# A TRIGONOMETRY BASED LINEAR FORMFINDING METHOD FOR IRREGULAR MULTILAYER PRISM TENSEGRITY 

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# A TRIGONOMETRY BASED LINEAR FORMFINDING METHOD FOR IRREGULAR MULTILAYER PRISM TENSEGRITY 

by

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## LIST OF SYMBOLS

| [C] | Incidence matrix |
| :---: | :---: |
| [D] | Force density matrix |
| [D] | Damping matrix |
| [G] | Matrix of generalized coordinate of struts |
| [K] | Stiffness matrix |
| [M] | Mass matrix |
| [T] | Matrix of tension force of cables |
| [4] | matrix of change in the length of cables related to struts |
| [ $\delta L]$ | matrix of change in length of cables |
| $b$ | Distance between the scaling base point to the mid-point of the member of the first conjunction polygon |
| c | Compression diagonal member |
| $\dot{c}$ | Number of cables |
| $d$ | Member of the second ring of the previous layer |
| $d^{\prime}$ | Member of the second ring of the subsequent layer |
| $e$ | Elongation of member |
| $\bar{F}$ | Vector of force |
| ${ }^{\text {i }}$ | Joint number of the first or previous layer |
| I | Reference factor |
| $i^{\prime}$ | Joint number of the subsequent layer |
| j | Joint of the first ring of the previous layer |
| $j^{\prime}$ | Joint of the second ring of the previous layer |
| $j^{\prime \prime}$ | Joint of the first ring of the subsequent layer |
| $k$ | Coefficient for the force ratio of the members of the first ring |
| $k^{\prime}$ | Coefficient for the force ratio of the members of the second ring |
| $k^{\prime \prime}$ | Adjusting factor |


| $l$ | Member of the first ring of the previous layer |
| :---: | :---: |
| $l^{\prime}$ | Member of the first ring of the subsequent layer |
| $i$ | Length of cable |
| $\bar{L}$ | Vector of length |
| $n$ | Number of vertices |
| $p$ | Member of the connection polygon |
| POL | Polygon |
| $q$ | Force density |
| $r$ | Radius of circle |
| $s$ | Scale ratio of the second ring |
| $s^{\prime}$ | Scale ratio of the first conjunction polygon |
| $\dot{s}$ | Number of struts |
| $\ddot{s}$ | Number of parameters of generalized coordinates of strut |
| $s^{\prime \prime}$ | Scale ratio of the second conjunction polygon |
| $t$ | Initial force of member |
| $t$ | Tension diagonal member |
| $u$ | Vectors of displacement |
| $\dot{u}$ | Vectors of velocity |
| $\ddot{u}$ | Vectors of acceleration |
| $x$ | Distance between a joint in the first conjunction polygon and the same joint in the scaled first conjunction polygon |
| $x^{\prime}$ | Distance between the mid-point of the member of the firs conjunction polygon to mid-point of the member of the scaled firs conjunction polygon |
| $\alpha$ | Internal angle of the first ring |
| $\alpha^{\prime}$ | Azimuth angle of the member of the first ring |
| $\alpha$ | Twisting angle between upper and lower polygon |
| $\beta$ | Internal angle of the second ring |

Azimuth angle of the member of the second ring
Central angle of upper polygon
Polar angle of the diagonal tension member
Polar angle of the diagonal compression member
Virtual displacement
Angle between the member of the scaled second conjunction polygon and the member of the scaled first conjunction polygon

Azimuth angle of the diagonal tension member
Azimuth angle of the diagonal compression member

## LIST OF ABBREVIATIONS

| $3 D$ | Three dimensions |
| :--- | :--- |
| All. | Allowable |
| ASTM | American Society for Testing and Materials |
| AutoCAD | Autodesk® AUTOCAD ® |
| Ave. | Average |
| $C C W$ | Counter clockwise |
| $C P R$ | Centroid point of the second ring |
| $C W$ | Clockwise |
| Def. | Deflection |
| Dis. | Displacement |
| $G P a$ | Kilo Newton Pascal |
| $k N$ | Meter |
| $m$ | Maximum |
| $M a x$ | Millimetre |
| $m m$ | Mega Pascal |
| $M P a$ | Rotational direction of the second conjunction polygon |
| $R D C$ | Autodesk® Robot TM Structural Analysis |
| $R o b o t$ | Rotational direction of the second ring of the previous layer |
| $S J P$ | Unsymmetrical |
| $S Y M$ |  |

# KAEDAH PENENTUAN-BENTUK LELURUS BERDASARKAN TRIGONOMETRI UNTUK STRUKTUR TENSEGRITI PRISMA PELBAGAI LAPIS TAK-SERAGAM 


#### Abstract

ABSTRAK

Tensegriti prisma adalah sejenis sistem tensegriti yang sesuai dipertimbangkan untuk penggunaan dalam bidang kejuruteraan awam dan senibina atas sebab ciri khasnya. Proses penentuan-bentuk untuk satu tensegriti prisma tak-seragam adalah satu proses tak-lelurus secara kebiasaannya atas sebab bilangan persamaan keseimbangan daya adalah kurang daripada bilangan parameter yang tidak diketahui. Peninjauan kajian lepas berkaitan kaedah penentuan-bentuk yang telah dikemukan menunjukkan bahawa tiada di antara kesemua kaedah adalah praktikal untuk tujuan penentuan-bentuk tensegiti tak-seragam dengan jumlah anggota yang besar. Atas sebab di atas, kajian ini telah dijalankan dengan matlamat untuk mengemukakan satu kaedah penentuan-bentuk yang praktikal yang cepat, tepat dan boleh memuaskan kehendak pereka. Tambahan lagi, adalah diketahui bahawa berat sendiri dan beban luaran mempunyai kesan ke atas bentuk akhir tensegriti prisma yang biasanya diabaikan dalam analisa penentuan-bentuk. Tensegriti prisma yang digunakan dalam bidang kejuruteraan awam dan senibina perlu mematuhi keperluan praktikal anjakan struktur dan ubahbentuk anggota. Justeru, kajian ini juga disasarkan untuk mengkaji kesan tegasan awal dan bentuk ke atas ubahbentuk tensegriti prisma di bawah berat sendiri. Dalam kajian ini, hubungan trigonometri digunakan untuk mengaitkan persamaan hubungan panjang dan persamaan keseimbangan daya untuk mendapatkan satu kaedah lelurus untuk penentuan-bentuk yang dikenali sebagai kaedah trigonometri. Pertama sekali, kaedah trigonometri baru untuk penentuan-bentuk tensegriti prisma satu lapis diperkenalkan. Selepas itu, konsep poligon penghubung


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