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The probability of multidimensional poverty in the European Union

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Abstract. This paper evaluates multidimensional poverty in European countries introducing two main novelties compared with the previous literature: first, the dimensions of poverty are selected on the basis of the shared values included in the Charter of Fundamental Rights of the European Union; second, the whole space of feasible weights is used to summarise the multidimensional information, in order to remain agnostic about the importance given to the different deprivations. Using data from four waves of EU-SILC, the methodological innovations introduced here have allowed to produce a family of measures that capture the individual probability of being multidimensionally poor. Individual probabilities are then used to analyse the within and between distribution of multidimensional poverty in ten countries. Finally, they get combined with the generalised Lorenz dominance techniques in order to derive socially preferred distributions with the minimum load of value judgments. The novel methods proposed in this analysis allow to move from a dual definition of poverty, where poor and non-poor individuals are classified in a mutually exclusive context, to a continuous measure of deprivation, which allows to capture both the extensive and intensive margin of multidimensional poverty.

Keywords: Multidimensional Poverty; Charter of Fundamental Rights; Hierarchy Stochastic Multicriteria Acceptability Analysis; Povertà multidimensionale, Carta dei Diritti Fondamentali; Analisi Stocastica di Accettabilità Multicriterio

JEL codes I3; D63; C43

1. Introduction

There is widespread agreement on the need to conceptualise poverty as a multidimensional phenomenon. Low consumption or income is surely at the heart of the notion of poverty but several other domains, like poor human health, limited access to education and powerlessness, are systematically concerned by inadequate living standards (Ferreira and Lugo, 2013).

Since the pioneering works of Tsui (2002) and Bourguignon and Chakravarty (2003), a number of approaches were developed to measure deprivation in multiple dimensions (see among others Alkire and Foster, 2011; Chakravarty *et al.*, 1998; Cheli and Lemmi, 1995; Chiappero-Martinetti, 1994; Deutsch and Silber, 2005; Maasoumi and Lugo, 2008). However, multidimensional poverty measures are far from being universally welcomed. One of the main debates around them concerns the degree of arbitrariness used to define suitable dimensions and indicators of poverty, to set poverty thresholds, and to specify a system of weights to aggregate the various dimensions.

To begin with, which dimensions matter and who should be selecting them are questions that repeatedly raise issues of ethics and legitimacy. The method most used for selecting dimensions is drawing on a list generated by public consensus.[1] However, retrieving information on shared societal values is not straightforward when the analysis is carried out at international or even at the global level (Alkire, 2007).

The identification of deprivation indicators and poverty thresholds – to be set both within and across indicators – requires further sensitive decisions, although they end up being data-driven in most cases, especially when the poverty analysis is performed in the 'counting of deprivations' framework (Alkire *et al.*, 2015).

Relative weights attached to attributes of different nature are also a matter of concern. In the income-centred framework, prices are commonly used to aggregate components of consumption expenditure (or the incomes used to finance such consumption). They are then used to compose an index of aggregate consumption to be compared with an aggregate poverty line defined in the same space. Ideally, such an aggregation includes not only market goods and services, but also imputed values for non-market commodities, like public goods (Ravallion, 2011). Even though there exist different reasons why prices might not be ideal welfare weights[2], they provide a clear understanding of the effects of the weighting scheme (Maasoumi and Lugo, 2008) as they explicitly address the issue of trade-offs between different goods and services, or the rate at which consumers are willing to trade one unit of an expenditure component for another (i.e., the marginal rate of substitution – MRS – between two goods). Moreover, MRSs play the important role of informing on whether two commodities, for every individual, are complementary, independent or substitutes – that is, whether, if we increase the quantity of one good, the final utility of the other increases, remains constant or decreases (Schultz, 1935).

[1] See for instance the Sustainable Development Goals experience (Fukuda-Parr, 2016).

[2] Among these: (i) the existence of externalities and missing or highly imperfect markets; (ii) the fact that price data are often geographically coarse, so actual price variation in space is missing from the information available to the researcher; and (iii) the regular need for imputing prices for market as well as for non-market goods (Bourguignon and Chakravarty, 2003; Ferreira and Lugo, 2013).

Similarly, in a multidimensional setting relative weights play the central role of determining trade-offs between dimensions. They reflect value judgments and possibly the very structure of social preferences. For these reasons, the setting of a weighting system is inevitably subject to the formulation of strong normative assumptions and ethical considerations on what a 'good life' is, and should be made as explicitly as possible.

The literature provides an array of methods – normative, statistical, or hybrid – to set relative weights in a multidimensional context (Decancq and Lugo, 2013), although in practice, because well-being dimensions are deemed equally important from an ethical point of view, weights are often distributed equally among dimensions, as in the case of the Human Development Index (UNDP, 1990). Moreover, quantifying how many units of, say, education an individual would give up to compensate one extra year of life is a rather complicated task. In the first place, such an evaluation would require an amount of information that might not be easy or possible to retrieve. Second, the MRS between any two dimensions – that is, the amount of the first dimension that an individual is willing to give up for the second one while maintaining the same level of well-being (Decancq and Lugo, 2013) – could vary from an individual to another on the basis of the actual levels of the considered achievements, as in the case of age.

This has relevant implications whenever one wants to compare not only individuals but also different territorial entities, like European Union (EU) countries. Empirically, assuming one specific vector of weights to be attached to a given set of dimensions may heavily affect both interpersonal comparisons and country rankings (Foster *et al.*, 2013), leading to less robust results.

This paper shows that it is possible to minimize the degree of arbitrariness commonly used to choose dimensions and weights in order to compare selected EU countries on the basis of a multidimensional poverty index. To this purpose, it first employs a normative approach to derive relevant dimensions from an expression of public consensus, that is the Charter of Fundamental Rights of the European Union (European Parliament, Council of the European Union, and European Commission, 2000). Then, drawing on the data on income and living conditions in the EU made available by Eurostat (EU-SILC), it addresses the issue of weighting dimensions by applying Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma and Salminen, 2001), which allows to embody unknown preferences on the weights assigned to each dimension. Such an approach was previously used to investigate health outcomes, both in Italy and the US (Lagravinese *et al.*, 2019a, 2019b). In this article, SMAA techniques are used for the first time to construct a robust composite poverty index based on individual-level data for all feasible sets of weights.

The article is organised as follows: in Section 2, the identification strategy to select the various dimension of poverty in EU countries is discussed, along with the choice of deprivation indicators and poverty thresholds. Section 3 introduces the SMAA methodology, while Section 4 presents the results from both a cross-country and a diachronic perspective, and analyses the overall inequality in the distribution of the probability of being multi-dimensionally poor, within and between countries, according to the Analysis of Gini (ANOGI) methodology (Liberati, 2015), adapted to a multidimensional setting as proposed by Lagravinese *et al.* (2019a). Finally, in Section 5 the Generalised Lorenz dominance technique is used to perform pairwise country comparisons of the distribution of probabilities to rank them from a social perspective with the minimum load of value judgments. Section 6 concludes.

2. Assessing multidimensional poverty in the European Union

Poverty measurement implies the accomplishment of two fundamental tasks: the first is to identify the poor among the total population; the second is to aggregate the information about the poor, either through the use of a poverty index (Sen, 1976) or by using dominance ordering (see, for example, Deaton, 1997). When performed in a multidimensional setting, the identification step requires to make several choices, including defining suitable dimensions and indicators, setting poverty thresholds whenever appropriate, and defining a system of weights.

The information about the poor can be then aggregated either first across individuals and then across dimensions (e.g. the HPI, Anand and Sen, 1997), or first across dimensions and then across individuals (e.g. the Global MPI, Alkire and Santos, 2014). Each type of aggregation order has empirical advantages and disadvantages. Aggregating first across dimensions and then across people typically imposes a restricted choice of the usable data, which has to come from the same source for the studied population. Yet, poverty measures based on this kind of aggregation are very appealing as they are able to account for people's simultaneous deprivations in different spheres of life.

In the context of the EU, such kind of statistical source is represented by the European Union Statistics on Income and Living Conditions (EU-SILC), which was launched in 2003 on the basis of an agreement between Eurostat and a number of Member States with the aim of providing timely and comparable annual data on variables such as income, social exclusion, material deprivation, health, education and labour at both household and individual level. Although the EU-SILC does not cover all the domains that could be of interest for a multidimensional poverty analysis, it is still wide enough to assess deprivations over multiple facets of life. Moreover, since 2010 it is also used for monitoring poverty and social exclusion in the EU in accordance with the Europe 2020 Strategy, reason why it appears to be an appropriate and sound basis of information to measure multidimensional poverty in the EU.

2.1 *Dimensions selection*

In order to produce reliable statistics, procedures for selecting life domains in a multidimensional setting should minimize the degree of arbitrariness. Alkire (2007) suggests different methods to select poverty dimensions, summarized in five main processes: (i) relying on existing data or convention; (ii) making normative assumptions; (iii) drawing on a list generated by consensus; (iv) using an ongoing deliberative participatory process; (v) using empirical evidence regarding people's values.

One method that is widely used within institutions at the global level is the public consensus one (see, e.g., the Human Development Index experience). As put by Alkire (2007), this method consists in identifying a set of dimensions that have been established through some consensus-building process at one point in time and are relatively stable. In some countries, it has been used to justify the exploitation of National Constitutions and laws to retrieve information on publicly agreed values – see the National Council for Evaluation of Social Development Policy experience in Mexico (CONEVAL, 2010) and some scholarly initiatives (Burchi *et al.*, 2014).

Retrieving information on shared societal values is however not unequivocal when it comes to a supranational entity like the EU. One possible source of such a piece of information is the Charter of Fundamental Rights of the European Union, a document containing the declaration of the common values of the peoples of Europe (European Parliament, Council of the European Union and European Commission, 2000). The Charter was incorporated into the Treaty of Lisbon in 2009 and has since then come into legal force in Member States. It conveys a shared understanding of social justice and states the principles according to which the Union commits itself to fight poverty and social exclusion. The great majority of its articles deal with the domain of civil and political liberties and different kinds of freedoms (e.g., the right to life and the protection of human dignity, the right to the integrity of the person, the prohibition of slavery, forced labor, torture and degrading treatment, the right to security of the person, the respect for private and family life, the freedom of thought, conscience and religion, the freedom of expression, assembly and association).

In addition, the Charter recalls other valuable life dimensions. The first one is decent work, or the right to employment opportunities for productive work and the possibility to deliver a fair income in conditions of freedom, equity, security and human dignity (ILO, 1999). Social solidarity appears as another possible dimension to value, which includes the right to the provision of social and economic protection, for instance through the access to services of general economic interest, consumer protection, the entitlement to social security benefits in the case of loss of employment, maternity, illness, dependency or old age and through the right to property. Finally, the broader concept of human development – or the right to the human flourishing of individuals in a just and protected environment – emerges from the Charter, through the right of education, the freedom of arts and sciences, the protection of human health and the environment.

Even though it might not be considered as definitive, we start from this list of dimensions (summarized in Table 1) as a base of shared societal values to be used to inform the multidimensional assessment of poverty for EU countries.

2.2. *Deprivation indicators*

Regarding the choice of the indicators, the EU-SILC does not cover all the dimensions identified by the Charter. Variables accounting for the first dimension, *Political and civil liberties*, are completely missing in the database, the reason why this dimension will not be considered in the following analysis.[3]

For the *Decent work* dimension, we follow the review of EU-SILC labour-related indicators provided in Tosi (2015) to select two relevant indicators: *Activity status* (PX050) and *Low work intensity* (RX050),

[3] Acknowledging that it is not possible to retrieve in the data all the information that is considered theoretically essential does not constitute a shortcoming of the proposed approach. As also recommended by Robeyns (2003), an explicit and openly discussed selection of suitable life domains is a step that needs to be performed before endeavoring any kind of empirical assessment, so as to avoid relying only upon the available information and, possibly, to stimulate a more specifically targeted data collection.

respectively accounting for employment conditions and (quasi-) joblessness, as conceived by Eurostat as part of the composite indicator *At Risk of Poverty and Social Exclusion* rate (AROPE).

Regarding the *Social solidarity* dimension, different indicators in the EU-SILC allow to capture the level of social protection offered to European citizens, e.g., through the variables Family/Children related allowances, Social exclusion not elsewhere classified, and Housing allowances. In fact, because all the policies just mentioned sustain people's standard of living by integrating their income through the channel of monetary transfers, it appears reasonable to choose an income poverty indicator as a general proxy for this dimension. The variable *Monetary poverty* (after transfers) (HX080) is thus used to account for deprivations in the Social solidarity dimension.

