A Two-sided Matching System Design for Dynamic Labor Markets

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Abstract

A Two-sided Matching System Design for Dynamic Labor Markets

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This thesis designs an automatic two-sided matching system for dynamic labor markets with large scale of data. Such markets consist of a group of vacancies and applicants, a matching function, a set of events causing transitions of the state of the market. Due to the dynamic nature of the labor markets, matching systems based on the classical deferred acceptance algorithm are not sufficient for producing stable matching solutions. Therefore, the central theme of this thesis is to address the effectiveness and efficiency of generating matching results in dynamic large labor markets.

The main contribution of this thesis consists of three dynamic matching algorithms and a agent-based matching system design. The dynamic matching algorithms are extensions of the classical deferred acceptance algorithm. The first algorithm generates a vacancy-optimal stable matching result without considering locking or break-up constrains. The second algorithm considers locking period constraints in the matching process and the third algorithm computes applicant-optimal stable matchings with the consideration of break-up penalties in dynamic environments. To verify the effectiveness and efficiency of the proposed matching algorithms, theoretical proofs and experimental results are presented as well. The results indicate that the designed system can be used as an efficient and effective tool for recruitment management in today's dynamic and internet based labor markets to reduce administrative work load of human resource departments and produce stable job allocations.

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Chapter 1

Introduction

There are a lot of markets in which transactions are completed bilaterally. Such as marriage markets with single men and single women, college admission markets with students and colleges and labor market with job-seekers and employers. These markets are called two-sided matching markets that each individual on one side looks for a matching counterpart on the other side. This thesis focuses on labor markets in which there are two joint groups: applicants and vacancies.

1.1 Labor markets as two-sided matching markets

In general, labor market is regarded as a workplace that can allocate applicants into appropriate vacancies. Unlike commodity markets which focus primarily on the supply and demand of resources, the critical factors in the processes of allocation in labor market are "personal desires"[1]. In fact, labor markets are much more complex than commodity markets, because each individual in the market has his or her own desire rather than simple transaction. Each applicant should signal his or her passion, credentials and drive while employer of each job position offers salaries, perks and prospects for advancement.

There are two groups of participants determine the supply and demand in labor markets, namely vacant job positions and job applicants. They form two disjoint sets and the process of labor forces allocation is a bilateral selection process. These are the major characteristics of two-sided matching market, "two-sided" and "matching" [2]. The term "two-sided" refers to the fact that participants in such markets belong to one of two disjoint sets, such as firms or workers as well as men or women; and "matching" refers to the bilateral nature of exchange in these markets, such as if I work for some firm, then the firm employs me and if I marry a girl, then the girl marries me [3].

1.2 Challenges of two-sided matching in labor markets

In their seminal paper [4], Gale and Shapley proposed a criterion, stability, to measure the quality of outcome of a two-sided matching market. Later, stability is regarded as one of the most important standards of matching market. Stability of the matching results is also a critical criterion for evaluating the quality of a labor market. A labor market that produces stable job allocations will reduce employers' recruiting and training costs, therefore, improve their market competitiveness. From system design perspective, in this thesis, if a matching system produces stable matchings, we say the matching system is effective. It is known that the deferred acceptance algorithm can obtain stable matches between job position and applicants in static two-sided markets which assume complete strict preferences and unchanging environments [5]. However, real-world labor markets are full of imperfect and asymmetric information, which is difficult for each side in the labor market to acquire complete and real-time information over the opposite sides [6]. Therefore, a strict preference list of participants, which is a critical information for operating deferred acceptance algorithm, seems to be impossible to be offered.

In addition to imperfect information, today's labor markets are more dynamic, which means they are more flexible and complex environments which involve more frequent variation and more unpredictable behaviour. David Andolfatto concludes that labor market has some dynamic characteristics [7]: 1) persistence and variability; 2) cyclical movement and 3) negative correlation between vacancies and unemployment. In such dynamic labor markets, a matching system should consider various uncertainties before giving a solution of matches between workers and employers. In addition to imperfect information and dynamic environments, large scale of data is becoming a third challenge facing labor market designers. In recent years, Web, as a worldwide information resource, are widely applied for conducting online search for information about jobs' and applicants [8]. Entor analyzed the condition of employment in both the United States and Western Europe, and gave the conclusion that computer plays an important role in labor market to help employees obtain higher salaries [9]. In addition, early recruitment-related activities on the internet also affect decision of each job seeker's applications, it involves activities—publicity, sponsorship, word-of-mouth endorsements, and advertising, all the four activities are the major approaches for all applicants to acquire information of firms or vacancies [10]. Besides sufficient information and high-speed server, internet of information technology brings greater scale of data [8].

Given those challenges, matching systems based on the static deferred acceptance algorithm cannot operate in many modern labor markets with impact information and dynamic environments. Current manual processes which are still used by many companies do not have the ability to handle large scale data produced by numerous social media accounts and internet job search engines. Although job search engines have become important tools for any human resource departments, they only provide job position and candidate information. They currently do not provide automated two-sided matching with stable outcomes.

1.3 Proposed solution and contributions

From two-sided market perspective, we can see that the internet can help both vacancy side and applicant side exchange their preference information. Therefore, it has the potential to improve matching stability. On the other hand, since internet data can be processed more efficiently using automated computer systems, we now have the opportunity to design matching systems which can produce effective matchings in a more efficient way in large scale and dynamic market environments.

In this thesis, I design a two-sided matching system for dynamic labor markets with large scale of data. The core matching algorithms proposed for the system are extensions of the classical deferred acceptance algorithms. I considered three dynamic settings: 1) free matching, 2) matching with locking period constraints, and 3) matching with break-up penalties. In the free matching setting, when dynamic events occur, matching system can re-do the matching freely to achieve new stable matchings. The setting of matching with locking period constraints is motivated by the study from Clark and Summers [11]. They show that people frequently entering or leaving labor market will cause higher unemployment. In our setting, every matching has a locking period within which the two sides are locked in. They cannot rematch to others. We study how the adjustment of locking periods can affect the stability of the market in dynamic environments. The motivation of the setting of matching with break-up penalties is from the observation by Kugler in [12]. The author found that firing costs determines the matching decision of a labor market. Our system will also consider firing costs when making matching decisions in dynamic labor market environments.

1.4 Outline of the thesis

The rest of the thesis is organized as follows. Chapter 2 identifies a critical conflict of labor market by analyzing a core requirement based on the Environment Based Design methodology. Chapter 3 reviews the literature of deferred acceptance algorithms and present its detailed application in static labor market. Chapter 4 proposes three extended algorithms from classical deferred acceptance algorithms for solving dynamic matching problems. Chapter 5 proposes a multiagent system to achieve effective and efficient operation of matching function. Finally, in Chapter 6, we conclude the work and presents future research directions.

Chapter 2

Requirement analysis using EBD

As mentioned in Chapter 1, current manual operations of *deferred acceptance algorithm* and researching engines cannot provide stable matching results for dynamic large-scale labor markets. Due to the limitation of the current matching approaches, we will design a system to solve the challenges existed in labor markets. In the chapter, an effective methodology, *Environment Based Design*, is introduced to analyze the environment of labor markets. In addition, we apply the analyzing processes of the EBD to identify critical conflicts and core environment components for the design of the system.

2.1 Environment Based Design

Environment Based Design (EBD) is a design methodology developed over the last 20 years [13], which originally stems from the observation that the purpose of design is changing an existing environment to a desired environment by generating a new object, such as products or services.As we know that design is dependent to environment, it starts from environment, functions for environment and changes environment. The processes of environment changing accompanied by the recursive evolution of design problems, design knowledge and design solution. Therefore, some rules for regulating the changing of environment should be given. There are three basic activities involved in EBD: environment analysis, conflict identification and solution generation. The analyzing processes for dynamic labor market is based on the first two activities: the environment analysis and conflict identification.

In the following sections, by applying EBD, I will analyze the requirements of current labor markets and generate critical conflicts and core design environment components by following the sequence of environment analysis and conflicts identification. Some related analyzing approaches like recursive objects model (ROM) diagram and performance network will be applied as well.

2.2 Environment analysis

The objective of environment analysis is identifying the environment that how the desired products or services work in the environment. It is usually divided into three types, natural environment, built environment and human environment. The processes of analyzing a environment should include the components and its relationships. In order to define the components as well as the relationships, a recursive object model (ROM), a linguistic tool in design, is helpful for generating the definition of each objects in a design problem.

The design problem can be described as:

- Design an effective and efficient system for dynamic large-scale labor market.

A ROM diagram, based on the problem statement, is shown in Fig.2.1:

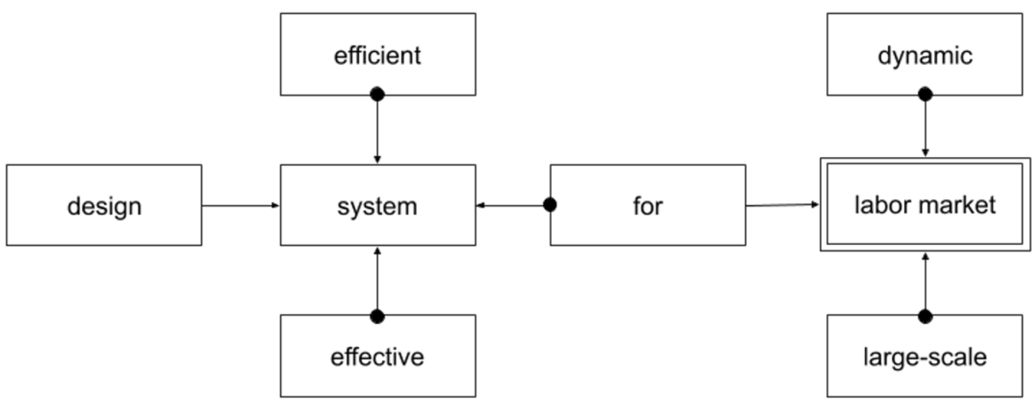


Figure 2.1: ROM Diagram

The processes of generating the definition of each objects in ROM diagram is helping designers better understand the design problem through linguistic analysis using ROM. We have gotten the initial problem statement, further questions of each objects should be asked for collecting information that may have significant affect on our design problem. We can observe that the core object of the design is “sysetm”, for defining the object, “dynamic labor market", "effective" and "efficient" should be defined firstly. The question generating process is shown as follows:

- What is "dynamic"?

"Dynamic" means constant change.

- What is "labor market"?

"Labor market" is a marketplace that match vacancies and applicants.

- What do you mean by "effective"?

"Effective" refers to the stability in labor market.

- What do you mean by "efficient"?

"Efficient" refers to do something well and thoroughly with less waste of time.

- Who "design an effective and efficient approach for dynamic labor market"?

The human resources manager in labor market design an effective and efficient approach for dynamic labor market.

- Why design such an approach for dynamic labor market?

Because the current system of labor market does not behave well.

According to the answers of each object in initial problem statement, the updated one is

- Human resources manager designs a system to rapidly form stable and changed pairs between a large number of applicants and vacancies,

with the ROM diagram as Fig. 2.2:

2.3 Conflict identification

Conflicts are considered as the driving force in EBD analysis process. It is helpful for designers to develop problem solution for satisfying the requirement of design problem by identifying properly all conflicts. From the ROM diagram Fig. 2.2, there are some conflicts and potential conflicts between the relationships of each objects.

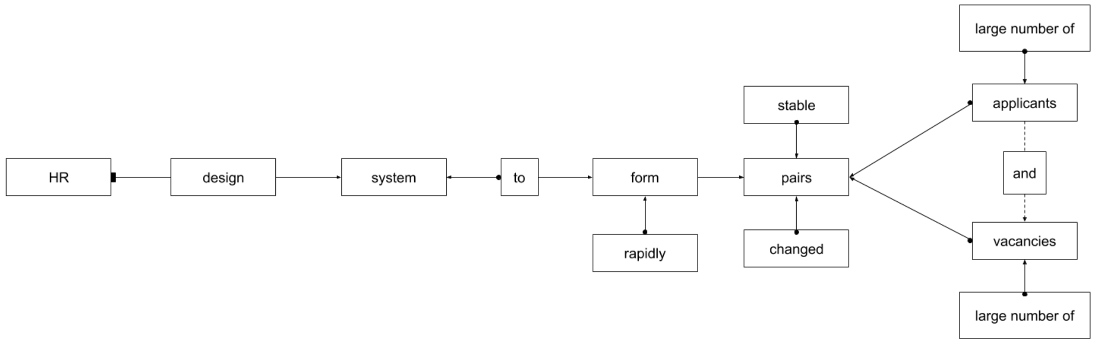


Figure 2.2: Revised ROM Diagram

Each action of the renewed problem statement in the diagram is called interaction. In order to understand the internal relationships among all the interactions, performance network helps us figure out conflicts for each interaction. The critical conflict of design problem is behind the root constrains. The 1st performance network table is shown in Table 2.1. It is obvious that I_3 and I_4 constrain I_2, while I_2 constrains I_1, the internal

Interaction	Description
I_1	Human resources designs a system
I_2	System matches applicants with positions
I_3	System achieve stability
I_4	System within less time

Table 2.1: Interaction Description 1st Round

relationships are shown below in Fig 2.3.

It is found that there exists potential conflicts in Interaction I_3 and I_4. Two more question is generated based on these two interactions.

- How to achieve stability?

Each applicant has no tendency to break up with assigned vacancy. And each vacancy has no tendency to break up with assigned applicant.

- why do you need match within less time?

Current matching function are operated by manual.

