. We propose, to this end, a structuring method in two steps: first all attributes, state of the alternatives and consequences are identified using previous questions and their interactions formulated using a meta-matrix analysis and secondly the strengthens of these interactions are elicitated that result in a Bayesian network.

Variables identification To identify and define all the variables we propose the following scheme:

- first of all, the analyst, expert or decision maker must identify all the risk/opportunity attributes, in fact all the events that may have a negative/positive impact on the performance of the alternative with regard to the objective by answering the first question of risk/opportunity assessment process, that is "*what can go wrong/better ?* "; this leads to the sets $\mathcal{A}_R(o, A)$ of risk attributes and $\mathcal{A}_O(o, A)$ of opportunity attributes respectively;
- the second stage consist in assessing the variables defining the state of the alternative by answering question of the form "*what actual features of the alternative may lead to its performance degradation/improvement with regard to the considered objective ?*"; this permit to identify $\mathcal{S}_R(o, A)$, the set of state variables that act toward risk and $\mathcal{S}_O(o, A)$ the set of state variables acting in the sense of opportunity;
- finally consequences variables (negative in the case of risk and positive for opportunity) are identified by answers to questions such as "*given the state of the alternative what will happen to the objective in the case of occurrence of a particular attribute ?* "

obtaining then the sets $\mathcal{C}_R(o, A)$ and $\mathcal{C}_O(o, A)$ (set of risk consequences and opportunity consequences respectively for objective o).

Interactions identification: meta-matrix analysis How attributes and state of the alternative interact to produce consequences may be very complex and furthermore within a same group of variables there may be interaction: for instance attributes that interact directly with consequences may be not directly measurable so that one needs to decompose them until measurable attributes (that constitute the sets $\mathcal{A}_R(o, A)$ and $\mathcal{A}_O(o, A)$) are obtained; the same process may apply to state variables as well as to consequences. To structure this process we propose a meta-matrix analysis to identify the chains of interactions. Each entry of this matrix will be a directed graph giving the chains of influences. The model of this matrix is given on Figure 3 and the signification of each graph is explained below.

	State	Attributes	Consequences
State	S-S graph		S-C graph
Attributes		A-A graph	A-C graph
Consequences			C-C graph

Fig. 3. Meta - matrix to elicitate influence beween variables for risk and opportunity assessment

- **Attributes graph (A-A graph)**: this graph defines causal relationships that may exist between attributes from primary attributes to that directly influencing consequences; to identify these relationships one must answer questions such as "*how a low level attribute is obtained ?*".
- **State graph (S-S graph)** represents potential influence that may exist among the variables defining the state of the alternative; a question similar to that of former point need to be answered.
- **Consequences graph (C-C graph)** defines relationships between consequences, going from consequences directly influenced by attributes and state variables until measurable consequences are obtained.
- **Sate-Consequences graph (S-C graph)**: this graph shows how state variables influence consequences ones.
- **Attributes-Consequences graph (E-C graph)** defines how attributes influence consequences.

This process leads to a meta-graph as shown on Figure 4.

This meta - graph can be used in two directions according to how severities/opportunities are measured. If they are measured by conditional probability of failure/success of the objective then this meta-graph will be considered to be a meta Bayesian network whereas if they are measured by some values then this meta-graph is used in the sense of AHP analysis.

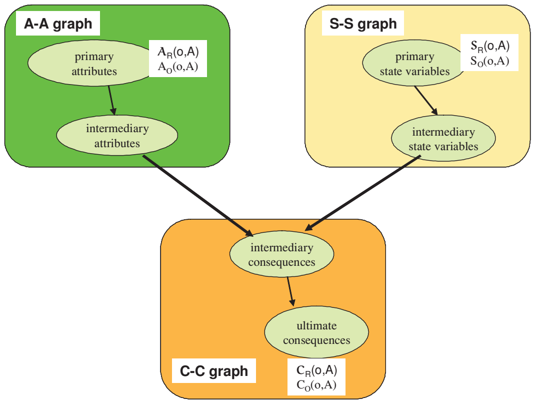

Fig. 4. Meta - graph for risk/opportunity elicitation process

2.2.4.1. Meta Bayesian network Defining a Bayesian network (graphical model of probabilistic relationships between variables of a knowledge domain based on ideas of (3)) consists in two things (13): defining the structure of the network (a directed acyclic graph) and defining the parameters that are conditional probability tables.