Finally, in the EU-SILC there are different variables that can be used to construct deprivation indicators in the last dimension, *Human development*. Some of them relate to human health, while some others refer to the educational attainment or to the quality of the living environment. The nine selected variables and the corresponding modalities are extensively commented in Tosi (2015) and outlined, along with all the chosen indicators, in Table 2.

Table 1 – An application of the overlapping consensus method to the Charter of Fundamental Rights of the European Union

Dimension	Values and principles	Articles of the Charter
POLITICAL AND CIVIL LIBERTIES	Human dignity, Right to life, Right to the integrity of the person, Prohibition of torture, slavery and forced labour, Right to security, Protection of personal data, Respect for private life and the right to marry, Freedom of thought, conscience and religion, Freedom of expression, assembly and association, Freedom of the arts and sciences, Right to asylum, Protection in the event of removal, expulsion or extradition, Equality before the law, Right to non-discrimination, Protection of cultural, religious and linguistic diversity, Right to vote and to stand as a candidate at elections, Right to good administration, Right to petition, Freedom of movement and residence, Right to a fair trial, Presumption of innocence and right of defence	1–13, 18–22, 39–50
DECENT WORK	Freedom to choose an occupation and right to engage in work, Equality of employment, work and pay for women and men, Right to information and consultation within the undertaking, Right of collective bargaining and action, Right of access to placement services, Protection in the event of unjustified dismissal, Fair and just working conditions, Prohibition of child labour, Protection from dismissal for a reason connected with maternity and right to parental leave	15, 23, 27–33
SOCIAL SOLIDARITY	Freedom to conduct a business, Right to property, Social, economic and legal protection of the family, Right to social security and social assistance, Integration of persons with disabilities, Health care and protection, Access to services of general economic interest, Consumer protection	16, 17, 26, 33, 34, 35, 36, 38
HUMAN DEVELOPMENT	Freedom of the arts and sciences, Right to education, Health care and protection, Rights of the child, Rights of the elderly, Environmental protection	13, 14, 24, 25, 35, 37

Note: Articles 13, 33 and 35 fall in more than one dimension.
Source: Authors

Table 2 – Identification strategy for a multidimensional poverty assessment in the EU

Dimensions	Indicators	Variables	Cut-offs
DECENT WORK	Unemployment	Activity status (PX050)	2=Employee 3=Employed persons except employees 4=Other employed **5=Unemployed** 6=Retired 7=Inactive 8=Other
	Low work intensity	Low work intensity (RX050)	0=No low work intensity **1=Low work intensity** 2=Not applicable
SOCIAL SOLIDARITY	Income poverty	Monetary poverty (HX080)	0=when HX090 >= at risk of poverty threshold (60% of Median HX090) **1=when HX090 < at risk of poverty threshold (60% of Median HX090)**
HUMAN DEVELOPMENT	Low educational attainment	Highest ISCED level attained (PE040)	**0=Pre-primary education** **1=Primary education** 2=Lower secondary education 3=Upper secondary education 4=Post-secondary education 5=First stage of tertiary education (not leading directly to an advanced research qualification) 6=Second stage of tertiary education (leading to an advanced research qualification)
	Bad self-reported health	General health (PH010)	1=Very good 2=Good 3=Fair **4=Bad** **5=Very Bad**
	Chronic illness	Suffers from chronic illness or condition (PH020)	**1=Yes** 2=No
	Unmet medical needs	Unmet medical need for medical examination or treatment (PH040) + Main reason for unmet medical need (PH050)	**1=Yes, there was at least one occasion when the person really needed examination or treatment but did not** 2=No, there was no occasion when the person really needed examination or treatment but did not 1=Could not afford to (too expensive) **2=Waiting list** **3=Could not take time because of work, care for children or for others** **4=Too far to travel/no means of transportation** 5=Fear of doctor/hospital examination/treatment 6=Wanted to wait and see if problem got better on its own 7=Did not know any good doctor or specialist 8=Other
	Poor quality of dwelling	Leaking roof, damp walls/floor/foundation or rot in window frames/floor (HH040)	**1=Yes** 2=No
	Inadequate sanitation facilities	Bath/shower in dwelling (HH080/HH081) + Indoor flushing toilet for sole use of the household (HH090/HH091)	1=Yes, for sole use of the household 2=Yes, shared **3=No** 1=Yes, for sole use of the household 2=Yes, shared **3=No**
	Noise	Noise from the neighbours or from the street (HS170)	**1=Yes** 2=No
	Pollution	Pollution, grime or other environmental problems (HS180)	**1=Yes** 2=No
	Crime	Crime, violence or vandalism in the area (HS190)	**1=Yes** 2=No

Modalities of the EU-SILC variables indicating deprivation are highlighted in bold.
Source: Authors' elaborations

2.3 *The issue of relative weights*

One powerful critique to multidimensional poverty indices concerns the second issue described above, i.e. how to aggregate the different dimensions of poverty and thus how to set the weights attached to attributes of different nature. In their thorough investigation on weights in multidimensional indices of well-being, Decancq and Lugo (2013) explain that, in order to study how small changes in the achievements of different well-being dimensions can or cannot compensate each other, one needs to look precisely at the structure of weights. To this purpose, they introduce the MRS between two dimensions j_1 and j_2 as the amount of dimension 2 an individual is willing to give up for an extra unit of dimension 1, while maintaining the same level of well-being. Formally, they define the MRS between dimensions j_1 and j_2 as:

$$MRS_{j_1,j_2} = \frac{\partial I(x)}{\partial x_{j1}} / \frac{\partial I(x)}{\partial x_{j_2}}$$

where $I(x)$ is the well-being index and x is the vector of achievements for all j dimensions. In fact, there exists different approaches to set relative weights in a multidimensional poverty analysis. Decancq and Lugo (2013) distinguish three classes: data-driven, normative, and hybrid. Data-driven approaches – like frequency-based weights, statistical weights (Krishnakumar and Nadar, 2008) and most-favorable weights (Melyn and Moesen, 1991) – are a function of the distribution of the achievements in the society and are not based on value judgements about trade-offs between different life domains.

Frequency-based weights often assign an inverse relation between the frequency of deprivation in a dimension and the weight of that dimension (e.g., Deutsch and Silber, 2005). The motivation behind such a relation lies in the idea that less frequent deprivations should have a higher weight because individuals would attach a higher importance to the shortfalls in dimensions where the majority in their society do not fall short, reason why some have also interpreted such weights as the "objective measures of the subjective feelings of deprivation." (Desai and Shah, 1988, p. 52)

Statistical weights, on the other hand, are often classified into two broad sets: multivariate statistical methods, among which the most commonly used technique is based on the Principal Component Analysis (Klasen, 2000; Noorbakhsh, 1998), and explanatory models based on the idea of the latent variable, like Factor Analysis (Noble et al., 2006), the Rasch model (Fusco and Dickens, 2008), multiple indicator and multiple causes models (MIMIC) (Di Tommaso, 2006), and structural equation models (Kuklys, 2005; Krishnakumar, 2007; Krishnakumar and Ballon, 2008).

Finally, the most-favorable weights technique, which has been widely used to set weights in well-being indices (see e.g., Despotis, 2005a, 2005b; Mahlberg and Obersteiner, 2001; Zaim et al., 2001) is a particular case of the data envelope analysis proposed by Melyn and Moesen (1991) and considers weights as individual-specific and endogenously determined, i.e., the highest relative weights are given to dimensions in which the person performs best.

Conversely, normative approaches depend on value judgements about the MRSs. Weights can either be set in an equal or unequal way, although in any case they are assigned arbitrarily, that is according to particular considerations about specific trade-offs among dimensions. Arbitrariness could be overcome by following an 'expert opinion approach', that is, letting experts or well-informed persons decide which particular weighting scheme to attach to different poverty attributes (see for instance Chiappero-Martinetti and von Jacobi, 2012). This latter method includes the Budget Allocation Technique (Moldan and Billharz, 1997; Chowdury and Squire, 2006; Mascherini and Hoskins, 2008), where experts are asked to distribute a budget of points to the different attributes, and the Analytic Hierarchy Process (Saaty, 1987), which compares dimensions pairwise and assigns for each round a score of importance.

Lastly, hybrid approaches, like stated preference weights (Mack and Lansley, 1985; Halleröd, 1995a, 1995b; de Kruijk and Rutten, 2007; Guio et al., 2009; Bossert et al., 2009) and hedonic weights (Schokkaert, 2007; Ferrer-i-Carbonell and Freijters, 2004; Nardo et al., 2008; Fleurbaey, 2009) are a mix of the former two.

As we will see in the next section, this paper tries to overcome the arbitrary choice of the set of weights, introducing a new methodology to measure multidimensional poverty. The aim will not be that of defining a specific poverty index for each individual, but that of estimating the probability that she/he will be below a given threshold for different vectors of weights.

3. Measuring multidimensional poverty: an alternative approach

Irrespective of the way of setting relative weights, all the above-mentioned approaches (i.e. data-driven, normative, and hybrid) use a single weight vector for all units (or in the case of most-favourable weights technique, a different weight vector for each unit) to reduce multidimensionality into a composite indicator. But the uniqueness of the vector of weights does not allow to take into account that, in a differentiated society, each individual may assign a different importance to each dimension. With only one vector of weights, representativeness may be valid only for a very small portion of the population. Since weights are likely to change according to individual preferences and needs, and since a "social" vector of weights could not be unanimously agreed upon, some studies have recently proposed to take into account the whole space of feasible vectors of weights in the evaluation process (Greco *et al*. 2018; Lagravinese *et al*. 2019a, 2019b).

In particular, Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma and Salminen, 2001) has been shown to be the appropriate tool to make comparisons in a multidimensional framework, while remaining agnostic about the weighting schemes. This methodology has an appealing application in all cases where the individual characteristics, like poverty dimensions, have to be aggregated to obtain either social norms or rankings.

Formally, using selected indicators from EU-SILC, the set of individuals A $(a_1, \ldots, a_m)$, where $m = 176{,}518$ in 2008, $m = 178{,}904$ in 2010, $m = 181{,}864$ in 2012, and $m = 182{,}912$ in 2014, is evaluated on three dimensions $(g_1, \ldots, g_n)$: 1. *Decent Work*; 2. *Social Solidarity*; and 3. *Human Development*. The composite indicator can be seen as the average of the three dimensions weighted by the weights (w) associated to each of them:

(1) $$CI(a_k, w) = \sum_{i=1}^{n} w_i g_i(a_k)$$

where w_i reflects the importance given to the dimension i, and $g_i(a_k)$ the achieved result of individual a_k for dimension i. As shown in Section 2.3, Decancq and Lugo (2013) list several procedures to set w, but as the order of importance given to different indicators is a subjective choice, one single vector of w for summarising multidimensional poverty does not exist.

3.1 *Stochastic Multicriteria Acceptability Analysis*

In order to embody unknown preferences on the weights assigned to each dimension and to reduce the degree of arbitrariness in aggregating dimensions, SMAA considers the probability distributions $f_W(w)$ in the set of the feasible weights W (Lahdelma and Salminen, 2001):

(2) $$W = \{(w_1, \dots, w_n) \in R_+^n, \qquad w_1 + \dots + w_n = 1\}$$

The set of feasible weights is a $(n-1)$ dimensional simplex. In the absence of knowledge about the importance given to the different dimensions, a uniform weight distribution can be assumed in the set of feasible weights W. Defining ξ_{ik} as the value of dimension g_i for individual a_k, from the probability distributions $f_\chi(\xi)$ on χ, where χ is the evaluation space (in our case the space of the values assumed by the dimension g_i in G), Lahdelma and Salminen (2001) introduce a ranking function attached to the individual a_k based on counting the dimensions in which a person is deprived:

(3) $$rank(k, \xi, w) = 1 + \sum_{h \neq k} \rho[CI(\xi_h, w) > CI(\xi_k, w)]$$

where $\rho(true) = 1$, and $\rho(false) = 0$. Hence, the rank of individual a_k, for a given vector of weights w, is one plus how many times the weighted average of multidimensional poverty of a_k ($CI(\xi_k, w)$) is dominated by the weighted average of multidimensional poverty of the other individuals ($CI(\xi_h, w)$). Thus, the value assumed by the variable $rank(k, \xi, w)$ in equation (3) is one plus the number of individuals that are more multidimensional poor than the individual a_k. Therefore, the lower the value of $rank(k, \xi, w)$ the higher the poverty of the individual a_k.