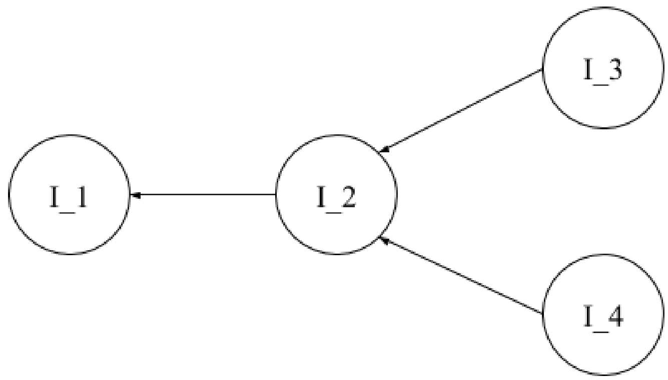


Figure 2.3: Performance Network 1st Round

The conflict in I_3 is the requirements of both applicants and vacancies, and the conflict in I_4 is the requirement of efficiency is constrained by manual matching. Then renewed interactions and Performance Network are given in Table 2.2 and Fig 2.4.

Interaction	descriptions
I_5	Applicant has no tendency to break up
I_6	Vacancy has no tendency to break up
I_7	Automatic matching improve efficiency.

Table 2.2: Interaction Description 2nd Round

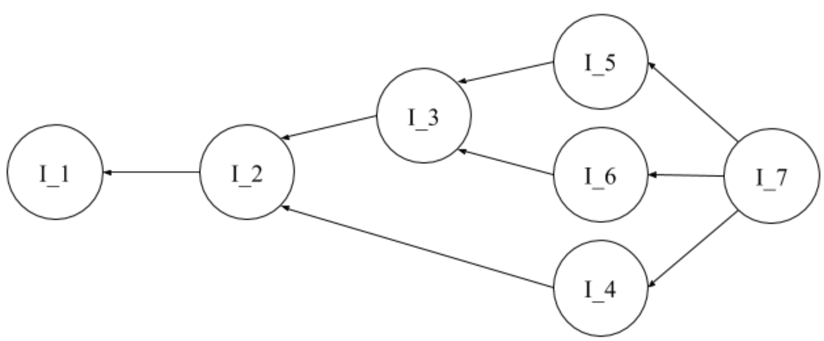


Figure 2.4: Performance Network 2nd Round

Therefore, after analyzing the initial problem statement, we finally found the most critical conflict, that is, “manual operation constrains the efficiency and effective of a matching system in labor market”. In addition, the core requirement of the design problem is concluded as “Design an automatic system to produce stable matching between applicants and vacancies even though the changes occur constanely”. And based on the definition of conflict, an insufficiency of resources for an object to produce a desired action on its environment or to accommodate the object's action on its environment.

We conclude four points: 1) environment: dynamic labor markets with large scale of data; 2) object: two-sided matching system; 3) desired action: generate stable matchings with fast response time. 4) Insufficiency of resources: Manual matching process is insufficient to produce stable matchings in large markets and classical deferred acceptance algorithms cannot be applied directly to dynamic labor market environments.

2.4 Summary

Based on the EBD requirement analysis, two core requirements of design problem for the limitation of current approaches in dynamic labor market are identified as: 1) effectiveness: the matching results generated by the designed system should be stable; 2) efficiency: the system should automated generate matching with fast response time. Associated dynamic matching algorithms will be proposed in Chapter 4, followed by a multiagent system to achieve matching automation in Chapter 5.

Chapter 3

Static matching model and Deferred Acceptance Algorithm

In Chapter 1, we know that labor markets are the marketplaces where applicants offer their skills to vacancies in exchange for wages, salaries, and other forms of compensation. The major function of labor markets is a special allocation where "personal desires" are considered. The allocating processes are defined as "matching", and labor markets are defined as "two-sided matching markets". To solve the matching problem in two-sided matching markets, a classical algorithm named deferred acceptance algorithm was proposed. In the chapter, I will review the literature related to the evolution of two-sided matching theory and the application of deferred acceptance algorithm in different domains. Furthermore, the specific application in static labor markets will be presented as well, which is the basis of dynamic matching algorithm design.

3.1 Evolution and application of Deferred Acceptance Algorithm

Gale and Shapley [4] firstly proposed the classical deferred acceptance algorithm, which has a profound influence on two-sided matching market. In the paper, they proposed two models, one is college admission model, which is a typical many-to-one two-sided matching market, and the second one, the marriage model is adopted to illustrate how deferred acceptance algorithm works in realistic applications. Besides, a concept of stability is also proposed by describing unstable situation that if an agent in the market prefers being unmatched to being matched with assigned mate and if a pair of agents prefer being matched with each other to being matched with assigned mate. Also, the proof of stability of matching results produced by deferred acceptance algorithm is given in the paper. Wilson [14] compared the matching results produced by classical assignment algorithm with the matching results generated by deferred acceptance algorithms under the criterion of stable marriage [4]. The comparisons showed the result based deferred acceptance algorithm is better than other algorithm for solving the classical assignment problem. Roth [15] considered some games in a matching problem that each agent in both disjoint groups has preference over opposite side would like to be matched with front choice in his or her preference list. The presented paper mainly focuses on how to give agents incentive to tell truth about their preferences in order to produce a stable matching result, and demonstrates two principal results that a matching procedure exists to yield a stable outcome and give agents incentive to tell truth about their preferences. And Roth [16] proposed a two-sided matching model of hospitals and medical students. Each agent in both sides has its preference list, besides hospital has a quota which is the number of job positions for the students. And this paper gave the proof of stability for many-to-one matching labor market. In addition, it is mentioned that the agents in the market are prepared to act on the incentives to truthfully tell about their preferences. Three years later, Roth [3] did some more works in the domain of two-sided matching, he studied two-sided matching processes as a game with incomplete information. He pointed that if each agent in the market does not know the preference of all agents except himself. all equilibria of incomplete information need not occur. After that Roth [17] pointed out that the failures of market in which the markets are addressed by centralized and deterministic matching procedures, seem to be associated with instability of the outcomes. But in decentralized environment, like labor market, the failures are not observed yet. The paper demonstrated that stable matching can be arise with positive probability from a situation that all agents are unmatched or some blocking pairs are randomly chosen to match. And he believed it is possible that the matching results can be global optimality. On the basis of Roth's work, Zhou [18] proved that there exists an equilibria through G-S algorithm in marriage model, if the preference profile is true. Mongell and Roth [19] studied the history and organization of recruitment processes of sororities. They analyzed the influence on stability of matching result and longevity of the system caused by the incentives to stategic behaviours under a centralized matching procedure. Sasaki and Toda [20] built a two-sided matching model with externality, and they proposed a new concept of stability and prove that the stability exits generally. Furthermore, if the stability satisfy Pareto optimality was given as well.Dutta and Masso [21] studied the two-side matching problems of labor market. That is the matching between individuals and institutions, and each of them has a preference list over all the agents over the opposite side. But some changes may occur if an individual's preference is effect by others, the writer studied the consequences for stability of the case. Sonmez [22] studied the two-sided matching problem between hospitals and medical residents in the United States, and hospital-optimal stable matching solution had been applied before. The author proposed a manipulation via pre-arranged matches in centralized environment and proved there is no solution that is both stable and non-manipulable. Balinski [23] explored a new class of matching problem in college admissions. It is not the same as GS's [4] college admission model, it is centralized and via standardized tests. In addition, the concrete application is also presented by Turkey college admission. After detailed analysis of Turkey college admission, the author pointed out the current mechanism Turkey applies has a lot of serious deficiencies, especially, it is inefficiency, vulnerability to manipulation, furthermore the potential of penalizing students for improved test scores. Hence, deferred acceptance algorithm is an alternative mechanism to improve the performance college admission in Turkey and get an student optimal matching result. Roth [24] reported on the design of a new clearinghouse to match new physicians with provided positions. It is found that simple matching theory shows a good approximation in the labor market, and a fact that in the market, each applicant can interview for only a small fraction of positions to explain the reason why the opportunities for strategic manipulation in the market is small. Ausubel [25] proposed an direct revelation mechanism called "ascending proxy auction", it is a kind of "deferred acceptance algorithm" and is similar to two-sided matching problem. As analyzed, the author pointed out the auction algorithm is also an allocation of the package exchange economy associated to some preference lists of agents in the bidding market. Hiroyuki Adachi [26] concludes the definition of two-sided matching, "two-sided" means there are two disjoint sets of agents in one market, for example, in labor market, firms and workers. "matching" refers to the fact that it is a bilateral nature of exchange process. For example, if a worker works for a firm, at the same time, the firm pays the worker. And this paper proposed a search model of two-side matching with consideration of nontransferable utility to achieve an equilibrium which tells both sides of agents will complete mating upon meeting. Ehlers [27] consider another aspects in the theory of two-sided matching, the author gave a definition of strategic choice for a worker in labor market, which is used to provide suggestions for the worker when she offers her preference lists. It works for producing three types of choices that firms would never accept her proposal, the firms may accept her proposal and the rest firms who must accept her proposal. Based on these, an effective preference lists would be given. Abdulkadirouglu [28] found that in Boston school matching problem, it is costly to list a first choice that a student do not succeed in admitted because, when other students are assigned their first choices, they can not be displaced even by a student with higher priority. Deferred acceptance algorithm is applied for the problem and avoid this situation. Satterthwaite [29] proposed a decentralized dynamic two-sided matching market with incomplete information over other agents. In the market, time is discrete and utility of each agent is considered as well. In the market, after trade is done, the buyers and sellers exit the market with their real utility. The paper gave us a direction to investigate dynamic labor market, of which time is discrete and utility of each agent determine the matching results, meaning trades or contracts. Korkmaz [30] studied a two-sided matching problem in military, assigning personnel to positions. The author proposes a new kind of agents, detailers, who is charge on producing matches to satisfy needs and preferences of commands and personnel in both sides. Also, the effects of the length of preference lists are examined in the paper. Roth [31] pointed out a centralized marketplace with deferred acceptance algorithms works well because it solves the problem of thickness of market and reducing congestion when the market is thick. In the environment, each agent can tell truth about his or her preference. Furthermore, the algorithm is widely used in labor market and school choice system, he mentioned. Kojima and Pathak [32] proposed a many-to-one matching market models of labor markets and student placement systems. In a large scale, they found misrepresentation of preference widely exists under some regularity conditions. But the conclusion is that if each participant in the market truthfully tells about their preference, an approximate equilibrium will be found under the student-optimal stable mechanism. Abdulkadiroglu [33] redesigned New York City high school matching market. In the previous market, some schools strategical rank students in their preference lists, however it is necessary to tell truth to produce stable matching results. The author adopts a student-proposing deferred acceptance mechanism and break indifference. After analyzing the data from the recent redesign of NYC high school match, it is found that potential efficiency loss is substantial. Alpern [34] gave us another aspects on two-sided matching problem, the author assumed that the preference lists of each side over other side are the same, and proved that the kind of constrains are realistic and can provide useful solution for designers of matching mechanisms. Chakraborty [35] studied a two-sided matching market between colleges and students in the environment of incomplete information. The author pointed out that if one side of the market can observe entire matching results, the stable matching can not be generated, while one side of the market observe only their own matches and other sided has identical preferences, the stability can be achieved. Hałaburda [36] believed that similarity of preferences can drive unravelling by investigating the reasons and welfare of unravelling in two-sided matching markets. Chen [37] reviewed the work of other application-rejection school choice mechanisms including [28], and the author compared the DA mechanism with Shanghai college admission mechanism, and got a conclusion by comparing the efficiency, DA is significantly more stable than Shanghai, which is more stable than Boston. Xu [38] applied two-sided matching theory in VM migration in Cloud computing. The stable matching problem involving thousands of VMs and servers. Ackermann [39] analyzed GS's [4] two-sided matching model, and proposed a question of convergence time. In the paper, they proved that the random best response dynamics, what the author extended from better response dynamics, in two-sided matching markets, converges to a stable matching with probability one. Chen [40] proposed a two-sided matching model between banks and firms in a loan market, it is a many-to-one two-sided matching problem, which is similar to college admission model [4]. The paper proposed two main points of obtaining information, one is geographically closer and the other is prior loans. In a word, the more detailed information each agent in the market can obtain, the more accurate a preference list of each agent can be made. Gu [41] adopted two-sided matching theory to manage resource in wireless network. The author found that after application of deferred acceptance mechanism, the results show that the performance of resource allocation has higher effectiveness and much more efficient. Azevedo and Leshno [42] developed a framework to a matching market by applying supply and demand analysis. They found that the processes of finding a stable matching is similar to achieve equation of demand and supply. And they viewed the agents in the market as continuum in order to achieve simplification of matching in large market.

3.2 Modeling and computing for static labor markets

A labor market consists a set of agents and their preferences. The agents typically involve applicants and vacancies. The agents have kinds of individual behaviors, dependencies and interactions, which are complicated to captured. Therefore, we replace two basic components of labor market by two well-designed agents, in order to simply represent complex individual characteristics.

- Job Applicant (JA) Agent

JA agent acts as a job applicant in labor market. The behaving rules of each JA agent involve: 1) post personal information; 2) rank all vacancies in the labor market into a strict order list by evaluating the qualification of each vacancy; 3) apply for the preferred vacancy according to preference list; and 4) make decision which offer proposed by one vacancy he or she should accept.

- Vacant Job Position Agent

VJP agent acts as a vacancy in labor market. Differentiated with firm-worker labor market, each vacancy only employs one applicant in the vacancy-applicant labor market. The behaviours rules of each VJP agent involve: 1) post detailed vacancy information; 2) rank all applicants in the labor market into a strict order list by interviewing and evaluating professional skills of each applicant; 3) send offer to the preferred applicant according to preference list; and 4) make decision which applicant, applying for the vacancy, should be employed.