2.2.4.1.1. Structure Defining the structure is straightforward from the previous meta-graph; indeed, this graph constitute the meta Bayesian network structure. Of course the model of Figure 4 is a meta-model; for a particular problem each graph will consist in a certain number of nodes and arcs describing influence between real factors. These factors can be *discrete* (the values they are allowed to take belong to a discrete finite set) or *continuous* (the allowed state of the variable belong to a continuous set such as an interval of real number). In this paper we consider only discrete nodes so that each continuous variable will be discretized by an appropriate procedure. Our motivation to consider discrete variables is due to the fact that many (if not the majority) of problems the developed approach will be addressing concern high level decisions making and planning where human experts are needed for relationships and parameters elicitation so that each variables should be evaluated over a limited number of modalities or attributes and should have a reduced number of parents also; having a reduced number of parents can be easy by a modeling artefact using aggregation and hierarchy for instance.

2.2.4.1.2. Parameters The second task to fully define a real Bayesian network will be to characterize the parameters of the network in terms of conditional probability tables by either a learning process or experts knowledge or both of them. For many practical problems, there will be no data for learning so that one will rely on expertise. The problem of elicitating conditional probabilities by experts have received a great interests by researchers in different domains including computer science, psychology, decision sciences to name few. To this end, a variety of methods have been developed including frequency estimation (see (1; 15)), gamble-like method (see (15)), hierarchical methods that allow using qualitative or quantitative information ((12)) or an AHP approach ((14)), the consideration of multiple experts (by aggregation for instance) is also used to attempt to increase the accuracy (see (6)). Here we suggest a straightforward approximate approach based on an AHP analysis that leads directly to a consistent elicitation by multiple experts; indeed elicitating conditional probability table for a node X with a great number of parent nodes $Pa(X)$ is a tremendous task. So we propose to use the influence model approach (see (2)) to approximate the conditional probability table $\Pr(X/P(X))$ by the following equation (3)

$$\Pr(X/Pa(X)) \simeq \sum_{Y \in Pa(X)} \alpha_Y \Pr(X/Y) \tag{3}$$

$$\text{with} \sum_{Y \in Pa(X)} \alpha_Y = 1 \tag{4}$$

where α_Y is a weight measuring the relative influence of a particular parent node Y on the node X. Elicitating the conditional probability table $\Pr(X/P(X))$ is reduced to elicitating marginal conditional probability tables $\Pr(X/Y)$ for $Y \in Pa(X)$ and the influence weights α_Y. To elicitate $\Pr(X/Y)$ we propose to use the systematic version of AHP. To do so let us suppose that the node X has n_X modalities $\{x_1,\ x_2,\ ...,\ x_{n_X}\}$ then for each modality y of the parent node Y, ask each experts k to choose a pivot modality x_p and supply a weight $\nu^k_{ip}(y)$ that measures how likely is the modality x_i in comparison to the pivot modality x_p in the opinion of the expert k given that the parent node Y takes the modality y. This weight will be derived using the standard scale of AHP (see (17)). From these supplied weights, a consistent [1] $n_X \times n_X$ comparison matrix $C^k(X/Y=y)$ is constructed as shown by equations (5)-(7)

$$C^k(X/Y=y)_{ii} = 1;\ C^k(X/Y=y)_{ip} = \nu^k_{ip}(y); \tag{5}$$

$$C^k(X/Y=y)_{ip} = \frac{1}{\nu^k_{ip}(y)} \tag{6}$$

and the rest of coefficients are deduced using the consistence relationship (7)

$$C^k(X/Y=y)_{il} = C^k(X/Y=y)_{ij} C^k(X/Y=y)_{jl}. \tag{7}$$

The conditional probability $\Pr^k(X=x_i/Y=y)$ according to expert k can then be estimated as given by the following equation (8)

$$\overset{k}{\Pr}(X=x_i/Y=y) = \frac{1}{n_X} \sum_{j=1}^{n_X} \left(\frac{C^k(X/Y=y)_{ij}}{\sum_{l=1}^{n_X} C^k(X/Y=y)_{lj}} \right). \tag{8}$$

The final conditional probability $\Pr(X=x_i/Y=y)$ can be considered to be the mean value over experts, see equation (9)

$$\Pr(X=x_i/Y=y) = \frac{\sum_{k=1}^{N} \Pr^k(X=x_i/Y=y)}{\sum_{j=1}^{n_X} \left\{ \sum_{k=1}^{N} \Pr^k(X=x_j/Y=y) \right\}}. \tag{9}$$

where N is the number of experts.