Accordingly, for each individual a_k and for each value that can be taken by the three poverty dimensions $\xi \in \chi$, SMAA computes the set of weights for which individual a_k assumes rank r:

(4) $$W_k^r(\xi) = \{w \in W : rank(k, \xi, w) = r\}$$

From equation (4), one can then compute the rank acceptability index:

$$b_k^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_k^r(\xi)} f_W(w)\, dw d\xi \tag{5}$$

Equation (5) indicates the probability that the individual a_k has the r-th position in the ranking, b_k^r, which is given by the ratio of the number of the vector of weights by which individual a_k gets rank r to the total number of vector of weights considered.

3.2 *Hierarchy Stochastic Multi-Objective Acceptability Analysis*

The structure of the multidimensional poverty assessment presented in Section 2 is hierarchical: dimensions are in the first level and the different indicators are in the second level. In the SMAA context, the inclusion of a hierarchical structure has been proposed by Angilella *et al*. (2016) and De Matteis *et a*l. (2018). In our poverty measure, each dimension $g_i \in G$ is given by the weighted sum of indicators $q_{ij} \in Q_i$:

$$g_i = \sum_{j=1}^{s_i} v_{ij} q_{ij} \tag{6}$$

In this case, the composite index of multidimensional poverty becomes the weighted average of dimensions, which are the weighted average of EU-SILC indicators. The new value function to aggregate the evaluations of an individual, from A with respect to the g_i dimensions from G, with respect to the indicators from Q_i, is a double weighted average. For each individual $a_k \in A$, we can estimate the following *CI*:

$$CI(a_k, w, v_k) = \sum_{i=1}^{n} w_i \sum_{j=1}^{s_i} v_{ij} q_{ij}\,(a_k) \tag{7}$$

where w_i is the weight given to the dimension i, and v_{ij} is the weight given to the EU-SILC indicator j. The Hierarchy Stochastic Multi-Objective Acceptability Analysis (HSMAA) allows to take into account of: (1) the uncertainty with respect to the weights assigned to the dimensions (as in the standard SMAA); and within dimensions (2) the uncertainty with respect to the weights assigned to the EU-SILC indicators.

To this purpose, the HSMAA considers three probability distributions: $f_W(w)$, $f_V(v)$; $f_\chi(\xi)$ on W, V; and χ (De Matteis *et al.* 2017), respectively, where:

$$\begin{gathered} W = \{(w_1, \ldots, w_n) \in R_+^n, \quad w_1 + \ldots + w_n = 1\} \\ V = \{(v_{i1}, \ldots, v_{is}) \in R_+^s, \quad v_{i1} + \ldots + v_{is} = 1, \quad i = 1, \ldots n\} \end{gathered} \tag{8}$$

and χ is the space of the value that can be taken by the EU-Silc indicators $q_{ij} \in Q_i (i = 1, \dots, n)$. We introduce a ranking function relative to the individual a_k:

(9)
$$rank(k, \xi, w, v) = 1 + \sum_{h \neq k} \rho\big(u(\xi_h, w, v_h) > u(\xi_k, w, v_k)\big)$$

where $\rho(true) = 1$, and $\rho(false) = 0$. Then, for each individual a_k, for each evaluation of individuals $\xi \in \chi$, and for each rank $r = 1, \dots, m$, HSMAA computes the set of weights of dimensions for which individual a_k assumes rank r:

(10)
$$W_k^r(\xi, v) = \{w \in W: rank(k, \xi, w, v) = r\}$$

HSMAA evaluation is based on the computation of the rank acceptability index, which is the relative measure of the set of weight vectors for which the individual a_k gets rank r:

(11)
$$b_k^r = \int_{w \in W_k^r(\xi)} fW(w) \int_{\xi \in \chi} f\chi(\xi) \int_{v \in V} fV(v)\, dv d\xi dw$$

where b_k^r is the probability that individual a_k gets the r-th position in the ranking. From a computational perspective, the multidimensional integrals defining the index are estimated using Monte Carlo simulations. In our application, we consider uniform probability distributions $f_W(w)$ on W and $f_V(v)$ on V. As Tervonen and Ladhelma (2007) show that, to rank individuals, 10,000 extractions are a sufficient number to get an error limit of 0.01 with a confidence interval of 95%, we apply the HSMAA technique to 10,000 extractions of w and v vectors.

We use the previously defined rank acceptability index b_k^r to calculate a multidimensional measure of poverty. For each individual, we take the downward cumulative rank acceptability index of rank l, i.e. the probability that the individual a_k has a rank l or lower (Angilella *et al*. 2016). In symbols:

(12)
$$b_k^{\leq l} = \sum_{s=1}^{l} b_k^s$$

Taking a specific threshold in the poverty ranking (we consider $l = 20\%$, $l = 10\%$, and $l = 5\%$), $b_k^{\leq l}$ measure the individual probability to be below that threshold, considering the whole space of feasible weights assigned to each dimension and indicator. It is worth noting that, to some extent, this approach can be interpreted as a generalization of the deprivation count approach recently developed by Aaberge *et al*. (2019), where the distributions of the deprivation count are separately considered over the space of dimensions of poverty. In our approach, the many dimensions of deprivations can instead be aggregated over the set of all possible weights and transformed to get the average probability of each individual to be within a given percentage of the poorest population regardless of the specific number of deprivations. In other words, our generalization allows to estimate a robust probability of poverty that is not loaded with a specific method to aggregate the dimensions of deprivation.

4. Results

4.1. *The probability of being multidimensional poor*

The results obtained from the application of the methodology above described are reported in Table 3. In particular, it shows the descriptive statistics of the individual probabilities of being among the poorest 20% of the European population by year, summarized by country. Of significant relevance, in almost all countries, is that given any set of weights, the median probability of being multidimensional poor is equal to zero, with the exceptions of Greece, Spain, Portugal and Italy. This outcome is consistent both with the endeavor to provide a robust estimation of multidimensional poverty in the EU, where living conditions are on average among the highest in the world, and with the indicators chosen to inform the analysis, that aim at reflecting acute poverty.

The highly-skewed shape of the probability distributions is illustrated for each country by means of the box plots (Figure 1). Due to the large outliers, country mean probabilities lie outside the interquartile range in most cases. However, for some Southern European countries – Greece, Spain, and Portugal – probability distributions are extremely sparse: even though country means are included in the interquartile range, extreme values attain the value of 1, as visually described by the overlapping of the maximum of the box plot and the upper bound of the probability distribution.

In these countries, multidimensional poverty is more widespread than elsewhere in Europe, as there are some individuals who have 100% probability of being among the poorest 20% of the population regardless of the weighting scheme applied to the set of multidimensional poverty assessment. Belgium and Italy also feature quite sparse distributions, with an average maximum probability exceeding 50% (Belgium in 2010 and 2012) and 90% (Italy in 2012) of being among the poorest 20%. Conversely, in Austria, France, Germany, Luxembourg and the UK, probability distributions are narrower and very close to zero, suggesting a greater robustness of the individual probabilities of being in the group of the multidimensional poorest 20% to changes of the weighting scheme attached to different poverty dimensions.

With regard to the diachronic perspective, it is worth noting that, within countries, the steadiness of country means across the years suggest that the overall probabilities of being multidimensional poor do not vary drastically over time. In some cases, however, changes from one year to the next appear to be more meaningful when even small variations of the means are associated to a substantial increase (or decrease) of the interquartile range. This is the case, for instance, of Belgium, where the probability of being poor durably increase after 2008 due the increased sparsity of individual probabilities in the range between 0 and 25%.

Greece and Italy, on the other hand, show a larger variability in the probability of falling into the poorest 20% in 2012 compared to the previous years, while Portugal see its probability distribution becoming even sparser in 2014. In all these cases, the discontinuity appears to be also driven by an enlargement of the proportion of individuals who have non-zero probability to be in the lowest quintile of the distribution: it increases by almost 7 percentage points in Greece and by 4 percentage points in Italy between 2010 and 2012; and it grows by 62 to 65% in Portugal between 2012 and 2014 (see the last column in Table 3).

The contribution in terms of densities to the shaping of the overall probability distributions is even clearer looking at the violin plot (Figure 2), which combines a box plot with the information conveyed by a kernel density plot for all probability distributions by country and year. From the graph, it is easy to see how two apparently similar distributions (as per the interpretation of the box plot) can differ in terms of concentration of the observations along the vertical line representing the possible values taken by each individual observation.[4] From the violin plot, it is observed that Greece, Spain (particularly in years 2008 and 2010), Italy (in 2008), and Portugal show a less flat probability of individuals taking non-zero values, corresponding to a higher average probability for their observations of being in the poorest quintile.

Back to Figure 1, also the UK shows a spike in 2014 indicating greater sparsity of individual probabilities compared with the 2008–2012 period, sustained by both a growing proportion of the population who has a non-zero probability of being poor (given all possible sets of weights assigned to poverty dimensions), and the rise – from 6 to 7 – in the average number of joint deprivations experienced by the same share of population. Conversely, in Luxembourg the probability of being multidimensional poor diminishes from 2010 onwards, due to both the reduction of the number of individuals who report a non-zero probability of being poor and the reduction in the average number of deprivations experienced by those who fall into the group of the poorest 20%. The last group of countries – Austria, Germany, and France – shows a constant or diminishing overall probability of being among the multidimensional poorest 20% over time. For those countries, within country variations are only imputable to a greater concentration around zero of the distribution of individual probabilities.

Concerning the two other sets of probabilities computed in this analysis (tables are reported in Appendix), all countries have some proportions of individuals with non-zero probability of being among the poorest 10% increase over time. In the case of Greece, that proportion is the second highest one in Europe (44% on average in the period 2008-2014) after Portugal. A distinctive pattern is observed for France, where the proportion of individuals with non-zero probability of being into the poorest 10% for any set of weights increase in 2014. That reverses the downward trend observed for the probability distribution relative to the bottom quintile.

Finally, the UK confirms the same pattern also when considering the probability to fall into the poorest 10% and the poorest 5% of the population. The proportion of individuals who have non-zero probability to be in the bottom tenth percentile and the bottom fifth percentile of the distribution increases between 2012 and 2014 (respectively, from 27.6% to 35.1% when considering the 10% threshold and from 18.6% to 23.3% for the 5% threshold). Moreover, in the same period the average number of deprivations increases by 1 for those in the last decile and by 2 – from 7 to 9 – for those who are in the bottom 5%.

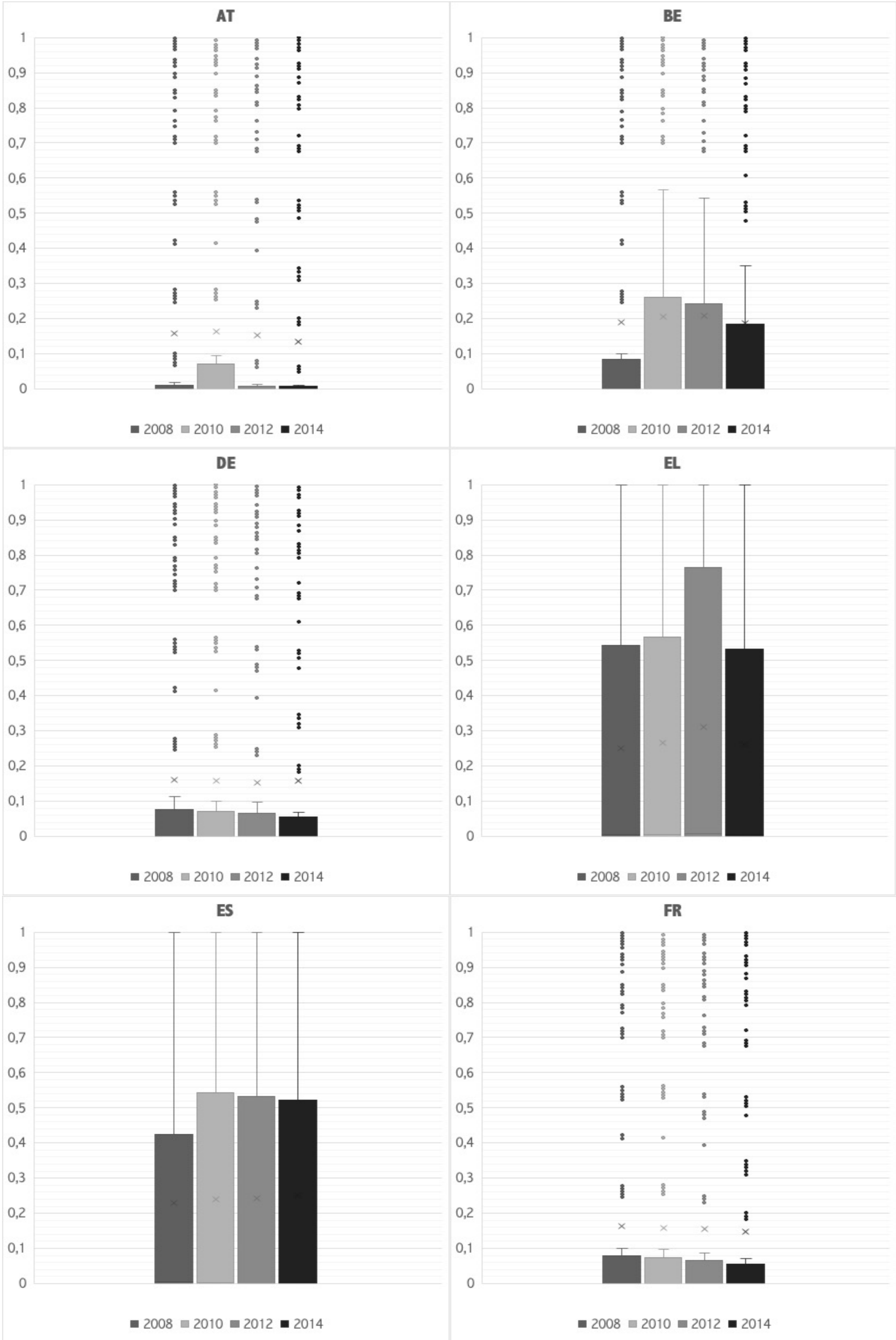
[4] One example is given by Italy, featuring quite different density plots for years 2008 and 2010 in Figure 2 while being seemingly not distinguishable in the box plot shown in Figure 1.