For simplification, in the following contexts of this thesis, we replace vacancies and applicants by VJP agents and JA agents to behave as the concrete objects.

3.2.1 The two-sided matching model of static labor markets

Deferred acceptance algorithm (DAA) is GS[4] designed originally for two-sided matching problem in College Admission and Marriage Market, they are many-to-one and one-to-one matching problem respectively. Two groups of unique individuals and bilateral choices are the factors of a matching market. Compared with commodity market, matching market mainly concentrates on the information exchange and manage rather than just value exchange driven by price. Labor market, as what we analyzed, is kind of typical matching market involving JA agents and VJP agents, outcome of the market is not simply driven by the salaries or profits, information collection and comparison is important as well. Hence, we can regard labor market as a two-sided matching market as the same as marriage model for the reason that JA agent and VJP agent are one-to-one matching relationships. Besides, it is more flexible and dynamic for labor market than marriage one, the variations including like JA agents or VJP agents entering, parts of matches locking or contracts breaking after a time period. All the behaviours we mentioned before can affect the stability of a matching result, simply repeating deferred acceptance algorithm can not handle the problems caused by the changes, some improvement for algorithms as well as modeling approach are necessary to help us solve the dynamic matching problem in labor market. In the section, two-sided matching model will be built to describe labor market with the introduction of deferred acceptance algorithm. Three improved algorithms for solving matching problem under dynamic environment of labor market will be proposed following the discrete event system model for describing all possible changes over the market.

Labor market is a typical one-to-one two-sided matching market in which there are two disjoint groups of agents, including JA and VJP respectively. VJP agent wants to employ a good JA agent among all JA agents, while each JA agent would like to be matched with a suitable VJP agent that the agent satisfy his or her basic requirement like salaries. In the section, we will mainly focus on the matching processes in static environment.

There are two sets of agents, VJP agents and JA agents, denoted by $V = \{v_0, v_1, ..., v_m\}$ and $A = \{a_0, a_1, ...a_n\}$ respectively, in the labor market.

Each element in one set has a complete and transitive preference over the agent in the opposite set. The preference of each agent can be represented as an rank order list of the form below:

$$P(v_i) = [a_o, a_p, ..., v_i, ..., a_q], o, p, q \in [0, m], i \in [0, n] \quad (1)$$

where a_o is the best choice of v_i and a_o is a better choice than a_p for v_i, the relationship can be denoted by a in-equation as below:

$$a_o >_{v_i} a_p \quad (2)$$

In real-world there exists a special case that part of agents over the opposite side is unacceptable for a JA agent or a VJP agent. The case can be represented as an in-equation like the form as below:

$$v_i >_{v_i} a_q \tag{3}$$

That means v_i prefers being unmatched to being matched with a_q. For the sake of reducing workload, we usually ignore the unacceptable choices. Besides, if a JA agent or a VJP agent is indifferent between any two acceptable mates, or between being unmatched and being matched with an acceptable mate, we have to break up the ties arbitrarily in order to get a strict preference list.

The outcome of labor market is a matching, denoted by a function μ: $V \cup A \longrightarrow V \cup A$, such that $a = \mu(v)$ if and only if $v = \mu(a)$, and for all a and v either $\mu(a)$ is in V or $\mu(a) = a$, and either $\mu(v)$ is in A or $\mu(v) = v$. It is that an outcome matches agents on one side to the agents on the other side or to themselves, and if v is matched to a, then a is matched to v.

For commodity market, quality is one of the most significant criterion to measure if a service or product is good or not. As what we analyzed in 2, in labor market, stability is what we should consider to achieve high effectiveness. Before giving the definition of stability in matching market, I will introduce two special cases.

- Blocked by Individual

If an agent k prefers being matched with himself to being matched with his assigned mate $\mu(k)$ made by market. The case can be represented as follow:

$$k >_k \mu(k) \tag{4}$$

- Blocked by Pair

There are two agents v and a in labor market, which belongs to the set of VJP agents and set of JA agents respectively. Both of them form a new pair, denoted by (v, a). Each of them has been assigned before, the matched mates are denoted by $\mu(a)$ and $\mu(v)$, but for v it prefers being matched with a to $\mu(v)$, while a prefers being matched with v to being matched with $\mu(a)$, the relationships are represented as two in-equations as below, the case is called blocked by pair (v, a).

$$v >_a \mu(a) \tag{5}$$

$$a >_v \mu(v) \tag{6}$$

Definition 3.2.1. *A matching μ is stable if it is not blocked by any individuals or pairs.*

The efficiency of one matching market is measured by stability and the core of a matching market is that matching results satisfy each agent's least requirement and no one complains or has tendency to break assigned mate.

3.2.2 Classic Deferred Acceptance Algorithm

GS [4] proposed a well-known algorithm to solve college admission and marriage two-sided matching problems. The objective of a labor market is assigning appropriate JA agent into an acceptable VJP agent to achieve high profits for companies and harvest salary for JA agents. In addition, recruitment can be viewed as a information and value exchanging process, which is the features of matching market, as same as marriage market. So deferred acceptance algorithm can be applied as well in labor market. The processes of getting a matching result by Deferred Acceptance Algorithm are shown below:

Notation	Meaning
V	VJP Agents Set
A	JA Agents Set
v	Any VJP Agent
a	Any JA Agent
$P(A)$	All JA agents' Preference Lists
$P(V)$	All VJP agents' Preference Lists
$P(a)$	Agent a's Preference List
$P(v)$	Agent v's Preference List
X	Matching Result
$First(P(v))$	Getting 1st Choice of v
W_k	JA agent k's Waiting List

Table 3.1: Explanations for Deferred Acceptance Algorithm

- Step 0:

After information of VJP agents and JA agents has been adequately exchanged. Each agent uploads a preference list over the opposite side.

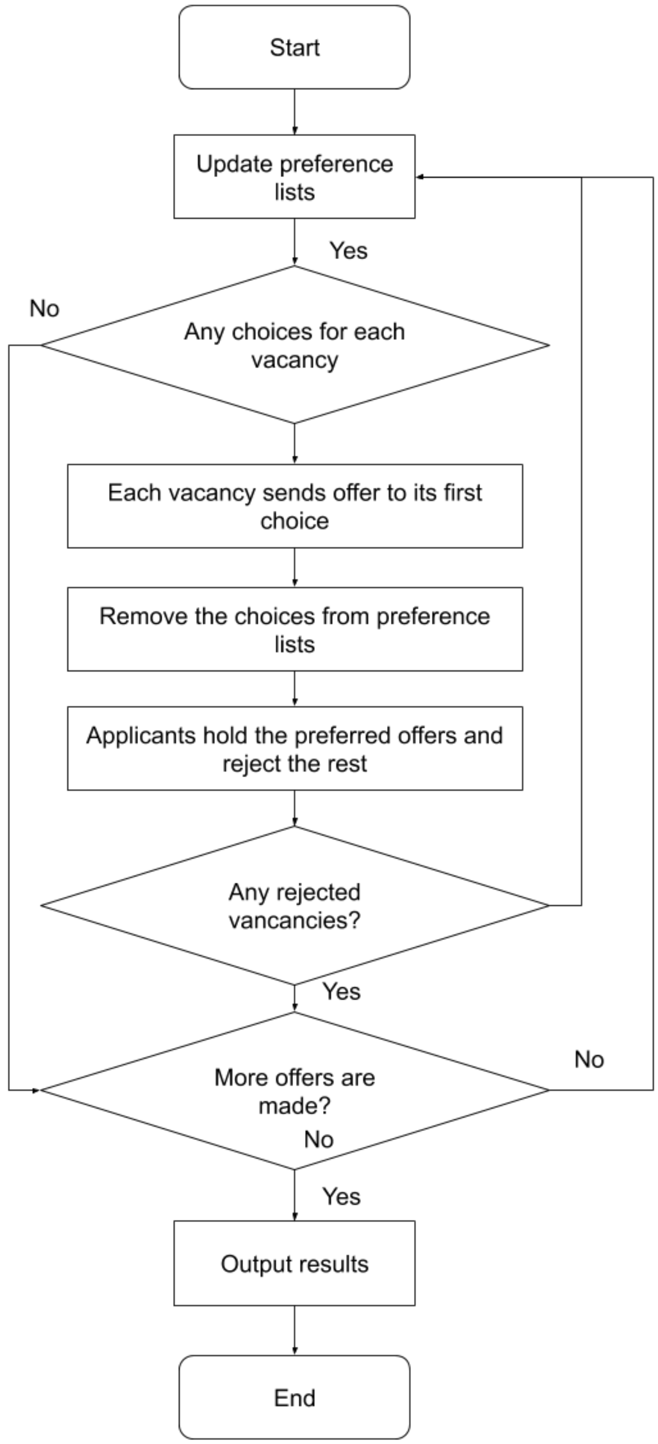


Figure 3.1: Flow chart of Deferred Acceptance Algorithm

- Step 1:

a. Each VJP agent v sends offer to its first choice (If the agent has any acceptable choices).
b. Each JA agent rejects any unacceptable offers and if more than one acceptable offer is received, "holds" the most preferred and rejects the rest.

- Step k:

a. Any VJP agent which are rejected at step k-1 send offer to their most preferred acceptable JA agents who have not rejected them yet. (If there is no acceptable choices of the rejected VJP agent, the agent makes no offer)
b. Each JA agent holds his or her most preferred acceptable offer and rejects the rest.

- Stop:

if there is no further offers are made, match each JA agent to the VJP agent (if any) whose offer the JA agent is holding.

Algorithm 1 Deferred Acceptance Algorithm

```
Input: P(A), P(V);
Output: X;
for v_i ∈ V do
    a_k = First(P(v_i));
    P(v_i) remove a_k;
    W_k ⟵ v_i;
end for
while exists length(W_k) ≥ 2 do
    for a_j ∈ A do
        for v_i ∈ W_j do
            if a_j rejects v_i then
                W_j remove v_i
                if length(P(v_i)) ≠ 0 then
                    a_k = First(P(v_i));
                    P(v_i) remove a_k;
                    W_k ⟵ v_i;
                end if
            end if
        end for
        X ⟵ W_j
    end for
end while
return X
```

The processes of Deferred Acceptance Algorithm is shown in Fig.3.1. The corresponding notations and functions are shown in Table 3.1. And the pseudo-code of Deferred Acceptance Algorithm is shown in Algorithm 1.

The major part of the algorithm is combined with two sub-algorithm, proposing and decision making for VJP agents and JA agents respectively. Then how we could guarantee that the results generated by DAA are stable? The theorm and proof of stability is represented as the form of observation:

Theorem 1. *The matching results generated by deferred acceptance algorithm is stable*

Proof. From the algorithm, no employer ever send an offer for a VJP agent to an unacceptable JA agent and no JA agent ever apply and send resume to an unacceptable VJP agent. Hence results generated by DAA are not blocked by any individuals

If some VJP agents prefer to be matched with an JA agent other than the assigned one, the employer of the VJP agent must, according to DAA, have already sent offer the that JA agent, and the JA agent has rejected the offer, which means the JA agent has an offer he or she strictly prefers, therefore, the results generated by DAA cannot form a blocking pair □

3.3 Summary

In the chapter, I reviewed the evolution of two-sided matching theory and application of deferred acceptance mechanism in different domains. In addition, a two-sided matching model of considered labor market is presented and the matching process of the deferred acceptance algorithm in the model are introduced in detail. Based on the contexts, the chapter builds a foundation for solving matching problems in dynamic and flexible labor markets, in next chapter, three dynamic matching algorithms, which are the extensions of classical deferred acceptance algorithm, will be proposed followed by an automaton model for describing common dynamics in labor markets.

Chapter 4

Two-sided matching system for dynamic labor markets

As mentioned in Chapter 3, the classical deferred acceptance algorithm can generate stable matching in static labor markets. However, in reality, due to the dynamic nature of the considered labor markets, the classic deferred acceptance algorithm under-performs both in solution quality and computational efficiency, especially on large labor markets with thousands of participants. To solve the problems, I propose three dynamic matching algorithms, which are the extensions of the classical deferred acceptance algorithm in the chapter, and dynamic labor markets are modeled as an automaton.

4.1 Discrete event system modeling for dynamic labor market

In the day-to-day life of technological and increasingly internet-dependent world, two things should be noticed. Firstly, some of the quantities we usually deal with are "discrete", typically like counting integer numbers, such as inventories level, how many cars are on a highway or how many students in a classroom. Secondly, some instantaneous "event", such as pushing a button, opening your computer or traffic light turning red. In fact, many of the things human have invented are "event-driven": manufacturing facilities, execution of computer program and the most important, dynamic labor market are typical examples. The section, we mainly focus on the introduction of discrete event system and its application on the considered dynamic labor market.