Elicitation of influence parameters α_Y, $Y \in Pa(X)$, can be done in the same way but the process here is more complex as nodes effects will more or less combine. To take this into account we suggest to use analytic network process (see (17)) with inner loop on parent nodes to measure the strength of pairwise effect combination (see Figure 5). In

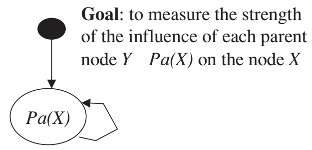

Fig. 5.

this case we have a network weighting matrix W as shown by equation (10) where $\mathbf{1}$ is a vector with appropriate dimension and all entries equal to 1 and $|Pa(X)|$ is the cardinality of the set $Pa(X)$. The matrix W_a is the inner comparison matrix for cluster $Pa(X)$; to obtain it one will proceed as in the AHP case by answering a question of the form "*how well the effect of a particular parent node $Y \in Pa(X)$ on the node X does depend on the another parent node $Z \in Pa(X)$?*" and finally the vector α is obtained as the normalized last $|Pa(X)|$ elements of the first column of the limiting matrix Ω of equation (10), see (17) for more details,

$$W = \begin{bmatrix} 0 & 0 \\ \dfrac{\mathbf{1}}{|Pa(X)|} & W_a \end{bmatrix} \text{ and } \Omega = \lim_{k \to \infty} W^k. \tag{10}$$

This procedure will lead to the fact that a node that effect influence the effect of many nodes will receive a strong weight.

Remark 1. We do consider that this approximation is more convenient for experts use instead of deriving comparison for any combination of parent node.

As in the case of benefit and cost case, risk and opportunity measures $R(a, A)$ and $O(a, A)$ will be normalized to obtain $r^o_A(a)$ and $o^o_A(a)$ respectively. To ultimately evaluate an alternative A, we need to aggregate its benefit measures $b^o_A(a)$, cost measures $c^o_A(a)$, opportunity measures $o^o_A(a)$, and risk measures $r^o_A(a)$. To this end, we suggest to use satisficing game approach.

3. AGGREGATION

Given the duality nature of attributes (benefit and opportunity on one hand and cost and risk on other hand), we will develop, in this section, an aggregation method that leads to two measures known as selectability and rejectability measures in the framework of satisficing game theory as done in previous works of authors, see (20; 21; 22).

3.1 Satisficing game

In its simplest version (see (19) for more details) a satisficing game is given by the following definition.

Definition 4. A satisficing game consists in the triplet $(\mathcal{A}, \mu_R, \mu_S)$ where

- $\mathcal{A}$ is the set (discrete) of alternatives;
- μ_R and μ_S represent mass functions or measures defined from $\mathcal{A}$ onto the interval $[0, 1]$ where μ_S measures the selectability degree and μ_R that of rejectability; a function p is said to be a mass function

[1] A comparison matrix M is said to be consistent if it verifies: $M_{ii} = 1$, $M_{ji} = \frac{1}{M_{ij}}$ and $M_{ik} = M_{ij} M_{jk}$.

over a discrete set $\mathcal{A}$ if it possesses a probability structure that is it verifies $p(A) \geq 0$ for any element A of $\mathcal{A}$ and the sum of $p(A)$ over $\mathcal{A}$ is one.

The interesting alternatives that can be qualified as satisficing ones are those for which the selectability measure does exceed the rejectability one as given by the following definition.

Definition 5. The satisficing alternatives set $\Sigma_q \subseteq \mathcal{A}$ with the index of boldness q is the set of alternatives defined by equation (11)

$$\Sigma_q = \{A \in \mathcal{A} : \mu_S(A) \geq q\mu_R(A)\}. \tag{11}$$

The boldness index q can be used to adjust the aspiration level: increase q if Σ_q is too large or on the contrary decrease q if Σ_q is empty for instance.