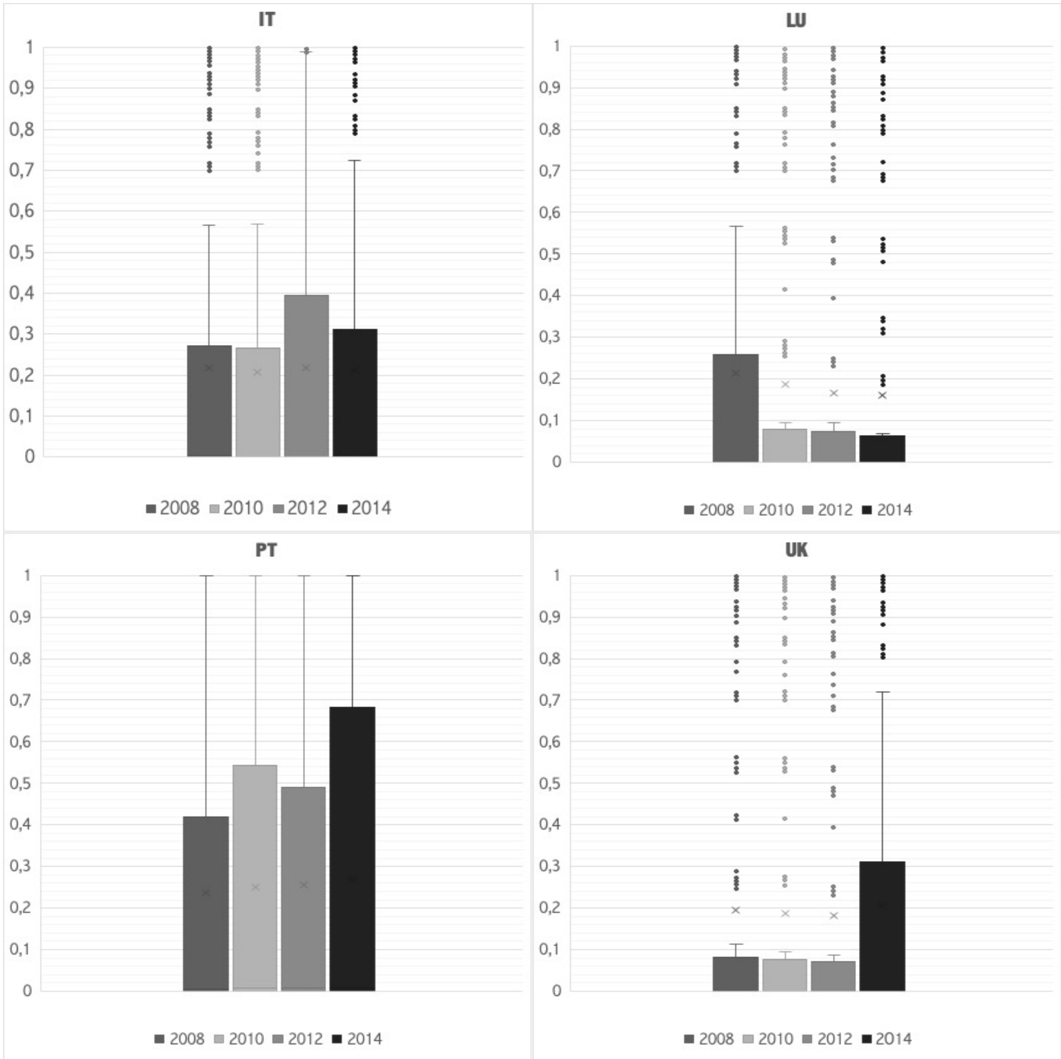
Table 3 – Probabilities of being among the poorest 20% of the population by year and country

Year	mean	sd	p25	p50	p75	% population with Prob20%>0	Average N of deprivations if Prob20%>0
				AT			
2008	0.157	0.32	0	0	0.011	40.4	7
2010	0.164	0.32	0	0	0.071	39.7	7
2012	0.153	0.32	0	0	0.009	40.3	6
2014	0.135	0.30	0	0	0.007	37.3	6
				BE			
2008	0.191	0.35	0	0	0.083	46.3	6
2010	0.204	0.36	0	0	0.261	44.3	6
2012	0.207	0.36	0	0	0.242	46.7	7
2014	0.186	0.35	0	0	0.184	44.0	7
				DE			
2008	0.161	0.32	0	0	0.076	44.8	6
2010	0.160	0.32	0	0	0.071	42.9	6
2012	0.155	0.32	0	0	0.065	43.4	6
2014	0.158	0.32	0	0	0.056	42.4	7
				EL			
2008	0.251	0.38	0	0.006	0.544	56.7	7
2010	0.266	0.38	0	0.006	0.568	57.0	6
2012	0.310	0.40	0	0.009	0.764	63.7	6
2014	0.262	0.38	0	0.005	0.532	58.7	6
				ES			
2008	0.230	0.37	0	0.005	0.424	53.6	6
2010	0.241	0.37	0	0.004	0.543	52.3	6
2012	0.243	0.38	0	0	0.532	50.8	6
2014	0.251	0.38	0	0.003	0.521	52.5	6
				FR			
2008	0.162	0.32	0	0	0.078	46.7	7
2010	0.158	0.32	0	0	0.074	45.2	7
2012	0.155	0.32	0	0	0.065	44.6	6
2014	0.146	0.31	0	0	0.055	41.3	7
				IT			
2008	0.218	0.36	0	0.004	0.273	52.6	7
2010	0.207	0.35	0	0	0.266	48.7	7
2012	0.217	0.36	0	0	0.395	52.2	6
2014	0.212	0.34	0	0.004	0.310	53.1	7
				LU			
2008	0.213	0.36	0	0	0.259	47.9	7
2010	0.187	0.34	0	0	0.079	43.1	6
2012	0.167	0.32	0	0	0.074	43.4	6
2014	0.160	0.31	0	0	0.062	42.0	6
				PT			
2008	0.236	0.37	0	0.006	0.413	58.9	7
2010	0.249	0.37	0	0.007	0.543	58.3	7
2012	0.256	0.38	0	0.007	0.490	62.1	6
2014	0.269	0.37	0	0.007	0.683	65.0	7
				UK			
2008	0.195	0.35	0	0	0.080	49.4	6
2010	0.188	0.35	0	0	0.075	43.1	6
2012	0.182	0.34	0	0	0.071	44.6	6
2014	0.206	0.34	0	0	0.310	47.6	7

Source: Authors' elaborations on EU-SILC data (2008–2014)

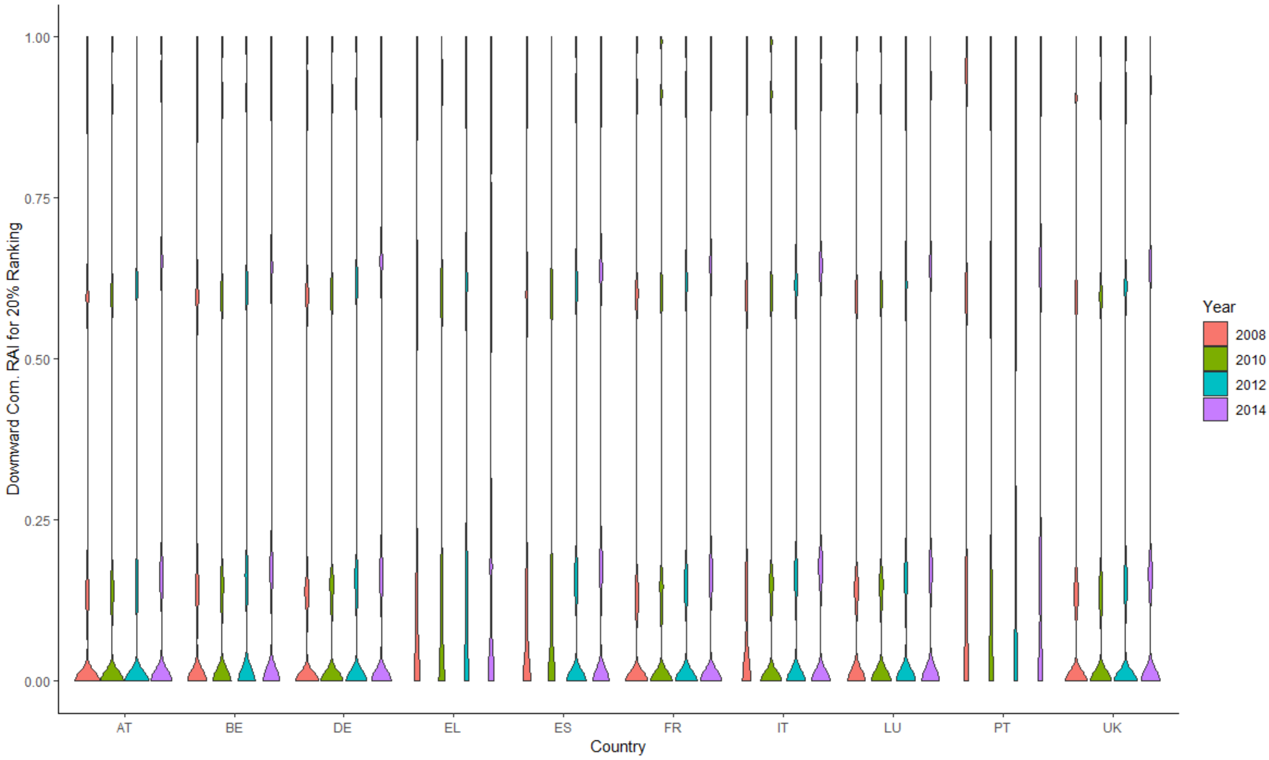
Figure 1 – Distributions of the probability of being among the poorest 20% of the population by country and year (box plots)





Source: Authors' elaborations on EU-SILC data (2008–2014)

Figure 2 – Distributions of the probability of being among the poorest 20% of the population by country and year (violin plot)



Note: The wider sections at the bottom of the plot indicate a higher probability of individuals taking the value of zero, while thinner sections correspond to lower probabilities.
Source: Authors' elaborations on EU-SILC data (2008–2014)

4.2. *The multidimensional ANOGI*

As shown in Greco *et al.* (2018) and Lagravinese *et al.* (2019a), for any given rank (l), the downward cumulative rank acceptability indices ($b_k^{\leq l}, k = 1, \dots, m$), defined in equation (12), can be used to estimate the multidimensional generalization of the Gini index, and the multidimensional generalization of the Analysis of Gini (ANOGI) as formalised by Yitzhaki (1994) and extended in Liberati (2015).

Using the three specific thresholds in the poverty ranking and $b_k^{\leq l}$ as a measure of the individual probability of being poor, the multidimensional generalization of the Gini index and the multidimensional generalization of the ANOGI is estimated by first transforming $b_k^{\leq l}$ in an outcome that can be used to approximate the usual ranking from the poorest to the richest individual. To this purpose, and for convenience of interpretation, we take the complement of $b_k^{\leq l}$, which is the individual probability of being non-poor. Formally:

(13)
$$b_k^{>l} = \sum_{s=l+1}^{m} b_k^s$$

where $b_k^{>l}$ is now the upward cumulative rank acceptability index. Thus, for any given l, $b_k^{>l}$ measures the individual probability of being above l, i.e. the probability of being non-poor.