4.1.1 Discrete event system and its modeling approach

The intuitive definition of system is given as follows [43]:

Definition 4.1.1. *A combination of components that act together to perform a function not possible with any of the individual parts has three core elements, involving input, output and their relationships.*

Based on these, system modeling process starts by defining a set of measurable variables related to a given system. We collect data of the system by measuring these variables over a period of time $[t_0, t_f]$. Then a subset of the variables are assumed that we have ability to vary them over time. The definition of input variables is given by a set of time function as below:

$$\{u_1(t), ..., u_p(t)\}, t_0 \leq t \leq t_f \tag{7}$$

Next, another set of variables which can be directly measured while varying $u_1(t), ..., u_p(t)$ are defined as output variables:

$$\{y_1(t), ..., y_m(t)\}, t_0 \leq t \leq t_f \tag{8}$$

This can be thought of as describing the "response" to the "stimulus" provided by the input functions. Otherwise, some variables, which are not involved in input and output, are referred to as suppressed output variables. We present input and output variables through column vector $\mathbf{u}(t)$ and $\mathbf{y}(t)$ respectively.

$$\mathbf{u}(t) = [u_1(t), ..., u_p(t)]^T \tag{9}$$

$$\mathbf{y}(t) = [y_1(t), ..., y_m(t)]^T \tag{10}$$

And some mathematical relationship between input and output are existed in one system as the form of functions:

$$y_1(t) = g_1(u_1(t), ..., u_p(t)), ..., y_m(t) = g_m(u_1(t), ..., u_p(t)) \tag{11}$$

We convert it into the mathematical form as:

$$\mathbf{y} = \mathbf{g}(\mathbf{u}) = [g_1(u_1(t), ..., u_p(t)), ..., y_m(t) = g_m(u_1(t), ..., u_p(t)]^T \tag{12}$$

This is the simplest modeling process of system. As for the modeling process of dynamic system, it is much more complex. It was defined as one system where the output generally depends on past values of the input, compared with static system, what we input determines the output at current time. In a word, "memory" of system is significant in dynamic system, hence we give a new concept of "state", which can describe its behaviour in some measurable way. Then we give the definition of state:

Definition 4.1.2. *The state of a system at time t_0 is the information required at t_0 such that the output $\boldsymbol{y}(t)$, for all $t \geq t_0$, is uniquely determined from this information and from $\boldsymbol{u}(t), t \geq t_0$*

In general, the state equations is based on the form of differential equations:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \tag{13}$$

where $\dot{\mathbf{x}}(t)$ denotes the change of state at current time t. Combine the definitions of state, input, output and the function among the three elements, we can obtain a new model as the form below:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t),\ \mathbf{x}(t_0) = x_0 \tag{14}$$

$$y(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t), t) \tag{15}$$

Obviously, the elements in state set $\mathbf{x}$ is continuous. However, in dynamic labor market, the state of system is generally defined as matches between vacancies and applicants, which are discrete ordered pairs. The dynamic labor is defined as a discrete-state system. However, the "activities" that be thought of as occurring instantaneously and leading to the transitions from one state to another are named as "Events", denoted as E. Based on the concept of "Event" and "State", a definition of Discrete Event System (DES) will be provided as below [43]:

Definition 4.1.3. *A Discrete Event System (DES) is a discrete-state, event-driven system, that is, its state evolution depends entirely on the occurrence of asynchronous discrete events* *over time.*

Then, for generating the languages formed by the strings of events in dynamic labor market according to well-defined rules, a device is proposed as a timed automaton model.

$$G = (X, E, f, \Gamma, x_0) \tag{16}$$

where:

X is the state space.

E is the set of events associated with G.

$f : X \times E \to X$ is the transition function: $f(x, e) = x'$ means that the state x will be changed to x' if event e occurs.

$\Gamma \ : X \to 2^E$ is the feasible event function. $\Gamma(x)$ is the set of all possible events in state x.

x_0 is the initial state.

This is the same as what we introduced in section 1.3, in addition, the clock structure associated with automaton G is a set:

$$\mathbf{V} = \{\mathbf{v}_i : i \in E\} \tag{17}$$

of clock sequences as:

$$\mathbf{v}_i = \{v_{i,1}, v_{i,2}, ...\}, i \in E, v_{i.k} \in R^+, k = 1, 2, ... \tag{18}$$

Many system, particularly "human-made" ones, are in fact discrete event system. The dynamic labor market is a typical example with some required characteristics, for example, a event of agent entering could drive state transition of discrete ordered pairs. For detailed describe all possible behaviours of dynamic labor market, in the following, a timed automaton model will be presented.

4.1.2 Automaton model of dynamic labor market

Compared with static labor markets, dynamic labor markets exist more unpredictable variation. Some specific events cause the state transition, especially, the transition of matching results. As proposed in the definition of stability, if an agent prefer being matched with another agent to being matched with assigned mate, the matching is unstable. However, in dynamic labor market, there always are more excellent agents entering the market which are better than the previous ones, each agent of previous matching result inevitably has tendency to break up contracts signed before to obtain better mate, then a new stable matching should be generated. But it should be noted that frequent transition of matching results is unrealistic, in order to reduce congestion and the rate of unemployment, market always stipulate some rules to regulate behaviours of each agent. We conclude three typical events that cause the state transition of dynamic labor market, and based on these events, an automaton model will be proposed as well.

- A New Group of Agents Enter the Market.

In a labor market, each vacancy wants to find a suitable applicant, while each applicant would like to find a good job. In order to satisfy the requirements of each agent, a classical deferred acceptance mechanism is adopted to produce stable matches for both sides. In such a stable labor market, new agents are attracted to enter. Therefore, a large number of agents try to enter the market and ask for stable matching services. There is a problem of congestion occurring while thickness of a market was solved.

- Partial "Locked-In"

For solving the congestion in large market, and it is normal for partial parts of agents to sign contract within fixed time. The behaviours of “Signing contracts” are named “Partial locked-in”, the parts of “locked-in” matches temporarily leave the market and stop participating in matching activities, but the market still keep their matching results for future work.

- Unlock

Business cycle widely exists, so does in labor market, the “locked-in” agents have tendency to return to the market and match with better choice that just enter the market. The behaviours of returning to the labor market for previous “locked-in” agents are named as an event “Unlock”.

Based on the definition of discrete event system [43], and features of dynamic labor markets. We can define a state automaton model for this system in Fig.4.1

$$G = (X, E, f, V) \tag{19}$$

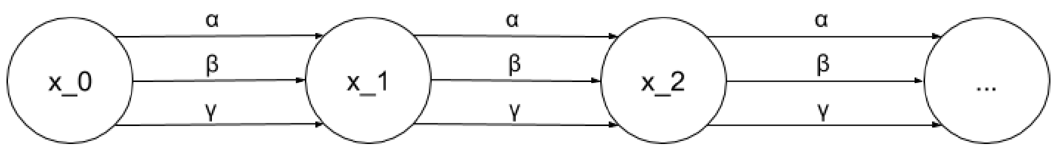


Figure 4.1: State Transition Diagram of Main System

X denotes a set of matching results which involve three subsets: locked set (denoted by L), unlocked set (denoted by U) and free set (denoted by F).

E denotes a set of events occurred in a dynamic labor market. Three types of events are defined in the model, denoted by α, β and γ.

- α: arrival of a set of vacancies and applicants to the market.
- β: a set of matched pairs are locked
- γ: a set of locked pairs are unlocked

f denotes a state transition function: $f : X \times E \longrightarrow X$

V denotes the clock structure of events occurring time.

There is a general example proposing to illustrate the processes of state transition. Shown in Fig. 4.1.

$$\begin{aligned}
&E = \{\alpha, \beta, \gamma\} && X = \{x_0, x_1, ...\} \\
&x_i = \{L, U, F\} \\
&L = \{(a_1, v_2).(a_2, v_3), ...\} \\
&U = \{(a_0, v_4), (a_3, v_0), ...\} \\
&F = \{(a_4, a_4), (a_5, v_5), (v_4, v_4), ...\} \\
&f(x_i, \alpha) = x_{i+1} && f(x_i, \beta) = x_{i+1} \\
&f(x_i, \gamma) = x_{i+1}
\end{aligned} \tag{20}$$

As well as the clock structure V, it is illustrated in the sample path of the example (shown in Fig. 4.2)

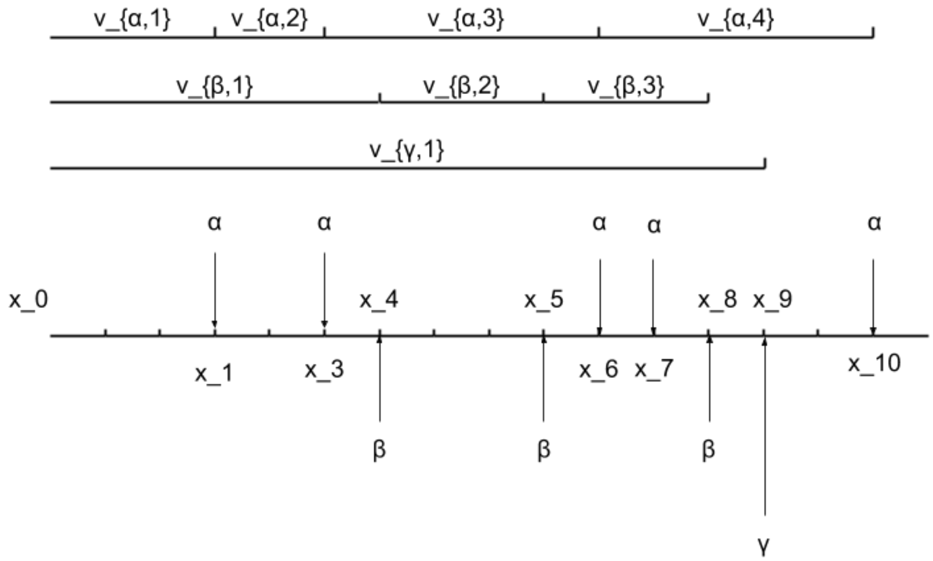


Figure 4.2: Sample Path of Dynamic Two-sided Matching System

As the sample path shown, at first there is an existing matching result in the dynamic labor market x_0, then a set of applicants and vacancies enter the market and trigger the re-matching function of the market, then the matching results, the state of the market, is transferred from x_0 to x_1. The time periods between x_0 and x_1 are constrained by clock structure $v_{\alpha,1}$. Another set of agents enter the market after $v_{\alpha,2}$ and trigger the re-matching function to generate x_3, followed by event β, in fact, the event β does not affect the matching results in labor market, but as what we defined, a state of market is combined by three subsets, L, U, F, β cause the parts of elements transferred from F to L, then after a strings of events $\beta\alpha\alpha\beta$, event γ is trigger by clock structure $v_{\gamma,1}$ occurs and because of the occurring, some parts of elements in L set of x_8 are transferred to U set, which means the parts if matches are free to re-match with other agent, but if they want to break up with previous mate, they have to pay for the penalty. Finally, another set of agents arrives following the event α and make matching results transferred from x_9 to x_10.

4.2 Dynamic matching with locking periods constraints

As the development of Internet technology, the thickness of labor market has been increasing within time. Each JA or VJP agent can enter one labor market in real-time to seek for job or employ new staffs. Based on the definition of stability, if there are some JA and VJP agents who are acceptable choices for other agents in the market, the previous matching results may be blocked if the ranking of new JA or VJP agents are higher than the current mates. In order to acquire a new stable matching result, DAA has to be operated whenever new acceptable JA agents or VJP agents enter the market. We call the algorithm as *Re-matching Algorithm* (RMA).

In fact, the algorithm is not efficient, and more congestion are made in the case, because DAA has to be operated frequently whenever new agents arrive in market, which improves the workload for managers to allocate applicants into job positions. In addition, it is not realistic that labor market is open all the time to frequently re-run DAA. Due to this, in the following, we propose a *Re-matching algorithm with locking periods* (RAL) to efficiently finding a stable matching result in the dynamic labor market.

Matching in labor market under the environment of advanced information technologies is a dynamic process. Each agent has his or her particular behaviours to satisfy his or her specific requirements, the manager of the market has his or her requirement as well. For example, JA agent has tendency to begin their career as soon as possible, while VJP agent wants to firstly employ the most excellent worker before more competitive VJP agent enter the market. In addition, the manager of the market tries to reduce workload and congestion of matching. So efficiency is one of the most important characteristics in dynamic labor market, the algorithm we need to design must satisfy the purposes. RAL is proposed for that:

- Step 0:

The market obeys the first-in-first-out (FIFO) rule that previous matched pairs should sign contract within fixed time and temporarily "locked-in". And according to the evaluation of new agents, update preference list of each agent in the market.