The values of index of caution q belong to the interval $[\, q_{\min},\, q_{\max}\,]$ where $q_{\min}$ is the value below which all alternatives are declared satisficing and $q_{\max}$ is the value above which no alternative is satisficing; these values are given by the following equation (12)

$$q_{\min} = \min_{A \in \mathcal{A}} \left(\frac{\mu_S(A)}{\mu_R(A)}\right) \text{ and } q_{\max} = \max_{A \in \mathcal{A}} \left(\frac{\mu_S(A)}{\mu_R(A)}\right). \tag{12}$$

But for a satisficing alternative there can exist other satisficing alternatives that are better (that dominate its in the sense of Pareto); let denoted by $\mathcal{D}(A)$ alternatives that dominates A, then we define an equilibrium alternative to be an alternative that is not dominated and the set $\mathcal{E}$ of non dominated alternatives is given by equation 13

$$\mathcal{E} = \{A \in \mathcal{A} : \mathcal{D}(A) = \varnothing\} \tag{13}$$

and *non dominated or good enough* alternatives at boldness index of q set $\mathcal{S}_q$ is given by (14)

$$\mathcal{S}_q = \mathcal{E} \cap \Sigma_q. \tag{14}$$

Notice that this set can always be rendered non empty as one can always adjust the size of Σ_q and as the equilibria set $\mathcal{E}$ cannot be empty by construction.

3.2 Aggregation procedure in satisficing game framework

The stepping stones of satisficing game theory are satisfiability (selectability and rejectability) measures; thus, formulating a decision analysis problem as a satisficing game return to deriving a procedure, from problem specifications, to compute these measures. Building a satisficing game based on the BOCR analysis considered previously is straightforward because benefit and opportunity will participate to constructing the selectability measure μ_S whereas cost and risk will contribute to rejectability measure μ_R. The global benefit, cost, risk and opportunity measure of objective o for alternative A are given by equations (15)-(16)

$$\psi_B(o, A) = \sum_{a \in \mathcal{B}(a,A)} \alpha_a b_A^o(a),\ \psi_C(o, A) = \sum_{a \in \mathcal{C}(a,A)} \beta_a c_A^o(a), \tag{15}$$

$$\psi_R(o, A) = \sum_{a \in \mathcal{R}(a,A)} \gamma_a r_A^o(a),\ \psi_O(o, A) = \sum_{a \in \mathcal{O}(a,A)} \delta_a o_A^o(a), \tag{16}$$

respectively where parameters α_a, β_a, γ_a, and δ_a measure relative importance of an attribute within its category; the aggregate measures that work towards achieving decision maker goal, $\Psi_S(A)$ and against decision maker goal, $\Psi_R(A)$ for an alternative A are given by equations (17)-(18)

$$\Psi_S(A) = \omega_B \left(\sum_o \varpi_o \psi_B(o, A)\right) + \omega_O \left(\sum_o \varpi_o \psi_O(o, A)\right) \tag{17}$$

$$\Psi_R(A) = \omega_C \left(\sum_o \varpi_o \psi_C(o, A)\right) + \omega_R \left(\sum_o \varpi_o \psi_R(o, A)\right) \tag{18}$$

where ϖ_o is the relative importance of objective o among all decision maker objectives; ω_B and ω_O are relative importance of benefit and opportunity for the selectability of alternative with the condition $\omega_B + \omega_B = 1$;the same explanation and condition apply to ω_C and ω_R. The satisfiability measures in the framework of satisficing game are then given by the following definition.

Definition 6. The selectability measure $\mu_S(A)$ and the rejectability measure $\mu_R(A)$ of decision analysis problem considered are given by equation (19)

$$\mu_S(A) = \frac{\Psi_S(A)}{\sum_{X \in \mathcal{A}} \Psi_S(X)} \text{ and } \mu_R(A) = \frac{\Psi_R(A)}{\sum_{X \in \mathcal{A}} \Psi_R(X)} \tag{19}$$

4. CONCLUSION

A framework for decision analysis that integrates multiplicity of objectives, attributes, and actors as well as uncertainty has been developed in this paper using BOCR (benefit, opportunity, cost, and risk) analysis paradigm. This framework permit to overcome one of the main limitation encountered by existing decision analysis approaches: the fact for alternatives to be characterized by same attributes. Our opinion is that for complex and large scale decision problems this assumption may fails and the important thing to consider is the adequacy between attributes and the pursued objectives or goals. The proposed framework is very general to be adapted to particular situations.

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