Given (13), the Gini index of the upward cumulative rank acceptability index of rank l can be estimated as follows:

(14)
$$G^{>l} = \frac{\sum_{h=1}^{m}\sum_{k=1}^{m}\left|b_h^{>l} - b_k^{>l}\right|}{2ml}$$

where $G^{>l}$ measures how the probabilities of attaining a rank higher than l are concentrated among individuals. For each threshold in the poverty ranking (l), the higher $G^{>l}$, the more concentrated the probabilities to be above this threshold, which would suggest that probabilities of being non-poor are heavily concentrated in a small number of individuals. If these probabilities were the same for all individuals $G^{>l}$ would be zero.

The ANOGI decomposition of $G^{>l}$, according to the extension developed in Liberati (2015), can be obtained as follows:

(15)
$$G^{>l} = \sum_i \underbrace{s_i p_i G_i^{>l}}_{Standard\ WI} + \underbrace{\sum\nolimits_i s_i G_i^{>l} \sum\nolimits_{j \neq i} p_j\, O_{ji}^{>l}}_{Impact\ of\ overlapping\ on\ WI} + \underbrace{G_{Bp}^{>l}}_{Standard\ BI} + \underbrace{\left(G_B^{>l} - G_{Bp}^{>l}\right)}_{Impact\ of\ overlapping\ on\ BI}$$

The first term is the within-country inequality (*WI*) in the absence of overlapping, where $G_i^{>l}$ is the Gini within country i, s_i is the share of the probabilities within country i of being above the rank l, and p_i is the share of population of country i. The second term is the impact of overlapping on within inequality, driven by the contribution of the overlapping index of each country with all other countries ($O_{ji}^{>l}$) weighted by their population shares.

The last two terms of equation (15) deal with the between-country inequality (*BI*). The term $G_{Bp}^{>l} = \frac{2cov\left(\overline{b^{>l}}_i, \bar{F}_i(b^{>l})\right)}{\overline{b^{>l}}}$ is the between-country inequality as defined in Pyatt (1976), with the covariance of the mean probability of each country $\overline{b^{>l}}_i$ and its rank in the distribution of the mean probabilities of all countries $\bar{F}_i\left(b^{>l}\right)$. This definition implies that $G_{Bp}^{>l} = 0$ when all the country-level mean probabilities are equal. Instead, according to Yitzhaki and Lerman (1991), one can define $G_B^{>l} = \frac{2cov\left(\overline{b^{>l}}_i, \bar{F}(b^{>l})\right)}{\overline{b^{>l}}}$, which is based on the covariance between the mean probability of each country $\overline{b^{>l}}_i$ and the average rank of all individual probabilities in the country in the overall distribution of probabilities $\bar{F}\left(b^{>l}\right)$. In this case, $G_B^{>l} = 0$ implies that the average rank of all individual in the overall distribution would be equal for all countries.

It is easy to see that the difference between the two formulas lies in the rank that is used to represent the country. Under Pyatt's approach, that rank is the rank of the country-level mean probability, while under the approach by Yitzhaki and Lerman it is the mean of the ranks of probabilities of individuals belonging to the country. These two approaches yield the same ranking in the case of perfect stratification. This implies that in the absence of overlapping of probabilities, between-inequality would be uniquely defined by $G_B^{>l}$. With overlapping, instead, $G_B^{>l} - G_{Bp}^{>l} < 0$, which can be interpreted as the reduction in between inequality caused by the overlapping of probabilities.

Finally, the term $O_{ji}^{>l}$ is a measure of how the distribution of probabilities in country i overlaps with the distribution of probabilities in another country j. If no individuals in country j lies in the range of the distribution of probabilities in country i, this latter would be a perfect stratum and $O_{ji}^{>l} = 0$. Thus, if all countries were perfect stratums, the second term on the right-hand side of (15) would collapse to zero. This would suggest that all countries have a within distribution of probabilities that is not within the range of any other country. On the other hand, since $O_{ji}^{>l} \leq 2$, the maximum value is achieved when all probabilities associated to country j that are located in the range of i are concentrated around the mean of the distribution i. This implies that the probabilities of country j would split the probabilities of country i that are below the average from those that are above the average. It is worth noting that the higher $O_{ji}^{>l}$, the lower will be $O_{ij}^{>l}$, which is obtained by switching the country used as a baseline.

Formally, the overlapping coefficient is defined as follows:

(16)
$$O_{ji}^{>l} = \frac{\operatorname{cov}\left(b_i^{>l}, F_j\left(b^{>l}\right)\right)}{\operatorname{cov}\left(b_i^{>l}, F_i\left(b^{>l}\right)\right)}$$

where the numerator is the covariance between the upward cumulative rank acceptability indices for rank l in country i, and their ranking in the distribution of the upward cumulative rank acceptability indices in country j. The denominator, instead, is the covariance between the same upward cumulative rank acceptability indices and their ranking within the country i.

Table 4 shows the calculation of the multidimensional ANOGI as in equation (15), which give information on how the individual probabilities of being above the poorest 20% are concentrated, both within and between the ten European countries here considered. As shown in the second column, the total inequality of the distribution of probabilities has not changed significantly between 2008 and 2014. Yet, some changes can be observed in the individual components of the Gini index. In particular, the *standard within* component has a slight monotonic decrease (from 0.027 in 2008 to 0.024 in 2014). Furthermore, while the impact of *overlapping* on within inequality remains quite constant, the intensity of the *between* component of total inequality increased from 0.021 to 0.032. This suggests that the average probabilities of being poor, among countries, are increasingly more dispersed, even though they are more intertwined, as suggested by the increasing negativity of the impact of overlapping on between inequality (from -0.019 to -0.027). These time trends are robust and confirmed when moving the threshold of poverty both at 10% and at 5% (see Tables A3 and A4 in the Appendix).

Table 4 – Multidimensional ANOGI of the probabilities of not being among the poorest 20% (2008-2014)

Year	Total Inequality	Standard WI	Impact of overlapping on WI	Standard BI	Impact of overlapping on BI
2008	0.196	0.027	0.167	0.021	-0.019
2010	0.197	0.025	0.169	0.025	-0.022
2012	0.197	0.024	0.169	0.031	-0.027
2014	0.197	0.024	0.168	0.032	-0.027

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table 5, instead, shows detailed statistics of inequality by country. Lower levels of inequality are observed in Austria, Germany, and France, which have Gini coefficients of 0.153, 0.158, and 0.158 respectively. Higher inequality is instead observed in Greece, Portugal, and Spain, in which Gini coefficients are 0.241, 0.226, and 0.222 respectively. This outcome is consistent with the fact that the first three countries, compared with the latter three, have a higher frequency of persons that are not classified as poor, so that their distribution of probabilities is less concentrated. It follows that countries where the probabilities of being non-poor are lower are also those countries with the highest inequality of the distribution of probabilities.

The same outcome occurs when moving the poverty threshold to 10% (see Table A5 in Appendix), while in the case of 5%, there are significant changes in the ranking. Although the overall negative correlation between Gini and the average probability at country level remains significant, considering the poverty threshold at 5%, the three countries with the highest Gini are Belgium, Italy and Portugal (0.060, 0.056, and 0.056 respectively), while the three countries with the lowest Gini are Austria, the UK, and France (0.031, 0.040, and 0.042 respectively) (see Table A6 in Appendix). In both cases, however, the sizes of the Gini coefficients are smaller than in the previous cases, suggesting that the corresponding distributions, as expected, are closer to the equidistribution line in the sense that there are a greater number of persons having a 100% probability of not being among the poorest part of the population.

Table 5 –Detailed statistics of inequality of the probabilities of not being among the poorest 20%, by country (2008)

Country	N	p	mean	s	G	O
AT	10,846	0.061	0.843	0.065	0.153	1.056
BE	10,073	0.057	0.809	0.058	0.186	1.044
DE	22,834	0.129	0.839	0.136	0.158	1.031
EL	13,486	0.076	0.749	0.072	0.241	0.954
ES	27,784	0.157	0.770	0.152	0.222	0.988
FR	19,493	0.110	0.838	0.116	0.158	1.006
IT	42,532	0.241	0.782	0.236	0.210	0.979
LU	7,486	0.042	0.787	0.042	0.207	1.034
PT	8,505	0.048	0.764	0.046	0.226	0.928
UK	13,479	0.076	0.805	0.077	0.190	1.009

Note: N = Number of observations; p = Share of population; mean = Average Upward cumulative rank acceptability index; s = Share of probability of being above 20% of poverty; G = Gini coefficients; O = Average overlapping.
Source: Authors' elaborations on EU-SILC data (2008)

Table 6 reports the matrices of $O_{ji}^{>20\%}$ for 2008, obtained by the decomposition of the general overlapping index, with rows indicating the baseline country i and columns reporting each country j. By construction, each element of the main diagonal of this matrix equals one. If no person in country j lies in the range of the distribution of probabilities of persons in country i, country i could be defined a perfect stratum and $O_{ji}^{>20\%} = 0$. Table 6 reports no cases in which cells equal zero, which means that it is always the case that there are individuals in country j that lie in the distribution of probabilities of individuals in country i. As a matter of fact, none of the 10 European countries considered in this study can be considered a perfect stratum, as in each country there are people that have some probability of being among the poorest 20% of the European population, and these probabilities overlap among countries. Considering the average overlapping (i.e., $mean_{j \neq i}(O_{ji}^{>20\%})$, not reported in matrix), lower indices – i.e., a higher stratification – is found in Portugal, Greece, and Italy (0.928, 0.954, and 0.979 respectively). On the opposite side, higher indices, i.e. a lower stratification, are found in Austria, Belgium, and Luxembourg, with average overlapping of 1.056, 1.044, and 1.034 respectively. The combination of these two results suggests that there are relatively more people from poorest countries in the range of the distribution of less poor countries, than there are people in richer countries in the range of the distribution of the poorest countries. As an example, we can consider the relationship between Austria and Portugal. Taking Austria as a baseline, in 2008 $O_{AP}^{>20\%} = 1.118$, while taking Portugal as a baseline $O_{PA}^{>20} = 0.860$. This means that there are relatively more people in Portugal overlapping the distribution of people in Austria than there are people in Austria overlapping the distribution of Portugal.

Overall, however, the matrix of overlapping, to some extent as expected from the fact that we are considering European countries with similar economic structures, shows that the distribution of the probabilities among countries are significantly intertwined. The same outcome occurs when moving the threshold of poverty to 10% and 5% (See Table A7 and Table A8 in Appendix).[5]

Table 6 – Overlapping matrix of the probabilities of not being among the poorest 20% (2008)

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.016	1.033	1.086	1.062	1.059	1.074	1.016	1.118	1.049
BE	0.979	1	1.012	1.088	1.057	1.036	1.065	1.012	1.114	1.036
DE	0.967	0.988	1	1.074	1.043	1.023	1.052	0.998	1.099	1.024
EL	0.892	0.910	0.921	1	0.966	0.943	0.976	0.923	1.022	0.947
ES	0.923	0.942	0.955	1.034	1	0.978	1.010	0.955	1.058	0.979
FR	0.942	0.963	0.977	1.046	1.016	1	1.026	0.972	1.074	0.996
IT	0.916	0.935	0.947	1.023	0.991	0.970	1	0.947	1.047	0.971
LU	0.974	0.989	1.003	1.075	1.044	1.026	1.053	1	1.098	1.030
PT	0.860	0.882	0.891	0.980	0.942	0.912	0.952	0.900	1	0.918
UK	0.946	0.965	0.980	1.047	1.018	1.006	1.029	0.969	1.077	1

Source: Authors' elaborations on EU-SILC data (2008–2014)

[5] To give a synthetic indication of the closeness of the outcome, one can consider that the rank correlation between $mean_{j \neq i}(O_{ji}^{>20\%})$ and $mean_{j \neq i}(O_{ji}^{>10\%})$ is 0.903 and rank correlation between $mean_{j \neq i}(O_{ji}^{>10\%})$ and $mean_{j \neq i}(O_{ji}^{>10\%})$ is 0.951.

Moving the analysis to 2014, Table 7 reports the corresponding statistics of inequality in the distribution of probabilities. Countries with lower Gini coefficient in 2014 are Austria, France, and Luxembourg, with values of 0.132, 0.143, and 0.154 respectively. Comparing these three values with values in the same countries in 2008, a significant decrease in all the three Gini coefficients can be noted, in particular in Luxembourg in which the Gini coefficient decreases from 0.207 to 0.154. As a general tendency in those countries, a reduction of the Gini index implies that the distribution of the probabilities is less unequal, i.e. that the concentration of the probabilities of being non-poor has further reduced. The decrease of the Gini coefficient in Luxembourg is even more evident when moving the poverty threshold to 10% and to 5% (Table A9 and Table A10 in the Appendix).