- Step 1:

1) Each VJP agent sends offer to to its 1st choice (If it has any acceptable and unlocked

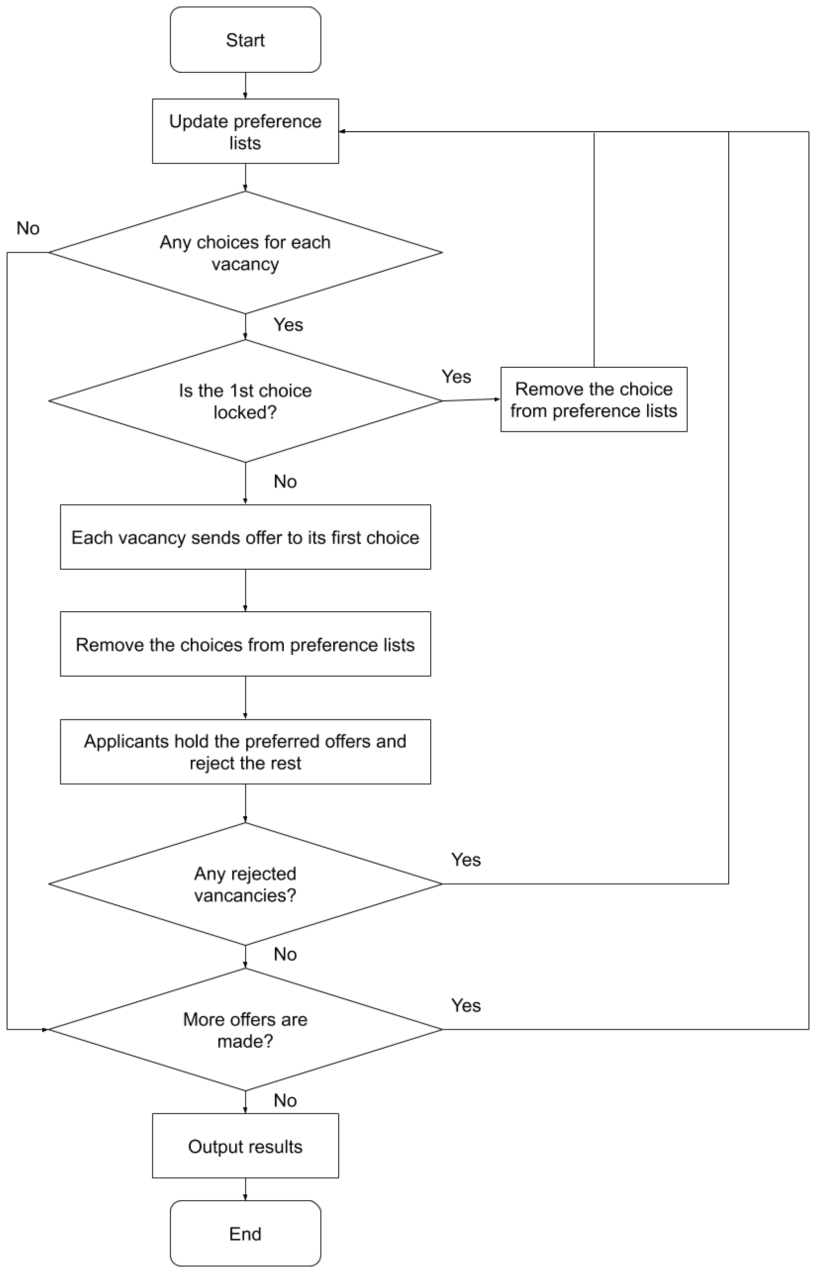


Figure 4.3: Re-matching algorithm with locking periods

Algorithm 2 Re-matching algorithm with locking periods

Input: $P(A)$, $P(V)$, A_{new}, V_{new}, X_k, A_{free}, V_{free}
Output: X_{k+1};

```
for v_i ∈ V_new do
    a_k = First(P(v_i));
    if a_k ∈ A_free then
        P(v_i) remove a_k;
        W_k ⟵ v_i;
    else
        P(v_i) remove a_k;
    end if
end for
while exists length(W_k) ≥ 2 do
    for a_j ∈ A do
        for v_i ∈ W_j do
            if a_j rejects v_i then
                W_j remove v_i;
                if length(P(v_i)) ≠ 0 then
                    a_k = First(P(v_i));
                    if a_k ∈ A_free then
                        P(v_i) remove a_k;
                        W_k ⟵ v_i;
                    else
                        P(v_i) remove a_k;
                    end if
                end if
            end if
        end for
        X ⟵ W_j
    end for
end while
return X
```

choices).

2) If more than one acceptable offers are received, each unlocked JA agent "holds" the most preferred and rejects the rest.

- Step k:

1) Any VJP agent rejected at step $k-1$ makes new offer to its most preferred acceptable mate who has not rejected the VJP agent. (if no acceptable and unlocked choices remain, the VJP agent makes no offer)

2) Each unlocked JA agent holds his or her most preferred acceptable offer and rejects all others.

- Stop:

when no more offers are made, match each JA agent to the VJP agent (if any) whose offer the JA agent is holding.

Notation	Meaning
A_{new}	New JA Agents Set
V_{new}	New VJP Agents Set
X_k	Matching Results in Cycle k
A_{free}	Unlocked JA Agents Set
V_{free}	Unlocked VJP Agents Set
X_{k+1}	Matching Results in Cycle k+1

Table 4.1: Explanations for Re-matching algorithm with locking periods

The pseudo-code is shown in Algorithm 2. The notations are shown in Table 4.1 And the flow chart of RAL is shown in Fig.4.3) as well. The flow chart is combined with two sub-algorithm of JA and VJP agents decision making processes. The left one is for VJP agent and the right one if for JA agent. Compared with DAA, even thought the background of Partial Matching Algorithm has been changed, the stability, representing effectiveness in labor market, is still one of the most significant features in labor market. The theorem and proof of stability is shown as follow:

Theorem 2. *The matching results generated by Re-matching algorithm with locking periods is stable*

Proof. We have stipulated that no more unacceptable agents are involved in any preference lists, which means matching results will never be blocked by any individuals. If some VJP agents would prefer to be matched with a JA agent other than assigned mate. The VJP agent must, according to the algorithm, have already sent offer to the JA agent, or the preferred JA agent has been locked before the open day. Hence the results of the market can not form blocking pairs. □

4.3 Dynamic matching with break-up penalties

Free choices are the part of natural rights, each JA agent can reject or accept any offer from VJP agents, VJP agents also have tendency to employ better staff who can create more profits. However, “invisible hand” can not always work, especially under the condition of the market economy witch allowing unrestrained freedom. A market designer should consider some constraints on freely choosing, chaos is a kind of the worst scenes in labor markets, it leads to higher unemployment. But if a labor market restrict agents flows, the competition and vitality will be decreased. Therefore, manager of a labor market always set up some mechanisms, like punishment mechanism on firing. The *Re-matching algorithm with Breaking-up Penalty* (RABP) is a solution for the condition that any VJP agents can unlock previous contracts if any better enough JA agents apply for a locked VJP agent. And any JA agent can break previous contract if any better enough offers of some VJP agents. If profits, represented as reservation utility, taken from new JA agent, can cover both reservation utility of previous assigned JA agent and firing cost, VJP agent can break previous signed contract with assigned JA agent and re-sign a new one with new JA agent.

- Step 0:

Estimate utility u of each JA agent and firing cost c of each VJP agent. Unlock pairs which were "locked-in" before made by RAL.

- Step 1:

1) Each VJP agent sends offer to its 1st choice (if it has any acceptable choices).
2) Each JA agents, “holds” the most preferred offer that profits of the offer can cover both profits of previous offer or contract and penalty of previous contract, if more than one acceptable offers are received.

- Step k:

1) Any VJP agents rejected at step k-1 makes new offer to its most preferred acceptable mate who has not yet rejected it. (If no acceptable choices remain, it makes no offer).
2) Each JA agent, “holds” his or her most preferred acceptable offer that profits of the offer can cover both profits of previous offer or contract and penalty of previous contract and rejects the rest.

- Stop:

when no further offers are made, match each JA agent to the VJP agent (if any) whose offer he or she is holding.

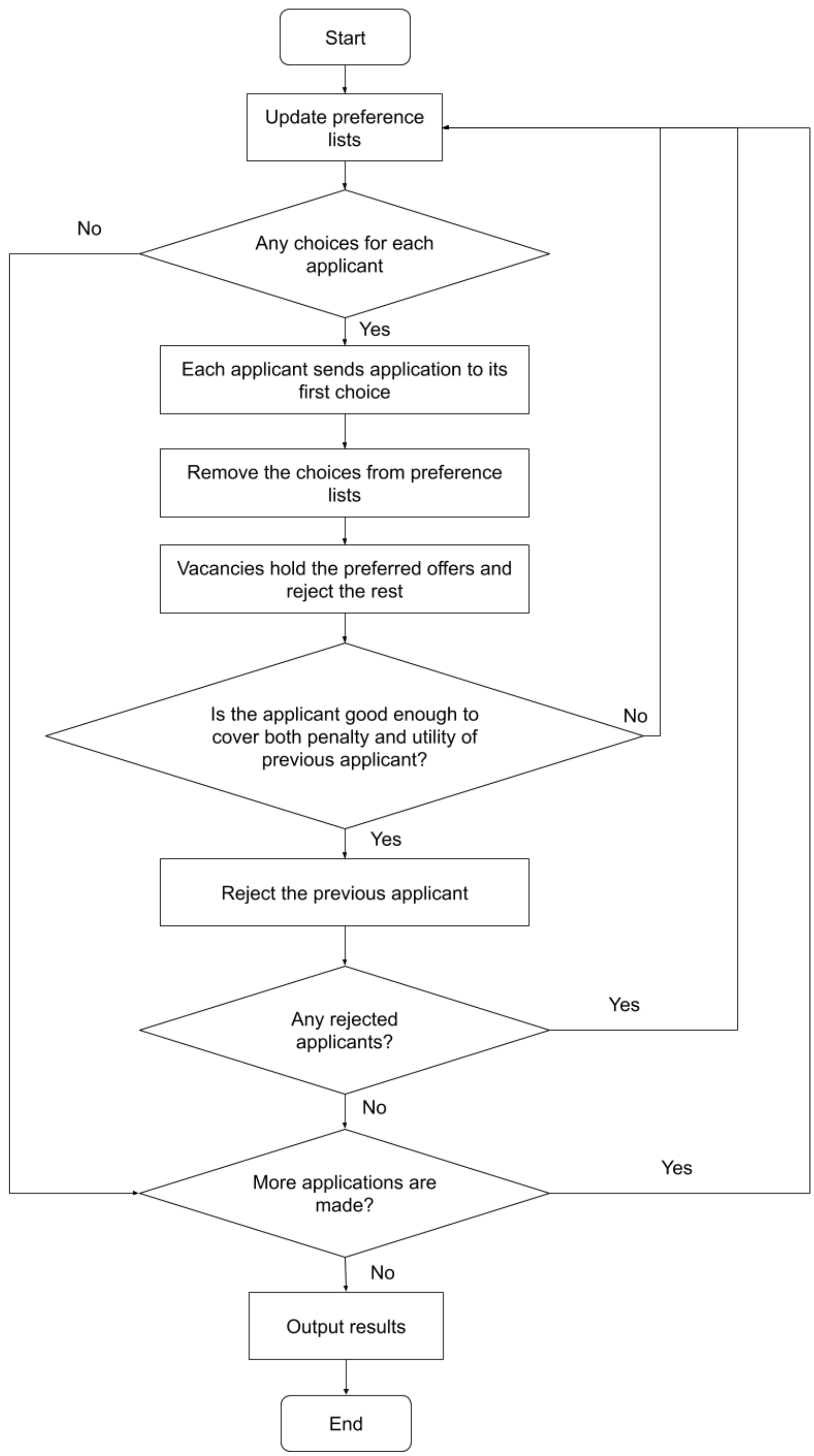


Figure 4.4: Re-matching algorithm with Breaking-up Penalty

RABP's flow chart, pseudo-code and related notations are shown in Fig 4.4. Algorithm 3 and Table 4.2

Notation	Meaning
$U(A)$	All JA agents' Utilities
$C(V)$	All VJP agents' Penalty Costs
W_k	VJP agent k's waiting list
$R(a_i)$	a_i's Ranking in v_j's Preference List
$a_{j,hold}$	v_j's Old Employee
$U(a)$	a's Utility
$C(v)$	v's Penalty Cost

Table 4.2: Explanations for Re-matching algorithm with Breaking-up Penalty

The theorem and proof of stability is shown below:

Theorem 3. *The matching results generated by Re-matching algorithm with Breaking-up Penalty is stable.*

Proof. No blocked individuals are made according to estimation of acceptability. And if some VJP agents v would prefer to be matched to a JA agent a other than its assigned mate $\mu(v)$, it must have already send offer to a and a has rejected it, meaning a has a contract or offer a strictly prefers or the profits of v is not good enough to cover both penalty and profits of previous contract or offer. Hence there will be no blocking pairs in the algorithm. □

4.4 Summary

In the chapter, I introduced a timed automaton model for dynamic labor market to describe some common behaviours of each agent. Three core dynamic matching algorithms are proposed following that model. It should be noted that only one of the events, new agents arrival, can drive operation of algorithms, while other events play roles of activating specific algorithms and transferring specific matching pairs from one subset to another one. Besides, the stability, which was defined as effectiveness of required system, is proved as well in the chapter. And in order to improve performance and efficiency of current online job searching engine by applying proposed dynamic matching algorithms, in Chapter 5, an automated matching system will be presented in detail.

Algorithm 3 Re-matching algorithm with Break-up Penalty

Input: $P(A)$, $P(V)$, A_{new}, V_{new}, X_k, $U(A)$, $C(V)$
Output: X_{k+1};

for $a_i \in A_{new}$ **do**
    $v_k = First(P(a_i))$;
    $P(a_i)$ remove v_k;
    $W_k \longleftarrow a_i$;
end for
while exists $length(W_k) \geq 2$ **do**
    for $v_j \in V$ **do**
        for $a_i \in W_j$ **do**
            if $R(a_i) > R(a_{j,hold})$ **then**
                W_j remove a_i
            else
                if $U(a_i) - U(a_{j,hold}) \leq C(v_j)$ **then**
                    W_j remove a_i
                    if $length(P(a_i)) \neq 0$ **then**
                        $v_k = First(P(a_i))$;
                        $P(a_i)$ remove v_k;
                        $W_k \longleftarrow a_i$;
                    end if
                end if
            end if
        end for
        $X \longleftarrow W_j$
    end for
end while
return X

Chapter 5

Dynamic two-sided matching system design

In Chapter 4, I proposed three dynamic matching algorithms for solving three common dynamics in current labor markers. The three algorithms are proved as stability for matching results, meaning that the algorithms can be operated effectively in dynamic labor markets. And an automated matching system which is responsible for selecting appropriate algorithms to generate stable matchings according to realistic requirements will be presented, followed by the simulation and experiments to verify the efficiency.

5.1 Components of multiagent system

Agent-based modeling approaches involving four main components, which are agent encapsulation, coordination and negotiation protocols, system architectures and decision schemes for individual agents.

- Encapsulation

Functional decomposition and Physical decomposition are two distinct approaches for a dynamic matching system, Functional decomposition uses agents to encapsulate modules assigned to functions such as matching and scheduling. Physical decomposition uses agents to represent entities in real world, such as job applicants, vacant job positions and matchmakers.