Countries with higher within Gini coefficient in 2014 are Portugal, Greece, and Spain with a value of 0.255, 0.250, and 0.243 respectively. These three countries were also the three countries with the highest Gini in 2008, but, unlike the previous case, all these countries have experienced an increase of the Gini coefficients from 2008 to 2014, which means that the concentration of the probabilities of being non-poor is even more concentrated than before.

Overall, however, the ranking of countries has not significantly changed, as the rank correlation between country-level Gini coefficients in 2008 and country-level Gini coefficients in 2014 is 0.906. Moving the poverty threshold to 10% and to 5%, the ranking in terms of Gini is still significantly correlated, with two main deviations represented by the above-mentioned Luxembourg and the UK (Table A9 and Table A10 in the Appendix).

Table 7 –Detailed statistics of inequality of the probabilities of not being among the poorest 20%, by country (2014)

Country	N	p	mean	s	G	O
AT	10651	0.058	0.865	0.063	0.132	1.065
BE	11236	0.061	0.814	0.063	0.182	1.070
DE	21462	0.117	0.842	0.124	0.155	1.058
EL	17768	0.097	0.738	0.090	0.250	0.944
ES	26049	0.142	0.749	0.134	0.243	1.027
FR	20659	0.113	0.854	0.121	0.143	1.048
IT	38604	0.211	0.788	0.209	0.202	0.944
LU	7891	0.043	0.840	0.046	0.154	1.009
PT	14579	0.080	0.731	0.073	0.255	0.867
UK	14013	0.077	0.794	0.076	0.197	1.000

Note: N = Observation; p = Share of population; mean = Average Upward cumulative rank acceptability index; s = Share of probability of being above 20% of poverty; G = Gini coefficients; O = Average overlapping.
Source: Authors' elaborations on EU-SILC data (2014)

Finally, Table 8 shows the overlapping matrix for 2014. As in 2008, there are no cases in which cells equal zero, which means that also in this case none of the 10 European countries can be considered a perfect stratum in 2014. On average, higher levels of overlapping are in Belgium, Austria, and Germany, while Portugal, Greece and Italy are more stratified in their distribution of probabilities. Yet, the rankings in terms of average overlapping between 2008 and 2014 are positively correlated (0.843). The main changes in the ranking are for

Luxembourg, which means that the country has experienced an increasing stratification process from 2008 to 2014, as it increased the average probability of being non-poor and it reduced its concentration at country level.

As in 2008, the ranking of countries in average overlapping does not change significantly moving the threshold of poverty to 10% and 5% in 2014 (See Table A11 and Table A12 in the Appendix). Rank correlations of average overlapping, indeed, remains high, being 0.945 between 20% and 10% poverty thresholds and 0.977 between 10% and 5% poverty thresholds.

Table 8 – Overlapping matrix of the probabilities of not being among the poorest 20% (2014)

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.001	1.018	1.104	1.024	1.024	1.121	1.051	1.181	1.065
BE	0.987	1	1.010	1.123	1.041	1.014	1.127	1.044	1.202	1.072
DE	0.979	0.989	1	1.107	1.025	1.005	1.115	1.035	1.186	1.059
EL	0.859	0.874	0.879	1	0.918	0.885	1.002	0.920	1.076	0.949
ES	0.940	0.955	0.962	1.082	1	0.967	1.085	1.001	1.162	1.031
FR	0.975	0.981	0.995	1.092	1.011	1	1.105	1.030	1.171	1.050
IT	0.873	0.879	0.890	0.988	0.907	0.897	1	0.931	1.059	0.945
LU	0.951	0.949	0.967	1.043	0.965	0.974	1.060	1	1.112	1.007
PT	0.778	0.794	0.800	0.925	0.839	0.804	0.929	0.845	1	0.871
UK	0.937	0.936	0.952	1.037	0.957	0.960	1.056	0.990	1.111	1

Source: Authors' elaborations on EU-SILC data (2014).

The main findings of the ANOGI can thus be summarised as follows. From 2008 to 2014, the total inequality of the individual probabilities of being non-poor in the 10 EU countries considered here has not changed significantly, yet there has been a non-negligible increase of inequality between countries. This increase can be partially explained by an increase of the average probability of being non-poor in countries having higher average probability of being non-poor in 2008 (in particular Austria, Luxembourg and France), and a decrease of the average probability of being non-poor in countries having lower average probabilities of being non-poor in 2008 (i.e. Portugal, Spain, and Greece). This process has increased the distance between countries with lower average probabilities and countries with higher average probabilities of being non-poor. Furthermore, there is evidence of an increase of the inequality of the probabilities within those countries with low average probabilities of being non-poor, and a reduction of the inequality of the same probabilities in countries with high average probabilities of being non-poor. In some cases, as in Luxembourg, this has driven an increase of the degree of stratification, which means that the probabilities of being non-poor in Luxembourg are less shared by individuals living in other EU countries in 2014 with respect to 2008.

5. Dominance conditions of the probability of being poor

5.1 *Extending dominance criteria to the probabilities of being non-poor*

In the previous section, the analysis of poverty, which is typically developed on a distribution censored by the poverty line, has been extended to the overall distribution of probabilities of being non-poor by using ANOGI. Thus, our analysis differs from the standard way of dealing with inequality among poor, i.e. with inequality calculated among individuals that are below a poverty line, to favour a global view of the inequality of probabilities and thus a measure of the concentration of non-poor individuals.

In this section, using the same approach, a step further is done to investigate to what extent the ranking of countries according to the probability of being poor can be translated into wider social norms. To perform this task, we use the dominance criterion related to the generalised Lorenz dominance technique. To build this process, as before, we still use the complement of the estimated individual probability of being poor as an indicator of the position in the income distribution. In particular, individuals in each country can be ordered from the lowest to the highest probability of being *non*-poor, where the lowest probability of being non-poor will be 0 and the highest probability of being non-poor will be 1.

By ordering individuals according to this indicator, the outcome can be interpreted as an approximation of the usual ranking from the poorest to the richest individual. As a consequence, the dominance of the generalised Lorenz (GL) curve of the probabilities of individuals in country A over the generalised Lorenz curve of the probabilities of individuals in country B would mean that individuals in country A have less (probability of) poverty than individuals in country B for any fraction of the population. As in the previous section, the focus is on the whole distribution of probabilities – and thus on the total number of individuals – and not only on the distribution of probabilities of individuals below a given poverty line.

As it is well known, however, the GL dominance is not a synthetic measure, neither of inequality nor of poverty, which means that uncertain outcomes between countries may occur whenever the GL curves cross. In order to combine GL dominance with more general social prescriptions in the analysis of multidimensional poverty, recourse has been made to an extension of the well-established correspondence between classes of social welfare functions and dominance conditions.[6]

In our case, the GL dominance of a given distribution of the probabilities of being non-poor may be thought as more socially preferred, as the dominating distribution implies a higher probability of being non-poor for any fraction of the population. To this purpose, define a class of social norms $W(z)$ that satisfies $W'(z) > 0$ and $W''(z) < 0$. These two conditions only require that the social preference is increasing in the argument (i.e., it increases when the probability of being non-poor increases) and concave, which means that a "transfer" of the probability of being non-poor from a higher to a lower probability would increase the social preference.[7] Thus, GL dominance would allow general conclusions when comparing social preferences without the need to specify an exact functional form for $W(z)$.

[6] See for all Lambert (1993) and Deaton (1997).

[7] It is worth noting that this second condition is simply a restatement of the principle of transfers that holds when income is the argument of a social welfare function, and that fundamentally embodies aversion to inequality.

As described above, however, GL dominance may not occur; rather, GL curves may cross. This outcome would prevent to draw unanimous conclusions about which distribution of probabilities should be socially preferred. Yet, some conclusions may be achieved with the additional requirement that $W'''(z) > 0$. This feature corresponds to the principle of diminishing transfer, which means that an increase of the probability of being non-poor at lower levels of this probability increases social preference more than an increase of the same probability at higher levels.

In this case, the focus is shifted on the dominance in the lowest part of the distribution, that in our case would mean to focus on the part of the population where the lowest probabilities of being non-poor are concentrated. In particular, if $GL_x >_A GL_y$, where the symbol $>_A$ means that the distribution x intersect the distribution y from above at a given point, the distribution x will be socially preferred if two conditions are met (mean-variance condition):

(17) $$\mu_x < \mu_y$$

(18) $$\sigma_x^2 < \sigma_y^2 - (\mu_y - \mu_x)(2v - \mu_y - \mu_x)$$

where v is the maximum probability of being non-poor, which is equal to 1. Condition (17) simply states that the mean of the distribution x (μ_x) must be lower than the mean of the distribution y (μ_y). Condition (18) requires that the variance of the distribution x (σ_x^2) must be *sufficiently* lower than the variance of the distribution y (σ_y^2). It is also worth noting that if the mean level of the two distributions were equal, the only relevant condition would be $\sigma_x^2 < \sigma_y^2$, i.e. that the variance of x is lower than the variance of y.

When either of the two conditions does not hold, no general conclusions in terms of social preference would be possible. When both hold, instead, one can go a step further to measure the robustness of the social ranking to the degree of inequality-poverty aversion. This can be done by calculating a lower limit of that aversion below which social unanimous prescriptions obtained by GL no longer hold. This lower bound is given by:

(19) $$b = \frac{v(\mu_y - \mu_x)}{(\sigma_y^2 - \sigma_x^2) - (\mu_y - \mu_x)(2v - \mu_y - \mu_x)}$$

The calculation of b is potentially important to understand the robustness of the ranking in terms of consensus across different decision makers with different (and unknown) degrees of inequality-poverty aversion. To this purpose, the larger the difference $[(\sigma_y^2 - \sigma_x^2) - (\mu_y - \mu_x)(2v - \mu_y - \mu_x)]$, the nearer to zero will be the lower bound of equation (3). Since $b = 0$ would connote inequality neutrality, the larger the gap, the greater is the class of $W(z)$ for which the result will hold.

This methodology represents a novel approach to combine poverty analysis and dominance criteria, as it combines a value-free method of estimating the probabilities of being non-poor with a value-free way of determining social preferences that directly connect poverty levels and the inequality in their distributions.

With the exception of a recent contribution by Aaberge *et al*. (2019), this is also the first attempt to apply dominance criteria to the issue of multidimensional poverty. Our contribution, however, differs from that by Aaberge *et al*. (2019), as in that case the analysis is based on a deprivation count distribution where no attempt is made to aggregate the count into a synthetic multidimensional poverty index at individual level. In our analysis, instead, the deprivation count distribution is the baseline to calculate the probability of each individual to be below a given poverty threshold. This difference allows us to apply dominance criteria directly considering the whole distribution of probabilities obtained by aggregating the dimensions of poverty with 10,000 different vectors of weights; while in Aaberge *et al*. (2019), the dominance is *sequentially* applied (either downward or upward) by progressively adding fractions of populations with a different number of deprivations.

5.2 *GL dominance and GL crossings*

The outcome of the Lorenz dominance for the probability of being among the poorest 20% of the European population is reported in Table 9 for all years. Each panel can be easily read by rows. For example, in 2008, Austrian individuals have always a lower probability ("Lower") of being among the poorest 20% of the European population than any other country, with the exception of France. For Italian individuals, instead, this probability is lower only compared to Portugal, while crossings occur with Belgium, Greece, Spain, and Luxembourg. At the same time, individuals from Greece and Portugal have the highest probability of being widely represented in the poorest 20%, as a lower probability does not appear in any comparison.

The analysis is replicated in each year, and gives evidence of the changes occurred in the ranking of probabilities among countries. In particular, in the panel of year 2014, changes with respect to 2008 are highlighted. Changes occur in each country, with a slight improvement of the relative position only in Italy, Luxembourg, and Portugal. A slightly worse comparative outcome can instead be traced in Austria, Belgium, Germany, Greece, Spain, and France. Finally, in the UK, a relative improvement occurs with respect to Austria, while the relative position worsens with respect to Luxembourg.

In terms of social preferences, the conclusions are readily obtained. By considering the last year of the analysis, 2014, since "Lower" corresponds to all cases where the GL curve of the probabilities of being *non-poor* in the country in row dominates the GL curve of the same probabilities in the country in column, the social preference as measured by any member of the class $W = \{W: W'(z) > 0; W''(z) < 0\}$ is always for the distribution of probabilities in the country in row. It is worth noting that the dominance also implies that the social preference will be higher regardless of any specific poverty line below the income level corresponding to the richest individual among the lowest 20% of the distribution. The opposite holds in the case where the matrix is filled by "Higher".