- Coordination and negotiation protocols

Most multiagent systems use negotiation protocols for scare resources allocation. In dynamic matching system, deferred acceptance algorithm is the core protocol to assign appropriate applicant into acceptable job position. The protocol stipulated the rules that each agent in the system should tell truth about his or her preference.

- Architectures

Architecture provides a framework to present how agents are designed and constructed. And architectures for multiagent dynamic matching system fall into three categories: autonomous agent, federated and hierarchical. According to the specific demand of system, choose suitable category as the system architecture. Hierarchical architecture is usually used in a situation that each agent represents a function or a department in a traditional manufacturing system. And federated architecture solved the problem suffered by hierarchical architectures due to centralization. Finally, autonomous agent architecture is well suited for developing multiagent system consisting of small number of agents.

- Decision Schemes

Each agent in one system needs compete negotiate or bargain with other agents, the decision of each agent mainly depends on two factors: the coordination or negotiation mechanisms and each agent's local decision-making mechanisms with sufficient knowledge.

The following sections develop a multiagent dynamic matching system within the four major components.

5.2 Multi-agent system architecture

Multi-agent system architecture provides designers a framework for describing how agents are designed and constructed. For improving stability of labor market, matchmakers are necessary for a multi-agent system architecture to allocate JA agents into VJP agents and stipulate rules to support the preface proceeding of market. In addition, each JA agent or VJP agent can be regarded as an autonomous agent who is not controlled by any other agents and can communicate and interact directly with other agent in the other side. And each autonomous agent know other agent's information and is familiar with the environment of the system. Besides each autonomous agent has their own objects and motivations. For the reasons, the multi-agent system of dynamic labor market consists of three major agents,

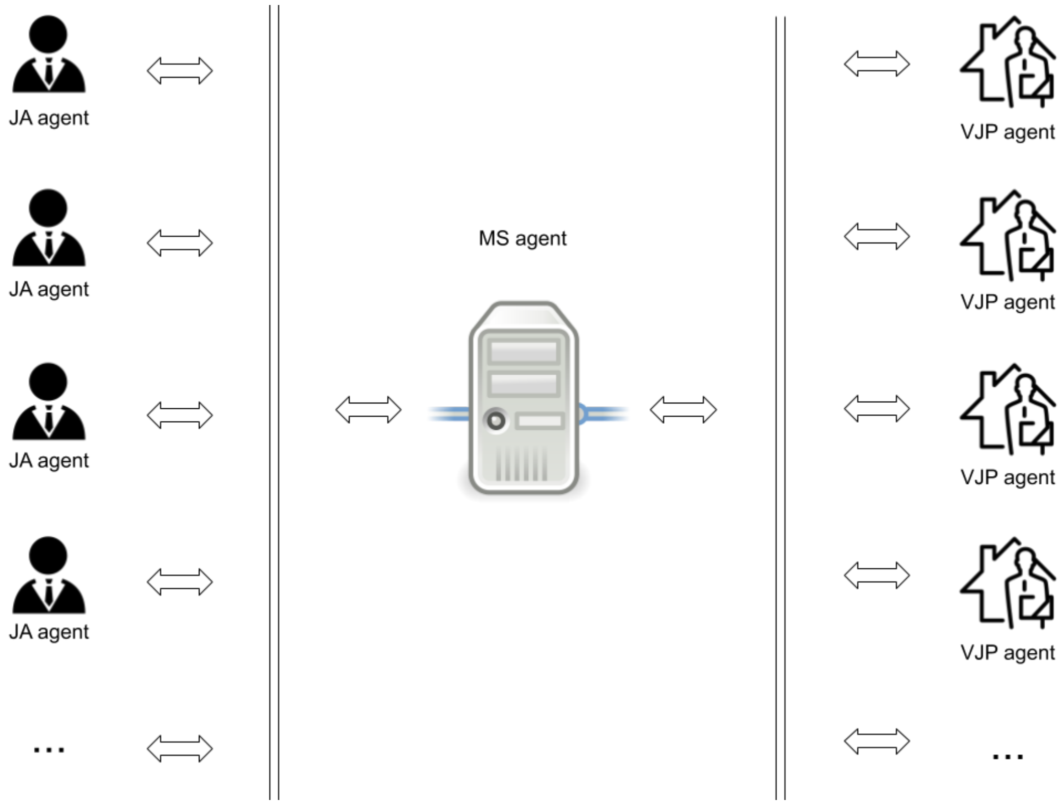


Figure 5.1: Architecture of MAS for Dynamic Labor Market

Job Applicant Agent (JA), Matchmaker Server Agent (MS) and Vacant Job Position Agent (VJP), as shown in Fig.5.1

The functionalities of these agents are described as follows:

- Job Applicant (JA) Agent

JA agent acts as an applicant, which has knowledge of applicant's professional skills and requirements of vacancies, and has ability to generate preference list by reviewing information of all VJP agents. Furthermore, each JA agent must upload personal information to MS agent in order to get widely checked and approved by VJP agents. Each JA agent should obey the market rules stipulated by MS agent for reducing congestion and keeping market thick. As for the decision making process of each JA agent, estimating if an offer made by MS or VJP agents is the preferred to hold temporarily until MS agent announce market closes, and rejection should be made immediately after decision making completed.

- Matchmaker Server Agent

MS agent acts as a server, which can solve operation problem of dynamic labor market and stipulate rules to keep market stable, efficient and effective. It has the knowledge of each JA and VJP agents' information, and is responsible for processing the data to be visualized forms. Then the visualized forms will be provided to each JA or VJP agent to acquire preference lists according to the direction of sending message. Also, MS agent takes charges on the scheduling date of market opening, locking and unlocking, besides, approve the contracts made by VJP agent is MS agent's responsibility as well. The decision making process of MS agent is if the matching is over and inform both JA and VJP agent the final results. Finally, the utilities of JA agents should be estimated based on the information provided by JA agents within unlocked period.

- Vacant Job Position Agent

VJP agent acts as a vacancy, which has knowledge of the information and requirements of the job position, has ability to make preference list by reviewing information of all JA agents. The information and requirements of each VJP agent should be uploaded to MS agent for making all JA agents can browse and filter. Every VJP agents in labor market must obey the rules stipulated by MS agent. And each VJP is responsible for making contract with assigned JA agent under the supervision by MS agent. It should be noted that, firing behaviour of each VJP agent can be happened according to the schedules made by MS agent. And penalty costs of each VJP agents are involved in contracts.

5.3 Interaction protocol

Within the architecture of multi-agent system model, matching service can be seen as a coordination process between JA agent and VJP agent with the assistance of MS agent. The matching negotiation processes are automated based on some protocols. The processes consist of three stages: proposal sending (PS), decision making (DM) and date scheduling (DS). The negotiation protocols are used in each of the stages. And matching processes repeat until all the applicants or vacancies are allocated.

- Proposal Sending (PS) Protocol

The protocol contains the following three steps:

1. **Preference Lists Uploading**, each proposer (depends on if the market is driven by offers or applications) upload their strict preference lists before starting the matching service.

2. **Proposal Authorizing**, after uploading proposers' preference lists, each proposer (depends on if the market is driven by offers or applications) should authorize MS agent to exercise the power of proposing in order to reduce workload.

3. **Proposal Sending**, MS agent who is behalf of proposer send proposal to the agents in the opposite side. If any proposal are rejected, the repeating to send is executed by MS agent as well.

- Decision Making (DM) Protocol

The protocol contains three steps as well:

1. **Preference Lists Uploading**, each each receiver (depends on if the market is driven by offers or applications) upload their strict preference lists before starting the matching service.

2. **Decision Authorizing**, after uploading receivers' preference lists, each receiver (depends on if the market is driven by offers or applications) should authorize MS agent to exercise the power of decision making in order to reduce workload.

3. **Decision Making**, MS agent who is behalf of receiver make decision if a proposal is rejected or not, and inform MS agent to repeat proposing process if any proposal is rejected and the proposer still has any choices which has not been rejected.

- Date Scheduling (DM) Protocol

The protocol contains three steps:

1. **Open Day**, for the sake of reducing congestion, making market thick and getting a stable matching results, MS agent should make a schedule for stipulating the date allowing JA and VJP agents enter in market.

2. **Applicant Protecting**, in order to reduce unemployment and keep stable for labor market, MS agent set up protection period for applicant that no VJP agent can break contracts with any JA agents during the periods.

3. **Protection Relieving**, after a period of protection for JA agents, the contract breaking behaviours are relived so that VJP agent can employ better JA agent. However the previous JA agent who is matched with the VJP agent is still be protected by the form

of penalty, meaning that if VJP agent wants to break previous contract, it will have to pay the penalty costs to that JA agent. Hence MS agent should make a schedule of relieving protection for JA agents.

5.4 Decision scheme of agents

Each agent in dynamic labor market has own decision scheme in the processes of coordination and negotiation. In the section, the decision scheme of agents will be introduced respectively.

- Job Applicant Agent's Decision Making

JA agent's decision making process happens after a matching result is made by MS agent. Those who has not received any offers or new entering JA agents can decide if apply for a VJP agent with the help of MS agent or just wait for next cycle of open period. The first decision works only in the period of protection relieving, because MS agent allows JA agent to apply for VJP agents only in that period.

- Vacant Job Position Agent's Decision Making

VJP agent's decision making process happens after a matching result is made by MS agent. Those who has not received any applications or new entering VJP agent can decide if send offer to a JA agent or just wait for next cycle of protection relieving. Because, only in the period of protection relieving, the JA agent can apply for VJP agent with the help of MS agent.

- Matchmaker Server Agent's Decision Making

MS agent's decision making process happens after all preference lists of JA and VJP agents. At first, MS agent should make decision of open day on the basis of scale of JA and VJP agents. Then during the open day, MS agent should make decision if a proposal should be held or rejected according to the preference list of agent it is acting. In addition, the duration of applicant protection should be decided according to the environment of the labor market, while the date of relieving protection depends on the environment as well.

The sequences how the market works based on the protocols and decision scheme of each agent are shown as a flow chart below in Fig.5.2.

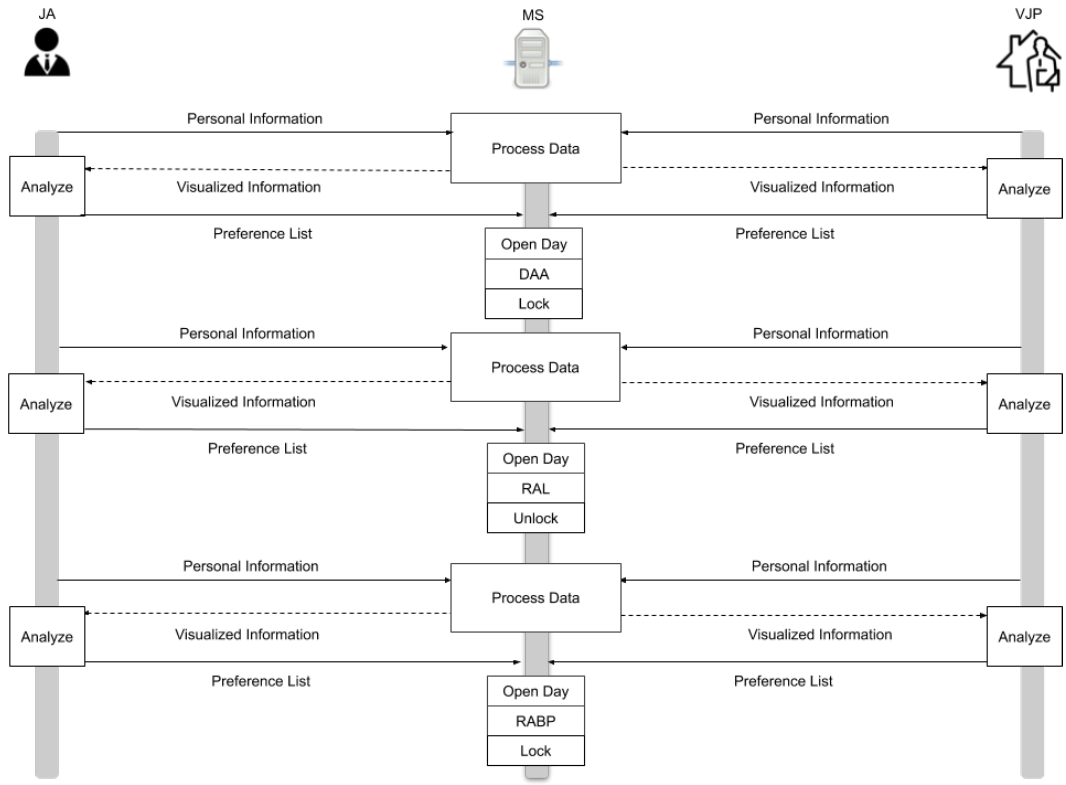


Figure 5.2: Sequence Diagram of Dynamic Labor market

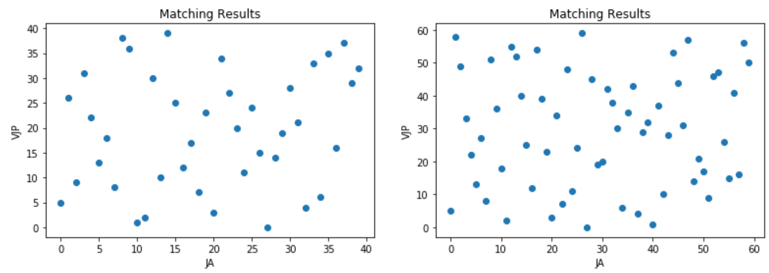
5.5 System simulation and results evaluation

A simulation for multiagent system of dynamic labor market is given in the section, some visualized matching results and comparison of correlative parameters, which measure efficiency and stability, are presented as well.