Uncertain outcomes, instead, occur when GL curves cross ("Crossing"). To solve this uncertainty, we first identify the comparisons between countries where the dominance occurs in the lowest part of the distribution (i.e. before the intersection, from above). This happens in the following cases: $GL_{IT} >_A GL_{BE}$; $GL_{IT} >_A GL_{DE}$;

$GL_{IT} >_A GL_{FR}$; $GL_{LU} >_A GL_{AT}$; $GL_{LU} >_A GL_{DE}$; $GL_{LU} >_A GL_{FR}$; $GL_{PT} >_A GL_{BE}$; $GL_{PT} >_A GL_{DE}$; $GL_{PT} >_A GL_{EL}$; $GL_{PT} >_A GL_{ES}$; $GL_{UK} >_A GL_{AT}$; $GL_{UK} >_A GL_{BE}$; $GL_{UK} >_A GL_{DE}$; $GL_{UK} >_A GL_{FR}$.

In all comparisons, both conditions (17) and (18) are satisfied, which means that the dominating distribution is socially preferred for any member of the restricted class $W = \{W: W'(z) > 0; W''(z) < 0; W'''(z) > 0\}$. More importantly, as shown in Table 10, the values of b, as in equation (19), are calculated. For example, the dominance of Italy over France will embody a social preference for degrees of inequality-poverty aversion higher than 0.779. As can be easily seen, some crossings correspond to a higher social preference only for degrees of inequality-poverty aversion greater than 1, as in the cases of Portugal vs. Greece, Portugal vs. Spain, and Greece vs. Spain.

It is worth stressing, at this point, that this outcome is particularly important in the analysis of poverty, as it allows a double stronger conclusion with respect to the existing literature. The first derives from the fact that the probabilities of being *multidimensional* poor are estimated without making recourse to a specific set of weights; the second derives from the fact that the social welfare implications are not constrained by a specific functional form of the social preference. In other terms, the ranking among countries according to their level of poverty that is here obtained is loaded by the minimum set of arbitrary choices, in terms of weighting the various dimensions of poverty, to estimate the distribution of the individual probabilities of being non-poor, and in terms of transforming this distribution of individual probabilities to derive social preferences.

Table 9 – The probability of being among the poorest 20% using GL dominance

2008	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Lower	Lower	Lower	Crossing	Lower	Lower	Lower	Lower
BE	Higher		Higher	Crossing	Crossing	Higher	Crossing	Crossing	Lower	Crossing
DE	Higher	Lower		Crossing	Crossing	Crossing	Lower	Crossing	Lower	Crossing
EL	Higher	Crossing	Crossing		Higher	Higher	Crossing	Higher	Crossing	Higher
ES	Higher	Crossing	Crossing	Lower		Higher	Crossing	Crossing	Crossing	Higher
FR	Crossing	Lower	Crossing	Lower	Lower		Lower	Lower	Lower	Crossing
IT	Higher	Crossing	Higher	Crossing	Crossing	Higher		Crossing	Lower	Higher
LU	Higher	Crossing	Crossing	Lower	Crossing	Higher	Crossing		Lower	Higher
PT	Higher	Higher	Higher	Crossing	Crossing	Higher	Higher	Higher		Higher
UK	Higher	Crossing	Crossing	Lower	Lower	Crossing	Lower	Lower	Lower	

2010	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Crossing	Lower	Lower	Crossing	Lower	Lower	Lower	Lower
BE	Higher		Higher	Crossing	Crossing	Higher	Crossing	Higher	Lower	Higher
DE	Crossing	Lower		Crossing	Lower	Higher	Crossing	Crossing	Lower	Crossing
EL	Higher	Crossing	Crossing		Crossing	Higher	Higher	Higher	Crossing	Higher
ES	Higher	Crossing	Higher	Crossing		Higher	Higher	Higher	Lower	Higher
FR	Crossing	Lower	Lower	Lower	Lower		Lower	Lower	Lower	Lower
IT	Higher	Crossing	Crossing	Lower	Lower	Higher		Higher	Lower	Higher
LU	Higher	Lower	Crossing	Lower	Lower	Higher	Lower		Lower	Crossing
PT	Higher	Higher	Higher	Crossing	Higher	Higher	Higher	Higher		Higher
UK	Higher	Lower	Crossing	Lower	Lower	Higher	Lower	Crossing	Lower	

2012	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Crossing	Lower	Lower	Lower	Lower	Lower	Lower	Lower
BE	Higher		Higher	Lower	Lower	Higher	Crossing	Higher	Lower	Higher
DE	Crossing	Lower		Lower	Lower	Crossing	Lower	Crossing	Lower	Crossing
EL	Higher	Higher	Higher		Higher	Higher	Higher	Higher	Higher	Higher
ES	Higher	Higher	Higher	Lower		Higher	Higher	Higher	Lower	Higher
FR	Higher	Lower	Crossing	Lower	Lower		Lower	Crossing	Lower	Lower
IT	Higher	Crossing	Higher	Lower	Lower	Higher		Higher	Lower	Higher
LU	Higher	Lower	Crossing	Lower	Lower	Crossing	Lower		Lower	Lower
PT	Higher	Higher	Higher	Lower	Higher	Higher	Higher	Higher		Higher
UK	Higher	Lower	Crossing	Lower	Lower	Higher	Lower	Higher	Lower	

2014	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Lower	Lower	Lower	Lower	Lower	Crossing	Lower	Crossing
BE	Higher		Higher	Crossing	Lower	Higher	Crossing	Higher	Crossing	Crossing
DE	Higher	Lower		Lower	Lower	Higher	Crossing	Crossing	Crossing	Crossing
EL	Higher	Crossing	Higher		Crossing	Higher	Higher	Higher	Crossing	Higher
ES	Higher	Higher	Higher	Crossing		Higher	Higher	Higher	Crossing	Higher
FR	Higher	Lower	Lower	Lower	Lower		Crossing	Crossing	Lower	Crossing
IT	Higher	Crossing	Crossing	Lower	Lower	Crossing		Higher	Lower	Higher
LU	Crossing	Lower	Crossing	Lower	Lower	Crossing	Lower		Lower	Lower
PT	Higher	Crossing	Crossing	Crossing	Crossing	Higher	Higher	Higher		Higher
UK	Crossing	Crossing	Crossing	Lower	Lower	Crossing	Lower	Higher	Lower	

Source: Authors' elaborations

Table 10 – Lower bound of inequality aversion

2014	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT										
BE										
DE										
EL					1.026					
ES										
FR										
IT		0.830	0.793			0.779				
LU	0.709		0.733			0.721				
PT		0.919	0.873	1.066	1.041					
UK	0.758	0.822	0.786			0.772				

Source: Authors' elaborations

6. Conclusions

This paper proposes a multidimensional poverty analysis in 10 European countries which introduces two main innovations compared with the previous literature: first, the dimensions are defined on the basis of the Charter of Fundamental Rights of the European Union with the aim of avoiding any exogenous definition of poverty; second, the whole space of feasible (positive) weights is used to summarise the multidimensional information, in order to remain agnostic about the importance given to the different dimensions.

From a methodological perspective, this paper exploited the Hierarchy Stochastic Multi-Objective Acceptability Analysis (HSMAA) which has four main advantages comparted with other techniques: it allows to explore the whole set of feasible weights by Monte Carlo generation; it allows to quantify the volume of vectors of weights by which each individual get a specific position in ranking; it does not suffer the curse of dimensionality; and it allows to consider nested features in multidimensional poverty measures. In other words, by means of HSMAA the uncertainty in weights is dealt with, but the probability that each dimension be important into the composition of the final index is independent from the number of indicators used to measure it.

Using data from four waves of EU-SILC (2008, 2010, 2012, 2014), the methodological innovations introduced here has allowed to produce a family of measures of multidimensional deprivation which capture the individual probability of being among the poorest 20%, the 10% and the 5% of the EU population. These probabilities are analysed at country level, in terms of both average levels and inequality, and are finally combined with the generalised Lorenz dominance techniques in order to derive socially preferred distributions with the minimum load of value judgments.

Results show that the individual probability of being among the poorest 20% have median zero for all countries with the exceptions of Greece, Spain, Portugal and Italy. In particular, in Greece, Spain, and Portugal, there is a significant number of individuals who have 100% probability of being among the poorest 20% of the population regardless of the weighting scheme applied to the set of multidimensional deprivation indicators. On the contrary, in Austria, France, Germany, Luxembourg and the UK the probability distributions of being among the poorest 20% are shorter, and the presence of large outliers means that the weights attached to the deprivation indicators can significantly change the probability of being considered multidimensional poor.

The multidimensional generalization of the Analysis of Gini (ANOGI) shows that from 2008 to 2014, total inequality among individuals in the 10 EU countries considered here has not changed significantly, but it is also shown that this outcome is the result of conflicting paths of a decreasing within inequality and of an increasing between inequality of the probabilities of being non-poor.

The increase of between inequality can be partially explained by an increase of the average probability of being non-poor in countries having a higher average probability of being non-poor in 2008 (Austria, Luxembourg and France in particular), and a reduction of the average probability of being non-poor in countries having a lower average probability of being non-poor in 2008 (i.e. Portugal, Spain, and Greece). Furthermore, as an overall tendency, there is evidence of an increase of inequality within countries with low average probabilities of being non-poor, and a reduction of inequality in countries with a high average probability of being non-poor.

Finally, the outcome of the pairwise Lorenz dominance show that in 2008 Austrian individuals have always a lower probability of being among the poorest 20% of the European population than any other country, with the exception of France. On the contrary, in the same year, the distribution of probabilities in Greece and Portugal never dominates other countries.

Overall, the methodology here applied can shed new light on the multidimensional poverty analysis by moving from a dual definition of poverty, where poor and non-poor individuals are classified in a mutually exclusive context, to a continuous measure of deprivation given by the probability of being poor. These methodological innovations allow to capture both the extensive and intensive margin of multidimensional poverty. The framework proposed here can be applied in any setting where either count, or continuous data are available.

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APPENDIX

Table A1 – Probabilities of being among the poorest 10% of the population

Year	mean	sd	p25	p50	p75	% population with Prob10%>0	Average N of deprivations if Prob10%>0
				AT			
2008	0.070	0.21	0	0	0	22.5	7
2010	0.075	0.22	0	0	0	24.7	7
2012	0.064	0.20	0	0	0	23.0	7
2014	0.062	0.20	0	0	0	27.1	7
				BE			
2008	0.104	0.26	0	0	0.003	29.7	7
2010	0.111	0.26	0	0	0.003	32.4	7
2012	0.113	0.27	0	0	0.003	33.2	7
2014	0.112	0.27	0	0	0.003	34.7	8
				DE			
2008	0.086	0.24	0	0	0.002	28.2	7
2010	0.085	0.24	0	0	0.002	27.3	7
2012	0.079	0.23	0	0	0	27.8	7
2014	0.083	0.23	0	0	0.002	30.8	7
				EL			
2008	0.124	0.27	0	0	0.031	38.7	7
2010	0.131	0.27	0	0	0.084	42.1	7
2012	0.160	0.30	0	0	0.141	48.2	7
2014	0.144	0.30	0	0	0.066	46.9	7
				ES			
2008	0.118	0.27	0	0	0.027	35.4	7
2010	0.127	0.28	0	0	0.028	36.9	7
2012	0.129	0.28	0	0	0.075	36.0	7
2014	0.152	0.31	0	0	0.066	42.9	7
				FR			
2008	0.081	0.23	0	0	0.003	29.3	7
2010	0.077	0.22	0	0	0.002	29.3	7
2012	0.076	0.22	0	0	0	28.0	7
2014	0.077	0.22	0	0	0	32.4	7
				IT			
2008	0.111	0.26	0	0	0.006	35.2	7
2010	0.102	0.25	0	0	0.003	33.9	7
2012	0.108	0.26	0	0	0.016	36.6	7
2014	0.090	0.21	0	0	0.017	41.5	7
				LU			
2008	0.110	0.26	0	0	0.004	32.2	8
2010	0.091	0.24	0	0	0.002	28.9	8
2012	0.078	0.22	0	0	0.002	29.1	8
2014	0.063	0.18	0	0	0	30.7	7
				PT			
2008	0.122	0.27	0	0	0.031	42.5	8
2010	0.132	0.28	0	0	0.083	43.6	8
2012	0.137	0.29	0	0	0.066	46.7	7
2014	0.130	0.25	0	0	0.117	54.5	7
				UK			
2008	0.090	0.23	0	0	0.002	29.4	7
2010	0.088	0.23	0	0	0.002	28.0	7
2012	0.083	0.23	0	0	0	27.6	7
2014	0.081	0.20	0	0	0.003	35.1	8