5.5.1 System simulation

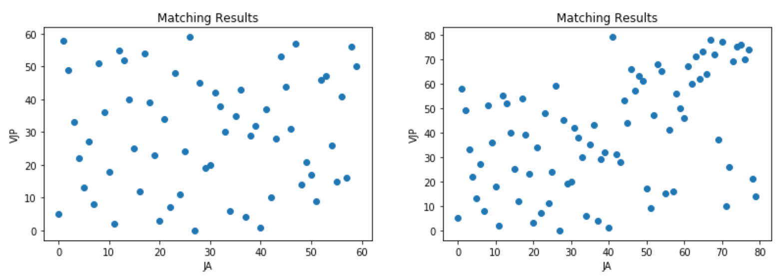
It is assumed that there is a labor market, which is applying proposed multiagent dynamic matching system. Then a sequence of events are scheduled by MS agent, they follow as the form: $\alpha\beta\alpha\gamma\alpha$. each state X after each event occurs is denoted as coordinates in each scatter diagrams.

As what we defined in Chapter 4, the matching results are denoted by ordered pairs, the coordinates are a suitable to present matching results shown in scatter diagrams, from Fig.5.3 to Fig.5.5. At first re-matching algorithm (RMA) is operated after event α (20*20 agents enter the market) occurs (X_0 denotes the initial matching results (40*40) in labor



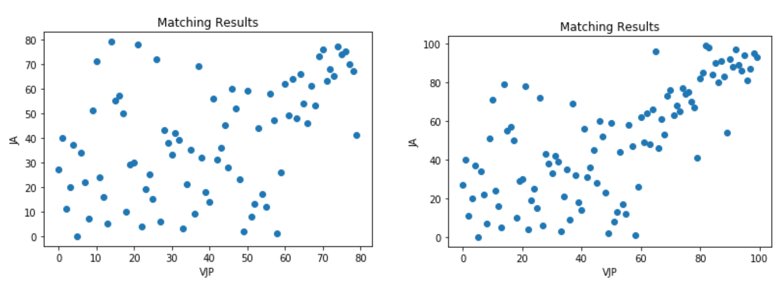
(a) X_0 (b) X_1

Figure 5.3: RMA: $X_0 \xrightarrow{\alpha} X_1$



(a) X_1 (b) X_2

Figure 5.4: RAL: $X_1 \xrightarrow{\beta\alpha} X_2$



(a) X_2 (b) X_3

Figure 5.5: RABP: $X_2 \xrightarrow{\gamma\alpha} X_3$

market) to get a new result, X_1. Then in order to reduce unemployment and chaos, contracts between parts of VJP and JA agents are signed (Event β occurring), meaning parts of matches are locked temporarily. Then Re-matching Algorithm with Locking Periods (RAL) is activated until new event α (20*20 new agents enter the market) occurs, the algorithm is operated and X_2 is the new result computed by RAL. In the following, event γ announces that all matched pairs are unlocked, which means all agents can break previous ties and re-match with a better choice, RABP is activated now. Finally, an event α (20*20 agents enter the market) drives RABP to get X_3. But it should be noted that each event has their unique clock structure ($\mathbf{v}$) to stipulate occurrence time in advance by MS agent.

5.5.2 Consistency comparison

Some obvious features can be observed between three matching results from X_1 to X_3. Matching results are changed differently based on different algorithms they operated. As what we analyzed, a labor market prefers being stable to being chaos, and for reducing unemployment, each VJP agent prefers employing employed JA agent to employing new one. Hence a concept of consistency is proposed as a parameter to measure the matching difference: Ehlers [44] proposed a concept of consistency in allocation problem of house. the definition the author gave is "a condition of stability when the set of agents and resources may change". We can find a same condition happens in dynamic labor market, hence I will propose a similar definition of consistency as follows:

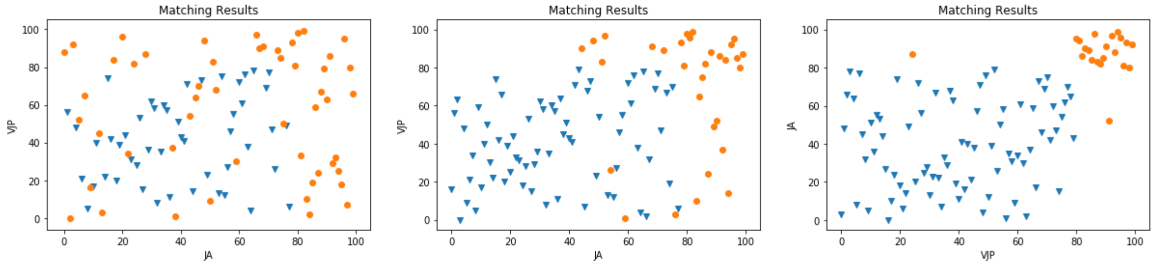
$$consistency = \frac{n_s}{n_p}$$

n_s denotes the number of same pairs between previous matching results and new ones, and n_p denotes the number of pairs in previous matching results.

For not losing generality, ten experiments are made, and consistency of each experiment computed by different algorithm is shown from Table.5.1 to Table.5.3 and from Fig.5.6 to Fig.5.15. In each figure, triangle denotes the same pairs compared with last result, while circle denotes different ones.

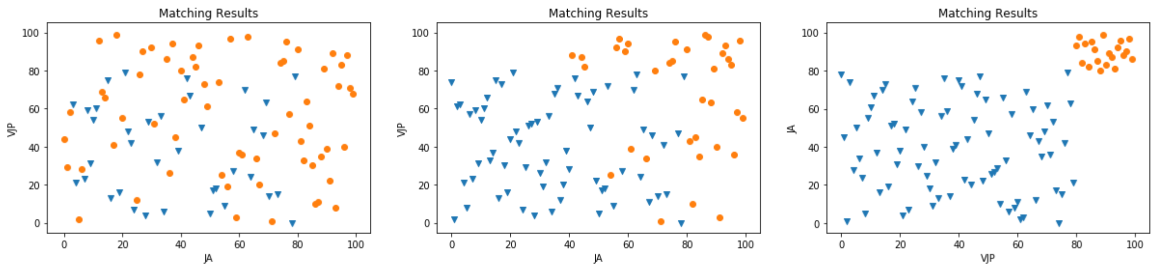
Experiment	1	2	3	4	5	6	7	8	9	10
Consistency	61.25%	46.25%	60.00%	45.00%	56.25%	53.75%	51.25%	41.25%	58.75%	52.5%

Table 5.1: Consistency of RMA



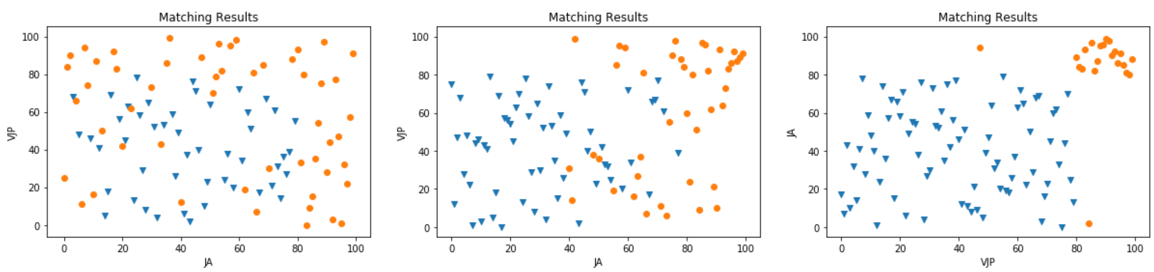
(a) RMA (b) RAL (c) RABP

Figure 5.6: Consistency comparison



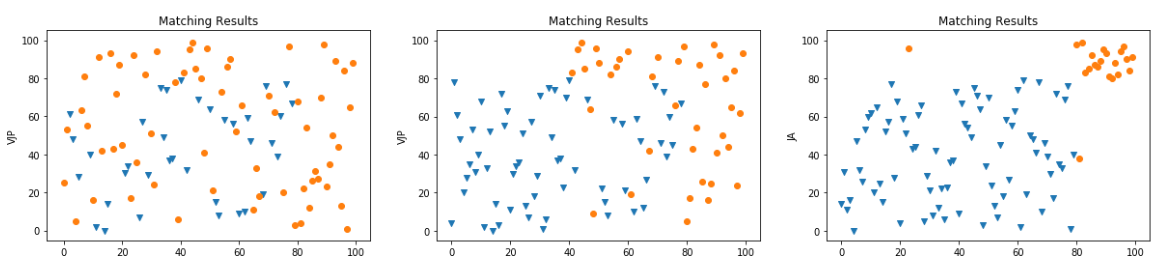
(a) RMA (b) RAL (c) RABP

Figure 5.7: Consistency comparison



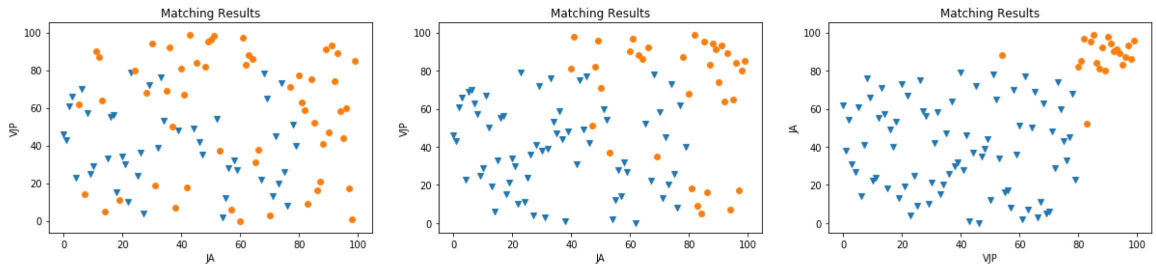
(a) RMA (b) RAL (c) RABP

Figure 5.8: Consistency comparison



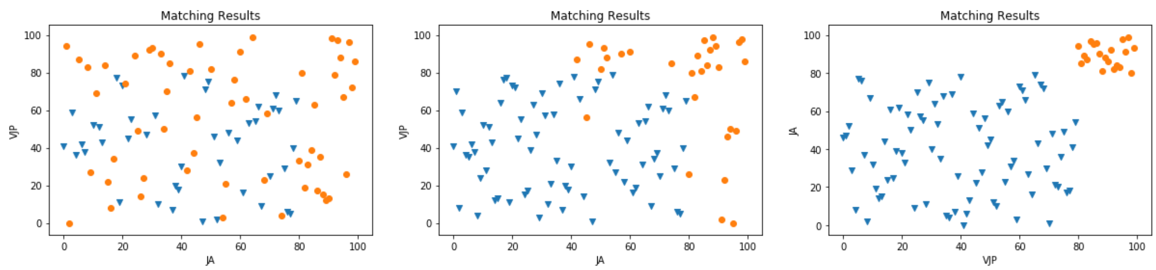
(a) RMA (b) RAL (c) RABP

Figure 5.9: Consistency comparison



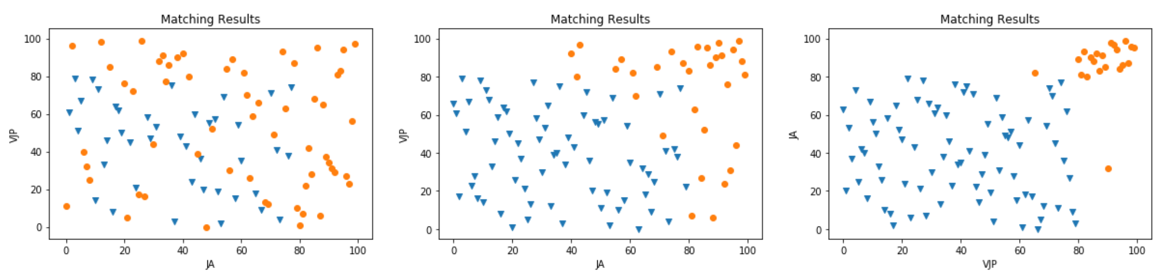
(a) RMA (b) RAL (c) RABP

Figure 5.10: Consistency comparison



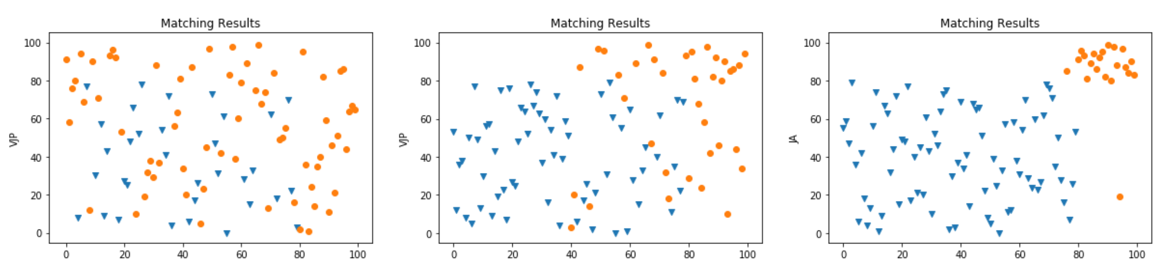
(a) RMA (b) RAL (c) RABP

Figure 5.11: Consistency comparison



(a) RMA (b) RAL (c) RABP

Figure 5.12: Consistency comparison



(a) RMA (b) RAL (c) RABP

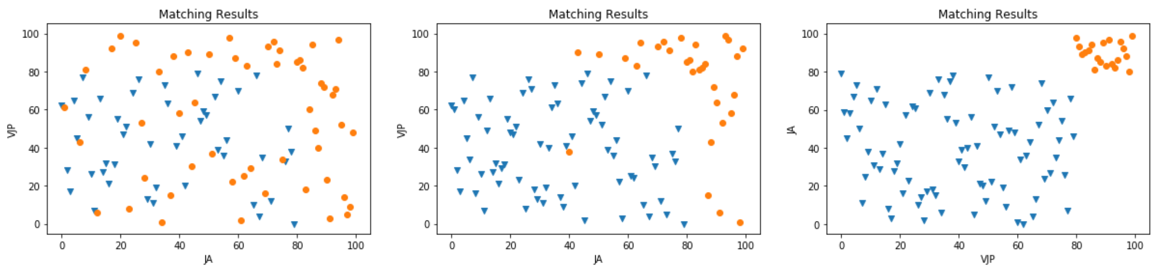
Figure 5.13: Consistency comparison

Experiment	1	2	3	4	5	6	7	8	9	10
Consistency	86.25%	81.25%	80.0%	73.75%	76.25%	82.50%	88.75%	80.00%	87.50%	87.50%

Table 5.2: Consistency of RAL

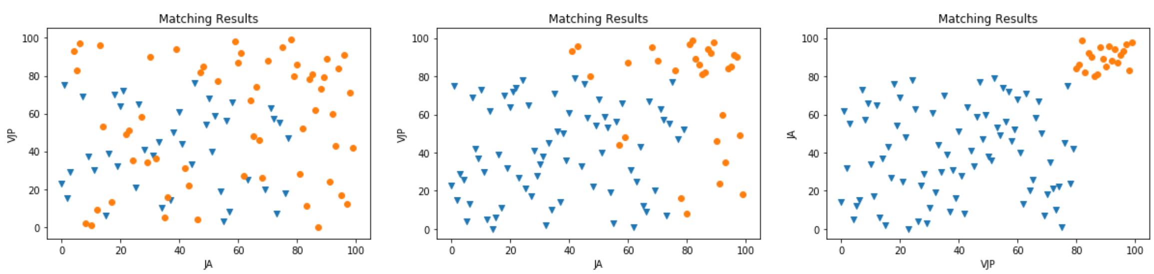
Experiment	1	2	3	4	5	6	7	8	9	10
Consistency	98.75%	98.8%	100%	98.75%	98.75%	100%	98.75%	98.75%	100%	100.0%

Table 5.3: Consistency of RABP



(a) RMA (b) RAL (c) RABP

Figure 5.14: Consistency comparison



(a) RMA (b) RAL (c) RABP

Figure 5.15: Consistency comparison

After observing the results in three tables. We can realize that RMA got the lowest consistency, meaning that each α leads to big scale of contract breaking, which cause the increasing of higher costs and time wasting. And it is easy to find that RABP has higher consistency, compared with RAL and RMA, to keep market consistent, because of involving utility functions and firing costs which are the most important things a labor market concerns. But it should be noted that, RAL has a good consistency as well, which should be applied under some specific scenes, especially in a short-term matching market.

5.5.3 Evaluation of scalability

The criterion of dynamic matching system are effectiveness and efficiency. The effectiveness, represented as stability of matching has been proved that three dynamic algorithms can guarantee. And in the chapter the efficiency is represented as three parameters, running time, times of message and times of decisions, running time records the time interval between start and end of producing matching results. Times of message records proposing behaviours of each algorithm, that is each proposer send offers or application to receiver, the parameter records the times of the offers and applicants. And times of decision records the decision making times of each receiver who receive more than one proposal.
The data of the three parameters are shown in Table5.4 to Table5.6:

Scale	Running Time	Times of Message	Times of Decision
100*100	997	368	266
200*200	9975	1324	1116
300*300	18949	1876	1559
400*400	17953	2012	1587
500*500	44867	3266	2737
600*600	40885	3149	2517
700*700	89760	4644	3908
800*800	86737	4817	3962
900*900	173537	7100	6137
1000*1000	220390	7885	6817

Table 5.4: Re-matching Algorithm

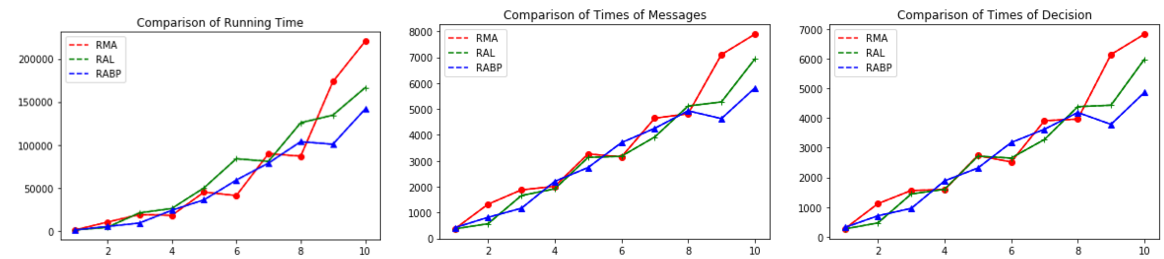
Scale	Running Time	Times of Message	Times of Decision
100*100	997	370	264
200*200	3982	571	467
300*300	20917	1658	1441
400*400	25932	1916	1600
500*500	49867	3130	2713
600*600	83789	3175	2651
700*700	80785	3918	3273
800*800	125664	5114	4379
900*900	144533	5271	4428
1000*1000	166555	6926	5971

Table 5.5: Re-matching Algorithm with Locking Periods

We can conclude that the running time of RMA is longer than the other two algorithms, especially, with the data scale is increasing, RAL works better in efficiency while RABP got

Scale	Running Time	Times of Message	Times of Decision
100*100	970	417	313
200*200	4987	817	707
300*300	8977	1166	956
400*400	23937	2205	1882
500*500	35905	2738	2314
600*600	58849	3700	3171
700*700	78789	4253	3619
800*800	103718	4929	4184
900*900	100736	4628	3784
1000*1000	141621	5806	4854

Table 5.6: Re-matching Algorithm with Breaking-up Penalty



(a) Running Time (Microsecond) (b) Times of Message (Time) (c) Times of Decision (Time)

Figure 5.16: Comparison of Parameters

the highest efficiency. The units of running time are recorded as microsecond, the results show that even though the scale of data is increased to 1000*1000, the running time is still below 1 second. In addition, it is found that as the increasing of scale of data, all three parameters are grown as exponential (Shown in Fig.5.16), which shows the space of improving performance in time complicity of algorithms.

5.6 Summary

This chapter proposes a multiagent system to solve automated matching problem in dynamic labor market. Three dynamic matching algorithms (Chapter 4) are integrated and selected by the system to produce stable matches in an effective and efficient way. A new concept, consistency, was defined for represented as the percentage of same matches compared between previous matching results and current results, which verifies the degrees of stability the three proposed algorithms can improve in dynamic labor markets. In addition, I defined three parameters for evaluating the running time and complexity as well as tested 10 groups of different scale of data from size: 100 to size: 1000. The results shows that as the increasing of data scale, running time and complexity are increasing exponentially. But the running time is still at microsecond level, which verifies the efficiency of the automation system.

Chapter 6

Conclusions and future work

In this thesis, I found a limitation, which exists in current labor market, that current manual operation of deferred acceptance algorithm and researching engine can not provide an efficient solution for generating stable matching for dynamic labor markets when the scale of data grows large. Then I presented a comprehensive application of two-sided matching theory, which was established as a field over fifty years ago and has developed a solid theoretical foundation, in labor markets. However, there still exists a significant barrier of utilizing these models to real-world matching problems, because, as abstractions and simplifications of real world matching problems, common static matching models do not address many issues which are of importance for real world application domains. So after providing the fundamental concepts of discrete event system and discussing the dynamics of labor market, the thesis investigates a matching problem model derived from labor market, in which dynamic and flexible environment has a significant influences on matching results. Finally, I designed an automated matching system which is responsible for selecting appropriate dynamic algorithms according to requirements of labor markets and stipulate rules to regulate behaviours of each agent. And in this chapter, we have four parts, based on core concepts or theories I applied, to present.

6.1 Contribution and uniqueness

The main contribution and uniqueness of the paper are divided into four parts:

- EBD Analysis

Environment Based Design is a logical and recursive process that aims to provide designers the right direction for solving a design problem [45]. It is what the thesis applied that provides a methodology to recursively obtain the requirements of a customer to design a product or service by analyzing the environment of human, built and natural. As what we can imagine a system that can produce desired matches efficiently and effectively is what we need in labor market. But it is not detailed for a designer to reference, we need more accurate requirement problem statement. ROM analysis helps us expand each words in initial problem statement and has a whole understanding of design requirements. And performance network plays a significant role to figure out a critical conflict in the relationships between objects in requirements. By obtaining the conflicts, the core environment components can be figured out for further research and design. After a series of processes, in the paper, the core requirement problem statement, *Design an automatic system to produce stable matching between applicants and vacancies when changes occur constantly*, is obtained by EBD. And the critical conflict is *manual operation constrains the efficiency and effective of a matching system in labor market*. Furthermore, the concepts of effectiveness and efficiency in the thesis are defined (Chapter 2) as stability and automation. The following contributions are based on the requirement analysis of EBD.

- Multiagent System Modeling

In the thesis, we focus on systems that involve large number of active objects (vacancies, employers, applicants and matchmakers and like timing, event strings or other kind of behaviours related to them). Agent-based approach is helpful to capture more realistic phenomena than other approach that not make much sense being less efficient, harder to develop pr simply not matching the nature of problem [46]. For a system design for dynamic labor market, timing, event ordering or other kind of individual behaviours are what we should consider that simple agent based modeling cannot simulate the complex environment efficiently. Therefore, Multi-agent system is applied to view each individual in labor market as an concrete agent to behave. Because in real world, the behaviours of each individual like applicant and firm are complex, agents can primely replace them to be involved in labor market. In the paper, we introduced three main agents, vacant job position (VJP), job applicant (JA) and matchmaker server (MS) agents to behave in order to get a stable matching results under the dynamic environment. VJP agent acts as an vacancy that he or she wants to employ an appropriate applicant in labor market according to his or her preference list. JA agent behaves as an applicant in the market, who would like to find an acceptable job for some personal desires, which is presented as preference lists as well. MS agent is a server or manager of labor market, who can make schedules for some special events and integrate associated algorithms to produce stable matches based on the information obtained from JA and VJP agents. In addition, all internal protocols and decision schemes are proposed as well, which is helpful to stipulate rules of dynamic labor market, in order to make it stably and orderly operate.

- Discrete Event System Modeling

Dynamic labor market, as what we analyzed, is a typical discrete event system, can be modeled as a timed automaton, served as an approach to describe all possible behaviours of each agent in market and generate languages based on the strings formed by events. We conclude three behaviours to drive state transitions in labor market, agents entering (α), pairs locking (β) and unlocking all pairs (γ), the three events, especially agents enter (α) occurring causes instability of matching results. The problem leads to the redesign of classic deferred acceptance to satisfy dynamic requirements. In addition, matching results are views as the state of dynamic labor market, the state transitions are triggered by the three events we proposed before. Each state set of matching results are combined by three subsets, locked set, unlocked set and free set, each of them represents the matched pairs that have signed contracts, finished contracts but firing costs should be considered and free agents that can sign contracts at any time without constrains respectively. Except event α which totally changes the number of matched pairs, event β and γ just cause the pairs transition from one subset to another one. Each state set of matching results are combined by three subsets, locked set, unlocked set and free set, each of them represents the matched pairs that have signed contracts, finished contracts but firing costs should be considered and free agents that can sign contracts at any time without constrains respectively. And based on the algorithms we proposed in Chapter 4, different stable matching can be generated followed by each event.

- Dynamic two-sided matching algorithms design

Some special events mentioned in automaton model, influence the stability of labor market. Based on the variations, agents arrival, locking and unlocking, three related algorithms were proposed to produce stable matching results when the events occur. They are dynamic matching by re-running deferred acceptance algorithm (RMA), dynamic matching with locking periods (RAL) and dynamic matching with breaking-up penalty (RMBP) respectively. RMA mainly focus on a special case that there is no any constrains about matching results, no matter who enters the market, rerun the classical deferred acceptance algorithm. RAL solve a case that if some of pairs signed contracts and temporarily left the labor market. Finally, RABP provides a solution for a labor market that if the previous locked pairs return to the market again and have tendency to break previous contracts and gain higher profits from new matching results.

Furthermore, some experiments are designed to prove the efficiency of each algorithm with scalability increasing. It is found that the RABP has the lowest time complexity, while RAL can be applied in some special cases, and RMA is a general solution for some simple matching environment. And the consistency, as a criterion to measure stability of a market, of RABP is the best among the three dynamic algorithms. All the three algorithms are scheduled by the proposed system to achieve effective and efficient matching processes.

6.2 Limitation and future work

In the thesis, preference lists are assumed to be obtained directly after a simple analysis of personal information. In fact, generating strict preference lists cannot be achieved by current matchmaker servers. On the other hand, in my system design, it is assumed that each agent truthfully tell their preferences, however, in order to obtain more benefits in matching results, strategical decision could be made by each individual in labor markets. Therefore, it is necessary to have a mechanism to guarantee each agent truthfully tell their preferences.

And, there are a lot of other variations affecting the stability of a market, in the paper, the variations are not involved because of length. In the thesis, I verified efficiency of the system by comparing running time for matching 100 pairs of vacancies and applicants to 1000 pairs of vacancies and applicants. However, larger scale of data exists in labor markets, which should be verified as well in the future. In addition, the line chart of running time grows exponentially, meaning that time complexity could be reduced by some improvement. What's more, I assume that preference of each agent is available and strict, the assumption is not validity in real world, hence there could be more research on the direction to automated produce preference list for each agent in labor market. These will be the future works.

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