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A2 – Probabilities of being among the poorest 5% of the population

Year	mean	sd	p25	p50	p75	% population with Prob5%>0	Average N of deprivations if Prob5%>0
				AT			
2008	0.032	0.15	0	0	0	15.4	8
2010	0.036	0.16	0	0	0	17.1	8
2012	0.028	0.14	0	0	0	15.0	8
2014	0.029	0.14	0	0	0	16.6	7
				BE			
2008	0.061	0.20	0	0	0	23.6	8
2010	0.061	0.20	0	0	0	24.3	8
2012	0.061	0.20	0	0	0	24.4	8
2014	0.068	0.22	0	0	0	23.7	8
				DE			
2008	0.051	0.19	0	0	0	19.2	8
2010	0.049	0.19	0	0	0	19.2	8
2012	0.042	0.17	0	0	0	18.1	8
2014	0.044	0.18	0	0	0	19.6	8
				EL			
2008	0.052	0.17	0	0	0	31.7	8
2010	0.052	0.17	0	0	0	32.4	8
2012	0.077	0.22	0	0	0.002	36.6	7
2014	0.085	0.23	0	0	0.002	32.2	8
				ES			
2008	0.053	0.18	0	0	0	28.5	7
2010	0.065	0.21	0	0	0	27.9	8
2012	0.072	0.22	0	0	0	25.6	8
2014	0.097	0.26	0	0	0.001	30.5	7
				FR			
2008	0.043	0.17	0	0	0	22.2	8
2010	0.038	0.16	0	0	0	20.8	8
2012	0.037	0.16	0	0	0	18.1	7
2014	0.039	0.16	0	0	0	19.9	8
				IT			
2008	0.057	0.19	0	0	0	28.1	8
2010	0.051	0.18	0	0	0	25.0	8
2012	0.052	0.18	0	0	0	25.7	8
2014	0.032	0.13	0	0	0	25.3	8
				LU			
2008	0.050	0.18	0	0	0	25.8	8
2010	0.040	0.16	0	0	0	21.4	8
2012	0.035	0.15	0	0	0	20.7	8
2014	0.019	0.10	0	0	0	18.3	7
				PT			
2008	0.058	0.19	0	0	0	35.4	8
2010	0.062	0.20	0	0	0.002	32.2	8
2012	0.065	0.20	0	0	0.002	35.0	8
2014	0.051	0.16	0	0	0.003	38.1	8
				UK			
2008	0.040	0.16	0	0	0	18.7	8
2010	0.042	0.17	0	0	0	19.0	8
2012	0.040	0.16	0	0	0	18.6	7
2014	0.023	0.11	0	0	0	23.3	9

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A3 – Multidimensional ANOGI of Upward cumulative rank acceptability index for the top 10% EU poverty

Year	Total Inequality	Standard WI	Impact of overlapping on WI	Standard BI	Impact of overlapping on BI
2008	0.099	0.014	0.085	0.010	-0.009
2010	0.099	0.013	0.086	0.012	-0.011
2012	0.099	0.012	0.085	0.016	-0.014
2014	0.100	0.012	0.085	0.019	-0.016

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A4 – Multidimensional ANOGI of Upward cumulative rank acceptability index for the top 5% EU poverty

Year	Total Inequality	Standard WI	Impact of overlapping on WI	Standard BI	Impact of overlapping on BI
2008	0.049	0.007	0.042	0.004	-0.004
2010	0.051	0.006	0.043	0.006	-0.005
2012	0.050	0.006	0.043	0.009	-0.008
2014	0.051	0.006	0.043	0.015	-0.013

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A5 – Detailed statistics of inequality in Upward cumulative rank acceptability index for the top 10% EU poverty for subgroups (2008)

Country	N	p	mean	s	G	O
AT	10846	0.061	0.930	0.064	0.068	1.059
BE	10073	0.057	0.896	0.057	0.101	1.034
DE	22834	0.129	0.914	0.132	0.084	1.038
EL	13486	0.076	0.876	0.075	0.119	0.952
ES	27784	0.157	0.882	0.155	0.114	0.984
FR	19493	0.110	0.919	0.113	0.079	1.015
IT	42532	0.241	0.889	0.239	0.108	0.982
LU	7486	0.042	0.890	0.042	0.106	1.010
PT	8505	0.048	0.878	0.047	0.118	0.921
UK	13479	0.076	0.910	0.077	0.087	1.017

Note: N= Number of observations; p = share of population; mean = average Upward cumulative rank acceptability index; s = share of probability of being above 20% of poverty; G = Gini coefficients; O = average overlapping.
Source: Authors' elaborations on EU-SILC data (2008)

Table A6 – Detailed statistics of inequality in Upward cumulative rank acceptability index for the top 5% EU poverty for subgroups (2008)

Country	N	p	mean	s	G	O
AT	10846	0.061	0.968	0.063	0.031	1.044
BE	10073	0.057	0.939	0.056	0.060	1.012
DE	22834	0.129	0.949	0.129	0.050	1.037
EL	13486	0.076	0.948	0.076	0.051	0.964
ES	27784	0.157	0.947	0.157	0.052	0.977
FR	19493	0.110	0.957	0.111	0.042	1.018
IT	42532	0.241	0.943	0.239	0.056	0.985
LU	7486	0.042	0.950	0.042	0.049	1.010
PT	8505	0.048	0.942	0.048	0.056	0.951
UK	13479	0.076	0.960	0.077	0.040	1.026

Note: N = Number of observations; p = Share of population; mean = Average Upward cumulative rank acceptability index; s = Share of probability of being above 20% of poverty; G = Gini coefficients; O = Average overlapping.
Source: Authors' elaborations on EU-SILC data (2008)

Table A7 – Overlapping matrix of Upward cumulative rank acceptability index for the top 10% EU poverty (2008)

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.027	1.03	1.097	1.069	1.05	1.075	1.043	1.129	1.044
BE	0.966	1	0.997	1.081	1.05	1.017	1.052	1.024	1.111	1.015
DE	0.967	1.004	1	1.088	1.055	1.021	1.058	1.028	1.119	1.017
EL	0.889	0.919	0.914	1	0.969	0.936	0.97	0.943	1.027	0.935
ES	0.92	0.951	0.947	1.031	1	0.968	1.002	0.975	1.06	0.967
FR	0.949	0.982	0.979	1.063	1.031	1	1.034	1.005	1.093	0.996
IT	0.916	0.948	0.943	1.031	0.999	0.964	1	0.973	1.058	0.963
LU	0.948	0.977	0.975	1.056	1.026	0.995	1.028	1	1.084	0.995
PT	0.852	0.887	0.879	0.974	0.94	0.9	0.941	0.914	1	0.901
UK	0.954	0.984	0.984	1.06	1.029	1.005	1.034	1.002	1.091	1

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A8 – Overlapping matrix of Upward cumulative rank acceptability index for the top 5% EU poverty (2008)

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.031	1.009	1.08	1.067	1.027	1.059	1.034	1.092	1.017
BE	0.964	1	0.976	1.048	1.035	0.993	1.028	1.002	1.061	0.984
DE	0.987	1.025	1	1.074	1.061	1.017	1.054	1.026	1.088	1.008
EL	0.922	0.951	0.929	1	0.988	0.948	0.978	0.954	1.011	0.938
ES	0.933	0.964	0.941	1.013	1	0.96	0.991	0.966	1.024	0.95
FR	0.971	1.006	0.982	1.055	1.042	1	1.034	1.008	1.067	0.99
IT	0.94	0.972	0.948	1.022	1.009	0.967	1	0.974	1.034	0.958
LU	0.965	0.997	0.974	1.048	1.034	0.993	1.026	1	1.059	0.983
PT	0.906	0.939	0.915	0.988	0.976	0.934	0.967	0.941	1	0.924
UK	0.983	1.012	0.99	1.063	1.05	1.01	1.041	1.016	1.075	1

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table A9 – Detailed statistics of inequality in Upward cumulative rank acceptability index for the top 10% EU poverty for subgroups (2014)

Country	N	p	mean	s	G	O
AT	10651	0.058	0.938	0.061	0.06	1.081
BE	11236	0.061	0.888	0.061	0.11	1.065
DE	21462	0.117	0.917	0.12	0.081	1.076
EL	17768	0.097	0.856	0.093	0.139	0.955
ES	26049	0.142	0.848	0.135	0.148	1.01
FR	20659	0.113	0.923	0.116	0.075	1.046
IT	38604	0.211	0.91	0.214	0.085	0.943
LU	7891	0.043	0.937	0.045	0.06	1.016
PT	14579	0.08	0.87	0.077	0.121	0.849
UK	14013	0.077	0.919	0.078	0.077	0.996

Note: N = Number of observations; p = Share of population; mean = Average Upward cumulative rank acceptability index; s = Share of probability of being above 20% of poverty; G = Gini coefficients; O = Average overlapping.
Source: Authors' elaborations on EU-SILC data (2008)

Table A10 – Detailed statistics of inequality in Upward cumulative rank acceptability index for the top 5% EU poverty for subgroups (2014)

Country	N	p	mean	s	G	O
AT	10651	0.058	0.971	0.06	0.029	1.057
BE	11236	0.061	0.932	0.06	0.067	1.023
DE	21462	0.117	0.956	0.118	0.043	1.046
EL	17768	0.097	0.915	0.094	0.083	0.967
ES	26049	0.142	0.903	0.136	0.095	0.991
FR	20659	0.113	0.961	0.114	0.038	1.037
IT	38604	0.211	0.968	0.215	0.031	0.971
LU	7891	0.043	0.981	0.045	0.019	0.999
PT	14579	0.08	0.949	0.08	0.049	0.909
UK	14013	0.077	0.977	0.079	0.023	0.987

Note: N = Number of observations; p = Share of population; mean = Average Upward cumulative rank acceptability index; s = Share of probability of being above 20% of poverty; G = Gini coefficients; O = Average overlapping.
Source: Authors' elaborations on EU-SILC data (2008)

Table A11 – Overlapping matrix of Upward cumulative rank acceptability index for the top 10% EU poverty **(2014)**

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.015	1.011	1.114	1.054	1.038	1.138	1.053	1.23	1.081
BE	0.972	1	0.987	1.108	1.049	1.013	1.121	1.027	1.225	1.062
DE	0.985	1.011	1	1.117	1.058	1.027	1.131	1.04	1.232	1.073
EL	0.858	0.89	0.875	1	0.943	0.9	1.01	0.916	1.114	0.954
ES	0.914	0.947	0.93	1.056	1	0.956	1.063	0.968	1.169	1.006
FR	0.96	0.98	0.973	1.083	1.023	1	1.102	1.014	1.197	1.044
IT	0.868	0.876	0.876	0.969	0.912	0.903	1	0.921	1.081	0.947
LU	0.95	0.951	0.955	1.036	0.978	0.983	1.073	1	1.148	1.02
PT	0.76	0.781	0.773	0.885	0.828	0.8	0.907	0.819	1	0.852
UK	0.928	0.93	0.934	1.017	0.959	0.962	1.052	0.979	1.128	1

Source: Authors' elaborations on EU-SILC data (2014)

Table A12 – Overlapping matrix of Upward cumulative rank acceptability index for the top 5% EU poverty (2014)

	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT	1	1.031	1.013	1.083	1.052	1.02	1.082	1.042	1.153	1.059
BE	0.963	1	0.977	1.054	1.024	0.984	1.046	1.004	1.121	1.021
DE	0.984	1.025	1	1.081	1.052	1.006	1.066	1.024	1.141	1.041
EL	0.906	0.946	0.921	1	0.971	0.927	0.989	0.946	1.063	0.964
ES	0.928	0.972	0.944	1.028	1	0.95	1.011	0.967	1.087	0.985
FR	0.98	1.013	0.994	1.065	1.035	1	1.061	1.021	1.132	1.038
IT	0.925	0.939	0.934	0.982	0.951	0.94	1	0.968	1.059	0.983
LU	0.955	0.965	0.963	1.007	0.975	0.97	1.031	1	1.089	1.014
PT	0.858	0.879	0.869	0.926	0.897	0.876	0.937	0.9	1	0.916
UK	0.942	0.955	0.951	0.997	0.966	0.958	1.017	0.986	1.075	1

Source: Authors' elaborations on EU-SILC data (2014)